Calcon 2013 Tutorial 1: Basics and Applications of Spectroradiometry

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Motivations

1. Develop a calibration plan (SI-traceable) for a satellite sensor
   a) Calibration requirements
   b) Calibration approach
   c) Use of on-board calibrators
   d) Perform system-level end-to-end calibration (validation)
2. Elements of the plan should
   a) Answer how the calibration requirements will meet the mission and instrument requirements
   b) Develop a sensor design and radiometric model (measurement equation)
   c) Characterize subsystems (uncertainty analysis)
   d) Compare model predictions and validate system level calibrations
   e) Establish pre-launch radiometric uncertainties

From: NISTIR 7637 (2009), "Best Practice Guideline…" R. Datla et al.

Outline of the Tutorial

1. Basics of Radiometry
2. Detector-based Radiometry
3. Source-based Radiometry
4. Tools of Spectroradiometry
   a) Detectors
   b) Filter Radiometers
   c) Spectroradiometers
5. Measurement Equation and Uncertainty Analysis
6. Applications from the NIST Short Course

Possible Sources of Error

	Max. Potential Error
1.Spectral scattering	> 100 %
2.Spectral distortion	> 100 %
3.Nonlinearity	20%
4.Directional and positional effects	20%
5.Polarization effects	5%
6.Size-of-source effect (radiance)	5%
7.Wavelength instability	100 % / nm
8.Detector instability	10%
9.Uncertainty of the standard	
10.Instability of the standard	
11.Instability of the quantity being-measured	
12.Noise in the measurement data	1 % to 5 %
(*Reliable Radiometry*, p. 422)	

1. Basics of Radiometry

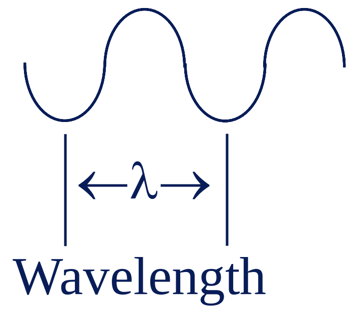
Basics of Spectroradiometry: Outline

1. Definitions of radiometric terms
    a) Flux, radiant intensity, irradiance, radiance
    b) Property modifiers
    c) The concept of solid angle
2. Radiometry basics
    a) Point source
    b) Extended source
    c) Flux transfer and throughput
3. Applications
    a) Calibration of radiometers
    b) Some numerical examples

Electromagnetic Radiation

Name	Wavelength ranges
UV-C	100 nm to 280 nm
UV-B	280 nm to 315 nm
UV-A	315 nm to 400 nm
VIS	360 nm to 800 nm
NIR	800 nm to 1400 nm
SWIR	1.4 µm to 3 µm
MWIR	3 µm to 5 µm

$$c = n\lambda\nu$$



c = speed of light
λ = wavelength
n = index of refraction
ν = frequency

The wavelength is determined by the speed of light and measurements of the frequency by comparison to the atomic standards.

For example: λ = 555 nm, then $\nu = 540 \times 10^{12}$ Hz.

Property modifiers

1.	**Spectral:**	having a dependence on wavelength; within a very narrow region of wavelength
2.	**Total:**	integrated or summed over all wavelengths
3.	**Exitent**:	leaving a surface
4.	**Incident:**	arriving at a surface
5.	**Directional :**	having a dependence on direction; within a very small solid angle
6.	**Hemispherical :**	averaged over all solid angles passing through a hemisphere centered over the surface element

Radiant quantities

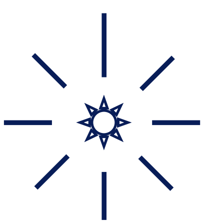
1) Radiant energy, Q [J]

E.g. deposited laser energy at 650 nm 5.3 mJ (0.25 s)

2) Radiant flux, Φ [W], [J/s]

E.g. laser power meters, cryogenic radiometer (beam underfills the detector)

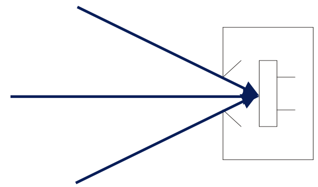
3) Radiant intensity, I [W/sr]



E.g. point sources such as stars observed at great distances

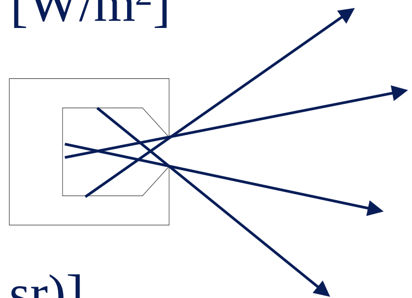
Radiant quantities, continued

4) Irradiance, E [W/m^2]



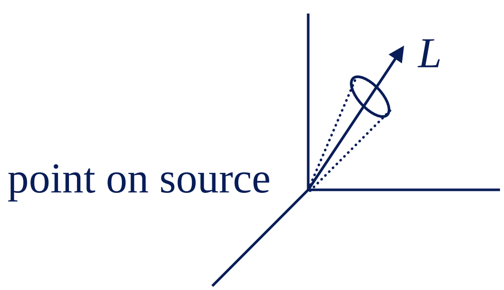
E.g. solar terrestrial irradiance

5) Radiant exitance, M [W/m^2]



E.g. blackbody, the Stefan-Boltzmann law

6) Radiance, L [W/(m^2 sr)]



E.g. real sources (blackbodies, surface of the earth, moon, and other objects)

Comparison of Radiometric and Photometric Units

Radiometric	Symbol	Unit	Unit	Symbol	Photometric
Radiant Energy	Q	J	lm s	Q_V	Luminous energy
Radiant flux (power)	Φ	W	lm	Φ_V	Luminous flux
Irradiance	E	W/m^2	$lm/m^2 = lx$	E_V	Illuminance
Radiance	L	$W/(m^2\ sr)$	$lm/(m^2\ sr)$	L_V	Luminance
Radiant exitance	M	W/m^2	lm/m^2	M_V	Luminous exitance
Radiant intensity	I	W/sr	lm/sr=cd	I_V	Luminous intensity
Radiance Temperature	T	K	K	T	Color Temperature

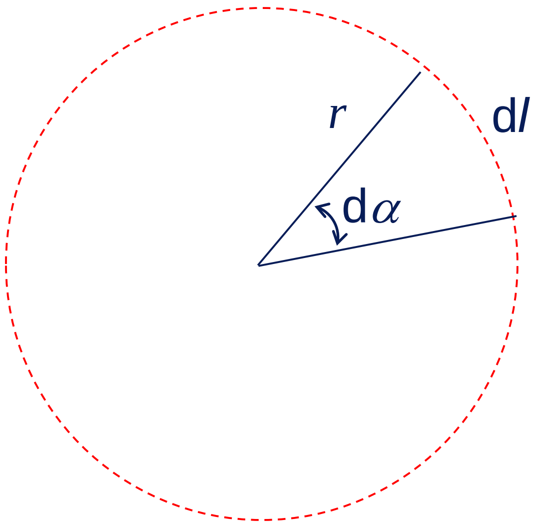
Thermodynamic (equivalence of heat and radiant flux)

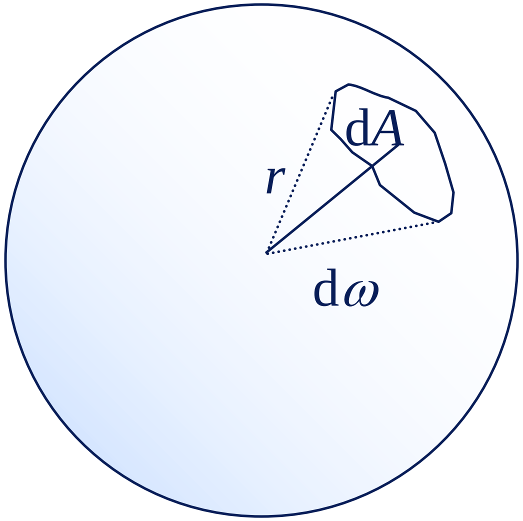
Specialized (human visual response for detector model)

Spectral modifier—"spectroradiometry"

Quantity	Symbol	Unit
Spectral Radiant Energy	Q_λ	J/nm
Spectral Radiant flux (power)	Φ_λ	W/nm
Spectral Irradiance	E_λ	W/m^2/nm
Spectral Radiance	L_λ	W/(m^2 sr)/nm
Spectral Exitance	M_λ	W/m^2/nm

Plane and solid angles





r	Radius of circle
$\mathrm{d}l$	Arc length along circumference
$\mathrm{d}\alpha$	Plane angle subtended by arc at center

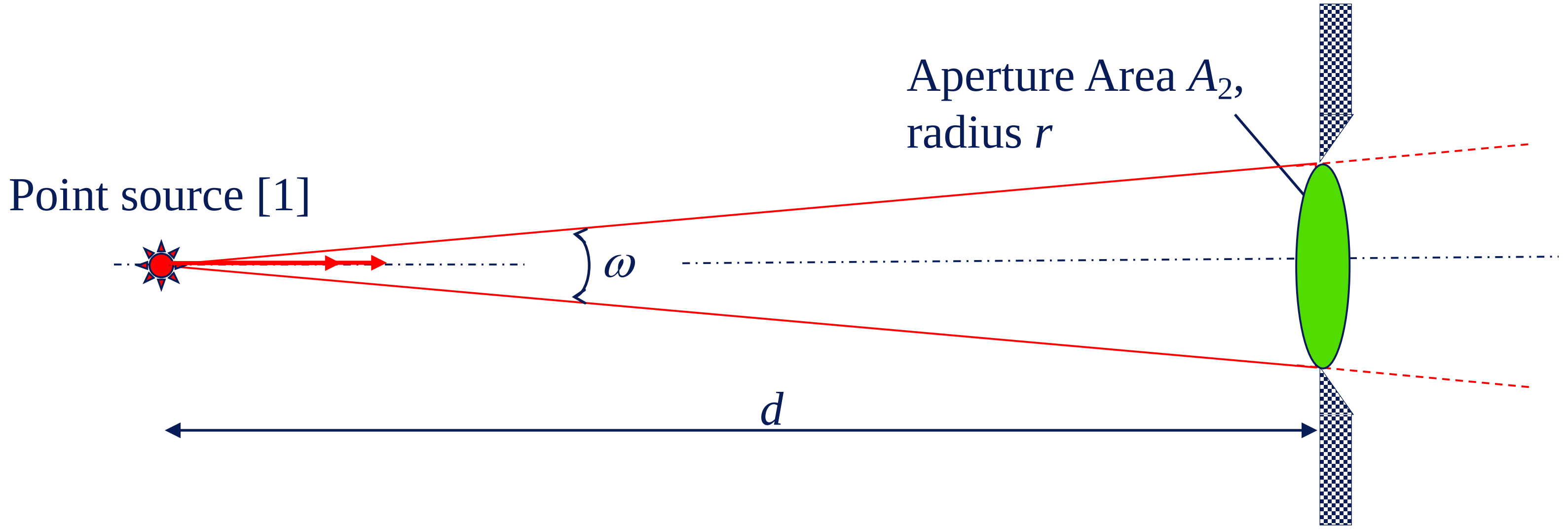
r	Radius of sphere
$\mathrm{d}A$	Area of sphere segment
$\mathrm{d}\omega$	Solid angle subtended by area at center

$$\mathrm{d}\alpha = \frac{\mathrm{d}l}{r} \quad \text{Unit : radian (rad)}$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} \quad \text{Unit : steradian (sr)}$$

Solid angle: perpendicular surface

Point Source and Circular Aperture



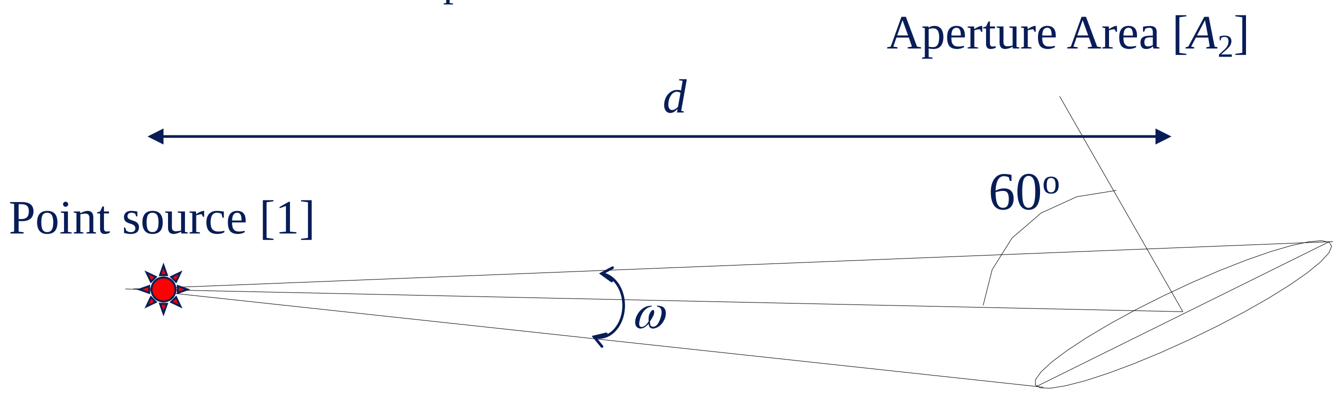
A_2	Area of the aperture
d	Distance between the aperture and the source
ω	Solid angle subtended by [A_2] at the source

$$\omega \approx \frac{A_2}{d^2}$$

Note: Distance d is large compared to aperture dimensions so that $d^2 >> A_2$.

Solid angle: tilted surface

Point Source and Circular Aperture



A_2	Area of the aperture
d	Distance between the aperture and the source
ω	Solid angle subtended by [A_2] at the source

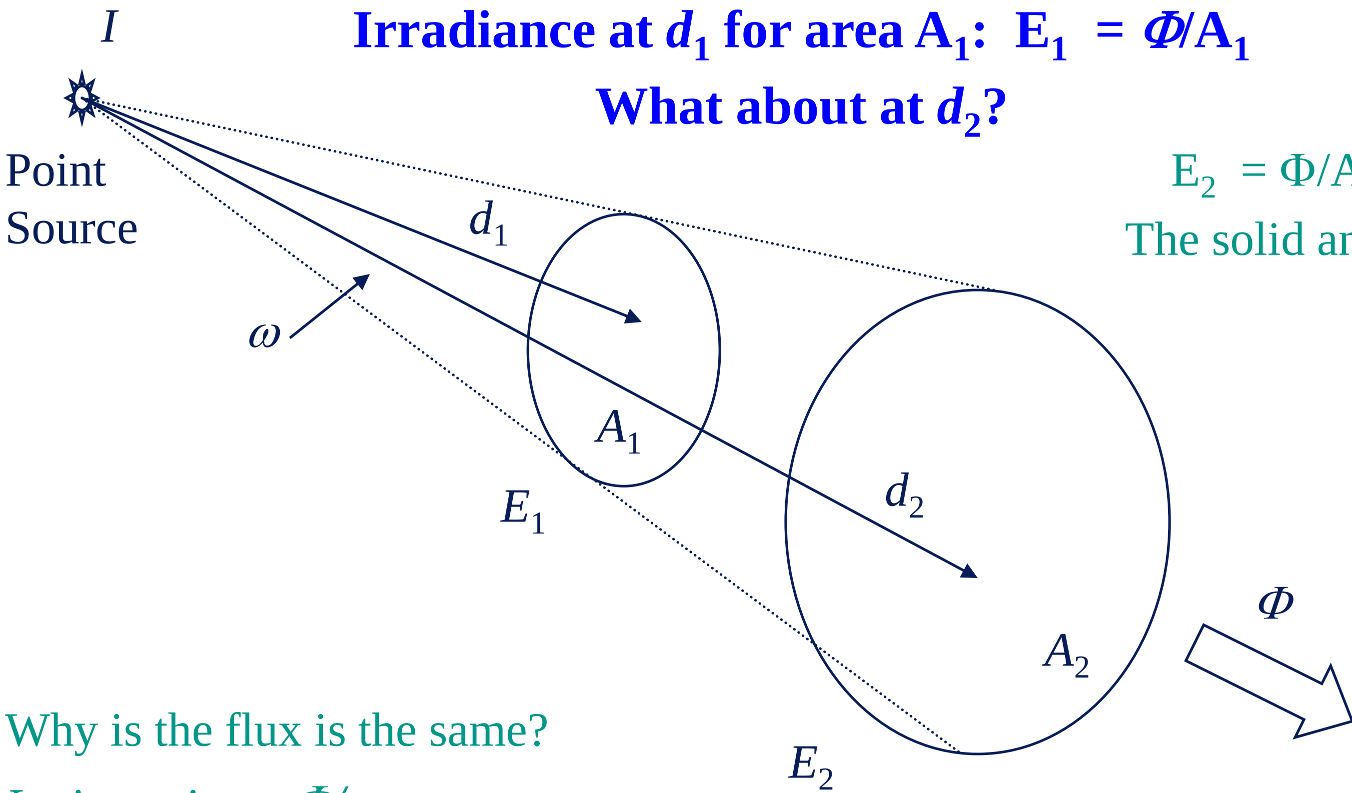
$$\omega \approx \frac{A_2 \cos\theta}{d^2} = \frac{A_2 \cos 60^o}{d^2} = \frac{A_2}{d^2}\frac{1}{2}$$

Note: Distance d is large compared to aperture dimensions so that $d^2 >> A_2$.

Radiometry of point sources (Irradiance, E)

Irradiance at d_1 for area A_1: $E_1 = \Phi/A_1$

What about at d_2?



$E_2 = \Phi/A_2$ (flux is the same)

The solid angle is also constant (by geometry):

$$\omega = \frac{A_1}{d_1^2} = \frac{A_2}{d_2^2}$$

$$E_2 = E_1 \frac{d_1^2}{d_2^2}$$

Why is the flux is the same?

I = intensity = Φ/ω

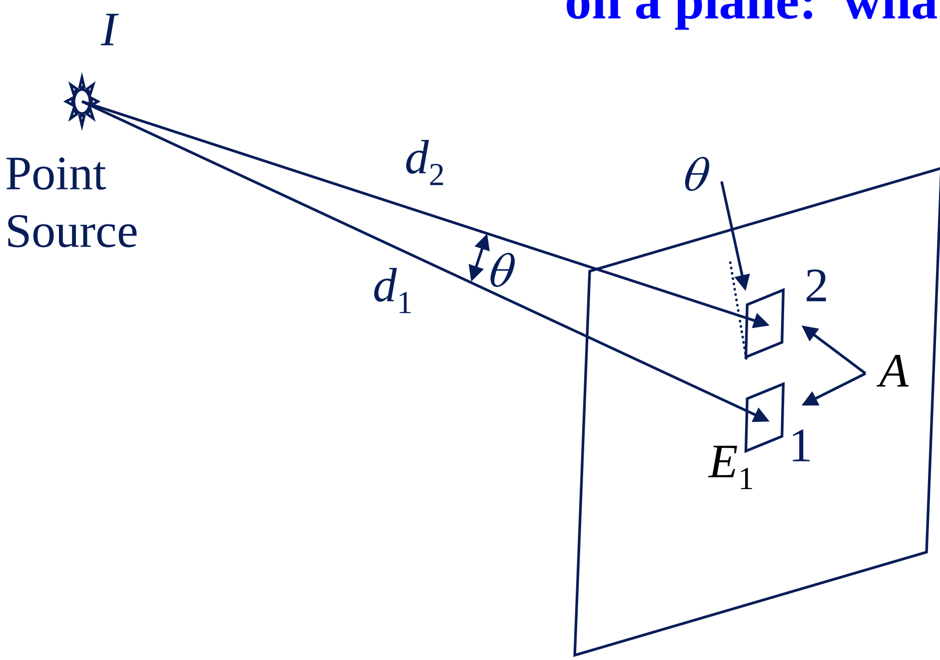
For a point source, I is independent of direction (isotropic).

$$I = \frac{\Phi}{\omega} = \Phi \frac{d_1^2}{A_1} \text{ or } E_1 = \frac{I}{d_1^2}$$

The irradiance from an ideal point source falls off as $1/d^2$. How well must the distance be measured?

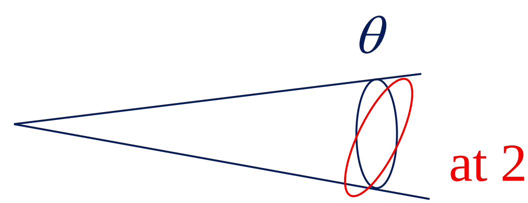
Irradiance distribution, point source on a plane

Irradiance at r_1 for different locations on a plane: what is E_2/E_1?



Point 1 is “on-axis”

At Point 2, the distance is greater by $r_2 = r_1/\cos\theta$ and the area is tilted by θ.



$$E_2 = E_1 \left[\frac{A}{A/\cos\theta} \right] \left[\cos^2\theta \right]$$

$$E_2 = E_1 \cos^3\theta$$

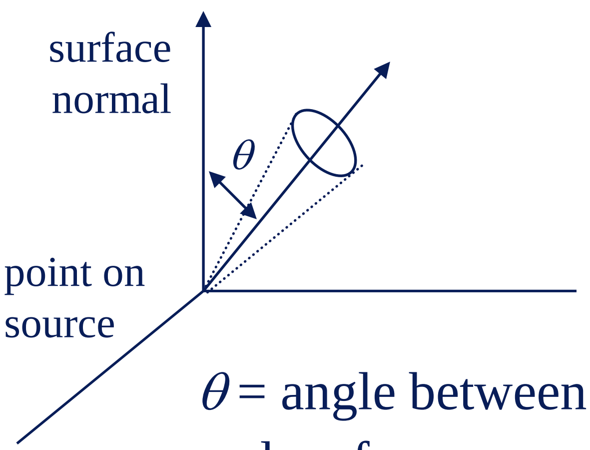
The “off-axis” irradiance from an ideal point source in a plane falls off as $\cos^3\theta$. Keep angles small if uniformity is critical!

Radiometry of extended sources (Radiance, *L*)

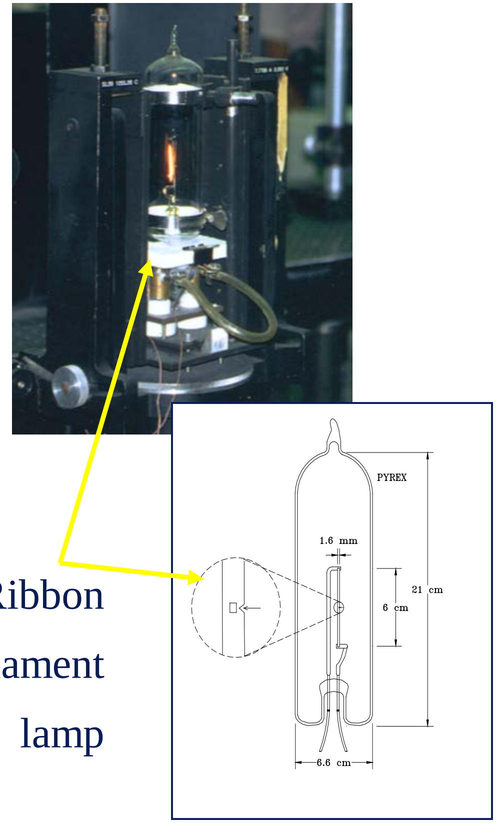
Real Sources

1) have finite size and 2) the flux depends on view direction, target size, location on source, and solid angle.

Radiance: $$L = \frac{\Phi}{A \cos\theta\,\omega}$$

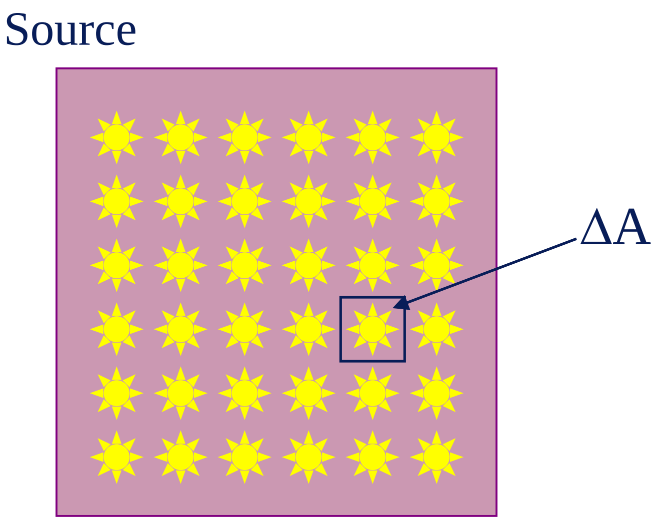


θ = angle between view direction and surface normal, ω = solid angle, A = source area



Ribbon filament lamp

Lambert's law



Trial model: A collection of point sources, uniformly spaced across the source area

Lambert's Law: $L(\theta) = L(0)$

Lambertian or "diffuse" source

Radiance from one of the point sources:

Viewed perpendicular $$L(0) = \frac{\Phi}{\Delta A\, \omega}$$

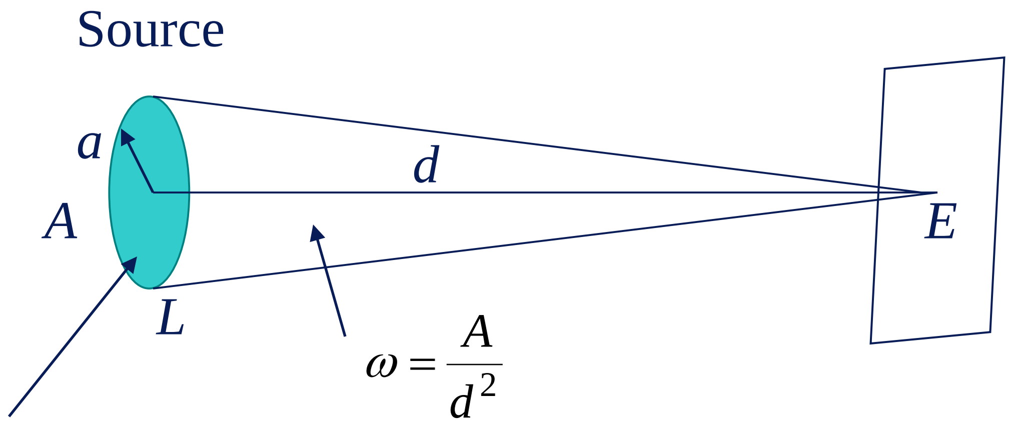
Viewed tilted $$L(\theta) = \frac{\Phi}{\Delta A \cos\theta\, \omega}$$

We conclude $$L(\theta) = \frac{L(0)}{\cos\theta}$$

But real sources don't increase in brightness when viewed off-axis, most in fact remain constant or become dimmer

Model not a collection of ideal point sources ($I = \Phi/\omega$), but a collection of pseudo point sources ($I = \Phi \cos\theta/\omega$)

Irradiance from an extended source (on-axis)



$$\omega = \frac{A}{d^2}$$

Extended source as a collection of point sources

Irradiance at the plane from each point source (for $d >> a$) is I/d^2.

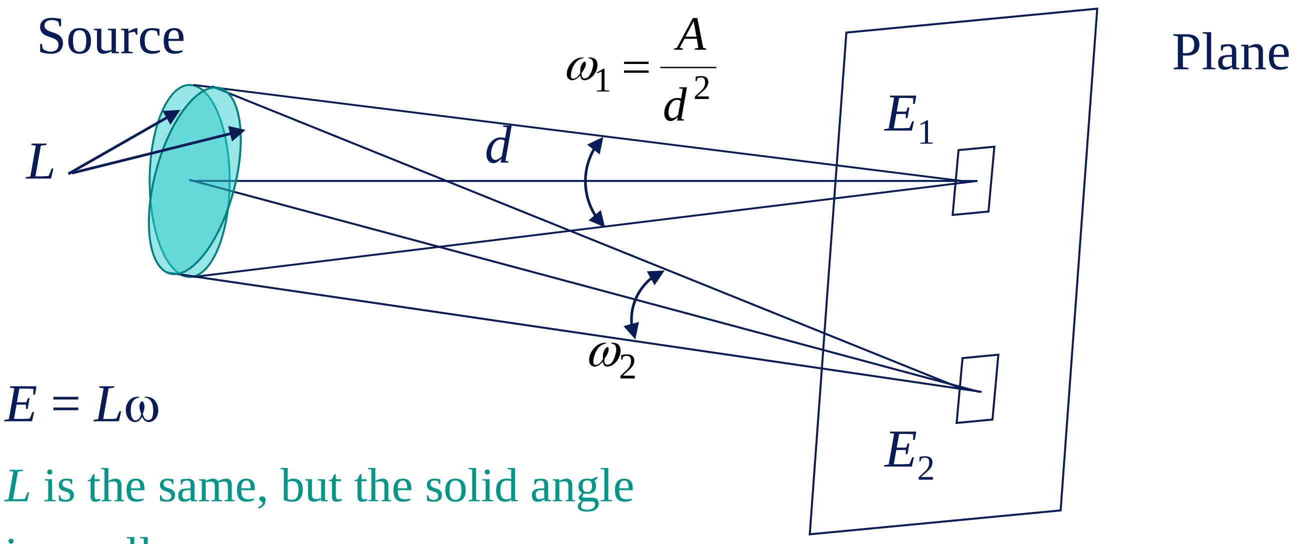
The intensity is $I = L\,\Delta A$, where ΔA is the “area” of each point source.

Sum over all point sources

$$E = \frac{L \sum \Delta A}{d^2} = \frac{L\,A}{d^2} = L\,\omega$$

The irradiance depends only on *L* and the solid angle of the source from the point on the plane.

Irradiance from an extended source (off-axis)



$E = L\omega$

L is the same, but the solid angle is smaller:

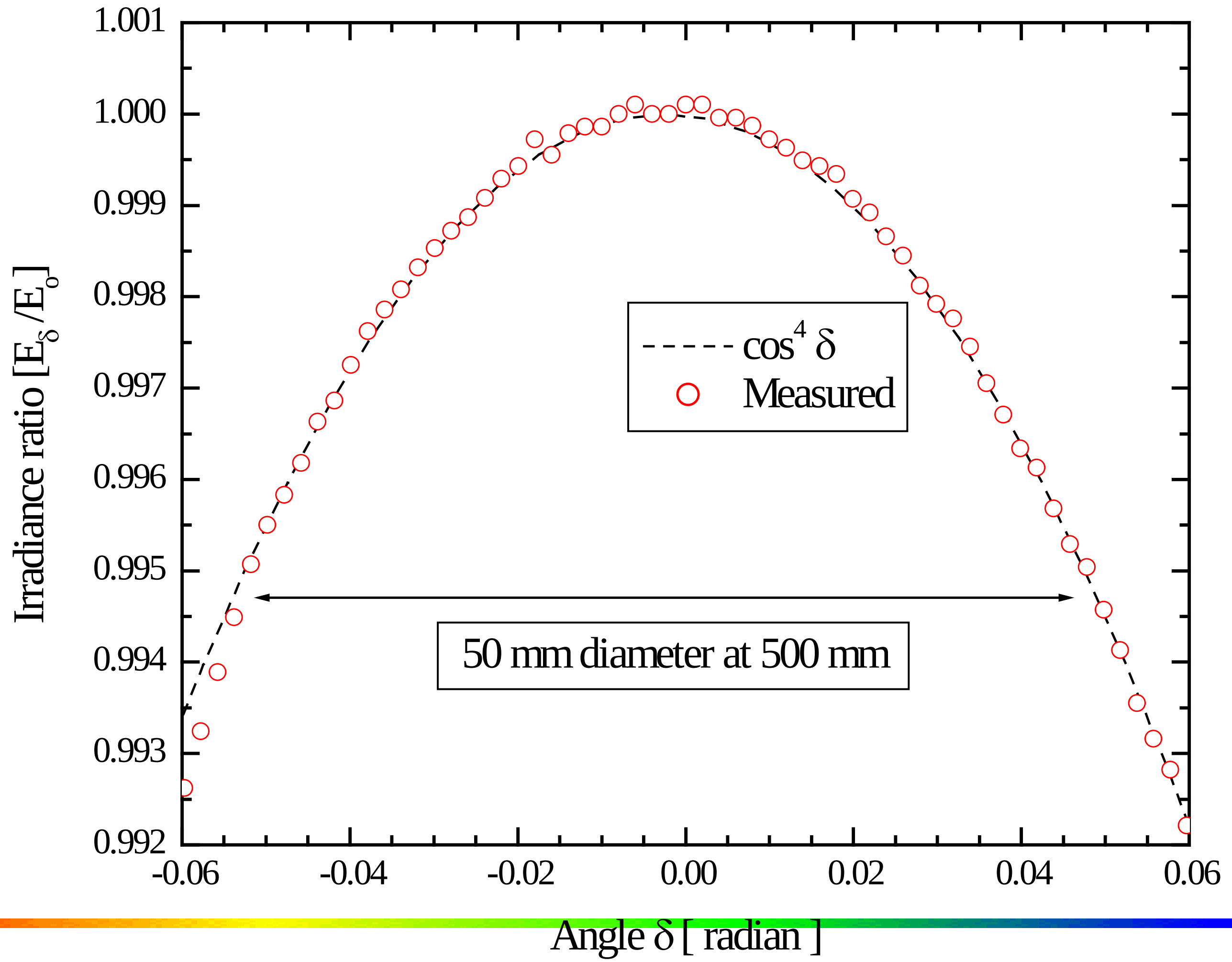
$$\omega_2 = \frac{A\cos\theta}{d^2 / \cos^2\theta} = \omega_1 \cos^3\theta$$

As before, we have to remember the area on the plane is tilted by θ:

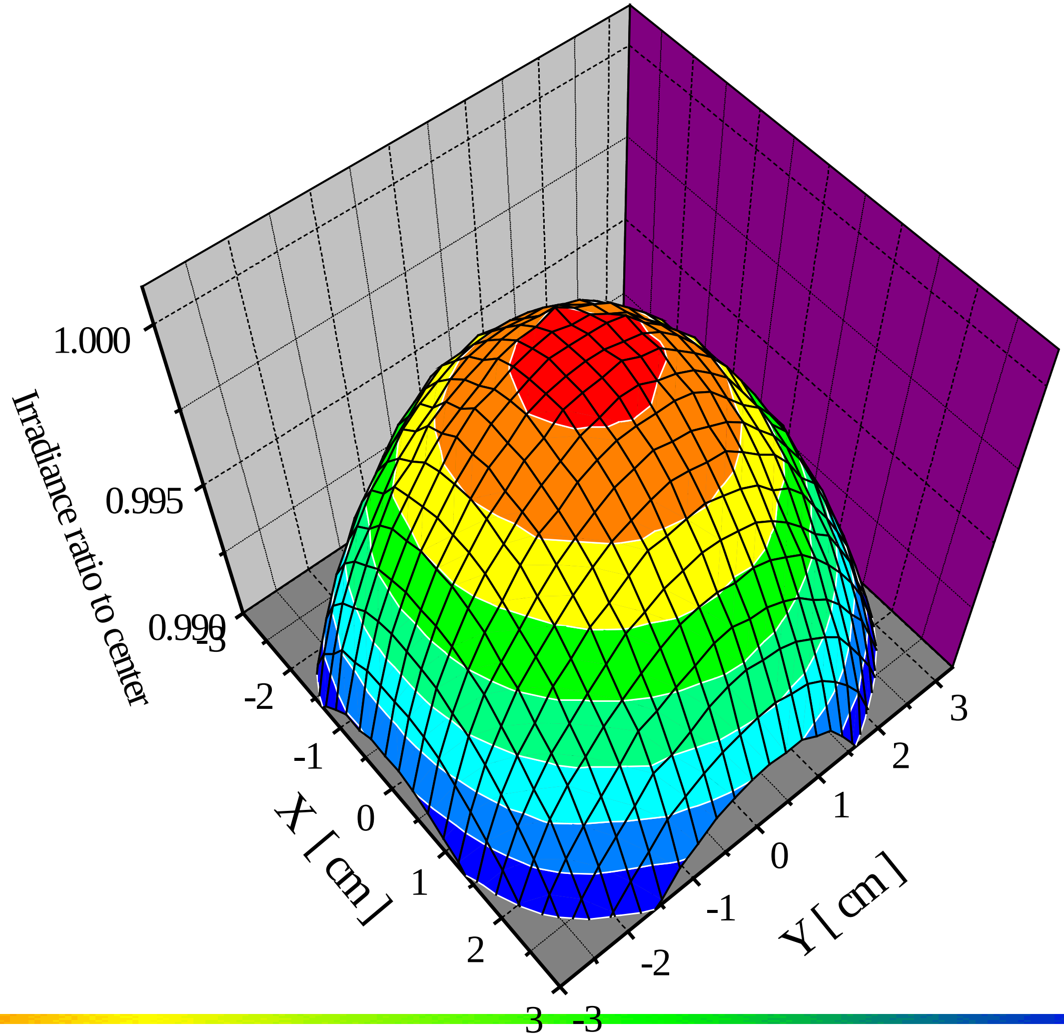
$$E_2 = E_1 \left[\cos^3\theta\right]\left[\frac{A}{A / \cos\theta}\right] = E_1 \cos^4\theta$$

The irradiance distribution in a plane from an extended Lambertian source drops faster than from a point source (∝ $\cos^3\theta$). Known as the "cosine fourth law"

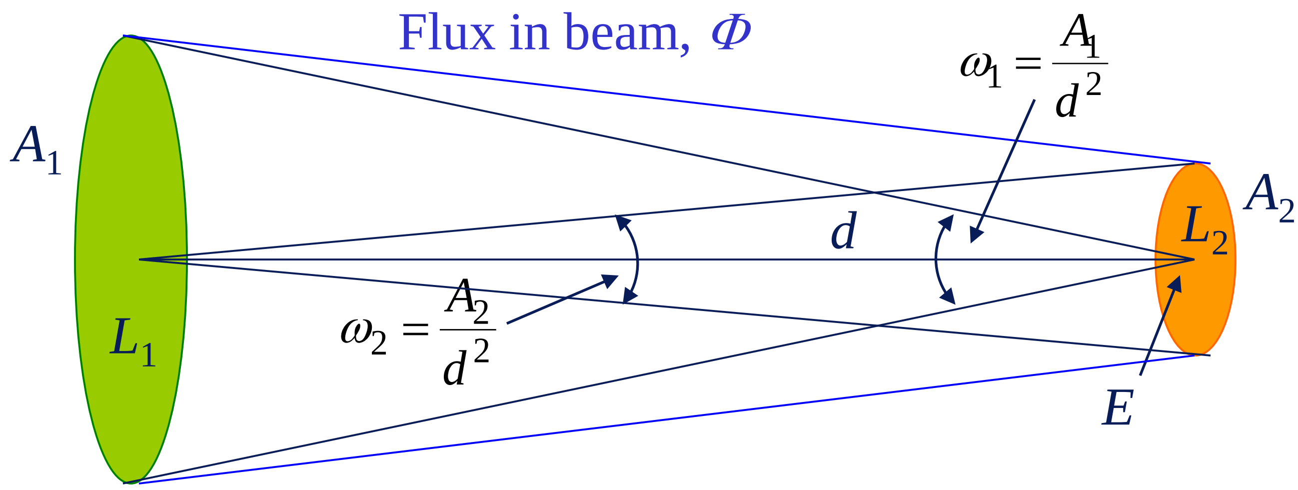
Angular $\cos^4$ output of the NIST 308 mm diameter integrating sphere



Spatial scan of the 308 mm sphere irradiance



Invariance of radiance



From before,

$$E = L_1 \omega_1$$

$$\Phi = E A_2$$

$$L_1 = \frac{E}{\omega_1} = \frac{\Phi}{A_2 \omega_1}$$

It also must be true (from the definition of radiance)

$$L_2 = \frac{\Phi}{A_2 \omega_1} = L_1$$

Invariance of Radiance

$$L_1 = L_2$$

Note we could also have said

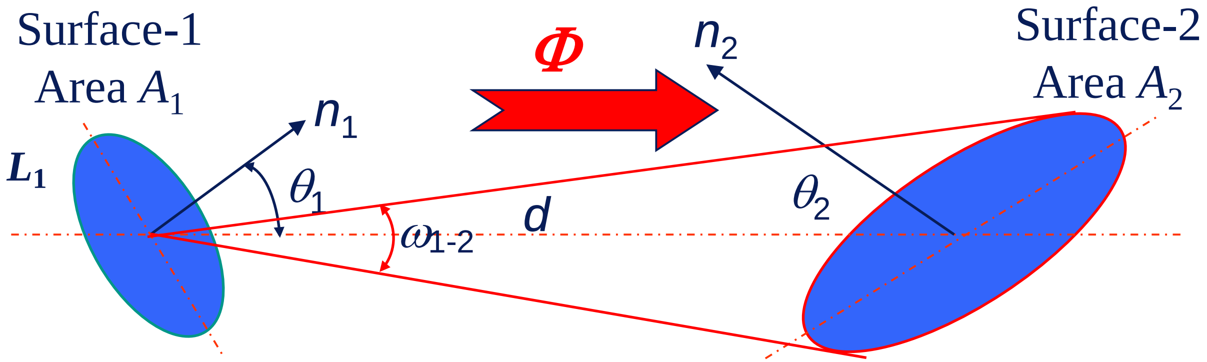
$$L_1 = \frac{\Phi}{A_1 \omega_2}$$

Throughput

$$A\omega = A_1 \omega_{1-2} = A_2 \omega_{2-1}$$

$$\left(\omega_{1-2} = \frac{A_2}{d^2}, \quad \omega_{2-1} = \frac{A_1}{d^2} \right)$$

Radiant flux transfer, arbitrary orientation



ω_{1-2} = Solid angle subtended by A_2 with respect to A_1 $\quad \omega_{1-2} = \dfrac{A_2 \cdot \cos\theta_2}{d^2}$

$$
\begin{aligned}
\Phi &= \text{Radiant flux in the beam} \\
&= [\text{Radiance} \cdot \text{Projected area}]_1 \cdot [\text{Solid angle}]_{1-2} \\
&= [L_1 \cdot A_1 \cdot \cos\theta_1] \cdot [\omega_{1-2}] \\
&= [L_1 \cdot A_1 \cdot \cos\theta_1] \cdot \left[\frac{A_2 \cdot \cos\theta_2}{d^2}\right]
\end{aligned}
$$

For real surfaces, divide the two areas into many sub-areas and carry out two dimensional integration.

