Calcon 2013 Tutorial 1: Basics and Applications of Spectroradiometry

Howard W. Yoon
Sensor Science Division, NIST
Motivations

1. Develop a calibration plan (SI-traceable) for a satellite sensor
   a) Calibration requirements
   b) Calibration approach
   c) Use of on-board calibrators
   d) Perform system-level end-to-end calibration (validation)

2. Elements of the plan should
   a) Answer how the calibration requirements will meet the mission and instrument requirements
   b) Develop a sensor design and radiometric model (measurement equation)
   c) Characterize subsystems (uncertainty analysis)
   d) Compare model predictions and validate system level calibrations
   e) Establish pre-launch radiometric uncertainties

Outline of the Tutorial

1. Basics of Radiometry
2. Detector-based Radiometry
3. Source-based Radiometry
4. Tools of Spectroradiometry
   a) Detectors
   b) Filter Radiometers
   c) Spectroradiometers
5. Measurement Equation and Uncertainty Analysis
6. Applications from the NIST Short Course
Possible Sources of Error

<table>
<thead>
<tr>
<th>Possible Source of Error</th>
<th>Max. Potential Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Spectral scattering</td>
<td>&gt; 100 %</td>
</tr>
<tr>
<td>2. Spectral distortion</td>
<td>&gt; 100 %</td>
</tr>
<tr>
<td>3. Nonlinearity</td>
<td>20%</td>
</tr>
<tr>
<td>4. Directional and positional effects</td>
<td>20%</td>
</tr>
<tr>
<td>5. Polarization effects</td>
<td>5%</td>
</tr>
<tr>
<td>6. Size-of-source effect (radiance)</td>
<td>5%</td>
</tr>
<tr>
<td>7. Wavelength instability</td>
<td>100 % / nm</td>
</tr>
<tr>
<td>8. Detector instability</td>
<td>10%</td>
</tr>
<tr>
<td>9. Uncertainty of the standard</td>
<td></td>
</tr>
<tr>
<td>10. Instability of the standard</td>
<td></td>
</tr>
<tr>
<td>11. Instability of the quantity being-measured</td>
<td></td>
</tr>
<tr>
<td>12. Noise in the measurement data</td>
<td>1 % to 5 %</td>
</tr>
</tbody>
</table>

*(Reliable Radiometry, p. 422)*
1. Basics of Radiometry
Basics of Spectroradiometry: Outline

1. Definitions of radiometric terms
   a) Flux, radiant intensity, irradiance, radiance
   b) Property modifiers
   c) The concept of solid angle

2. Radiometry basics
   a) Point source
   b) Extended source
   c) Flux transfer and throughput

3. Applications
   a) Calibration of radiometers
   b) Some numerical examples
## Electromagnetic Radiation

<table>
<thead>
<tr>
<th>Name</th>
<th>Wavelength ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>UV-C</td>
<td>100 nm to 280 nm</td>
</tr>
<tr>
<td>UV-B</td>
<td>280 nm to 315 nm</td>
</tr>
<tr>
<td>UV-A</td>
<td>315 nm to 400 nm</td>
</tr>
<tr>
<td>VIS</td>
<td>360 nm to 800 nm</td>
</tr>
<tr>
<td>NIR</td>
<td>800 nm to 1400 nm</td>
</tr>
<tr>
<td>SWIR</td>
<td>1.4 µm to 3 µm</td>
</tr>
<tr>
<td>MWIR</td>
<td>3 µm to 5 µm</td>
</tr>
</tbody>
</table>

\[ c = n \lambda \nu \]

- **c** = speed of light
- **\( \lambda \)** = wavelength
- **n** = index of refraction
- **\( \nu \)** = frequency

The wavelength is determined by the speed of light and measurements of the frequency by comparison to the atomic standards.

For example: \( \lambda = 555 \) nm, then \( \nu = 540 \times 10^{12} \) Hz.
Property modifiers

1. **Spectral:** having a dependence on wavelength; within a very narrow region of wavelength
2. **Total:** integrated or summed over all wavelengths
3. **Exitent:** leaving a surface
4. **Incident:** arriving at a surface
5. **Directional:** having a dependence on direction; within a very small solid angle
6. **Hemispherical:** averaged over all solid angles passing through a hemisphere centered over the surface element
Radiant quantities

1) Radiant energy, $Q$ [ J ]
   E.g. deposited laser energy at 650 nm 5.3 mJ (0.25 s)

2) Radiant flux, $\Phi$ [ W ], [ J/s ]
   E.g. laser power meters, cryogenic radiometer (beam underfills the detector)

3) Radiant intensity, $I$ [ W/sr ]
   E.g. point sources such as stars observed at great distances
Radiant quantities, continued

4) Irradiance, $E$ [W/m²]  
   E.g. solar terrestrial irradiance

5) Radiant exitance, $M$ [W/m²]  
   E.g. blackbody, the Stefan-Boltzmann law

6) Radiance, $L$ [W/(m² sr)]  
   E.g. real sources (blackbodies, surface of the earth, moon, and other objects)
### Comparison of Radiometric and Photometric Units

<table>
<thead>
<tr>
<th>Radiometric</th>
<th>Symbol</th>
<th>Unit</th>
<th>Unit</th>
<th>Symbol</th>
<th>Photometric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiant Energy</td>
<td>$Q$</td>
<td>J</td>
<td>lm s</td>
<td>$Q_V$</td>
<td>Luminous energy</td>
</tr>
<tr>
<td>Radiant flux (power)</td>
<td>$\Phi$</td>
<td>W</td>
<td>lm</td>
<td>$\Phi_V$</td>
<td>Luminous flux</td>
</tr>
<tr>
<td>Irradiance</td>
<td>$E$</td>
<td>W/m$^2$</td>
<td>lm/m$^2$ = lx</td>
<td>$E_V$</td>
<td>Illuminance</td>
</tr>
<tr>
<td>Radiance</td>
<td>$L$</td>
<td>W/(m$^2$ sr)</td>
<td>lm/(m$^2$ sr)</td>
<td>$L_V$</td>
<td>Luminance</td>
</tr>
<tr>
<td>Radiant exitance</td>
<td>$M$</td>
<td>W/m$^2$</td>
<td>lm/m$^2$</td>
<td>$M_V$</td>
<td>Luminous exitance</td>
</tr>
<tr>
<td>Radiant intensity</td>
<td>$I$</td>
<td>W/sr</td>
<td>lm/sr=cd</td>
<td>$I_V$</td>
<td>Luminous intensity</td>
</tr>
<tr>
<td>Radiance Temperature</td>
<td>$T$</td>
<td>K</td>
<td>K</td>
<td>$T$</td>
<td>Color Temperature</td>
</tr>
</tbody>
</table>

**Thermodynamic (equivalence of heat and radiant flux)**

**Specialized (human visual response for detector model)**
Spectral modifier—“spectroradiometry”

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral Radiant Energy</td>
<td>$Q_\lambda$</td>
<td>J/nm</td>
</tr>
<tr>
<td>Spectral Radiant flux (power)</td>
<td>$\Phi_\lambda$</td>
<td>W/nm</td>
</tr>
<tr>
<td>Spectral Irradiance</td>
<td>$E_\lambda$</td>
<td>W/m²/nm</td>
</tr>
<tr>
<td>Spectral Radiance</td>
<td>$L_\lambda$</td>
<td>W/(m² sr)/nm</td>
</tr>
<tr>
<td>Spectral Exitance</td>
<td>$M_\lambda$</td>
<td>W/m²/nm</td>
</tr>
</tbody>
</table>
**Plane and solid angles**

- $r$: Radius of circle
- $dl$: Arc length along circumference
- $d\alpha$: Plane angle subtended by arc at center

\[ d\alpha = \frac{dl}{r} \quad \text{Unit: radian (rad)} \]

- $r$: Radius of sphere
- $dA$: Area of sphere segment
- $d\omega$: Solid angle subtended by area at center

\[ d\omega = \frac{dA}{r^2} \quad \text{Unit: steradian (sr)} \]
Solid angle: perpendicular surface

Point Source and Circular Aperture

\[ \omega \approx \frac{A_2}{d^2} \]

Note: Distance \( d \) is large compared to aperture dimensions so that \( d^2 >> A_2 \).
Solid angle: tilted surface

Point Source and Circular Aperture

Point source [1]  

Aperture Area \([A_2]\)

\[d\]

\[60^\circ\]

\[\omega\]

\[A_2\] Area of the aperture
\[d\] Distance between the aperture and the source
\[\omega\] Solid angle subtended by \([A_2]\) at the source

\[\omega \approx \frac{A_2 \cos \theta}{d^2} = \frac{A_2 \cos 60^\circ}{d^2} = \frac{A_2}{d^2 \cdot 2}\]

Note: Distance \(d\) is large compared to aperture dimensions so that \(d^2 \gg A_2\).
Radiometry of point sources (Irradiance, \(E\))

Irradiance at \(d_1\) for area \(A_1\):
\[E_1 = \frac{\Phi}{A_1}\]

What about at \(d_2\)?

\[E_2 = \frac{\Phi}{A_2}\] (flux is the same)

The solid angle is also constant (by geometry):
\[\omega = \frac{A_1}{d_1^2} = \frac{A_2}{d_2^2}\]

Why is the flux is the same?

\[I = \text{intensity} = \frac{\Phi}{\omega}\]

For a point source, \(I\) is independent of direction (isotropic).

The irradiance from an ideal point source falls off as \(1/d^2\). How well must the distance be measured?
Irradiance distribution, point source on a plane

Irradiance at \( r_1 \) for different locations on a plane: what is \( E_2/E_1 \)?

Point 1 is “on-axis”
At Point 2, the distance is greater by \( r_2 = r_1/\cos \theta \) and the area is tilted by \( \theta \).

The “off-axis” irradiance from an ideal point source in a plane falls off as \( \cos^3 \theta \). Keep angles small if uniformity is critical!

\[
E_2 = E_1 \left[ \frac{A}{A/\cos \theta} \right] \cos^2 \theta
\]

\[
E_2 = E_1 \cos^3 \theta
\]
Radiometry of extended sources (Radiance, $L$)

Real Sources
1) have finite size and 2) the flux depends on view direction, target size, location on source, and solid angle.

Radiance:

$$L = \frac{\Phi}{A \cos \theta \omega}$$

$\theta$ = angle between view direction and surface normal, $\omega$ = solid angle, $A$ = source area
Lambert’s law

Radiance from one of the point sources:

Viewed perpendicular

\[ L(0) = \frac{\Phi}{\Delta A \omega} \]

Viewed tilted

\[ L(\theta) = \frac{\Phi}{\Delta A \cos \theta \omega} \]

We conclude

\[ L(\theta) = \frac{L(0)}{\cos \theta} \]

But real sources don’t increase in brightness when viewed off-axis, most in fact remain constant or become dimmer

Lambert’s Law: \( L(\theta) = L(0) \)

Lambertian or “diffuse” source

Model not a collection of ideal point sources \((I = \Phi/\omega)\), but a collection of pseudo point sources \((I = \Phi \cos \theta/\omega)\)

Trial model: A collection of point sources, uniformly spaced across the source area

\[ \Delta A \]
Irradiance from an extended source (on-axis)

The intensity is $I = L \Delta A$, where $\Delta A$ is the “area” of each point source.

The irradiance depends only on $L$ and the solid angle of the source from the point on the plane.

Irradiance at the plane from each point source (for $d \gg a$) is $I/d^2$.

