A SPECTRAL ANALYSIS OF SINGLE ANTENNA INTERFEROMETRY

Craig Stringham

Microwave Earth Remote Sensing Laboratory
Brigham Young University
459 CB, Provo, UT 84602
March 18, 2013

ABSTRACT

This paper analyzes the potential of performing interferometry with a single antenna system on an aircraft with significant motion. The spectral analysis of [1, 2] is extended to low altitude imaging scenes. We then use this spectral analysis to find the collection geometry requirements to achieve coherence between SAR images. Using these requirements, we show that the motion of the aircraft during CASIE-09 is insufficient to achieve coherence.

1. INTRODUCTION

SAR interferometry is a powerful tool that exploits the coherence of two complex images created from slightly differing aspect angles to infer further information of the scene such as topography or the detection of moving targets. Traditionally SAR interferometry has used images collected using narrow beam antennae with a linear flight track from high altitude platforms. These assumptions make it easy to analyze the SAR operation in the spectral domain, which has enabled the development of computationally efficient algorithms for image compression [1, 3, 4]; however, with the recent availability of raw computational power from GPUs it has become desirable to forgo the computational efficiency of Fourier based methods and use time domain backprojection in order to properly focus the images; however we wondered if the motion of the aircraft could be used to infer the statistics of the sea ice, possibly by using subaperture images to create an interferometric estimate of a portion of the sea ice. This was the basis of my original proposal. However by extending the spectral analysis developed for traditional SAR method [1, 2] to backprojected images, potentially from aircrafts at lower altitudes and non-linear paths, we show that the motion of the aircraft during the CASIE mission is insufficient for this purpose. We also clarify the requirements required for coherence between two SAR images.

2. ROUGH SURFACE MODEL AND COHERENCE

Interferometric estimation of topography relies on the assumption that the imaging scene \( g(x, y, z) \) can be described as a rough surface

\[
g(x, y, z) = r(x, y)\delta(z - h(x, y)),
\]

where \( h(x, y) \) is the topography of the surface and \( r(x, y) \) is the surface reflectivity function, and \( \delta \) is the Dirac delta function. It is also assumed that \( h(x, y) \) is smooth compared to the resolution of the SAR. The surface reflectivity function, \( r(x, y) \) is modeled as a wide sense stationary (WSS) random process, where each resolution cell contains multiple independently positioned point scatterers. This results in an exponential distribution of
the received power, which is typical of natural surfaces [1, 6]. It is apparent that in order to achieve the coherence between SAR images needed for interferometry the pixels must be spatially co-located and of similar shape; however, the pixels also need to be generated from the same portions of the surface spectrum $R(\omega_x, \omega_y)$. Because $r(x, y)$ is a WSS random process its Fourier transform $R(\omega_x, \omega_y)$ is also WSS, and samples of $R(\omega_x, \omega_y)$ are statistically independent. Therefore disjoint portions of the spectrum used in generating the images lead to incoherence. Image registration or stereometric techniques can be used to achieve overlap spatially, so the following develops the tools need to determine the amount of spatial overlap and how this information can be used to improve the coherence of the SAR images.

3. SPECTRAL SAMPLING OF A ROUGH SURFACE IN 2 DIMENSIONS

We first begin by investigating a two dimensional imaging scene described by a single pulse, then this development will be expanded to the traditional three dimensional SAR imaging scene of stationary surfaces. Consider the two dimensional imaging scenario described in Fig. 2. The imaging cell scene is made up of multiple point scatterers as described in Section 2. We assume that the radar has a narrow azimuth beam and that the scene is invariant in the azimuth direction. Using the Born approximation the radar return is described as a convolution of the radar signal with the imaging scene within the beamwidth of the antenna, modelled mathematically as

$$s_r(r) = \int_0^{R_{\text{max}}} s_t(r - \tau) \int_{B_\psi} g(\tau \sin(\psi + l) + x, \tau \cos(\psi + l) + z) dl \, d\tau$$

$$s_t(r) * \int_{B_\psi} g(r \sin(\psi + l) + x, r \cos(\psi + l) + z) dl,$$

where the antenna location is given by $(x, z)$, $r$ is the distance from the antenna, $R_{\text{max}}$ is the maximum imaging range, $\psi$ is the angle of the scene to the an-
tenna, and $B_\psi$ is the antenna elevation beamwidth. Because we are interested in pixel to pixel correlation of images, we can constrain our analysis to a single range compressed sample. Thus we only consider a small portion of $B_\psi$ which we designate as $L$. The imaging scene is much smaller than the distance to the target. Using the small angle approximation for functions of $l$, yields

$$s_r(r) = s_l(r) \ast \int_L g(r (\sin \psi \cos l + \cos \psi \sin l) + x, r (\cos \psi \cos l - \sin \psi \sin l) + z) \, dl$$

Equation (3) describes an orthographic projection of $g$ onto a line with an elevation angle $\psi$. The reader may note that the orthographic projection is only valid for small angle ranges, thus this analysis may not be valid for an image formation procedure for airborne SAR where the range of incidence angle is large; however, this analysis is valid in evaluating which portions of the spectrum are used in creating a pixel value.