A review so far

1. Irradiance from a source on a plane
   a) Point source: $E(0) = I/d^2$ and $E(\theta) = E(0)\cos^3\theta$
   b) Extended source: $E(0) = L\omega$ and $E(\theta) = E(0)\cos^4\theta$
2. Radiance and extended sources
   a) Generally, $L(\theta) = \mathrm{L}(0)$ [Lambertian]
   b) Invariance of radiance: $L_1 = L_2$ (no absorption or scatter)
   c) Throughput = $A_1\cos\theta_1\,[A_2\cos\theta_2]/d^2$ for large distances
3. Flux transfer (the detector responds to flux)
   a) $\Phi = L *$ throughput
   b) $\Phi = E *$ "detector" area
4. Look at the units

Irradiance at a plane, radiance to the hemisphere

The plane reflects in a “diffuse” manner—so the radiance is the same in all directions (Lambertian)

Now we must divide into small areas and integrate. For the spherical element,

$$dA_{sp} = r^2 \sin\theta \, d\theta \, d\phi$$

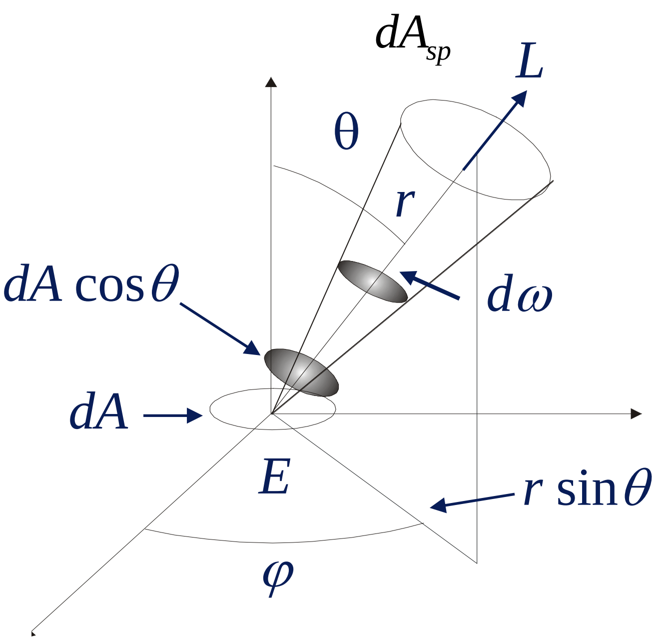
Then, for each flux element,

$$d\Phi = L \, dA \cos\theta \, \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2}$$

Integrate over the hemisphere

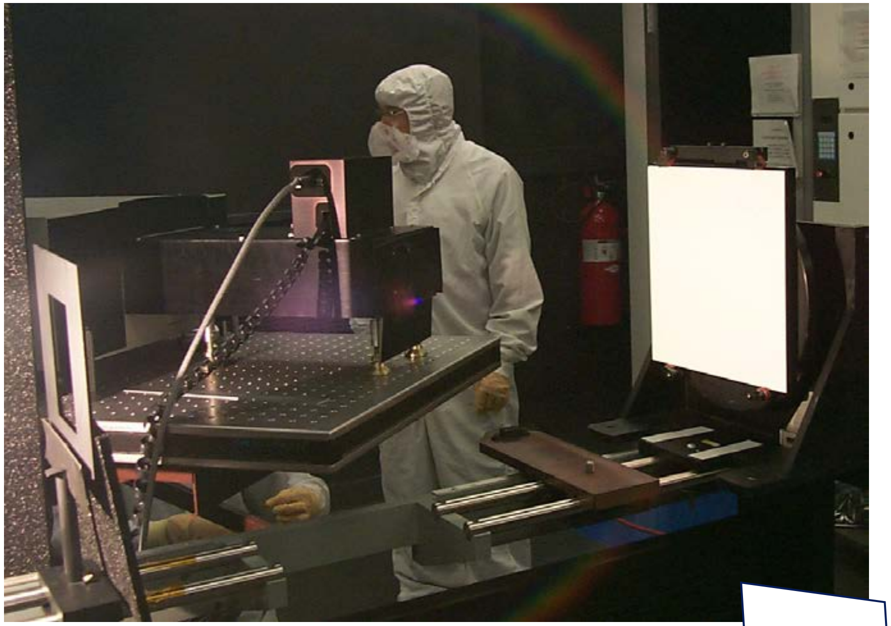
$$\frac{\Phi}{dA} = E = L \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

$$E = L \, 2\pi \, \frac{1}{2} \left(\sin^2\theta\right)\Big|_0^{\pi/2} = \pi L$$



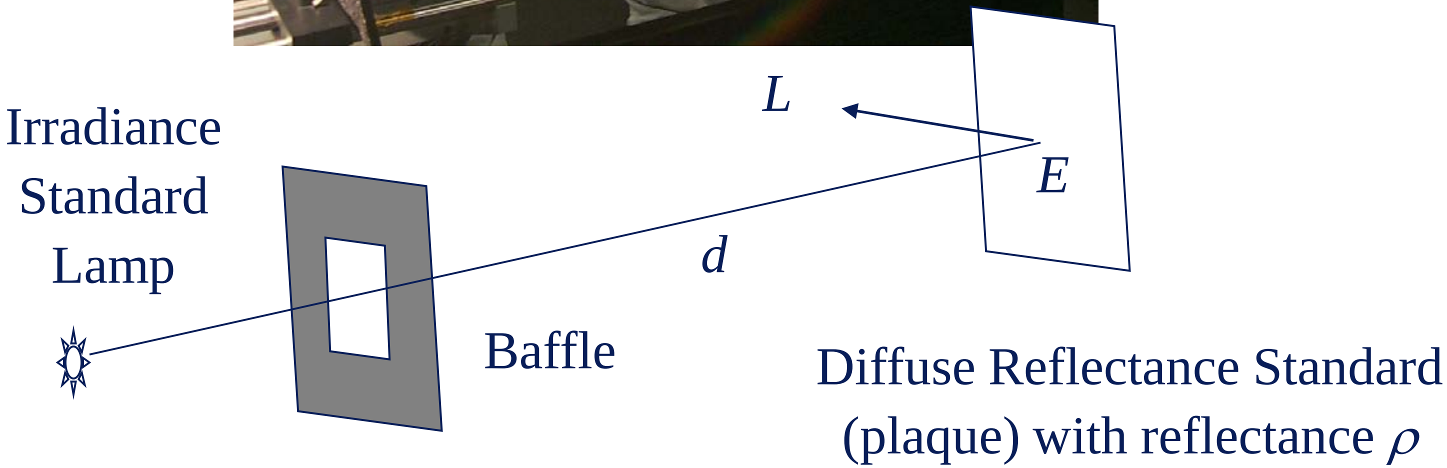
$$E = \pi L$$

“Lamp-Plaque Method”



Classic example of irradiance to radiance transfer, for producing a source of known radiance for instrument calibration.

$$L = \frac{E\,\rho}{\pi}$$

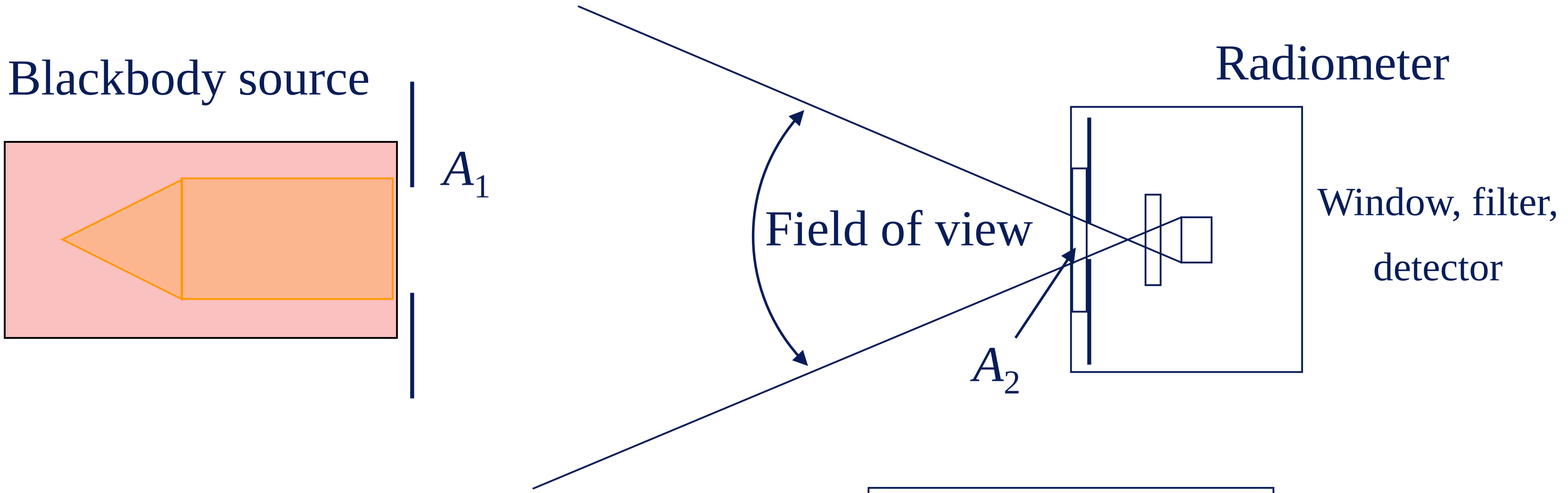


What matters?

1. Lamp-Plaque method
   a) Distance: $E \propto 1/d^2$. The relative uncertainty is then $\Delta E/E=2\Delta d/d$. Standard lamps (“FELs”) are calibrated at 500 mm; so a 1 mm uncertainty is 0.4% in irradiance.
   b) Correct lamp current and proper baffle placement.

2. Calibration methods
   a) A irradiance detector must have its field of view “underfilled” by the source; distance matters
   b) A radiance detector must have its field of view “overfilled” by the source; distance does not matter

Irradiance mode (see *E* from extended source)



$$\Phi = L \cdot \Theta$$

$$\Theta = \pi A_1 F_{D1-D2}$$

$$F_{D1-D2} = \frac{1}{2}\left(\frac{(r_2^2 + r_1^2 + d^2) - \left(\sqrt{\left(r_2^2 + r_1^2 + d^2\right)^2 - 4 r_1^2 r_2^2} \right)}{r_1^2} \right)$$

See: Siegel and Howell

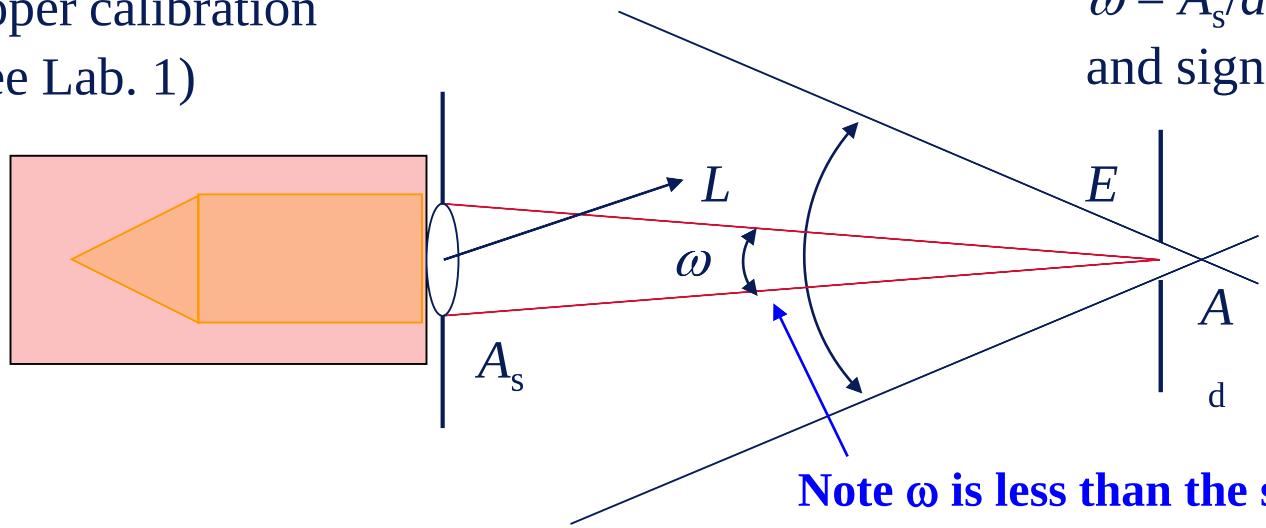
Irradiance mode, continued

$E = L\,\omega$

$\omega = A_s/d^2$

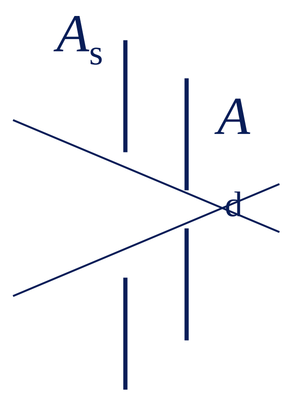
and signal $S \propto \Phi = E\,A_d$

Proper calibration
(See Lab. 1)



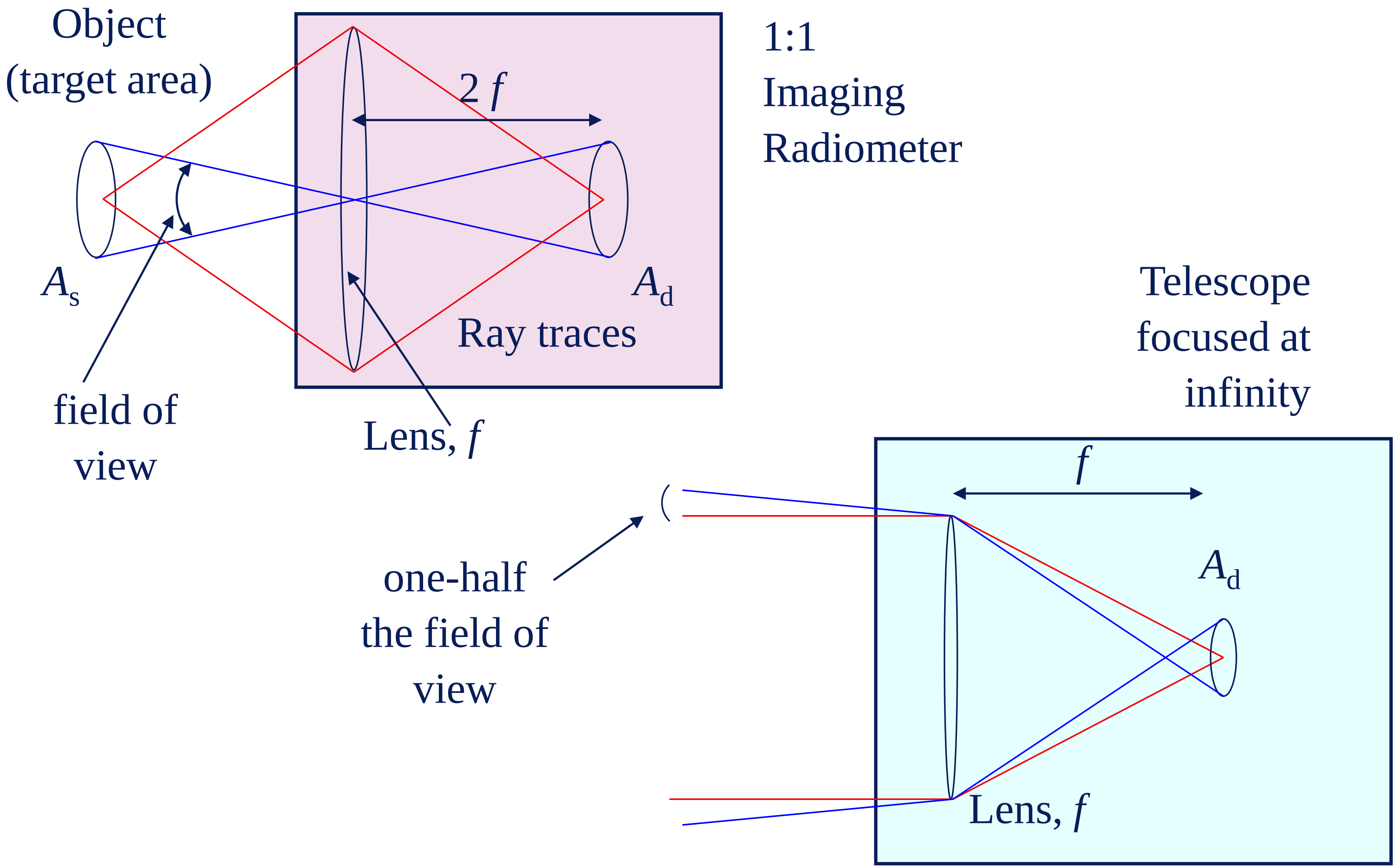
Note ω is less than the solid angle for the radiometer field of view

Incorrect calibration of irradiance detector (too close)



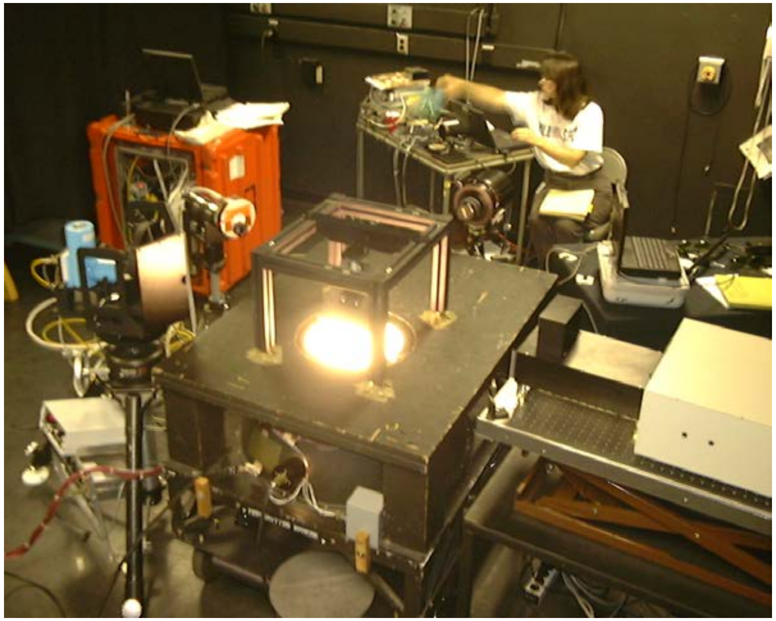
The ω we would calculate from A_s would be greater than the limit of the radiometer

Radiance mode (see invariance of radiance)



Integrating sphere examples

Calibration of sun photometers using an integrating sphere source (NASA GSFC).

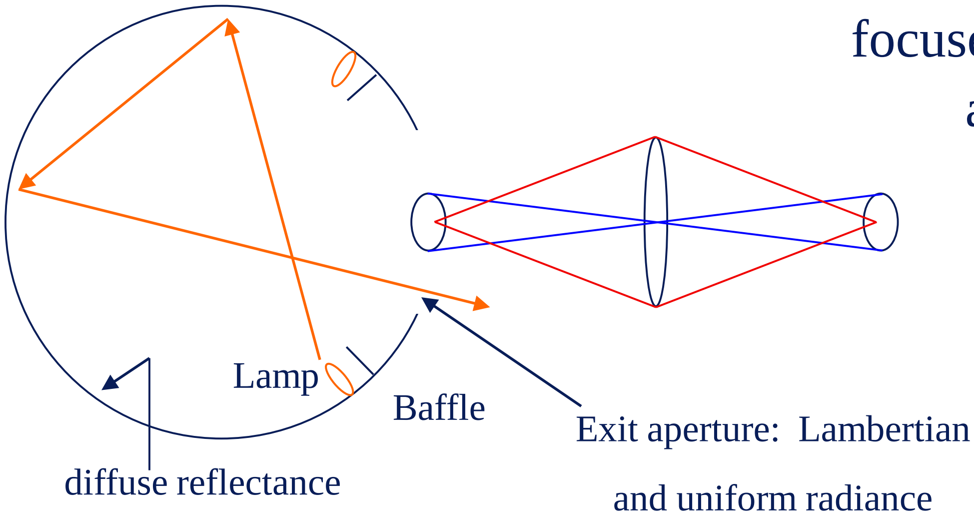


An up-looking sphere source for nadir-viewing, aircraft-deployed spectroradiometers (NASA Ames).

Radiance mode (see invariance of radiance)

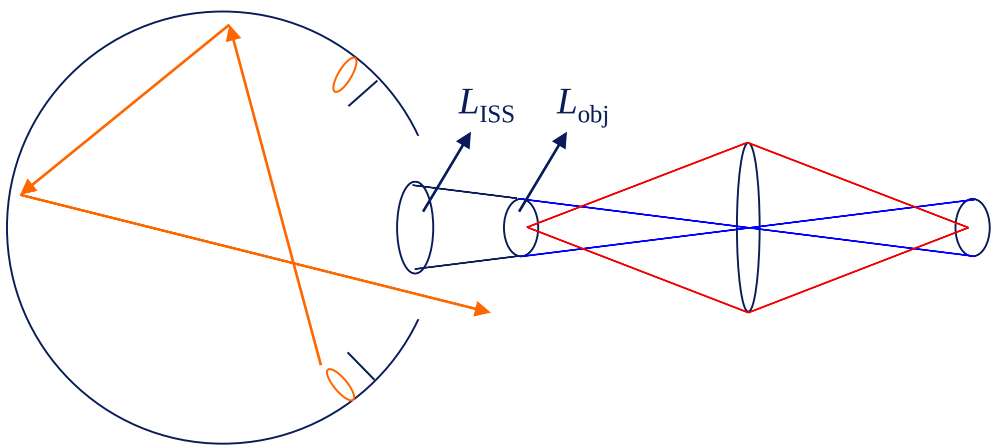
Integrating sphere source

1:1 imaging radiometer focused on the exit aperture



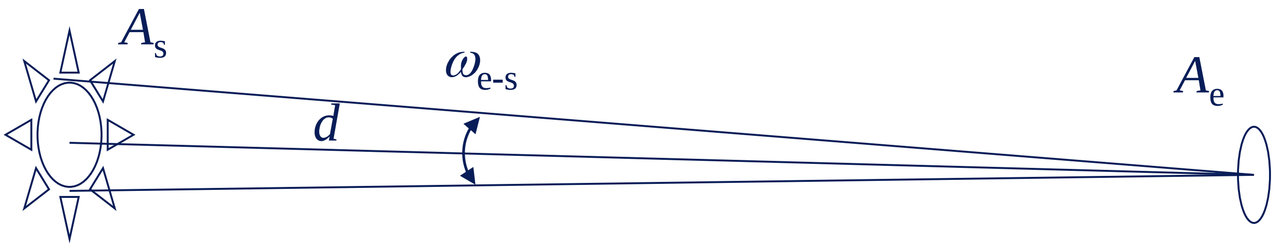
Either method is valid because of the invariance of radiance:

$$\boldsymbol{L_{\mathrm{ISS}} = L_{\mathrm{obj}}}$$



1:1 imaging radiometer focused in front of the exit aperture

Example: Solar constant (irradiance at Earth's surface)



Sun:

Blackbody at T = 5800 K;

Lambertian;

diameter = 6.96 x 10^8 m; Earth-sun distance d = 1.5 x 10^{11} m

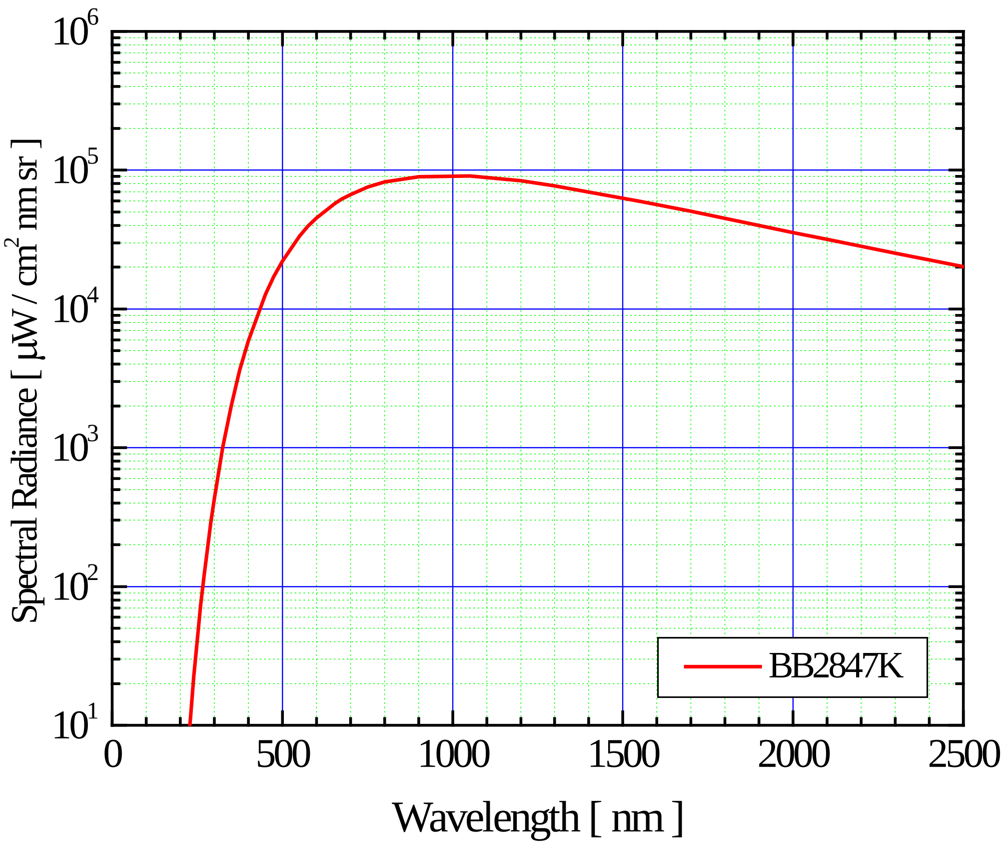
Earth:

Total irradiance (all wavelengths) is of interest for Earth's energy balance and solar physics research.

diameter = 6.38 x 10^6 m

Spectral aspects of radiometry

A blackbody source obeys Planck's law

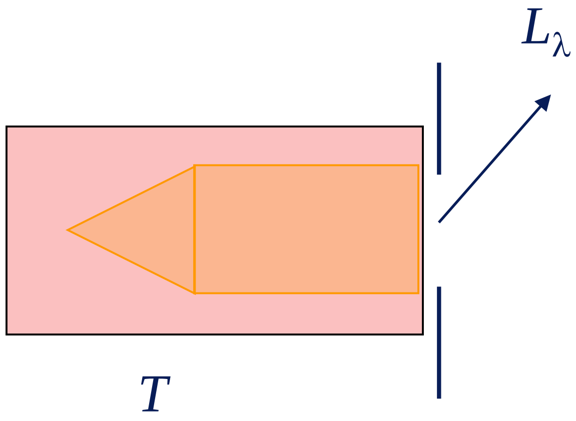


$$L_\lambda = \frac{c_{1L}}{\lambda^5 \exp\left[c_2 / (\lambda \cdot T) - 1\right]}$$

[μW / cm² nm sr]

The radiance drops very sharply below a particular wavelength. As the temperature increases, the radiance increases for all wavelengths and the peak moves to shorter wavelength ($\lambda_{max} \propto 1/T$).

Total exitance, *M*



Integrate Planck's radiance law over all wavelengths and the entire hemisphere above the exit aperture.

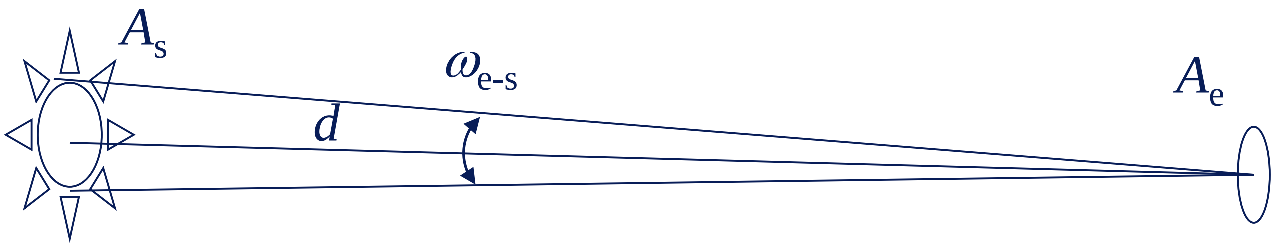
$$M(T) = \sigma \cdot T^4 \ \ [\mathrm{W/m^2}]$$

$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

Stefan-Boltzmann law
(total exitance)

The Stefan-Boltzmann relationship is useful when the detector responds over a wide range of wavelength with a nearly constant responsivity.

Example: Solar constant (irradiance at Earth's surface)



Sun:

Blackbody at T = 5800 K;

Lambertian;

diameter = 6.96 x 10^8 m; Earth-sun distance d = 1.5 x 10^{11} m

Earth:

Total irradiance (all wavelengths) is of interest for Earth's energy balance and solar physics research.

diameter = 6.38 x 10^6 m

$$M = \sigma T^4$$

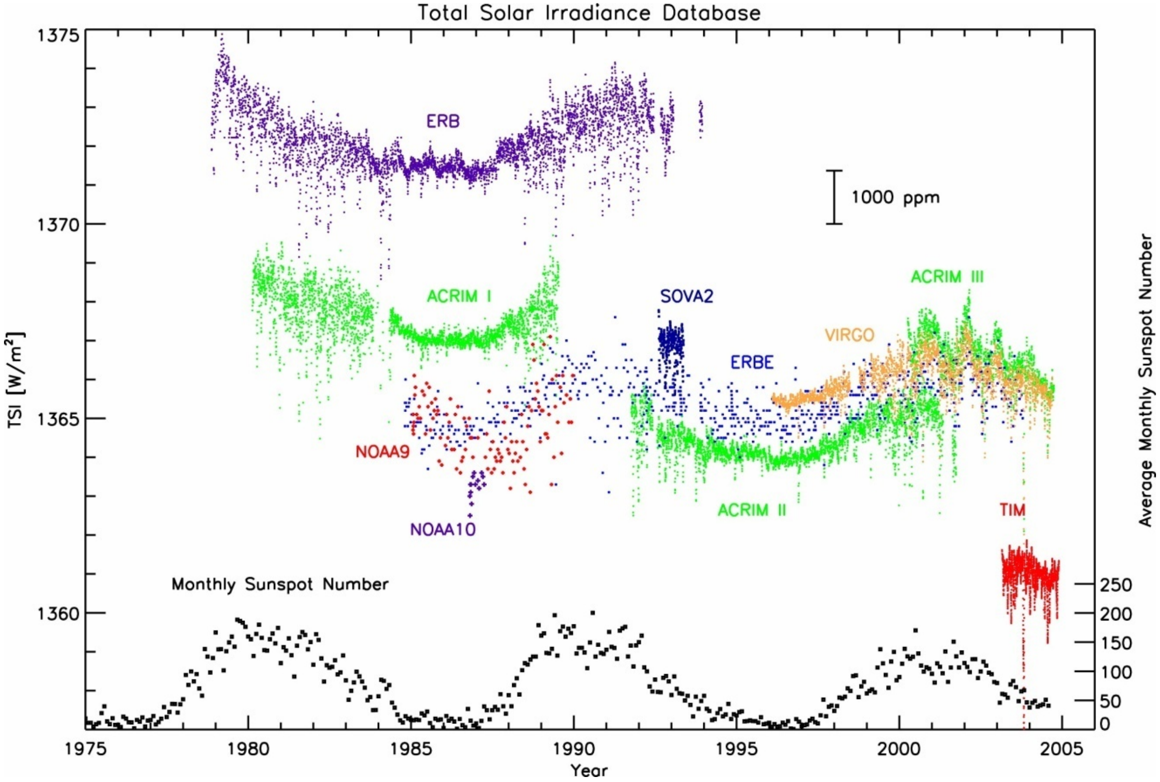
$$L_s = \frac{M}{\pi}$$

$$E_e = L_s \, \omega_{e-s}$$

$$= \frac{\sigma T^4}{\pi} \frac{A_s}{d^2}$$

We solved this problem using the point to hemisphere throughput derivation (Slide 22) and the irradiance on a plane from an extended source (Slide 16). The answer is E_e = 1389 W m^{-2}. We must assume sun is Lambertian.

Measurements of the solar constant



Exoatmospheric measurements using electrical substitution radiometers (ESRs)

http://spot.colorado.edu/~koppg/TSI

Latest instrument: TIM on SORCE http://lasp.colorado.edu/sorce/

Launched January 25, 2003

Measurement Equation Approach:

In general, we use the measurement equation approach for characterizing and calibrating sources and radiometers. A simplified measurement equation is:

$$I(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda} \int_{A} \int_{\omega} S_\phi(x,y,\theta,\phi,\lambda,\lambda_o) \cdot L_\lambda(x,y,\theta,\phi,\lambda,\lambda_o) \cdot \cos\theta \cdot \mathrm{d}\omega \cdot \mathrm{d}A \cdot \mathrm{d}\lambda$$

$I(A,\omega,\Delta\lambda,\lambda_o)$ - the measured current

S_ϕ - the spectral flux (power) responsivity of the detector at a position x,y

L_λ - the spectral radiance of the source

A - receiving area of the detector

ω - the solid angle of the source viewed by the detector

Linearity, polarization dependences not considered in this expression but can be added.

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2. Detector-based Radiometry

Outline

1. What is Detector-based Radiometry
2. Detector-based Scale Realizations
   a) Electrical Substitution Radiometers (ESR)
      - Cryogenic Radiometers
   b) Spectral Responsivity Measurement Facilities
      - Power, irradiance, and radiance responsivity
   c) Scale Transfer to Measurement Facilities
3. Application Example (SRSC Laboratory #2)
   - Photometry

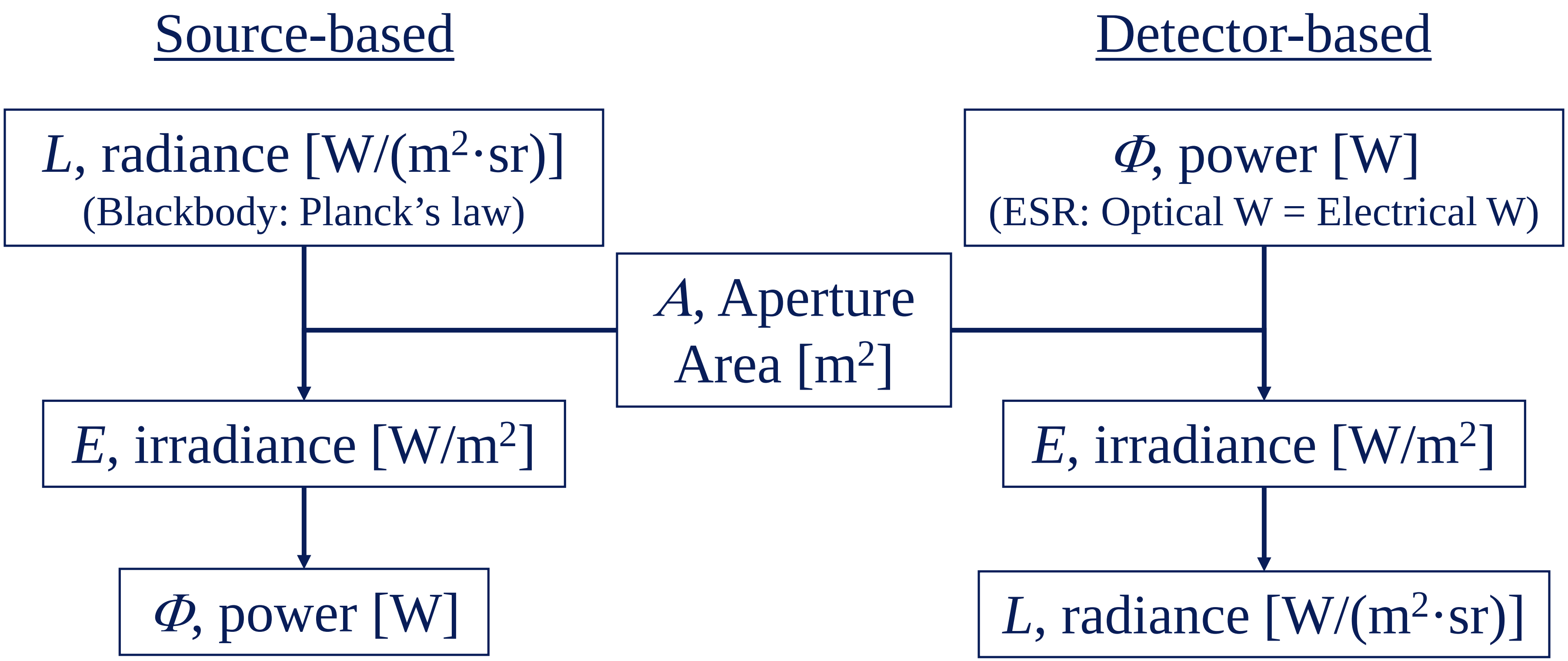
     Illuminance [lux]

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

What Is Detector-based Radiometry?

- Radiometric measurements using detectors whose calibration is traceable to a detector (primary) standard

Comparison of source and detector-based scales



Fundamental Radiometric Scales

Electrical Substitution Radiometry [W] (0.05 %)

Schwinger Equation (0.5 %)

$$P(E,R,\theta,\lambda)=\frac{4e^2cR}{3\lambda^4}\gamma^{-4}\left(1+\chi^2\right)^2\left[K_{2/3}^2(\xi)+\frac{\chi^2}{\left(1+\chi^2\right)}K_{1/3}^2(\xi)\right]$$

Planck Radiance (0.25 %)

$$L_\lambda=\frac{c_1}{n^2\lambda^5\left(e^{\left[\frac{c_2}{n\lambda T}\right]}-1\right)}$$

Definition of Traceability

"property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties."

International System of Units (SI)

1. Established in 1960, SI is the modern metric system of measurement used throughout the world.
2. SI defines three classes of units: basic, derived and supplementary. Examples
   a) Basic: Thermodynamic temperature kelvin [K]
   b) Derived: Area, square meter [m^2]
      Steradian [sr]
   c) Supplementary: Power, watt [W]

Radiometric Quantities (Review)

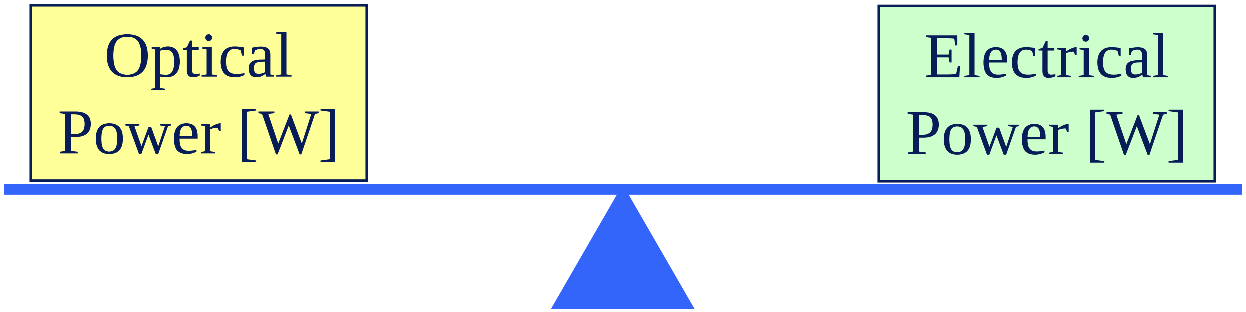
Radiometric quantities with their symbols and SI units

W = watt, m = meter, sr = steradian

Radiometric quantity	Symbol	Unit
Radiant flux (power)	P, Φ	W
Irradiance	E	W/m^2
Radiance	L	$W/(m^2 \cdot sr)$
Radiant intensity	I	W/sr

Electrical Substitution Radiometer (ESR)

The principle of electrical substitution radiometry is to balance the electrical and optical power [Watt] needed to create the same temperature rise in the ESR

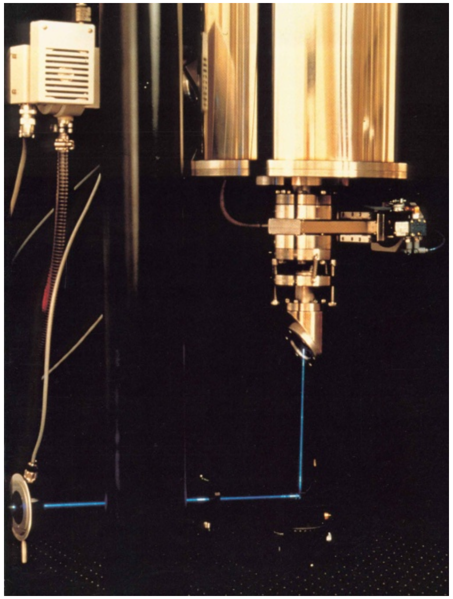


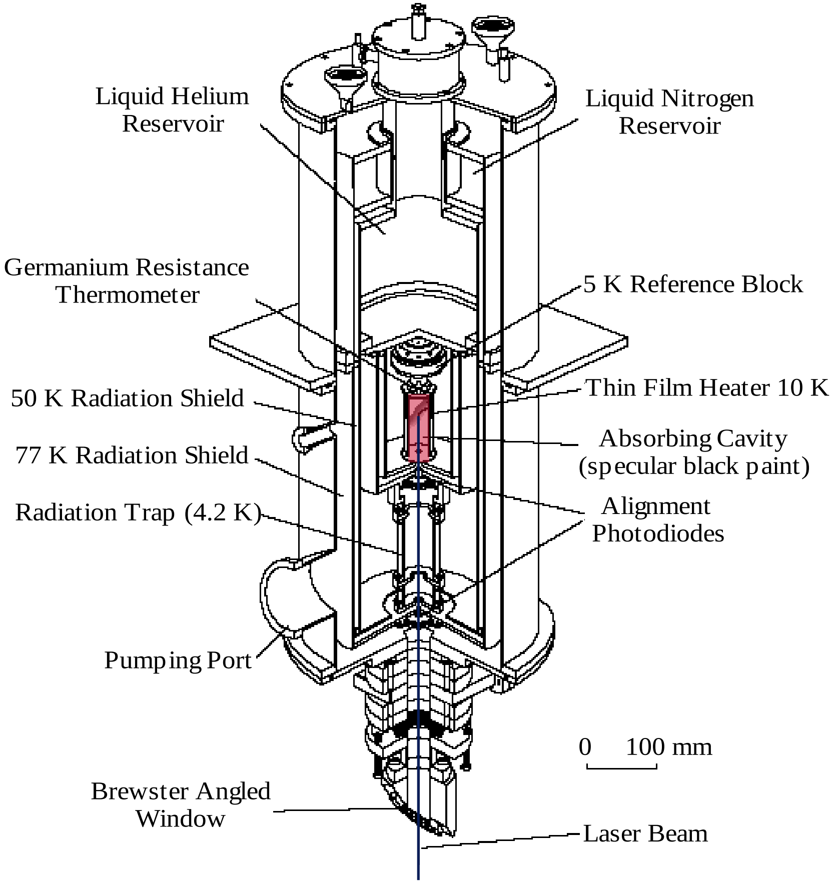
Optical Power [W] = Electrical Power [W]

NIST Cryogenic Radiometer

Cryogenic temperatures allow lower degree of non-equivalence:

1. Larger cavity due to increased heat capacity
2. Reduced lead heating due to superconducting leads
3. Reduced temperature gradients between electrical and optical heating
4. Reduced background radiation





NIST Cryogenic Radiometer

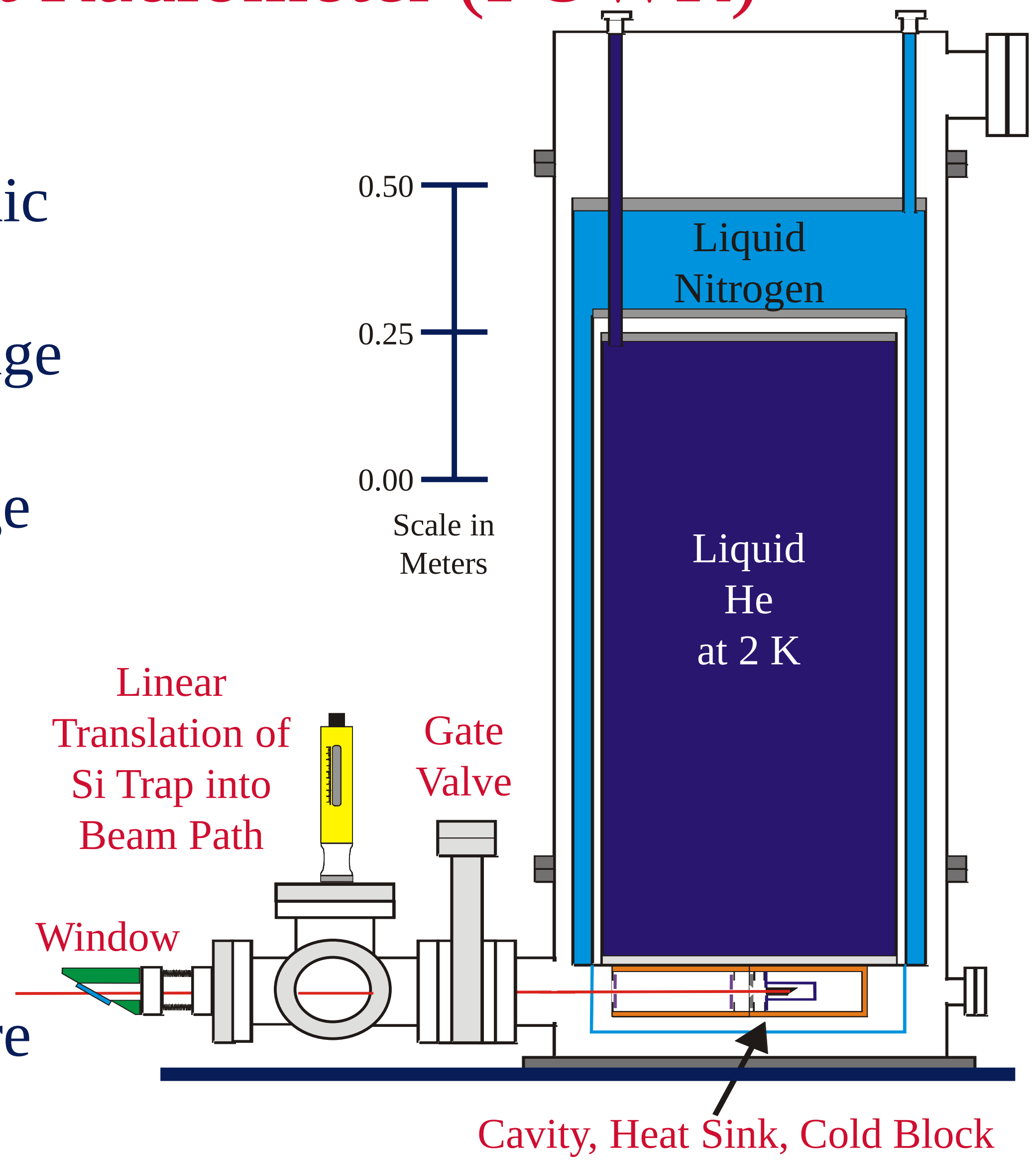
Primary Optical Watt Radiometer (POWR) is the U.S. primary standard for optical power.