Extended source as a collection of point sources

$$\omega = \frac{A}{d^2}$$

Sum over all point sources

$$E = \frac{L \sum \Delta A}{d^2} = \frac{LA}{d^2} = L \omega$$
Irradiance from an extended source (off-axis)

Source

\[ \omega_1 = \frac{A}{d^2} \]

\[ E = L \omega \]

$L$ is the same, but the solid angle is smaller:

\[ \omega_2 = \frac{A \cos \theta}{d^2 \cos^2 \theta} = \omega_1 \cos^3 \theta \]

As before, we have to remember the area on the plane is tilted by $\theta$:

\[ E_2 = E_1 \left[ \cos^3 \theta \right] \left[ \frac{A}{A/\cos \theta} \right] = E_1 \cos^4 \theta \]

The irradiance distribution in a plane from an extended Lambertian source drops faster than from a point source ($\propto \cos^3 \theta$). Known as the “cosine fourth law”
Angular $\cos^4$ output of the NIST 308 mm diameter integrating sphere

![Graph showing angular cosine to the fourth power output of the NIST 308 mm diameter integrating sphere. The x-axis represents the angle $\delta$ in radians, and the y-axis represents the irradiance ratio $E_{\delta}/E_0$. The graph includes a dashed line representing $\cos^4 \delta$ and red circles representing measured data. A box indicates a 50 mm diameter at 500 mm.]
Spatial scan of the 308 mm sphere irradiance
Invariance of radiance

Flux in beam, $\Phi$

From before, 
$E = L_1 \omega_1$
$\Phi = E A_2$
$L_1 = \frac{E}{\omega_1} = \frac{\Phi}{A_2 \omega_1}$

It also must be true (from the definition of radiance)
$L_2 = \frac{\Phi}{A_2 \omega_1} = L_1$

Invariance of Radiance
$L_1 = L_2$

Note we could also have said
$L_1 = \frac{\Phi}{A_1 \omega_2}$

Throughput
$A \omega = A_1 \omega_{1-2} = A_2 \omega_{2-1}$
$\left( \omega_{1-2} = \frac{A_2}{d^2}, \quad \omega_{2-1} = \frac{A_1}{d^2} \right)$
Radiant flux transfer, arbitrary orientation

\[ \omega_{1-2} = \text{Solid angle subtended by } A_2 \text{ with respect to } A_1 \quad \omega_{1-2} = \frac{A_2 \cdot \cos \theta_2}{d^2} \]

\[ \Phi = \text{Radiant flux in the beam} \]

\[ = [\text{Radiance} \cdot \text{Projected area}]_1 \cdot [\text{Solid angle}]_{1-2} \]

\[ = [L_1 \cdot A_1 \cdot \cos \theta_1] \cdot [\omega_{1-2}] \]

\[ = [L_1 \cdot A_1 \cdot \cos \theta_1] \cdot \left[ \frac{A_2 \cdot \cos \theta_2}{d^2} \right] \]

For real surfaces, divide the two areas into many sub-areas and carry out two dimensional integration.
A review so far

1. Irradiance from a source on a plane
   a) Point source: \( E(0) = \frac{I}{d^2} \) and \( E(\theta) = E(0) \cos^3 \theta \)
   b) Extended source: \( E(0) = L\omega \) and \( E(\theta) = E(0) \cos^4 \theta \)

2. Radiance and extended sources
   a) Generally, \( L(\theta) = L(0) \) [Lambertian]
   b) Invariance of radiance: \( L_1 = L_2 \) (no absorption or scatter)
   c) Throughput = \( A_1 \cos \theta_1 [A_2 \cos \theta_2]/d^2 \) for large distances

3. Flux transfer (the detector responds to flux)
   a) \( \Phi = L \times \) throughput
   b) \( \Phi = E \times \) “detector” area

4. Look at the units
Irradiance at a plane, radiance to the hemisphere

The plane reflects in a “diffuse” manner—so the radiance is the same in all directions (Lambertian)

Now we must divide into small areas and integrate. For the spherical element,

\[ dA_{sp} = r^2 \sin \theta \, d\theta \, d\phi \]

Then, for each flux element,

\[ d\Phi = L \, dA \cos \theta \frac{r^2 \sin \theta \, d\theta \, d\phi}{r^2} \]

Integrate over the hemisphere

\[ \frac{\Phi}{dA} = E = L \int_{0}^{2\pi} \int_{0}^{\pi/2} \cos \theta \sin \theta \, d\theta \, d\phi \]

\[ E = L \, 2\pi \frac{1}{2} \left( \sin^2 \theta \right)_{0}^{\pi/2} = \pi \, L \]
“Lamp-Plaque Method”

Classic example of irradiance to radiance transfer, for producing a source of known radiance for instrument calibration.

\[ L = \frac{E \rho}{\pi} \]
What matters?

1. Lamp-Plaque method
   a) Distance: \( E \propto 1/d^2 \). The relative uncertainty is then \( \Delta E/E = 2\Delta d/d \). Standard lamps (“FELs”) are calibrated at 500 mm; so a 1 mm uncertainty is 0.4% in irradiance.
   b) Correct lamp current and proper baffle placement.

2. Calibration methods
   a) A irradiance detector must have its field of view “underfilled” by the source; distance matters
   b) A radiance detector must have its field of view “overfilled” by the source; distance does not matter
Irradiance mode (see $E$ from extended source)

\[ \Phi = L \cdot \Theta \]

\[ \Theta = \pi A_1 F_{D1-D2} \]

\[ F_{D1-D2} = \frac{1}{2} \left( \frac{(r_2^2 + r_1^2 + d^2) - \left( \sqrt{r_2^2 + r_1^2 + d^2} \right)^2 - 4r_1^2r_2^2}{r_1^2} \right) \]

See: Siegel and Howell
Irradiance mode, continued

Proper calibration
(See Lab. 1)

Incorrect calibration
of irradiance detector (too close)

\[ E = L \omega \]
\[ \omega = \frac{A_s}{d^2} \]
and signal \( S \propto \Phi = E A_d \)

Note \( \omega \) is less than the solid angle for the radiometer field of view

The \( \omega \) we would calculate from \( A_s \) would be greater than the limit of the radiometer
Radiance mode (see invariance of radiance)

Object (target area) $A_s$

Field of view

Lens, $f$

Ray traces

$2f$

Imaging Radiometer

$A_d$

Telescope focused at infinity

One-half the field of view

$A_d$

Lens, $f$
Integrating sphere examples

Calibration of sun photometers using an integrating sphere source (NASA GSFC).

An up-looking sphere source for nadir-viewing, aircraft-deployed spectroradiometers (NASA Ames).
Radiance mode (see invariance of radiance)

Either method is valid because of the invariance of radiance: 
\[ L_{\text{ISS}} = L_{\text{obj}} \]

1:1 imaging radiometer focused on the exit aperture

1:1 imaging radiometer focused in front of the exit aperture

Integrating sphere source

Exit aperture: Lambertian and uniform radiance
Example: Solar constant (irradiance at Earth’s surface)

Sun:
Blackbody at $T = 5800$ K;
Lambertian;
diameter = $6.96 \times 10^8$ m; Earth-sun distance $d = 1.5 \times 10^{11}$ m

Earth:
Total irradiance (all wavelengths) is of interest for Earth’s energy balance and solar physics research.
diameter = $6.38 \times 10^6$ m
Spectral aspects of radiometry

A blackbody source obeys Planck’s law

\[ L_\lambda = \frac{c_{1L}}{\lambda^5 \exp[c_2/(\lambda \cdot T) - 1]} \]

[\mu W / cm^2 nm sr ]

The radiance drops very sharply below a particular wavelength. As the temperature increases, the radiance increases for all wavelengths and the peak moves to shorter wavelength (\( \lambda_{\text{max}} \propto 1/T \)).
Total exitance, $M$

Integrate Planck’s radiance law over all wavelengths and the entire hemisphere above the exit aperture.

$$M(T) = \sigma \cdot T^4 \ [W/m^2]$$

The Stefan-Boltzmann relationship is useful when the detector responds over a wide range of wavelength with a nearly constant responsivity.

$\sigma = 5.67 \times 10^{-8} \ W \ m^{-2} \ K^{-4}$
Example: Solar constant (irradiance at Earth’s surface)

**Sun:**
Blackbody at $T = 5800$ K;
Lambertian;
diameter $= 6.96 \times 10^8$ m; Earth-sun distance $d = 1.5 \times 10^{11}$ m

**Earth:**
Total irradiance (all wavelengths) is of interest for Earth’s energy balance and solar physics research.
diameter $= 6.38 \times 10^6$ m

\[
M = \sigma T^4
\]
\[
L_s = \frac{M}{\pi}
\]
\[
E_e = L_s \omega_{e-s}
\]
\[
= \frac{\sigma T^4}{\pi} \frac{A_s}{d^2}
\]

We solved this problem using the point to hemisphere throughput derivation (Slide 22) and the irradiance on a plane from an extended source (Slide 16). The answer is $E_e = 1389$ W m$^{-2}$. We must assume sun is Lambertian.
Measurements of the solar constant

Exoatmospheric measurements using electrical substitution radiometers (ESRs)

http://spot.colorado.edu/~koppg/TSI

Latest instrument: TIM on SORCE http://lasp.colorado.edu/sorce/

Launched January 25, 2003
Measurement Equation Approach:

In general, we use the measurement equation approach for characterizing and calibrating sources and radiometers. A simplified measurement equation is:

\[ I(A, \omega, \Delta \lambda, \lambda_o) = \int \int_{\Delta \lambda} \int_{A} \int_{\omega} S_\phi(x, y, \theta, \phi, \lambda, \lambda_o) \cdot L_\lambda(x, y, \theta, \phi, \lambda, \lambda_o) \cdot \cos \theta \cdot d\omega \cdot dA \cdot d\lambda \]

- \( I(A, \omega, \Delta \lambda, \lambda_o) \) - the measured current
- \( S_\phi \) - the spectral flux (power) responsivity of the detector at a position \( x, y \)
- \( L_\lambda \) - the spectral radiance of the source
- \( A \) - receiving area of the detector
- \( \omega \) - the solid angle of the source viewed by the detector

Linearity, polarization dependences not considered in this expression but can be added.
References:


Kostkowski, H. J., Reliable Spectroradiometry, Spectroradiometry Consulting, La Plata, MD 1997, Chapter 1.


2. Detector-based Radiometry
Outline

1. What is Detector-based Radiometry

2. Detector-based Scale Realizations
   a) Electrical Substitution Radiometers (ESR)
      • Cryogenic Radiometers
   b) Spectral Responsivity Measurement Facilities
      • Power, irradiance, and radiance responsivity
   c) Scale Transfer to Measurement Facilities

3. Application Example (SRSC Laboratory #2)
   • Photometry
     Illuminance [lux]
What Is Detector-based Radiometry?

- Radiometric measurements using detectors whose calibration is traceable to a detector (primary) standard

Comparison of source and detector-based scales

**Source-based**

- $L$, radiance $[W/(m^2 \cdot sr)]$
  (Blackbody: Planck’s law)

- $E$, irradiance $[W/m^2]$

- $\Phi$, power $[W]$

**Detector-based**

- $\Phi$, power $[W]$
  (ESR: Optical W = Electrical W)

- $A$, Aperture Area $[m^2]$

- $E$, irradiance $[W/m^2]$

- $L$, radiance $[W/(m^2 \cdot sr)]$
Fundamental Radiometric Scales

Electrical Substitution Radiometry [W] (0.05 %)

Schwinger Equation (0.5 %)

Planck Radiance (0.25 %)

\[
P(E, R, \theta, \lambda) = \frac{4e^2 c R}{3 \lambda^4} \gamma^{-4} (1 + \chi^2)^2 \left[ K_{2/3}(\xi) + \frac{\chi^2}{1 + \chi^2} K_{1/3}(\xi) \right]
\]

\[
L_\lambda = \frac{c_1}{n^2 \lambda^5} \left( e^{\frac{c_2}{n \lambda T}} - 1 \right)
\]
Definition of Traceability

"property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties."
International System of Units (SI)

1. Established in 1960, SI is the modern metric system of measurement used throughout the world.

2. SI defines three classes of units: basic, derived and supplementary. Examples
   a) Basic: Thermodynamic temperature
      kelvin [K]
   b) Derived: Area, square meter [m²]
      Steradian [sr]
   c) Supplementary: Power, watt [W]
Radiometric Quantities (Review)

Radiometric quantities with their symbols and SI units

\[ W = \text{watt}, \ m = \text{meter}, \ sr = \text{steradian} \]

<table>
<thead>
<tr>
<th>Radiometric quantity</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiant flux (power)</td>
<td>( P, \ \Phi )</td>
<td>( W )</td>
</tr>
<tr>
<td>Irradiance</td>
<td>( E )</td>
<td>( W/m^2 )</td>
</tr>
<tr>
<td>Radiance</td>
<td>( L )</td>
<td>( W/(m^2 \cdot sr) )</td>
</tr>
<tr>
<td>Radiant intensity</td>
<td>( I )</td>
<td>( W/sr )</td>
</tr>
</tbody>
</table>
Electrical Substitution Radiometer (ESR)

The principle of electrical substitution radiometry is to balance the electrical and optical power [Watt] needed to create the same temperature rise in the ESR.