The spectral sampling of the radar pulse given in Eq. (2) is defined by the Fourier transform of Eq. (2); however, by the projection slice theorem, this is approximately equivalent to taking a slice of the two dimensional Fourier transform of the two dimensional image spectrum. This is illustrated by looking at the slice of the spectrum taken along the $\omega_z$ axis

$$G(\omega_r, 0) = \int \int g(x, z) e^{-j2\pi \omega_r x} \, dx \, dz$$

$$= \int \left[ \int g(x, z) \, dz \right] e^{-j2\pi \omega_r x} \, dx$$

$$(4)$$

$$= \int p_x(x) e^{-j2\pi \omega_r x} \, dx$$

$$= \mathcal{F}\{p_x\},$$

where the $p_x$ is the orthographic projection of $g(x, z)$ onto the $x$ axis. This extends to a slice at an arbitrary angle because the Fourier transform of a rotated space is equal to the rotated Fourier transform of the non-rotated space. Note that the spectral slices always extend from the origin of the spectral space. The spectral sample is illustrated in Fig. 3 for two different incident angles. The background represents the spectrum of a surface contained on the $x$ axis, thus the spectrum is constant in $\omega_z$. The angle of observation angle of the radar determines the angle of the slice through the two dimensional spectrum. The center frequency, $f_c$ and the bandwidth, $B_{\omega_r}$ of the radar determines the sampled portion of the one dimensional slice. Note that the discrete samples illustrated by the colored dashes in Fig. 3 can be considered the raw data samples of a LFM pulse or the Fourier transform of the range compressed data. The spectrum in Fig. 3 is constant in the $\omega_z$ direction because the scatterers are contained on a line in the two dimensional imaging space. Likewise if the surface extended along and angle $\alpha$ the spectrum is rotated by $\alpha$ then it is constant along $\alpha + \frac{\pi}{2}$, or equivalently the slice is taken along $\psi + \alpha$ of the non rotated spectrum. Thus the portion of the surface spectrum sampled by the radar at angle $\psi$ is equivalent to the portion sampled by a radar with 0 degree incidence angle and a lowered center frequency and bandwidth.
3.1. Coherence in two dimensional cross-track interferometry

Topography estimation is performed using cross-track interferometry where two radars (or alternatively two simultaneous receivers and one radar transmitter) are placed at different elevation angles $\psi$ and $\psi'$ as illustrated in Fig. 3. In this example the radars have the same center frequency and bandwidth. The radars sample slightly different portions of the surface spectrum as illustrated by the projection of the samples onto the surface angle. To increase the coherence of the two images, the disjoint portions of the spectrum can be filtered off by applying windows in the spectral domain. For LFM-CW SAR the filtering may be done by windowing the dechirped data before range compression. Alternatively the decorrelation can be overcome by “tunable systems”, which can adjust their operating frequencies as suggested in [2].

4. SPECTRAL SAMPLING IN THREE DIMENSIONS

The previous analysis is easily extended to the three dimensions because the projection slice theorem is applicable to any number of dimensions. As explained in Section 3, the contribution of a single resolution cell to a radar pulse can be described by the convolution of the radar transmit signal with an orthographic projection of the imaging cell onto a line proceeding from the radar to the center of the imaging cell, and by the projection slice theorem the spectral contribution contained in this pulse is approximately the one dimensional slice of the three dimensional spectrum taken at the same angles as the projection. To illustrate this refer to the geometry shown in Fig. 4 of two squinted SAR collections from two different altitudes. Note that the aircraft at the higher altitude effectively has a somewhat narrower beamwidth because the target is viewed from only the same span of ground azimuth angles.

Figure 5 illustrates the spectral sampling of the SAR data collections illustrated in Figure 4. The sampling lines extend at the same angles, but sample a portion of a one dimensional slice as specified by the radar operating frequencies. Because the scene of interest is contained on the surface of the x-y plane the spectrum is constant in $\omega_z$ thus the two collections effectively sample the portions shown in Fig. 5b. Note that like the two dimensional imaging scenario, the difference in incidence angle shifts the sampling of the ground spectrum along the ground azimuth angle. However because the shift is only along the ground azimuth angle we also see that in order for the collections to have azimuth spectral overlap they must view the scene from the same range of azimuth angles.

5. IMPLICATIONS TO SINGLE APERTURE INTERFEROMETRY

Let us first consider the case of an aircraft approaching the imaging scene where scene of interest is to the right of the initial heading of the aircraft. To ease the analysis we first ignore the transmit frequency requirements outlined earlier and assume that the radar can arbitrarily adjust its transmit and receive frequencies, such that the length of the baseline and the altitude are not an important considerations. We choose the reference frame origin to proceed from the scene center and
to measure the ground azimuth angle between the projections of the pointing angle to the receive antenna and the angle orthogonal to aircraft heading pointing toward the aircraft as shown in Fig. 6.

Consider that the aircraft has a collection of pulses taken along a relatively straight flight path. In order for the aircraft to collect pulses from the same range of azimuth angles then the aircraft must either reverse direction or the projection of the heading onto the ground plane must cross to the other side of the imaging scene as illustrated in Fig. 6. For a horizontally flat surface this can be accomplished by a helicopter mounted SAR, but for an airplane, the heading crossing the scene is difficult to achieve except at high squint angles and requires that the antenna beam either be very wide and forward pointing (resulting in difficult ambiguities to resolve) or to be steered to point to other side of the aircraft. Furthermore, the length of the interferometric baseline is determined by the airplane turning radius, which in most imaging scenarios results in a large range in the incidence angle requiring a very wide range of transmit frequencies. However, if the surface rises away from the aircraft the requirements are lower.

6. CONCLUSION

By extending the spectral analysis of SAR data collections done by [1, 2] we have shown that SAR data collected as part of CASIE-09 cannot be used for single-pass interferometry.

7. REFERENCES


Fig. 6: Depiction of a flight geometry that allows for azimuth spectral overlap.