Cryogenic temperatures allow lower degree of non-equivalence:

- Larger cavity due to increased heat capacity
- Reduced lead heating due to superconducting leads
- Reduced temperature gradients between electrical and optical heating
- Reduced background radiation

Primary Optical Watt Radiometer (POWR)

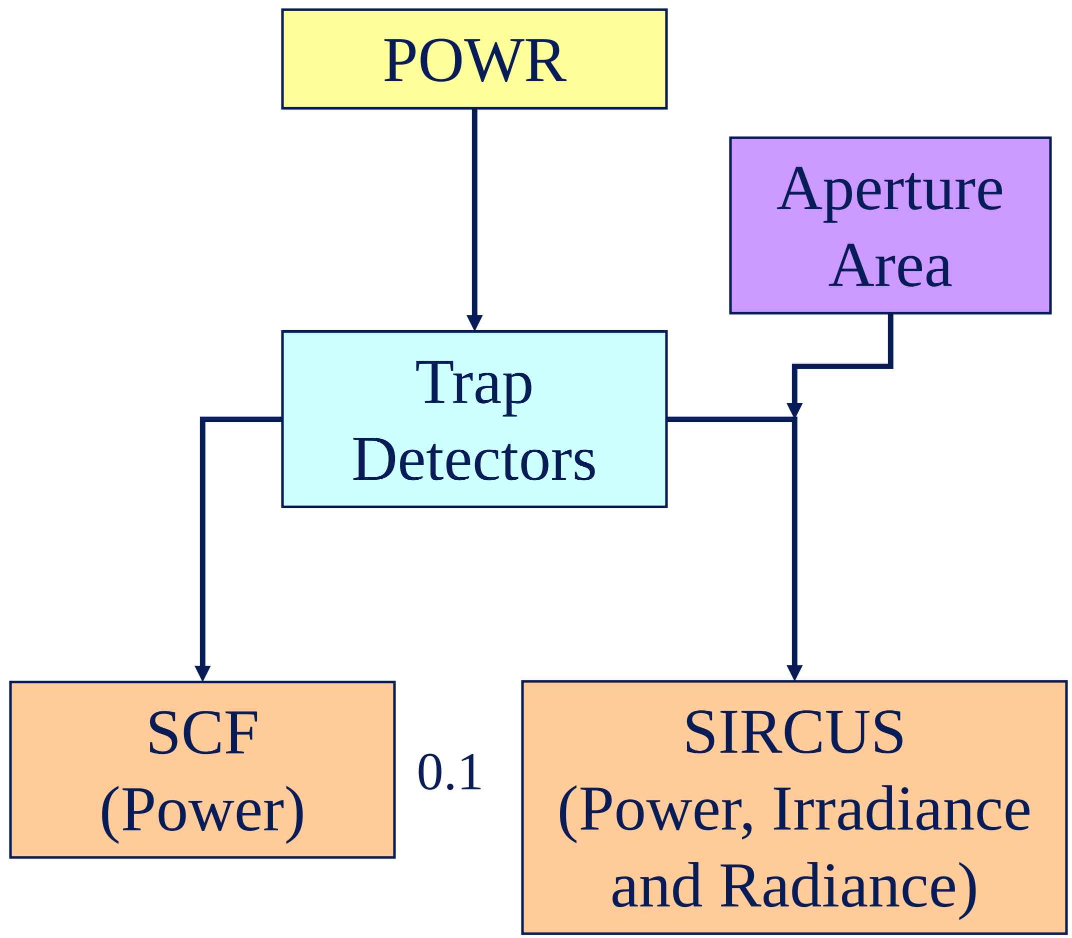
1. Shorter calibration chain
2. Greater power level dynamic range (μW to mW)
3. Continuous spectral coverage (200 nm to 20 μm)
4. Extend IR and UV coverage
5. Windowless transfer, decreasing transfer uncertainties
6. Explore irradiance measurements
7. Modular design allows for modifications to meet future requirements



Transfer to Measurement Facilities

Block diagram of POWR to SCF and SIRCUS

		Uncertainty (k=1)
Primary Standard	POWR	0.01
Transfer Standards	Trap Detectors	0.03
Working Standards	SCF (Power)	0.1
Working Standards	SIRCUS (Power, Irradiance and Radiance)	0.05



Measurement Facilities for Spectral Responsivity

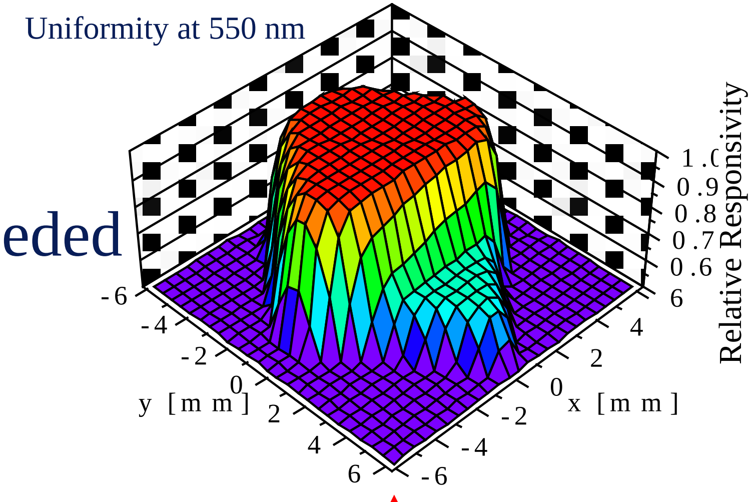
Two principal detector measurement facilities:

1. Spectral Comparator Facilities (SCF)
   a) Monochromator based
   b) UV SCF: 200 nm to 500 nm
   c) Visible to Near IR SCF: 350 nm to 1800 nm
2. Spectral Irradiance and Radiance Calibrations using Uniform Sources Facility (SIRCUS)
   a) Tunable laser based
   b) 210 nm to 1800 nm (UV-Vis-NIR SIRCUS)
   c) 1000 nm to 5000 nm (IR SIRCUS)
   d) Various source configurations tailored to the measurement (typically an integrating sphere)

When to Use Trap Detectors

Low uncertainty transfer standard from HACR

1. Advantages
   a) Uniform responsivity
   b) Polarization insensitive
   c) Reflection measurements not needed

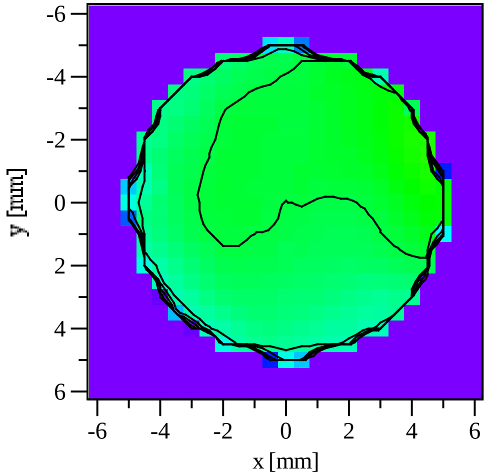


2. Drawbacks
   a) Limited field-of-view (FOV)
   b) “Impossible” to buy
   c) Hard to make
   d) Windowless diodes, potentially unstable
   e) Lower shunt resistance (diodes in parallel) limits gain to less than with a single photodiode

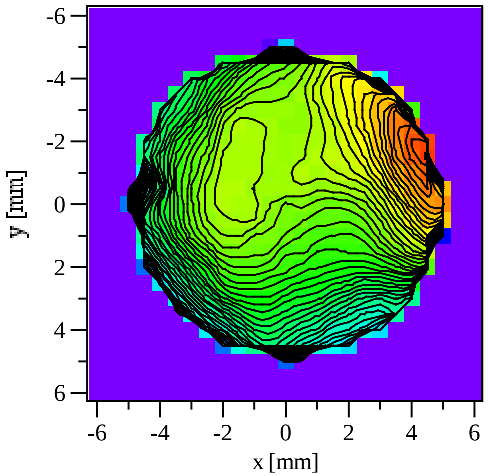
Trap Detector Examples and Uniformities

S2281 Diode at 500 nm

Contour Lines = 0.2 %

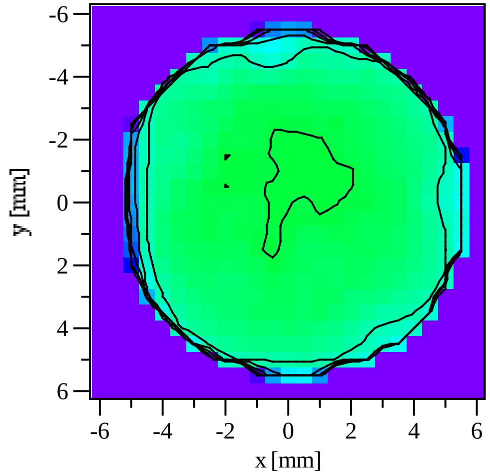


Contour Lines = 0.01 %

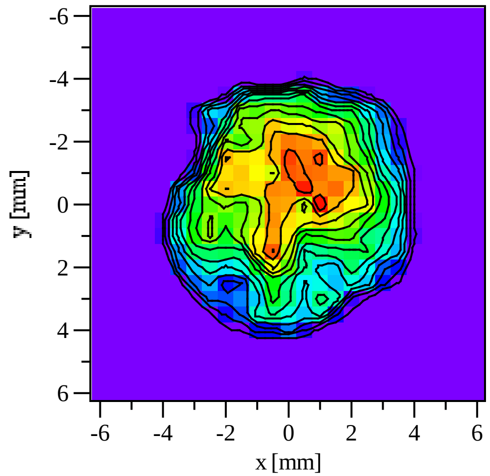


QED-200 at 500 nm

Contour Lines = 0.2 %

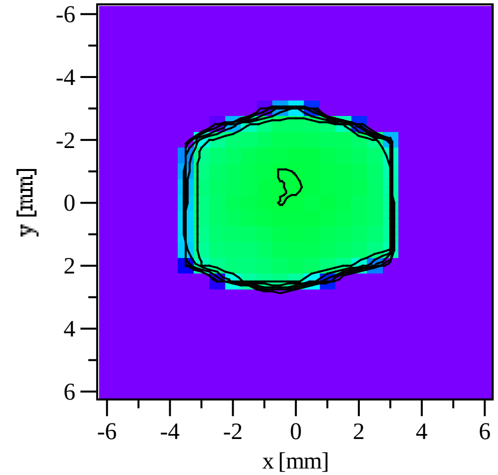


Contour Lines = 0.01 %

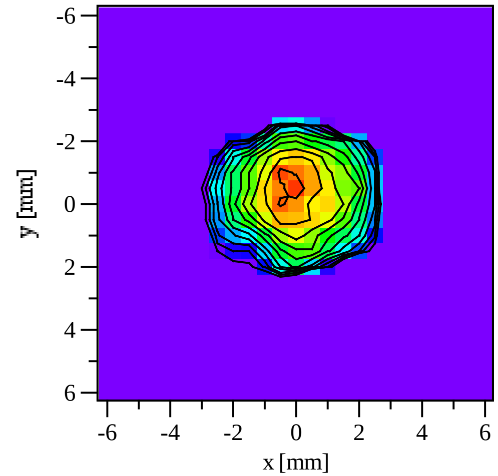


Tunnel Trap at 500 nm

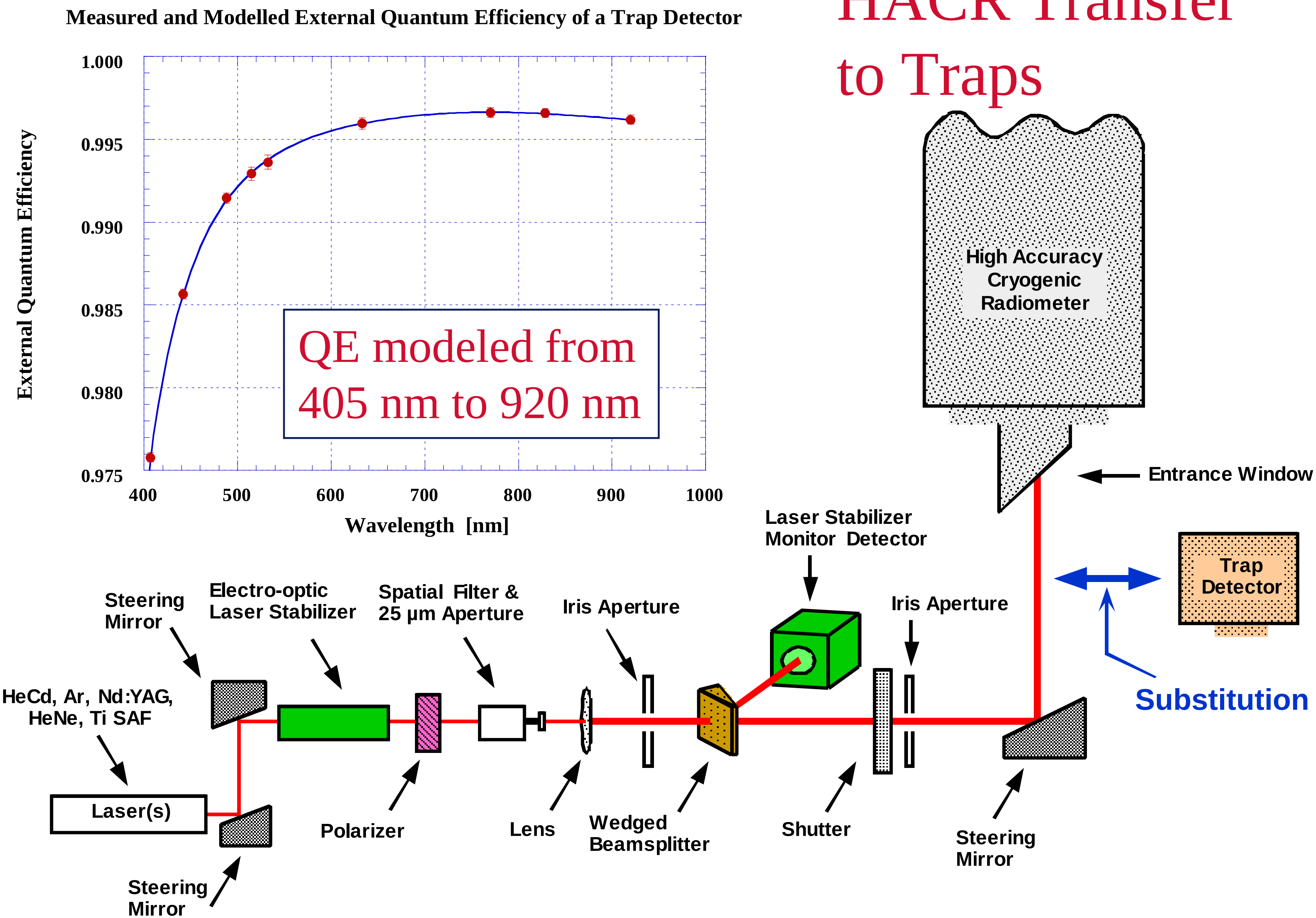
Contour Lines = 0.2 %



Contour Lines = 0.01 %

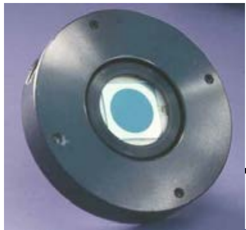


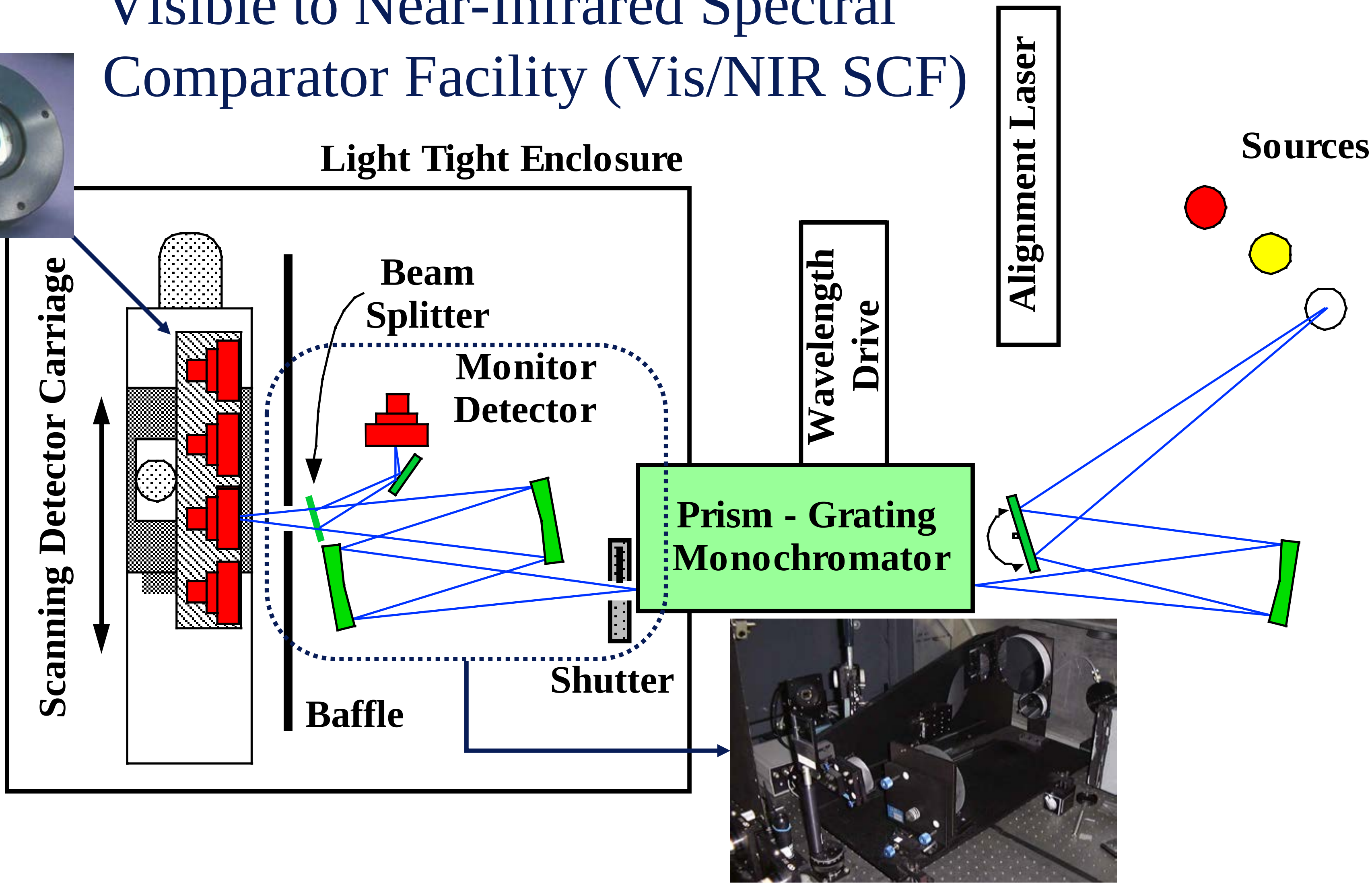
HACR Transfer to Traps



Spectral Power Responsivity

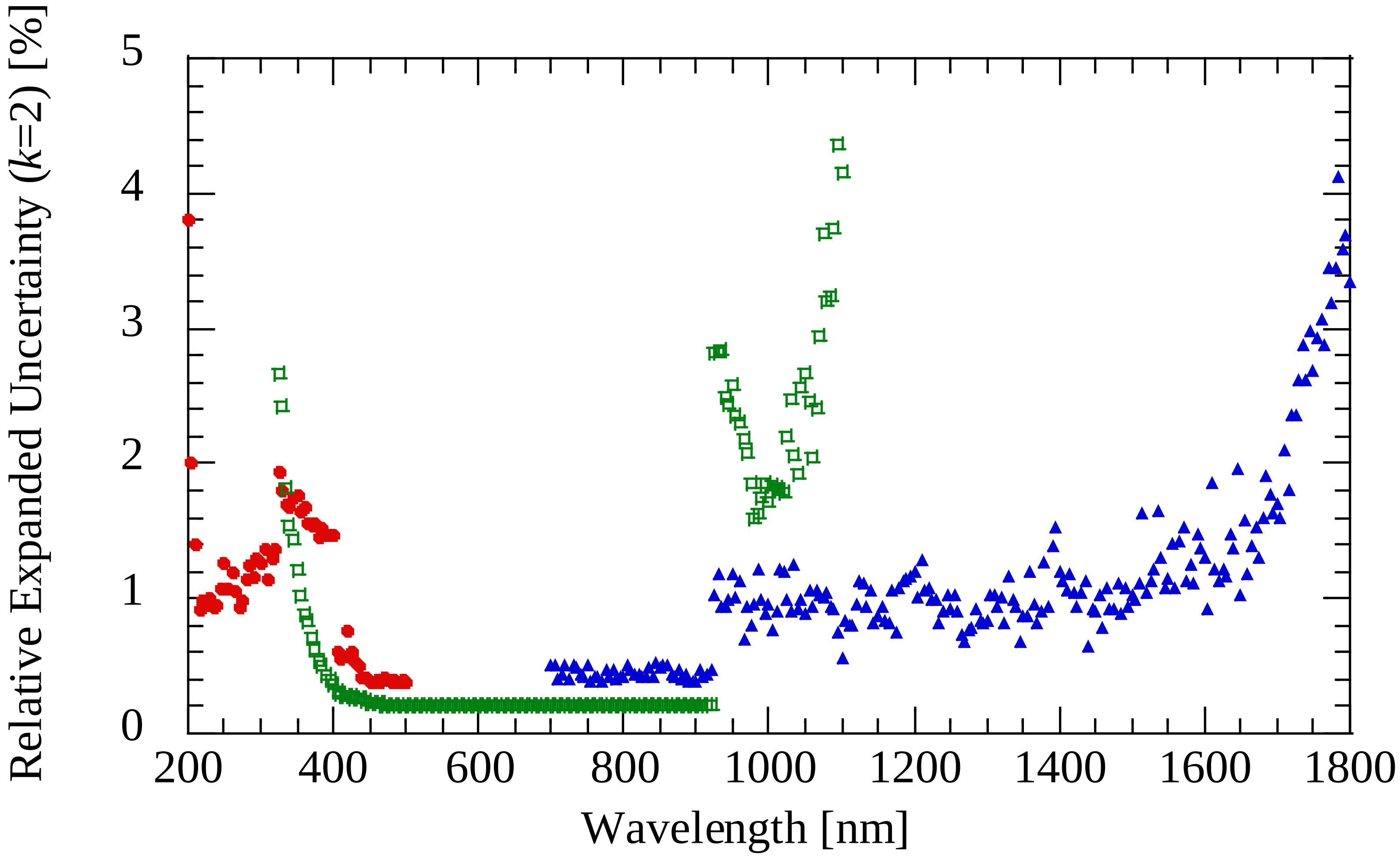
Visible to Near-Infrared Spectral Comparator Facility (Vis/NIR SCF)





SCF (Spectral Power Responsivity) Uncertainty

Current SCF uncertainty from 200 nm to 1800 nm



Radiometric Measurement Configurations

1. Radiant Flux (Power) Measurement Φ [W]

Underfills Aperture

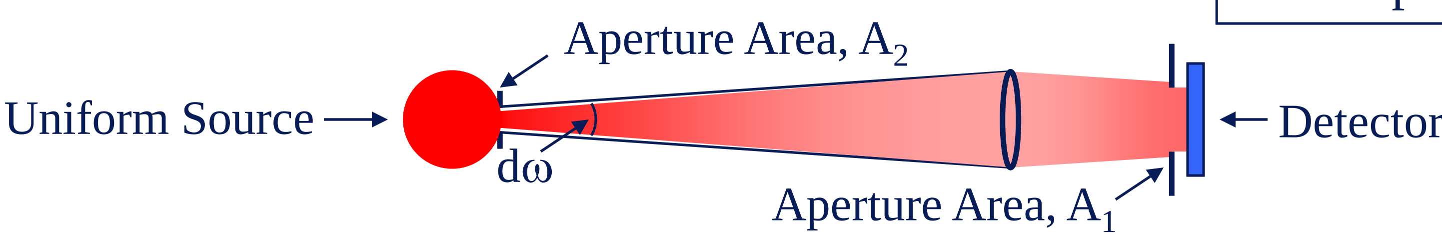
2. Irradiance Measurement E = flux/collector area = Φ/A_1 [W/m^2]

Overfills Aperture

3. Radiance Measurement L = flux/projected source area/solid angle

$$= \Phi/A_2 \cdot d\Omega \; [\mathrm{W/(m^2 \cdot sr)}]$$

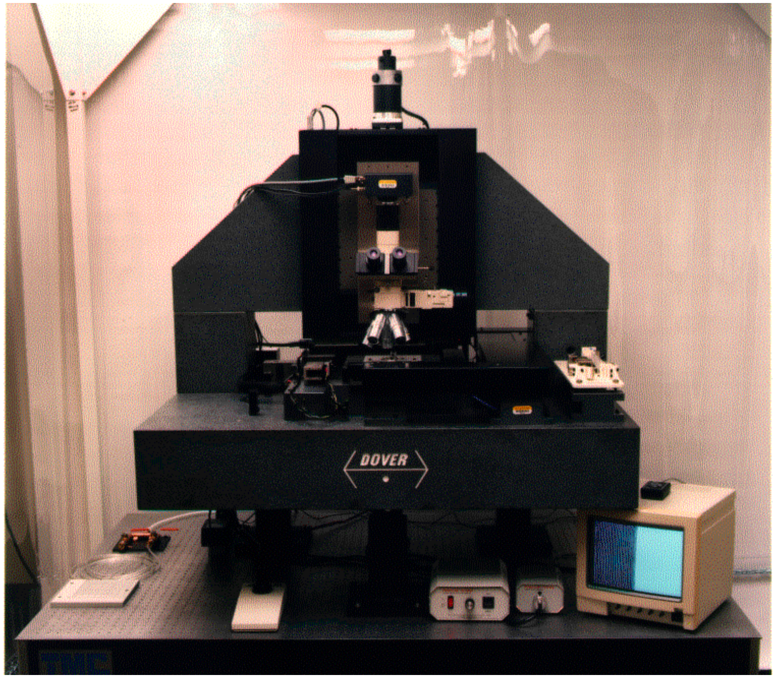
$d\omega$ < Aperture A_2

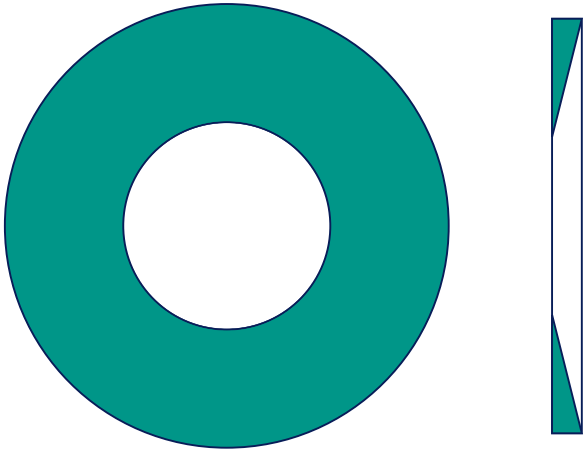


NIST
National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

NIST Aperture Area Measurement Facility

1. Measures the geometric area of high-quality circular apertures
2. Uncertainty (*k*=1) <0.01 % for aperture diameters ranging from 2 mm to 30 mm
3. Uses a precision microscope with stage position referenced to a laser interferometer
   - Standard uncertainty in relative stage position < 50 nm
4. A separate, flux-transfer instrument is used for measurements relative to a standard
5. Currently participating in an international intercomparison

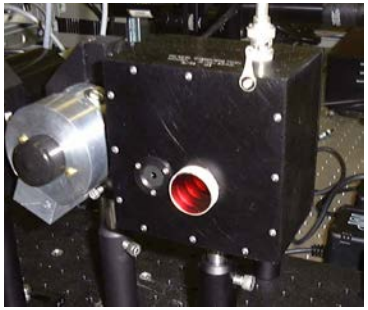


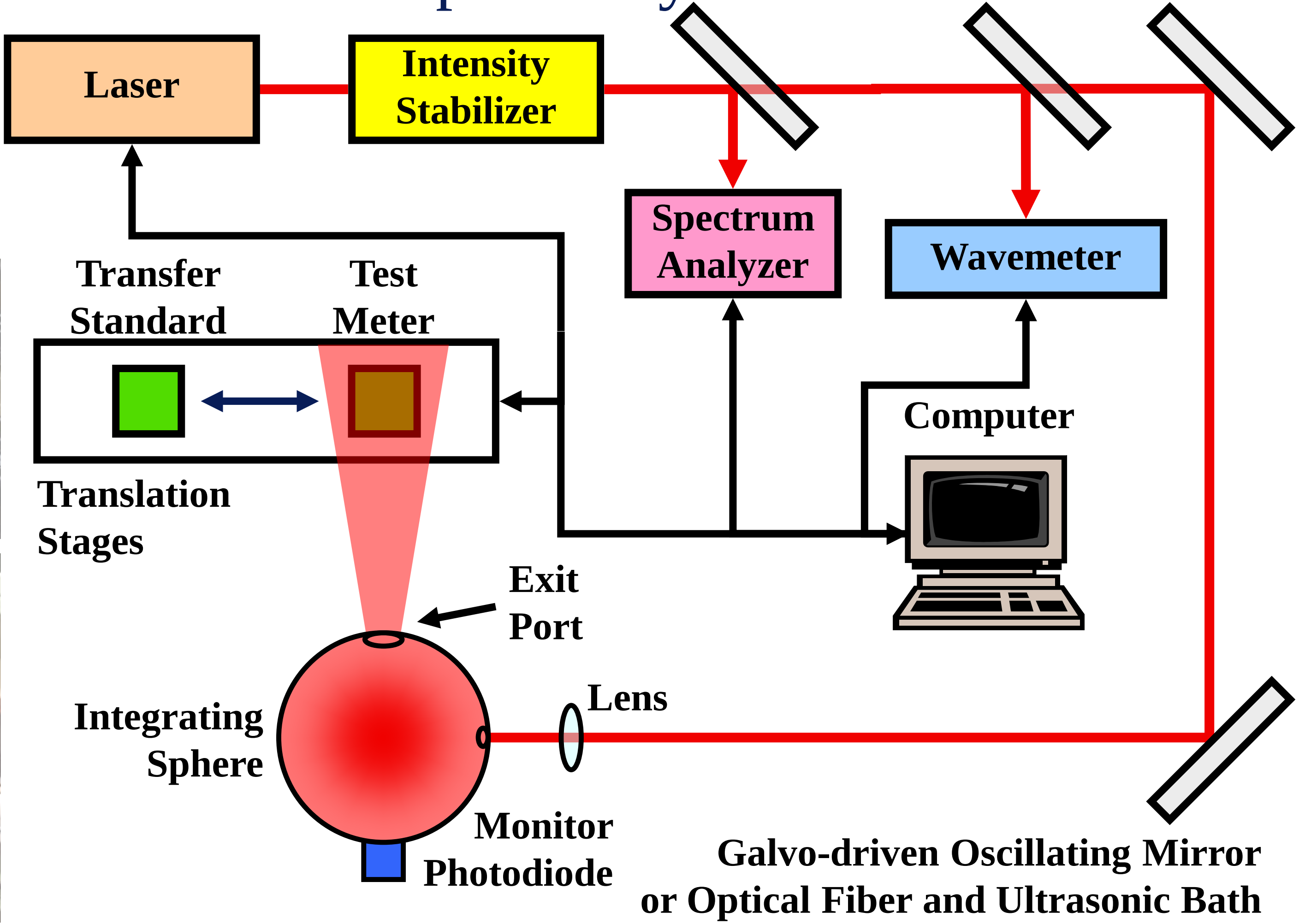


Spectral Irradiance and Radiance Calibrations using Uniform Sources (SIRCUS) Facility

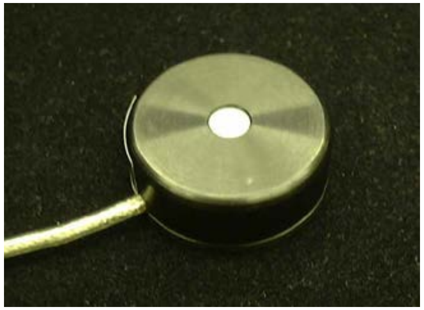
Radiance and Irradiance Responsivity

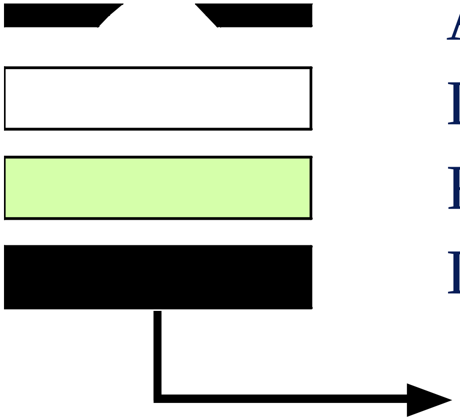
SIRCUS uses tunable lasers from 210 nm to 1800 nm





Filter Radiometer Example



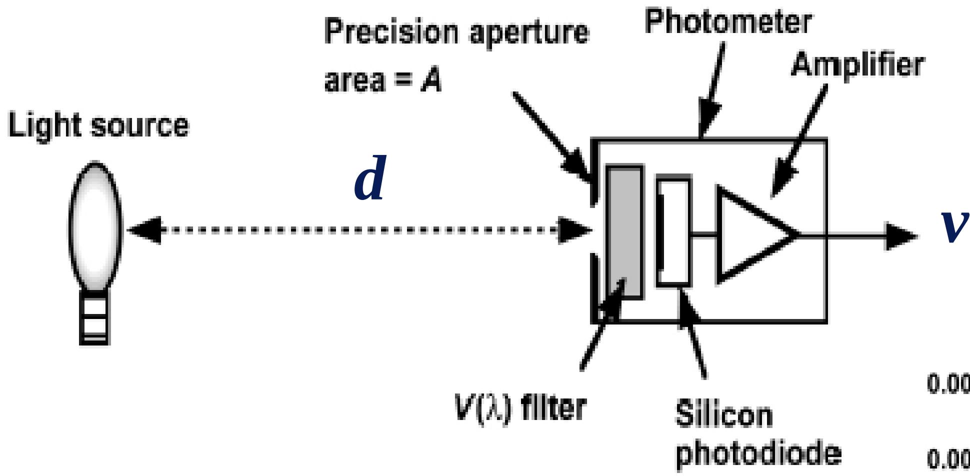


Component	Function
Aperture	Defines measurement area
Diffuser	Maintains cosine response (optional)
Filter	Spectral selection
Detector	Power measurement

$$i = A\int_{\lambda} E(\lambda)s(\lambda)\mathrm{d}\lambda$$

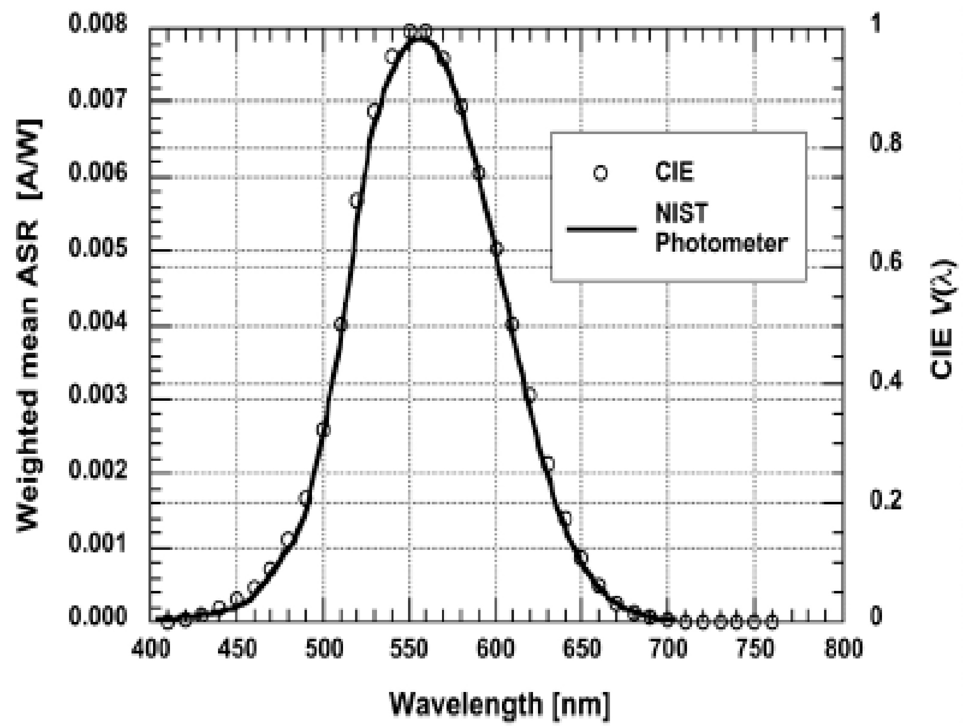
The signal *i* observed from such a radiometer is the aperture area *A* multiplied by the integral of the product of the spectral irradiance of the source at the aperture $E(\lambda)$ and the meter's spectral power responsivity $s(\lambda)$.

Example Photometer



Example photometer component layout

CIE $V(\lambda)$ function and NIST photometer spectral responsivity $s(\lambda)$



Conversion to Photometric Units

The luminous flux is related to the radiant flux by:

$$\Phi_{\mathrm{v}} = K_{\mathrm{m}} \int_{360\ \mathrm{nm}}^{830\ \mathrm{nm}} \Phi(\lambda) V(\lambda) \mathrm{d}\lambda$$

K_{m} maximum spectral luminous efficacy [683 lm/W]

$V(\lambda)$ spectral luminous efficiency function

The luminous flux can also be written:

$$\Phi_{\mathrm{v}} = A K_{\mathrm{m}} \int_{\lambda} E(\lambda) V(\lambda) \mathrm{d}\lambda$$

Note: for brevity the explicit notation of the photopic wavelength range is indicated by λ.

Luminous Flux and Illuminance Responsivity

The luminous flux responsivity [A/lm] of a photometer:

$$s_{\mathrm{v,f}} = \frac{\mathrm{signal}_{\mathrm{out}}}{\Phi_{\mathrm{v}}} = \frac{\int_\lambda E(\lambda)s(\lambda)\mathrm{d}\lambda}{K_{\mathrm{m}}\int_\lambda E(\lambda)V(\lambda)\mathrm{d}\lambda}$$

The illuminance responsivity [A/lx] of a photometer is:

$$s_{\mathrm{v,i}} = As_{\mathrm{v,f}} = A\frac{\int_\lambda E(\lambda)s(\lambda)\mathrm{d}\lambda}{K_{\mathrm{m}}\int_\lambda E(\lambda)V(\lambda)\mathrm{d}\lambda}$$

Given $s_{\mathrm{v,f}}$ is uniform over the aperture A

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2. T. R. Gentile, J. M. Houston, J. E. Hardis, C. L. Cromer, and A. C. Parr, "National Institute of Standards and Technology High-Accuracy Cryogenic Radiometer," *Appl. Opt.* **35**, 1056-1068 (1996).
3. T. R. Gentile, J. M. Houston, and C. L. Cromer, "Realization of a Scale of Absolute Spectral Response Using the National Institute of Standards and Technology High-Accuracy Cryogenic Radiometer," *Appl. Opt.* **35**, 4392-4403 (1996).
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9. T. C. Larason and C. L. Cromer, "Sources of Error in UV Radiation Measurements," *J. Res. Natl. Inst. Stand. Technol.*, **106**, 649–656 (2001)[1].

[1]Articles in the Journal of Research of the NIST since 1995 are available as .pdf files at http://www.nist.gov/jres.

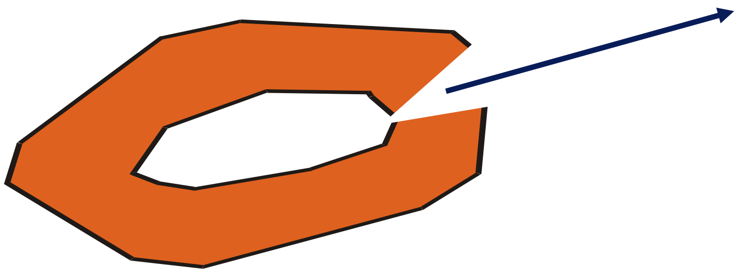
[2]Available as a .pdf file from NIST web pages.

3. Source-based Radiometry

Radiance and Irradiance

1. Radiance Sources
   a) Overfill the field-of-view of the radiometer
   b) Extended source that is spatially uniform
   c) Radiance is independent of view angle
   d) Radiance is independent of distance to radiometer
2. Irradiance Sources
   a) Underfill the field-of-view of the radiometer
   b) Approximate a point source (follows $1/d^2$ law)
   c) Uniform irradiance at the entrance pupil of the radiometer

Planck's Law



Ideal Blackbody

$$L_b(\lambda) = \frac{c_{1L}}{n^2 \lambda^5} \frac{1}{\exp\left(c_2 / \left(n \lambda T\right)\right) - 1}$$

c_{1L} = first radiation constant for spectral radiance

$n(\lambda)$ = index of refraction of medium

(1.00029 for air, 1.00028 for argon)

c_2 = second radiation constant

λ = wavelength of radiation

c_{1L} = 1.191 042 722(93) x 10^8 [W µm^4 m^{-2} sr^{-1}]

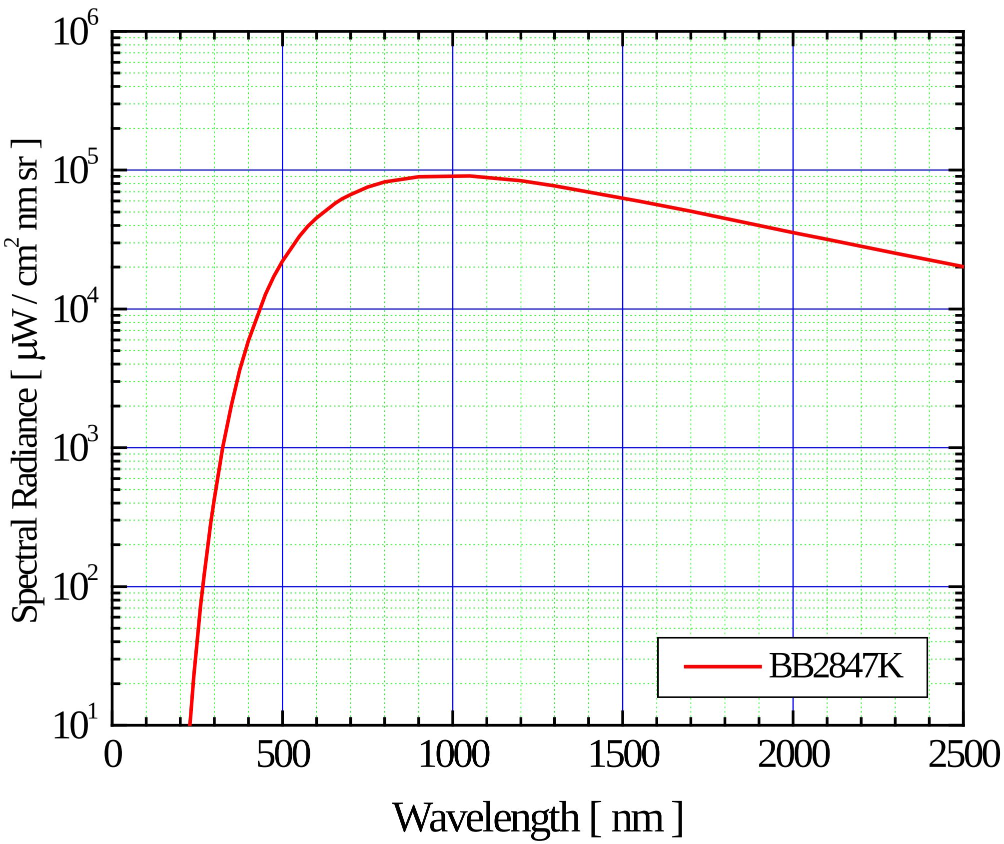
c_2 = 14 387.752 (25) [µm K]

Non ideal Blackbody: $L(\lambda) = L_b(\lambda)\ \varepsilon(\lambda)$

Note nonlinear relationship between Spectral Radiance and Blackbody Temperature

Spectral aspects of radiometry

A blackbody source obeys Planck's law



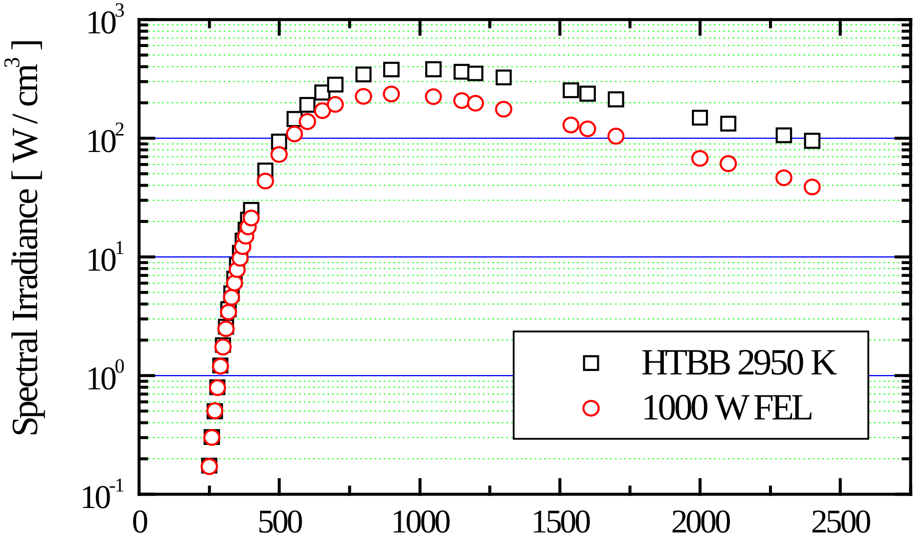
$$L_\lambda = \frac{c_{1L}}{\lambda^5 \exp[c_2 / (\lambda \cdot T) - 1]}$$

[μW / cm^2 nm sr]

The radiance drops very sharply below a particular wavelength. As the temperature increases, the radiance increases for all wavelengths and the peak moves to shorter wavelength ($\lambda_{max} \propto 1/T$).

