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Jodie Corbridge Gillespie
Utah State University

JR Dennison
Utah State Univesity

Alec M. Sim
Utah State University

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Density of State Models and Temperature Dependence of Radiation Induced Conductivity

Jodie Corbridge Gillespie, JR Dennison, and Alec M. Sim

***Abstract*—Expressions are developed for radiation induce conductivity (RIC) over an extended temperature range, based on density of states models for highly disordered insulating materials. A general discussion of the DOS of can be given using two simple types of DOS distributions of defect states within the bandgap for disordered materials are considered, one that monotonically decreases within the bandgap and one with a distribution peak within the band gap. Three monotonically decreasing models (exponential, power law, and linear), and two peaked models (Gaussian and delta function) are considered, plus limiting cases with a uniform DOS for each type. Variations using the peaked models are considered, with an effective Fermi level between the conduction mobility edge and the trap DOS, within the peaked trap DOS, and between the trap DOS and the valence band. The models are compared to measured RIC values over broad temperature ranges for two common materials, low density polyethylene (LDPE) and disordered silicon dioxide.**

***Index Terms*— radiation induced conductivity, conductivity, density of states, disordered materials, temperature**

I. Introduction

The conductivity of a material is the primary property for determining charge transport, and hence the dissipation rate, of accumulated charge within a material. A material's conductivity can be determined through straightforward measurements of current under an applied field, but it can have complex dependence on time, temperature, electric field, magnetic field, and the magnitude and rate of charge deposition [1-3]. Another way to increase conductivity is through the deposition of energy by incident high energy radiation which results in excitation of charge carriers into the conduction band (CB), a process called radiation induced conductivity (RIC). When the incident particle radiation is energetic enough, it penetrates completely through the material, thereby avoiding charge accumulation. The increased number of charge carriers, and hence the magnitude of the enhanced conductivity, is dependent on a number of factors including temperature and the spatial and energy dependence and occupation of the material's distribution of localized trap states within the band gap—or defect density of states (DOS). Expressions for RIC in terms of the filling of the DOS up to an effective Fermi level were largely developed by Rose [4-5], and were extended by Fowler [6-7], Vissenberg [8], and others. Under these conditions, the enhanced conductivity can be compared to a photoconductivity and is the starting point for understanding the mechanisms involved [4-7,9]. A number of useful reviews of the subject are available [2,4,10-13].

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Jodie Corbridge Gillespie (e-mail:JodGillespie@gmail.com) and JR Dennison (e-mail: JR.Dennison@usu.edu) are with the Materials Physics Group in the Physics Department at Utah State University in Logan, UT 84322 USA.

II. Density of States Distributions

A. Description of DOS Distributions

To obtain a generic model of temperature dependent RIC behavior, one must first develop a general discussion of the distribution of localized defect states (density of states or DOS) in relation to the CB. We consider two types of distributions: one that has a distribution peak within the band gap and one that monotonically decreases within the band gap (*i.e.*, has a peak at energies in the CB). In other words, the two types of distributions have a maximum amplitude at energies of either $E \leq 0$ or $E > 0$.

We consider seven specific DOS modes as shown in Table 1 and Fig 1. These are three monotonically decreasing models (exponential, power law, and linear) and two symmetric peaked models (Gaussian and delta function), plus a limiting case with a uniform DOS for each type. All DOS distributions, $n_A(E)$, are functions of energy E as measured from the CB edge, E_C, towards the valence band (VB) edge, E_V. Each distribution has an energy width and corresponding effective temperature associated with it of the form $E_o^A \equiv k_B T_o^A$ with: $A = X$ for exponential, $A = P$ for power law, $A = L$ for linear, $A = U$ for uniform, $A = G$ for Gaussian and $A = D$ for delta function distributions (see Table 1, column 3).

All distributions are normalized to the total defect density, N_t, by integrating over the entire bandgap (BG):

$$N_t \equiv \int_0^\infty n_A(E) dE. \quad (1)$$

Each distribution also has an energy centroid (or first moment) associated with it (see Table 1, column 4). This mean energy of all (both occupied and unoccupied) states of the distributions within the BG is

$$E_{centroid} = \frac{1}{N_T} \int_0^\infty E \, n_A(E) dE \quad . \quad (2)$$

For the decreasing distributions the centroid can be expressed in terms of the width, while for the symmetric peaked distributions there is an independent centroid, E_o^t, at the maximum of the trap distribution.

The mean energy and fraction of all *occupied* trap states within the BG are

$$\langle E_A(T) \rangle = \frac{1}{N_T} \int_0^\infty E \, f_{FD}(E) \, n_A(E) dE. \quad (3)$$

