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# Arced Labyrinth Weir Flow Characteristics

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#### **TECHNICAL PAPERS**

# **ARCED LABYRINTH WEIR FLOW CHARACTERISTICS**

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**ABSTRACT**: The increase in predicted extreme flood event magnitudes has required the modification of many existing reservoir spillway flow control structures with more hydraulically efficient designs. The arced labyrinth is a relatively new, highly efficient, nonlinear weir design specifically applicable to reservoir applications. Flow characteristics of eleven laboratory-scale arced labyrinth weir geometries were studied at the Utah Water Research Laboratory with sidewall angles  $\alpha = 12^{\circ}$  and  $20^{\circ}$ . Rating curves, flow conditions, and discharge efficiencies were documented for each configuration to increase the hydraulic database of arced labyrinth weirs.

Arced labyrinth weirs with  $\alpha = 20^{\circ}$  were found to have discharge efficiencies 5 to 10% higher than  $\alpha = 12^{\circ}$  weirs. They also generated more flow anomalies (i.e. unstable nappe aeration) due to approach flow interaction with cycle orientation and weir position. The hydraulic efficiency of projecting, traditional labyrinth weirs, as a function of abutment wall geometry, was also evaluated. No standardized design method currently exists for arced labyrinth weirs, and relatively little is known about their hydraulic characteristics; this discussion increases the hydraulic information available for the arced labyrinth weir design.

**Keywords**: Arced labyrinth weirs, approach flow, labyrinth spillways, discharge efficiency.

# **INTRODUCTION**

Weirs are hydraulic structures used for measuring discharge, controlling flood water, providing water storage, facilitating water flow diversions, and altering flow regime in a channel or river. Weirs are also commonly used as head-discharge control structures in reservoir (spillways) and channel applications. Predicted probable maximum flood (PMF) magnitude increases caused by

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improved datasets, land-use changes, and/or climate change have resulted in a growing need to increase existing dam discharge capacities. This might be done by replacing an existing linear weir with a more hydraulically efficient nonlinear weir or by adding an additional spillway.

Weir head-discharge relationships can be described empirically using a standard form of the weir equation [Eq. (1)] (HENDERSON 1966). In this equation,  $\hat{O}$  is the weir discharge,  $H_T$  is the total upstream head measured relative to the crest elevation,  $L$  is the weir length,  $C_d$  is the discharge coefficient, and *g* is the acceleration of gravity.

$$
Q = C_d \frac{2}{3} L H_T^{3/2} \sqrt{2g}
$$
 (1)

Per Eq. (1), the weir discharge is directly proportional to *L*. Since the width of the spillway channel is often restricted, one way to increase discharge capacity is to increase *L* by folding the weir (in plan view) into trapezoidal segments, or cycles, creating a nonlinear, labyrinth, or 3-D weir (FALVEY 2003). Figure 1 shows a photograph of a prototype labyrinth weir (Yahoola Dam, Georgia, USA.)



Figure 1 – Photograph of prototype labyrinth weir (photo courtesy of Schnabel Engineering)

KOCAHAN and TAYLOR (2000) suggested that the labyrinth shape allows more discharge than a linear ogee weir at the beginning of a flood. Labyrinth weirs also represent constructible alternatives to widening the spillway channel (TULLIS et al. 1995). Due to their increased discharge efficiency, labyrinth weirs require less upstream driving head for a given discharge. Replacing a linear weir with a labyrinth weir could result in more reservoir volume being utilized for water storage due to a reduction in required reservoir volume set aside for flood routing. For reservoir applications, the labyrinth weir cycles can be arranged in an arced configuration, taking better advantage of converging approach flow patterns. A labyrinth weir layout where the downstream apexes of each cycle follow the arc of a circle is termed an arced labyrinth weir.

Previous model studies have shown arced labyrinth weirs to be viable options for reservoir weir applications where approach flow conditions are non-channelized. For Maria Cristina Dam (Spain), the approach flow conditions and discharge capacity were improved by arcing 6 of the 7 labyrinth weir cycles within the limited footprint area of the spillway (CORDERO-PAGE et al. 2007).

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Based on laboratory experiments of labyrinth weirs in reservoir applications, CROOKSTON (2010) concluded that "the arced configurations were found to be the most efficient labyrinth weirs tested" and that "an arced cycle configuration can increase discharge efficiency as it improves the orientation of the cycle to the approaching flow  $(\sim 90^{\circ}$  to the weir centerline is desirable)." CROOKSTON and TULLIS (2012 a, b,  $\&$  c) further introduced arced labyrinth weir-specific geometric parameters (Figure 2) and tested several physical weir models. These tests and nomenclature are referred to and adopted in this study.



Figure 2 – Schematic of 10-cycle arced labyrinth weir including CROOKSTON and TULLIS (2012a) nomenclature.