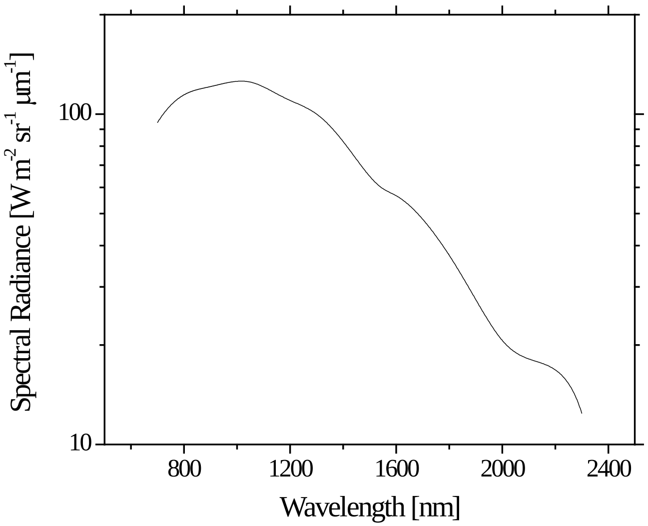
Blackbody sources are often used to calibrate spectroradiometers.

Lamps vs. blackbodies



If possible, match the temperature of the blackbody and the illumination geometries to result in similar signals. In this case, the goal is to assign irradiance values to FEL lamps.

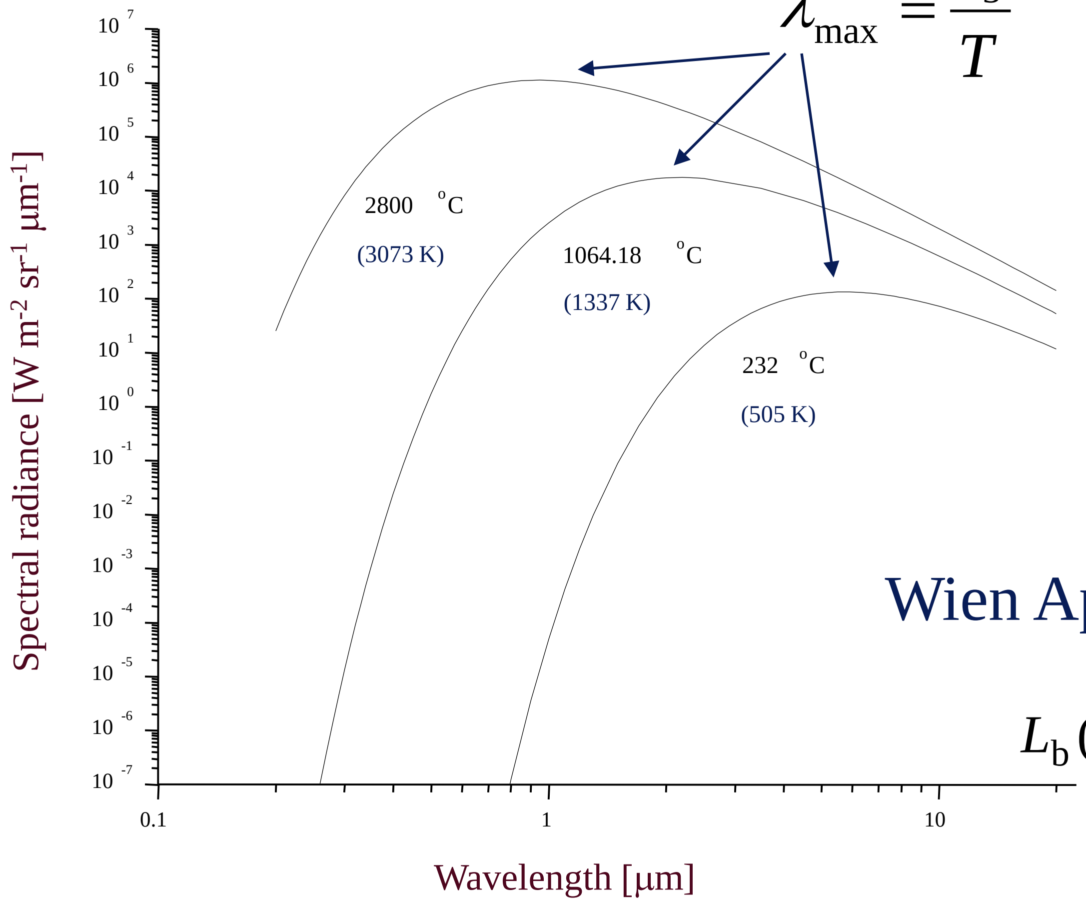
For lamp-illuminated integrating sphere sources and reflecting plaques, the spectral radiance is modified by the surface reflectance and atmospheric absorption.



Spectral Distribution, $L_b(\lambda)$

$$\lambda_{max} = \frac{c_3}{T}$$

c_3 = 2898 [μm K]



Wien Approximation ($\lambda < \lambda_{max}$):

$$L_b(\lambda) = \frac{c_{1L}}{n^2 \lambda^5} \exp\left(-c_2 / (n \lambda T)\right)$$

Stefan-Boltzmann Law

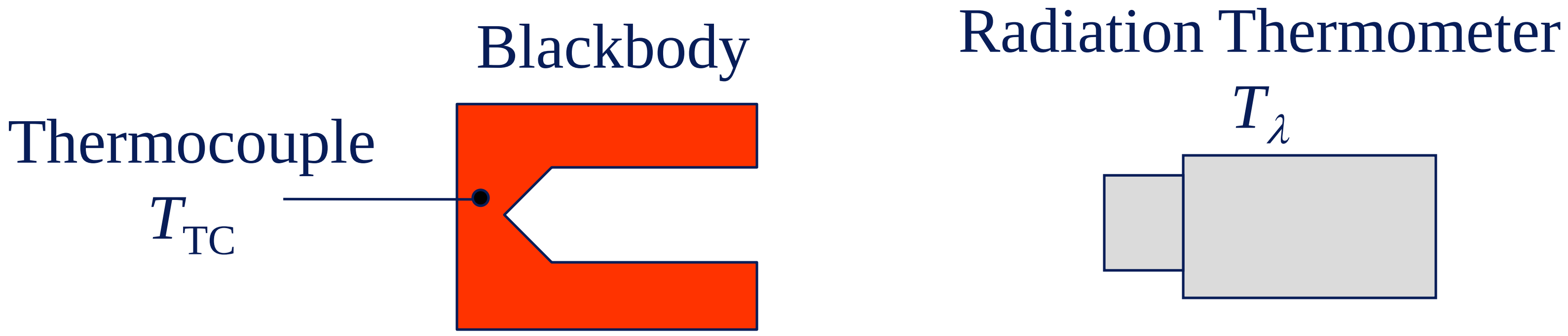
- Total exitance M: sum $L(\lambda)$ over all directions (into the hemisphere above the opening) **and** sum $L(\lambda)$ over all the electromagnetic spectrum (all wavelengths)
- For an ideal blackbody, the spectral radiance is lambertian
- With $\varepsilon(\lambda) \approx \varepsilon$ and $n(\lambda) \approx n$, the sums yield $M = \varepsilon n^2 \sigma T^4 \cong \varepsilon \sigma T^4 \quad (\text{with } n \approx 1)$
- σ = Stefan-Boltzmann constant

  $\sigma = 5.670\,400 \times 10^{-8}$ [W m^{-2} K^{-4}]

Problems with Blackbodies

1. Temperatures above 3000 K are very difficult to achieve
2. Expensive to produce accurate systems (testing and modeling)
3. Not very transportable
4. Slow time constants

Radiance Temperature vs. Bulk Temperature

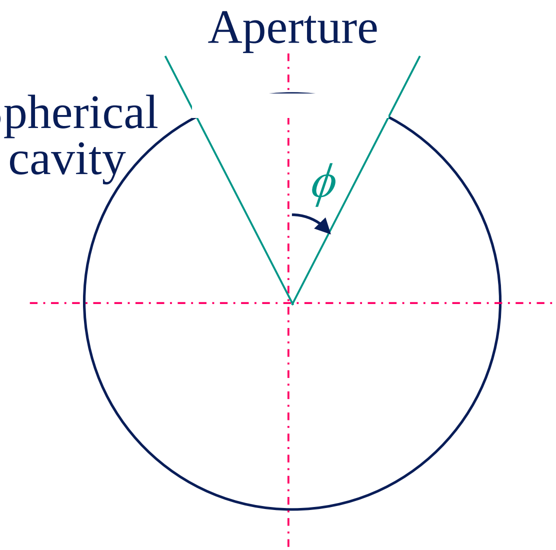


Question: What are the uncertainties associated with the comparison of T_{TC} with T_λ?

1. Accuracy of contact thermometer
2. Cavity design
3. Temperature gradients
4. Spectral and directional effects
5. Heat transfer losses
6. Diffraction losses
7. Reflected radiance

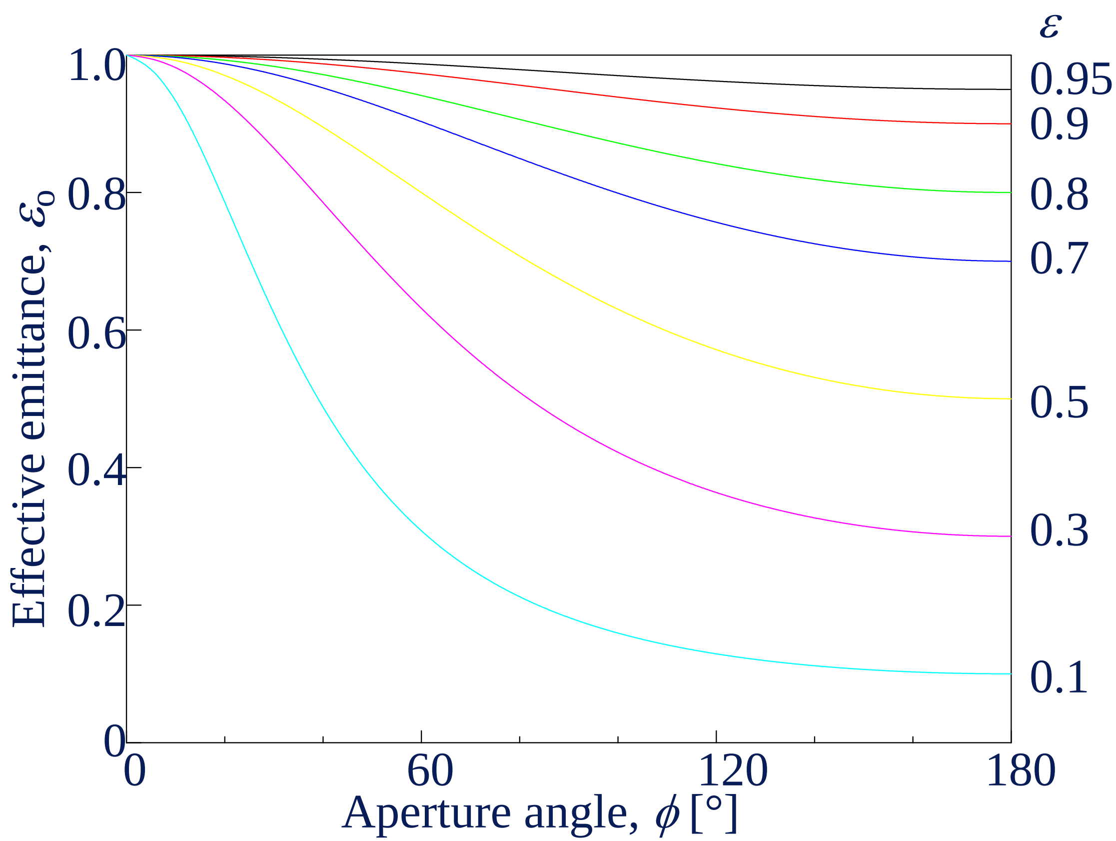
Cavity Design

Exact Solution for Effective Emittance of Spherical Cavity



$$\varepsilon_o = \frac{\varepsilon}{\varepsilon + (1-\varepsilon)\cdot(1-\cos\phi)/2}$$

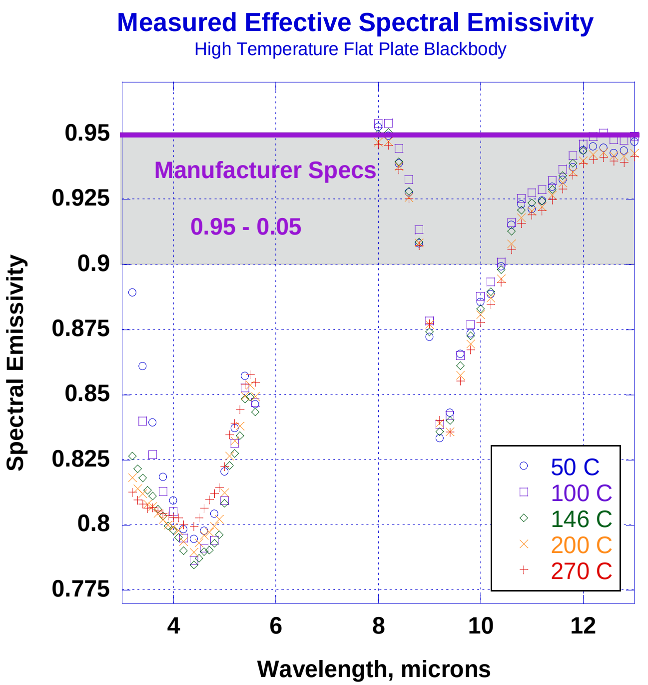
Due to Bedford



When the aperture angle ϕ is small, the effective emittance ε_o is close to unity, even for small values of cavity surface emittance ε.

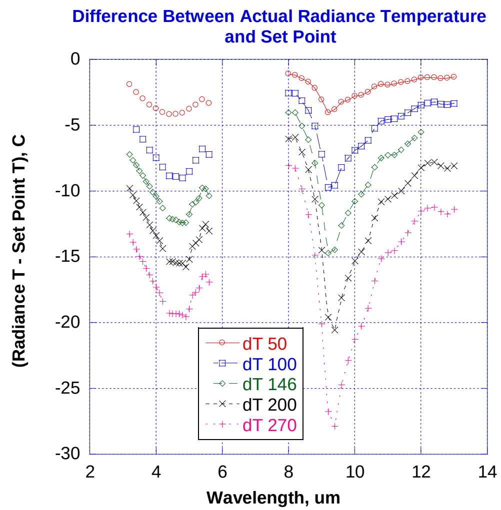
D&N-669

Example of Flat Plate BB Performance



Strongly selective spectral properties of used black paint (left figure) may lead to calibration errors up to 20 C because of difference between actual and expected (calculated using emissivity 0.95) radiance temperatures (right figure).

This BB is made by a major international manufacturer and quite common (NASA Transfer Standard)



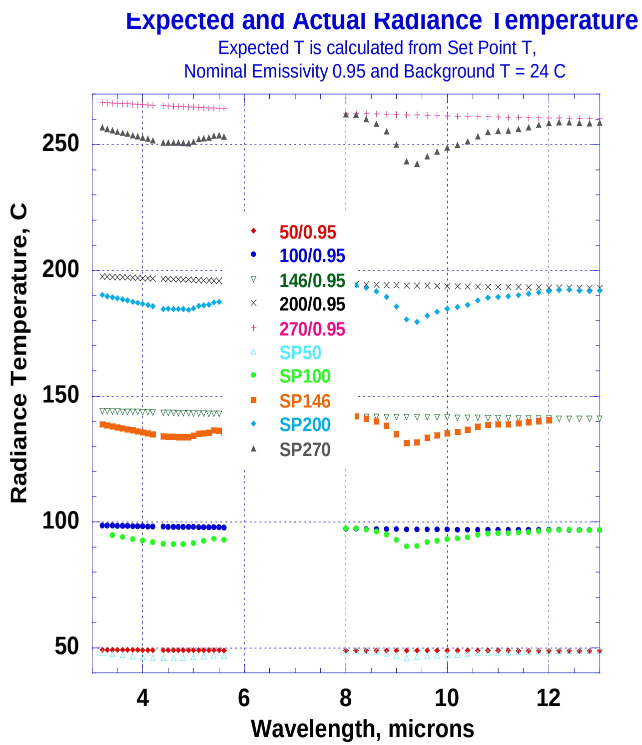


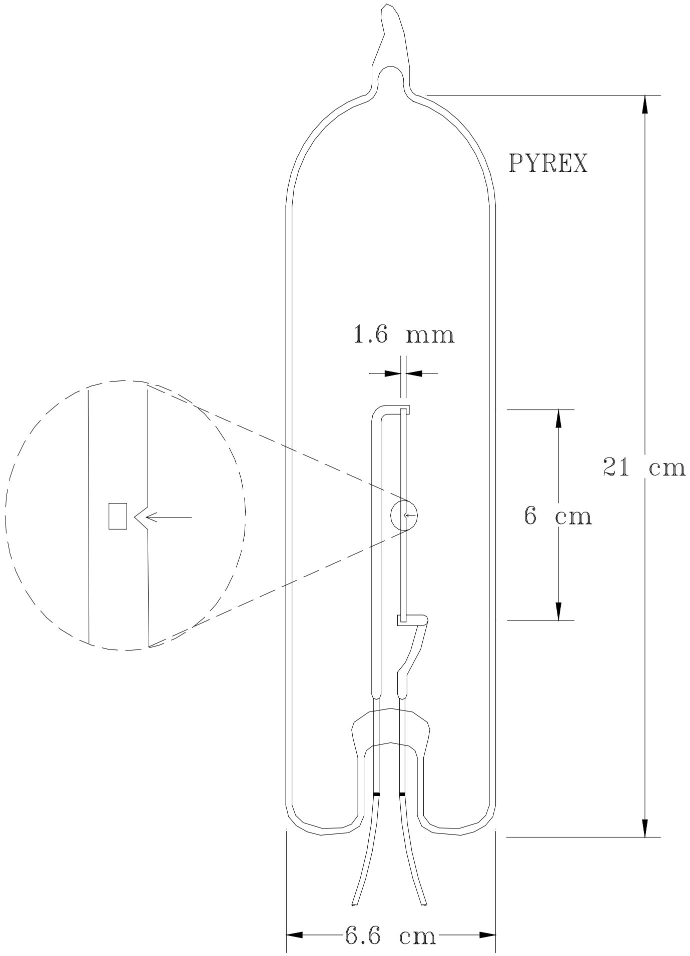
Figure on the Left:

Difference between actual temperature and the set point temperature.

Blackbody Alternatives

1. Lamps, arc sources (many types), heated refractories, light emitting diodes, lasers, synchrotron radiation
2. Examples:
   a) tungsten filament strip lamps
   b) tungsten quartz-halogen lamps
   c) deuterium (D_2) gas discharge lamps
   d) xenon arc lamps
   e) Nernst glower and Globar

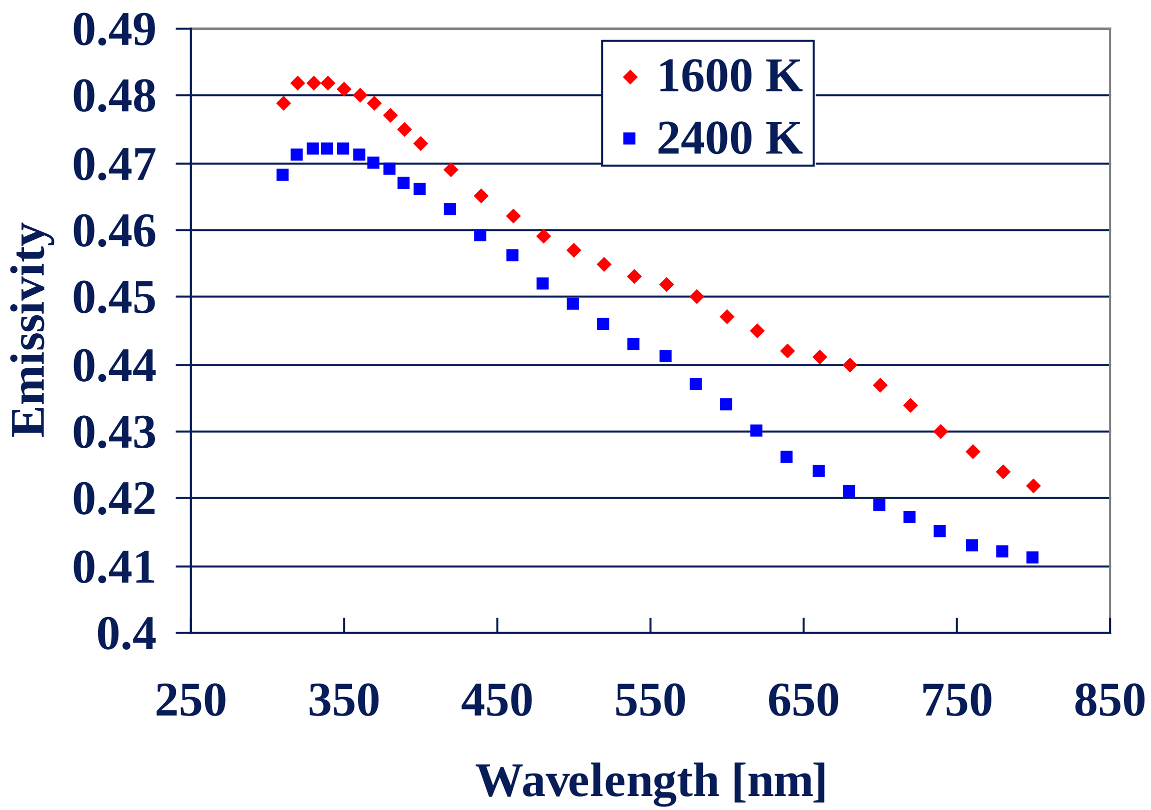
Tungsten strip lamp features



- Spectral Radiance or Radiance Temperature standards
- Vacuum or Gas-filled
- Quartz or glass windows

- Good stability (especially for the vacuum type)
- Small target area (0.6 mm wide by 0.8 mm tall)
- Careful alignment procedures required
- Calibrated by comparison to a blackbody or another strip lamp at 0.654 µm
- Suited for Devices Under Test with small field-of-views

Emittance of Tungsten



Spectral and temperature dependence of tungsten emissivity.

Radiance Temperature

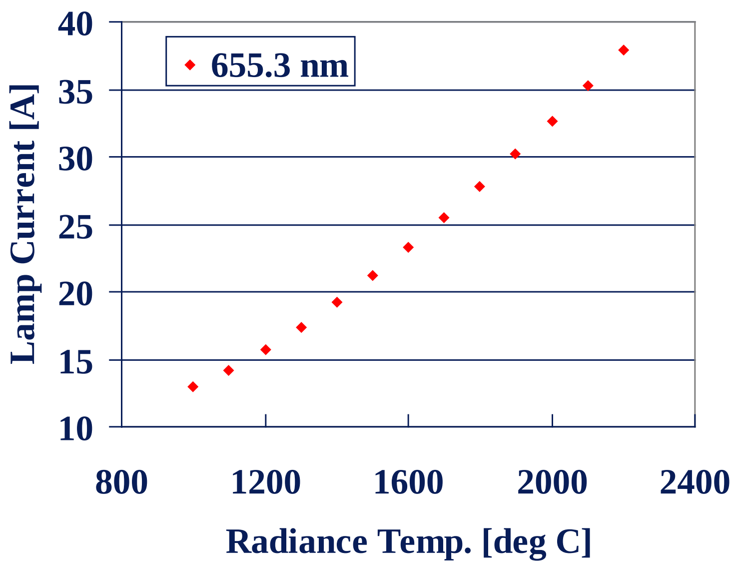
$$T_\lambda = \frac{1}{\frac{1}{T} - \frac{\lambda}{c_2}\ln(\varepsilon)}$$

=1510 K at 1600 K and 660 nm

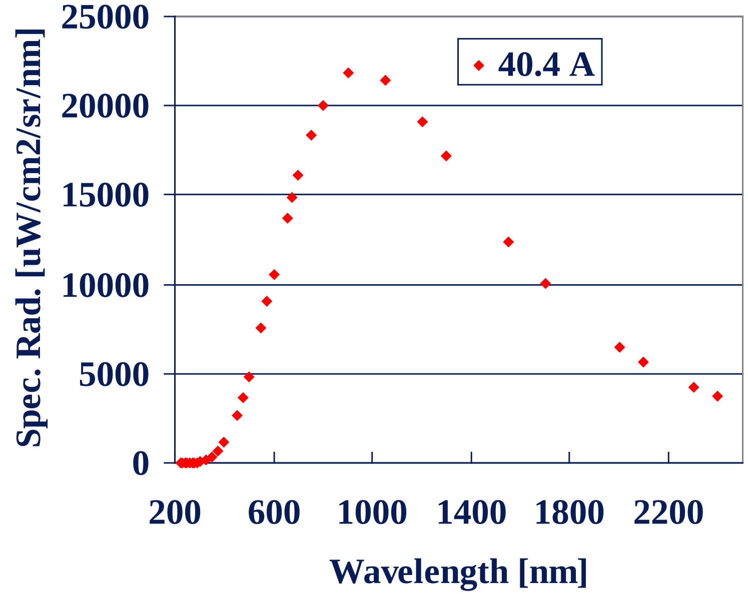
Tungsten strip lamp output

Gas-filled Lamps (to suppress tungsten evaporation)

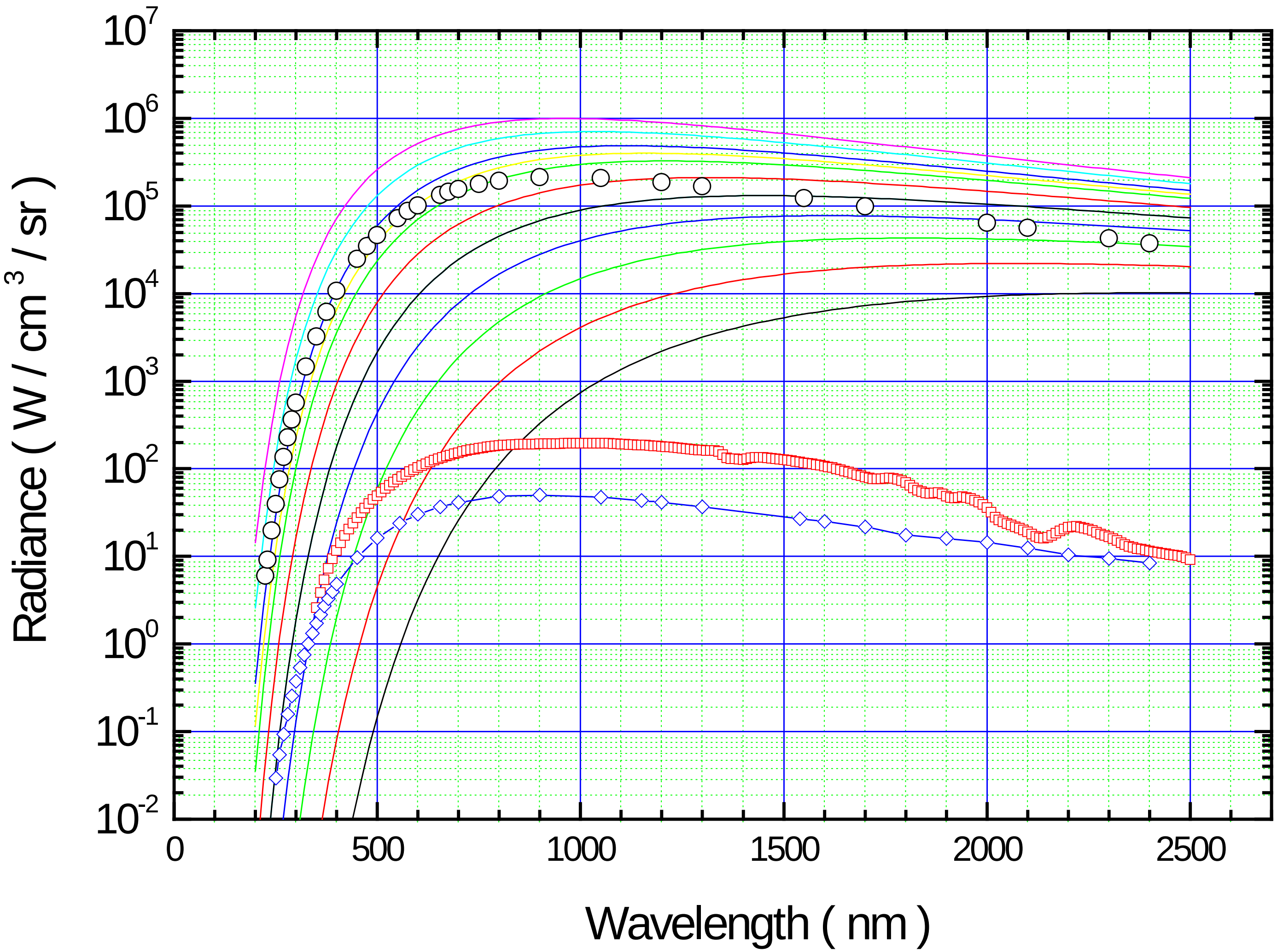
For Radiance Temperature



For Spectral Radiance



Comparison of blackbodies and tungsten strip lamps and integrating sphere sources



Integrating Spheres

1. Features:
   a) Spherical geometry
   b) Low absorbance
   c) Diffuse reflectance
2. Result
   a) Flux “averager”
3. Applications
   a) Radiance source (add lamp, laser, LED, etc)
   b) Irradiance collector
   c) Internal or external sources and detectors

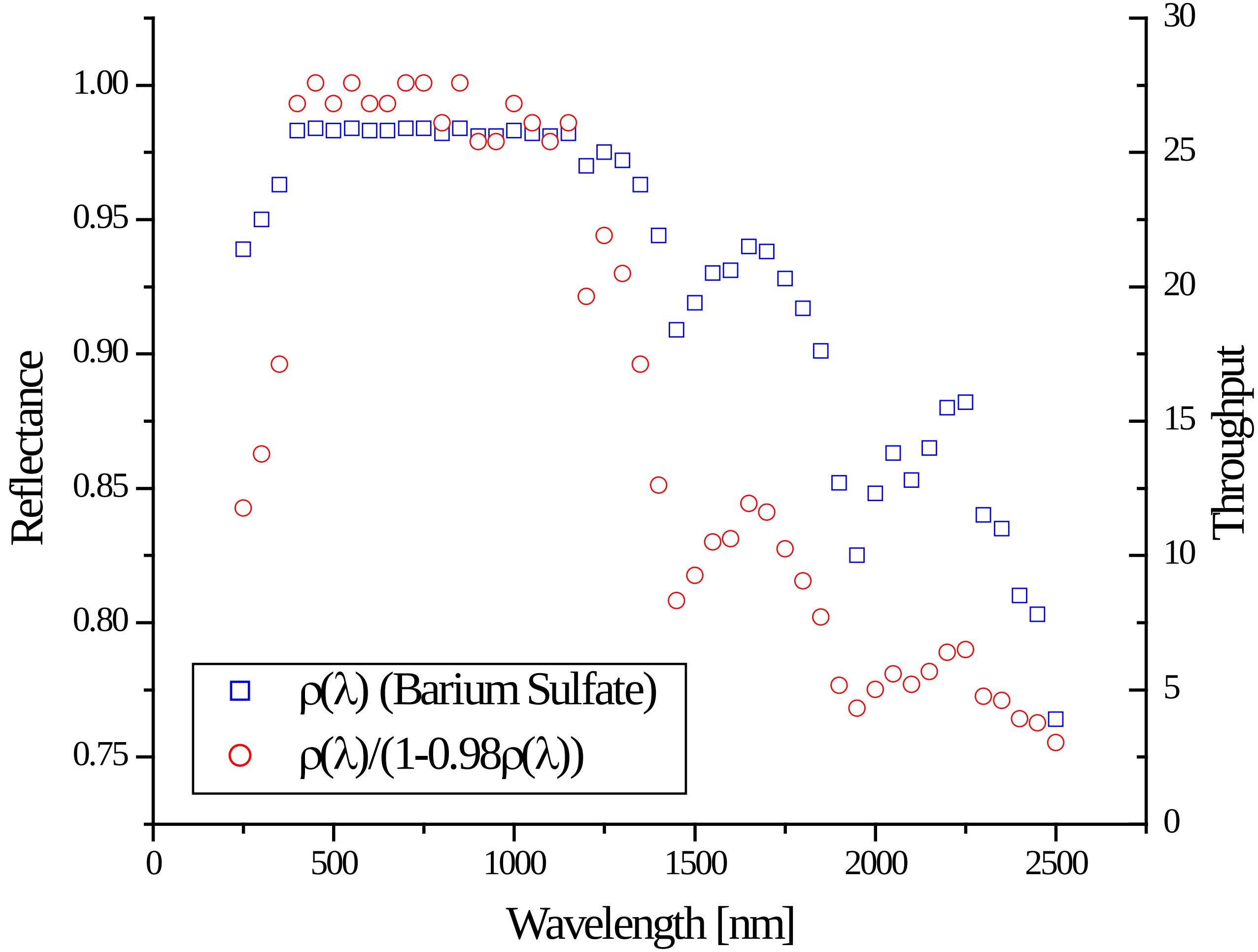
Sphere Performance

1. Flux transfer equations yield

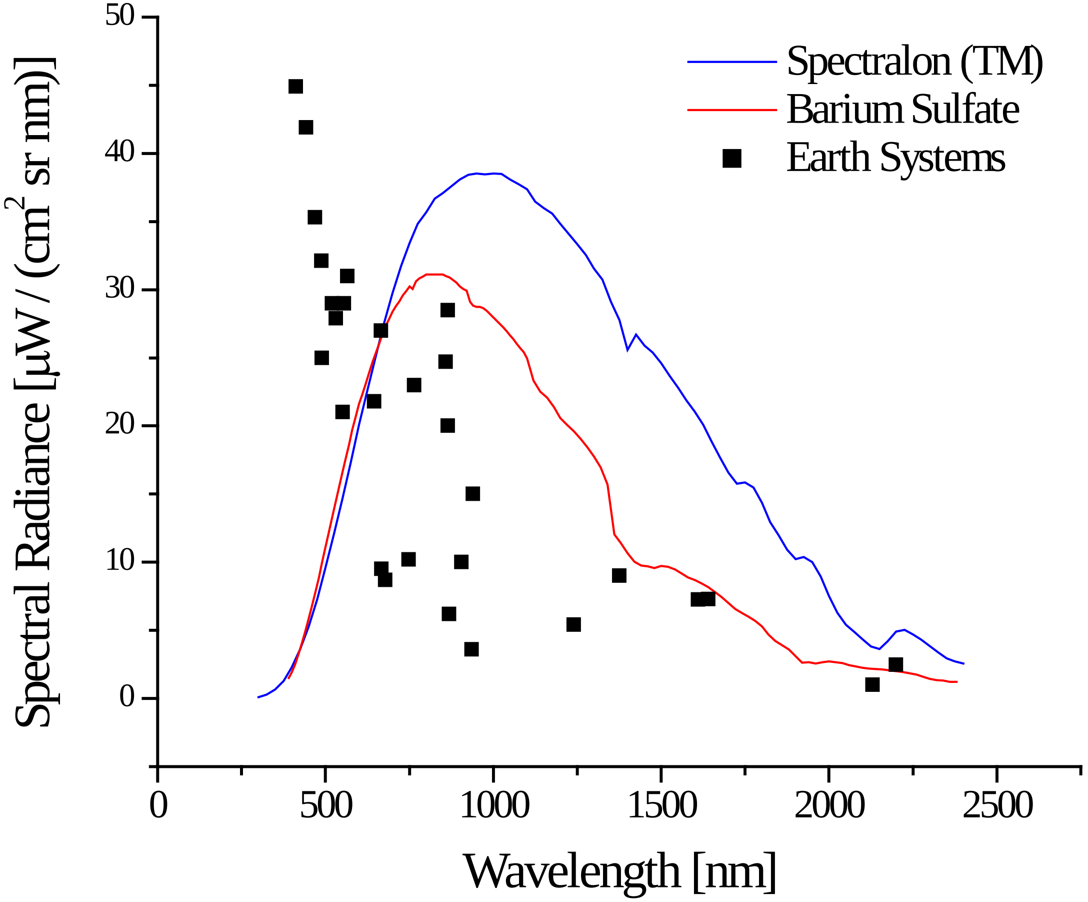
$$L(\lambda) = \frac{\rho(\lambda)\,\Phi(\lambda)}{\pi\, A\big(1-\rho(\lambda)(1-f)\big)} \qquad f = \frac{\sum \text{port areas}}{A}$$

2. Baffles to shield direct view of lamps
3. Integrated monitor detectors to record performance
4. Stable power supplies and reflectance of interior wall

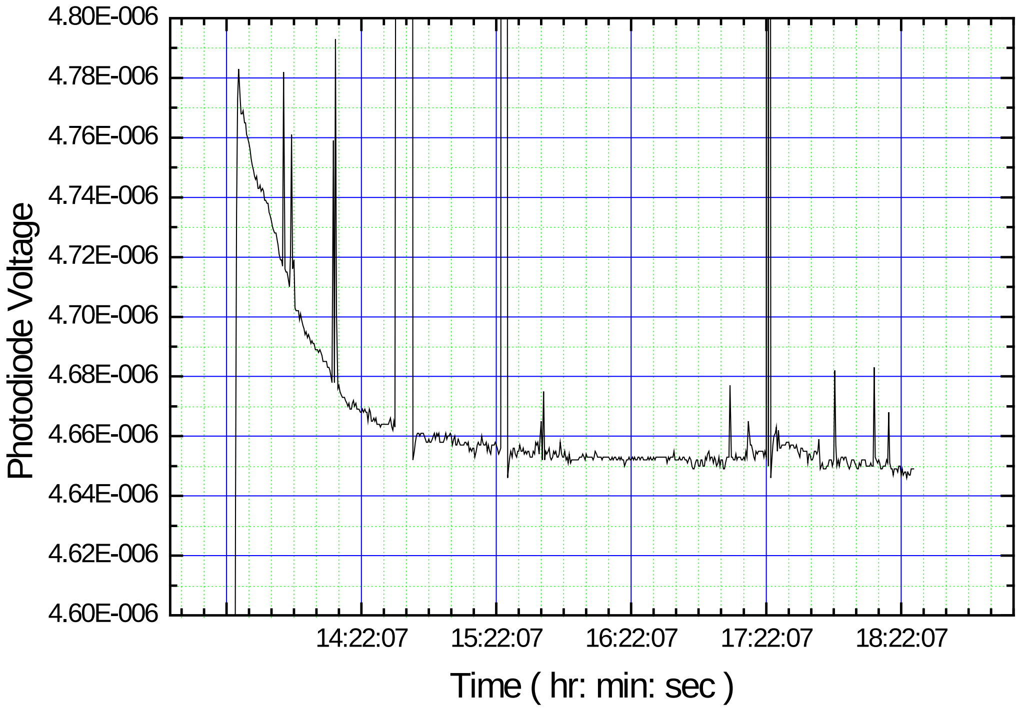
Reflectance and Throughput

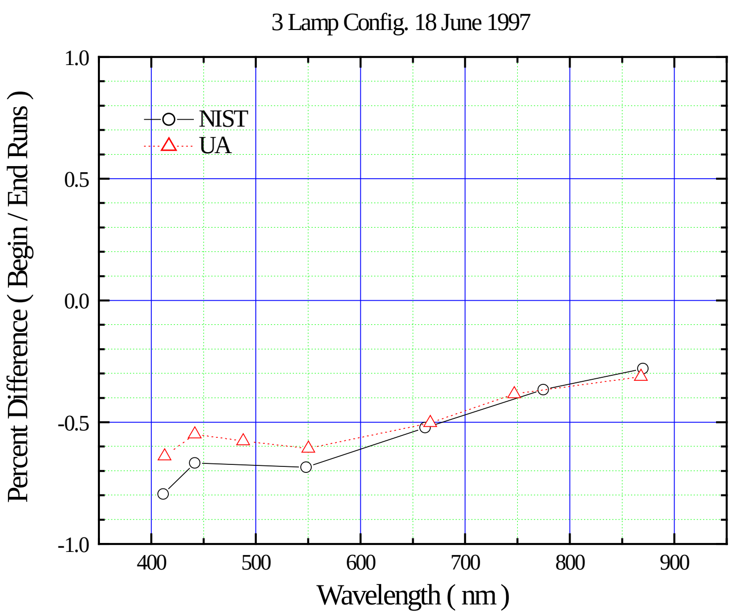


Radiance of Integrating Spheres



Temporal changes in the sphere output





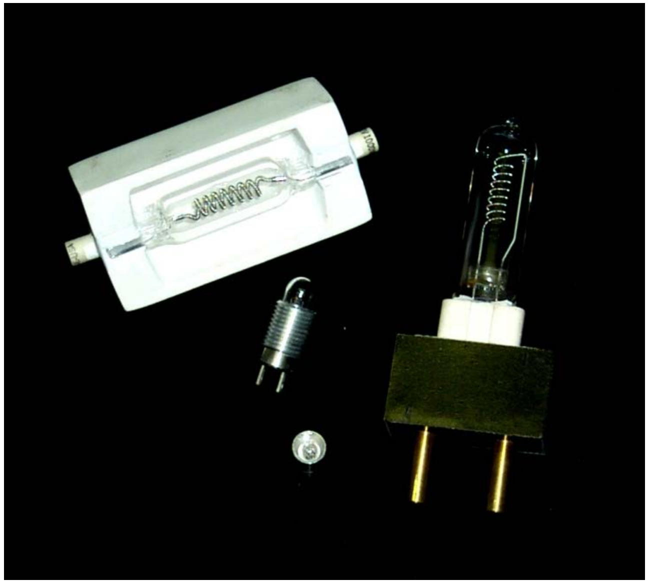
Photometer measurements

Changes at 400 nm are more pronounced

Sphere Source Protocols

1. Geometry for uniform illumination
   a) Lamps baffle
2. Document operation
   a) Lamp current, lamp voltage drop, monitor detector signals, Lamp operating hours
3. Keep coating clean
4. Recalibrate
5. Map spatial uniformity and dependence on view angle

Halogen Filament Lamps

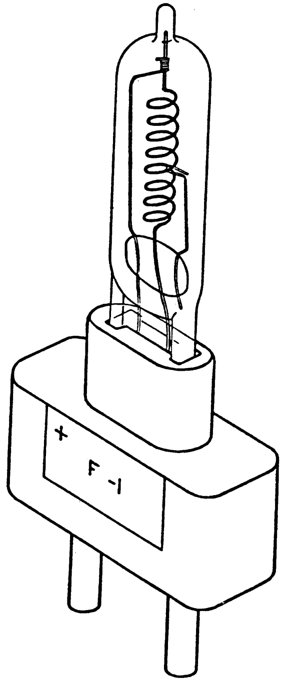


- Illumination, heating, & irradiance standards
- Wide commercial selection
- Select on features:
  - lifetime
  - color temperature
  - lumen efficacy
  - current or voltage
  - built in lens
  - base configuration
- Maximum wavelength range: 250 nm to 2.6 μm

