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The effects of model misspecification on linear regression coefficients as applicable to solar and linear terms

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Abstract

Determining atmospheric solar response from data is typically done by fitting a linear model to the data using a least squares approximation. These models typically include a solar proxy that follows the 11 year solar intensity variation, as well as a linear cooling trend. In this paper it is argued that such a regression model is flawed in that the atmospheric solar response might be out of phase with the solar input. And if so, the phase difference between solar input and atmospheric solar response can significantly bias the linear regression coefficient and attenuate the solar coefficient. This result is important because the sign of the solar response has been noted to change with altitude. Consequently, at some point between these two regions the solar response *must* go through zero, regardless of whether the actual solar response is zero at that altitude.

Introduction

Solar electromagnetic flux has an 11 year intensity variation. The solar ultraviolet output is of particular interest because of its significant impact on stratospheric and mesospheric temperatures. Overall solar intensity varies less than 1% W/m^2. In the shorter UV spectrum it varies approximately 10% W/m^2 (Donnelly, 1991; Donnelly et al. 1982). Various methods have been employed to determine how the atmosphere responds to solar input. One method involves deseasonalizing atmospheric temperatures and looking for elevated temperature levels at solar max and lower temperatures at solar min. This direct method makes no assumptions about how the atmosphere is responding to solar input and does not have the problem of coefficient bias or attenuation due to model misspecification. Two other methods employ a solar proxy: The deseasonalized temperature response can be checked for correlation with a solar proxy; or a model can be fit to the data using least squares regression. However, if the atmospheric solar response is 90° out of phase with the solar input the correlation is zero. In a least squares regression, including a solar proxy in the model has the implicit assumption of an in phase or out of phase atmospheric solar response. Depending on the phase of the solar proxy relative to the data and how the model is constituted, the solar response as indicated by the model can be zero when it is in fact nonzero.

Because of the importance of atmospheric solar heating, the atmospheric solar response is typically included in a least squares model of atmospheric temperatures. Various proxies have been employed as indicators of solar activity, such as sunspot number, F10.7 cm flux, or Mg II core-to-wing ratio. As mentioned above, by including a solar proxy in a least squares model an implicit assumption is made: The atmospheric solar response is exactly in phase or exactly out of phase with the solar proxy. In this paper the effects of this assumption being false are explored.

A typical least squares regression for atmospheric temperatures looks like this.

$$T_i = \alpha t_i + B_1\cos(2\pi\cdot t_i) + B_2\sin(2\pi\cdot t_i) + A_1\cos(4\pi\cdot t_i) + A_2\sin(4\pi\cdot t_i) + A\cdot SP_i + \varepsilon_i \,.$$

This model is mean centered so the intercept is omitted. T is a vector of atmospheric temperatures, t is the time, SP is the solar proxy, ε is the residual, α is the linear cooling rate, A is the magnitude of the atmospheric solar response; the other four terms are the annual and semi-annual oscillation respectively. The annual and semiannual terms affect the model coefficients very little. So I shall consider a simplified model.

$$T_i = \alpha' t_i + A' SP_i + \varepsilon_i' \,. \tag{1a}$$

(1a) is the proposed model to be fit to temperature data. Now suppose the true model is this,

$$T_i = \alpha_t t_i + A \cdot \sin(\omega t + \varphi) + \varepsilon_i \,, \tag{2}$$

where the atmospheric solar response has a phase offset φ. That is the atmospheric response to the solar input has phase φ. The coefficient A is the amplitude of the atmospheric solar response. As the phase offset must be measured from a reference point, the time center of the data set is selected. Furthermore, because the data may begin at any point in a solar cycle, the solar proxy in the proposed model must also have a phase offset relative to the time center of the data. The proposed model may be written as

$$T_i = \alpha' t_i + A' \sin(\omega t + \theta) + \varepsilon_i \,. \tag{1b}$$

For identification let the solar proxy SP = $\sin(\omega t + \theta)$ and the solar response SR = $\sin(\omega t + \varphi)$, where φ is the phase of the atmospheric solar response and θ is the phase of the solar proxy. As mentioned above, the phase is measured relative to the time center of the data set. For example, if the portion of the solar cycle coincident with the data is that shown in Figure 1, then the phase of the solar proxy is $\theta = 0$.

