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Diffusive Electron Heat Flow And Temperature Variance Along Magnetic Field Lines

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ABSTRACT

In this research we examine how electron heat moves along magnetic field lines and how this affects temperature variations in plasmas. Specifically we wrote FORTRAN code to solve the electron temperature equation numerically. We also solved the steady state electron temperature equation analytically using an integrating factor. We verified that the numerical and analytical solutions obtained the same result. Finally we calculated the standard deviation of temperature in our domain for the steady state. Gaussian legendre quadrature was used to integrate various functions. We represented our magnetic field and heat source with Fourier series. The sin and cosine coefficients for the heat source and the inhomogeneous magnetic field strength were given in an input file along with other initial conditions which our code read prior to each run. This allowed different numerical experiments to be run without the need to recompile the code for each one. The primary result that was found was that an increase in the initial background temperatures led to smaller variations in temperature. That is, as plasma collisionality decreases with increasing mean temperature, diffusive electron heat flow is capable of smoothing out temperature perturbations more effectively.

1. Introduction

In our world today the average human is becoming more dependent on energy consumption. As traditional methods like coal, oil, and natural gas lead to environmental concerns and alternative renewable sources like solar, hydro, and wind are either currently too inefficient or lack scalability, we are left with a promising candidate in nuclear power. Current nuclear power involves fission, the process of breaking large nuclear isotopes apart and harnessing the energy from the separation. This provides large energy gains with little environmental impact. The largest drawback to nuclear fission is the radioactive materials that are produced from the fission process. One solution to this is to use nuclear fusion, the joining of two light nuclei. In contrast to fission, the energy gains are more substantial per unit mass and little hazardous radioactive material is produced. The difficulty for fusion lies in the temperatures required to overcome the electrostatic repulsion of the two nuclei being smashed together. One current method for managing particles at these high temperatures is by confining them with magnetic fields. The tokamak, shown in the figure below, is the most common device used.

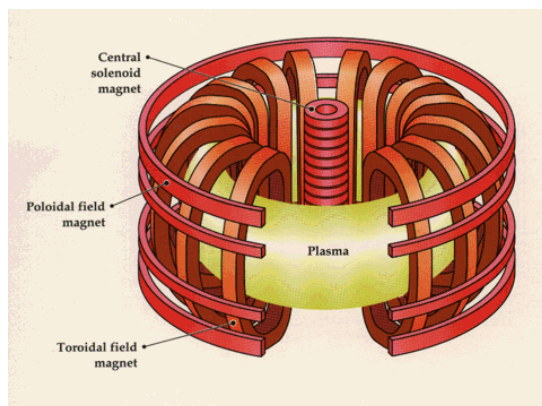


Figure 1: A simple tokamak confines high energy particles with toroidal (long way around the torus) and poloidal (short way around the torus) magnetic fields

An understanding of how tokamaks confine electron thermal energy is the motivation for this research. In this project we will examine how electron heat flow leads to different temperature variance along magnetic field lines.

2. Electron Motion in Magnetic Fields

The most basic electron motion involves a constant magnetic field and leads to circular motion around this field. If the electron has any initial velocity in the direction of the magnetic field then this circular motion becomes helical. This is shown in the figure below.

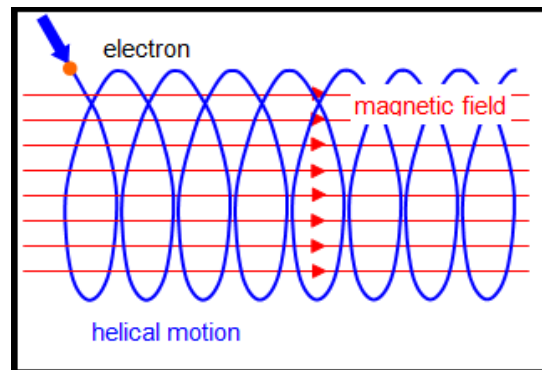


Figure 2: The magnetic part of the Lorentz force

leads to helical motion in a constant magnetic field

The tokamak takes this fairly simple motion one step farther by wrapping the magnetic field back around on itself and thus creating a magnetic loop for the electron or any ion to cycle around. Unlike the simple case above, the tokamak does not have a constant magnetic field since the toroidal field is proportional to the inverse of the major radius. This leads to particles with small velocities parallel to the magnetic field bouncing back and forth as they go from lower to higher field strengths. Particles with larger components for their parallel