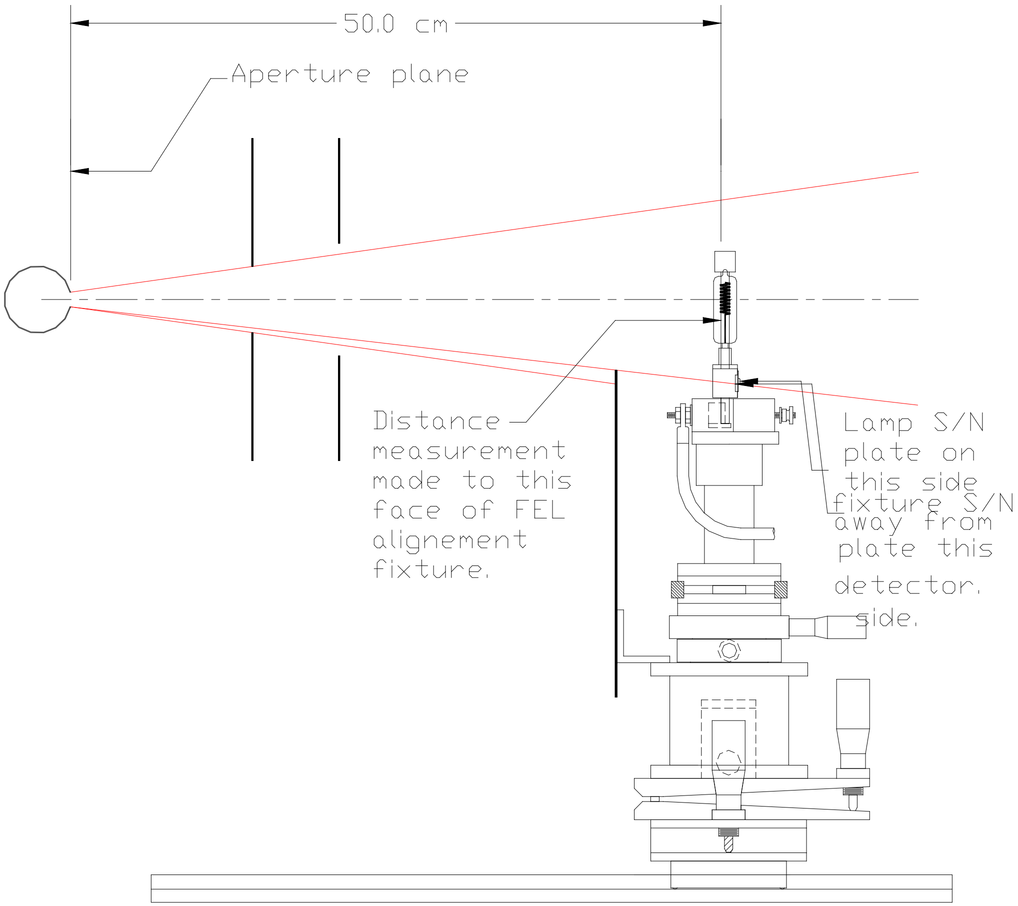
FEL Lamp Irradiance Standards

- 1000 W output
- Coiled-coil structure to increase emittance
- FEL type (a model number)
- Modified by addition of bipost base

- Calibrated by comparison to a high temperature blackbody
- 50 cm from front of post
- 1 cm^2 collecting area
- Selected and screened for undesirable features



FEL alignment system



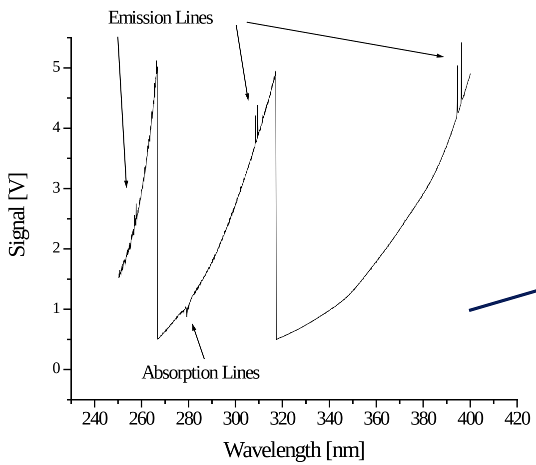
FEL Lamp Screening

1. Inspect, test, anneal, age, pot into base
2. Spectral line screening (currently 0 % pass rate)
   a) 250 nm to 400 nm in 0.1 nm steps with 0.04 nm bandpass (emission and absorption lines)
3. Temporal stability (90 % pass rate)
   a) <0.5 % before and after 24 h continuous operation at four wavelengths in UV to near infrared
4. Geometric (95% pass rate)
   a) < 1% in ± 1° at 655 nm

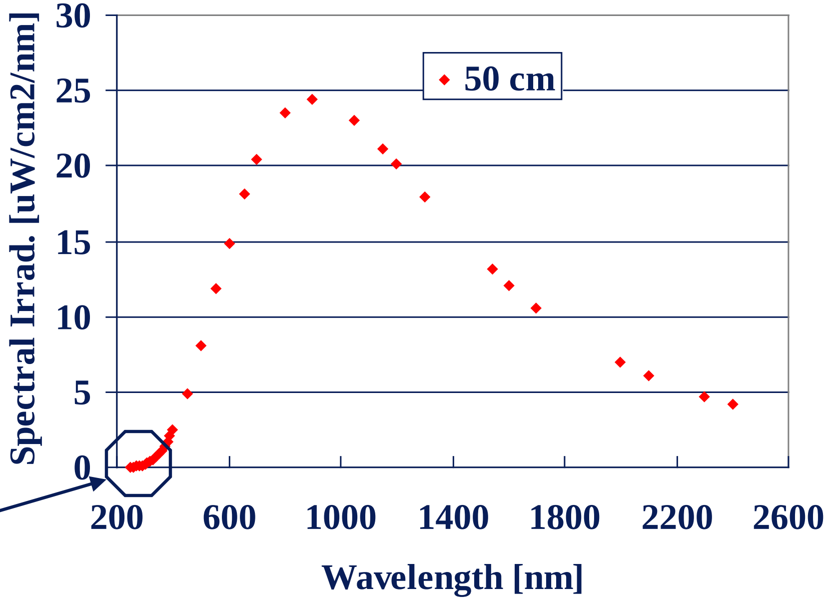
FEL Output

a.	256.97 nm	(256.80 nm)
b.	257.67 nm	(257.51 nm)
c.	308.48 nm	(308.22 nm)
d.	309.47 nm	(309.27 nm)
e.	394.57 nm	(394.40 nm)
f.	396.27 nm	(396.15 nm)

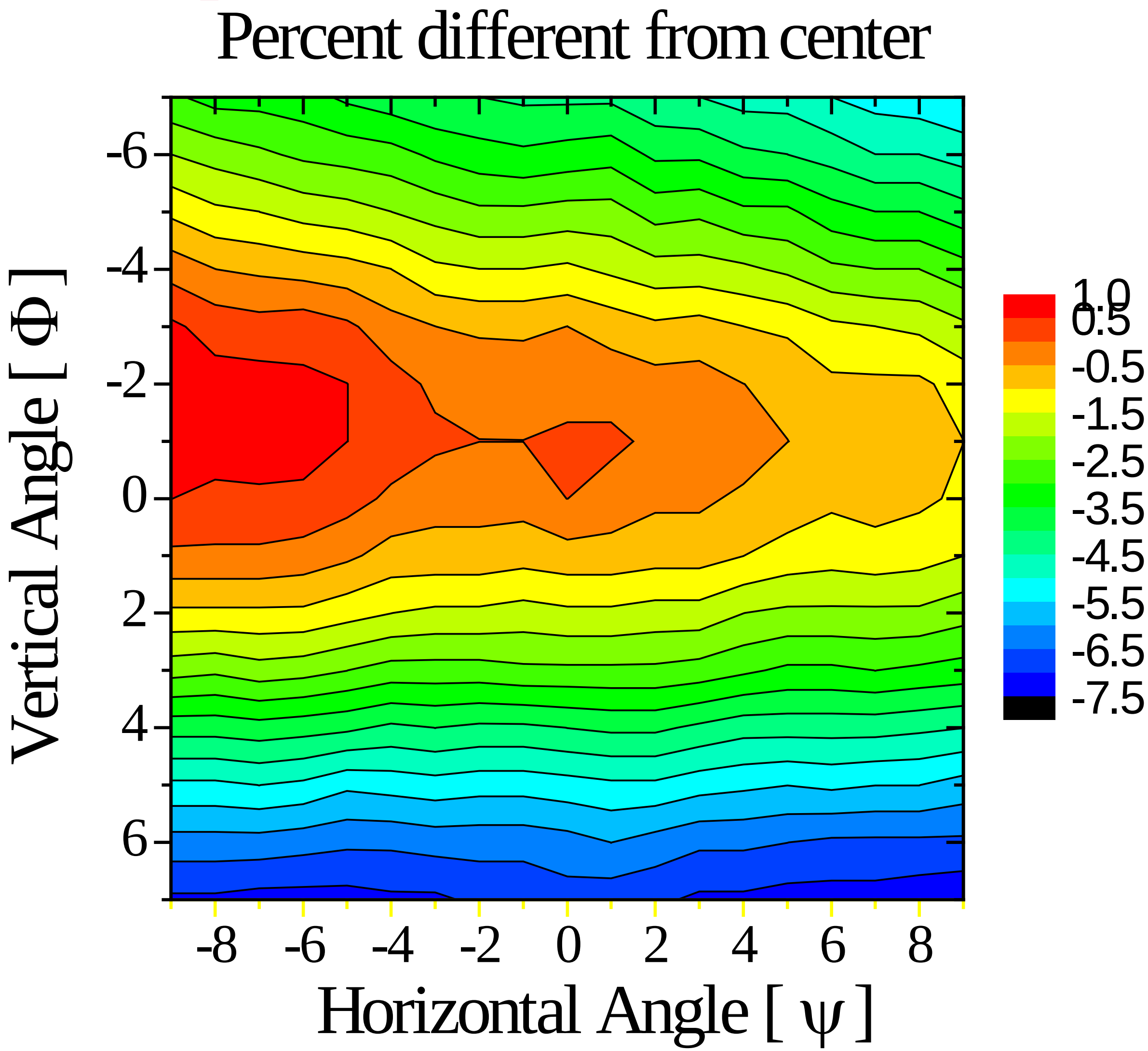
Undesirable Lines

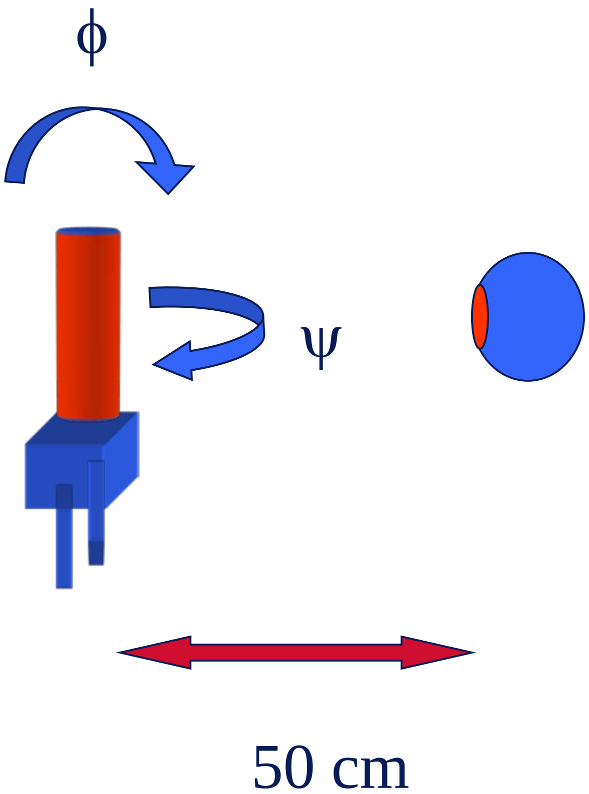


Calibration Data, FEL at 8.2 A

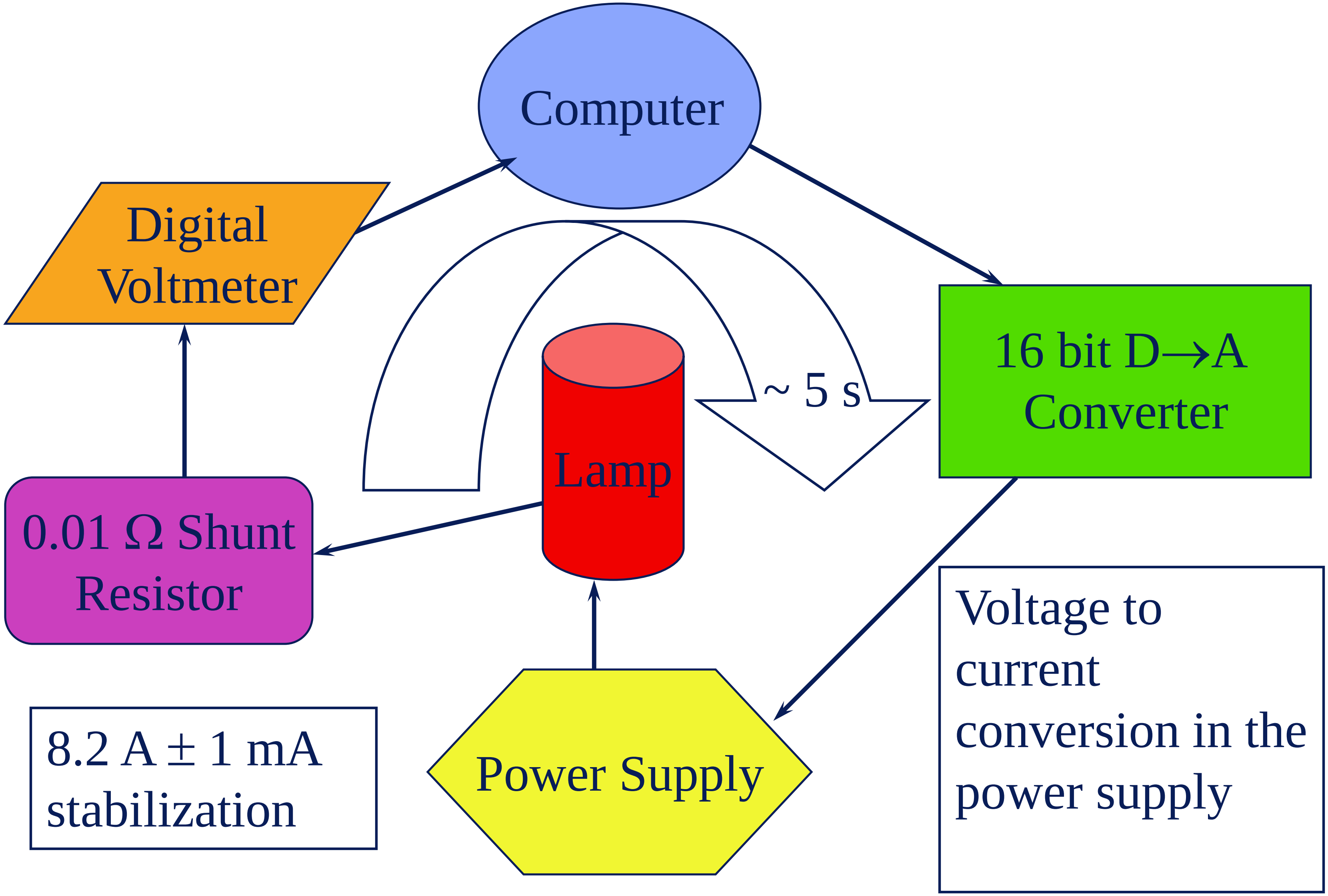


Dependence on horizontal and vertical angles

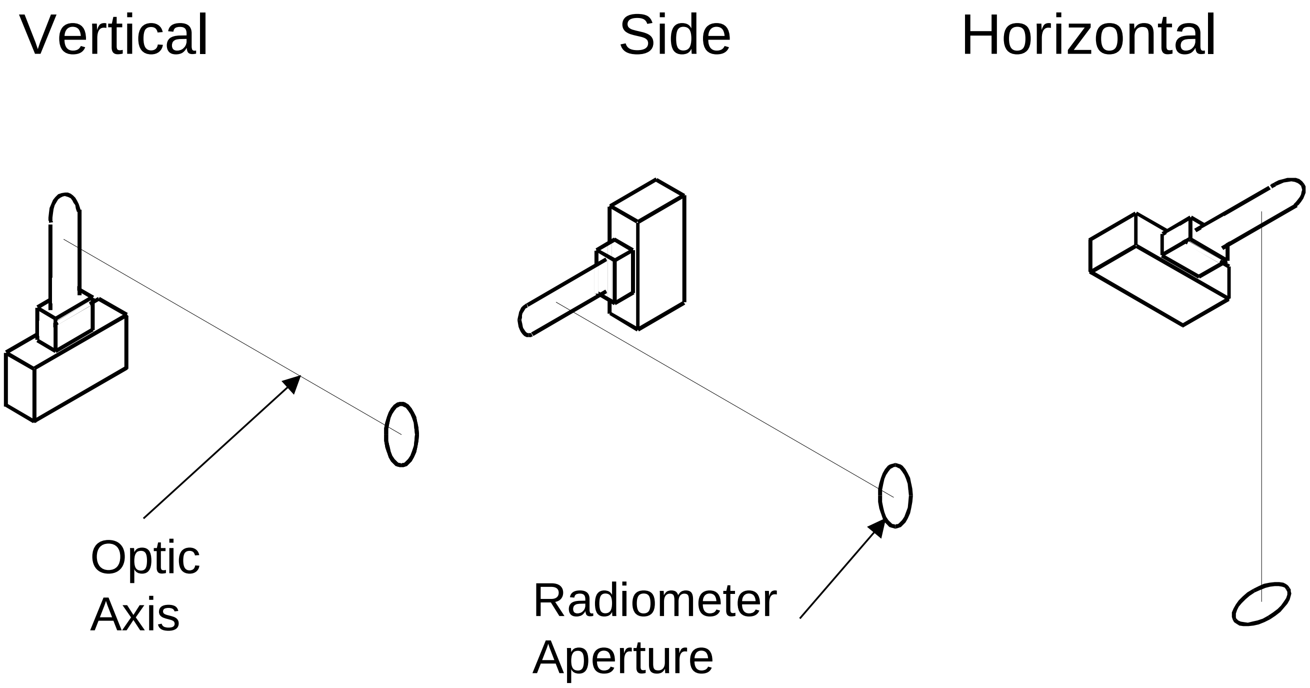




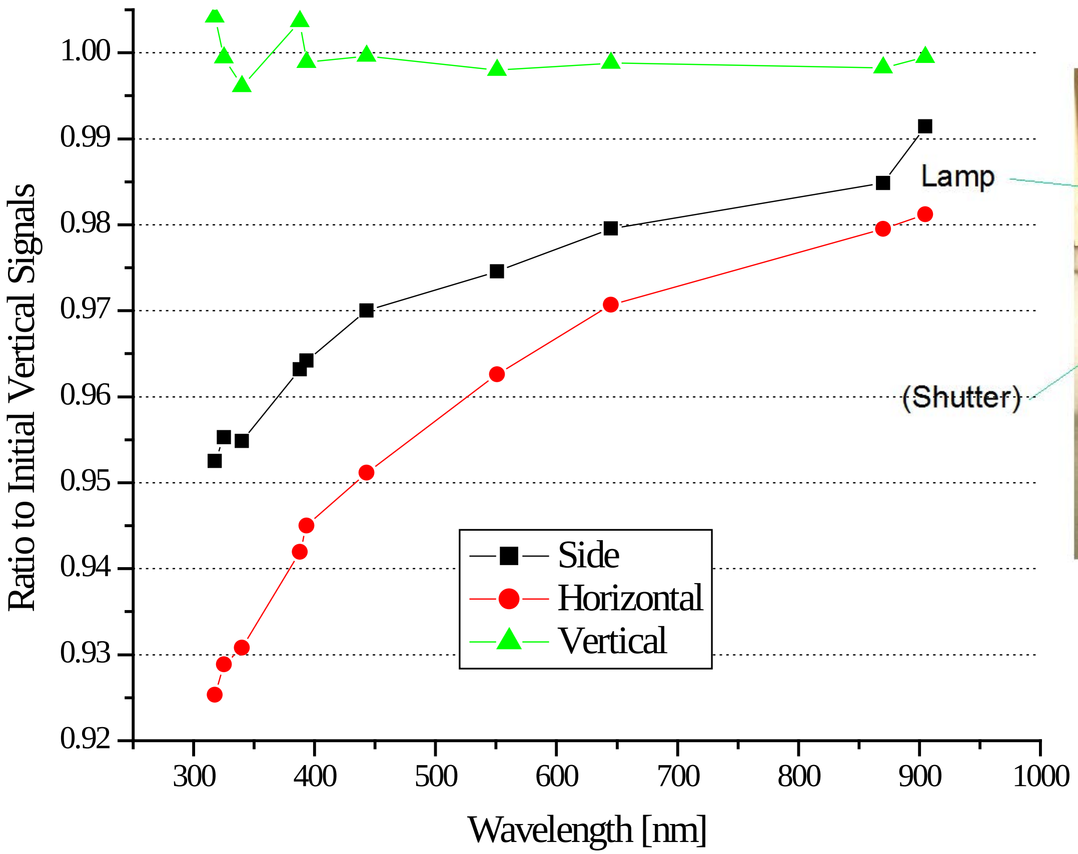
Power Supply Feedback Loop



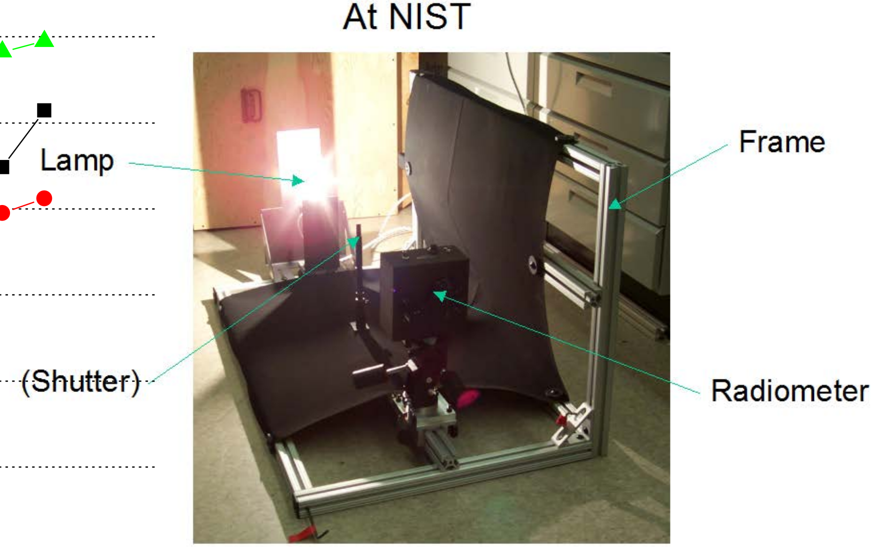
Lamp Orientations



Orientation dependence of the FEL



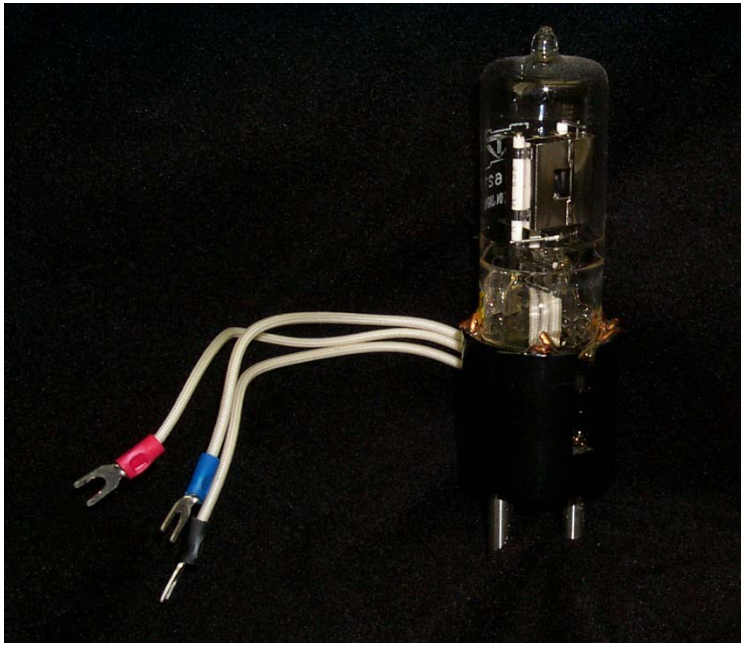
Frame, Lamp, and Radiometer



Protocols for FEL Standard Lamps

1. Orientation
   a) 50 cm from front of posts, entrance pupil diameter of 1 cm^2, use special alignment jig for FELs
2. Electrical
   a) maintain polarity, constant current, log voltage drop and burning hours
   b) Similar sensitivity to error in current as strip lamps
3. Operational
   a) 30 min warm-up; recalibrate every 50 h
   b) transfer to user working standards
   c) don't touch the envelope; don't enclose the lamp during operation; baffle properly

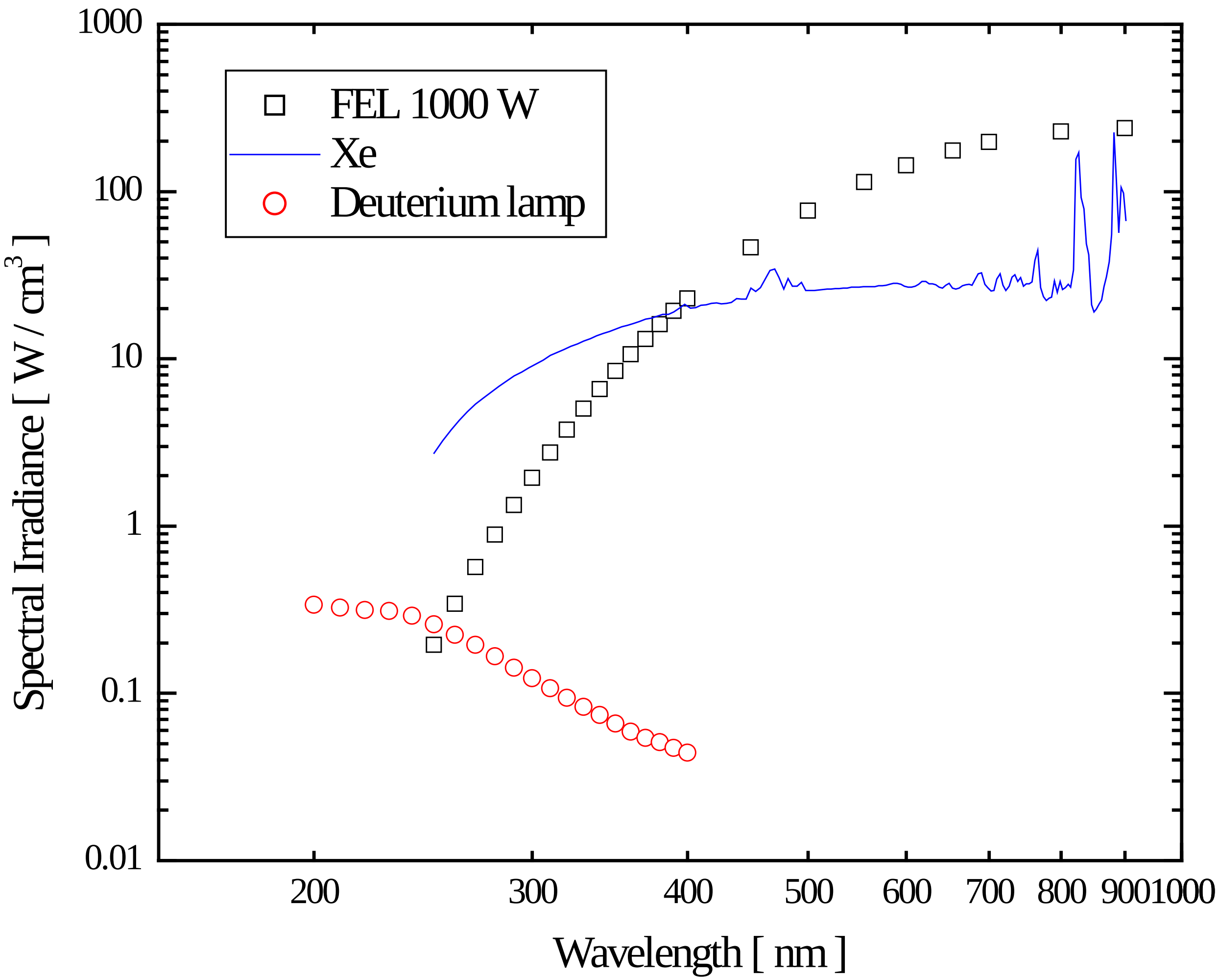
D_2 Irradiance Standards



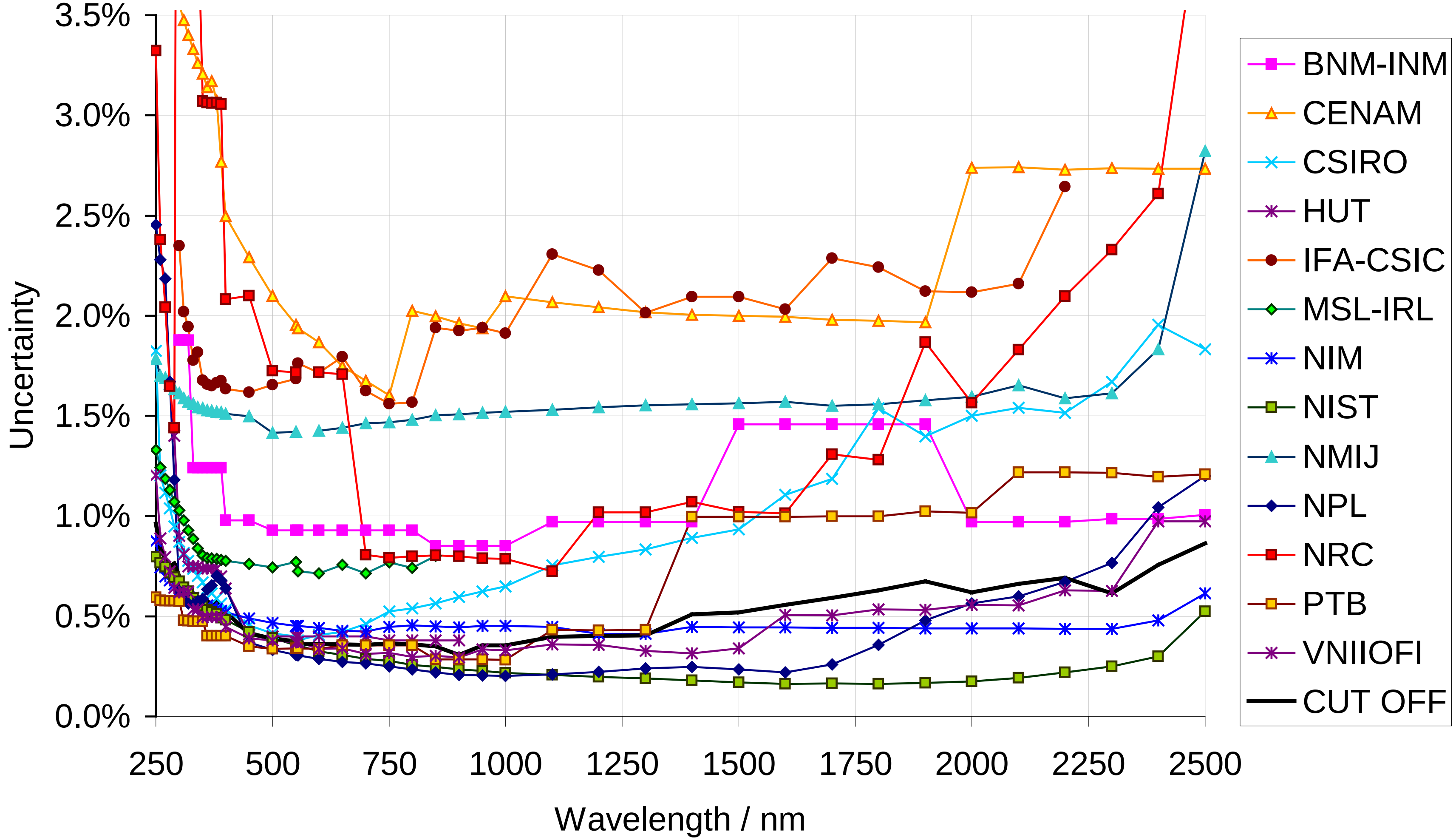
- 30 W output
- Stable relative spectral irradiance distribution
- 200 nm to 350 nm
- Modified by addition of bipost base (same as FEL)

- Calibrated by a) relative distribution from wall stabilized hydrogen arc and b) FEL at 250 nm
- 50 cm from front of post
- 1 cm^2 collecting area
- Selected and screened for undesirable features

Deuterium, Xe and FEL



NIST uncertainties (k=1) (lowest in the world)



4. Properties of Detectors

Outline

1. Radiometric characteristics of photodiodes
2. Electronic characteristics of photodiodes
3. Comparison of basic detector characteristics
4. PMTs
5. Selection of detectors for different applications
6. Selection of signal meters for different detectors

Radiometric characteristics of photodiodes

1. **Internal Quantum Efficiency (IQE),**
2. **External Quantum Efficiency (EQE), and**
3. **Spectral Responsivity s(λ) of Quantum Detectors**
4. **Noise Equivalent Power (NEP) and D***
5. **Radiometric Sensitivity, Photons/s**
6. **Response Linearity of Photodiodes**
7. **Spatial and Angular Responsivities**
8. **Temperature Dependent Responsivity**

IQE, EQE, and s(λ) of quantum detectors

$$\text{IQE} = \frac{\text{Number of collected electrons}}{\text{Number of absorbed photons}}$$

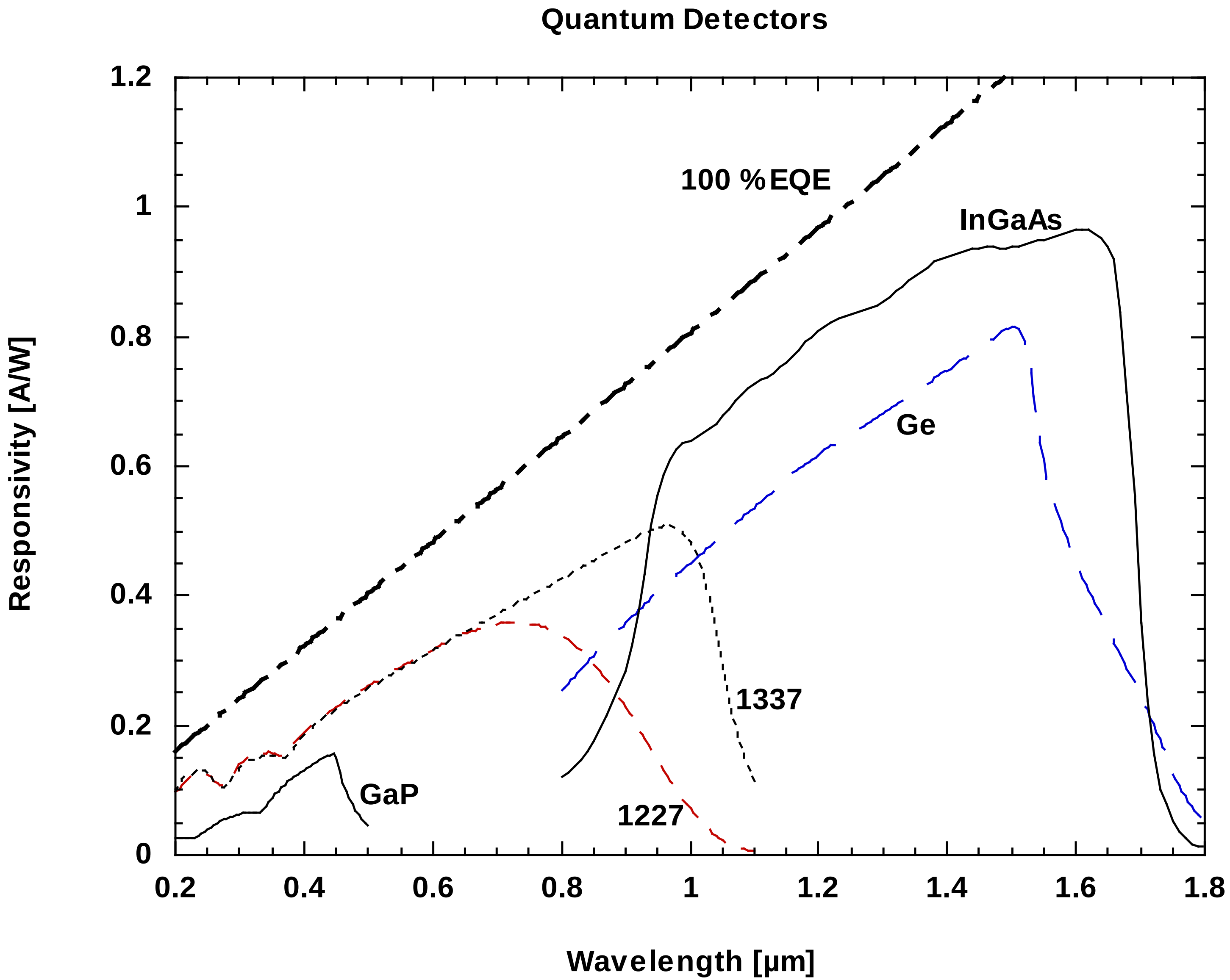
$$\text{EQE} = (1-\rho)\ \text{IQE}$$

where ρ is the reflectance;

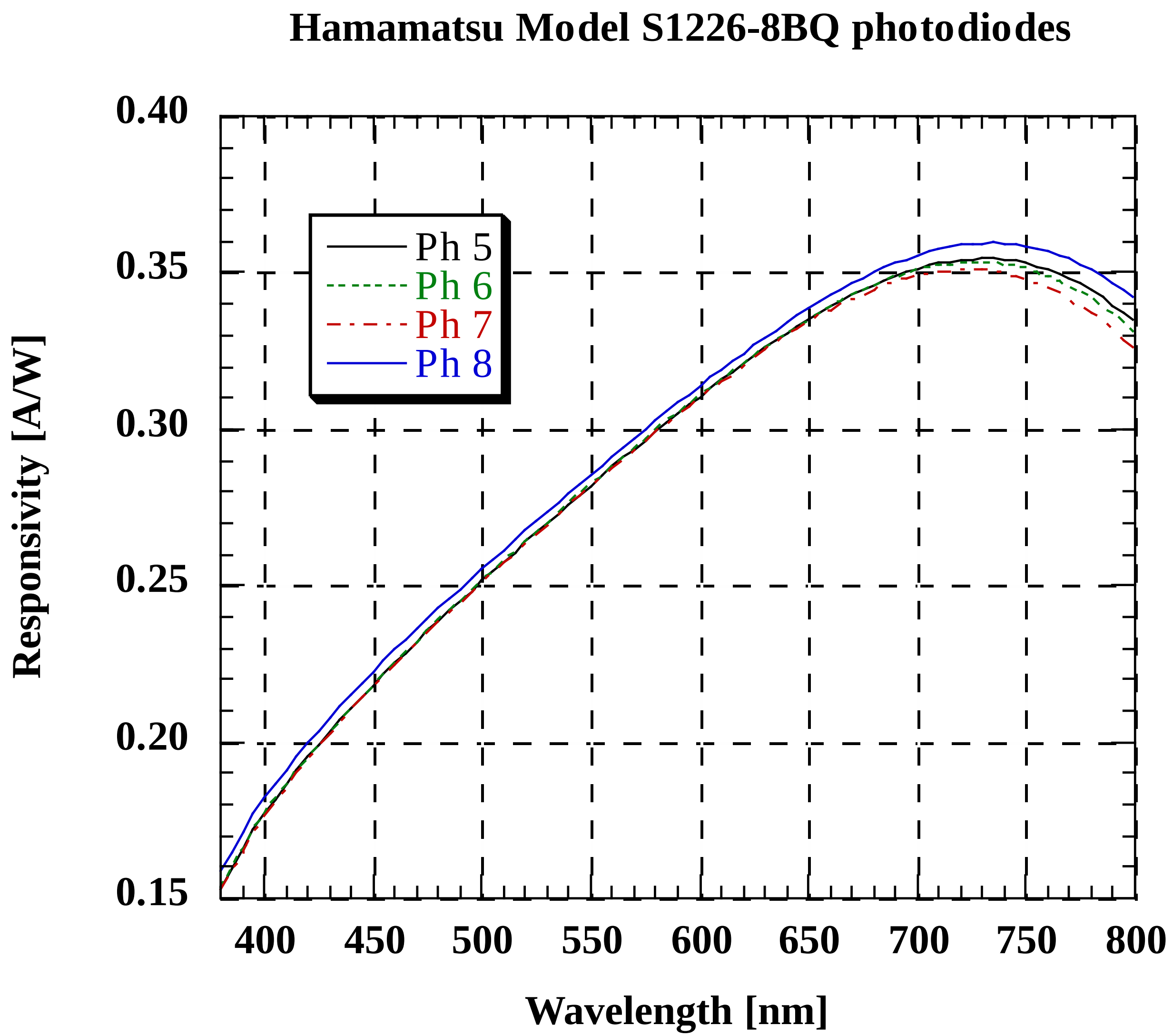
The power responsivity is:

$$s(\lambda) = \text{EQE}\,\frac{e\,\lambda}{h\,c} = \text{EQE} * \lambda * \text{const.}$$

Spectral power responsivity of frequently used photodiodes



Spectral responsivity variations within the same model



Noise Equivalent Power (NEP) and D* of detectors

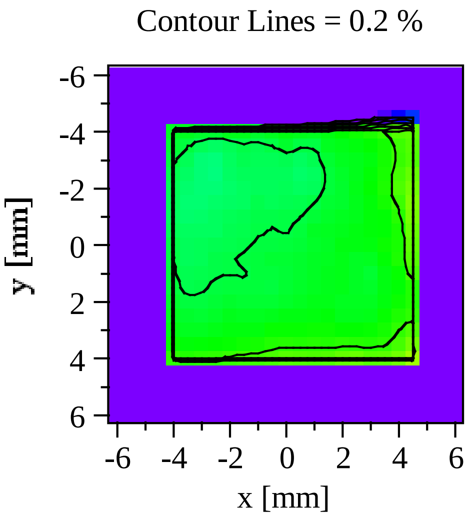
$$\mathbf{NEP} = \frac{\mathbf{P}}{\mathbf{S/N}_{(\Delta f=1)}} = \frac{\mathbf{N}}{\mathbf{S/P}} = \frac{\mathbf{N}}{\mathbf{R}} \quad [\mathbf{W/Hz^{1/2}}]$$

where, S is the detector output signal for
P incident radiant power,
R is the detector responsivity, and
N is the detector output noise.

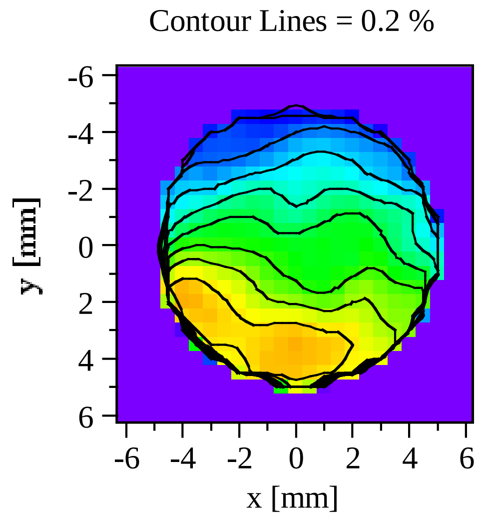
$$\mathbf{D^*} = \frac{\mathbf{A^{1/2}}}{\mathbf{NEP}} \quad [\mathbf{cm\ Hz^{1/2}/W}],$$

where A is the detector area.