In standard normal form the coefficients for the proposed model are

$$E(\boldsymbol{\beta}') = (X^T X)^{-1} X^T T,$$

where X = [t, SP] is the data space and T are measured atmospheric temperatures. Substituting (2) into this expression and solving for the coefficients α' and A' we get the following.

$$A' = A\{ (t^T t)(SP^T SR) - (SP^T t)(SR^T t)\} / \gamma$$
$$\alpha' = \alpha + A\{ (SP^T SP)(SR^T t) - (SP^T t)(SP^T SR)\} / \gamma$$

$t^T t$, $SP^T SR$, $SP^T t$, etc, are inner products and γ is the determinant of $X^T X$. These two equations indicate that the linear term can be biased and also that the amplitude of the solar response is an attenuation of the true atmospheric solar response.

For brevity the derivations for the following are omitted. The sine term in the solar proxy and atmospheric solar response may be broken into individual sine and cosine terms. By putting these terms into least squares models the effects of bias and attenuation can be determined for various combinations of the individual sine and cosine terms. For example, if the proposed model is $E(T) = \alpha' t + B_1' \sin \omega t$ and the true model is $E(T) = \alpha t + B_1 \sin \omega t + B_2 \cos \omega t$, then α', B_1', and B_2' will be unbiased. The results are shown in Table 1. What can be concluded is that if the proposed model has a sine-like solar proxy and the true model is one of the models shown in the top row of Table 1 then the linear term is unbiased. This is true regardless of the amplitude and phase of the atmospheric solar response.

Table 1: Proposed models and true models. There are only two cases with a biased linear term.

		True Models		
	E(P), E(T)	$\alpha t + B_1 \sin \omega t + B_2 \cos \omega t$	$\alpha t + B_1 \sin \omega t$	$\alpha t + B_2 \cos \omega t$
Proposed Models	$\alpha' t + B_1' \sin \omega t + B_2' \cos \omega t$	α', B_1', B_2' (unbiased)	α', B_1' (unbiased) $B_2' = 0$	α', B_2' (unbiased) $B_1' = 0$
	$\alpha' t + B_1' \sin \omega t$	α', B_1' (unbiased)	α', B_1' (unbiased)	α' (unbiased) $B_1' = 0$
	$\alpha' t + B_2' \cos \omega t$	α' (biased) B_2' (unbiased)	α' (biased) $B_2' = 0$	α', B_2' (unbiased)

This table should be used with a degree of caution. Though the sin ωt is mathematically orthogonal to cos ωt, this does not mean it has an orthogonal inner product with real data; there might be data gaps which prevent the sum from being zero. That is if $S_i = \sin \omega t_i$ and $C_i = \cos \omega t_i$, and there are significant data gaps then $\sum S_i \cdot C_i$ is unlikely to be zero. Also, I mentioned at the beginning of this paper that a proposed model can be $T_i = \alpha' t_i + A'\sin(\omega t + \theta) + \varepsilon_i$. If we break up the sine term into individual sine and cosine terms we get $A'\sin(\omega t)\cos(\theta) + A'\cos(\omega t)\sin(\theta)$. By matching these coefficients to those of the second proposed model in the above table we get $B_1' = A'\cos(\theta)$ and $B_2' = A'\sin(\theta)$. So while B_1' and B_2' are unbiased, they are attenuated versions of the true solar amplitude.

USU temperatures

Typically, when looking for a solar response, a solar proxy and linear term are included in the model, and this model is fit to the data using a least squares approximation. Because the phase θ of the solar proxy is fixed, if there is a delayed atmospheric response to the solar input it's possible that the problems of bias and attenuation will arise in the proposed model. Suppose (2) is the true model and (1b) the proposed model.

$$T_i = \alpha' t_i + A' \cdot \sin(\omega t + \theta) + \varepsilon_i \,, \tag{1b}$$

$$T_i = \alpha t_i + A \cdot \sin(\omega t + \varphi) + \varepsilon_i \,, \tag{2}$$

where θ is the phase of the solar proxy and φ the phase of the atmospheric solar response. α is the true linear term coefficient, and A the true amplitude of the atmospheric solar response. α' and A' are the estimates of α and A.

