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ABSTRACT

This study reveals certain systematic tendencies in consumer behavior in Taiwan from 1951 to 1990. The method of systems estimation and precision estimates revealed by significant t statistics and higher R^2 make the system of equations a useful tool to characterize broad tendencies in individual allocation of expenditure behavior. This analysis has important implications for government production strategies relating to price and subsidy policies and promotion of reasonable standards of social welfare and health. In general, this study represents our attempt to make a systematic estimation of consumer demand to identify the broad tendencies of individual demand behavior in Taiwan.

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I. Introduction

The economy of the Republic of China (ROC) on Taiwan has been undergoing rapid transformation for the past four decades. The role of consumer behavior in this process of transformation is extremely crucial. Any changing pattern of demand might have serious implications for trade and growth strategies in development. The role of prices and appropriate pricing policies have been given considerable weight in the setting of strategies for economic development in Taiwan as well as other developing countries. This change motivates an examination of the structure of demand. We know that commodity composition of the consumers' basket varies with their income and prices of goods. Thus, it follows that pragmatic policy decisions, especially with respect to tax and subsidy reforms, require knowledge of price and expenditure elasticities. The Taiwan (ROC) government in both the past and the present has pursued various programs in the form of price and income policies to promote consumer welfare among other objectives of development. In the absence of any systematic study of consumer behavior, these policies could not be based on any firm estimates of price and expenditure elasticities.

In this paper we attempt to provide a systematic estimate of aggregate consumer-demand behavior for Taiwan consumers. Elasticity estimates are based on an econometric analysis of time series budget survey data available for Taiwan from 1951-1990. All of the consumer decisions are modeled simultaneously using a Linear Expenditure System (LES). This model was originally developed by Richard Stone as early as 1954. Later it has been shown by Stone-Geary that this demand system can be derived from the Klein-Rubin utility index of cost of living. According to Parks (1969), the LES has the defect that for certain values of prices and income the predicted expenditure becomes negative. While this is clearly not satisfactory from a theoretical point of view, the system may still be useful for a rather wide range of price-income points.

The model that we use here specifies seven major categories of expenditures. The specific categories have been chosen to emphasize the role of basic needs in Taiwan. According to literature, the LES approach based on time-series data has been extensively used for developed countries since Stone (1954) first formulated a preliminary version of LES in 1954 and then used an improved version of LES with British data in 1964. Nevertheless, the application of LES for developing countries is rather limited owing to a lack of continuous cross-section or long time-series data.[1] Previous LES studies for such countries have been based on single-year cross-section data by making a priori assumptions about at least one of the parameters in the system. Needless to say, these elasticity estimates are unduly restrictive as a result of the arbitrary assumption used

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1.Lluch and Powell (1975) (henceforth L-P), in their studies "International Comparisons of Expenditure Patterns", used time-series data to analyze 19 countries' consumption patterns. The countries include developed, developing, and less-developed countries. However, the length of individual time-series ranges from 7 years in the case of Chile to 20 years in the case of the U.K. Taiwan (1955, 58-68) was also included in their studies.
The commodity groups they picked are the same as what we did in this study.

in deriving the elasticity parameters.[2] In contrast, we use annual data from the "Statistical Yearbook of the Republic of China" published by Directorate-General of Budget, Accounting and Statistics, Executive Yuan, Republic of China (1975, 1985, and 1991). As a result, elasticity estimates for Taiwan consumers are based on annual estimates of consumption behavior. The paper is organized as follows. In section II the formulation and estimation of the system demand equations are discussed. Section III describes the data used for the estimation of the model. Section IV contains empirical results and section V summarize our research results.

II. Model

The methodology used here in determining the parameters of the demand system is derived from the neoclassical theory of consumer choice. According to Pollak (1971), this methodology postulates that a representative consumer's utility function is an additively separable function of the form $U(x_1, x_2,..., x_n)$ and can be represented, after a monotonic transformation, as the sum of a set of individual partial utility functions that takes the following form:

$$U=\sum_{i=1}^{n} \beta_i \ln(x_i-\gamma_i) \qquad (1)$$

where x_i = consumption of the ith good, β_i and γ_i are parameters of the utility function with $\beta_i \succ 0$ and $x_i \succ \gamma_i$, and the requirement that $\sum \beta_i=1$ ensures that the preference structure implied by (1) is well behaved. We can find the behavioral relations by maximizing U subject to a budget constraint. The budget constraint is the total expenditure on all commodities, which takes the following form:

$$\sum_{i=1}^{n} P_i x_i=M \qquad (2)$$

where P_i = price of the ith good, and M = total expenditure on n goods. We can solve the constrained maximization to give us a system of behavioral demand equations of the following form:

2.Deaton (1988), has developed and implemented an appropriate methodology to utilize information from a single-year cross-section budget survey that permits determination of price elasticities by utilizing information on the spatial distribution of prices.

$$P_i x_i = P_i \gamma_i + \beta_i (M - \sum_{j=1}^{n} P_j \gamma_j) \qquad (3)$$

for i = 1, 2, 3,...n

The adding-up condition demands that parameters β_i are all positive and that the parameters add up to one to satisfy the budget constraint. Since P_i is positive, the regularity condition (that is that the implied Slutsky substitution matrix is symmetric and negative semidefinite [see Yoshihara (1969)]) will hold only if $(x_i - \gamma_i)$ and $(\beta_i - 1)$ are of opposite sign for every i [see Parks (1969) and Brown and Deaton (1972)]. This can only be true if $x_i \succ \gamma_i$. Hence, the regularity condition suggests that $x_i \succ \gamma_i$. A positive β_i implies that the marginal utility of good i is decreasing [see Goldberger (1987)]. Thus these demand functions satisfy the condition of homogeneity in prices and total expenditure, the adding-up restriction (that the estimated or predicted expenditure for the different commodities equal total expenditure in any period), and the regularity condition, which implies quasi concavity of the utility function.

The parameter γ_i is called the subsistence parameter. This implies that the individual first purchases γ_i units of good i at a cost of $P_i\gamma_i$ which is called "committed" or "subsistence" consumption. The total cost of subsistence is $\sum P_j\gamma_j$. This leaves $M - \sum P_j\gamma_j$ as "supernumerary expenditure". The β_i's denote how a consumer allocates his supernumerary expenditure over different commodities. The parameter γ_i may be positive or negative. Positive γ_i implies inelastic demand and negative γ_i implies elastic demand [see Pollak and Wales (1969)]. The subsistence parameters γ_i's are no longer valid when some of them are negative, hence it is not appropriate to regard the intercept terms as "subsistence" quantities unless they are assumed to be positive.[3] However, if we restrict the γ_i's to being positive, then all commodities would be price elastic, a condition that would not be a realistic assumption empirically. Therefore, I let the signs of the γ_i be determined empirically.

We also can calculate the (noncompensated) own price, cross price, and expenditure elasticities of commodity i from (3).[4] They can be expressed as follows:

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3. According to Pollak (1971), if the γ_i's are all negative it makes no sense to describe the individual as purchasing a necessary (negative) collection of goods and dividing his supernumerary income in constant proportions (β_1, β_2,...β_n) among the goods.

4. Compensated Own Price Elasticity and Compensated Cross Price Elasticity can be calculated as follows:

$$\eta_{ii}=\frac{\partial x_i P_i}{\partial P_i x_i}=-1+\frac{\gamma_i(1-\beta_i)}{x_i} \qquad (4)$$

$$\eta_{ij}=\frac{\partial x_i P_j}{\partial P_j x_i}=-\beta_i\frac{P_j\gamma_j}{x_i P_i} \qquad (5)$$

$$\eta_m=\frac{\partial x_i M}{\partial M x_i}=\frac{\beta_i M}{x_i P_i} \qquad (6)$$

Given that $0 \prec \beta_i \prec 1$, when γ_i is positive, the absolute value of the own price elasticity of commodity i will always be smaller than one, so commodity i will be price inelastic. Similarly, when γ_i is negative, the absolute value of the own price elasticity will always be greater than one; hence commodity i will be price elastic. Clearly from (5), we can see that when commodity j is price elastic, the cross price elasticity is positive. When the γ_i's are positive, the cross price effects are negative, a fact implying that the income effects are stronger than the substitution effects. From (6), we find that expenditure elasticity can be obtained on the basis of the equation $w_i = P_i x_i/M$, the budget share of commodity i, and an estimate of the marginal expenditure share parameter β_i. We can combine them with the additivity restriction of $\sum\beta_i = 1$ and find that $\sum\eta_i w_i = 1$, which is a very reasonable restriction stating that the sum of the expenditure elasticities weighted by budget shares should be equal to one. However, the assumption that $\beta_i \succ 0$ rules out the possibility of having inferior goods in the model [see Parks (1969)].