[Diagram showing the balance between optical and electrical power with a laser, shutter, resistive heater, and ESR cavity.]
NIST Cryogenic Radiometer

Cryogenic temperatures allow lower degree of non-equivalence:
1. Larger cavity due to increased heat capacity
2. Reduced lead heating due to superconducting leads
3. Reduced temperature gradients between electrical and optical heating
4. Reduced background radiation

---

[Image: Diagram of NIST Cryogenic Radiometer with labeled parts such as Liquid Helium Reservoir, Germanium Resistance Thermometer, 50 K Radiation Shield, 77 K Radiation Shield, Radiation Trap (4.2 K), 5 K Reference Block, Thin Film Heater 10 K, Absorbing Cavity (specular black paint), Alignment Photodiodes, Pumping Port, Brewster Angled Window, Laser Beam, 0 100 mm scale.]
NIST Cryogenic Radiometer

Primary Optical Watt Radiometer (POWR) is the U.S. primary standard for optical power.

Cryogenic temperatures allow lower degree of non-equivalence:

- Larger cavity due to increased heat capacity
- Reduced lead heating due to superconducting leads
- Reduced temperature gradients between electrical and optical heating
- Reduced background radiation
Primary Optical Watt Radiometer (POWR)

1. Shorter calibration chain
2. Greater power level dynamic range (µW to mW)
3. Continuous spectral coverage (200 nm to 20 µm)
4. Extend IR and UV coverage
5. Windowless transfer, decreasing transfer uncertainties
6. Explore irradiance measurements
7. Modular design allows for modifications to meet future requirements
Transfer to Measurement Facilities

Block diagram of POWR to SCF and SIRCUS

Primary Standard

Transfer Standards

Working Standards

POWR

Trap Detectors

Aperture Area

SCF (Power)

SIRCUS (Power, Irradiance and Radiance)

Uncertainty ($k=1$)

0.01

0.03

0.1

0.05
Measurement Facilities for Spectral Responsivity

Two principal detector measurement facilities:

1. Spectral Comparator Facilities (SCF)
   a) Monochromator based
   b) UV SCF: 200 nm to 500 nm
   c) Visible to Near IR SCF: 350 nm to 1800 nm

2. Spectral Irradiance and Radiance Calibrations using Uniform Sources Facility (SIRCUS)
   a) Tunable laser based
   b) 210 nm to 1800 nm (UV-Vis-NIR SIRCUS)
   c) 1000 nm to 5000 nm (IR SIRCUS)
   d) Various source configurations tailored to the measurement (typically an integrating sphere)
When to Use Trap Detectors

Low uncertainty transfer standard from HACR

1. Advantages
   a) Uniform responsivity
   b) Polarization insensitive
   c) Reflection measurements not needed

2. Drawbacks
   a) Limited field-of-view (FOV)
   b) “Impossible” to buy
   c) Hard to make
   d) Windowless diodes, potentially unstable
   e) Lower shunt resistance (diodes in parallel) limits gain to less than with a single photodiode
Trap Detector Examples and Uniformities

S2281 Diode at 500 nm
Contour Lines = 0.2 %

QED-200 at 500 nm
Contour Lines = 0.2 %

Tunnel Trap at 500 nm
Contour Lines = 0.2 %

Contour Lines = 0.01 %
Measured and Modelled External Quantum Efficiency of a Trap Detector

![Graph showing the external quantum efficiency of a trap detector. The graph plots wavelength against external quantum efficiency. The efficiency reaches values of 0.975 to 1.000.]

HACR Transfer to Traps

QE modeled from 405 nm to 920 nm

HeCd, Ar, Nd:YAG, HeNe, Ti SAF

Laser(s)

Steering Mirror

Electro-optic Laser Stabilizer

Spatial Filter & 25 µm Aperture

Iris Aperture

Laser Stabilizer Monitor Detector

Iris Aperture

Shutter

Steering Mirror

Polarizer

Lens

Wedges Beamsplitter

Entrance Window

Substitution

High Accuracy Cryogenic Radiometer

Calcon Tutorial 2013: Spectroradiometry

2013 - Basics: Page 57
Spectral Power Responsivity

Visible to Near-Infrared Spectral Comparator Facility (Vis/NIR SCF)
SCF (Spectral Power Responsivity) Uncertainty

Current SCF uncertainty from 200 nm to 1800 nm

Relative Expanded Uncertainty ($k=2$) [%]

Wavelength [nm]

200 400 600 800 1000 1200 1400 1600 1800
Radiometric Measurement Configurations

1. Radiant Flux (Power) Measurement $\Phi$ [W]
   - Collimated
   - Convergent

   Light flux $\Phi$ in beam

   Underfills Aperture

   Detector

2. Irradiance Measurement $E = \frac{\text{flux}}{\text{collector area}} = \frac{\Phi}{A_1}$ [W/m$^2$]
   - Collimated
   - Point Source

   Uniform Irradiance

   Overfills Aperture

   Detector

3. Radiance Measurement $L = \frac{\text{flux}}{\text{projected source area/solid angle}}$
   \[= \frac{\Phi}{A_2 \cdot d\Omega} \text{ [W/(m}^2\cdot\text{sr)}]}\]

   Uniform Source

   d$\omega < $ Aperture $A_2$

   Detector

   Aperture Area, $A_1$
NIST Aperture Area Measurement Facility

1. Measures the geometric area of high-quality circular apertures
2. Uncertainty ($k=1$) $<$0.01 % for aperture diameters ranging from 2 mm to 30 mm
3. Uses a precision microscope with stage position referenced to a laser interferometer
   - Standard uncertainty in relative stage position $<$ 50 nm
4. A separate, flux-transfer instrument is used for measurements relative to a standard
5. Currently participating in an international intercomparison
Spectral Irradiance and Radiance Calibrations using Uniform Sources (SIRCUS) Facility

Radiance and Irradiance Responsivity

SIRCUS uses tunable lasers from 210 nm to 1800 nm
The signal $i$ observed from such a radiometer is the aperture area $A$ multiplied by the integral of the product of the spectral irradiance of the source at the aperture $E(\lambda)$ and the meter’s spectral power responsivity $s(\lambda)$.

$$i = A \int \lambda E(\lambda) s(\lambda) d\lambda$$
Example Photometer

CIE $V(\lambda)$ function and NIST photometer spectral responsivity $s(\lambda)$
Conversion to Photometric Units

The luminous flux is related to the radiant flux by:

\[ \Phi_v = K_m \int_{360 \text{ nm}}^{830 \text{ nm}} \Phi(\lambda)V(\lambda) \, d\lambda \]

- \( K_m \): maximum spectral luminous efficacy [683 lm/W]
- \( V(\lambda) \): spectral luminous efficiency function

The luminous flux can also be written:

\[ \Phi_v = AK_m \int_{\lambda} E(\lambda)V(\lambda) \, d\lambda \]

Note: for brevity the explicit notation of the photopic wavelength range is indicated by \( \lambda \).
Luminous Flux and Illuminance Responsivity

The luminous flux responsivity \([A/\text{lm}]\) of a photometer:

\[
S_{v,f} = \frac{\text{signal}_{\text{out}}}{\Phi_v} = \frac{\int_{\lambda} E(\lambda)s(\lambda)d\lambda}{K_m\int_{\lambda} E(\lambda)V(\lambda)d\lambda}
\]

The illuminance responsivity \([A/\text{lx}]\) of a photometer is:

\[
S_{v,i} = A S_{v,f} = A \frac{\int_{\lambda} E(\lambda)s(\lambda)d\lambda}{K_m\int_{\lambda} E(\lambda)V(\lambda)d\lambda}
\]

Given \(s_{v,f}\) is uniform over the aperture \(A\)
References


2 Available as a .pdf file from NIST web pages.
3. Source-based Radiometry
Radiance and Irradiance

1. Radiance Sources
   a) Overfill the field-of-view of the radiometer
   b) Extended source that is spatially uniform
   c) Radiance is independent of view angle
   d) Radiance is independent of distance to radiometer

2. Irradiance Sources
   a) Underfill the field-of-view of the radiometer
   b) Approximate a point source (follows $1/d^2$ law)
   c) Uniform irradiance at the entrance pupil of the radiometer
Planck’s Law

\[ L_b(\lambda) = \frac{c_{1L}}{n^2 \lambda^5} \frac{1}{\exp\left(\frac{c_2}{n\lambda T}\right)} - 1 \]

- \( c_{1L} \) = first radiation constant for spectral radiance
- \( n(\lambda) \) = index of refraction of medium
- (1.00029 for air, 1.00028 for argon)
- \( c_2 \) = second radiation constant
- \( \lambda \) = wavelength of radiation

\[ c_{1L} = 1.191 \, 042 \, 722(93) \times 10^8 \, [W \, \mu m^4 \, m^{-2} \, sr^{-1}] \]
\[ c_2 = 14 \, 387.752 \, (25) \, [\mu m \, K] \]

Non ideal Blackbody: \( L(\lambda) = L_b(\lambda) \, \varepsilon(\lambda) \)

Note nonlinear relationship between Spectral Radiance and Blackbody Temperature
Spectral aspects of radiometry

A blackbody source obeys Planck’s law

$$L_\lambda = \frac{c_{1L}}{\lambda^5 \exp[c_2/(\lambda \cdot T) - 1]}$$

[\mu W / cm^2 nm sr ]

The radiance drops very sharply below a particular wavelength. As the temperature increases, the radiance increases for all wavelengths and the peak moves to shorter wavelength ($\lambda_{\text{max}} \propto 1/T$).

Blackbody sources are often used to calibrate spectroradiometers.
Lamps vs. blackbodies

If possible, match the temperature of the blackbody and the illumination geometries to result in similar signals. In this case, the goal is to assign irradiance values to FEL lamps.

For lamp-illuminated integrating sphere sources and reflecting plaques, the spectral radiance is modified by the surface reflectance and atmospheric absorption.
Spectral Distribution, $L_b(\lambda)$

\[ \lambda_{\text{max}} = \frac{c_3}{T} \]

$c_3 = 2898 \, [\mu m \, K]$ 

Wien Approximation ($\lambda < \lambda_{\text{max}}$):

\[ L_b(\lambda) = \frac{c_1 L}{n^2 \lambda^5} \exp\left(-c_2 / (n\lambda T)\right) \]
Stefan-Boltzmann Law

- Total exitance $M$: sum $L(\lambda)$ over all directions (into the hemisphere above the opening) and sum $L(\lambda)$ over all the electromagnetic spectrum (all wavelengths)
- For an ideal blackbody, the spectral radiance is lambertian
- With $a(\lambda) \approx \varepsilon$ and $n(\lambda) \approx n$, the sums yield $M = \varepsilon n^2 \sigma T^4 \approx \varepsilon \sigma T^4$ (with $n \approx 1$)
- $\sigma = \text{Stefan-Boltzmann constant}$
  $\sigma = 5.670 \ 400 \times 10^{-8} \ [\text{W m}^{-2} \text{K}^{-4}]$
Problems with Blackbodies

1. Temperatures above 3000 K are very difficult to achieve
2. Expensive to produce accurate systems (testing and modeling)
3. Not very transportable
4. Slow time constants
Radiance Temperature vs. Bulk Temperature

Blackbody

Thermocouple $T_{TC}$

Radiation Thermometer $T_\lambda$

Question: What are the uncertainties associated with the comparison of $T_{TC}$ with $T_\lambda$?

1. Accuracy of contact thermometer
2. Cavity design
3. Temperature gradients
4. Spectral and directional effects
5. Heat transfer losses
6. Diffraction losses
7. Reflected radiance
When the aperture angle $\phi$ is small, the effective emittance $\varepsilon_o$ is close to unity, even for small values of cavity surface emittance $\varepsilon$.

