Annual Solar Motion And Spy Satellites

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Introduction

A topic often taught in introductory astronomy courses is the changing position of the Sun in the sky as a function of time of day, and season. The relevance and importance of this motion is explained in the context of seasons and the impact it has on human activities such as agriculture. The geometry of the observed motion in the sky is usually reduced to graphical representations and visualizations that can be difficult to render and grasp. Sometimes students are asked to observe the Suns changing motion and record their data, but this is a long-term project requiring several months to complete. This poster describes an activity for introductory astronomy students that takes a modern approach to this topic, namely determining the Suns location in the sky on a given date through the analysis of satellite photography of the Earth.

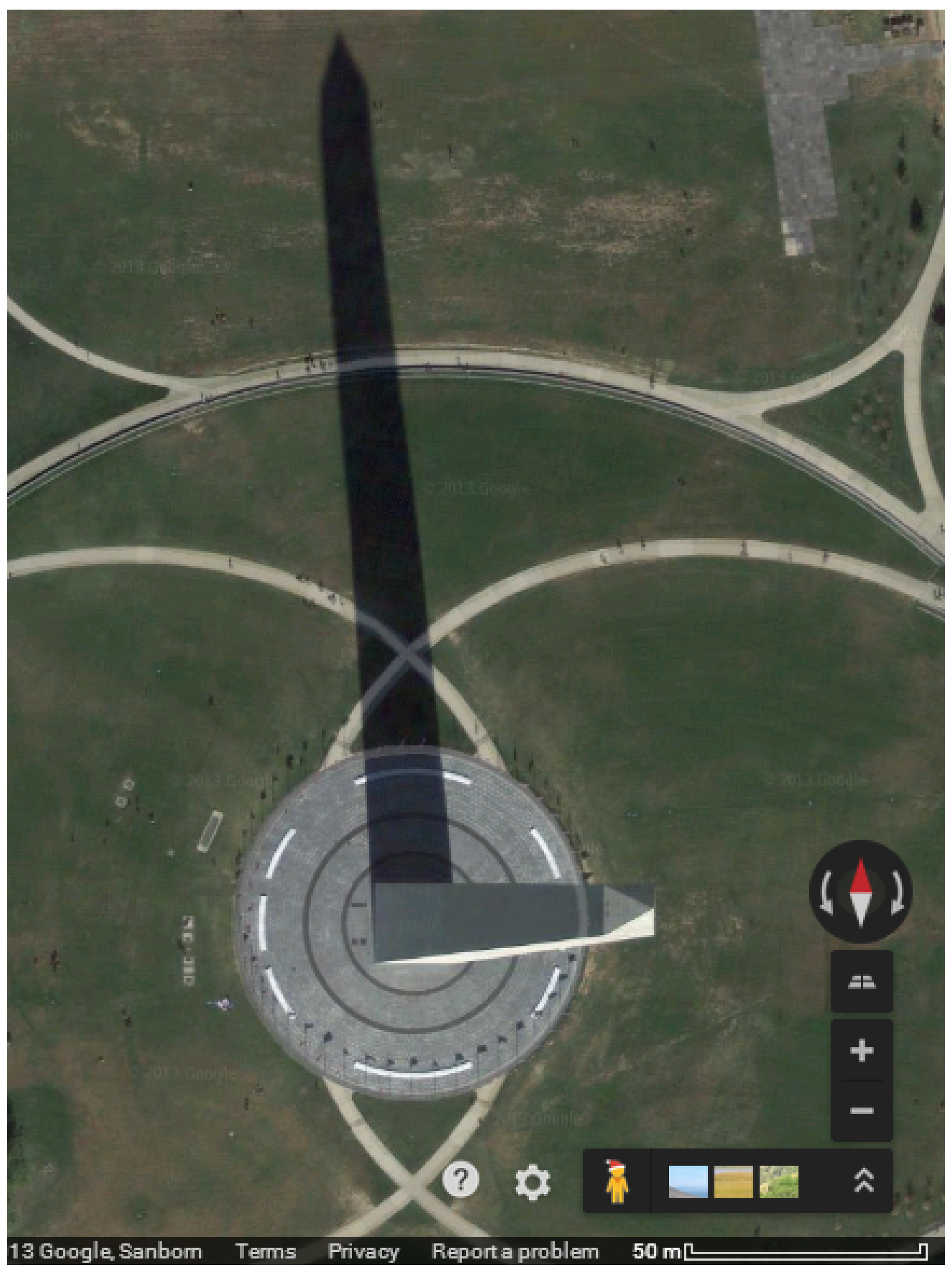

Fig.1: A satellite image of the Washington Monument, retrieved from Google Maps.