The derivation for the equations for bias and attenuation are as follows. We can write the expected values of these two equations:

$$E(T) = \alpha' t + A' \sin(\omega t + \theta) \tag{3}$$

$$E(T) = \alpha t + A \sin(\omega t + \varphi) \,, \tag{4}$$

Using least squares to solve for the coefficients of (3) we get

$$E(\boldsymbol{\beta}') = (X^T X)^{-1} X^T E(T),$$

where $\boldsymbol{\beta}' = (\alpha', A')^T$, $X = (t, SP)$ and $SP = \sin(\omega t + \theta)$. Substituting (4) into the above expression we get

$$E(\boldsymbol{\beta}') = (X^T X)^{-1} X^T (\alpha t + A \cdot SR),$$

where $SR = \sin(\omega t + \varphi)$. Solving this expression for A' and α' we get.

$$A' = A \; \{ (t^T t)(SP^T SR) - (SP^T t)(SR^T t) \} \; / \; \gamma$$

$$\alpha' = \alpha + A \{ (SP^T SP)(SR^T t) - (SP^T t)(SP^T SR) \} \; / \; \gamma \,,$$

where γ is the determinant of $X^T X$. The inner products are summations and can be treated as integrals. Because we are assuming mean centered data the integrals are evaluated from $-t_0$ to $+t_0$, where t_0 is the maximum time of the time-centered time regressor. For example $t^T t = \sum t_i t_i \sim \int t^2 \, dt$. Evaluating this integral from $-t_0$ to $+t_0$ we get $2/3 \; t^3$. Evaluating the other inner-product terms in this manner gives the following

expressions for A' and α'.

$$A' = A\ \{\ [2t_0^3/3][\ t_0\cos(\varphi-\theta) - s_2\cos(\varphi+\theta)/2\omega - 2s_1^2\sin(\varphi)\sin(\theta)/t_0\omega^2] - [2\cdot\cos(\theta)(s_1 - \omega t_0 c_1)/\omega^2][2\cdot\cos(\varphi)(s_1-\omega t_0 c_1)/\omega^2]\}/\gamma \quad (5)$$

$$\alpha' = \alpha + A\{[t_0 - s_2\cos(2\theta)/2\omega - 2s_1^2\sin^2\theta/t_0\omega^2][2\cdot\cos(\varphi)(s_1-\omega t_0 c_1)/\omega^2] - [2\cdot\cos(\theta)(s_1 - \omega t_0 c_1)/\omega^2][t_0\cos(\varphi-\theta) - s_2\cos(\varphi+\theta)/2\omega - 2s_1^2\sin(\varphi)\sin(\theta)/t_0\omega^2]\}/\gamma \quad (6)$$

where $\gamma = (t^Tt)(SP^TSP) - (SP^Tt)^2 = \{\ [2t_0^3/3]\cdot[t_0 - s_2\cos(2\theta)/2\omega - 2s_1^2\sin^2\theta/t_0\omega^2] - [2\cdot\cos(\theta)(\ s_1 - \omega t_0 c_1)/\omega^2]^2\ \}$, $s_1 = \sin\omega t$, $c_1 = \cos\omega t$, and $s_2 = \sin 2\omega t$. A' is an attenuated true solar response coefficient A, and α' is biased from the true cooling trend α. Depending on the phase of the solar cycle and the phase of the atmospheric response α' might equal α, and A' might equal to A.