Due to Bedford

$$\varepsilon_o = \frac{\varepsilon}{\varepsilon + (1 - \varepsilon) \cdot (1 - \cos \phi)/2}$$

Cavity Design
Exact Solution for Effective Emittance of Spherical Cavity
Expected and Actual Radiance Temperature

- Expected T is calculated from Set Point T, Nominal Emissivity 0.95 and Background T = 24 C

### Measured Effective Spectral Emissivity

High Temperature Flat Plate Blackbody

- Manufacturer Specs
  - 0.95 - 0.85

### Spectral Emissivity

- 0.775
- 0.8
- 0.825
- 0.85
- 0.875
- 0.9
- 0.925
- 0.95

- 4 6 8 10 12

### Wavelength, microns

- 4 6 8 10 12

### Difference Between Actual Radiance Temperature and Set Point

- dT 50
- dT 100
- dT 146
- dT 200
- dT 270

### Radiance Temperature, C

- 50 C
- 100 C
- 146 C
- 200 C
- 270 C

Strongly selective spectral properties of used black paint (left figure) may lead to calibration errors up to 20 C because of difference between actual and expected (calculated using emissivity 0.95) radiance temperatures (right figure).

This BB is made by a major international manufacturer and quite common (NASA Transfer Standard).

Figure on the Left:
Difference between actual temperature and the set point temperature.
Blackbody Alternatives

1. Lamps, arc sources (many types), heated refractories, light emitting diodes, lasers, synchrotron radiation

2. Examples:
   a) tungsten filament strip lamps
   b) tungsten quartz-halogen lamps
   c) deuterium ($D_2$) gas discharge lamps
   d) xenon arc lamps
   e) Nernst glower and Globar
Tungsten strip lamp features

- Spectral Radiance or Radiance Temperature standards
- Vacuum or Gas-filled
- Quartz or glass windows

- Good stability (especially for the vacuum type)
- Small target area (0.6 mm wide by 0.8 mm tall)
- Careful alignment procedures required
- Calibrated by comparison to a blackbody or another strip lamp at 0.654 µm
- Suited for Devices Under Test with small field-of-views
Emittance of Tungsten

Spectral and temperature dependence of tungsten emissivity.

Radiance Temperature

\[ T_\lambda = \frac{1}{\frac{1}{T} - \frac{\lambda}{c_2} \ln(\varepsilon)} \]

= 1510 K at 1600 K and 660 nm
Tungsten strip lamp output

Gas-filled Lamps (to suppress tungsten evaporation)

For Radiance Temperature

For Spectral Radiance

- Lamp Current [A]
- Radiance Temp. [deg C]
- Wavelength [nm]
- Spec. Rad. [uW/cm²/sr/nm]

- 655.3 nm
- 40.4 A
Comparison of blackbodies and tungsten strip lamps and integrating sphere sources
Integrating Spheres

1. Features:
   a) Spherical geometry
   b) Low absorbance
   c) Diffuse reflectance

2. Result
   a) Flux “averager”

3. Applications
   a) Radiance source (add lamp, laser, LED, etc)
   b) Irradiance collector
   c) Internal or external sources and detectors
Sphere Performance

1. Flux transfer equations yield

\[ L(\lambda) = \frac{\rho(\lambda) \Phi(\lambda)}{\pi A(1 - \rho(\lambda)(1 - f))} \]

\[ f = \sum_{\text{port areas}} \frac{\text{port areas}}{A} \]

2. Baffles to shield direct view of lamps

3. Integrated monitor detectors to record performance

4. Stable power supplies and reflectance of interior wall
Reflectance and Throughput

Reflectance

Wavelength [nm]

Throughput

ρ(λ) (Barium Sulfate)

ρ(λ)/(1-0.98ρ(λ))
Radiance of Integrating Spheres

Spectral Radiance [$\mu$W / (cm$^2$ sr nm)]

Wavelength [nm]

- Spectralon (TM)
- Barium Sulfate
- Earth Systems
Temporal changes in the sphere output

Photometer measurements

Changes at 400 nm are more pronounced
Sphere Source Protocols

1. Geometry for uniform illumination
   a) Lamps baffle

2. Document operation
   a) Lamp current, lamp voltage drop, monitor detector signals,
      Lamp operating hours

3. Keep coating clean

4. Recalibrate

5. Map spatial uniformity and dependence on view angle
Halogen Filament Lamps

• Illumination, heating, & irradiance standards
• Wide commercial selection
• Select on features:
  • lifetime
  • color temperature
  • lumen efficacy
  • current or voltage
  • built in lens
  • base configuration
• Maximum wavelength range: 250 nm to 2.6 µm
FEL Lamp Irradiance Standards

- 1000 W output
- Coiled-coil structure to increase emittance
- FEL type (a model number)
- Modified by addition of bipost base

- Calibrated by comparison to a high temperature blackbody
- 50 cm from front of post
- 1 cm² collecting area
- Selected and screened for undesirable features
FEL alignment system

- Distance measurement made to this face of FEL alignment fixture.
- Lamp S/N plate on this side fixture, S/N away from plate this detector side.
- Aperture plane
- 50.0 cm
FEL Lamp Screening

1. Inspect, test, anneal, age, pot into base

2. Spectral line screening (currently 0 % pass rate)
   a) 250 nm to 400 nm in 0.1 nm steps with 0.04 nm bandpass (emission and absorption lines)

3. Temporal stability (90 % pass rate)
   a) <0.5 % before and after 24 h continuous operation at four wavelengths in UV to near infrared

4. Geometric (95% pass rate)
   a) < 1% in ± 1° at 655 nm
FEL Output

- a. 256.97 nm (256.80 nm)
- b. 257.67 nm (257.51 nm)
- c. 308.48 nm (308.22 nm)
- d. 309.47 nm (309.27 nm)
- e. 394.57 nm (394.40 nm)
- f. 396.27 nm (396.15 nm)

Undesirable Lines

![Diagram showing absorption and emission lines with specific wavelengths and signal voltages.]

Calibration Data, FEL at 8.2 A

![Graph showing spectral irradiance in uW/cm²/nm against wavelength in nm with specific calibration data points at 50 cm.]
Dependence on horizontal and vertical angles

Percent different from center

- Vertical Angle $[\Phi]$
- Horizontal Angle $[\psi]$

-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

-7.5 -6.5 -5.5 -4.5 -3.5 -2.5 -1.5 -0.5 0 0.5 1.0

50 cm
Power Supply Feedback Loop

- **Digital Voltmeter**
- **0.01 Ω Shunt Resistor**
- **16 bit D→A Converter**
- **Lamp**
- **Computer**
- **Power Supply**

Voltage to current conversion in the power supply

- **8.2 A ± 1 mA stabilization**
- ~5 s
Lamp Orientations

Vertical

Side

Horizontal

Optic Axis

Radiometer Aperture

NIST
Orientation dependence of the FEL

Frame, Lamp, and Radiometer

At NIST

Ratio to Initial Vertical Signals

Wavelength [nm]

- ● Side
- ▲ Horizontal
- ▲ Vertical

Frame

Radiometer
Protocols for FEL Standard Lamps

1. Orientation
   a) 50 cm from front of posts, entrance pupil diameter of 1 cm², use special alignment jig for FELs

2. Electrical
   a) maintain polarity, constant current, log voltage drop and burning hours
   b) Similar sensitivity to error in current as strip lamps

3. Operational
   a) 30 min warm-up; recalibrate every 50 h
   b) transfer to user working standards
   c) don’t touch the envelope; don’t enclose the lamp during operation; baffle properly
D$_2$ Irradiance Standards

- 30 W output
- Stable relative spectral irradiance distribution
- 200 nm to 350 nm
- Modified by addition of bipost base (same as FEL)

- Calibrated by a) relative distribution from wall stabilized hydrogen arc and b) FEL at 250 nm
- 50 cm from front of post
- 1 cm$^2$ collecting area
- Selected and screened for undesirable features
Deuterium, Xe and FEL

![Graph showing spectral irradiance vs wavelength for FEL, Xe, and Deuterium lamp.](image)
NIST uncertainties (k=1) (lowest in the world)
4. Properties of Detectors
Outline

1. Radiometric characteristics of photodiodes
2. Electronic characteristics of photodiodes
3. Comparison of basic detector characteristics
4. PMTs
5. Selection of detectors for different applications
6. Selection of signal meters for different detectors
Radiometric characteristics of photodiodes

1. Internal Quantum Efficiency (IQE),
2. External Quantum Efficiency (EQE), and
3. Spectral Responsivity $s(\lambda)$ of Quantum Detectors
4. Noise Equivalent Power (NEP) and $D^*$
5. Radiometric Sensitivity, Photons/s
6. Response Linearity of Photodiodes
7. Spatial and Angular Responsivitie{s}
8. Temperature Dependent Responsivity
IQE, EQE, and $s(\lambda)$ of quantum detectors

Number of collected electrons

$$\text{IQE} = \frac{\text{Number of collected electrons}}{\text{Number of absorbed photons}}$$

$$\text{EQE} = (1-\rho) \text{IQE}$$

where $\rho$ is the reflectance;

The power responsivity is:

$$e^\lambda$$

$$s(\lambda) = \frac{e^\lambda}{hc} = \text{EQE} \times \lambda \times \text{const.}.$$
Spectral power responsivity of frequently used photodiodes

Quantum Detectors

Responsivity [A/W] vs. Wavelength [µm]

- GaP
- Ge
- InGaAs

100 % EQE
Spectral responsivity variations within the same model

Hamamatsu Model S1226-8BQ photodiodes

Responsivity [A/W] vs. Wavelength [nm]
Noise Equivalent Power (NEP) and $D^*$ of detectors

\[
\text{NEP} = \frac{P}{S/N_{(\Delta f=1)}} = \frac{N}{S/P} = \frac{N}{R} \quad \text{[W/Hz}^{1/2}\text{]}
\]

where, \( S \) is the detector output signal for \( P \) incident radiant power,
\( R \) is the detector responsivity, and \( N \) is the detector output noise.

\[
D^* = \frac{A^{1/2}}{\text{NEP}} \quad \text{[cm Hz}^{1/2}/\text{W}],
\]

where \( A \) is the detector area.
Spatial response of large-area photodiodes

![Contour Lines = 0.2 %](image1)

Si S1337 @ 500 nm

![Contour Lines = 0.2 %](image2)

UV100 @ 500nm

![Contour Lines = 0.2 %](image3)

GaP @ 340 nm

![Contour Lines = 0.2 %](image4)

Ge @ 1225 nm

![Contour Lines = 0.5 %](image5)

Thin oxide Nitrided Si, AXUV-100G

![Contour Lines = 2 %](image6)

InGaAs @ 1250 nm

Calcon Tutorial 2013:Spectroradiometry

2013 - Basics: Page 110
Angular response of a 1337 Si photodiode

With the permission of L. P. Boivin.

From: Metrologia, 32, Fig. 3, p.567
Temperature dependent responsivity of photodiodes

Response Temperature Coefficients of Si, Ge, and InGaAs Photodiodes

Si and Ge responsivity change %/°C

InGaAs responsivity change %/°C

Wavelength (nm)

Si and Ge responsivity change %/°C

InGaAs responsivity change %/°C

Wavelength (nm)
Fundamental electronic characteristics of detectors and photocurrent meters

1. Photodiode shunt resistance
2. Linear photocurrent measurements
3. Noise and drift
4. Settling time
5. Stability
Photodiode (PV) shunt-resistance

The shunt resistance has a major influence for linearity and voltage-gain of noise and drift!