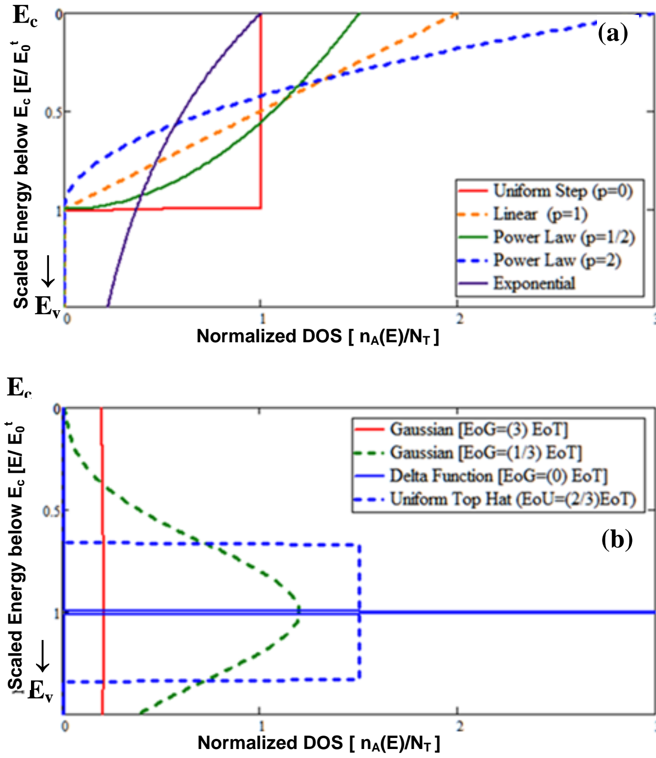

Fig. 1. Density of states (DOS) models. The graphs plot the normalized energy below the conduction band edge as a function of the normalized DOS, $n_A(E)$ / N_T. (a) Monotonically decreasing DOS models, including the exponential, power law, and linear models, as well as the limiting case uniform step model. Power law distributions are shown for two cases, p = ½ < 1 and p = 2 > 1. The energies are normalized by dividing by the width of the distributions, E_o^A. (b) Symmetric peaked DOS models, including the Gaussian and delta function models. Gaussian distributions are shown for two cases, (E_o^G/E_o^t) = ⅓ < 1 and (E_o^G/E_o^t) = 3 > 1; the later approaches the limiting case uniform top hat model. The energies are normalized by dividing by the center or peak of the distributions, E_o^A or E_o^t, respectively.

$$f_A(T) \equiv \frac{n_t(T)}{N_T} = \frac{1}{N_T}\int_0^\infty f_{FD}(E,T)\, n_A(E)dE \quad . \tag{4}$$

B. Temperature Dependence of Occupied States

The temperature dependence of E_A is contained in the Fermi-Dirac distribution function, $f_{FD}(E)$, that describes occupation of the trap states:

$$f_{FD}(E,T) = \begin{cases} \left\{1+exp\left[-\left(E-E_F^{eff}\right)/k_BT\right]\right\}^{-1} & ; \quad \text{exact, all } T \\ \Theta\left(E-E_F^{eff}\right) & ; \quad T=0\text{ K} \\ \begin{cases} 0, & \text{if } \left(E_F^{eff}-E\right) > 2k_BT \\ \frac{1}{2}\left\{1+\left[\left(E-E_F^{eff}\right)/2k_BT\right]\right\}, & \text{otherwise} \\ 1, & \text{if } \left(E-E_F^{eff}\right) > 2k_BT \end{cases} & ; \quad \text{low T} \\ exp\left[\left(E-E_F^{eff}\right)/k_BT\right] & ; \quad \text{high T} \end{cases} \tag{5}$$

Figure 2 shows a comparison of the exact distribution with the zero and low T approximations, and the associated absolute errors. The absolute error of the low T approximation is ≲11% independent of T (see Fig. 2(b); the error in the integral expression for $n_t(E)$, Eq. (4), has a maximum error of ±6% when $E_o^A > 4k_BT$, and is typically much less for slowly varying DOS near E_F^{eff}. Further, since this approximation is

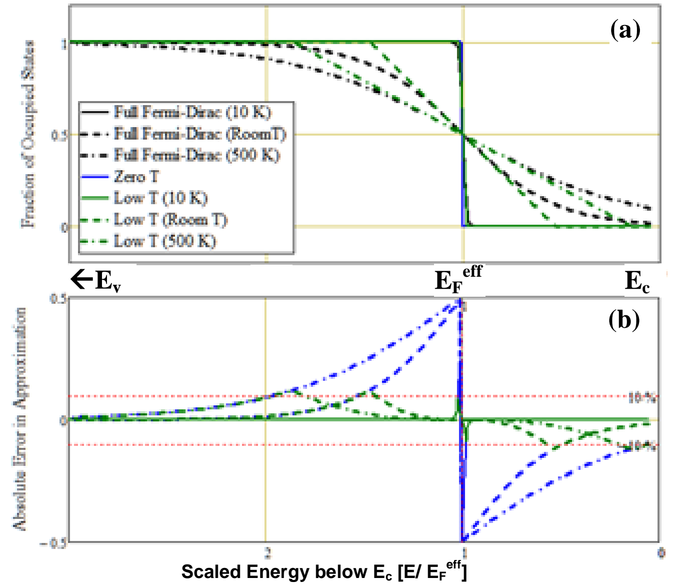

Fig. 2. Fermi Dirac distribution function approximations. **(a)** Fraction of occupied states versus a scaled energy, $[E/E_F^{eff}(T)]$ from E_C≡0 to $3{\cdot}E_F^{eff}$ (taken here as 0.3 eV) at three temperatures: (i) a low temperature, 10 K, which is below typical spacecraft operating environments and temperatures at which RIC is measured; (ii) room temperature; and (iii) a high temperature, 500 K, above which most polymeric materials melt or disassociate and few spacecraft operate. **(b)** Absolute error versus scaled energy, for the zero and low temperature approximations. The relative error peaks at $\pm[2k_BT/E_F^{eff}(T)]$ are consistent at ~11%, independent of T.

antisymmetric about E_F^{eff}, only deviations from a symmetric DOS contribute to the integral of interest in Eqs. (3) and (4). Note the high temperature approximation is the Maxwell-Boltzmann distribution.

From Eqs. (3) and (4), as T→0 K (or for $n_A(E)$ symmetric about E_F^{eff}),

$$\langle E_A\rangle_0 = \int_{\left(E_F^{eff}\right)}^\infty E\ n_A(E)d\varepsilon \qquad \text{and} \tag{6}$$

$$f_{A0} = \frac{1}{N_T}\int_{\left(E_F^{eff}\right)}^\infty n_A(E)dE \quad . \tag{7}$$

We can thus expect the low T approximations for $f_A(T)$ can be effectively expressed as expansions in terms of $[E_o^t - E_F^{eff}(T)]$..

The fraction of occupied states at T = 0 K for each distribution is listed in column 5 of Table 1. For the decreasing distributions f_{A0} can be expressed in terms of the width, E_o^A, and E_F^{eff}; the symmetric peaked distributions required an additional independent centroid, E_o^t. References [2] and [14] offer additional details for the specific DOS models, and discuss their use in various electron transport processes and their appearance in the related literature.

III. Equations for RIC

A. Conductivity Equations

The RIC contribution to the total conductivity, σ_{total}, adds to the material's dark current conductivity, σ_{DC} [17]:

TABLE 1. Density of states models.