The demand system from (3) has a total of 2n structural parameters, of which 2n-1 are independent parameters in view of the adding-up restriction. The independent parameters are n-1 β's and n γ's. In order to specify the stochastic form of the equation, a disturbance term ε_i was

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$$\eta_{ii}=(\beta_i-1)(1-\frac{\gamma_i}{x_i}),\eta_{ij}=-(\frac{\beta_i P_j}{P_i x_i})(x_J-\gamma_j)$$

introduced into each share equation.

$$P_i x_i = P_i \gamma_i + \beta_i (M - \sum_{j=1}^{n} P_j \gamma_j) + \varepsilon_i \qquad (7)$$

for i = 1, 2, 3,...n

We assume that the ε's follow a normal distribution with a mean of zero and a variance-covariance matrix Ω for all i. Since these LES equations are highly interrelated, a systematic approach was used to estimate the parameters of the model. The estimates are obtained by utilizing Zellner's (1962) method for "iterative seemingly unrelated regression".[5]

III.Data

We use annual Taiwan time series data for 1951-1990 to estimate the LES model. The primary source for these data series is the "Statistical Yearbook of the Republic of China" published by Directorate-General of Budget, Accounting and Statistics, Executive Yuan, Republic of China (1975, 1985, and 1991). In this study, seven commodity groups are used to estimate the model. They are as follows :

Group (1): Food, beverage & tobacco.
Group (2): Clothing & footwear.
Group (3): Gross rent, fuel & power, furniture, furnishing & household equipment & operations.
Group (4): Medical care & health expense.
Group (5): Transport & communication.
Group (6): Recreation, entertainment, edu.& cultural services.
Group (7): Miscellaneous goods & services.

The series on expenditure at current prices were deflated by end-year population estimates to convert them into per capita figures.[6] Following Lluch and Powell (1975) and Hassan and

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5.Since the expenditure shares of the seven different groups of commodities always add up to one, the sum of the disturbances across the seven equations is zero at each observation. This implies that the covariance matrix of the disturbance term will be singular. In order to ensure a nonsingular covariance matrix, we can drop any one equation from the system and then minimize the residual covariance matrix with respect to the parameters and obtain maximum likelihood estimates of the system.

6.Average per capita income for the past four decades in Taiwan are $ 50,931 National Taiwan dollars and the average total expenditure on seven commodity groups are $ 27,100

Johnson (1980), the price series on the groups were obtained by dividing expenditure in current dollars by expenditure in constant dollars and are called implicit deflators which means that the weights of the various components shift with the changing composition of commodities or services over time.

IV. Empirical Results

The parameter estimates of the Linear Expenditure System are reported in Table 1 along with the values of the t statistics and R^2 coefficient. The t values for all of the estimates are statistically significant as shown in Table 1. The regularity conditions have been satisfied at all observations. The R^2 values for the individual equations have been reported and are all particularly significant because the data are time series and a large number of parameters are used in the equations [see Pollak and Wales (1969)]. From Table 1, we see that the negative minimum or committed expenditure γ_i in the LES system can only be measured for commodity group (5) [Transport & communication] alone. In other words, this is the only commodity expenditure that is price elastic in the individual budget. This is different from L-P's (1975) findings (see Table 2). One justification for L-P's results may be that their chosen sample periods (1955, 58-68) were based on data from Taiwan's early development period. During that period, almost all of the transport & communication were controlled (monopoly) by the government, people had less choice than what they might have right now.[7] The γ_i values for all other commodities are positive, implying that they are price inelastic commodities. As expected, the expenditure level for commodity group (1) [Food, beverage & tobacco] is higher than that for other commodity groups. This result is the same as those reported in L-P's findings. The parameters β_i's shown in Table 1 indicate that the commodity group (3) [such as gross rent, fuel & power, furniture, furnishing & household equipment & operations] had the highest estimated marginal budget share of 0.237. The second highest marginal budget share, 0.212, was the commodity group (1) [Food, beverage & tobacco].[8]

National Taiwan dollars. This means that about 53.21 percent of total income spent on these seven commodity groups. For the past four decades, the average saving rates are 25.5 percent.