The shunt resistance is temperature dependent! For Si, the increase with decreasing temperature is ~11%/°C.

\[ R_s = 2.7 \, \text{M} \Omega \]

Linear PV photo-current measurement

\[ R_S \text{ has to be selected to a minimum value to obtain a linear relationship between V and } I_P: \]

\[ R_S \gg R_I = \frac{R}{A} \]

1. Example: For \( R=10 \ \text{G}\Omega \) and open-loop gain \( A=10^6 \), \( R_I=10 \ \text{k}\Omega \).

\( R_S=10 \ \text{M}\Omega \) is needed to obtain 0.1 % non-linearity.
Detector noise sources

1. **Photon noise**: noise contained in the signal and noise due to background radiation

2. **Detector-generated noise**:
   - **Johnson**: thermal motion of charged particles and thermal current fluctuations in resistors
   - **Shot**: in (PV) detectors with P-N junction (variance in the rate of photoelectron generation)
   - **G-R**: in PC detectors produced by fluctuations in the generation and recombination of current carriers
   - **1/f**: caused by non-perfect conductive contact and bias current or voltage in detectors

3. **Preamplifier noise**: Johnson, Shot, G-R, 1/f and
   - **Phonon**: from temperature changes not caused by the detected radiation
Equivalent PV circuit showing the main noise components

The feedback impedance, $R$ and $C$, of operational amplifier, OA, converts the photocurrent $I_P$ of photodiode P into a voltage $V$. $R_S$ and $C_J$ are the photodiode impedance. Single circles illustrate voltage sources and double circles illustrate current sources. One signal (the photocurrent) source and three noise sources (voltage noise $V_N$, current noise $I_N$, and resistor noise $R_N$) are shown in the circuit.
Output total-noise measured in dark

Dark noise with S1226-8BQ photodiode ($R_S=7 \ \Omega$) and OPA128LM.
The integration time of the DVM at the I-V output is 1.7 s.
Settling time of a Si photodiode current meter using Model S1226 ($R_S=7$ GΩ) and OPA128LM

The settling time depends on the magnitude of the signal change as well!
Long term stability of Si photometers

![Graph showing long term stability of Si photometers](image)
Comparison of typical characteristics of radiometric quality
detectors within the 200 nm to 20 µm range

<table>
<thead>
<tr>
<th>Type</th>
<th>Wavelength range [nm]</th>
<th>Diameter [mm]</th>
<th>Spatial response non-uniformity [%]</th>
<th>NEP [pW Hz(^{-1/2})]</th>
<th>(Shunt) resistance [M Ω]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrided Si</td>
<td>to 320</td>
<td>5 - 10</td>
<td>1</td>
<td>0.1 - 1</td>
<td>10 - 100</td>
</tr>
<tr>
<td>Silicon</td>
<td>200 - 1000</td>
<td>5 - 18</td>
<td>0.3</td>
<td>(2-10) x 10(^{-4})</td>
<td>100 - 10000</td>
</tr>
<tr>
<td>Ge</td>
<td>800 - 1800</td>
<td>5 - 13</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>InGaAs</td>
<td>800 - 1800</td>
<td>5 - 10</td>
<td>0.5</td>
<td>0.02</td>
<td>1</td>
</tr>
<tr>
<td>Extended InGaAs</td>
<td>1000 - 2550</td>
<td>2 - 3</td>
<td>1 (?)</td>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>InSb</td>
<td>1600 - 5500</td>
<td>4 - 7</td>
<td>1</td>
<td>1</td>
<td>0.1 - 1</td>
</tr>
<tr>
<td>HgCdTe PV or PC</td>
<td>2000 - 26000</td>
<td>2 - 4</td>
<td>10 - 90</td>
<td>PV: 30</td>
<td>PV: 200 Ω</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PC: 300</td>
<td>PC: 15 Ω</td>
</tr>
<tr>
<td>Pyroelectric</td>
<td>200 - 20000</td>
<td>5 - 12</td>
<td>0.1 - 5</td>
<td>10(^4) - 10(^5)</td>
<td>–</td>
</tr>
<tr>
<td>Thermopile</td>
<td>200 - 20000</td>
<td>5 - 10</td>
<td>0.2 - 3</td>
<td>2 x 10(^4)</td>
<td>N/A</td>
</tr>
<tr>
<td>Bolometer (cryogenic)</td>
<td>200 - 20000</td>
<td>5 - 10</td>
<td>1</td>
<td>40</td>
<td>1 - 2 at 4.5 K</td>
</tr>
</tbody>
</table>
Photomultiplier Tubes (PMT)

Advantages:
- Multiplication of secondary-electrons:
  - Extremely high responsivity
  - Exceptionally low noise
- Large photosensitive area
- Fast time response
- Virtually ideal constant-current source (very high shunt resistance)

Disadvantages:
- Poor spatial response uniformity
- Temperature dependent responsivity
- Fatigue and hysteresis (overshoot or undershoot for high-voltage and light)
- High-voltage, temperature, illumination, and time dependent dark-current
- Very stable high-voltage is required
- Affected by magnetic fields
- Drift and aging
- Linear and stable operation only at low signal levels
DC and AC PMT measurements

The current of the R voltage dividers must be much larger than $I_a$!
Comparison of PMT to Si photodiode

<table>
<thead>
<tr>
<th></th>
<th>PMT</th>
<th>Si</th>
<th>PMT/Si</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Responsivity [A/W]</strong></td>
<td>$5 \times 10^5$</td>
<td>$5 \times 10^{-1}$</td>
<td>$10^6$</td>
</tr>
<tr>
<td><strong>Noise [pA]</strong></td>
<td>54</td>
<td>$6 \times 10^{-4}$</td>
<td>$10^5$</td>
</tr>
<tr>
<td><strong>NEP [W]</strong></td>
<td>$10^{-16}$</td>
<td>$10^{-15}$</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Signal/Noise</strong></td>
<td></td>
<td>10x</td>
<td></td>
</tr>
</tbody>
</table>

$$N_{PMT} = \sqrt{2eI_{ad}K\Delta f} = 54 \text{pA}$$  \quad [W]$$

where $e$ is the elementary electron charge, $I_{ad}$ is the PMT anode dark current [A], $K$ is the PMT current amplification, and $\Delta f$ is the electrical bandwidth [Hz].
Selection of detectors for different applications

- Radiant power measurement:
  - Detectors with high spatial-response uniformity are needed
- Irradiance and radiance measurements:
  - Spatially non-uniform detectors can be used with uniform sources
- Photometric and color measurements:
  - Si photodiodes should be used
- UV measurements:
  - Passivating Nitrided Oxides or Pt-Silicide front layers eliminate UV damage
- Scale extension to UV and IR:
  - Pyroelectric detectors and bolometers with high spatial-response uniformity
- SW-IR measurements (1 µm to 5 µm):
  - NIR photodiodes, extended InGaAs and InSb photodiodes are preferred
- LW-IR measurements (5 µm to 20 µm):
  - HgCdTe detectors, pyroelectric detectors, and cryogenic bolometers
Scheme of optical radiation measurements

Matching preamplifier to a selected photodiode will dominate the performance (signal-to-noise ratio) of the overall measurement!
Frequency dependence of photodiodes

- The **internal speed** depends on the
  - Time to convert the accumulated charge into current

- The **maximum frequency** depends on the
  - Area of the photodiode
  - Type of material

- The **internal capacitance** $C_j$ depends on the
  - Active area
  - Resistivity (can change from 1 $\Omega$cm to 10 k$\Omega$cm for Si)
  - Reverse voltage
Frequency dependence of photodiodes (cont.)

• **Time constant** of a photodiode
  (with one dominating internal capacitance $C_j$):
  \[ \tau = C_j R_L \]
  where $R_L$ is the load-resistance

• **Rise time** (for photodiodes with multiple time constants):
  The current changes from 10 % to 90 %
Frequency dependence of photodiodes (cont.)

If the photodiode (shunt) resistance is much larger than $R_L$, the voltage on $R_L$ is: $V = I \frac{R_L}{1 + j\omega C_j R_L}$

The **upper roll-off frequency** is $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} C_j R_L$

where $\omega_0 = \frac{1}{\tau}$, and $I$ is the photocurrent.

For $C_j = 1$ nF and $R_L = 1$ kΩ, $f_0 = 160$ kHz
AC (chopped) radiation measurement

Chopping is needed to tune out measurement from the $1/f$ noise range (close to 0 Hz) and the eliminate DC background signal in infrared measurements.

Chopped measurements need partial frequency compensations!

- $\tau_1 = RC$ must be small to keep the roll-off higher than the signal frequency
- Photodiode with small $C_j$ is needed to decrease $\tau_2$, e.g. Hamamatsu S5226-8BQ
- Wide band (high open loop gain and low noise) OPA is needed, e.g. OPA627.
AC (chopped) radiation measurement (cont.)

- **Signal gain curves (measured).** The 3 dB roll-off frequencies for all gains are 80 Hz or higher except for gain $10^{10}$ V/A.

Partial compensations were made for all the signal gains shown here. No compensation was made for $10^{10}$ V/A. The operating point should be on the flat parts of the curves at 10 Hz chopping (or frequency stabilized chopper is needed)!
AC measurements with chopper and lockin

- **Chopper:**
  1. tunes out the signal (by modulation) from $1/f$ noise and drift
  2. Separates the signal to be measured from the DC background signal

  ✓ **Frequency:** needs to be stable if the operating point is on the slope of the signal-gain versus frequency curve

- **Lockin:** phase controlled rectifier + low-pass filter
  ✓ **Phase control:** synchronized from chopper
  ✓ **Low-pass filter:** smoothes out the rectified (structured but DC) signal
  ✓ **Output:** in-phase and quadrature ($X$ and $Y$) components of the signal (in rectangular form), or magnitude $M=(X^2+Y^2)^{1/2}$ and phase $\Phi$ (in polar form)
  ✓ **Input:** sine or square wave. The sine wave measurement selects the fundamental frequency component of the chopped waveform.
Sine wave lockin measures square wave
Calibration of the lockin reading against a DVM.

- Signal to be measured:

- Theoretical reading of sine-wave lockin: \[ S_1 = \frac{H}{\frac{4}{\sqrt{2}} \pi} = 0.9003H \]

- Reference reading of a DVM in DCV mode (with large S/N): \( S_2 = H \) with running chopper, or \( S_2 = 2H \) if the chopper is stopped

- The real correction factor is the ratio of the lockin reading to the DVM reading: \( S_1/S_2 \)
Selection of commercial signal meters for detectors

• DC or AC photocurrent from photodiodes:
  ✓ Electrometers, current preamplifiers, and picoammeters can be used
  ✓ The typical shunt resistance of a DVM in DC-I mode is 1 kΩ in the lowest (300 µA f.s.) range. The input shunt resistance can be higher for DMMs. A “Burden” voltage of about 0.2 V can develop on this resistance causing an error in the measured current. The lower the detector resistance the larger the error.

**DO NOT DO THIS:**

• V-measurement on detectors or load resistors:
  ✓ Non-linearity with biased PC detectors
  ✓ High non-linearity with photodiodes (measurement along the V-axis of the I-V curve)

• Photodiode shunt resistance measurement with ohm-meters
  (A large current would be forced through the photodiode!)
5. Determining Measurement Uncertainties
Outline

1. Measurement Uncertainty, Measurement Error
2. Accuracy & Precision
3. Measurement Equation
4. Measurement Steps
5. Direct Methods for Uncertainty Propagation
6. GUM Supplement 1
Measurement Equation Approach:

In general, we use the measurement equation approach for characterizing and calibrating sources and radiometers. A simplified measurement equation is:

\[ I(A, \omega, \Delta \lambda, \lambda_o) = \int \int \int_{\Delta \lambda} S_{\phi}(x, y, \theta, \phi, \lambda, \lambda_o) \cdot L_{\lambda}(x, y, \theta, \phi, \lambda, \lambda_o) \cdot \cos \theta \cdot d\omega \cdot dA \cdot d\lambda \]

- \( I(A, \omega, \Delta \lambda, \lambda_o) \) - the measured current
- \( S_{\phi} \) - the spectral flux (power) responsivity of the detector at a position \( x,y \)
- \( L_{\lambda} \) - the spectral radiance of the source
- \( A \) - receiving area of the detector
- \( \omega \) - the solid angle of the source viewed by the detector
Measurement Uncertainty

Measurement result is complete only when a quantitative estimate of the uncertainty in the measurement is stated.

The “true value” of the measurand is the value of the measurand.

**Formal definition**

Uncertainty of measurement is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

“Expressed as a standard deviation (u)”
Why do you need an uncertainty budget?

Traceability- “Property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons, all having stated uncertainties.”

ISO International Vocabulary of Basic and General Terms in Metrology, 2nd ed., 1993, definition 6.10

Uncertainty budget will enable one to identify the dominant terms in the uncertainties to reduce those terms.
Repeatability and Reproducibility

Closeness of agreement between the results of successive measurements of the same measurand carried out under same conditions of measurement

Repeatability

under same conditions of measurement

Same
principle
method
observer
location
instrument
time

Reproducibility

under changed conditions of measurement

Different
principle
method
observer
location
instrument
time
Accuracy and Precision

Accuracy
Closeness of agreement between the result of a measurement and the value of the measurand.

Precision
Closeness of agreement between the results of measurements of the same measurand.