DOS Type	Normalized DOS Function, $n_A(E)$	Width, E_o^A [a]	Centroid, $E_{centroid}$ [b]	Fraction of Occupied Traps at T→0, f_{A0} [c]
Monotonically decreasing DOS models with $E_o^t \leq 0$.				
Exponential	$n_X(E;E_o^X) = N_T \left[\frac{1}{e \cdot E_o^X}\right] \exp\left(\frac{E_o^X - E}{E_o^X}\right)\Theta(E)$	E_o^X $\left(\frac{1}{e}\text{ width}\right)$	$E_o^X \equiv k_B T_o^X$	$\exp\left(\frac{-E_F^{eff}}{E_o^X}\right)$
Power Law	$n_P(E;E_o^P) = N_T \left[\frac{(p+1)}{E_o^p}\left(\frac{E_o^p - E}{E_o^p}\right)^p\right] \Theta(E_o^p - E)\,\Theta(E)$	E_o^P	$\left(\frac{1}{p+2}\right) E_o^P$	$\left(\frac{E_o^p - E_F^{eff}}{E_o^p}\right)^{(P+1)}$
Linear (Power Law, *p* = 1)	$n_L(E;E_o^L) = N_T \left[\frac{2}{E_o^L}\left(\frac{E_o^L - E}{E_o^L}\right)\right] \Theta(E_o^L - E)\,\Theta(E)$	E_o^L	$\left(\frac{1}{3}\right) E_o^L$	$\left(\frac{E_o^L - E_F^{eff}}{E_o^L}\right)^2$
Uniform Step (Limit of Top Hat, $E_1^U \to 0$) (Limit of Power Law, *p* = 0)	$n_{US}(E;E_o^U) = N_T \left[\frac{1}{E_o^U}\right] \Theta(E_o^U - E)\,\Theta(E)$	E_o^U	$\frac{1}{2} E_o^U$	$\left(\frac{E_o^U - E_F^{eff}}{E_o^U}\right)$
Peaked DOS models with $E_o^t > 0$.				
Gaussian	$n_G(E;E_o^G,E_o^t) = N_T \left[1 + erf\left(\frac{2\,E_o^t}{\sqrt{2}\,E_0^G}\right)\right]^{-1} \times \left[\frac{2}{\sqrt{2\pi}\cdot E_o^G}\right] \exp\left[-\frac{1}{2}\left[\frac{(E_o^t - E)}{E_o^G}\right]^2\right]\Theta(E)$	$2\,E_o^G$ (2X Standard Deviation)	Centroid: $E_o^t + \frac{2}{\sqrt{2\pi}\cdot E_o^G}\left\{\frac{e^{-\frac{1}{2}\left(E_o^t/E_o^G\right)^2}}{\left[1 + erf\left(\frac{2\,E_o^t}{\sqrt{2}\,E_0^G}\right)\right]}\right\}$ Mean Energy: $\left[1 + 2erf\left(\frac{E_o^t - E_F^{eff}}{\sqrt{2}\cdot E_o^G}\right)\right] \Big/ \left[1 + erf\left(\frac{2\,E_o^t}{\sqrt{2}\,E_0^G}\right)\right]$	
Delta Function (Limit of Gaussian, $E_o^G \to \infty$)	$n_D(E;E_o^t) = N_T\ \delta(E_o^t - E)$	$E_o^G \to 0$	E_o^t	1
Uniform Top Hat (Limit of Constant, $E_1^U = 0$ and $E_2^U \to \infty 0$) (Limit of Uniform Step, $E_1^U > 0$) (Limit of Gaussian, $E_o^G \to \infty$)	$n_{UT}(E;E_1^U,E_2^U) = N_T \left[\frac{1}{E_2^U - E_1^U}\right] \Theta(E_2^U - E)\,\Theta(E - E_1^U)$	$E_o^G \to \infty$ $E_o^U \equiv E_2^U - E_1^U$	$E_o^t = \frac{1}{2}\,(E_2^U + E_1^U)$	$\left(\frac{E_o^U - E_F^{eff}}{E_o^U}\right)$

$\Theta(E)$ is a Heaviside step function, equal to 0 at $E < 0$ and 1 at $E > 0$.
$\delta(E)$ is the Dirac delta function, equal to infinity at E and zero elsewhere.
erf(E) is the error function evaluated at *E*.
[a] From Eq. (6).
[b] Mean energy of trap state within band gap, from Eq. (2).
[c] From Eq. (7).

$$\sigma_{total} = \sigma_{DC} + \sigma_{RIC}. \quad (8)$$

RIC conductivity, σ_{RIC}, has been found to follow a simple power law, both theoretically [4,6,10,12] and experimentally [1,7,9,11,13,15]

$$\sigma_{RIC} = k_{RIC}(T)\,\dot{D}^{\Delta(T)}. \quad (9)$$

To develop a temperature-dependent expression for σ_{RIC} based on material-dependent parameters, we begin with an analogy to a semiconductor system, with dopant states at a single energy, E_d. The effective Fermi level is the energy at which 50% of the states are occupied and may depend on temperature, dose rate, and charge distribution. We assume a reservoir of trapped electrons pinned to the effective Fermi level, E_F^{eff}; that is, with nearly constant excitation energies such that E_d in the semiconductor system is replaced by E_F^{eff} in highly disordered insulating materials.

The fundamental equation for conductivity

$$\sigma = \sum_i^{carrier\ types} q_i\,\mu_i\,n_i \to q_e\,\mu_e\,n_c(T) \ . \quad (10)$$

requires expressions for the carrier charge, q_i, the carrier mobility, μ_i, and the density of occupied carrier states, n_i. We restrict the discussion here to electron conduction so $q_i \to q_e$ and $n_i \to n_e \equiv n_c(T)$ in a single term in the sum of Eq. (10) (Refer to [2,4,10,12] for discussion of other carrier types.) As argued by Rose [5] for volume-excited photocurrents, we also assume that the mobility is constant and determined only by the free (or nearly free) electron mobility, $\mu_i \to \mu_e$. The effective mobility of nearly free electrons in a dielectric medium can be approximated by: (i) replacing the electron mass, m_e, with its effective mass, m_e^*, to model the weak uniform binding potential experienced by electrons traveling in the conduction band, and (ii) including the relative dielectric constant of the material, ε_r, to account for screening or polarization of the trap center charge by the charge background of the medium. That is, $\mu_e^{NF} \approx \mu_e[m_e/\,m_e^*]\varepsilon_r^{\,2}$. Using these assumptions for fixed values for CB electrons, q_e and μ_e [14], it is only $n_c(T)$ that controls the temperature dependence of $\sigma_{RIC}(T)$.