Due to their infinite variability in possible geometric configurations, arced labyrinth weirs can provide unique challenges to designers. The objective of this report is to help expand the hydraulic database for arced labyrinth weirs and provide additional insight for their implementation in reservoirs. Since limited information is currently available in the literature, including a limited number of physical model tests, a thorough study of arced labyrinth weir hydraulics is needed. The information presented herein is intended to increase that knowledge base. Additional information and insights can also be found in CHRISTENSEN (2012).

# **EXPERIMENTAL SETUP**

All arced labyrinth weir testing was performed in an elevated head box (7.3m x 6.7m x 1.5m) located at the Utah Water Research Laboratory (UWRL), at Utah State University. Eleven laboratory-scale models were built with geometries intended to expand the work of CROOKSTON and TULLIS (2012a, 2012b). These physical models were selected for detailed analysis of flow anomalies and characteristics not easily observable with numerical or analytical simulations.

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The configurations tested included: sidewall angles (*α*) of 12 and 20 degrees; cycle arc angles (*θ*) of 0º, 10º, 20º, and 30º; and cycle numbers (*N*) equal to 5, 7, and 10. All weirs were fabricated using high-density polyethylene (HDPE) sheeting with wall thicknesses  $(T_w)$  of 254 mm, wall heights of 0.305 m, and half-round crest shapes. Each weir was installed on a level HDPE apron in the model reservoir. All arced labyrinth weirs were installed projecting into the reservoir, as shown in Figure 2. Piezometric head (±0.15mm) was measured upstream of each weir at a location where the velocity head was negligible (i.e., total head equals piezometric head). The weir discharge was measured using a calibrated flow meter  $(\pm 0.25\%)$  in the upstream supply piping.  $C_d$  values were calculated using total head, measured discharge, and Eq. (1).

Discharge efficiency was determined based on head-discharge relationships. Because the weir length was not maintained constant for all arced labyrinth weir configurations tested, the discharge efficiency, as quantified by the relative value of  $C_d$ , represents the weir unit discharge (discharge divided by the weir length) at a given upstream head condition. 2-D flow velocities were measured upstream using a Sontec® flow-tracker device. Velocity vector fields were then digitized to characterize the nature of the approach flow field. These relationships were compared to previous data presented by CROOKSTON and TULLIS (2012 a, b,  $\&c$ ).

#### **RESULTS & DISUSSION**

CROOKSTON (2010) tested six arced labyrinth weir models ( $\theta$  = 10°, 20°, 30°,  $\alpha$  = 6°, 12°). In this study,  $\alpha = 12^{\circ}$  data were repeated, and  $\alpha = 20^{\circ}$  data were added to extend CROOKSTON's (2010) findings. These variations are categorized into geometrically similar and geometrically comparable weirs. Geometric similarity refers to a condition where all geometric parameters for one labyrinth weir are uniformly scaled in producing the geometry of a second labyrinth weir. Arced labyrinth weirs with geometrically similar cycles, but arranged with different arc angles  $(\theta)$ , are referred to as geometrically comparable.

*C<sub>d</sub>* data were collected from  $0.1 \leq H_T/P \leq 0.9$  for *N*=5 configurations and  $0.1 \leq H_T/P \leq 0.5$  for  $N > 5$ .  $C_d$  vs.  $H_T/P$  trend lines were developed using data-fitting software. Figure 3 shows the experimental  $C_d$  vs.  $H_T/P$  data for  $\alpha = 12^\circ$  and 20° sidewall angled weirs.

#### Cycle Arcing Effects

Based on the data presented in Figure 3, several observations were made. For both the  $\alpha = 12^{\circ}$ and  $20^\circ$  sidewall angled weirs, the discharge efficiency, as characterized by  $C_d$ , increased with increasing  $\theta$ , particularly for relatively lower heads. Increasing the cycle arc angle, which subsequently reduces the arc radius, splays out the labyrinth weir cycles to better accommodate 180° converging approach flow conditions. At relatively high heads  $(H_T/P > 0.4$  to 0.6),

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however, the discharge efficiency gains from the larger *θ* values tended to be lost. As local submergence formed in the outlet cycles at higher heads, the approach flow streamlines in the reservoir began to orient themselves more with the downstream channel centerline and less with the inlet cycle centerlines. The distal cycles were also exposed to greater flow separation and turbulence at the weir/training wall boundary. The combined effects resulted in increased head loss and decreased hydraulic efficiency.



Figure 3 – Sidewall angle  $\alpha = 12^{\circ}$  (left) & 20° (right) discharge data with trend lines

As local submergence increased in the arced labyrinth weir and the approach flow streamline orientations adjusted, the head-discharge control point began to transition from the weir crest to the points farther downstream. At very high heads, the weir was fully submerged and the control point shifted to the discharge channel inlet (contracting sidewalls downstream of the labyrinth weir). Velocity data were collected in the reservoir upstream of each arced weir configuration, and velocity vector fields were digitized to show the changes in flow alignment (caused by a control point shift). The velocity vectors in Figure 4 illustrate changes in approach flow alignment with increasing upstream head.