Value of the Measurand

Measurement

Accuracy

Precision

Note: The ISO Guide to Uncertainty in Measurements (GUM) discourages the use of the terms, but are still used and confused in common usage.
Accuracy and Precision - Example

Precision

Accuracy

High

Low

High

Low
Error of Measurement

Result of a measurement **minus** the value of the measurand.
(Sum of random and systematic errors)

**Random error**
Result of a measurement **minus**
the mean that would result from an infinite number of
measurements of the same
measurand carried out under repeatability conditions

\[ x_{i,k} - x_i \]

**Systematic error**
Mean that would result from an infinite number of
measurements of the same measurand carried out under
repeatability conditions **minus**
the value of the measurand.

\[ x_i - \bar{x} \]
Error of Measurement - Illustration

Measurements

Random error

Systematic error

Mean

Value of the Measurand
Classify the Uncertainty Components

Due to random effects (Type A)
Give rise to possible random error in the unpredictable result of the current measurement process.

Usually decrease with increasing number of observations.

Due to systematic effects (Type B)
Give rise to possible systematic error in the result due to recognized effects in the current measurement process.
Correction and Correction Factor

**Correction**

Value *added algebraically* to the uncorrected result of a measurement to compensate for systematic error.

\[
\text{Correction} = - (\text{systematic error})
\]

**Correction Factor**

Numerical factor by which the uncorrected result of a measurement is *multiplied* to compensate for systematic error.

E.g. Linearity, offset, shunt resistance, drift, stray light
Standard Uncertainty

Measurand ($y$) determined from $m$ input parameters $x_i$ through functional relationship

$$f(x_1, x_2, \ldots, x_i, \ldots, x_m)$$

**Example: Radiometer signal measurement**

$$v \cong \Gamma \cdot G \cdot s(\lambda) \cdot \tau(\lambda) \cdot L(\lambda, T) \cdot \Delta \lambda$$

Input parameters are throughput ($\Gamma$), gain ($G$), responsivity ($s$), transmittance ($\tau$), radiance ($L$), wavelength ($\lambda$), bandwidth ($\Delta \lambda$) and source temperature ($T$)

**Standard uncertainty**

Estimated standard deviation associated with each input estimate $x_i$, denoted $u(x_i)$

Example: $u(\Gamma)$, $u(\lambda)$, $u(T)$, etc.

Standard uncertainty $u(x_i)$ determined from probability distribution ($P$) of parameter ($x_i$)
Normal Probability Distribution

Probability that $x$ lies between $(x_i - \sigma)$ and $(x_i + \sigma)$ is $68\%$.
For large number of observations, about $68\%$ of the values lie in this range, OR
a value deviating more than $\sigma$ from mean $x_i$ will occur about once in $3$ trials.

Probability that $x$ lies between $(x_i - 2\sigma)$ and $(x_i + 2\sigma)$ is $95.5\%$.
For large number of observations, about $95\%$ of the values lie in this range, OR
a value deviating more than $2\sigma$ from mean $x_i$ will occur about once in $20$ trials.
# Evaluation of Uncertainty

## Two Categories: Type A and Type B

<table>
<thead>
<tr>
<th>Type A</th>
<th>Type B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluated using statistical methods for analyzing the measurements.</td>
<td>Evaluated by methods other than statistical.</td>
</tr>
</tbody>
</table>

**Examples:**
- Standard deviation of a series of independent observations,
- Least squares fit
- Scientific judgment, experience, manufacturer’s specification, data from other sources (reports, handbooks)
Statistical Parameter – Sample Mean

Mean

\[ x_i = \frac{1}{n} \sum_{k=1}^{n} x_{i,k} \]

Sum of all the sample values \( (x_{i,k}) \) divided by the size of the sample \( (n) \)

Example

Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0

Size of the sample = 5

Sample mean = \( \frac{(0.9 + 1.2 + 1.1 + 0.8 + 1.0)}{5} = 1.0 \) [V].
Statistical Parameter – Sample Variance

Variance: \( \sigma^2(x_{i,k}) = \frac{1}{n-1} \sum_{k=1}^{n} (x_{i,k} - x_i)^2 \)

Sum of the squares of the deviations of the sample values \( (x_{i,k}) \) from the mean value \( (x_i) \), divided by \( (n - 1) \).
Measures the spread or dispersion of the sample values, and is positive.

Variance of the mean \( \sigma^2(x_i) = \frac{\sigma^2(x_{i,k})}{n} \)

Example: Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0; Sample mean = 1.0 [V]

Variance \( = [(0.9-1.0)^2 + (1.2-1.0)^2 + (1.1-1.0)^2 + (0.8-1.0)^2 + (1.0-1.0)^2 ]/(5-1) = 0.025 \ [V^2] \)

Variance of the mean \( = 0.025/5 = 0.005 \ [V^2] \)
Type A Evaluation of Standard Uncertainty

\[
\text{Standard deviation} = (\text{Variance})^{1/2} = \sigma(x_{i,k})
\]

(Positive square root of the sample variance)

\[
\text{Standard deviation of the mean: } \sigma(x_i) = \sigma(x_{i,k}) / n^{1/2}
\]

Standard uncertainty \( u(x_i) = \sigma(x_i) \)

(Standard deviation divided by the square root of the number of samples)

Relative standard uncertainty \( = u(x_i)/x_i \)

Example: Five voltage readings: 0.9, 1.2, 1.1, 0.8, 1.0
Sample mean = 1.0 [V], Variance = 0.025 [V^2], Variance of the mean = 0.005 [V^2]

\[
\text{Standard deviation} = (\text{Variance})^{1/2} = 0.025^{1/2} = 0.158 \text{ [V]}
\]

\[
\text{Standard uncertainty} = \text{Standard deviation of the mean} = 0.158/5^{1/2} = 0.071 \text{ [V]}
\]

Relative standard uncertainty \( = 0.071/1.0 = 0.071 \)
Type B Evaluation of Standard Uncertainty

Evaluated based on scientific judgment, experience, manufacturer’s specification, data from other sources (reports, handbooks)

**Examples**

Convert a quoted uncertainty (with a stated multiple) to a standard uncertainty by dividing by the multiple

Convert a quoted uncertainty (with a specified confidence level, such as 95 % or 99 %) to a standard uncertainty by dividing by the appropriate factor for a normal distribution

**Computational methods**

Model the quantity by an assumed probability distribution such as normal, rectangular or triangular.
Type B Calculation – Normal Distribution

Center of the limits
= \( \frac{a_+ + a_-}{2} \)

Half width of interval
\( a = \frac{a_+ - a_-}{2} \)

Estimated the lower limit \( (a_-) \), and the upper limit \( (a_+) \) of the quantity.
Best estimated value of the quantity (mean) = center of the limits

50.0 % probability, value lies in the interval \( a_- \) to \( a_+ \), then \( u(x_i) = 1.48 \, a \)

67.7 % probability, value lies in the interval \( a_- \) to \( a_+ \), then \( u(x_i) = a \)

99.7 % probability, value lies in the interval \( a_- \) to \( a_+ \), then \( u(x_i) = a/3 \)
Type B Calculation – Rectangular Distribution

Equal probability the value lies in the interval $a_-$ and $a_+$ is 100 % and zero outside
(Reasonable default model in the absence of any other information)

Center of the limits

$= (a_+ + a_-)/2$

Half width of interval

$a = (a_+ - a_-)/2$

Best estimated value of the quantity (mean) = center of the limits with

$u(x_i) = a/3^{1/2}$ or $[\text{max-min}]/(12)^{1/2}$
Type B Calculation – Triangular Distribution

Probability the value lies in the interval $a_-$ and $a_+$ is 100% and zero outside

Center of the limits
$$ = (a_+ + a_-)/2$$

Half width of interval
$$a = (a_+ - a_-)/2$$

Best estimated value of the quantity (mean) = center of the limits
with
$$u(x_i) = a/6^{1/2} \text{ or } [\text{max-min}]/(24)^{1/2}$$
Expressing Measurement Uncertainty

Functional relationship between measurand and input parameters

\[ y = f(x_1, x_2, ..., x_i, ..., x_m) \]

Combined standard uncertainty, \( u_c(y) \)

Represents the estimated standard uncertainty of the measurand \( y \).

given by

Law of Propagation of Uncertainty

\[
\begin{align*}
    u_c^2(y) &= \sum_{i=1}^{m} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial f}{\partial x_i} & : \text{sensitivity coefficient,} \\
    u(x_i) & : \text{standard uncertainty of } x_i \\
    \frac{u(x_i)}{x_i} & : \text{relative standard uncertainty of } x_i \\
    u(x_i, x_j) & : \text{covariance of } x_i \text{ and } x_j \\
    r(x_i, x_j) & : \text{correlation coefficient} \\
    r = 0, \text{ if uncorrelated} & [-1 \leq r \leq 1]
\end{align*}
\]
Expressing Measurement Uncertainty - Example

Additive function (Two independent random variables $x_1$ and $x_2$)
Use standard uncertainties to calculate combined standard uncertainty

$$y = a \cdot x_1 + b \cdot x_2$$
$$\frac{\partial y}{\partial x_1} = a \quad \frac{\partial y}{\partial x_2} = b$$
$$u_c^2(y) = a^2 \cdot u^2(x_1) + b^2 \cdot u^2(x_2)$$

Multiplicative function (Two independent random variables $x_1$ and $x_2$)
Use relative standard uncertainties to calculate combined standard uncertainty

$$y = a \cdot x_1 \cdot x_2$$
$$\frac{\partial y}{\partial x_1} = a \cdot x_2 \quad \frac{\partial y}{\partial x_2} = a \cdot x_1$$
$$u_c^2(y) = a^2 \cdot x_2^2 \cdot u^2(x_1) + a^2 \cdot x_1^2 \cdot u^2(x_2)$$
$$\frac{u_c^2(y)}{y^2} = \frac{u^2(x_1)}{x_1^2} + \frac{u^2(x_2)}{x_2^2}$$
Example: Frequency of pendulum

\[ \omega = \sqrt{\frac{g}{\ell}} \]

then

\[ \frac{\partial \omega}{\partial \ell} = -\frac{1}{2} \sqrt{\frac{g}{\ell^{3/2}}} = -\frac{1}{2} \omega \]

\[ \frac{\partial \omega}{\omega} = -\frac{1}{2} \frac{\partial \ell}{\ell} \]

The relationship between frequency, \( \omega \) and length is given by the sensitivity coefficient, \( \frac{1}{2} \).
Expanded Uncertainty

Measure of uncertainty defining an interval about the result $y$ within which the measurand is confidently believed to lie.

Expanded uncertainty ($U$) = Coverage factor ($k$) $\times$ Combined uncertainty $u_c(y)$

<table>
<thead>
<tr>
<th>Coverage factor $k$</th>
<th>Confidence level for a normal probability distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>68.27 %</td>
</tr>
<tr>
<td>1.645</td>
<td>90.00 %</td>
</tr>
<tr>
<td>1.960</td>
<td>95.00 %</td>
</tr>
<tr>
<td><strong>2.000</strong></td>
<td><strong>95.45 %</strong></td>
</tr>
<tr>
<td>2.576</td>
<td>99.00 %</td>
</tr>
<tr>
<td>3.000</td>
<td>99.73 %</td>
</tr>
</tbody>
</table>

1 out of 20 times you should fall outside your uncertainty budget
Uncertainty Evaluation Procedure – Summary

1. Express functional relationship between the measurand \( y \) and input parameters \( x_i \).

\[
y = f(x_1, x_2, \ldots, x_i, \ldots, x_m)
\]

2. Determine values of input parameters \( x_i \) [Statistical analysis or other means].

3. Evaluate standard uncertainty \( u(x_i) \) of each input \( x_i \) (Type A or Type B technique).

4. Calculate the value of measurand \( y \) from the functional relationship [Step 1].

5. Determine the combined standard uncertainty \( u_c(y) \) from the standard uncertainties associated with each input parameter \( x_i \). [Step 3].

6. Calculate the expanded standard uncertainty \( U \) as the combined standard uncertainty \( u_c(y) \) times the coverage factor \( k \).