We can now develop an expression for the temperature-dependent density of (nearly) free electrons in the CB, $n_c(T)$. We assume that there are no interactions between electrons in trap states, or equivalently that the mean spatial separation of defects is larger than their interaction range. Then $n_c(T)$ is

given by Ashcroft and Mermin [16] as

$$n_c(T) = N_c e^{-E_F^{eff}(T)/k_B T} + n_c^o \quad , \tag{11}$$

where N_c is the total density of accessibly energy states that can be thermally excited into the CB (from within a few $k_B T$ of the CB edge—in this case, electrons in shallow traps such that $n_c(T) \approx N_c$); n_c^o is the density of free electrons in the CB at $T = 0$—in this case, solely from electrons excited into the CB by the incident high energy radiation; T is the absolute temperature; and $E_F^{eff}(T)$ is the excitation energy from the effective Fermi level to the CB. Under these assumptions, the density of available states has been shown to be [16]

$$N_c \approx n_c(T) = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2}\left(\frac{m_e^* m_h^*}{m_e m_h}\right)^{3/4}. \tag{12}$$

m_e, m_h, m_e^*, and m_h^* are the electron and hole masses and effective masses, respectively.

We assume that the number of trapped electrons exceeds the number of free electrons (*i.e.*, trap states have relatively long lifetimes); then the density of primary VB positive centers (fixed holes) created by the high energy radiation, n_p, is approximately equal to the density of occupied traps, n_t:

$$n_p = n_t + n_c \approx n_t. \tag{13}$$

That is, almost all electrons excited from the VB into the CB by high energy radiation as electron-hole pairs spend most time in trapped states and not the CB. (When this assumption is no longer valid, RIC will exhibit saturation effects.) If $n_t >> n_c$, then even at low T, $n_t >> n_c^o$ and Eq. (11) becomes

$$n_c(T) \approx N_c e^{-E_F^{eff}(T)/k_B T}. \tag{14}$$

Solving for E_F^{eff} yields

$$E_F^{eff}(n_c, T) \approx k_B T \cdot \ln\left[N_c / n_c(T)\right]. \tag{15}$$

This is a variation of the familiar law of mass action for semiconductors dopant defects [16]. For further discussion on these assumptions and their resultant implications for disordered materials see [2,4,7,10,12,14].

At finite temperatures $E_F^{eff}(T)$ moves towards E_c as increased charge is stored in trapped states, the excitation energy is reduced, and more electrons can be thermally excited into the CB. It is this temperature dependence of the resulting balance of trap charge buildup from radiation excitations and thermal depletion of the trapped charge that principally determines the temperature dependence of RIC [2,4,7,10,12]. For the monotonically decreasing DOS and uniform DOS models, thermal depletion moves the effective Fermi level further from the CB, thereby increasing $E_F^{eff}(T)$, and we expect a resultant decrease in σ_{RIC}. By contrast, for a delta function DOS, E_F^{eff} is pinned to the single trap energy irrespective of the number of trapped electrons that are thermally excited. Hence, we expect no associated temperature decrease in its σ_{RIC} ; rather $\sigma_{RIC}(T)$ will be driven by the increase in carriers thermally excited into the CB.

In the steady-state condition for RIC, the rate of excitation of VB electrons into the CB by radiation equals the rate of recombination of conduction electrons with primary centers (VB holes),

$$\upsilon_{eh} = \upsilon_{he} = \frac{1}{\tau_e} = \upsilon_T s_c n_c n_p. \tag{16}$$

Here, υ_T is the thermal velocity of electrons; s_c is the capture cross section of primary centers for free electrons; and υ_{eh} is the rate of radiation excitations of electrons per unit volume, which is proportional to the dose rate $\dot{D}$ (or power deposited per unit mass).

$$\upsilon_{eh} = \frac{\dot{D}\,\rho_m}{E_{eh}} \tag{17}$$

Here ρ_m is the material mass density and E_{eh} is the mean energy required for an electron-hole pair to be created by radiation.

We can find an expression for υ_T by setting the thermal energy of a free electron equal to its kinetic energy:

$$\frac{3}{2} k_B T = \frac{1}{2} m_e \upsilon_T^2 \quad \text{or} \quad \upsilon_T = \sqrt{\frac{3k_B T}{m_e}} \tag{18}$$

For the nearly free electron case, $m_e \rightarrow m_e^*$ here. Solving Eq. (16) for n_c, we find

$$\begin{aligned} n_c(T) &= \frac{\nu_{eh}}{\upsilon_T s_C n_p} \\ &\approx \frac{\nu_{eh}}{\upsilon_T s_C n_t} = \frac{\nu_{eh}}{\upsilon_T s_C}\left[\int_0^\infty f_{FD}(E,T) n_A(E) dE\right]^{-1} \\ &\approx C_o\, \dot{D}\, T^{-1/2}\left[\int_0^\infty f_{FD}(E,T) f_A(E) dE\right]^{-1} \\ &\text{with} \quad C_o \equiv \rho_m\left[N_T s_C E_{eh}\sqrt{3k_B/m_e}\right]^{-1} \end{aligned} \tag{19}$$

where we have made substitutions for υ_{eh} and υ_T using Eqs. (17) and (18), respectively. Also, from Eq. (13), n_p is approximately equal to n_t, which in turn is expressed as an integral using Eq. (4).