velocities continue to cycle around the torus. The interactions between these trapped and passing particles leads to more complex motion. For our research we used the electron temperature equation to model heat flow for our charged particles. We then looked at the numerical solution to the electron temperature equation and compared this to the analytical solution for the steady state temperature equation.

3. Methods

In this section we will explain how we solved the steady state temperature equation analytically. We will also look at how we represented the magnetic and heat source terms in FORTRAN and the integration technique that was used when calculating the standard deviation of temperature. The code for both the numerical solution and analytical solution can be found in the appendix.

a. Analytical Solution

For the analytical solution we start with the following steady-state equation relating the heat source, S , to the parallel electron heat flow, q_{\parallel} :

$$1) \quad S = \vec{\nabla} \cdot (q_{\parallel} \hat{b}) = \hat{b} \cdot \vec{\nabla} q_{\parallel} + q_{\parallel} \vec{\nabla} \cdot \hat{b} = \frac{dq_{\parallel}}{dL} - q_{\parallel} \frac{d \ln B}{dL}$$

This is the steady-state form of the electron temperature evolution equation. Since this differential equation is of the form $y' + f(x)y = g(x)$ we know we can solve it using an integrating factor. The integrating factor is calculated as

$$2) \quad u(L) = e^{\int^L -\frac{d \ln B}{dL'} dL'} = e^{-\ln B} = \frac{1}{B}$$

Multiplying each term in our equation by this factor yields

$$3) \quad \frac{1}{B} \frac{dq_{\parallel}}{dL} - \frac{1}{B} \frac{d \ln B}{dL} q_{\parallel} = \frac{d}{dL} \left(\frac{q_{\parallel}}{B} \right) = \frac{S}{B}$$

Finally we can integrate both sides from 0 to L to get our final result

$$4) \quad \frac{q_{\parallel}(L)}{B(L)} - \frac{q_{\parallel}(0)}{B(0)} = \int_0^L dL' \frac{S}{B}$$

S and B are represented as Fourier series and we use Gauss-Legendre integration to calculate the right side integral.

b. Gauss Legendre Integration

When evaluating integrals we often require a numerical approximation scheme since the analytical solution may be unattainable. These approximation schemes are known as quadrature rules and typically they are given as a sum of weights at various points multiplied by the integrand evaluated at these same points. A well known example is the trapezoidal rule which uses the weights as the distance between two points multiplied by the integrand value at these points. This rule is shown in the equation below.

$$\int_a^b f(x) dx \approx \frac{1}{2} \sum_{i=1}^N (x_{i+1} - x_i) * (f(x_{i+1}) + f(x_i))$$

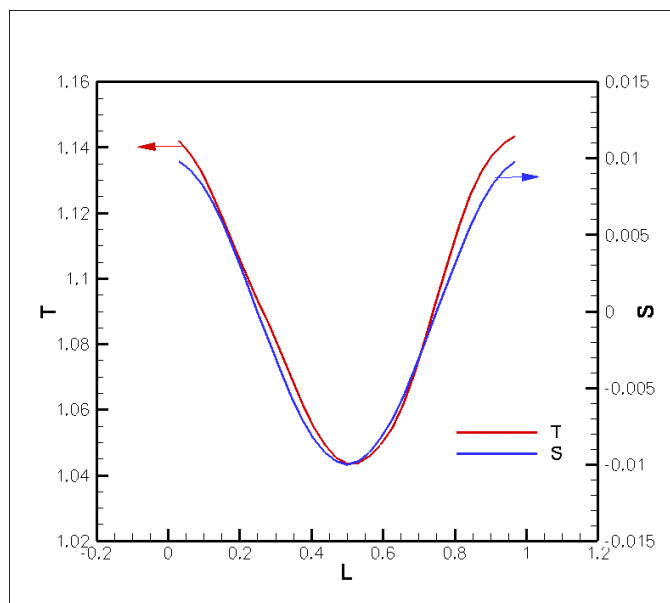
Gaussian Legendre quadrature is a method used to provide exact numerical results with n weights for a function represented as a polynomial of degree $2n - 1$. ^(Numerical Recipes, 1992) These weights are associated with the Legendre polynomials in the following way:

$$w_i = \frac{2}{(1-x_i^2)[P'_n(x_i)]^2}, \text{ where } P_n \text{ is the } n\text{th Legendre Polynomial, and}$$

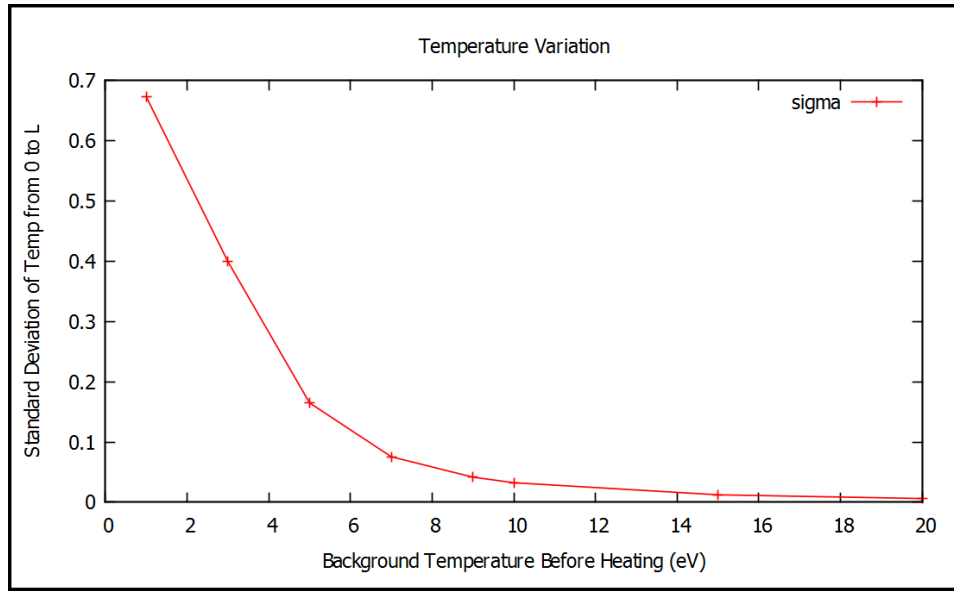
$$\int_{-1}^1 f(x) dx = \sum_{i=1}^n w_i f(x_i)$$

4. Results

In this section, we present results from numerical solutions of the electron temperature equation $\frac{3}{2}n\frac{dT}{dt} = -\vec{\nabla}\cdot(q_{\parallel}\vec{b})+S$. Comparing the numerical solution with the analytical steady state solution found that the two were consistent as long as $\int_0^L \frac{S}{B} = 0$. This periodicity restraint came from the fact that $\frac{q_{\parallel}(L)}{B(L)} - \frac{q_{\parallel}(0)}{B(0)} = 0$ and seems to indicate we are not free to choose any magnetic field and heat source. The next thing we examined was how the standard deviation of temperature varied as we increased the background temperature. The first graph shows what heat source was used for the calculations and the second graph shows how the temperature variation changed when we altered the initial background temperature. The decrease in temperature variation comes from the diffusive electron heat flow smoothing out temperature perturbations more effectively because of the decrease in plasma collisionality with the increased mean temperature.