One way to test for bias in α' and attenuation in A' is to get an approximation for α' and A' from another model-fit to the data. If this model provides better estimates for α and A they can be used in (5) and (6) to see if A' and α' obtained from (1a) match what is predicted by equations (5) and (6). Let the estimates of α and A be α" and A" respectively. These estimates are obtained by fitting this model to the data.

$$E(T) = \alpha''t + C_1\sin(\omega t) + C_2\cos(\omega t), \quad (7)$$

From this one obtains the amplitude $A'' = \sqrt{(C_1^2 + C_2^2)}$, phase $\varphi'' = \mathrm{atan}(C_2/C_1)$, and linear trend α". By fitting (1b) to the data α' and A' are obtained. The parameter θ is the phase of the solar proxy input and may be calculated from the solar proxy data, but is otherwise fixed. The bias and attenuation that results from fitting (1b) to the data when (2) is the true model can be calculated from the coefficient estimates obtained from fitting (1b) and (7) to the data. The bias is α' – α" and the attenuation A'/A". If these closely match the bias and attenuation predicted by equations (5) and (6) there exist grounds for arguing for the existence of a significant phase difference between the solar input and atmospheric solar response. This method was tested using the USU data.

Applied to the USU data

The USU data spans from 9/3/1993 to 8/5/2003. This covers the portion of the solar cycle shown in Figure 1. A fitted sine function is shown in gray. The phase of the solar proxy, as determined by the fit, is θ = -0.0151 rad (-0.86°). Note that the solar proxy is very nearly sine-like. Because of this sine-like solar input, according to the information in Table 1 there should be no bias. However, because of data gaps the columns might not be orthogonal and there could be unaccounted for bias in the linear and solar coefficients. This also assumes the model is not underspecified.

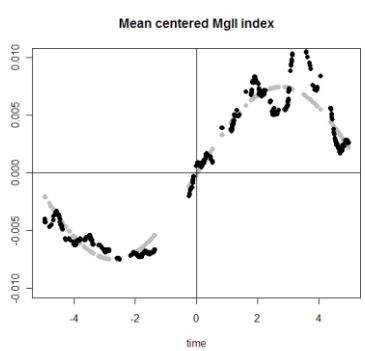

Figure 1: The mean centered MgII index plotted over the time covered by the USU data set. The sine function fit to the data is shown in **gray. The phase of the solar proxy is θ** = -0.0151 rad.

The linear trend bias and solar amplitude attenuation is obtained from fitting models (1b) and (7) to the data as described above. These are compared to the calculated bias and attenuation predicted by equations (5) and (6). This is shown in Figure 2. From this figure we can see that between 55 and 65 km there is no bias and the attenuation matches that predicted by the equations. However, above 65 km and below 45 km there is poor agreement in the bias.

This might be due to model under-specification, possibly a Pinatubo effect. She et al. (1998) found a 9 K episodic warming near the Mesopause that reaches a maximum mid-1993, which is when our USU data set begins. This

might be biasing our linear trend calculations. To remove any possible Pinatubo effects the first half-year of data was omitted and the calculations redone. (This involved removing 28 data points.) The bias and attenuation were recalculated. These are shown in Figure 5Figure 5. Notice the much better agreement from 45 to 55 km, and also above 65 km. The attenuation also has very good agreement. If the data for the entire first year is omitted from the data set, the bias and attenuation remain in good agreement (Figure 6). Notice that below 60 km there is no bias in the linear term. Above 65 km the agreement between the bias predicted by the equations and the bias determined by the models are in good agreement, as is the attenuation.

We can take a closer look at the attenuation on the solar response coefficient A' from model 1b. This is the model with a fixed solar proxy. The equations predict the coefficient A' will be attenuated according to the phase of the solar proxy and atmospheric solar response. This is shown in Figure 4 below. At most altitudes where the solar input phase is $\pi/2$ or $3\pi/2$ the amplitude of the solar proxy coefficient goes to zero. This is also true when the first half-year of the data is omitted from the analysis. See Figure 3.

The phase difference and amplitude of the atmospheric solar response are shown in Figure 7. The magnitude of the solar response from solar max to min is shown in Figure 7a. From 45 to 60 km the magnitude of the solar response is roughly 1.5 K. From 60 to 70 km it steadily increases to approximately 5 K. From 70 to 90 km it varies around 5K.