Motion of the sun

The apparent motion of the Sun in the sky results from the combination of two separate motions of the Earth: the daily spin of the Earth on its axis, and the annual revolution of the Earth around the Sun. The Earth's spin produces the daily east-to-west motion of the Sun in the sky.

The slow change in solar declination from North to South and back again occurs because as the Earth orbits the Sun, it is sometimes tipped toward the Sun (when the Sun is at high declinations) and at other times tipped away from the Sun (when the Sun is a t low declinations). This has an important effect on the geometry of shadows in satellite images.

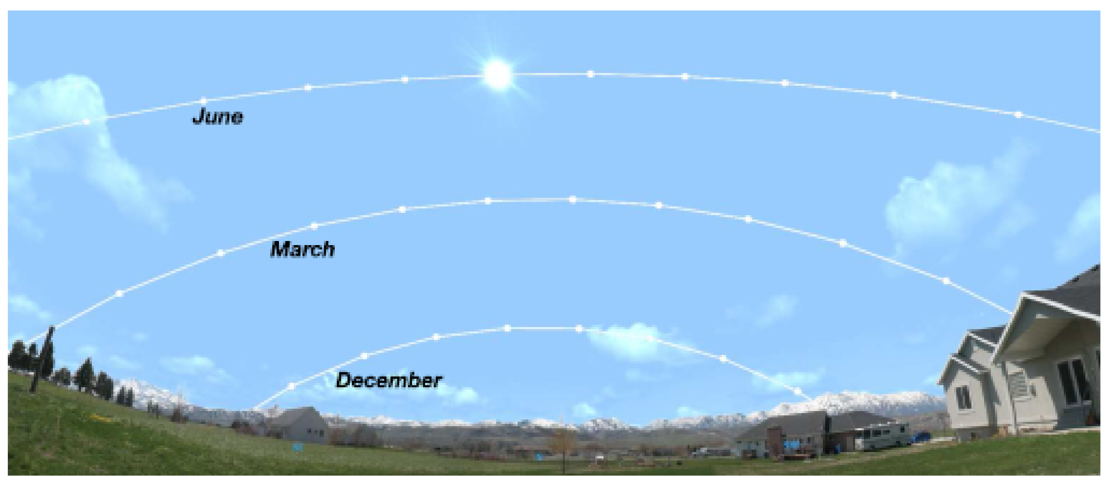

Fig. 2: Different solar tracks across the sky as a function of the time of year, as viewed by a northern hemisphere observer standing facing south.

These regular motions of the Sun means that the position of the Sun on the sky is a unique indicator of the date and time when the Sun is at a particular point.