Figure 4 – Change in approach flow velocity vector alignment with as a function of upstream head (5cycle,  $\alpha = 20^{\circ}$ ,  $\theta = 30^{\circ}$ ): H<sub>T</sub>/P = 0.3 (grey), H<sub>T</sub>/P = 0.6 (black).

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# Abutment Influences for Traditional Labyrinth Weirs in Reservoir Applications

CROOKSTON (2010) investigated the influence of abutment wall geometries on the discharge efficiency of traditional labyrinth weirs in reservoir applications. The same reservoir-specific labyrinth weir/abutment wall geometries were also evaluated in this study for  $\alpha = 20^{\circ}$  labyrinth weirs. The placement of the weir, either projecting into the reservoir or flush with the outlet, was also investigated. Three traditional labyrinth weir configurations were tested: projecting, flush (square-edged abutments), and rounded abutments (Figure 5).



Figure 5 – Weir Placement & Abutment Types

For  $\alpha = 20^{\circ}$  weirs, the rounded inlet was approximately 3% more efficient than the projecting and flush configurations for  $H_T/P > 0.2$ . Compared to the projecting weir (at similar heads,  $H_T/P$  $= 0.3$ ), the rounded inlet prevented unstable nappe formation on inner sidewalls and caused the nappe to remain in a clinging/non-aerated state for more time (Figure 6). For  $\alpha = 20^{\circ}$  weirs, the rounded inlet also eliminated flow separation and turbulent flow over the crest, allowing for nappe stabilization and improved discharge efficiency. The flush setup was consistently less efficient for all sidewall angles. Applying these results to arced labyrinth weirs may indicate that rounded inlets would help alleviate instability and flow separation concerns on some arced labyrinth weirs, especially for distal cycles on  $\alpha \geq 20^{\circ}$  weirs, further improving overall weir efficiency. Note that as the overall labyrinth weir length increases, the influence of abutmentinduced flow separation on hydraulic efficiency diminishes.



Figure 6 – Projecting weir (Left) vs. rounded inlet weir (Right) at  $H_T/P = 0.3$ 

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#### Sidewall Angle Effects

The investigation of the effect of the sidewall angle on flow can be seen in  $C_d$  data for  $\alpha = 12^{\circ}$ and  $\alpha = 20^{\circ}$  arced labyrinth weirs normalized to  $C_d$  data for non-arced projecting weirs. These data indicate that for the  $\alpha = 12^{\circ}$  and  $20^{\circ}$  configurations, arcing a labyrinth weir in a reservoir increases discharge efficiency by approximately 10 to 20% (Figure 7). The relative increase in discharge efficiency, compared to the projecting configuration with the same  $\alpha$ , is more significant for  $\alpha = 12^{\circ}$  than  $\alpha = 20^{\circ}$ .



Figure 7 – Arced weir compared to non-arced weir in reservoir for  $\alpha = 12^{\circ}$  (Right) & 20° (Left)

# **CONCLUSIONS**

The flow characteristics observed in this study provide additional insights into understanding arced labyrinth weir behavior. All data were compared to CROOKSTON (2010) data and highlight the importance of selecting effective configurations. Larger sidewall angles and cycle arc angles present various problems for efficiency, such as flow turbulence at weir/wall boundaries, local submergence, and nappe instability. Rounded inlet modifications may help alleviate some of these concerns, particularly for projecting, traditional labyrinth weirs (nonarced). Selection of appropriate  $\alpha$  and  $\theta$  geometries should be accomplished on a case-by-case basis.

This study will benefit future designers because it explains some of the flow characteristics that directly impact implementation of arced labyrinth weirs. Although no design method exists for this type of weir, this project takes multiple steps forward toward creating one.

#### **NOMENCLATURE**

- $\alpha$  = Sidewall angle (used for linear or arced configurations)
- *α*<sup> $\alpha$ </sup> = Upstream sidewall angle,  $\alpha' = \alpha + \theta/2$

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- $H_T$  = Total measured head relative to the crest
- $l_c$  = Centerline length of the sidewall

 $L_c$  *or*  $L =$  Centerline crest length of entire weir

- $L_{c\text{-cycle}}$  = Centerline crest length of one cycle
	- *N* = Number of cycles
	- *P* = Weir crest height
	- $Q =$  Flow
	- *R* = Arced radius,  $R = (W^2/4 + r^{2})^{1/2}$
	- $r'$  = Segment height from channel opening to perpendicular downstream apex
	- $r =$  Segment height from channel opening to center of imaginary arc circle
	- $\theta$  = Cycle arc angle,  $\theta$  =  $\Theta/N$
	- $\Theta$  = Central arc angle,  $\Theta = W'/R$
	- $t_w$  = Wall thickness at crest
	- *W* = Downstream channel width
	- $W'$  = Labyrinth weir arc length (through downstream apexes),  $W' = R\Theta$

 $w' = \text{Cycle}$  arc width,  $w' = W'/N$ 

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