Discussion of results

There are two principle arguments here. If the solar is positive at one altitude and negative at another then naturally the amplitude of the solar response goes through zero. This might or might not say something about the amplitude of the solar response at that altitude. Moreover, if the atmospheric solar response can be positive (in phase) at one altitude and negative (out of phase) at another, perhaps it can have a phase response other than 0 or π radians. This seems plausible.

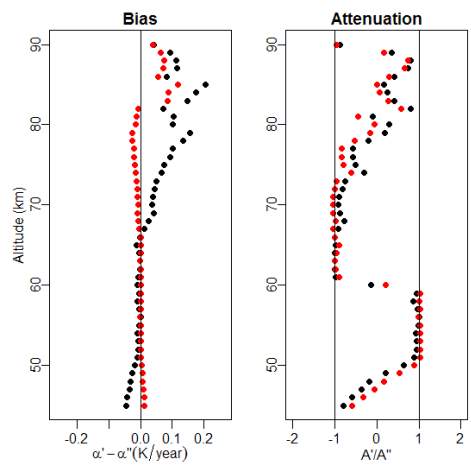

Figure 2: The bias and attenuation predicted from equations (11) and (12) compared to the bias and attenuation calculated from applying equations (1b) and (4) to the data.

The interpretation is more difficult. If the atmosphere responds in phase then when the solar UV is at a maximum atmospheric temperatures are elevated. If the atmosphere responds out of phase then if the solar UV is at a maximum atmospheric temperatures are at a minimum. But what does it mean for the atmosphere to response with a phase difference of 90º? If the phase offset is thought of as a time delay then a phase difference of 90º is equivalent to nearly 2.7 years. One doesn't normally think of the atmosphere lagging 5 years behind the solar input when the atmosphere has a negative temperature response. An out of phase response seems to point to a dynamical effect rather than a time delay. If at a given altitude the atmospheric response is 90º out of phase with the solar input, and assuming the time center of the data set is t = 0 and the solar input is sine-like, then at that altitude when the solar UV input is halfway between its max and min the atmosphere at that altitude is responding with a temperature maximum. If at another altitude the solar response is purely negative (180º out of phase) then when the solar UV input is at a minimum the atmospheric response is at a maximum. If the atmosphere has a maximum temperature response during solar minimum it seems possible for an atmospheric temperature max or min to occur halfway between solar max and min.

An analysis of the USU Mesospheric temperature data set exhibits an out of phase atmospheric solar response. Between 59 and 61 km the phase changes from being nearly in phase at 59 km to nearly out of phase at 61 km. This rapid transition suggests a zero temperature response at 60 km; the amplitude of the solar response at that altitude is 0.4 K from solar max to solar min. This result is consistent with findings from other researchers. Kubicki et al. (2008) shows an atmospheric temperature response to the solar

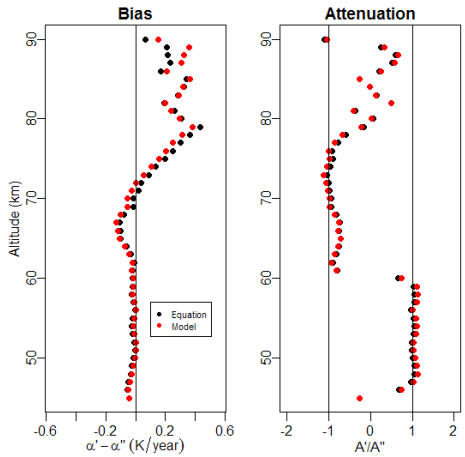

Figure 5: Same as Figure 2, except the first ½ year is omitted from the data set.

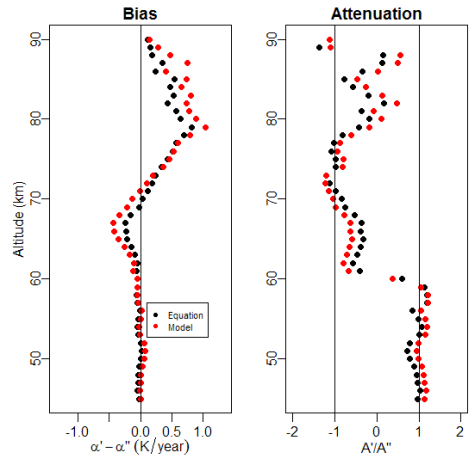

Figure 6: Same Figure 2, except the first year is omitted from the data set.