Shadow Geometry

The Sun traces a regular pattern in the sky as a function of time, a pattern that is captured in the length and direction of shadows on the ground. To reconstruct the position of the Sun in the sky, the two basic pieces of information from a satellite image need to be measured:

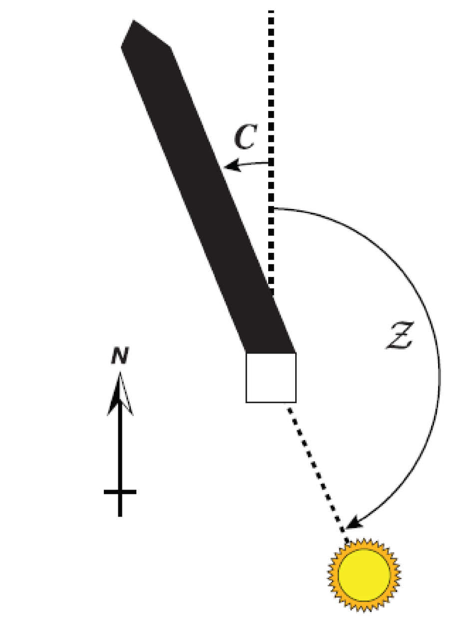

Fig 3. The shadow angle (C) and the solar azimuth angle,(Z). It is convenient to work in coordinates aligned to the cardinal directions, such that the y axis is aligned north-south, and the x axis is aligned east-west.

(1) The length of the shadow s, which is converted into the solar altitude angle (A) (angle of the Sun above the horizon), (2) the direction the shadow points, which is converted into the solar azimuth angle (Z), measured from due north clockwise toward the east.

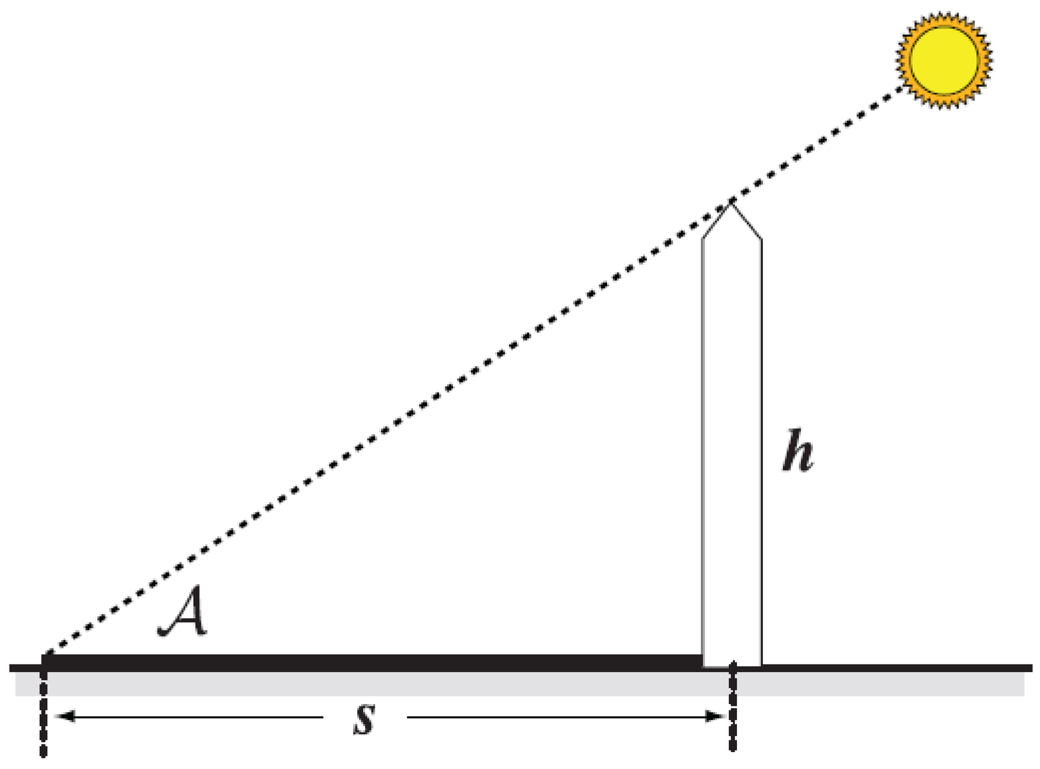

Fig. 4: The solar altitude angle (A) is derived from the shadow length s and target height h.

