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Problem Set 6

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PROBLEM SET 6

Problem 6.1

Use the method of separation of variables to find a nonzero solution of the 3-dimensional wave equation in the interior of a cube and which vanishes on the faces of a cube. (You can think of this as a mathematical model of sound waves in a room.)

Problem 6.2

Suppose that a function only depends upon the distance from the z-axis:

$$F(x, y, z) = f(\sqrt{x^2 + y^2}).$$

Consider the expression of F in cylindrical coordinates. Show that

$$\frac{\partial F}{\partial \phi} = 0,$$

(i) directly in cylindrical coordinates (easy!)

(ii) using the chain rule starting in Cartesian coordinates.

Problem 6.3

Show that spherical polar coordinates are identical to cylindrical coordinates when labeling points in the x-y plane.

Problem 6.4

Suppose that we are considering cylindrically symmetric solutions to the wave equation, $q = q(\rho, t)$. Starting from the wave equation in Cartesian coordinates, use the chain rule to derive the wave equation satisfied by $q(\rho, t)$.

Problem 6.5

If one looks for solutions to the wave equation that do not depend upon time, $q = q(\vec{r})$, then one must solve the Laplace equation

$$\nabla^2 q(\vec{r}) = 0.$$

Find the general form of the solution to this equation if one assumes

(a) cylindrical symmetry: $q = q(\rho)$

(b) spherical symmetry: $q = q(r)$.

Problem 6.6

Verify that the spherically symmetric functions

$$q_1(r,t) = \cos(kvt)\frac{\sin(kr)}{kr}, \quad q_2(r,t) = \cos(kvt)\frac{\cos(kr)}{kr}$$

solve the (three-dimensional) wave equation. Show that q_1 is well-defined at the origin while q_2 is not.

Problem 6.7

Find a non-zero spherically symmetric solution to the wave equation defined on the interior of a sphere of radius R and which vanishes on the surface of the sphere.

Problem 6.8

Using your favorite mathematical software, write a program to compute the spherical Bessel functions from the formula (13.22). Verify the results shown in (13.21).

Problem 6.9

Using your favorite mathematical software, write a program to display an animation depicting the cylindrically symmetric solution (12.38).