Using the low temperature Fermi-Dirac function approximation from Eq. (5) and assuming $E_F^{eff}(T) \gtrsim 2k_B T$, we can calculate the density of filled trap states, n_t, for the steady-state condition at low T by integrating an expression for the trap state density as a function of energy over all occupied states, or over all trap states in the distribution $n_A(E)$:

$$\begin{aligned} n_t &= \int_0^\infty f_{FD}(E,T)\, n_A(E)\, dE \\ &\approx \left\{\int_{E_F^{eff-}(T)}^{E_F^{eff+}(T)} \frac{1}{2}\left[1 + \frac{E - E_F^{eff}(T)}{2k_B T}\right] n_A(E) dE + \int_{E_F^{eff-}(T)}^{\infty} n_A(E) dE\right\} \end{aligned}$$

where $E_F^{eff\pm}(T) \equiv E_F^{eff}(T) \pm 2k_BT$. (20)

This expression is the only part of the RIC expression that contains information about the material, at least up to a proportionality constant. The first integral in this expression contains all of the temperature dependence of RIC.

B. Final Expression for Temperature-Dependent RIC

Inserting Eq. (19) into Eq. (10), we arrive at the final expression for temperature-dependent RIC:

$$\sigma_{RIC}(T) = q_e\,\mu_e\,n_c(T) \approx q_e\mu_e\,C_o\,\dot{D}\;T^{-1/2}\left[\int_0^\infty f_{FD}\,(E,T)f_A(E)dE\right]^{-1} \quad (21)$$

Table 2 column 2 shows expressions for $n_c(T)$ in the low T approximation from Eqs. (19) and (20), for all DOS listed in Table 1 evaluated with $E_F^{eff}(T)$ below, above, or within the distributions.

For $E_F^{eff}(T)$ between the trap distribution and the valence band ($E_F^{eff}(T) >> E_o^A$), $n_c(T) = 0$ as expected, since there are no electrons in the distribution to be excited into the CB. For $E_F^{eff}(T)$ between the CB and the distribution, at energies less than those within the distribution, $n_c(T) = C_o\dot{D}T^{-1/2}$.

Solving for $n_c(T)$ within the distributions in general requires numerical calculations, due to the implicit dependence of $E_F^{eff}(T;\,n_c(T))$ on $n_c(T)$, as seen in Table 2. Expressions for these distributions have all been solved previously in the zero temperature limit [4,9,14]. In addition, with the restrictions that the effective Fermi level has a small temperature-induced change and is small compared to the distribution energies, (i.e., $E_F^{eff}(T) - E_F^{eff}(T=0) \lesssim 2k_BT$ and $\left[E_o^A - E_F^{eff}(\mathrm{T})\right] \gg k_BT$), approximate expressions for most of the distributions have been determined [4,9,14].

C. RIC for Exponential Monotonically Decreasing DOS Model

An important case with an explicit solution is the exponential monotonically decreasing DOS with the exponential width, where the expression in Table 2 can be explicitly solved for $n_c(T)$ when $E_F^{eff}(T)$ is at least a few times $2k_BT$ [4,9,14]:

$$n_c(T) = \begin{cases} \left[C_o\dot{D}T^{-1/2}\right]^{\left(\frac{T_o^X}{T+T_o^X}\right)}[\mathrm{N_c}]^{\left(\frac{\mathrm{T}}{T+T_o^X}\right)} \\ ;\,T \to 0\,\mathrm{K} \\ \left[\left\{\frac{2T}{T_o^X}\right\} sinh\left(\frac{2T}{T_o^X}\right) C_o\dot{D}T^{-1/2}\right]^{\left(\frac{T_o^X}{T+T_o^X}\right)}[\mathrm{N_c}]^{\left(\frac{\mathrm{T}}{T+T_o^X}\right)} \\ ;E_F^{eff}(T) \gtrsim 2k_BT > 0 \end{cases} \quad (22)$$

The exponential monotonically decreasing DOS is commonly used to model shallow traps within the bandgap [2,4,14].

D. RIC for Gaussian Symmetric Peaked DOS Model

Finally, we consider a Gaussian-like distribution of traps, $n_G(E)$, within the bandgap,

$$n_G(E;E_o^G,E_o^t) = \frac{\left[\frac{N_T}{\sqrt{2\pi}E_0^G}\right]exp\left[-\frac{1}{2}\left[\frac{\left(E_0^t-E\right)}{E_0^G}\right]^2\right]}{\frac{1}{2}\left[1+erf\left(\frac{2\,E_0^t}{\sqrt{2}\,E_0^G}\right)\right]}\theta(E)\ , \quad (23)$$

with mean energy E_0^t and standard deviation E_0^G. The Heaviside step function, $\theta(E)$, truncates occupation to within the CB at $E<E_C \equiv 0$; the error function in the denominator corrects the normalization for this truncation such that N_T is still the total (occupied and unoccupied) trap state density; this correction term approaches unity when the peak is well within the bandgap, *i.e.* when $E_0^t \gg 0$ or $E_0^t \gg E_0^G$.

For this Gaussian DOS distribution, the density of conduction electrons is

$$n_c(T) = \frac{C_o\,\dot{D}\,T^{-1/2}\ \left[1+erf\left(\frac{2\,E_0^t}{\sqrt{2}\,E_0^G}\right)\right]}{\left\{1+\left[\frac{\sqrt{2}\,E_0^G)}{4k_BT}\right]\cdot\left[R_+\cdot erf(R_+)-R_-\cdot erf(R_-)+\frac{\left(e^{-(R_+)^2}-e^{-(R_-)^2}\right)}{\sqrt{2\pi}}\right]\right\}},$$

$$\text{where } R_\pm(n_c,T) \equiv \left\{\frac{\left[E_0^t-E_F^{eff}(n_c,T)\right]\pm 2k_BT}{\sqrt{2}\,E_0^G}\right\} . \quad (24)$$

By adjusting the width E_0^G, limiting cases of the uniform top hat and the delta function distributions are obtained (see Fig. 1 and Table 2). When the width of the distribution is large (i.e., $E_o^G \gg 0$), the limiting case of a uniform distribution is obtained. The first two terms in the square bracket will cancel each other and the exponential terms will disappear faster than the coefficient will blow up. In this case $n_c(T) \,\dot{\propto}\, C_oDT^{-1/2}$ as expected for $E_F^{eff}(T)$ within a uniform distribution. Similarly, when the width of the distribution is small (i.e., $E_o^G \approx 0$), the distribution approaches that of a delta function. $(R_\pm)^2 \gg 1$ and the exponential term in the square bracket will go to 0 much faster than E_0^G in the coefficient can blow up. $n_c(T) \doteq C_oDT^{-1/2}$ as expected for a delta function. Further details of these limiting cases and the effect the relative position of the effective Fermi level, E_F^{eff}, has on the temperature dependent RIC can be found in [14].