Recovering Date and time

For the purposes of the exercise described here, the most convenient sky coordinates to describe the location of the Sun are declination (coordinates lines equivalent to the Earths latitude lines projected onto the sky) and meridians (coordinate lines fixed on the sky, but parallel to the projected lines of the Earths longitude). The Sun's declination ($\delta_\odot$) and meridian ($\varphi_\odot$) can be determined from satellite imagery if one knows the latitude (L) of the object casting the shadow, the solar altitude (A) and the solar azimuth (Z) measured from the satellite image.

$$\delta_\odot = \sin^{-1}[\sin A * \sin L + \cos L * \cos Z * \cos A]$$

$$\varphi_\odot = \tan^{-1}[(\sin Z{*}\cos A)/\sin A{*}\cos L{-}\sin L{*}\cos Z{*}\cos A]$$

The day of the year the image was taken, (D), is determined solely from the solar declination ($\delta_\odot$):

$$D_1 = 81 + \frac{365}{2\pi} * \sin^{-1}(\frac{\delta_\odot}{23.44^\circ})$$

$$D_2 = 81 + \frac{365}{2\pi} * [\pi - \sin^{-1}(\frac{\delta_\odot}{23.44^\circ})]$$

The time the picture was taken depends on the reference time (t_N = noon), the rotational speed of the Earth ($\omega_\oplus$), the longitude of the observer (β) and the solar meridian ($\varphi_\odot$):

$$t_{UTC} = (t_N - t_\varphi) - \frac{\beta}{\omega_\oplus} = t_N - \frac{1}{\omega_\oplus}(\varphi_\odot + \beta)$$

Errors and Ambiguities

Why are there two solutions for the day of the year the picture was taken? The Suns annual motion crosses a particular declination in the sky twice each year once when it is heading northward through the sky, and once when it is heading southward.