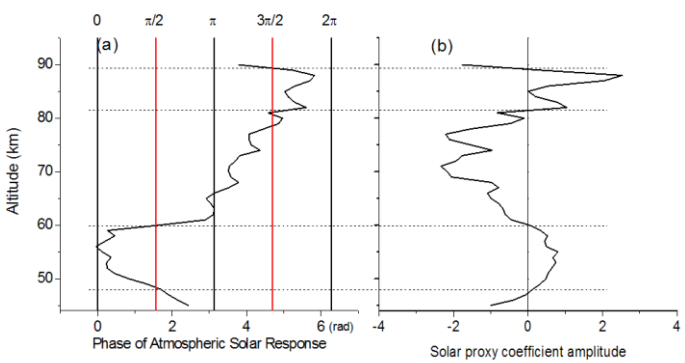

Figure 4: The phase difference between the atmospheric solar response and the solar proxy is shown in (a). The solar proxy coefficient is plotted in (b). At most of the points where the phase difference between the solar input and the atmospheric solar **response is π/2 or 3π/2 the amplitude of the solar**-proxy coefficient goes to zero. The exception is at 78 km. This could be due to effects that are not accounted for in the model.

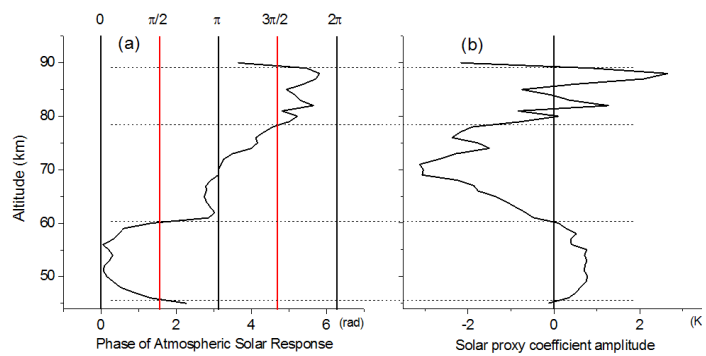

Figure 3: The phase difference between the atmospheric solar response and the solar proxy is shown in (a). The solar proxy coefficient is plotted in (b). This is with the first half-year of the data omitted from the analysis. At most points where the phase difference **between the solar input and the atmospheric solar response is π/2 or 3π/2 the amplitude o**f the solar-proxy coefficient goes to zero. Again the exception is at 78 km.

input transitioning from positive to negative at 59 km during winter and 52 km during summer. Keckhut and Kodera (1999) found a temperature change from positive to zero at 52 km for winter but a fairly uniform temperature response of 1 K from 30-55 km for summer. A similar temperature response at 50 km was found by Keckhut et al. (1995) as well as Cossart and Taubenheim (1987). Chanin et al. (1987) Figure 2 shows deseasonalized temperatures from 1979 to 1985 from 40 to 65 km, along with the 10.7 cm