IV. RESULTS

Figure 3 shows fits to measured RIC values as a function of temperature for two common spacecraft materials, disordered SiO_2 and low density polyethylene (LDPE). To obtain data over extended temperature ranges, three distinct data sets were compiled for each material and the data from the different studies were modestly scaled to agree at room temperature. Details about the materials and experiments [17] are found in the respective references.

Figure 3(a) shows the SiO_2 data (two data sets from USU [18] and one from Culler [19]) fit with a curve that would be expected for a material with an exponential monotonically decreasing DOS (see Eq. (22)) [9,14]. Data from the USU Data Set 2 shows a smaller decrease in RIC at the lowest temperatures, as predicted by the exponential monotonically decreasing DOS model. RIC for SiO_2 increases by only a

Table 2. Conduction band electron expressions for various DOS [14].

DOS Type	Density of Conduction Band Electrons, $n_c(T)$	Temperature Dependence
Monotonically decreasing DOS models with $E_o^t \leq 0$.		
Exponential $0 < E_F^{eff}$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left\{ \frac{E_o^X}{2k_BT} \left[\left(\frac{N_c}{n_c} \right)^{-\left(\frac{k_BT}{E_o^X}\right)} sinh\left(\frac{2k_BT}{E_o^X} \right) \right] \right\}^{-1}$	$T^{-1/2}$ when $\left(E_o^X - E_F^{eff}\right) \gg 2k_BT$; $T^{1/2}$ when $E_o^X \gg E_F^{eff} \gtrsim 2k_BT$
Power Law $0 < E_F^{eff} < E_o^P$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left\{ \frac{E_o^P}{4k_BT(p+2)} \{ (P_+)^{p+2} - (P_-)^{p+2} \} \right\}^{-1}$ $P_\pm(n_c,T) \equiv \left\{ \frac{\left[E_o^P - E_F^{eff}(n_c,T)\right] \pm 2k_BT}{E_o^P} \right\}$	$T^{-1/2}$ when $\left\|E_o^P - E_F^{eff}\right\| \gg 2k_BT$
Linear $0 < E_F^{eff} < E_o^L$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left\{ \left[\frac{E_o^L - E_F^{eff}(n_c,T)}{E_o^L} \right]^2 + \frac{1}{3} \left[\frac{2k_BT}{E_o^L} \right]^2 \right\}^{-1}$	$T^{-1/2}$ when $\left\|E_o^L - E_F^{eff}\right\| \gg 2k_BT$; $T^{-5/2}$ when $\left\|E_o^L - E_F^{eff}\right\| \ll 2k_BT$
Uniform Step $0 < E_F^{eff} < E_o^{US}$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left[\frac{E_o^{US} - E_F^{eff}(n_c,T)}{E_o^{US}} \right]^{-1}$	$T^{-1/2}$
Power Law, Linear, Uniform Step $0 < (E_o^A + 2k_BT) < E_F^{eff}$	$n_c = 0$	T-independent
Peaked DOS models with $E_o^t > 0$.		
Gaussian $0 < E_F^{eff}$	$n_c = \cdot C_o \dot{D} T^{-1/2} \cdot \left[1 + erf\left(\frac{2\,E_o^t}{\sqrt{2}\,E_0^G} \right) \right]$ $\times \left\{ 1 + erf(R_-) + \left[\frac{\sqrt{2}\,E_0^G)}{4k_BT} \right] \cdot \left[R_+ \cdot erf(R_+) - R_+ \cdot erf(R_-) + \frac{\left(e^{-(R_+)^2} - e^{-(R_-)^2} \right)}{\sqrt{2\pi}} \right] \right\}^{-1}$ $R_\pm(n_c,T) \equiv \left\{ \frac{\left[E_o^t - E_F^{eff}(n_c,T)\right] \pm 2k_BT}{\sqrt{2}\,E_o^G} \right\}$	$T^{-1/2}$ when $\left\|E_o^t - E_F^{eff}\right\| \gg 2k_BT$; $T^{-1/2}$ when $\left\|E_o^t - E_F^{eff}\right\| \ll 2k_BT$; 0 when $\left\|E_o^t - E_F^{eff}\right\| \gg 2k_BT$ $\left\|E_o^t - E_F^{eff}\right\| \geq E_o^G \geq 0$
Delta Function $0 < E_F^{eff} < E_o^t$	$n_c = C_o \dot{D} T^{-1/2}$	$T^{-1/2}$
Delta Function $\left\|E_o^t - E_F^{eff}\right\| \leq 2k_BT$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left\{ \frac{1}{2} + \left[\frac{E_o^t - E_F^{eff}(n_c,T)}{4k_BT} \right] \right\}^{-1}$	$T^{-1/2}$ when $\left\|E_o^t - E_F^{eff}\right\| \ll 2k_BT$; $T^{-1/2}$ when $\left\|E_o^t - E_F^{eff}\right\| \gg 2k_BT$
Uniform Top Hat $0 < E_F^{eff} < E_1^U$	$n_c = C_o \dot{D} T^{-1/2}$	$T^{-1/2}$
Uniform Top Hat $0 < E_1^U < E_F^{eff} < E_2^U$	$n_c = C_o \dot{D} T^{-1/2} \cdot \left\{ 1 - \left[\frac{E_1^U - E_F^{eff}(n_c,T)}{2(E_o^t - E_1^U)} \right] \right\}^{-1}$ $E_o^t = \frac{1}{2}(E_2^U + E_1^U)$	$T^{-1/2}$ when $\left(E_o^t - E_F^{eff}\right) \gg 2k_BT$
Delta Function, Uniform Top Hat $0 < \left[E_o^t + {}^1\!/_2 \cdot E_o^{width} + 2k_BT\right] < \left(E_F^{eff}\right)$	$n_c = 0$	T-independent

$C_o \equiv \frac{\rho_m}{N_T s_c E_{eh} \cdot \sqrt{3k_B/m_e}}$

□ **E_F^{eff} within the DOS distribution.**

■ **E_F^{eff} between the CB and the DOS distribution.**

■ **E_F^{eff} between the DOS distribution and the valence band.**

factor of ~4 from ~50 K to ~420 K, almost three orders of magnitude less than observed for LDPE over similar temperature ranges. A weak temperature dependence suggests a narrow distribution with a smaller E_o^X; here E_o^X≈20 meV. Cathodoluminescence for these SiO_2 materials have suggested the presence of an exponential shallow trap DOS and several fairly narrow (~10-50 meV wide) deep level trap DOS distributions centered between ~2 eV to 4.5 eV within the bandgap [18,20].