7. Report the value of the measurand \( y \) [Step 4] and specify the combined standard uncertainty \( u_c(y) \) [Step 5] or the expanded uncertainty \( U \) [Step 6].
6a. Applications from the NIST Short Course:

Photometer Responsivity Calibration
Objectives of Lab #2

1. Measure the spectral power responsivity of the NIST photometer by comparison to a NIST-traceable silicon photodiode standard (STD). (Calibration)

2. Use the previously calibrated NIST photometer to measure the 100 W QTH lamp at about 3 m distance, and compare the measured illuminance (lux) to those found using a separately calibrated commercial photometer. (Validation)
Spectral Responsivity \([A/W]\) (Lab #2, Step 1)

Lab #2 Spectral Responsivity Measurement Setup

Photometer Spectral Responsivity \([A/W]\) is given by:

\[
S_p = \frac{V_p}{V_s} \frac{G_s}{G_p} \frac{V_{mp}}{V_{ms}} S_s
\]

Alignment Laser

Alignment Mirror

Lamp

Monochromator

Shutter

Order Sorting Filter

Beamsplitter

Monitor

Detector Position (Photometer #118 or WS)
Realization of Detector-based Illuminance [lux]

1. Start with calibrated Si diode
2. Calibrate the spectral power responsivity of a photometer using monochromator system
3. Convert to illuminance responsivity using aperture (and calculation)
4. Calibrate Test illuminance meter with Photometer

Application Example from Photometry (Lab #2)

Traceability Chain

Optical Power \([\text{watt}]\) (Cryogenic radiometer)

Spectral Responsivity \([\text{ampere/watt}]\) (Silicon photodiode)

Illuminance Responsivity \([\text{ampere/lux}]\) (Photometer)

Illuminance Meter \([\text{lux}]\) (Test meter)
Uncertainty Calculation Example
Application to detector calibration (Lab #2)

Signal measurement equation

\[ S_x = \frac{V_x}{V_{mx}} \cdot \frac{G_s}{G_x} \cdot S_s \]

- \( S_x \): Spectral responsivity of test detector
- \( S_s \): Spectral responsivity of standard detector
- \( V \): Voltage from test detector (x) or standard detector (s)
- \( V_m \): Voltage from monitor detector
- \( G \): Amplifier gain
Illuminance Responsivity [A/lx] (Lab #2, Step 2)

\[ S_{v,i} = A \frac{\int_{\lambda} P(\lambda)s(\lambda) \, d\lambda}{K_m \int_{\lambda} P(\lambda)V(\lambda) \, d\lambda} \]

- \( S_{v,i} \): illuminance responsivity [A/lx] (Note: 1x = 1m/m²)
- \( A \): area of the photometer aperture [m²]
- \( P(\lambda) \): spectral power distribution of the light source (CIE Illuminant A with 2856 K Planck distribution)
- \( s(\lambda) \): photometer spectral power responsivity [A/W]
- \( K_m \): maximum spectral luminous efficacy [683 lm/W]
- \( V(\lambda) \): spectral luminous efficiency function
Illuminance Meter Calibration [lx] (Lab #2, Step 3)

The illuminance $E_{v,p}$ [lx] measured by the photometer is:

$$E_{v,p} = \frac{V_p}{S_{v,i} G_p}$$

- $V_p$: photometer signal [V]
- $S_{v,i}$: illuminance responsivity [A/lx]
- $G_p$: photometer gain [V/A]

The calibration factor for the test photometer is:

$$CF_{v,t} = \frac{E_{v,p}}{E_{v,t}}$$

- $E_{v,t}$: illuminance measured by the test photometer [lx]
Spectral Responsivity, \( V(\lambda) \), and Illuminant A

![Graph showing spectral responsivity and normalized responsivity.](image)

- CIE S x rel(\( \lambda \))
- Illuminant-A
- CIE V(\( \lambda \))
- Numerator P(\( \lambda \)) x s(\( \lambda \))
Uniformity of SRSC FR #117 with $V(\lambda)$ Filter

Responsivity uniformity

0.2 % contours at 555 nm;
1.1 mm beam size;
0.5 mm/Step

Surface Plot of Responsivity Relative to Center of Active Area
Linearity of the NIST standard photometers

<table>
<thead>
<tr>
<th>Decade</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10 – -9</td>
<td>1.00016</td>
</tr>
<tr>
<td>-9 – -8</td>
<td>1.00001</td>
</tr>
<tr>
<td>-8 – -7</td>
<td>0.99999</td>
</tr>
<tr>
<td>-7 – -6</td>
<td>1.00001</td>
</tr>
<tr>
<td>-6 – -5</td>
<td>0.99993</td>
</tr>
<tr>
<td>-5 – -4</td>
<td>1.00003</td>
</tr>
</tbody>
</table>
Stability of NIST Photometers

![Photometers Image]

**Relative Luminous Responsivity**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>B</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>C</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>D</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>E</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>F</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>G</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>H</td>
<td>0.985</td>
<td>0.987</td>
<td>0.989</td>
<td>0.991</td>
<td>0.993</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>1.001</td>
<td>1.003</td>
<td>1.001</td>
<td>0.999</td>
<td>0.997</td>
<td>0.995</td>
</tr>
</tbody>
</table>

**Legend:**
- A
- B
- C
- D
- E
- F
- G
- H
Stability of Silicon Photodiodes

Differences in the Vis SCF Silicon WS H626/H629 ratios from 1992 to 2002.

Scale Uncertainty ($k=1$): 0.1 %

- Unknown reason for shift, similar data for several months.
- Different SCF Channel (i.e., amplifier)
Stability of SRSC FR #117 with $V(\lambda)$ Filter
Factors Contributing to Uncertainty
Detector Calibration – Lab #2

### Examples of Sensitivity Coefficients

<table>
<thead>
<tr>
<th>Absolute</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_s :$</td>
<td>$dS_s = \left( \frac{V_x/V_{mx}}{V_s/V_{ms}} \cdot \frac{G_s}{G_x} \right) \cdot S_s$</td>
</tr>
<tr>
<td>$V_x :$</td>
<td>$dS_x = \left( \frac{1/V_{mx}}{V_s/V_{ms}} \cdot \frac{G_s}{G_x} \right) \cdot S_s$</td>
</tr>
<tr>
<td>$V_s :$</td>
<td>$dS_x = -\left( \frac{V_x/V_{mx}}{V_s^2/V_{ms}} \cdot \frac{G_s}{G_x} \right) \cdot S_s$</td>
</tr>
<tr>
<td>$\lambda:$</td>
<td>$dS_x = \frac{d(V_x/V_s)}{d\lambda} \cdot \frac{V_{ms}}{V_{mx}} \cdot \frac{G_s}{G_x} \cdot S_s$</td>
</tr>
</tbody>
</table>
Combined Standard Uncertainty
Detector Calibration – Lab #2

Law of Propagation of Uncertainties

Using absolute uncertainties

\[
    u^2_c(S_x) = \left( \frac{dS_x}{dS_s} \right)^2 u^2(S_s) + \left( \frac{dS_x}{dV_x} \right)^2 u^2(V_x) + \left( \frac{dS_x}{dV_s} \right)^2 u^2(V_s) + \cdots + \left( \frac{dS_x}{d\lambda} \right)^2 u^2(\lambda)
\]

Using relative uncertainties

\[
    \left( \frac{u_c(S_x)}{S_x} \right)^2 = \left( \frac{u(S_s)}{S_s} \right)^2 + \left( \frac{u(V_x)}{V_x} \right)^2 + \left( \frac{u(V_s)}{V_s} \right)^2 + \cdots + \left( \frac{d(V_x/V_s)}{d\lambda} \cdot \frac{u(\lambda)}{V_x/V_s} \right)^2
\]
# Uncertainty Summary

Detector Calibration (at 550 nm) – Lab #2

<table>
<thead>
<tr>
<th>Quantity (Symbol)</th>
<th>Unit</th>
<th>Value</th>
<th>Probability Distribution</th>
<th>Uncertainty limit</th>
<th>Standard uncertainty</th>
<th>Relative sensitivity</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Resp. ($S_s$)</td>
<td>A/W</td>
<td>0.2848</td>
<td>Normal</td>
<td></td>
<td>0.0003</td>
<td>1/0.2848</td>
<td>0.11 %</td>
</tr>
<tr>
<td>Std. Signal ($V_s$)</td>
<td>V</td>
<td>2</td>
<td>Normal</td>
<td></td>
<td>0.002</td>
<td>1/2</td>
<td>0.10 %</td>
</tr>
<tr>
<td>Test Signal ($V_x$)</td>
<td>V</td>
<td>1.8</td>
<td>Normal</td>
<td></td>
<td>0.004</td>
<td>1/1.8</td>
<td>0.22 %</td>
</tr>
<tr>
<td>Monitor Signal ($V_{ms}$)</td>
<td>V</td>
<td>1.1</td>
<td>Normal</td>
<td></td>
<td>0.001</td>
<td>1/1.1</td>
<td>0.09 %</td>
</tr>
<tr>
<td>Monitor Signal ($V_{mx}$)</td>
<td>V</td>
<td>1.09</td>
<td>Normal</td>
<td></td>
<td>0.001</td>
<td>1/1.09</td>
<td>0.09 %</td>
</tr>
<tr>
<td>Std. Gain ($G_s$)</td>
<td>A/V</td>
<td>10^{-6}</td>
<td>Normal</td>
<td></td>
<td>10^{-10}</td>
<td>1/10^{-6}</td>
<td>0.01 %</td>
</tr>
<tr>
<td>Test Gain ($G_x$)</td>
<td>A/V</td>
<td>10^{-6}</td>
<td>Normal</td>
<td></td>
<td>10^{-10}</td>
<td>1/10^{-6}</td>
<td>0.01 %</td>
</tr>
<tr>
<td>Wavelength ($\lambda$)</td>
<td>nm</td>
<td>550</td>
<td>Rectangular</td>
<td></td>
<td>1</td>
<td>0.6</td>
<td>1/9</td>
</tr>
</tbody>
</table>

| Test Resp. ($S_x$)       | A/W  | 0.2587| Combined uncertainty in the responsivity |                   |                      |                     | 0.30 %               |
|                         |      |       | Expanded uncertainty       |                   |                      |                     | 0.60 %               |
Findings: Difference from NIST Calibration Values

Spectral Power Responsivity of FR-118: Percent Difference from the SCF Calibration

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
<th>Instructor 1</th>
<th>Instructor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>-0.70</td>
<td>-0.60</td>
<td>-0.50</td>
<td>-0.40</td>
<td>-0.30</td>
</tr>
<tr>
<td>470</td>
<td>-0.60</td>
<td>-0.50</td>
<td>-0.40</td>
<td>-0.30</td>
<td>-0.20</td>
</tr>
<tr>
<td>490</td>
<td>-0.50</td>
<td>-0.40</td>
<td>-0.30</td>
<td>-0.20</td>
<td>-0.10</td>
</tr>
<tr>
<td>510</td>
<td>-0.40</td>
<td>-0.30</td>
<td>-0.20</td>
<td>-0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>530</td>
<td>-0.30</td>
<td>-0.20</td>
<td>-0.10</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>550</td>
<td>-0.20</td>
<td>-0.10</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>570</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>590</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
</tr>
<tr>
<td>610</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>630</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
</tr>
<tr>
<td>650</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
</tr>
</tbody>
</table>
## The Uncertainties - Spectral Responsivity

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Type</th>
<th>Relative Standard Uncertainty [%]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute responsivity scale</td>
<td>B</td>
<td>0.100</td>
<td>sp-250-41 Value</td>
</tr>
<tr>
<td>Working Standard Diode Signal</td>
<td>A</td>
<td>0.014</td>
<td>from data</td>
</tr>
<tr>
<td>Test Detector (Photometer) Signal</td>
<td>A</td>
<td>0.020</td>
<td>from data</td>
</tr>
<tr>
<td>Monitor Diode Signal with WS Diode</td>
<td>A</td>
<td>0.040</td>
<td>from data</td>
</tr>
<tr>
<td>Monitor Diode Signal with Test Detector (Photometer)</td>
<td>A</td>
<td>0.040</td>
<td>from data</td>
</tr>
<tr>
<td>Working Standard Diode Amplifier Gain</td>
<td>B</td>
<td>0.120</td>
<td>Doubled SP-250 data</td>
</tr>
<tr>
<td>Test Detector (Photometer) Amplifier Gain</td>
<td>B</td>
<td>0.100</td>
<td>Doubled SP-250 data</td>
</tr>
<tr>
<td>Monochromator Wavelength Calibration</td>
<td>B</td>
<td>0.040</td>
<td>Estimated using .2nm Error</td>
</tr>
</tbody>
</table>