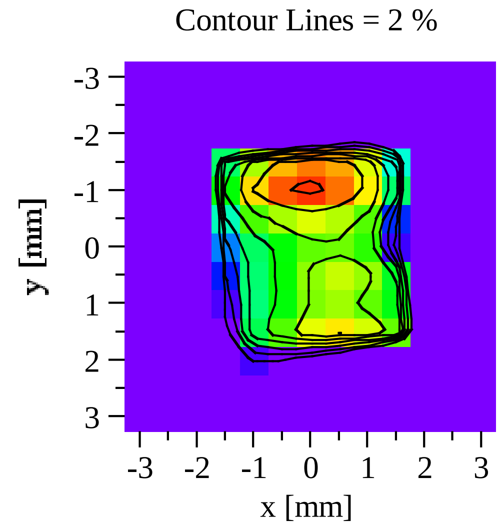
Spatial response of large-area photodiodes



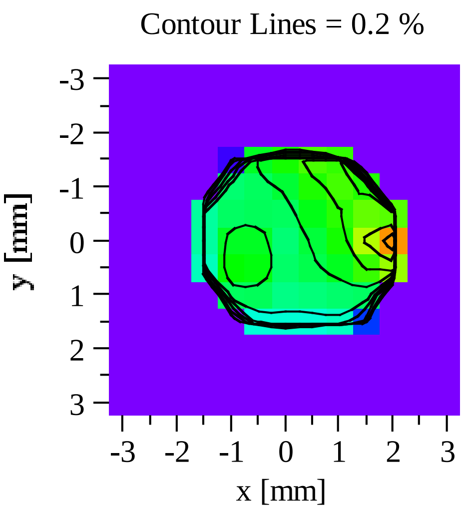
Si S1337 @ 500 nm



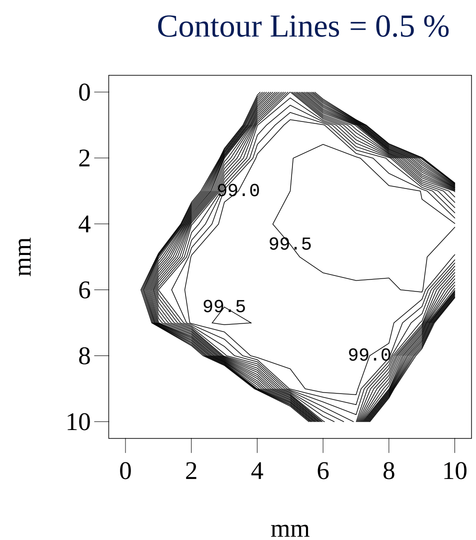
UV100 @ 500nm



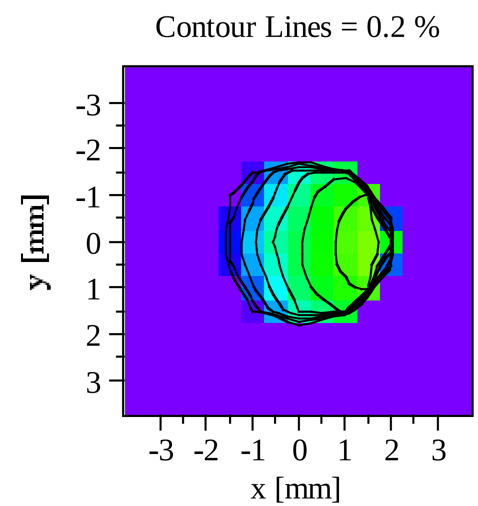
GaP @ 340 nm



Ge @ 1225 nm

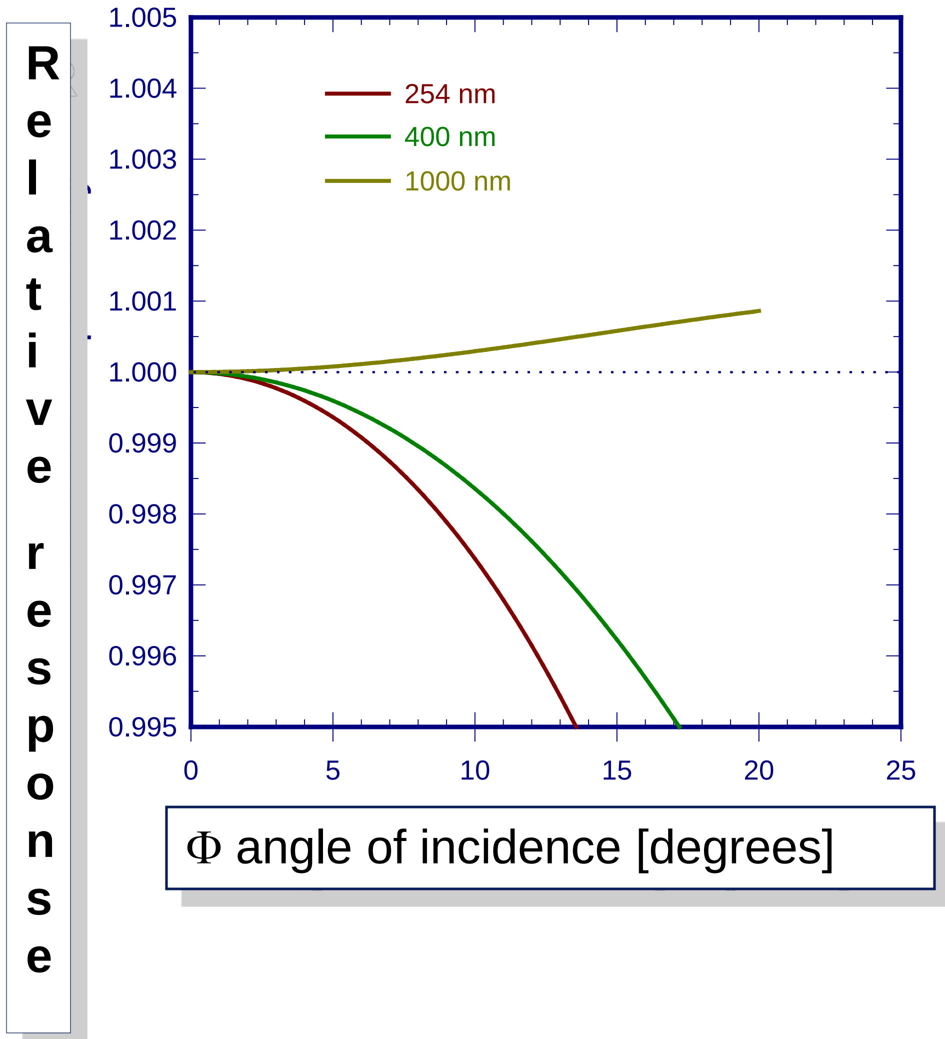


Thin oxide Nitrided Si, AXUV-100G



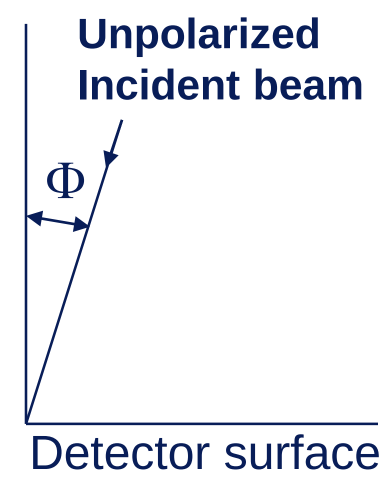
InGaAs @ 1250 nm

Angular response of a 1337 Si photodiode

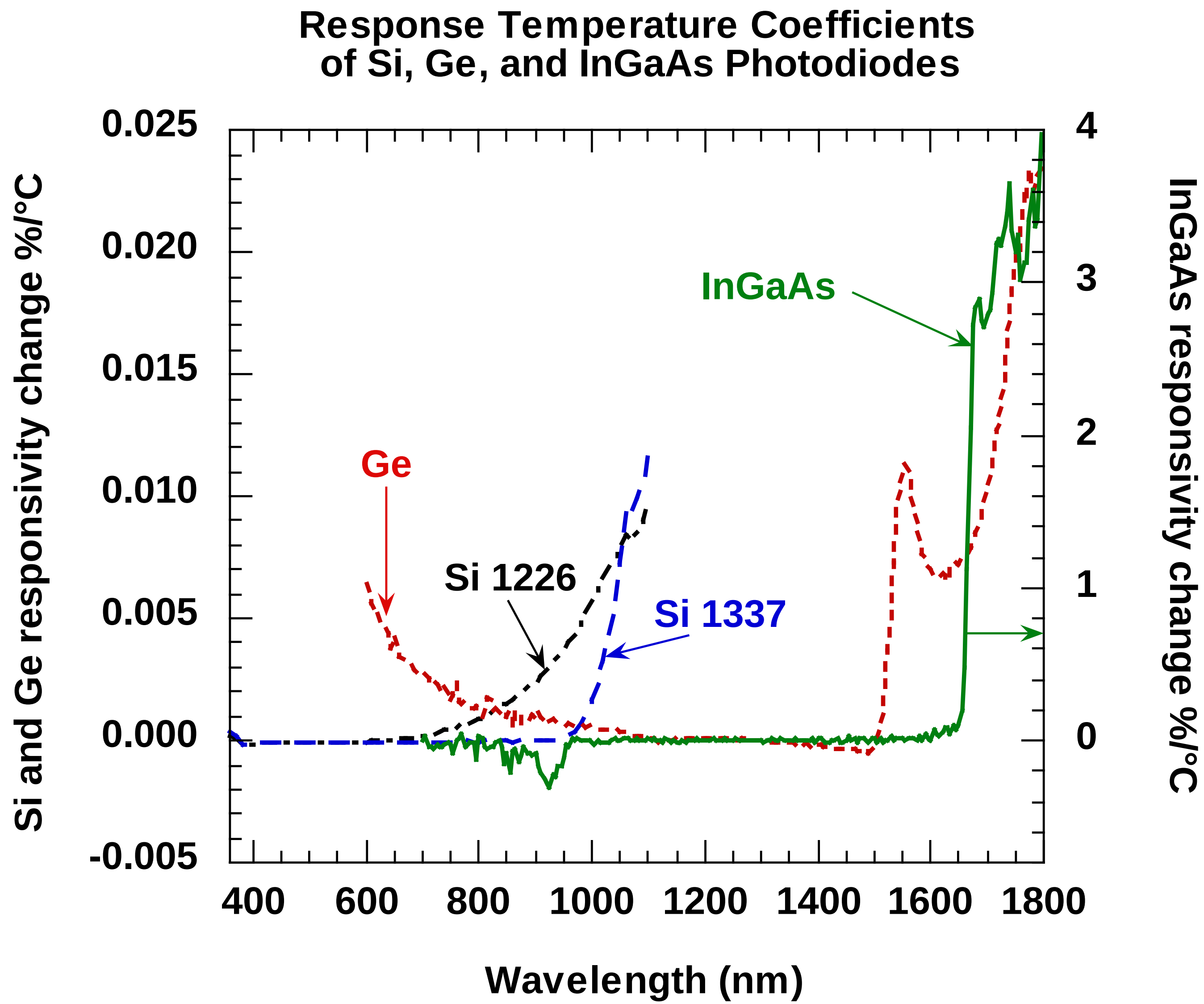


With the permission of L. P. Boivin.

From: Metrologia, **32**, Fig. 3, p.567



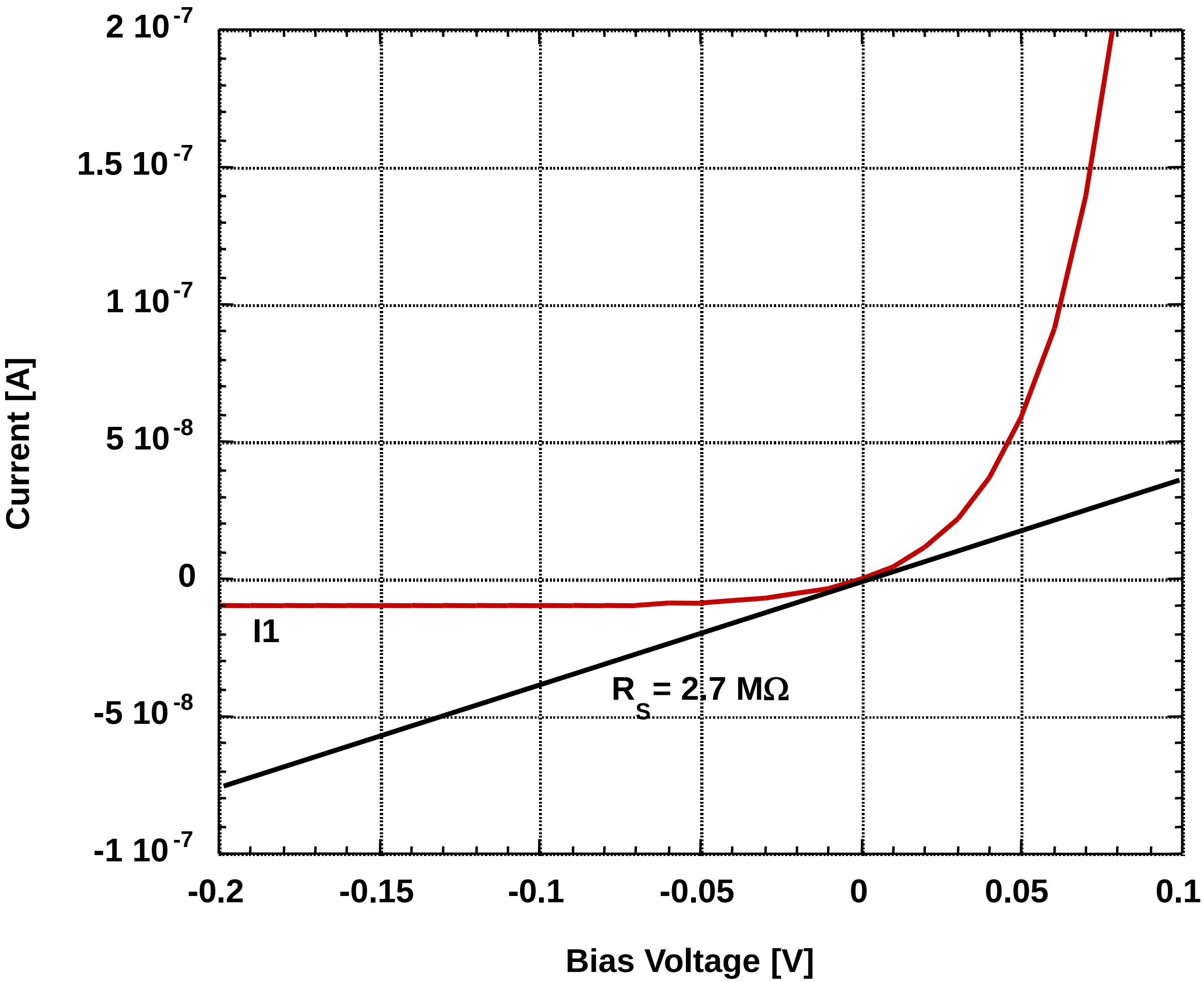
Temperature dependent responsivity of photodiodes



Fundamental electronic characteristics of detectors and photocurrent meters

1. Photodiode shunt resistance
2. Linear photocurrent measurements
3. Noise and drift
4. Settling time
5. Stability

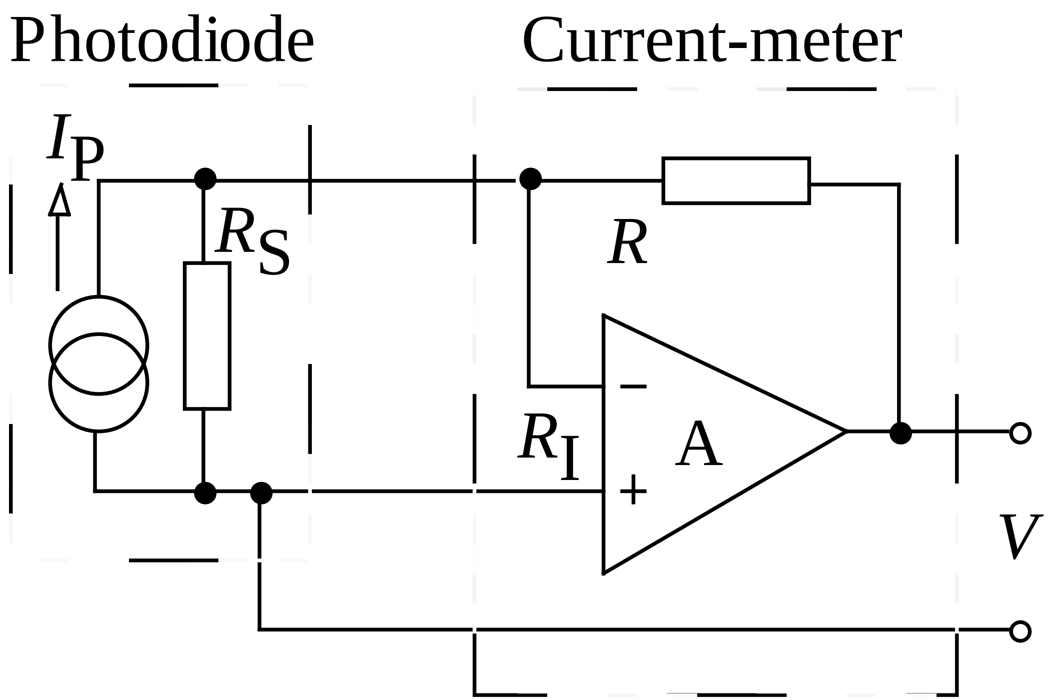
Photodiode (PV) shunt-resistance



The shunt resistance has a major influence for linearity and voltage-gain of noise and drift !

The shunt resistance is temperature dependent! For Si, the increase with decreasing temperature is ~11%/°C.

Linear PV photo-current measurement



R_S has to be selected to a minimum value to obtain a linear relationship between V and I_P:

$$R_S >> R_I = \frac{R}{A}$$

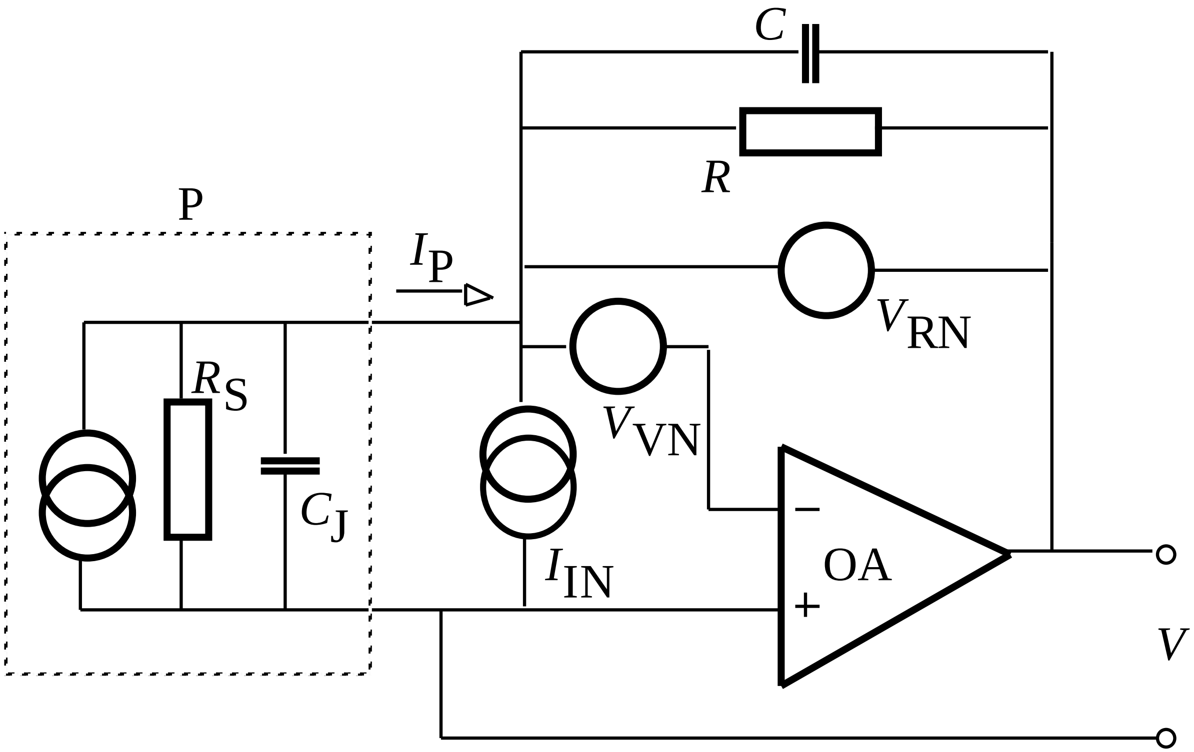
1. Example: For R=10 GΩ and open-loop gain A=10^6, R_I=10 kΩ. R_S=10 MΩ is needed to obtain 0.1 % non-linearity.

Detector noise sources

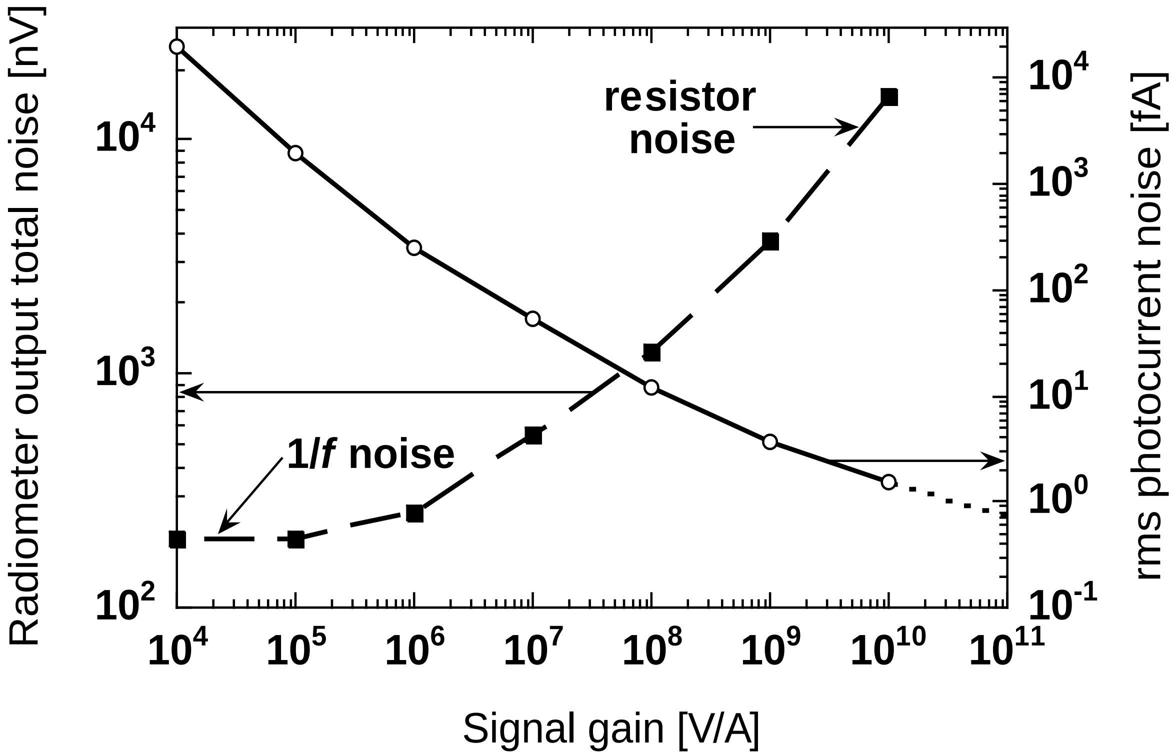
1. **<u>Photon noise:</u>** noise contained in the signal and noise due to background radiation
2. **<u>Detector-generated noise:</u>**
   - **Johnson:** thermal motion of charged particles and thermal current fluctuations in resistors
   - **Shot:** in (PV) detectors with P-N junction (variance in the rate of photoelectron generation)
   - **G-R:** in PC detectors produced by fluctuations in the generation and recombination of current carriers
   - **1/f:** caused by non-perfect conductive contact and bias current or voltage in detectors
3. **<u>Preamplifier noise:</u> Johnson, Shot, G-R, 1/f** and
   - **Phonon**: from temperature changes not caused by the detected radiation

Equivalent PV circuit showing the main noise components

The feedback impedance, *R* and *C*, of operational amplifier, OA, converts the photocurrent I_P of photodiode P into a voltage *V*. R_S and C_J are the photodiode impedance. Single circles illustrate voltage sources and double circles illustrate current sources. One signal (the photocurrent) source and three noise sources (voltage noise VN, current noise IN, and resistor noise RN) are shown in the circuit.



Output total-noise measured in dark

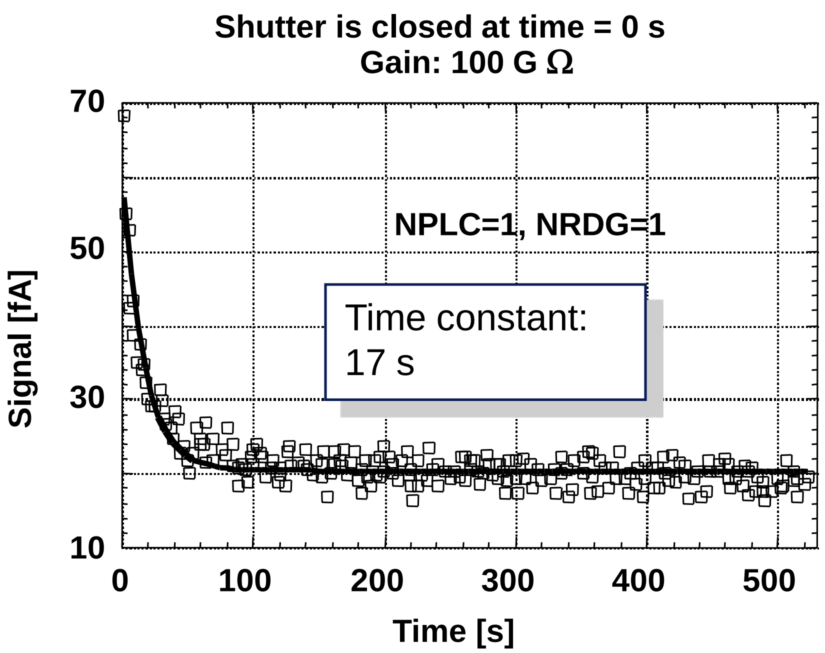


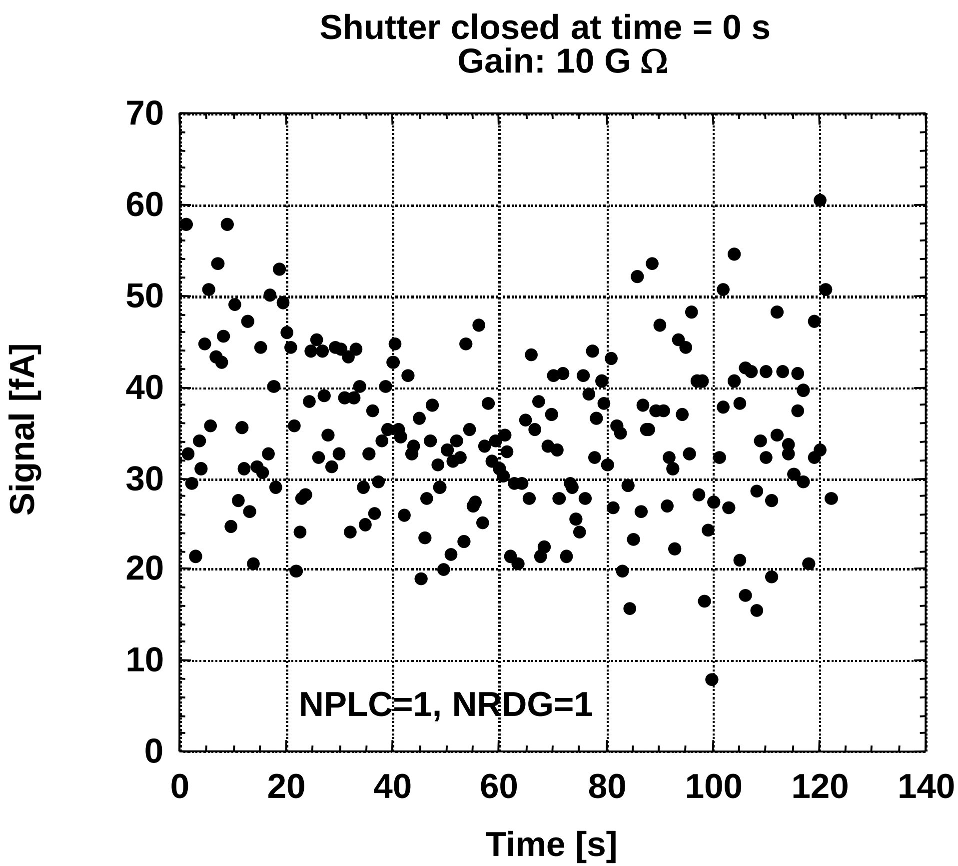
Dark noise with S1226-8BQ photodiode (R_S=7 GΩ) and OPA128LM.

The integration time of the DVM at the I-V output is 1.7 s.

Settling time of a Si photodiode current meter

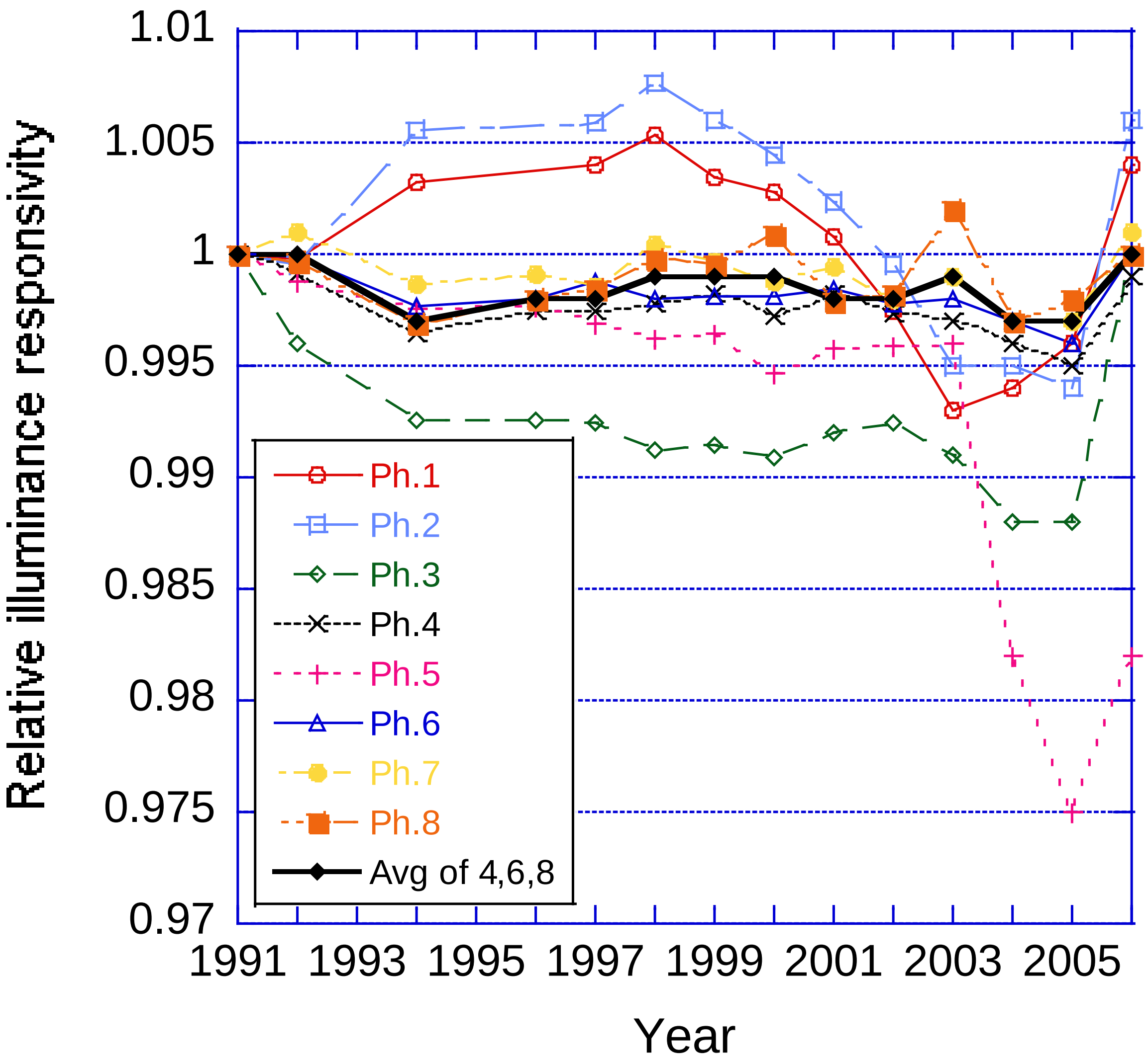
using Model S1226 (R_S=7 GΩ) and OPA128LM





The settling time depends on the magnitude of the signal change as well !

Long term stability of Si photometers



Comparison of typical characteristics of radiometric quality detectors within the 200 nm to 20 μm range

Type	Wavelength range [nm]	Diameter [mm]	Spatial response non-uniformity [%]	NEP [pW $Hz^{-1/2}$]	(Shunt) resistance [M Ω]
Nitrided Si	to 320	5 - 10	1	0.1 - 1	10 - 100
Silicon	200 - 1000	5 - 18	0.3	(2-10) x 10^{-4}	100 - 10000
Ge	800 - 1800	5 - 13	1	0.1	0.01
InGaAs	800 - 1800	5-10	0.5	0.02	1
Extended InGaAs	1000 - 2550	2 - 3	1 (?)	1	0.001
InSb	1600 - 5500	4 - 7	1	1	0.1 - 1
HgCdTe PV or PC	2000 - 26000	2 - 4	10 - 90	PV: 30 PC: 300	PV: 200 Ω PC: 15 Ω
Pyroelectric	200 - 20000	5 - 12	0.1 - 5	10^4 - 10^5	–
Thermopile	200 - 20000	5 - 10	0.2 - 3	2 x 10^4	N/A
Bolometer (cryogenic)	200 - 20000	5 - 10	1	40	1 - 2 at 4.5 K

Photomultiplier Tubes (PMT)

Advantages:

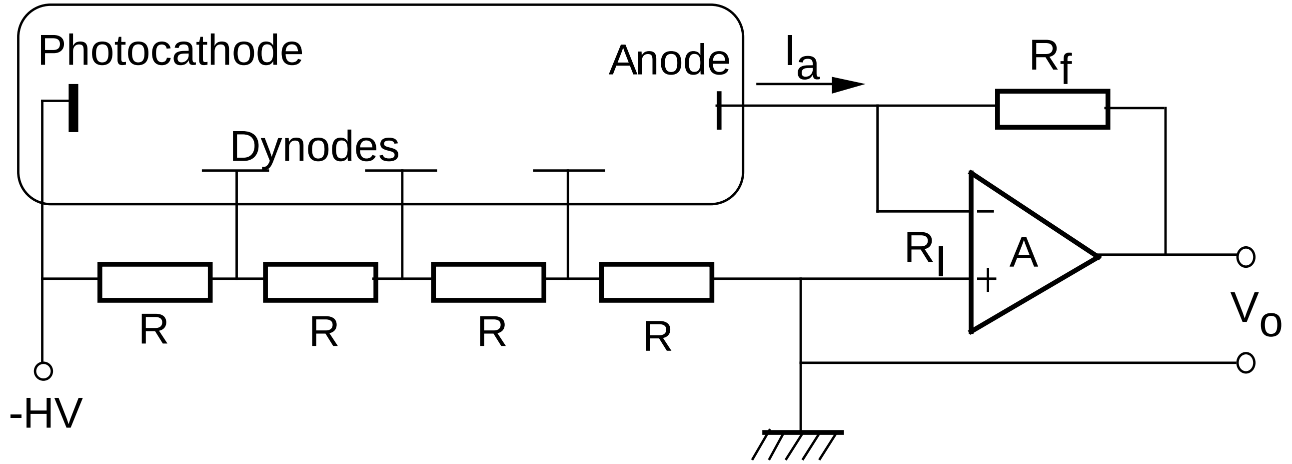
- Multiplication of secondary-electrons :
  - ✓ Extremely high responsivity
  - ✓ Exceptionally low noise
- Large photosensitive area
- Fast time response
- Virtually ideal constant-current source (very high shunt resistance)

Disadvantages:

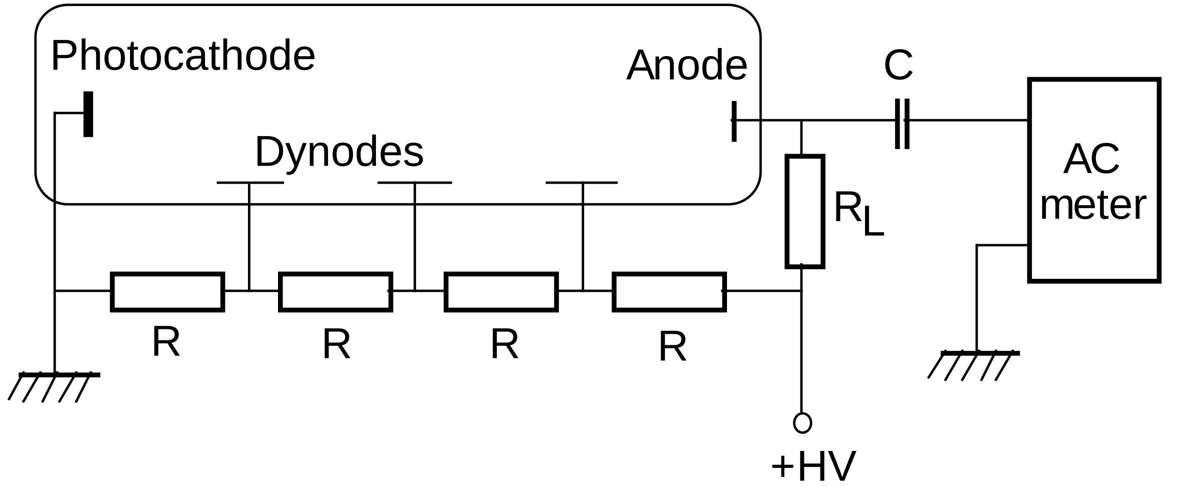
- Poor spatial response uniformity
- Temperature dependent responsivity
- Fatigue and hysteresis (overshoot or undershoot for high-voltage and light)
- High-voltage, temperature, illumination, and time dependent dark-current
- Very stable high-voltage is required
- Affected by magnetic fields
- Drift and aging
- Linear and stable operation only at low signal levels

DC and AC PMT measurements

DC:



AC:



The current of the R voltage dividers must be much larger than I_a !

Comparison of PMT to Si photodiode

Δf = 0.3 Hz (NPLC=100 on DVM)	PMT	Si	PMT/Si
Responsivity [A/W]	5×10^{5}	5×10^{-1}	10^{6}
Noise [pA]	54	6×10^{-4}	10^{5}
NEP [W]	10^{-16}	10^{-15}	0.1
Signal/Noise			10x

$$N_{PMT} = \sqrt{2eI_{ad}K\Delta f} = 54pA \qquad [W]$$

where e is the elementary electron charge,

I_{ad} is the PMT anode dark current [A],

K is the PMT current amplification, and

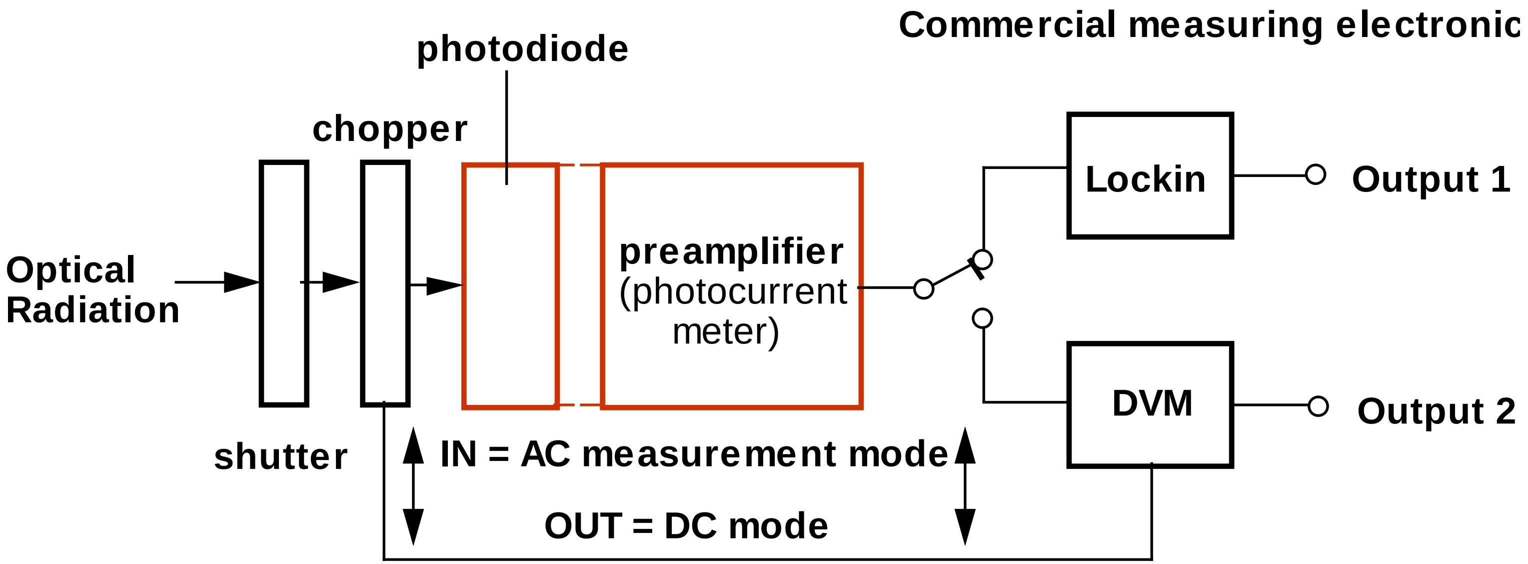
Δf is the electrical bandwidth [Hz].

Selection of detectors for different applications

- Radiant power measurement:
  - ✓ Detectors with high spatial-response uniformity are needed
- Irradiance and radiance measurements:
  - ✓ Spatially non-uniform detectors can be used with uniform sources
- Photometric and color measurements:
  - ✓ Si photodiodes should be used
- UV measurements:
  - ✓ Passivating Nitrided Oxides or Pt-Silicide front layers eliminate UV damage
- Scale extension to UV and IR:
  - ✓ Pyroelectric detectors and bolometers with high spatial-response uniformity
- SW-IR measurements (1 µm to 5 µm):
  - ✓ NIR photodiodes, extended InGaAs and InSb photodiodes are preferred
- LW-IR measurements (5 µm to 20 µm):
  - ✓ HgCdTe detectors, pyroelectric detectors, and cryogenic bolometers

Scheme of optical radiation measurements

Matching preamplifier to a selected photodiode will dominate the performance (signal-to-noise ratio) of the overall measurement !



Frequency dependence of photodiodes

- The **internal speed** depends on the
  - ✓ Time to convert the accumulated charge into current
- The **maximum frequency** depends on the
  - ✓ Area of the photodiode
  - ✓ Type of material
- The **internal capacitance** C_j depends on the
  - ✓ Active area
  - ✓ Resistivity (can change from 1 Ωcm to 10 kΩcm for Si)
  - ✓ Reverse voltage

Frequency dependence of photodiodes (cont.)

- **Time constant** of a photodiode

  (with one dominating internal capacitance C_j):

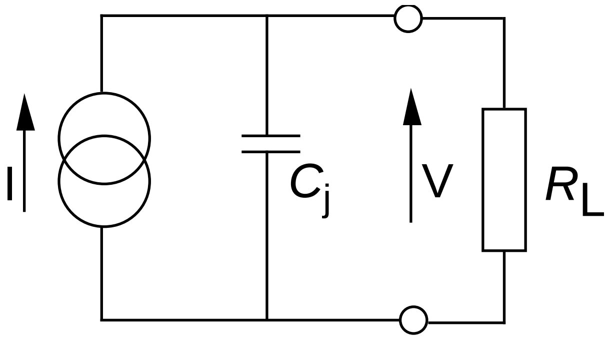
$$\tau = C_j R_L$$

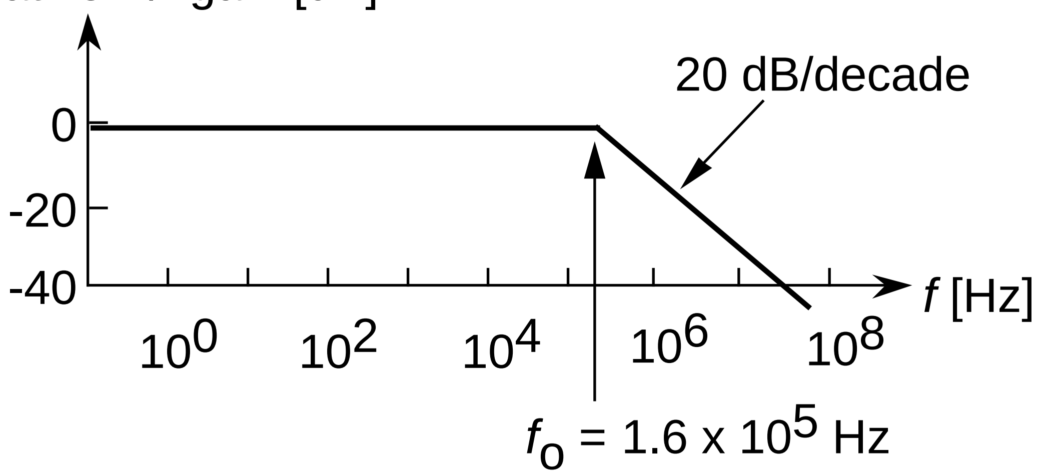
where R_L is the load-resistance

- **Rise time** (for photodiodes with multiple time constants):

  The current changes from 10 % to 90 %

Frequency dependence of photodiodes (cont.)





If the photodiode (shunt) resistance is much larger than R_L,
the voltage on R_L is: $V = I\, R_L / (1 + j\omega C_j R_L)$
The **upper roll-off frequency** is $f_0 = \omega_0/2\pi = 1/2\pi C_j R_L$
where $\omega_0 = 1/\tau$, and I is the photocurrent.