Figure 3(b) shows three normalized LDPE data sets (USU [9], Yagahi. [21], and Fowler [**7**]) fit with a curve predicted for an exponential monotonically decreasing DOS [9,14], with a much higher E_o^X≈140 meV. At temperatures below ~240 K, LDPE data exhibits a modest factor of ~3 increase in RIC. Such an increase at low temperatures is predicted for an exponential monotonically decreasing DOS of the form given by Eq. (22). However, for expected ranges of $E_o^X$and N_T, these increases are predicted at ~30-80 K (see, e.g., Fig. 3(a)). A better fit below ~240 K is found for a curve proportional to $T^{-1/2}$, as is expected for a peaked distribution (see Table 2).

The behavior observed in LDPE may be related to a LDPE structural phase transition observed at between 250 K and 262 K. This β transition is routinely observed in branched polyethylene, and has been associated with conformational changes along polymer chains in the interfacial matrix of disordered polymers between nanocrystalline regions in the bulk. Similar abrupt changes near ~250 K have been seen in prior studies of mechanical and thermodynamic properties and in dark current conductivity [1,23], RIC [1,9], electrostatic discharge [24], and other electronic properties.

V.CONCLUSION

We have calculated expressions for radiation induced conductivity applicable over an extended temperature range that encompasses most practical applications encountered for highly disordered insulating materials in spacecraft charging. Solutions were found for both monotonically decreasing and symmetric peaked DOS distributions of defect states within the bandgap. Variations were considered, with an effective Fermi level above the trap DOS, within the trap DOS, and below the DOS. Approximations were identified that led to analytic solutions for the distributions for many specific cases of energies and temperatures. When analytic solutions were not found, the general expressions for the occupied trap state distribution as a function of temperature can be found numerically by recursively solving the complex expressions for the occupied trap DOS as a function of temperature.

The derived expressions were used to fit measured RIC values over broad temperature ranges for two common materials, low density polyethylene (LDPE) and disordered silicon dioxide. Both LDPE and SiO2 were fit best with an exponential monotonically decreasing DOS model. Reasonable fits were obtained and the observed temperature dependence of RIC was successfully used to glean information about the nature of the distribution of defect states for the materials.

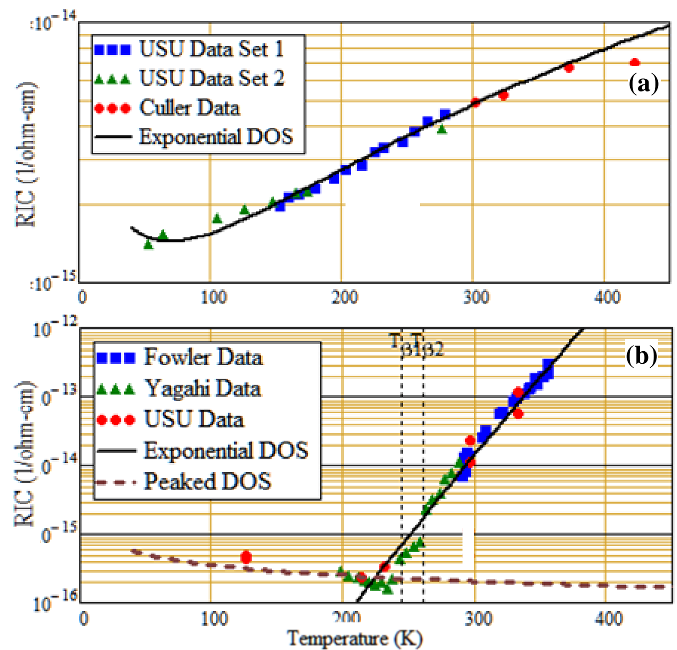

Fig. 3. Radiation induced conductivity versus temperature for: (a) disordered SiO_2, showing two data sets from USU [18] and Culler [19] with fits based on an exponential DOS, Eq. (22); (b) low density polyethylene (LDPE), showing data sets from USU [**9**], Yagahi. [21], and Fowler [**7**] with fits based on an exponential DOS, Eq. (22). and proportional to $T^{-1/2}$. Data from the different studies were scaled to normalize RIC near room temperature.

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REFERENCES

[1] J.R. Dennison, A.M. Sim, J. Brunson, S. Hart, J.C. Gillespie, J. Dekany, C. Sim and D. Arnfield, "Engineering Tool for Temperature, Electric Field and Dose Rate Dependence of High Resistivity Spacecraft Materials," Paper Number AIAA-2009-0562, *Proc. 47th Am. Inst. Aeronautics and Astronomics Meet. on Aerospace Sci.*, 2009.

[2] A.M. Sim and J.R. Dennison, "Comprehensive Theoretical Framework for Modeling Diverse Electron Transport Experiments in Parallel Plate Geometries," Paper Number, AIAA-2013-2827, *5th AIAA Atmospheric and Space Environ. Conf.*, (San Diego, CA, June, 2013).

[3] H.J. Wintle, "Conduction Processes in Polymers," *in Engineering Dielectrics – Volume IIA: Electrical Properties of Solid Insulating Materials: Molecular Structure and Electrical Behavior,* R. Bartnikas, Eds, (Am. Soc. Testing and Materials, Philadelphia, PA 19103, 1983).

[4] A. Rose, "An Outline of Some Photoconductive Processes," RCA Review **12**, 362-414, 1951.

[5] A. Rose, *Concepts in Photoconductivity and Allied Problem* (John Wiley and Sons, New York) , 1963.