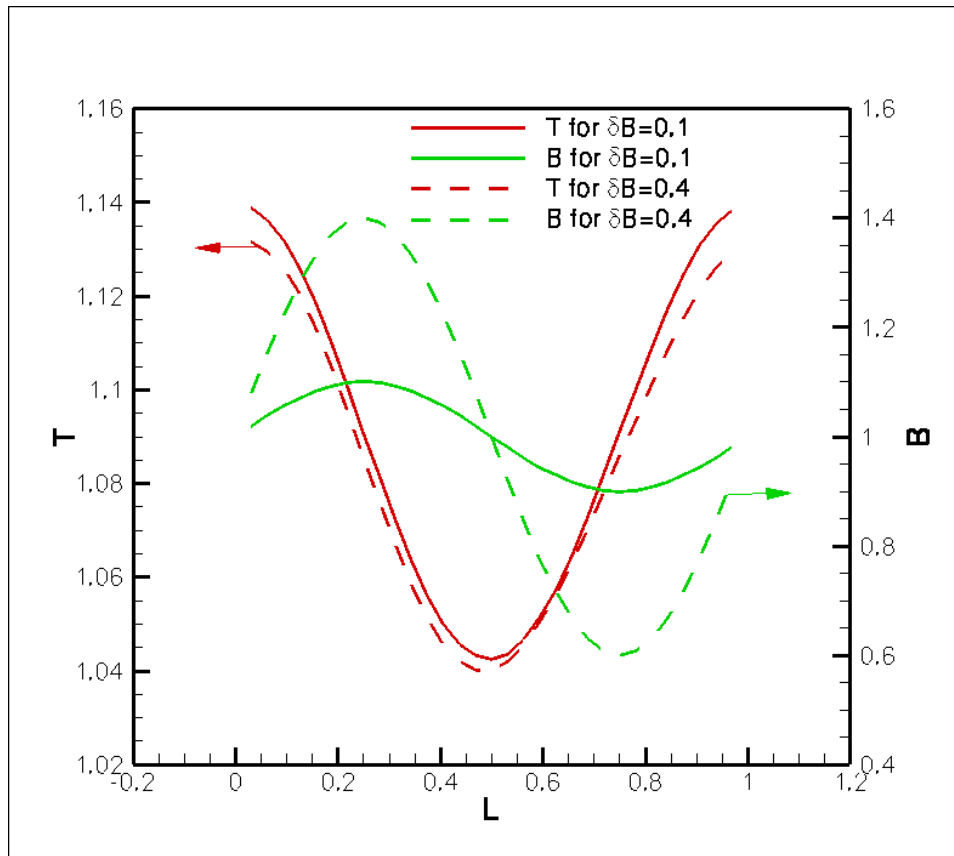


Graph 1: Constant heat source and the resulting steady-state temperature for 10 eV background temperature are shown. This corresponds to the 10 eV data point in Graph 2