For $C_j = 1$ nF and $R_L = 1$ kΩ, $f_0 = 160$ kHz

AC (chopped) radiation measurement

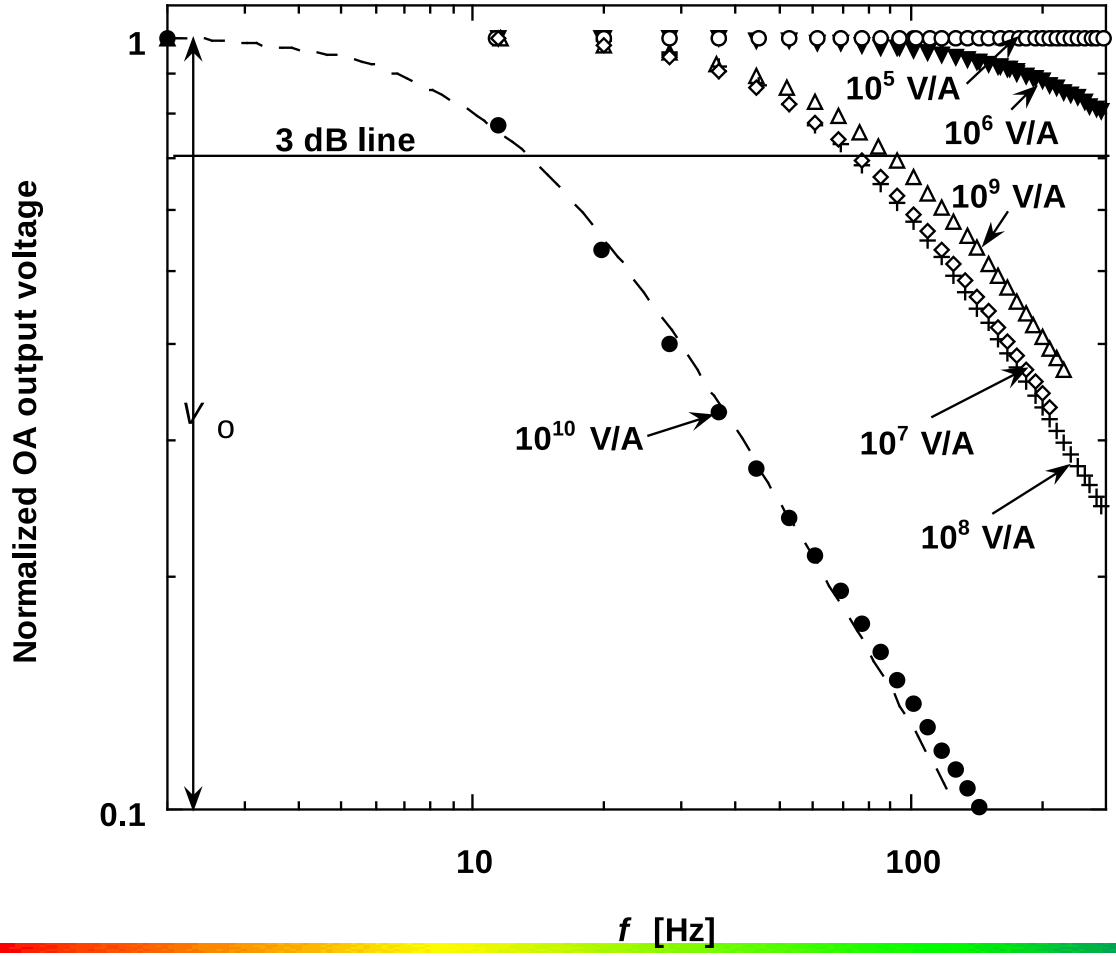
Chopping is needed to tune out measurement from the 1/*f* noise range (close to 0 Hz) and the eliminate DC background signal in infrared measurements.

Chopped measurements need partial frequency compensations !

- **$\tau_1 = RC$ must be small to keep the roll-off higher than the signal frequency**
- **Photodiode with small C_j is needed to decrease τ_2 , e.g. Hamamatsu S5226-8BQ**
- **Wide band (high open loop gain and low noise) OPA is needed, e.g. OPA627.**

AC (chopped) radiation measurement (cont.)

- **Signal gain curves (measured).** The 3 dB roll-off frequencies for all gains are 80 Hz or higher except for gain 10^{10} V/A.



Partial compensations were made for all the signal gains shown here. No compensation was made for 10^{10} V/A. The operating point should be on the flat parts of the curves at 10 Hz chopping (or frequency stabilized chopper is needed) !

AC measurements with chopper and lockin

- **Chopper:** 1. tunes out the signal (by modulation) from 1/*f* noise and drift

  2. Separates the signal to be measured from the DC background signal

  ✓ **Frequency:** needs to be stable if the operating point is on the slope of the signal-gain versus frequency curve

- **Lockin:** phase controlled rectifier + low-pass filter

  ✓ **Phase control:** synchronized from chopper

  ✓ **Low-pass filter:** smoothes out the rectified (structured but DC) signal

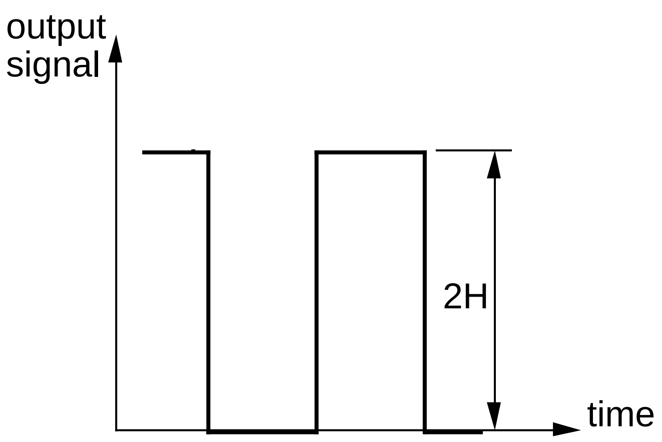
  ✓ ***Output:*** in-phase and quadrature (X and Y) components of the signal (in rectangular form), or magnitude $M=(X^2+Y^2)^{1/2}$ and phase Φ (in polar form)

  ✓ ***Input:*** sine or square wave. The sine wave measurement selects the fundamental frequency component of the chopped waveform.

Sine wave lockin measures square wave

Calibration of the lockin reading against a DVM.

- Signal to be measured:



- Theoretical reading of sine-wave lockin: $$S_1 = \frac{H}{\sqrt{2}}\frac{4}{\pi} = 0.9003H$$

- Reference reading of a DVM in DCV mode (with large S/N):

  $S_2 = H$ with running chopper, or $S_2 = 2H$ if the chopper is stopped
- The real correction factor is the ratio of the lockin reading to the DVM reading: S_1/S_2

Selection of commercial signal meters for detectors

- DC or AC photocurrent from photodiodes:
  - ✓ Electrometers, current preamplifiers, and picoammeters can be used
  - ✓ The typical shunt resistance of a DVM in DC-I mode is 1 kΩ in the lowest (300 µA f.s.) range. The input shunt resistance can be higher for DMMs. A "Burden" voltage of about 0.2 V can develop on this resistance causing an error in the measured current. The lower the detector resistance the larger the error.

DO NOT DO THIS:

- V-measurement on detectors or load resistors:
  - ✓ Non-linearity with biased PC detectors
  - ✓ High non-linearity with photodiodes (measurement along the V-axis of the I-V curve)
- Photodiode shunt resistance measurement with ohm-meters

(A large current would be forced through the photodiode!)

5. Determining Measurement Uncertainties

Outline

1. Measurement Uncertainty, Measurement Error
2. Accuracy & Precision
3. Measurement Equation
4. Measurement Steps
5. Direct Methods for Uncertainty Propagation
6. GUM Supplement 1

Measurement Equation Approach:

In general, we use the measurement equation approach for characterizing and calibrating sources and radiometers. A simplified measurement equation is:

$$I(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda}\int_{A}\int_{\omega} S_\phi(x,y,\theta,\phi,\lambda,\lambda_o)\cdot L_\lambda(x,y,\theta,\phi,\lambda,\lambda_o)\cdot \cos\theta \cdot \mathrm{d}\omega \cdot \mathrm{d}A \cdot \mathrm{d}\lambda$$

$I(A,\omega,\Delta\lambda,\lambda_o)$ - the measured current

S_ϕ - the spectral flux (power) responsivity of the detector at a position x,y

L_λ - the spectral radiance of the source

A - receiving area of the detector

ω - the solid angle of the source viewed by the detector

Measurement Uncertainty

Measurement result is complete only when a quantitative estimate of the uncertainty in the measurement is stated.

The “true value” of the measurand is the value of the measurand.

Formal definition

Uncertainty of measurement is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

“Expressed as a standard deviation (u)”

Why do you need an uncertainty budget?

Traceability- "Property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons, all having stated uncertainties."

ISO *International Vocabulary of Basic and General Terms in Metrology*, 2nd ed., 1993, definition 6.10

Uncertainty budget will enable one to identify the dominant terms in the uncertainties to reduce those terms.

Repeatability and Reproducibility

Closeness of agreement between the results of successive measurements of the same measurand carried out

Repeatability	Reproducibility
under same conditions of measurement	under changed conditions of measurement
Same	**Different**
principle	principle
method	method
observer	observer
location	location
instrument	instrument
time	time

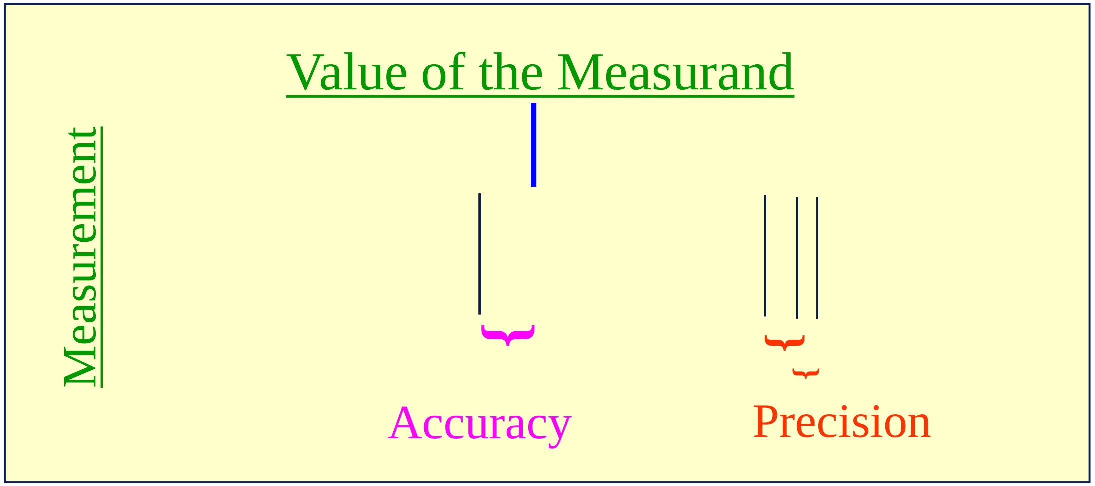
Accuracy and Precision

Accuracy

Closeness of agreement between the result of a measurement and the value of the measurand.

Precision

Closeness of agreement between the results of measurements of the same measurand.



Note: The ISO Guide to Uncertainty in Measurements (GUM) discourages the use of the terms, but are still used and confused in common usage.

Accuracy and Precision - Example

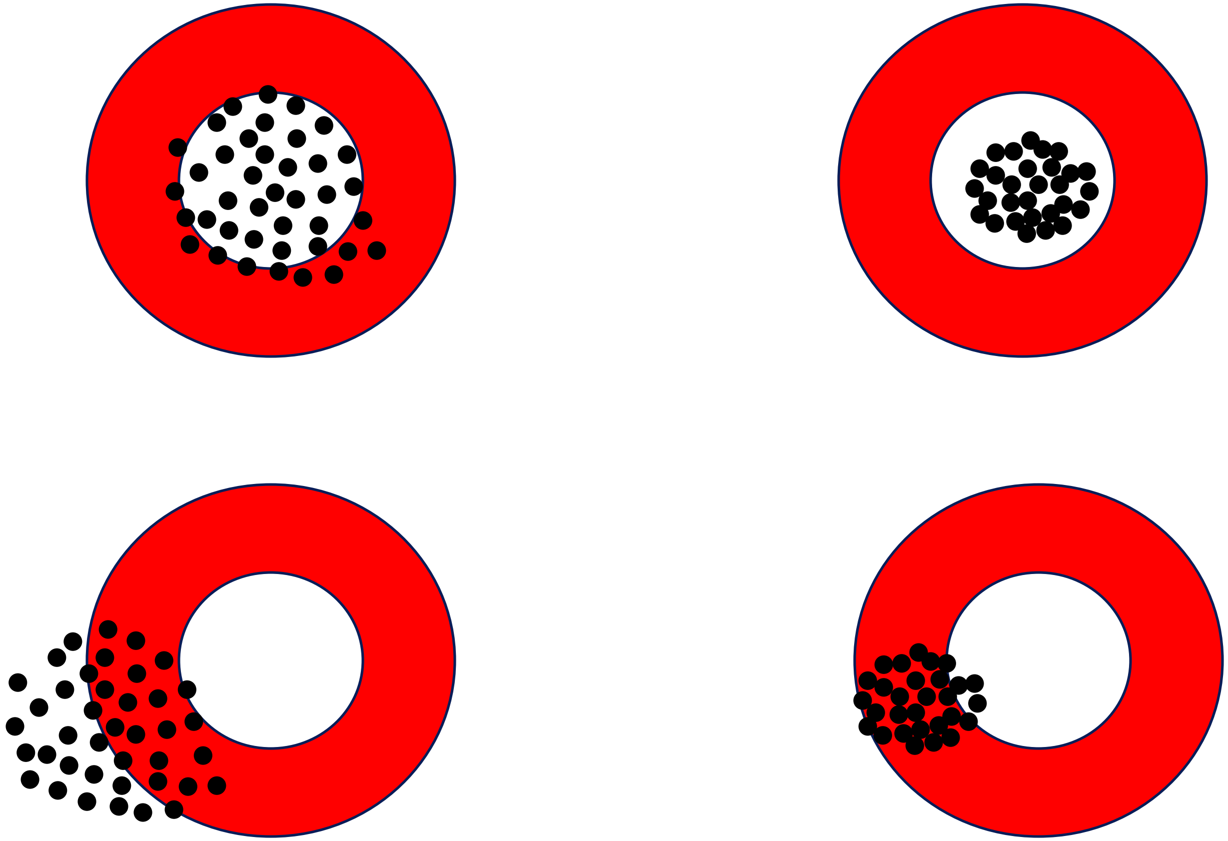
Precision

Low High

Accuracy

High

Low



Error of Measurement

Result of a measurement minus the value of the measurand.
(Sum of random and systematic errors)

Random error

Result of a measurement
minus
the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions

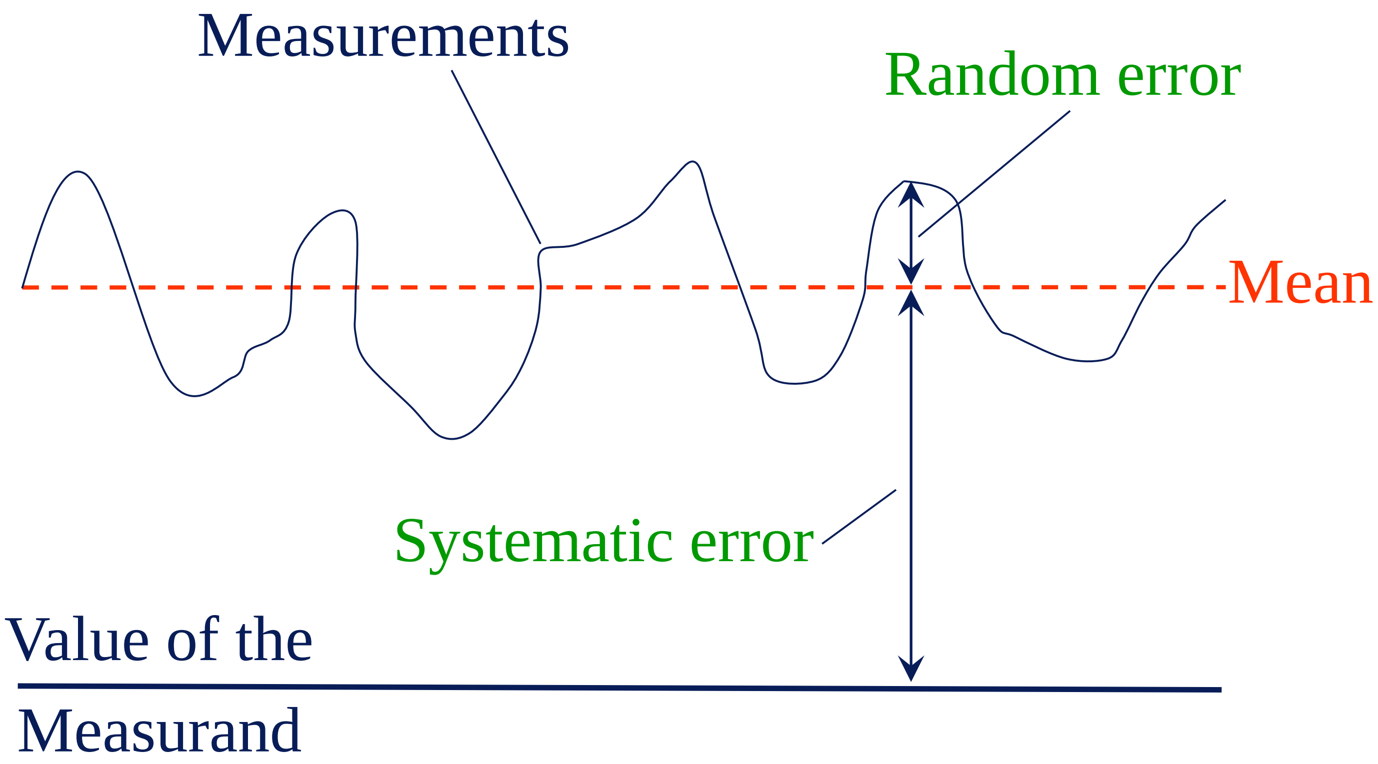
$$x_{i,k} - x_i$$

Systematic error

Mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions
minus
the value of the measurand.

$$x_i - x$$

Error of Measurement - Illustration



Classification of Uncertainty Components

Due to random effects (Type A)

Give rise to possible random error in the <u>unpredictable</u> result of the current measurement process.

Usually decrease with increasing number of observations

Due to systematic effects (Type B)

Give rise to possible <u>systematic</u> error in the result due to recognized effects in the current measurement process.

Correction and Correction Factor

Used to account for systematic error

Correction

Value added algebraically to the uncorrected result of a measurement to compensate for systematic error.

Correction = - (systematic error)

Correction Factor

Numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error.

E.g. Linearity, offset, shunt resistance, drift, stray light

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Standard Uncertainty

Measurand (y) determined from m input parameters x_i through functional relationship

$$f(x_1, x_2, \ldots, x_i, \ldots, x_m)$$

Example: Radiometer signal measurement

$$v \cong \Gamma \cdot G \cdot s(\lambda) \cdot \tau(\lambda) \cdot L(\lambda, T) \cdot \Delta\lambda$$

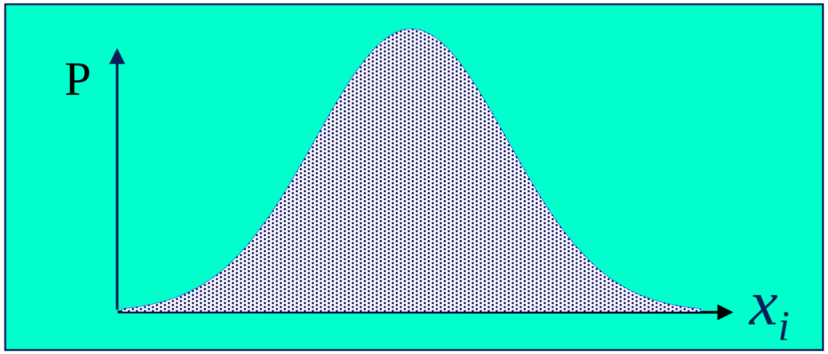
Input parameters are throughput (Γ), gain (G), responsivity ($\mathbf{s}$), transmittance (τ), radiance ($\mathbf{L}$), wavelength (λ), bandwidth ($\Delta\lambda$) and source temperature ($\mathbf{T}$)

Standard uncertainty

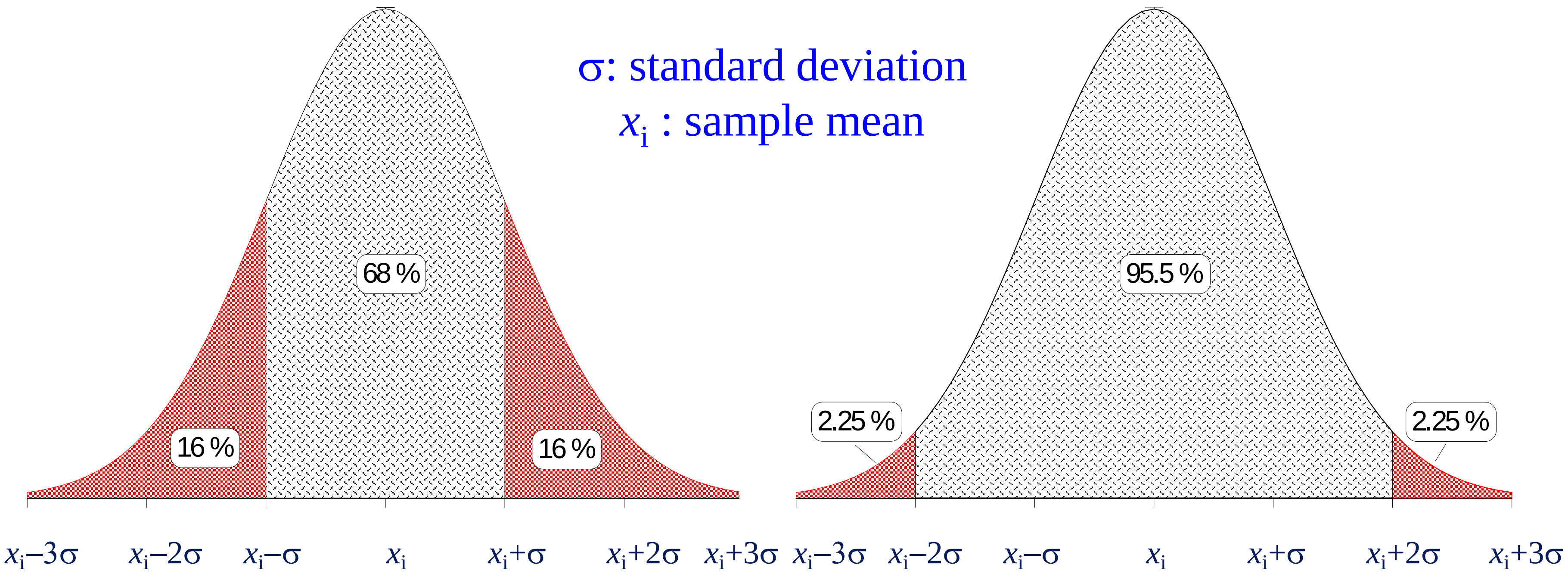
Estimated standard deviation associated with each input estimate x_i, denoted $u(x_i)$

Example: $u(\Gamma)$, $u(\lambda)$, $u(T)$, etc.

Standard uncertainty $u(x_i)$ determined from probability distribution (P) of parameter (x_i)



Normal Probability Distribution



Probability that x lies between $(x_i - \sigma)$ and $(x_i + \sigma)$ is 68 %.

For large number of observations, about 68 % of the values lie in this range, **OR**

a value deviating more than σ from mean x_i will occur about once in 3 trials.

Probability that x lies between $(x_i - 2\sigma)$ and $(x_i + 2\sigma)$ is 95.5 %.

For large number of observations, about 95 % of the values lie in this range, **OR**

a value deviating more than 2σ from mean x_i will occur about once in 20 trials.

Evaluation of Uncertainty

Two Categories: Type A and Type B

Type A

Evaluated using statistical methods for analyzing the measurements.

Examples: Standard deviation of a series of independent observations,
Least squares fit

Type B

Evaluated by methods other than statistical.

Examples: Scientific judgment,experience, manufacturer’s specification, data from other sources (reports, handbooks)

Statistical Parameter – Sample Mean

Mean

$$x_i = \frac{1}{n}\sum_{k=1}^{n} x_{i,k}$$

Sum of all the sample values ($x_{i,k}$) divided by the size of the sample (n)

Example

Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0

Size of the sample = 5

Sample mean = (0.9 + 1.2 + 1.1 + 0.8 + 1.0)/5 = **1.0 [V]** .

Statistical Parameter – Sample Variance

Variance: $$\sigma^2(x_{i,k}) = \frac{1}{n-1}\sum_{k=1}^{n}(x_{i,k} - x_i)^2$$

Sum of the squares of the deviations of the sample values ($x_{i,k}$) from the mean value (x_i), divided by (n - 1).

Measures the spread or dispersion of the sample values, and is positive.

Variance of the mean $$\sigma^2(x_i) = \frac{\sigma^2(x_{i,k})}{n}$$

Example: Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0; Sample mean = 1.0 [V]

Variance = $[(0.9-1.0)^2 + (1.2-1.0)^2 + (1.1-1.0)^2 + (0.8-1.0)^2 + (1.0-1.0)^2]/(5-1)$ = **0.025 [V²]**

Variance of the mean = 0.025/5 = **0.005 [V²]**

Type A Evaluation of Standard Uncertainty

Standard deviation = (Variance)$^{1/2}$ = $\sigma(x_{i,k})$

(Positive square root of the sample variance)

Standard deviation of the mean: $\sigma(x_i) = \sigma(x_{i,k}) / n^{1/2}$

Standard uncertainty $u(x_i)$ = $\sigma(x_i)$

(Standard deviation divided by the square root of the number of samples)

Relative standard uncertainty = $u(x_i)/x_i$

Example: Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0

Sample mean = 1.0 [V], Variance = 0.025 [V^2], Variance of the mean = 0.005 [V^2]

Standard deviation = (Variance)$^{1/2}$ = $0.025^{1/2}$ = **0.158 [V]**

Standard uncertainty = Standard deviation of the mean = $0.158/5^{1/2}$ = **0.071 [V]**

Relative standard uncertainty = 0.071/1.0 = **0.071**

Type B Evaluation of Standard Uncertainty

Evaluated based on scientific judgment, experience, manufacturer's specification, data from other sources (reports, handbooks)

Examples

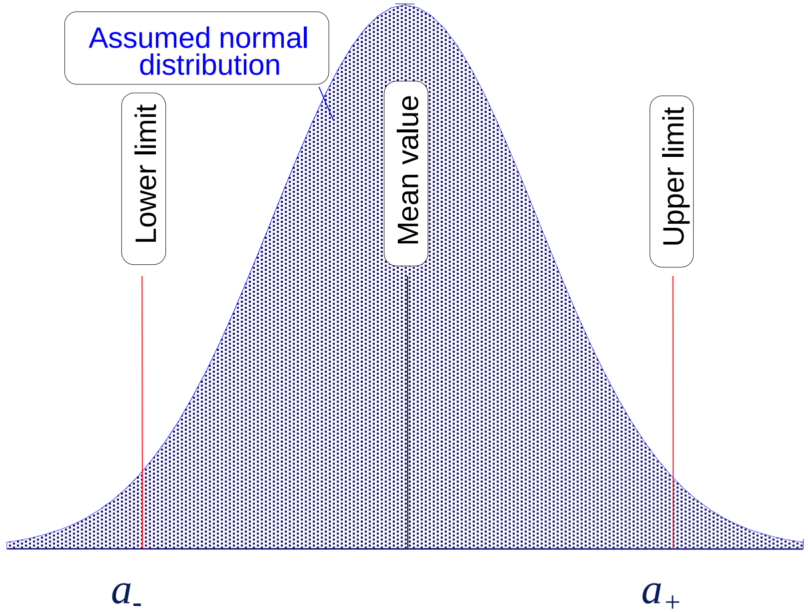
Convert a quoted uncertainty (with a stated multiple) to a standard uncertainty by dividing by the multiple

Convert a quoted uncertainty (with a specified confidence level, such as 95 % or 99 %) to a standard uncertainty by dividing by the appropriate factor for a normal distribution

Computational methods

Model the quantity by an assumed probability distribution such as normal, rectangular or triangular.

Type B Calculation – Normal Distribution



Center of the limits
$= (a_+ + a_-)/2$

Half width of interval
$a = (a_+ - a_-)/2$

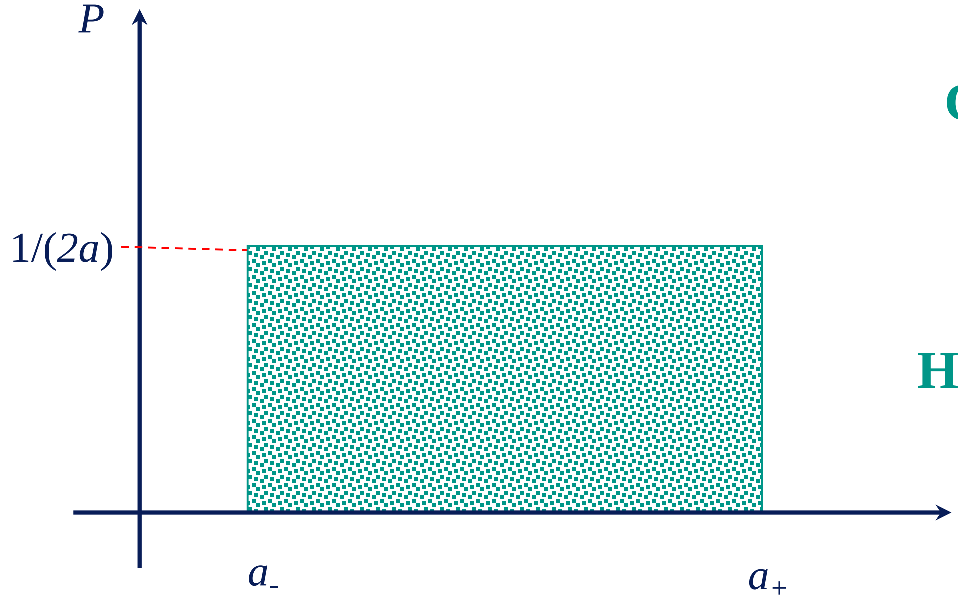
Estimated the lower limit (a_-), and the upper limit (a_+) of the quantity.
Best estimated value of the quantity (mean) = center of the limits

50.0 % probability, value lies in the interval a_- to a_+, **then $u(x_i) = 1.48\,a$**
67.7 % probability, value lies in the interval a_- to a_+, **then $u(x_i) = a$**
99.7 % probability, value lies in the interval a_- to a_+, **then $u(x_i) = a/3$**

Type B Calculation – Rectangular Distribution

Equal probability the value lies in the interval a_- and a_+ is 100 % and zero outside

(Reasonable default model in the absence of any other information)



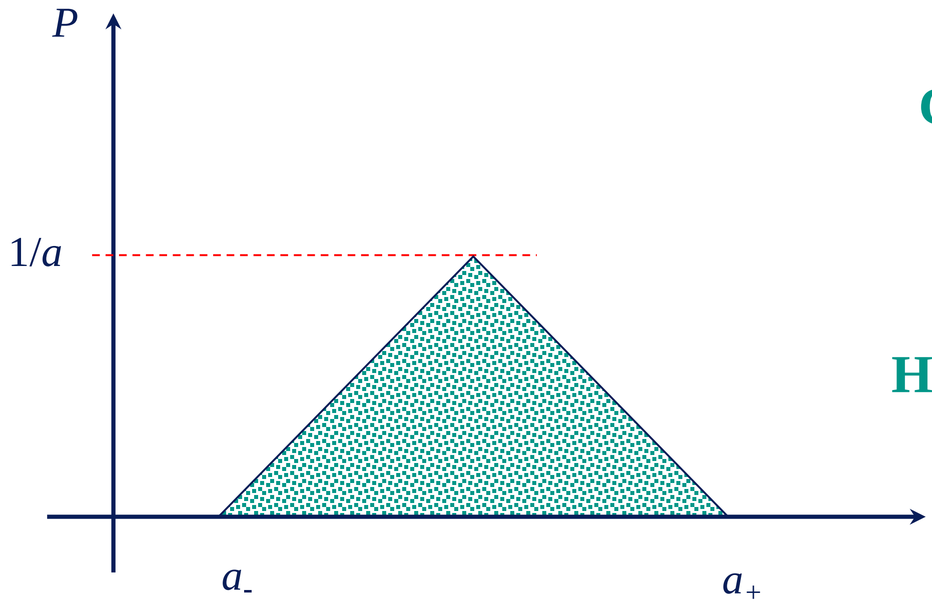
Center of the limits
$= (a_+ + a_-)/2$

Half width of interval
$a = (a_+ - a_-)/2$

Best estimated value of the quantity (mean) = center of the limits
with
$u(x_i) = a/3^{½}$ or $[\text{max-min}]/(12)^{1/2}$

Type B Calculation – Triangular Distribution

Probability the value lies in the interval a_- and a_+ is 100 % and zero outside



Center of the limits
$= (a_+ + a_-)/2$

Half width of interval
$a = (a_+ - a_-)/2$

Best estimated value of the quantity (mean) = center of the limits with

$u(x_i) = a/6^{1/2}$ or $[\text{max-min}]/(24)^{1/2}$

Expressing Measurement Uncertainty

Functional relationship between measurand and input parameters

$y = f(x_1, x_2, ..., x_i, ..., x_m)$

Combined standard uncertainty, $u_c(y)$

Represents the estimated standard uncertainty of the measurand y.

given by

Law of Propagation of Uncertainty

$$u_c^2(y) = \sum_{i=1}^{m} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

$\partial f / \partial x_i$: sensitivity coefficient, $u(x_i)$: standard uncertainty of x_i

$u(x_i)/x_i$: relative standard uncertainty of x_i

$u(x_i, x_j)$: covariance of x_i and x_j $= u(x_i) \cdot u(x_j) \cdot r(x_i, x_j)$

$r(x_i, x_j)$: correlation coefficient $r = 0$, if uncorrelated $[-1 \leq r \leq 1]$

Expressing Measurement Uncertainty - Example

Additive function (Two independent random variables x_1 and x_2)
Use standard uncertainties to calculate combined standard uncertainty

$$y = a \cdot x_1 + b \cdot x_2$$

$$\frac{\partial y}{\partial x_1} = a \qquad \frac{\partial y}{\partial x_2} = b$$

$$u_c^2(y) = a^2 \cdot u^2(x_1) + b^2 \cdot u^2(x_2)$$

$$y = a \cdot x_1 \cdot x_2$$

$$\frac{\partial y}{\partial x_1} = a \cdot x_2 \qquad \frac{\partial y}{\partial x_2} = a \cdot x_1$$

$$u_c^2(y) = a^2 \cdot x_2^2 \cdot u^2(x_1) + a^2 \cdot x_1^2 \cdot u^2(x_2)$$

$$\frac{u_c^2(y)}{y^2} = \frac{u^2(x_1)}{x_1^2} + \frac{u^2(x_2)}{x_2^2}$$

Multiplicative function (Two independent random variables x_1 and x_2)
Use relative standard uncertainties to calculate combined standard uncertainty

Example: Frequency of pendulum

$$\omega = \sqrt{\frac{g}{\ell}} \text{ then}$$

$$\frac{\partial \omega}{\partial \ell} = \frac{-1}{2}\frac{\sqrt{g}}{\ell^{3/2}} = \frac{-1}{2}\frac{\omega}{\ell},$$

$$\frac{\partial \omega}{\omega} = \frac{-1}{2}\frac{\partial \ell}{\ell}$$

The relationship between frequency,ω and length is given by the sensitivity coefficient, ½.

Expanded Uncertainty

Measure of uncertainty defining an *interval* about the result *y* within which the measurand is confidently believed to lie.

Expanded uncertainty (U) = Coverage factor (k) **x** Combined uncertainty $u_c(y)$

Coverage factor *k*	Confidence level for a normal probability distribution
1.000	68.27 %
1.645	90.00 %
1.960	95.00 %
2.000	**95.45 %**
2.576	99.00 %
3.000	99.73 %

1 out of 20 times you should fall outside your uncertainty budget

Uncertainty Evaluation Procedure – Summary

1. Express functional relationship between the measurand **(*y*)** and input parameters **(x_i).**

$$y = f(x_1, x_2, \ldots, x_i, \ldots, x_m)$$

2. Determine values of input parameters x_i [Statistical analysis or other means].

3. Evaluate standard uncertainty $u(x_i)$ of each input x_i (Type A or Type B technique).

4. Calculate the value of measurand **(*y*)** from the functional relationship [Step 1].

5. Determine the combined standard uncertainty $u_c(y)$ from the standard uncertainties associated with each input parameter **(x_i)**. [Step 3].

6. Calculate the expanded standard uncertainty **(*U*)** as the combined standard uncertainty $u_c(y)$ times the coverage factor **(*k*).**

7. Report the value of the measurand *y* [Step 4] **and** specify the combined standard uncertainty $u_c(y)$ [Step 5] **or the** expanded uncertainty *U* [Step 6].

6a. Applications from the NIST Short Course:

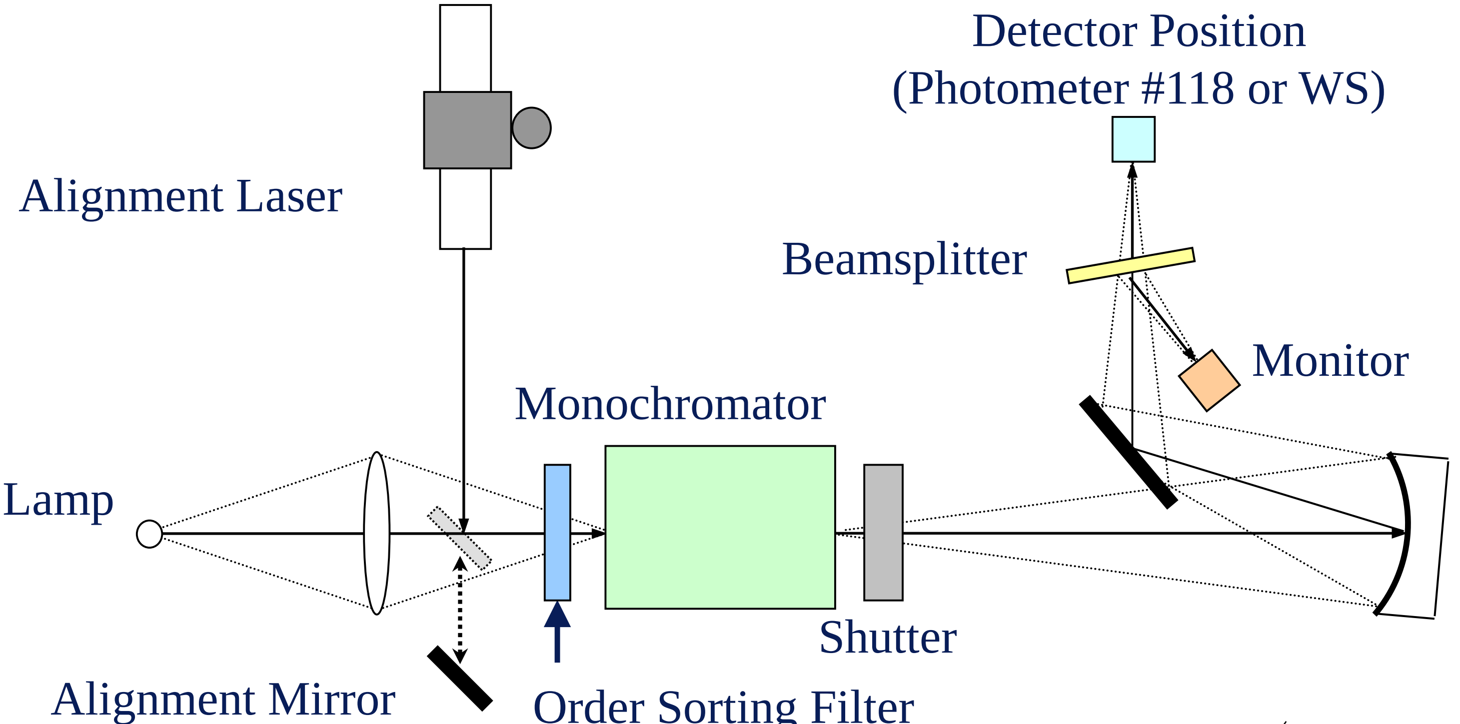
Photometer Responsivity Calibration

Objectives of Lab #2

1. Measure the spectral power responsivity of the NIST photometer by comparison to a NIST-traceable silicon photodiode standard (STD). (Calibration)
2. Use the previously calibrated NIST photometer to measure the 100 W QTH lamp at about 3 m distance, and compare the measured illuminance (lux) to those found using a separatelu calibrated commercial photometer. (Validation)

Spectral Responsivity [A/W] (Lab #2, Step 1)

Lab #2 Spectral Responsivity Measurement Setup



Photometer Spectral Responsivity [A/W] is given by:

$$s_p = \frac{V_p / V_{mp}}{V_s / V_{ms}} \frac{G_s}{G_p} s_s$$

Application Example from Photometry (Lab #2)

Traceability Chain

Optical Power [watt]
(Cryogenic radiometer)

Spectral Responsivity [ampere/watt]
(Silicon photodiode)

Calibrated Aperture

Illuminance Responsivity [ampere/lux]
(Photometer)

Illuminance Meter [lux]
(Test meter)

Realization of Detector-based Illuminance [lux]

1. Start with calibrated Si diode
2. Calibrate the spectral power responsivity of a photometer using monochromator system
3. Convert to illuminance responsivity using aperture (and calculation)
4. Calibrate Test illuminance meter with Photometer

Uncertainty Calculation Example

Application to detector calibration (Lab #2)

Signal measurement equation

$$S_x = \frac{V_x / V_{mx}}{V_s / V_{ms}} \cdot \frac{G_s}{G_x} \cdot S_s$$

S_x	Spectral responsivity of test detector
S_s	Spectral responsivity of standard detector
V	Voltage from test detector (x) or standard detector (s)
V_m	Voltage from monitor detector
G	Amplifier gain

Illuminance Responsivity [A/lx] (Lab #2, Step 2)

$$s_{\mathrm{v,i}} = A \frac{\int_\lambda P(\lambda) s(\lambda) \mathrm{d}\lambda}{K_\mathrm{m} \int_\lambda P(\lambda) V(\lambda) \mathrm{d}\lambda}$$

$s_{\mathrm{v,i}}$	illuminance responsivity [A/lx] (Note: lx = lm/m^2)
A	area of the photometer aperture [m^2]
$P(\lambda)$	spectral power distribution of the light source (CIE Illuminant A with 2856 K Planck distribution)
$s(\lambda)$	photometer spectral power responsivity [A/W]
K_m	maximum spectral luminous efficacy [683 lm/W]
$V(\lambda)$	spectral luminous efficiency function

Illuminance Meter Calibration [lx] (Lab #2, Step 3)

The illuminance $E_{v,p}$ [lx] measured by the photometer is:

$$E_{v,p} = V_p / s_{v,i} G_p$$

V_p photometer signal [V]

$s_{v,i}$ illuminance responsivity [A/lx]

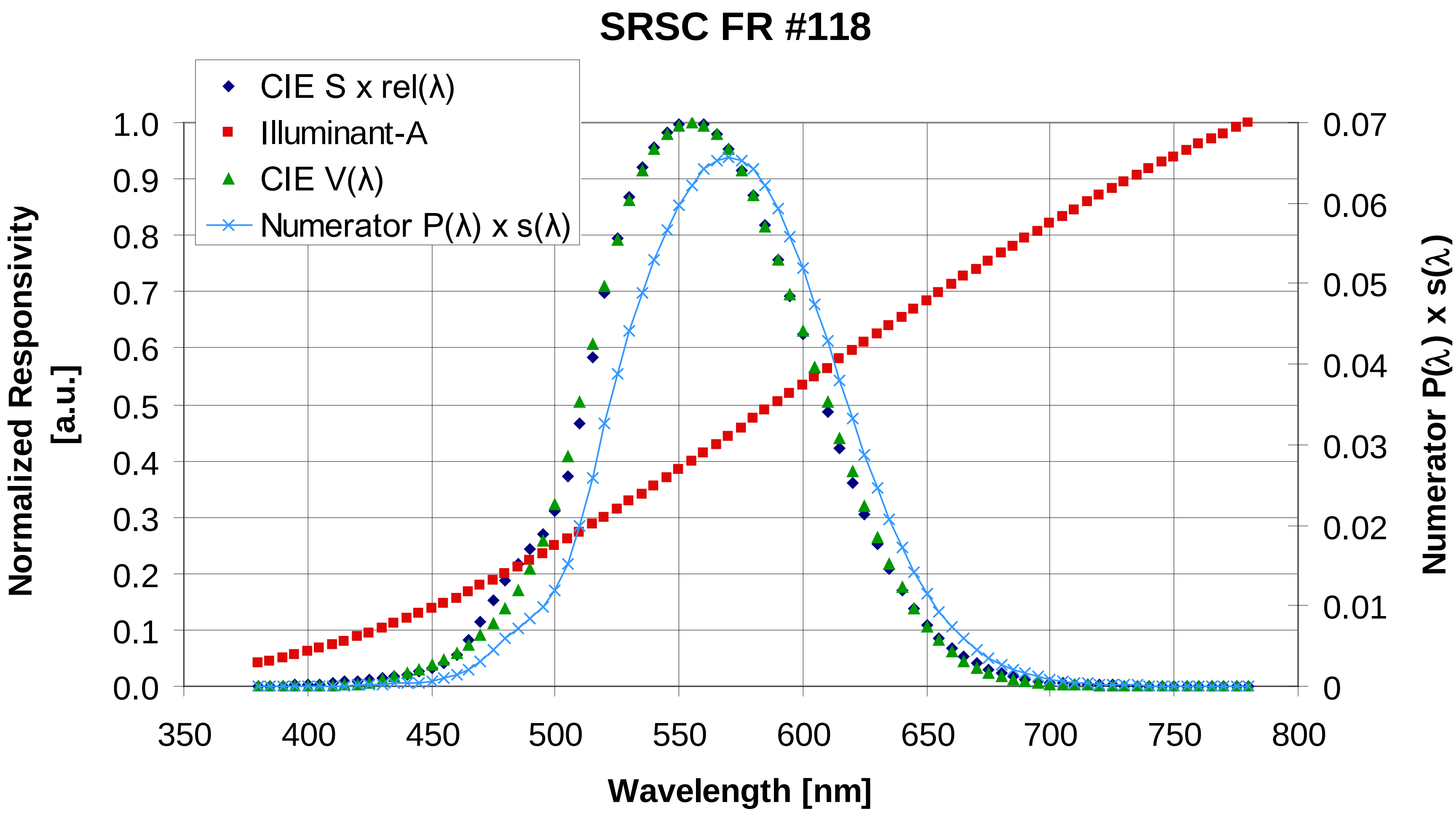
G_p photometer gain [V/A]