[6] J. F. Fowler, "X-ray induced conductivity in insulating materials," Proc. Royal Soc. London A, 236(1207), 464 (1956).

[7] J. F. Fowler, "X-Ray Induced Conductivity in Insulating Materials," M. S. thesis (University of London) 1956.

[8] M. C. J. M. Vissenberg, "Theory of the Field-Effect Mobility in Amorphous Organic Transistors," Phys. Rev. **57**, 12964-12968, 1998.

[9] J.R. Dennison, J. Gillespie, J.L. Hodges, R.C. Hoffmann, .J. Abbott, A.W. Hunt and R. Spalding, "Radiation Induced Conductivity of Highly-Insulating Spacecraft Materials," in *Application of Accelerators in Research and Industry*, Am. Instit. Phys. Conf. Proc. Series, Vol. 1099, ed. F.D. McDaniel and B. L. Doyle, (Am. Instit. of Phys., Melveille, NY, 2009), pp. 203-208.

[10] P. Moliníé, R. Hanna, T. Paulmier, M. Belhaj, B. Dirassen, D. Payan, and N. Balcon, "Photoconduction and radiation-induced conductivity on

insulators: a short review and some experimental results," *Proc.* 12*th Spacecraft Charging Techn. Conf.,* (Kitakyushu, Japan, May, 2012).
[11] A. Tyutnev, V. Saenko, and P. Evgeny, "Experimental and Theoretical Studies of Radiation-Induced Conductivity in Spacecraft Polymers," *Proc.* 13*th* *Spacecraft Charging Techn. Conf.,* (Pasadena, CA, June, 2014).
[12] S. A. Khatipov, "Radiation-Induced Electron Transport Processes in Polymeric Dielectrics (A Review)", *High Energy Chem.*, Vol. 35, No. 5, pp. 291–307, 2001.
[13] F.J. Campbell "Radiation Effects of the Electrical Properties of Solid Insulation," *in Engineering Dielectrics – Volume IIA: Electrical Properties of Solid Insulating Materials: Molecular Structure and Electrical Behavior, American Society for Testing and Materials,* R. Bartnikas, Editors, (American Society for Testing and Materials, Philadelphia, PA 19103, 1983).
[14] J.C. Gillespie, "Measurements of the Temperature Dependence of Radiation Induced Conductivity in Polymeric Dielectrics," MS Thesis (Utah State University) , 2013.
[15] A. R. Frederickson, "Radiation Induced Currents and Conductivity in Dielectrics," IEEE Trans. on Nucl. Sci. **NS-24**, 2532-2539,1977.
[16] N. W. Ashcroft, and N. D. Mermin, *Solid State Physics* (W. B. Saunders Company, New York, NY, 1976).
[17] J. Dekany, R.H. Johnson, G. Wilson, A. Evans and J.R. Dennison, "Ultrahigh Vacuum Cryostat System for Extended Low Temperature Space Environment Testing," *IEEE Trans. on Plasma Sci.*, **42**(1), 2014, 266-271.
[18] J.R. Dennison, J. Dekany, J.C. Gillespie, P. Lundgreen, A. Anderson, A.E. Jensen, G. Wilson, A.M. Sim, and R. Hoffmann, "Synergistic Models of Electron Emission and Transport Measurements of Disordered SiO_2," *Proc.* 13*th* *Spacecraft Charging Techn. Conf.,* (Pasadena, CA, June, 2014).
[19] V.E Culler and H.E. Rexford, "Gamma-radiation-induced conductivity in glasses," Proc. IEE, 112(7), 1462, 1965.
[20] A.E. Jensen and J.R. Dennison, "Defects Density of States Model of Cathodoluminescent Intensity and Spectra of Disordered SiO_2," *Proc.* 13*th* *Spacecraft Charging Techn. Conf.,* (Pasadena, CA, June, 2014).
[21] K. Yahagi and A. Danno, "Effect of Carrier Traps in Polyethylene under Gamma-Ray Irradiation," J. Appl. Phys., 34, 804 (1963).
[22] J.A Roth, R. Hoffmann, and J.R. Dennison, "Effects of Radiation Induced Conductivity on Electrostatic Discharge in Insulating Materials," *Proc. 1st AIAA Atmospheric and Space Environments Conf.* (San Antonio, TX, 2009).
[23] J.R. Dennison and J. Brunson, "Temperature and Electric Field Dependence of Conduction in Low-Density Polyethylene," *IEEE Trans. Plasma Sci*., 36(5), 2246-2252, 2008.
[24] A. Anderson, J.R. Dennison, A.M. Sim and C. Sim, "Electrostatic Discharge and Endurance Time Measurements of Spacecraft Materials: A Defect-Driven Dynamic Model," *Proc.* 13*th* *Spacecraft Charging Techn. Conf.,* (Pasadena, CA, June, 2014).

Jodie Corbridge Gillespie is a graduate student at Utah State University in Logan, UT pursing a PhD in physics. She received BS and MS degrees in physics from USU in 2003 and 2013, respectively. She has worked with the Materials Physics Group for eight years. Her work has been wide ranging, involving instrumentation, experimental and theoretical components, in areas including radiation induced conductivity, electron emission, photoyield, and materials characterization.

J. R. Dennison received the B.S. degree in physics from Appalachian State University, Boone, NC, in 1980, and the M.S. and Ph.D. degrees in physics from Virginia Tech, Blacksburg, in 1983 and 1985, respectively. He was a Research Associate with the University of Missouri—Columbia before moving to Utah State University (USU), Logan, in 1988. He is currently a Professor of physics at USU, where he leads the Materials Physics Group. He has worked in the area of electron scattering for his entire career and has focused on the electron emission and conductivity of materials related to spacecraft charging for the last two decades.

Alec Sim received a BS degree in physics from the University of California-San Bernardino, CA in 2004, an MS in physics from the University of Kentucky in Lexington, KY in 2008, and a PhD from Utah State University in Logan, UT in 2013. He worked with the Materials Physics Group for six years on electron emission measurements and theoretical studies of electron transport in highly disordered insulating materials. He is currently an Assistant Professor in the Department of Physical Sciences at Irvine Valley College, Irving CA.