7. During this time, Taiwan was still under less-developed status. According to Chang (1991), Taiwan has ranked as one of the developing countries since 1971.

8. From Table 2, we can see that the marginal budget share for commodity group (1) (Food, beverage & tobacco) (β_1), the L-P's findings ranked as the highest one compared with my findings is the second high. The commodity group (3) (Gross rent, fuel & power, furniture, furnishing & household equipment & operations) (β_3) jump to become the highest one in my findings compared with L-P's findings is the second high.

These results are similar to those found in Hassan and Johnson's (1980) study on U.S. consumer demand pattern. The third highest is the commodity group (6) [Recreation, entertainment, educational & cultural services]. Compared with the L-P's findings, this commodity group jump from the lowest (β_6=0.047) on L-P's findings to the third high (β_6=0.188) on our findings. One justification is that as Taiwan's economy has boomed, spare time for people in Taiwan has also increased. According to a survey [see Taiwan Sports (1991)] on weekly time allocation, people in Taiwan now enjoy an average of six hours and twenty minutes of spare time a day, which is about a quarter of the day. Groups with the lowest marginal budget shares are commodity group (2) [Clothing & footwear] and commodity group (4) [Medicare care & health expense]. One reason for low marginal budget shares for these two commodity groups may be that Taiwanese people are very conservative in their expenditures in these groups. Especially for commodity group (4), medical care and health expense, since traditional Taiwanese don't like to see the doctors very often, only when they are very ill. Sometimes they just go to the pharmacy buy some medicine and take care of their own illness.[9] This low expenditure level for commodity group (4) may have been the impetus behind a Legislative Branch proposal on Jan 17, 1990, to establish a ministry of social welfare and health, one major goal of which is to institute a comprehensive national health insurance program by 1994. The Six-Year National Development Plan (1991-1996) also promotes this social welfare and health.

Table 1:Estimated LES for Taiwanese Consumers. (1951-1990)

Commodity Group	Minimum required quantities (γ_i)	Marginal budget share (β_i)	R^2
Group (1)	14497 (41.764)	0.212 (58.369)	0.9989
Group (2)	957 (16.091)	0.050 (57.125)	0.9973
Group (3)	4238 (11.804)	0.237 (124.59)	0.9995
Group (4)	1080 (12.737)	0.049 (49.393)	0.9967
Group (5)	-1003 (-3.240)	0.179 (53.649)	0.9933
Group (6)	986 (2.435)	0.188 (64.159)	0.9982
Group (7)	1087 (4.982)	0.085 (29.001)	0.9888

*The number in parenthesis denotes t-statistics.

9.In Taiwan, people can go to the pharmacy to buy the medicine without any prescription from doctor. This medical system is totally different from that of U.S.

Table 2: A comparison of L-P and our findings.

Commodity Group	β_i		γ_i		η_{ii}		η_m	
	C-F	L-P	C-F	L-P	C-F	L-P	C-F	L-P
Group (1)	0.212	0.327	14497	8280	-0.454	-0.438	0.554	0.577
Group (2)	0.050	0.068	957	427	-0.589	-0.397	1.030	1.230
Group (3)	0.237	0.244	4238	1754	-0.719	-0.534	1.114	1.340
Group (4)	0.049	0.089	1080	587	-0.600	-0.493	1.046	1.551
Group (5)	0.179	0.052	-1003	75	-1.163	-0.856	1.999	2.962
Group (6)	0.188	0.047	986	257	-0.896	-0.548	1.498	1.835
Group (7)	0.085	0.174	1087	1062	-0.754	-0.601	1.197	1.804

L-P denotes Lluch and Powell's findings.
C-F denotes our own findings.