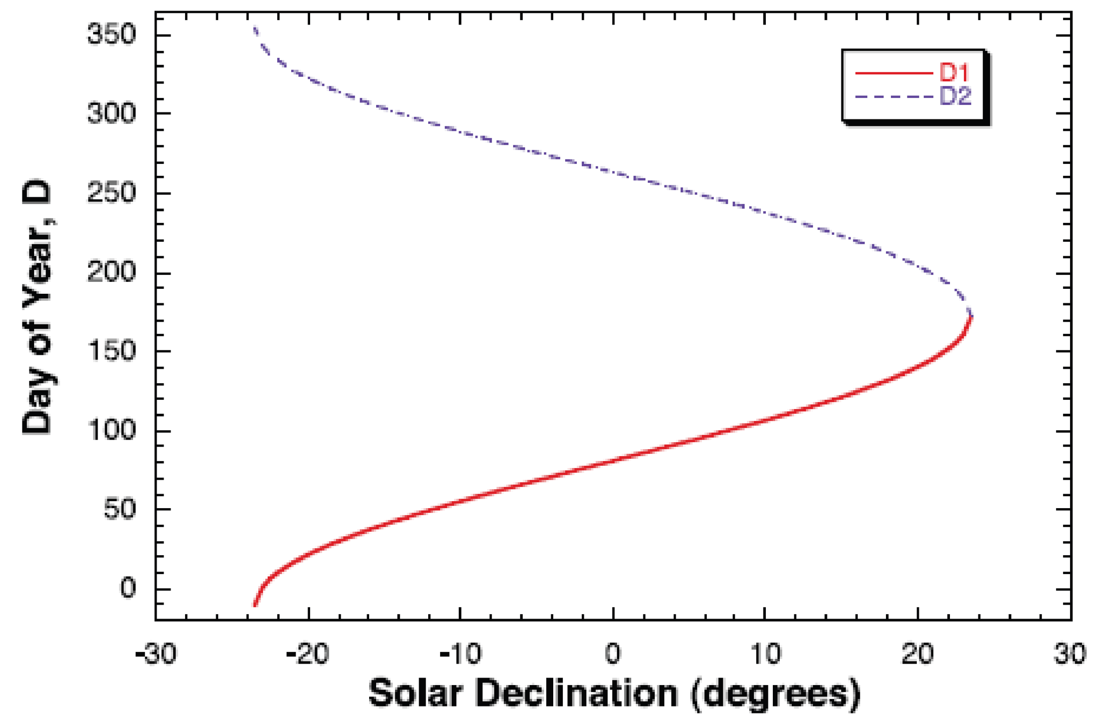

Fig. 5: The complete solution for day of the year (D1 or D2) as a function of the solar declination, $\delta_\odot$.

When attempting to decide which solution to use to determine the day of the year, you must rely on auxiliary data from the photograph, such as vegetative cover or other seasonal clues. An important source of error is the topography on which the shadow is projected in the satellite image. This can be corrected, if one knows the slope of the landscape.

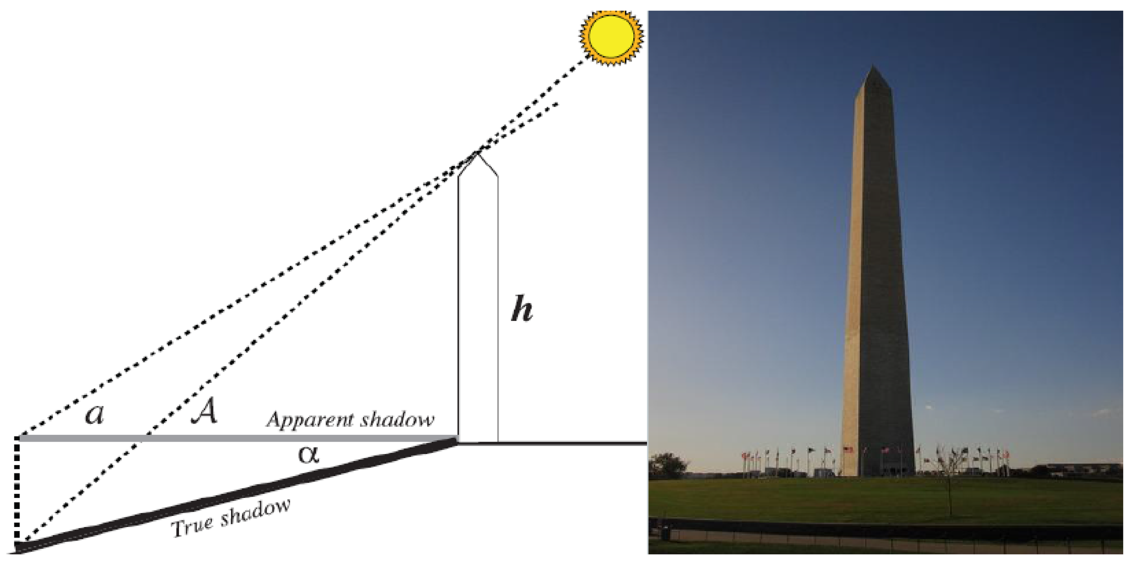

Fig 6. When the shadow is projected on a topographic slope of angle α, the apparent shadow length measured from an overhead satellite photograph is different than the true shadow length. The result is that the apparent solar altitude angle a will be different than the true solar altitude angle A.

Summary

By taking two simple indirect, or direct measurements from a single image, the reconstruction of a specific date and time the image was taken can be easily produced with straightforward mathematical formulae. Making science relevant to students and their lives is always a challenge, particularly in service level courses. In courses like introductory astronomy, the task is even more important since astronomy will for many students be the only science class they ever take. This project is amenable to the measure, plug and play type laboratory exercises that are common in such courses, reducing the work to making direct measurements that are then evaluated using algebraic formulae.

References

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