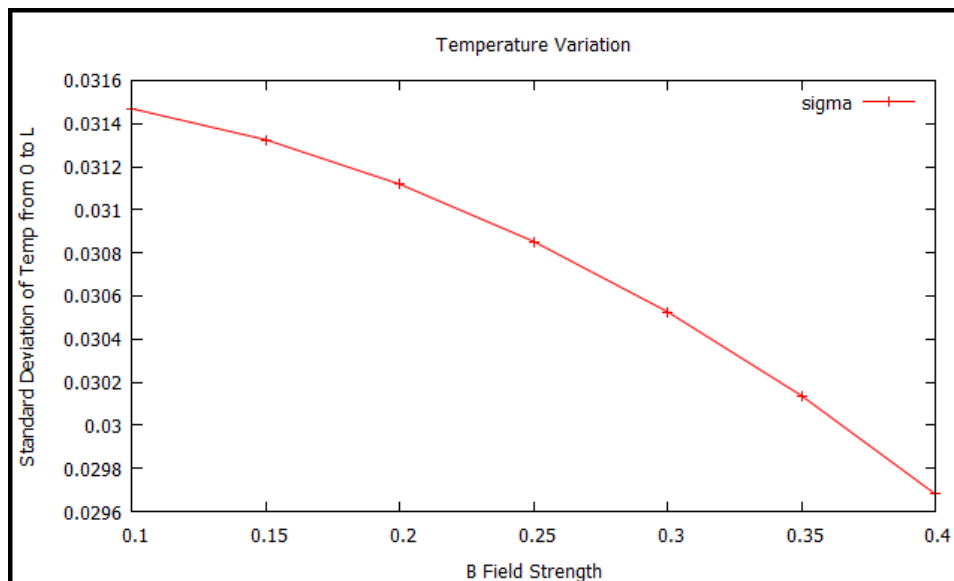


Graph 2: The standard deviation of temperature is shown to decrease as we increase our initial background temperature which lowers collisionality and makes diffusive heat transport more robust

Finally, we also examined how altering the magnitude of the sinusoidal perturbation in magnetic field strength affected temperature variation. In this experiment we held the initial background temperature at 30eV and altered our B source, that is the strength of the sinusoidal variation in $|B|$. The first graph shows the weaker and stronger sinusoidal terms that we used. The second graph shows how the temperature variation was affected by changing the strength of the sinusoidal variation in $|B|$.



Graph 3: The weak and strong sinusoidal terms are shown with the temperature



Graph 4: The standard deviation of temperature is shown to decrease as we increase the magnitude of the sinusoidal perturbation in magnetic field strength

5. Conclusion and Future Study

In conclusion, we were able to model electron temperature flow given a specific magnetic field and a heat source. We were able to verify that increasing the background temperature led to less temperature variations because of decreased plasma collisionality. Currently our numerical solution only solves for the linear case. For future study I would like to visit the nonlinear case and attempt to modify the code that simulates this.

APPENDIX

FORTRAN Code Used for Numerical and Analytical Solution

a. Numerical Solution

```
!-----  
!  initial conditions.  
!-----  
T=0._r8  
T(1)=1._r8 ! constant (n=0) norm. temperature at t=0  
F=0._r8 ! zero until heat source is turned on  
DO imode=1,nmodes  
  karr(imode)=(0,1)*(imode-1)*twopi  
ENDDO  
DO istep=1,nstep ! time loop  
  time = (istep-1)*dtm  
!-----  
!  Temperature equation.  
!-----  
T_old=T  
fac = (1. - exp(-(time/t_heat)))  
IF (linear) THEN  
  dT = karr*T_old  
  CALL fftx(work(1,:),dT(:,),1,'inverse',1,compt,compit,nphi,lphi)  
  work(1,:)=work(1,:)*redlnB(1,:)   
  CALL fftx(work(1,:),dTdB(:,),1,'forward',1,compt,  
$      compit,nphi,lphi)  
  DO imode=1,nmodes  
    qp(imode) = -kappa*dT(imode)  
    dT(imode) = -dtm*(karr(imode)*qp(imode)-S(imode))  
$      -dtm*kappa*dTdB(imode)  
  ENDDO  
  T=T_old+dT
```

b. Analytical

```
!-----  
! Compute qp from steady state differential equation.  
! Method below is pseudo-spectral where the Fourier series  
! of the integrand, S/B, is determined and the integral is  
! performed essentially analytically.  
!-----  
  ALLOCATE(SB(nmodes))  
  ALLOCATE(qp_ss(2,nphi))  
  ALLOCATE(int_SB(nphi))  
  ALLOCATE(reSB(1,nphi))  
  reSB = reS / reB !get S/B  
  CALL fftx(reSB,SB,1,'forward',1,compr,compi,nphi,lphi) !get fourier coefficients  
  DO i=1, nphi !From 0 to L  
    qp_ss(1,i) = REAL(SB(1))*(i-1._r8)/nphi  
    angle = twopi*(i-1._r8)/nphi  
    DO imode=2,nmodes  
      qp_ss(1,i) = qp_ss(1,i) + (REAL(SB(imode))*sin((imode-1)*angle)  
$   + AIMAG(SB(imode))*(cos((imode-1)*angle-1))/(pi*(imode-1))  
    ENDDO  
  ENDDO
```

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