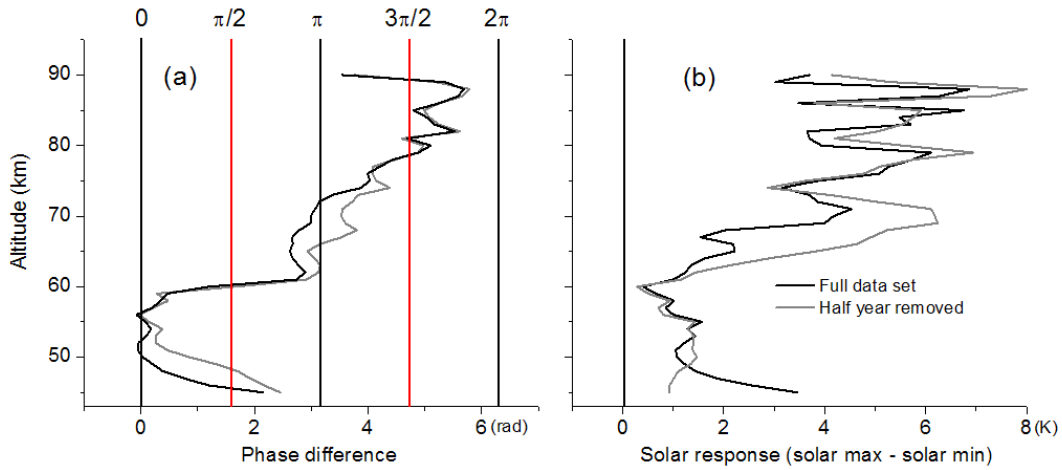

Figure 7: (a) Phase difference between the solar proxy (solar input) and the atmospheric solar response. (b) The magnitude of the atmospheric solar response from solar maximum to solar minimum.

solar flux for that time period. At 40 km there is a clear negative response, at 50 km the temperature response is zero and at 65 km it is positive. All this suggests the atmospheric temperature response in that altitude region is likely to be nearly zero. The more prominent question is about the phase at higher altitudes. According to an analysis of the USU data, between 80 and 90 km the phase varies around 3π/2, and the attenuation and bias predicted by an out of phase solar response seems to be present in our data. In analyzing data from the HALOE experiment Remsberg et al. (2002) found a phase lag of 2.3 years at 40º N at 0.05 hPa. This 2.3 year lag is of interest because a phase difference equivalent to one-quarter period is equivalent to approximately 2.7 years. (USU is located at 41.7º N.) They also report a lag of 1.9 and 1.5 years at 0.03 hPa and 0.02 hPa respectively at same latitude (Table 7). The data analysis in Remsberg included a solar phase offset in the regression analysis; instead of a solar proxy a sine function with a phase offset was employed. Our results don't exactly match those of Remsberg but a phase offset of 2.4 years indicates a significant phase difference can occur. The HALOE data in Remsberg covers approximately the same time period as the USU data set: 9.5 years from late 1991 to early 2001 for HALOE; late 1993 to late 2003 for USU. In an updated paper Remsberg and Deaver (2005) report an analysis of the HALOE data from 1991-2004 which shows a phase lag of 3.8 years at 0.05 hPa and 2.2 years at 0.03 hPa. This is confirmed again in Remsberg (2008) with a phase lag of 4.5 years at 69 km; they show a negative phase lag between 58 and 63 km. The difference in the height of the phase offset in our data might partly be due to a zonal asymmetry in the solar response. Simulations by Hampson et al. (2006) show zonal asymmetries in atmospheric solar response of up to 10 K (solar max to solar min) at 49 km. They also listed several important differences in atmospheric solar response profiles from six different data collection sites.

Conclusions

If a fixed phase solar proxy is employed in a least squares regression analysis of atmospheric temperatures to extract the amplitude of the atmospheric solar response, if the atmosphere is responding out of phase to the solar input the regression solar response amplitude can be attenuated and the cooling rate severely bias. An analysis of the USU temperature data indicates an atmospheric solar response of 1 K (max to min) between 45 and 60 km, and approximately 5 K (max to min) between 75 and 90 km. Had

a solar proxy been fit to the data we would not have found these results.

There is good evidence to indicate that the atmospheric solar response between 50 and 60 km is very small. The analysis by Remsberg et al. (2008) shows a significant phase lag in the atmospheric solar response. Remsberg shows a phase lag of 4.5 years at approximately 68 km at 40º N. At that altitude the USU data shows an out of phase atmospheric solar response. This difference might be due to zonal asymmetries in the atmospheric solar response.

An analysis of the bias and attenuation match those predicted by the models. Though more analysis is needed, it does seem reasonable that the atmosphere can have a response to the solar input that is in phase, out of phase, or any other phase offset to the solar input.

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