The calibration factor for the test photometer is:

$$CF_{v,t} = E_{v,p} / E_{v,t}$$

$E_{v,t}$ illuminance measured by the test photometer [lx]

Spectral Responsivity, *V*(*λ*), and Illuminant A



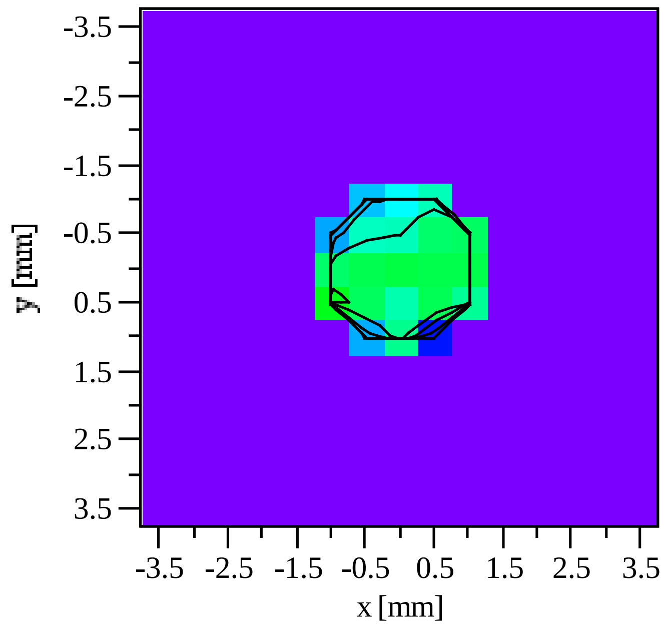
Uniformity of SRSC FR #117 with *V*(*λ*) Filter

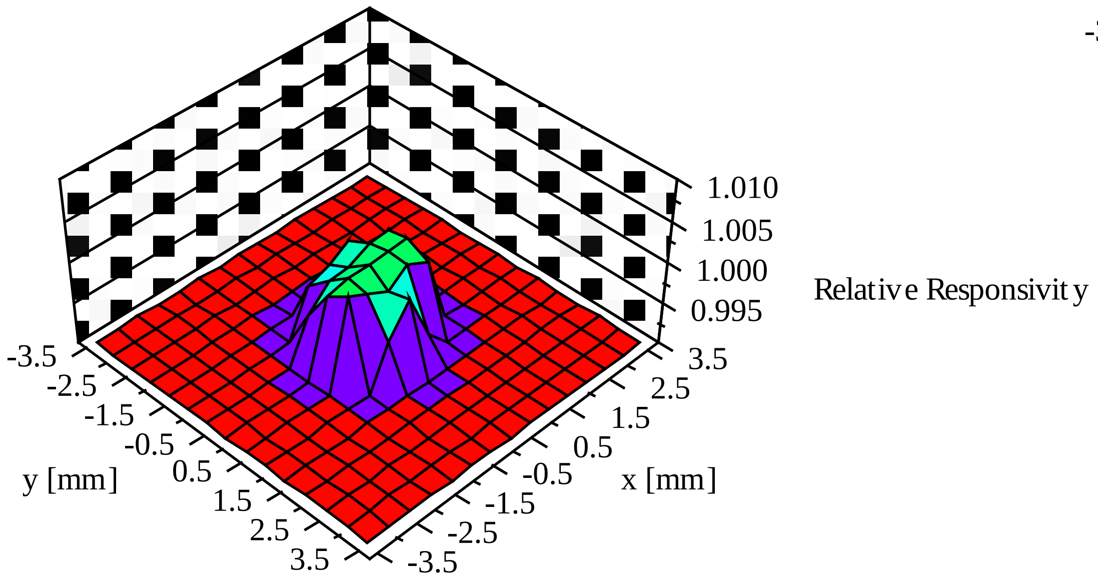
Responsivity uniformity

0.2 % contours at 555 nm;

1.1 mm beam size;

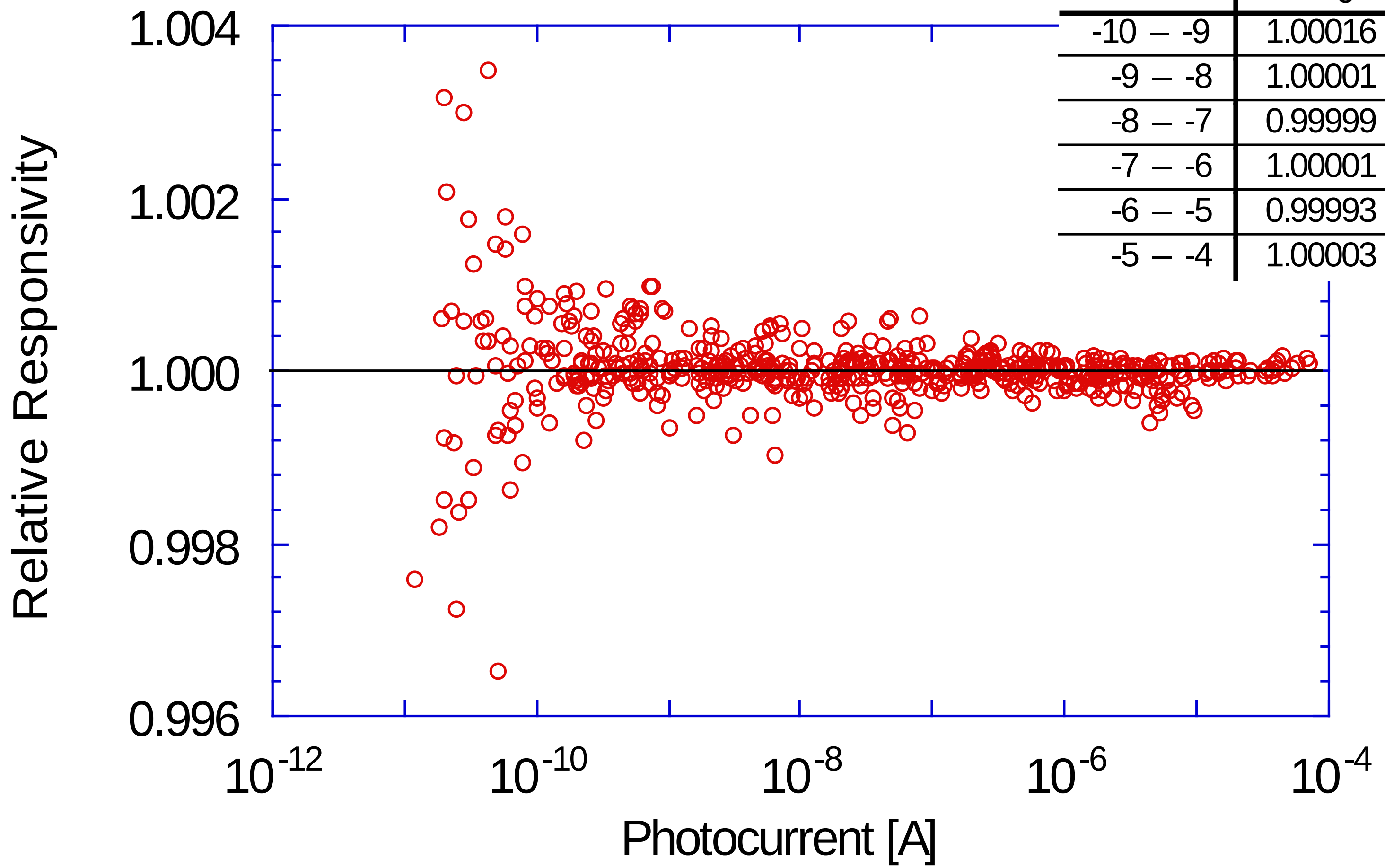
0.5 mm/Step





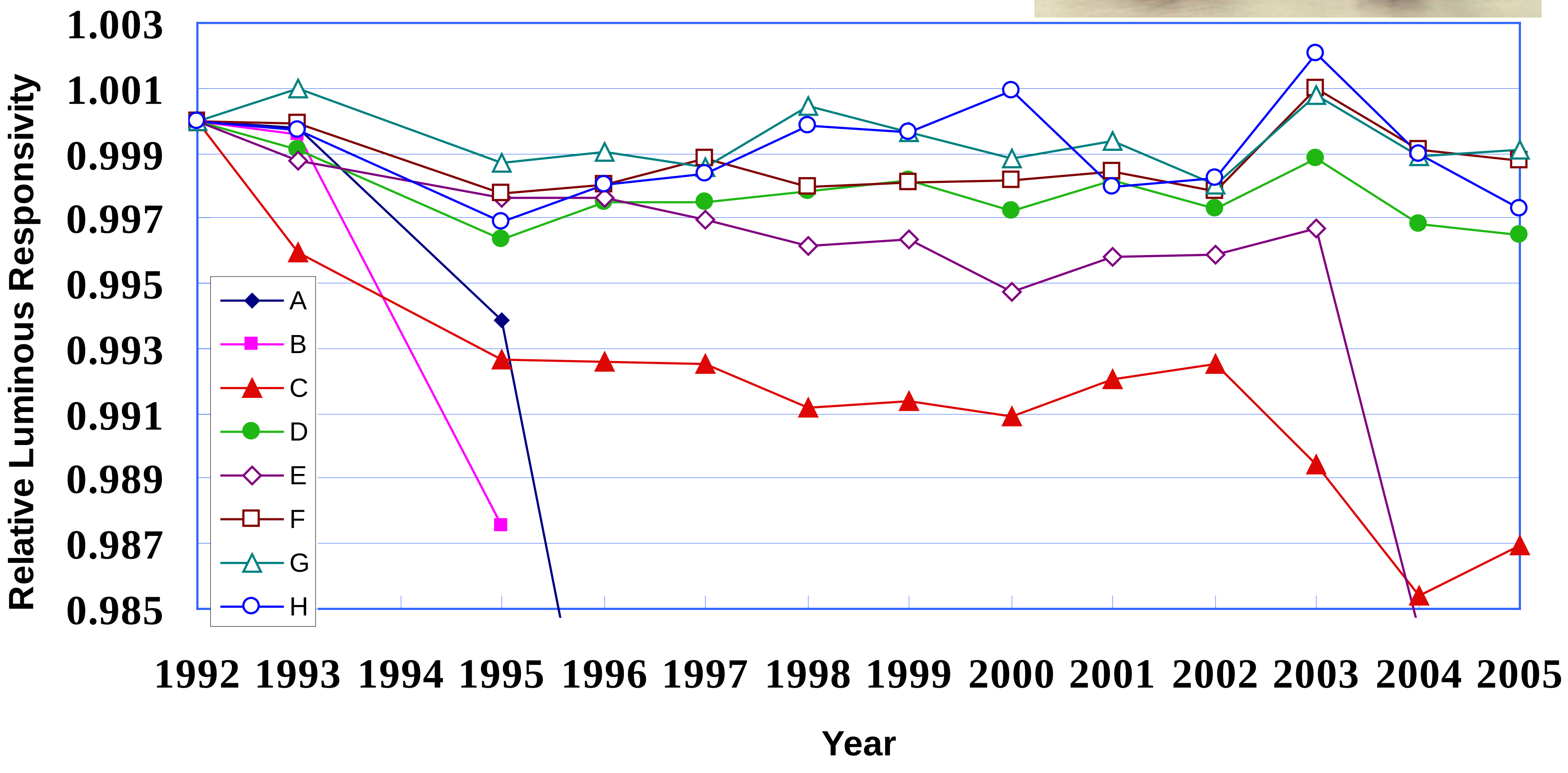
Surface Plot of Responsivity Relative to Center of Active Area

Linearity of the NIST standard photometers



Decade	Average
-10 – -9	1.00016
-9 – -8	1.00001
-8 – -7	0.99999
-7 – -6	1.00001
-6 – -5	0.99993
-5 – -4	1.00003

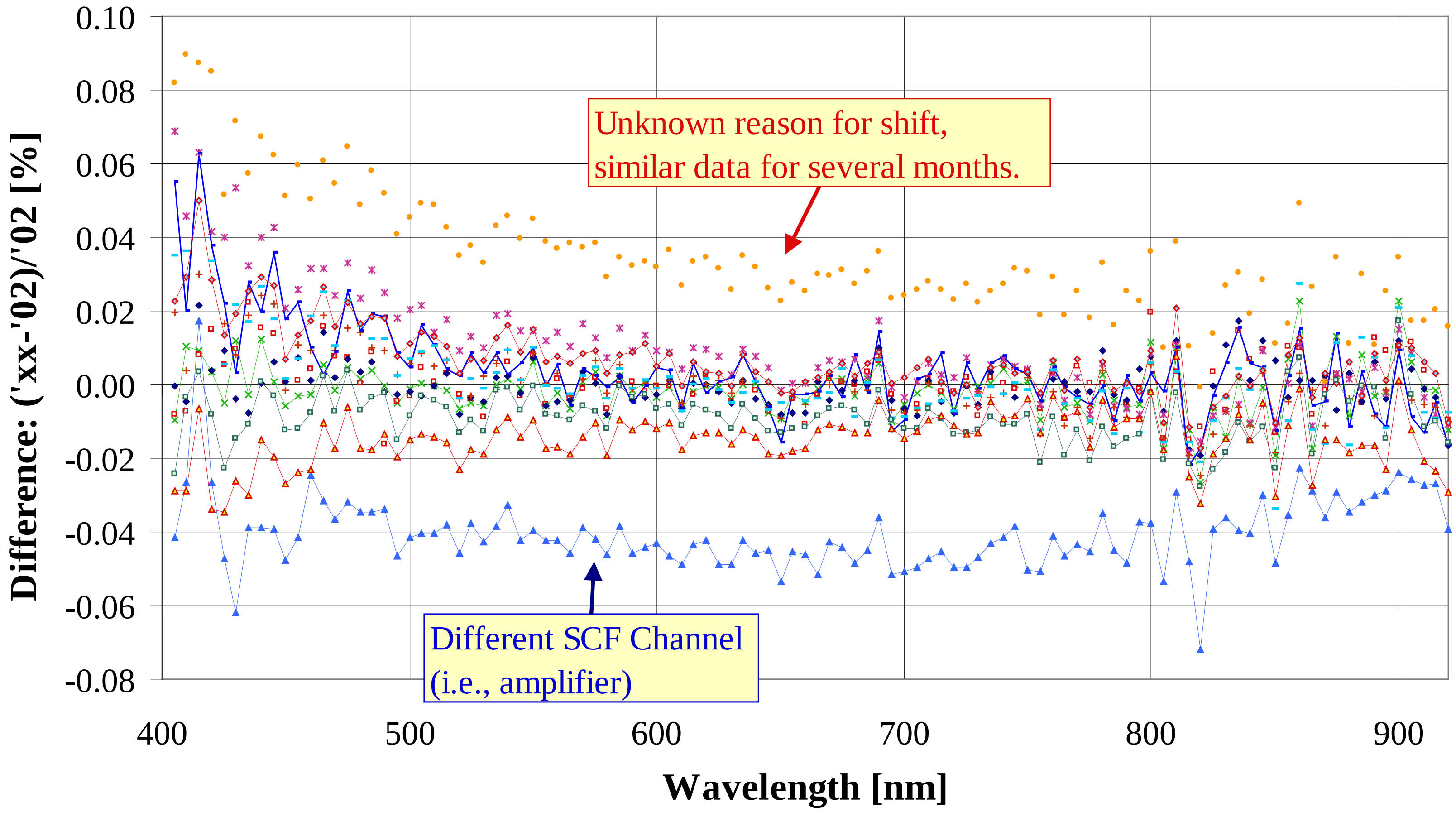
Stability of NIST Photometers



Stability of Silicon Photodiodes

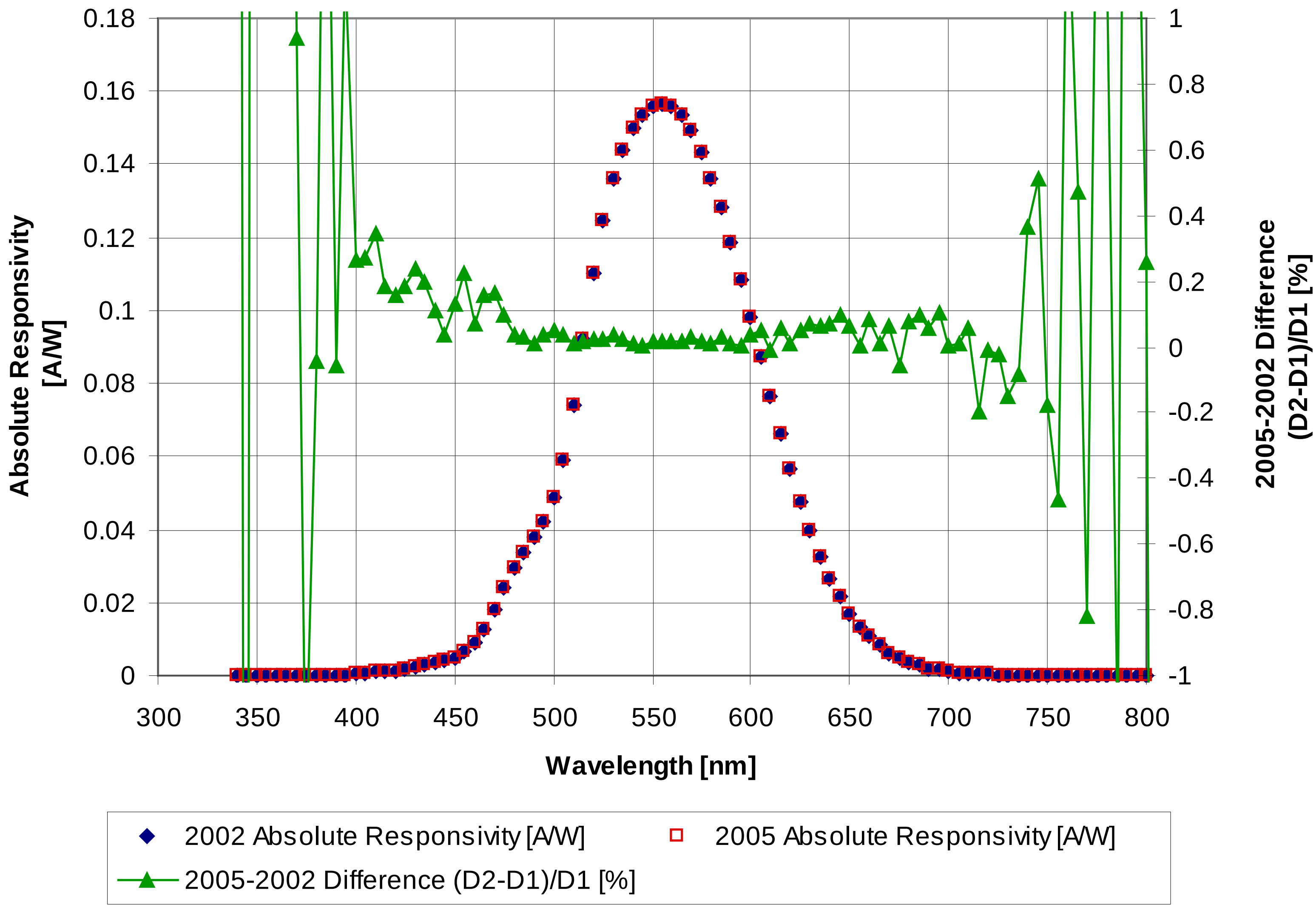
Scale Uncertainty (k=1): 0.1 %

Differences in the Vis SCF Silicon WS H626/H629 ratios from 1992 to 2002.



National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Stability of SRSC FR #117 with *V*(*λ*) Filter



Factors Contributing to Uncertainty
Detector Calibration – Lab #2

Examples of Sensitivity Coefficients

	Absolute	Relative
S_s :	$\frac{dS_x}{dS_s} = \left(\frac{V_x / V_{mx}}{V_s / V_{ms}} \cdot \frac{G_s}{G_x} \right)$	$\frac{dS_x}{S_x} = \frac{dS_s}{S_s}$
V_x :	$\frac{dS_x}{dV_x} = \left(\frac{1 / V_{mx}}{V_s / V_{ms}} \cdot \frac{G_s}{G_x} \right) \cdot S_s$	$\frac{dS_x}{S_x} = \frac{dV_x}{V_x}$
V_s :	$\frac{dS_x}{dV_x} = -\left(\frac{V_x / V_{mx}}{V_s^2 / V_{ms}} \cdot \frac{G_s}{G_x} \right) \cdot S_s$	$\frac{dS_x}{S_x} = \frac{-dV_s}{V_s}$
λ :	$\frac{dS_x}{d\lambda} = \frac{d(V_x / V_s)}{d\lambda} \cdot \frac{V_{ms}}{V_{mx}} \cdot \frac{G_s}{G_x} \cdot S_s$	$\frac{dS_x}{S_x} = \frac{d(V_x / V_s)}{d\lambda} \cdot \frac{d\lambda}{(V_x / V)_s}$

Combined Standard Uncertainty
Detector Calibration – Lab #2

Law of Propagation of Uncertainties

Using absolute uncertainties

$$u_c^2(S_x) = \left(\frac{dS_x}{dS_s}\right)^2 u^2(S_s) + \left(\frac{dS_x}{dV_x}\right)^2 u^2(V_x) + \left(\frac{dS_x}{dV_s}\right)^2 u^2(V_s) + \cdots + \left(\frac{dS_x}{d\lambda}\right)^2 u^2(\lambda)$$

Using relative uncertainties

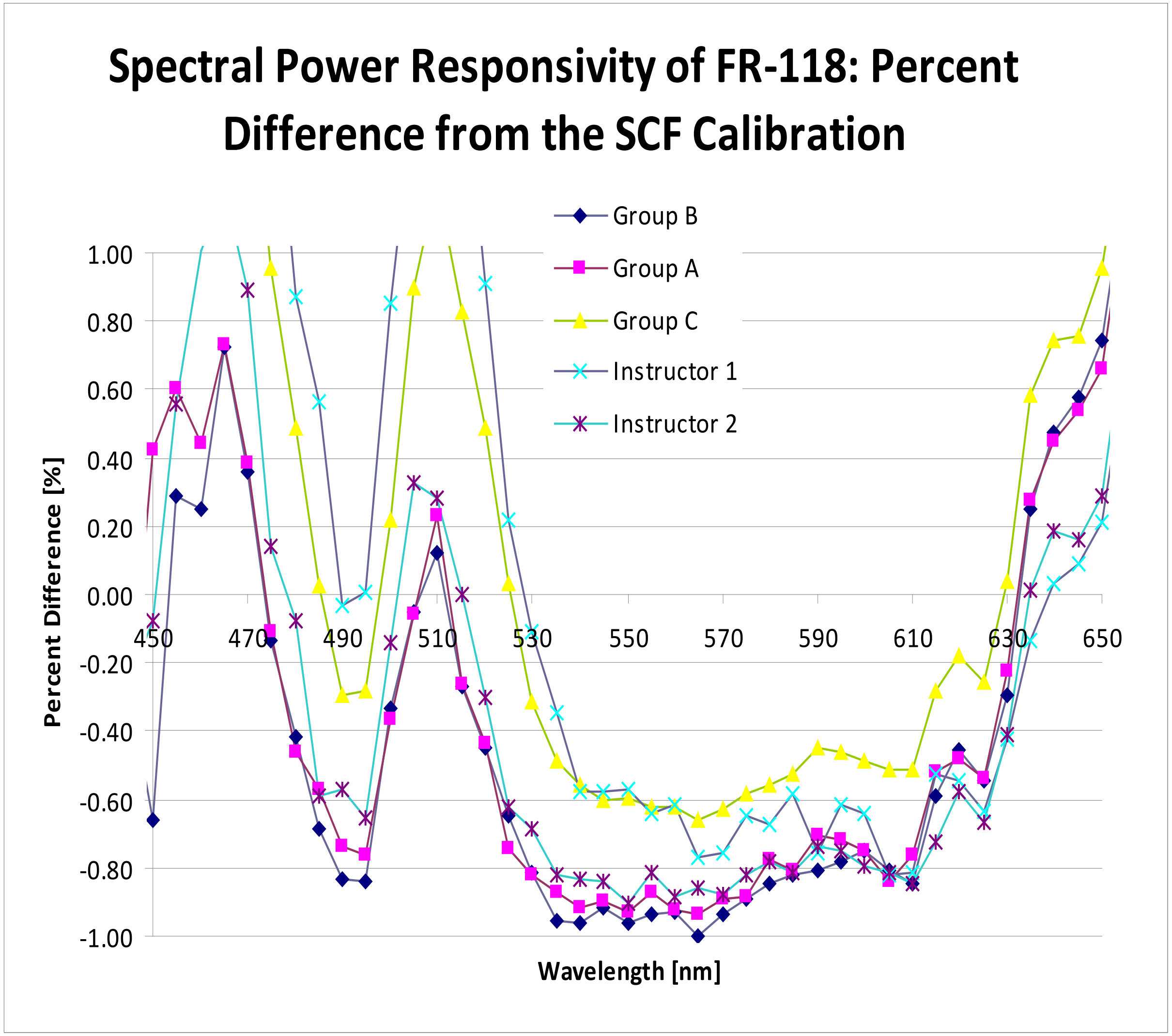
$$\left(\frac{u_c(S_x)}{S_x}\right)^2 = \left(\frac{u(S_s)}{S_s}\right)^2 + \left(\frac{u(V_x)}{V_x}\right)^2 + \left(\frac{u(V_s)}{V_s}\right)^2 + \cdots + \left(\frac{d(V_x/V_s)}{d\lambda} \cdot \frac{u(\lambda)}{(V_x/V_s)}\right)^2$$

Uncertainty Summary

Detector Calibration (at 550 nm) – Lab #2

Quantity (Symbol)	Unit	Value	Probability Distribution	Uncertainty limit	Standard uncertainty	Relative sensitivity	Relative uncertainty
Std. Resp. (S_s)	A/W	0.2848	Normal		0.0003	1/0.2848	0.11 %
Std. Signal (V_s)	V	2	Normal		0.002	1/2	0.10 %
Test Signal (V_x)	V	1.8	Normal		0.004	1/1.8	0.22 %
Monitor Signal (V_{ms})	V	1.1	Normal		0.001	1/1.1	0.09 %
Monitor Signal (V_{mx})	V	1.09	Normal		0.001	1/1.09	0.09 %
Std. Gain (G_s)	A/V	10^{-6}	Normal		10^{-10}	$1/10^{-6}$	0.01 %
Test Gain (G_x)	A/V	10^{-6}	Normal		10^{-10}	$1/10^{-6}$	0.01 %
Wavelength (λ)	nm	550	Rectangular	1	0.6	1/9	0.07 %
Test Resp. (S_x)	A/W	0.2587	Combined uncertainty in the responsivity				0.30 %
			Expanded uncertainty				0.60 %

Findings: Difference from NIST Calibration Values



The Uncertainties - Spectral Responsivity

Source of uncertainty	Type	Relative Standard Uncertainty [%]	
Absolute responsivity scale	B	0.100	sp-250-41 Value
Working Standard Diode Signal	A	0.014	from data
Test Detector (Photometer) Signal	A	0.020	from data
Monitor Diode Signal with WS Diode	A	0.040	from data
Monitor Diode Signal with Test Detector (Photometer)	A	0.040	from data
Working Standard Diode Amplifier Gain	B	0.120	Doubled SP-250 data
Test Detector (Photometer) Amplifier Gain	B	0.100	Doubled SP-250 data
Monochromator Wavelength Calibration	B	0.040	Estimated using .2nm Error
Relative combined standard uncertainty (RSS)	**[%]**	**0.20**	
Relative expanded uncertainty (*k* = 2)	**[%]**	**0.40**	

Goal 2: Calibration of the Illuminance meter using the Filter Radiometer

1. QTH source, close to illuminant A (2856 K)
2. Three detectors set up to 3 m from the source.
3. The filter radiometer was used to calibrate the two illuminance meters
4. A variation of baffling was used
5. Two different reference planes on the illuminance meter were used

The Uncertainties - Illuminance Responsivity

Source of uncertainty	Type	Relative Standard Uncertainty [%]	
Absolute responsivity scale	B	0.200	
Transfer of scale to photometer	A	0.080	Values from SP250-37
Wavelength calibration of monochromator	B	0.040	Estimated using .2nm Error
Numerical aperture of monochromator beam	B	0.100	Values from SP250-37
Area of photometer aperture	B	0.100	Values from SP250-37
Temperature variation	A	0.060	Values from SP250-37
Other factors	A	0.240	Values from SP250-37
Relative combined standard uncertainty (RSS)	**[%]**	**0.36**	
Relative expanded uncertainty (*k* = 2)	**[%]**	**0.72**	

The Uncertainties - Illuminance Meter

Source of uncertainty	Type	Relative Standard Uncertainty [%]	
Illuminance unit realization	B	0.360	
Long-term drift of the photometer	B	0.150	Values from SP250-37
Photometer temperature variation	A	0.030	Values from SP250-37
Spectral mismatch factor of photometer	B	0.020	Values from SP250-37
Illuminance meter alignment (distance and angle)	A	0.200	Estimate within 3mm error
Illuminance nonuniformity	B	0.050	Values from SP250-37
Lamp current regulation	A	0.040	Conservative estimates (no warmup stablization time)
Stray light in the “photometry bench”	B	0.100	Conservative estimates Double the SP250-37 value
Random noise (scatter by dust, lamp drift, etc.)	A	0.100	Values from SP250-37
Display resolution of the illuminance meter (1 in 199)	A	0.400	Conservative estimates
Inconsistency in responsivity between luminance levels	B	0.120	Values from SP250-37
Relative combined standard uncertainty (RSS)	**[%]**	**0.63**	
Relative expanded uncertainty (*k* = 2)	**[%]**	**1.26**	

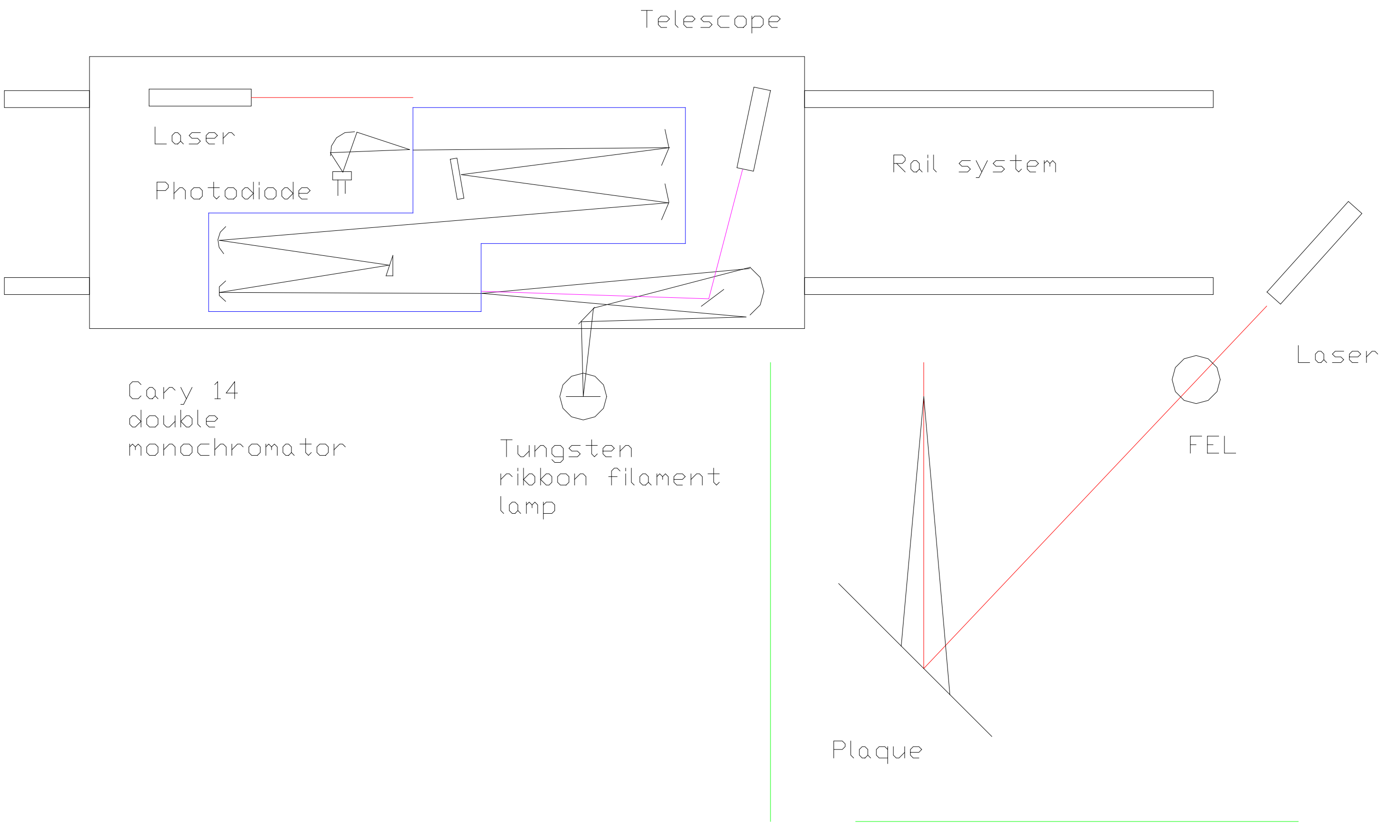
The Results of the Illuminance Meter Calibrations Day 3

Day 3: Group C					
Illuminance meter	FR-0118 Measured Value [Lx]	NIST Correction Factor	Photometer Based Value [Lx]	**NIST Calibrated Value [lx]**	Difference [%]
EOS	22.28	1.027	21.70	**22.29**	-0.03
Non EOS	22.28	1.147	19.30	**22.14**	0.64

6b. Applications from the NIST Short Course:

Spectral irradiance to spectral radiance transfer

Diagram of the lab setup



Calibration and Validation of spectral radiance

The spectral radiances of the FEL-plaque can be determined in two ways:

1. Using spectral irradiance to spectral radiance transfer knowing the 45°/0° reflectance factor of a plaque and the spectral irradiances of a standard source. (Calibration)
2. Using spectral radiance responsivities from the known spectral radiances of an argon-filled tungsten-strip lamp. (Validation)

Equations for spectral irradiance to radiance transfer

The irradiance at the plaque surface is given by

$$E_F(A,\omega,\lambda,t,d) = E(50\text{cm})\frac{(50+\delta)^2}{(d+\delta)^2} \qquad (1)$$

If the spectral irradiance is uniform then

$$L_{PE} = \frac{E(50\text{cm})}{\pi}\cdot\frac{(50+\delta)^2}{(d+\delta)^2}\cdot\rho(45/0,\lambda) \qquad (2)$$

Since the spectral radiance responsivities of the detector system did not change, then the spectral radiance of the plaque is also

$$L_{PR}(\lambda) = L_R(\lambda)\cdot\frac{G_R}{v_R}\cdot\frac{v_P}{G_P} \qquad (3)$$

Uncertainty Analysis – Experiment 3

Radiance of plaque L_P from two methods

1. Calibration from lamp irradiance and plaque reflectance factor

$$L_{PE} = \frac{E(50\text{cm})}{\pi} \cdot \frac{(50+\delta)^2}{(d+\delta)^2} \cdot \rho(45/0,\lambda)$$

2. Validation using radiance of strip lamp

$$L_{PR}(\lambda) = L_R(\lambda) \cdot \frac{G_R}{v_R} \cdot \frac{v_P}{G_P}$$

Where v_R = signal from strip lamp

v_P = signal from plaque

L_R = radiance of strip lamp

ρ = reflectance factor of plaque

E = irradiance of lamp

Uncertainties of Two Methods

1. Lamp irradiance transfer
   a. Lamp distance (spectrally flat)
   b. Lamp current (spectrally dependent)
   c. Plaque reflectance
   d. Plaque uniformity
   e. Signal to noise (depends on signal level (spectrally dependent))
   f. Wavelength (spectrally dependent)
2. Strip lamp radiance
   1. Strip lamp current (spectrally dependent)
   2. Signals
   3. Wavelength

Note: need to separate systematic and random effects since both lamps are traceable back to the same primary NIST standard

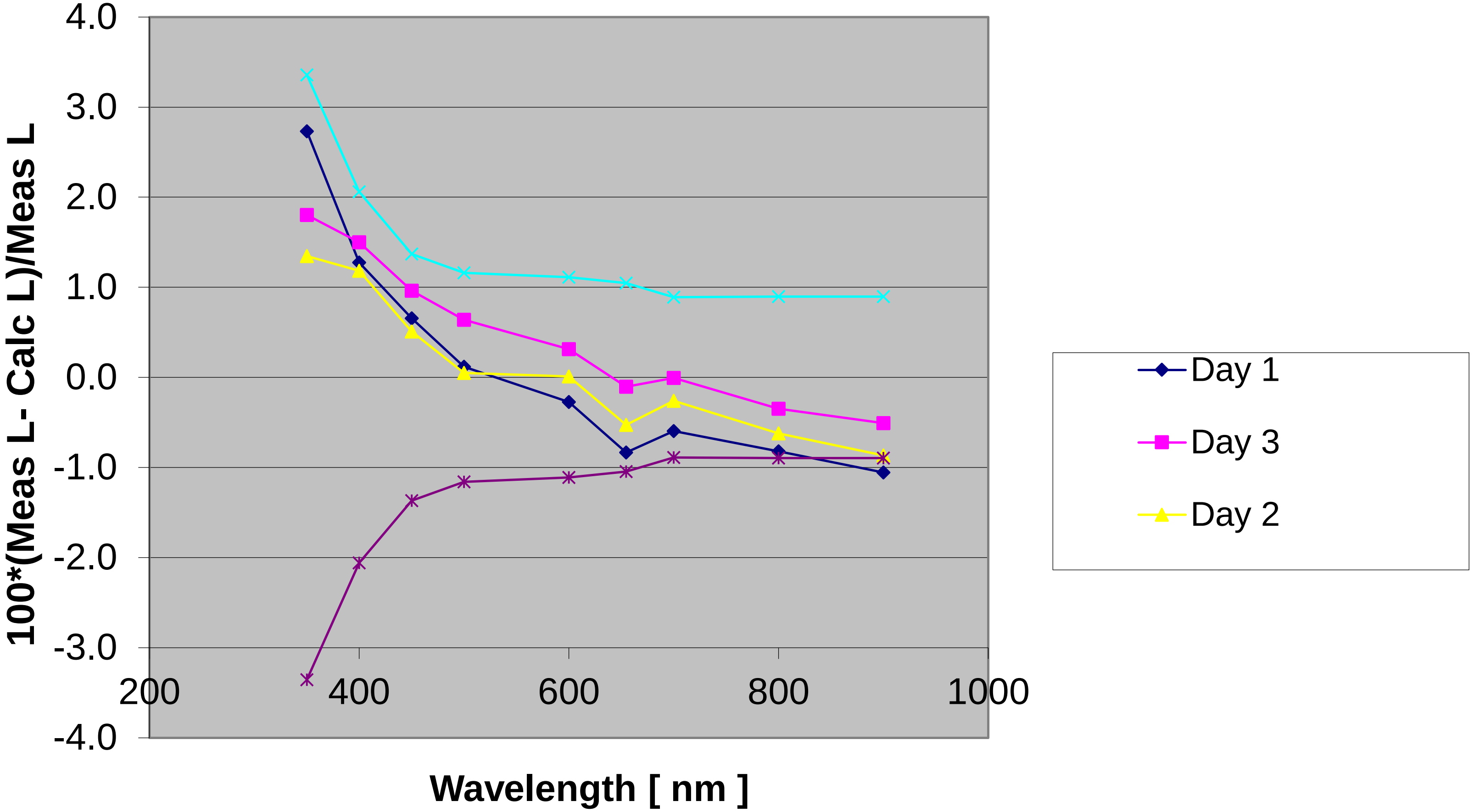
Uncertainties from the radiance lamp transfer

						Measured	(k=2)
Wavelength [nm]	RFL radiance	Net signal (RFL)	Gain(RFL)	Net signal (plaque)	Gain(plaque)	Plaque Radiance	total uncertainties
350	2972.07	0.00396	1.00E+06	0.00282	1.00E+09	2.12	4.58
400	9943.19	0.01714	1.00E+06	0.01021	1.00E+09	5.93	2.06
450	23405.37	0.04864	1.00E+06	0.02455	1.00E+09	11.82	1.37
500	43786.72	0.09614	1.00E+06	0.04264	1.00E+09	19.42	1.16
600	97489.44	0.20893	1.00E+06	0.07652	1.00E+09	35.70	1.11
654.575	129028.26	0.24759	1.00E+06	0.08397	1.00E+09	43.76	1.05
700	152685.80	0.25385	1.00E+06	0.08176	1.00E+09	49.18	0.89
800	191676.85	0.22930	1.00E+06	0.06750	1.00E+09	56.42	0.90
900	212010.44	0.28633	1.00E+06	0.07935	1.00E+09	58.75	0.90
1050	211363.32	0.24745	1.00E+06	0.06488	1.00E+09	55.42	0.90

Uncertainties of the FEL/Plaque transfer

Wavelength [nm]	FEL irradiance at 50 cm	FEL irradiance at d	Uncertainty of Spectral radiance	0/45 Reflectance Factor	Uncertainty of R factor	Spatial Uniformity	Plaque Radiance	Total uncertainty
350	9.024	6.711	0.1	0.985	0.6	0.5	2.104	0.79
400	24.443	18.176	0.1	1	0.43	0.5	5.786	0.67
450	49.024	36.454	0.1	1.008	0.4	0.5	11.697	0.65
500	80.519	59.875	0.1	1.01	0.35	0.5	19.249	0.62
600	149.318	111.034	0.1	1.013	0.35	0.5	35.803	0.62
654.575	182.219	135.499	0.1	1.013	0.35	0.5	43.691	0.62
700	204.738	152.245	0.1	1.013	0.35	0.5	49.091	0.62
800	236.210	175.647	0.1	1.013	0.33	0.5	56.637	0.61
900	244.919	182.124	0.1	1.014	0.32	0.5	58.783	0.60
1050	231.451	172.109	0.1	1.014	0.32	0.5	55.551	0.60

Comparison of the two different routes



References

1. ISO, *Guide to the Expression of Uncertainty in Measurement* (International Organization for Standardization, Geneva, Switzerland, 1993).
2. B. N. Taylor and C. E. Kuyatt, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297 (1994).
3. EAL, *Expression of the Uncertainty of Measurement in Calibration*, EAL-R2 (European Cooperation for Accreditation of Laboratories, 1997).
4. http://www.physics.nist.gov/cuu/Uncertainty/index.html
5. P.R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*,McGraw-Hill Publishing Co.

Conclusions:

1. A calibration-validation plan (SI-traceable) for a satellite sensor should
   a) Meet the calibration requirements
   b) Describe the calibration approach (calibration)
   c) Describe the use of on-board calibrators
   d) Describe the system-level end-to-end calibration performance (validation)
2. Elements of the plan should
   a) Answer how the calibration requirements will meet the mission and instrument requirements
   b) Develop a sensor design and radiometric model (measurement equation)
   c) Characterize subsystems (uncertainty analysis)
   d) Compare model predictions and validate system level calibrations
   e) Establish pre-launch radiometric uncertainties

From: NISTIR 7637 (2009), “Best Practice Guideline…” R. Datla et al.