**Relative combined standard uncertainty (RSS) [%]**  

0.20

**Relative expanded uncertainty (k = 2) [%]**  

0.40
Goal 2: Calibration of the Illuminance meter using the Filter Radiometer

1. QTH source, close to illuminant A (2856 K)
2. Three detectors set up to 3 m from the source.
3. The filter radiometer was used to calibrate the two illuminance meters
4. A variation of baffling was used
5. Two different reference planes on the illuminance meter were used
## The Uncertainties - Illuminance Responsivity

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Type</th>
<th>Relative Standard Uncertainty [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute responsivity scale</td>
<td>B</td>
<td>0.200</td>
</tr>
<tr>
<td>Transfer of scale to photometer</td>
<td>A</td>
<td>0.080</td>
</tr>
<tr>
<td>Wavelength calibration of monochromator</td>
<td>B</td>
<td>0.040 Estimated using .2nm Error</td>
</tr>
<tr>
<td>Numerical aperture of monochromator beam</td>
<td>B</td>
<td>0.100</td>
</tr>
<tr>
<td>Area of photometer aperture</td>
<td>B</td>
<td>0.100</td>
</tr>
<tr>
<td>Temperature variation</td>
<td>A</td>
<td>0.060</td>
</tr>
<tr>
<td>Other factors</td>
<td>A</td>
<td>0.240</td>
</tr>
</tbody>
</table>

### Relative combined standard uncertainty (RSS) [%]

0.36

### Relative expanded uncertainty ($k = 2$) [%]

0.72
## The Uncertainties - Illuminance Meter

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Type</th>
<th>Relative Standard Uncertainty [%]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illuminance unit realization</td>
<td>B</td>
<td>0.360</td>
<td></td>
</tr>
<tr>
<td>Long-term drift of the photometer</td>
<td>B</td>
<td>0.150</td>
<td>Values from SP250-37</td>
</tr>
<tr>
<td>Photometer temperature variation</td>
<td>A</td>
<td>0.030</td>
<td>Values from SP250-37</td>
</tr>
<tr>
<td>Spectral mismatch factor of photometer</td>
<td>B</td>
<td>0.020</td>
<td>Values from SP250-37</td>
</tr>
<tr>
<td>Illuminance meter alignment (distance and angle)</td>
<td>A</td>
<td>0.200</td>
<td>Estimate within 3mm error</td>
</tr>
<tr>
<td>Illuminance nonuniformity</td>
<td>B</td>
<td>0.050</td>
<td>Values from SP250-37</td>
</tr>
<tr>
<td>Lamp current regulation</td>
<td>A</td>
<td>0.040</td>
<td>Conservative estimates (no warmup stabilization time)</td>
</tr>
<tr>
<td>Stray light in the “photometry bench”</td>
<td>B</td>
<td>0.100</td>
<td>Conservative estimates Double the SP250-37 value</td>
</tr>
<tr>
<td>Random noise (scatter by dust, lamp drift, etc.)</td>
<td>A</td>
<td>0.100</td>
<td>Values from SP250-37</td>
</tr>
<tr>
<td>Display resolution of the illuminance meter (1 in 199)</td>
<td>A</td>
<td>0.400</td>
<td>Conservative estimates</td>
</tr>
<tr>
<td>Inconsistency in responsivity between luminance levels</td>
<td>B</td>
<td>0.120</td>
<td>Values from SP250-37</td>
</tr>
</tbody>
</table>

| Relative combined standard uncertainty (RSS) [%]   | 0.63 | |
| Relative expanded uncertainty (\(k = 2\)) [%]     | 1.26 | |
The Results of the Illuminance Meter Calibrations Day 3

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EOS</td>
<td>22.28</td>
<td>1.027</td>
<td>21.70</td>
<td>22.29</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>Non EOS</td>
<td>22.28</td>
<td>1.147</td>
<td>19.30</td>
<td>22.14</td>
<td>0.64</td>
</tr>
</tbody>
</table>
6b. Applications from the NIST Short Course:

Spectral irradiance to spectral radiance transfer
Diagram of the lab setup
Calibration and Validation of spectral radiance

The spectral radiances of the FEL-plaque can be determined in two ways:

1. Using spectral irradiance to spectral radiance transfer knowing the $45^\circ/0^\circ$ reflectance factor of a plaque and the spectral irradiances of a standard source. (Calibration)

2. Using spectral radiance responsivities from the known spectral radiances of an argon-filled tungsten-strip lamp. (Validation)
Equations for spectral irradiance to radiance transfer

The irradiance at the plaque surface is given by

\[ E_F(A, \omega, \lambda, t, d) = E(50\text{cm}) \frac{(50 + \delta)^2}{(d + \delta)^2} \]  \hspace{1cm} (1)

If the spectral irradiance is uniform then

\[ L_{PE} = \frac{E(50\text{cm})}{\pi} \cdot \frac{(50 + \delta)^2}{(d + \delta)^2} \cdot \rho(45/0, \lambda) \]  \hspace{1cm} (2)

Since the spectral radiance responsivities of the detector system did not change, then the spectral radiance of the plaque is also

\[ L_{PR}(\lambda) = L_R(\lambda) \cdot \frac{G_R}{v_R} \cdot \frac{v_P}{G_P} \]  \hspace{1cm} (3)
Uncertainty Analysis – Experiment 3

Radiance of plaque $L_p$ from two methods

1. Calibration from lamp irradiance and plaque reflectance factor

$$L_{PE} = \frac{E(50\text{cm})}{\pi} \cdot \frac{(50 + \delta)^2}{(d + \delta)^2} \cdot \rho(45/0, \lambda)$$

2. Validation using radiance of strip lamp

$$L_{PR}(\lambda) = L_R(\lambda) \cdot \frac{G_R}{v_R} \cdot \frac{v_P}{G_P}$$

Where

- $v_R = \text{signal from strip lamp}$
- $v_p = \text{signal from plaque}$
- $L_R = \text{radiance of strip lamp}$
- $\rho = \text{reflectance factor of plaque}$
- $E = \text{irradiance of lamp}$
Uncertainties of Two Methods

1. Lamp irradiance transfer
   a. Lamp distance (spectrally flat)
   b. Lamp current (spectrally dependent)
   c. Plaque reflectance
   d. Plaque uniformity
   e. Signal to noise (depends on signal level (spectrally dependent))
   f. Wavelength (spectrally dependent)

2. Strip lamp radiance
   1. Strip lamp current (spectrally dependent)
   2. Signals
   3. Wavelength

Note: need to separate systematic and random effects since both lamps are traceable back to the same primary NIST standard
### Uncertainties from the radiance lamp transfer

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>RFL radiance</th>
<th>Net signal (RFL)</th>
<th>Gain(RFL)</th>
<th>Net signal (plaque)</th>
<th>Gain(plaque)</th>
<th>Plaque Radiance</th>
<th>(k=2) total uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>2972.07</td>
<td>0.00396</td>
<td>1.00E+06</td>
<td>0.00282</td>
<td>1.00E+09</td>
<td>2.12</td>
<td>4.58</td>
</tr>
<tr>
<td>400</td>
<td>9943.19</td>
<td>0.01714</td>
<td>1.00E+06</td>
<td>0.01021</td>
<td>1.00E+09</td>
<td>5.93</td>
<td>2.06</td>
</tr>
<tr>
<td>450</td>
<td>23405.37</td>
<td>0.04864</td>
<td>1.00E+06</td>
<td>0.02455</td>
<td>1.00E+09</td>
<td>11.82</td>
<td>1.37</td>
</tr>
<tr>
<td>500</td>
<td>43786.72</td>
<td>0.09614</td>
<td>1.00E+06</td>
<td>0.04264</td>
<td>1.00E+09</td>
<td>19.42</td>
<td>1.16</td>
</tr>
<tr>
<td>600</td>
<td>97489.44</td>
<td>0.20893</td>
<td>1.00E+06</td>
<td>0.07652</td>
<td>1.00E+09</td>
<td>35.70</td>
<td>1.11</td>
</tr>
<tr>
<td>654.575</td>
<td>129028.26</td>
<td>0.24759</td>
<td>1.00E+06</td>
<td>0.08397</td>
<td>1.00E+09</td>
<td>43.76</td>
<td>1.05</td>
</tr>
<tr>
<td>700</td>
<td>152685.80</td>
<td>0.25385</td>
<td>1.00E+06</td>
<td>0.08176</td>
<td>1.00E+09</td>
<td>49.18</td>
<td>0.89</td>
</tr>
<tr>
<td>800</td>
<td>191676.85</td>
<td>0.22930</td>
<td>1.00E+06</td>
<td>0.06750</td>
<td>1.00E+09</td>
<td>56.42</td>
<td>0.90</td>
</tr>
<tr>
<td>900</td>
<td>212010.44</td>
<td>0.28633</td>
<td>1.00E+06</td>
<td>0.07935</td>
<td>1.00E+09</td>
<td>58.75</td>
<td>0.90</td>
</tr>
<tr>
<td>1050</td>
<td>211363.32</td>
<td>0.24745</td>
<td>1.00E+06</td>
<td>0.06488</td>
<td>1.00E+09</td>
<td>55.42</td>
<td>0.90</td>
</tr>
</tbody>
</table>
## Uncertainties of the FEL/Plaque transfer

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>FEL irradiance at 50 cm</th>
<th>FEL irradiance at d</th>
<th>Uncertainty of Spectral Radiance</th>
<th>0/45 Reflectance Factor</th>
<th>Uncertainty of R factor</th>
<th>Uniformity</th>
<th>Spatial Plaque</th>
<th>Total Plaque</th>
<th>Total uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>9.024</td>
<td>6.711</td>
<td>0.1</td>
<td>0.985</td>
<td>0.6</td>
<td>0.5</td>
<td>2.104</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>24.443</td>
<td>18.176</td>
<td>0.1</td>
<td>1</td>
<td>0.43</td>
<td>0.5</td>
<td>5.786</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>450</td>
<td>49.024</td>
<td>36.454</td>
<td>0.1</td>
<td>1.008</td>
<td>0.4</td>
<td>0.5</td>
<td>11.697</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>80.519</td>
<td>59.875</td>
<td>0.1</td>
<td>1.01</td>
<td>0.35</td>
<td>0.5</td>
<td>19.249</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>149.318</td>
<td>111.034</td>
<td>0.1</td>
<td>1.013</td>
<td>0.35</td>
<td>0.5</td>
<td>35.803</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>654.575</td>
<td>182.219</td>
<td>135.499</td>
<td>0.1</td>
<td>1.013</td>
<td>0.35</td>
<td>0.5</td>
<td>43.691</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>204.738</td>
<td>152.245</td>
<td>0.1</td>
<td>1.013</td>
<td>0.35</td>
<td>0.5</td>
<td>49.091</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>236.210</td>
<td>175.647</td>
<td>0.1</td>
<td>1.013</td>
<td>0.33</td>
<td>0.5</td>
<td>56.637</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>244.919</td>
<td>182.124</td>
<td>0.1</td>
<td>1.014</td>
<td>0.32</td>
<td>0.5</td>
<td>58.783</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>1050</td>
<td>231.451</td>
<td>172.109</td>
<td>0.1</td>
<td>1.014</td>
<td>0.32</td>
<td>0.5</td>
<td>55.551</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>
Comparison of the two different routes

![Comparison graph showing different routes on Days 1, 2, and 3.](image-url)
References


Conclusions:

1. A calibration-validation plan (SI-traceable) for a satellite sensor should
   a) Meet the calibration requirements
   b) Describe the calibration approach (calibration)
   c) Describe the use of on-board calibrators
   d) Describe the system-level end-to-end calibration performance (validation)

2. Elements of the plan should
   a) Answer how the calibration requirements will meet the mission and instrument requirements
   b) Develop a sensor design and radiometric model (measurement equation)
   c) Characterize subsystems (uncertainty analysis)
   d) Compare model predictions and validate system level calibrations
   e) Establish pre-launch radiometric uncertainties