Elasticity estimates for the LES model such as own price elasticities, both noncompensated and compensated, cross-price elasticities, and expenditure elasticities have also been calculated on the basis of the parameters estimates in Table 1 and presented in Table 3 and 4, respectively. All own price elasticities have the right sign, and all compensated price elasticities are, of course, smaller than those of the corresponding noncompensated price elasticities. The estimated own-price elasticities indicate relative inelastic demand for all commodity groups, except commodity group (5) [Transport & communication]. The higher price elasticity of commodity group (5) implies that the consumption of this commodity is more sensitive to price changes. These results are similar to those found on L-P's studies except commodity group (5) which is the only one commodity group has a higher price elasticity (-1,163) in our findings (see Table 2). The estimated commodity group expenditure elasticities for the LES model indicate that six of the seven commodities are relative luxurious; that is, commodity group expenditure elasticities are greater than one for commodity group (2)-(7), except commodity group (1) [Food, beverage & tobacco]. This confirms Engle's Law (decreasing share of expenditure on food when individual's income goes up). The η_m is only 0.5544 which indicates that the commodity group (1) appear to be necessities for Taiwanese. The elasticity of near 2 for commodity group (5) [Transport & communication], implies that as the general level of income rises, demand for such good will increase rapidly. The results of this analysis suggest that commodity groups such as (2)-(7) are highly preferred items in the consumer's budget, and their consumption is fairly sensitive to changes in income. These results may not be surprising for developing countries, where commodity group (2)-(7) are rather expensive commodities and may be viewed as relative luxurious. These results also tend to confirm L-P's findings on their studies. However, the η_m (estimated commodity group expenditure elasticities) are higher from those found by L-P's findings. A justification may be that during the sample period chosen by L-P, the income of Taiwanese was just taking off, so the income effect at this early period is much more stronger than what it is right now. This means that as income increases, the demand for such commodities increase more rapidly at the early period than at the recent period. The cross-price elasticities (η_{ij}, $i \neq j$) with respect to the price of each commodity group are presented in Table 4. As we mentioned before the positive η_{ij}'s indicate that all goods are net substitutes and the negative η_{ij}'s indicate that income effects swamp substitution effects so that all goods are gross complements. Clearly

from Table 4, the η_{ij}'s are only positive for the cross-price elasticities of commodity group (1)-(7) [except (5)] with respect to the price of commodity group (5).[10] This indicates that they are net substitutes. The rest of η_{ij}'s are all negative which means that they are gross complements.

Table 3:LES Expenditure Elasticities, Own Price Elasticities for Taiwanese Consumers. (1951-1990)

Commodity Group	Expenditure Elasticities	Noncompensated Own Price Elasticities	Compensated Own Price Elasticities
Group (1)	0.5544	-0.45395	-0.2232
Group (2)	1.0303	-0.58937	-0.5401
Group (3)	1.1138	-0.71883	-0.4819
Group (4)	1.0465	-0.60006	-0.5510
Group (5)	1.9995	-1.16284	-0.9838
Group (6)	1.4976	-0.89635	-0.7084
Group (7)	1.1974	-0.75358	-0.6960

Table 4:LES Elasticity Matrix for Seven Commodity Groups for Taiwanese Consumers. (1951-1990)

Commodity Groups	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	-0.4539	-0.0001	-0.0435	-0.1092	0.04645	-0.0089	-0.0106
(2)	-0.2730	-0.5893	-0.0808	-0.0203	0.0183	-0.0165	-0.0197
(3)	-0.2951	-0.0233	-0.7188	-0.0219	0.0198	-0.0178	-0.0213
(4)	-0.2773	-0.0219	-0.0821	-0.6000	0.0186	-0.0168	-0.0200
(5)	-0.5298	-0.0419	-0.1568	-0.0394	-1.1628	0.0320	-0.0382
(6)	-0.3968	-0.0314	-0.1174	-0.0295	0.0266	-0.0286	-0.0286
(7)	-0.3173	-0.0251	-0.0939	-0.0236	0.0213	-0.0192	-0.7535

*Evaluated at mean expenditure proportion.

V. Conclusion

This study reveals certain systematic tendencies in consumer behavior in Taiwan from 1951 to 1990. The method of systems estimation and precision estimates revealed by significant t statistics and higher R^2 make the system of equations a useful tool to characterize broad tendencies in individual allocation of expenditure behavior. Evidence from Taiwan aggregate budget data suggests that the so-called subsistence parameter (γ_i's) are positive for all commodity groups, except commodity group (5) [Transport & communication]. These imply that all commodity groups are price inelastic, except commodity group (5) is price elastic. Given the pattern of demand, the consumption on commodity group (5) will be growing over time; hence lower quality of these goods produced and supplied at home would entail larger imports of these goods. The relative higher price elasticities of commodity group (5) also indicate that price interventions for

10. Since the commodity group (5) are price elastic (because the so-called subsistence parameter is negative), the cross-price elasticity with respect to this commodity is positive.

these products can have a significant impact on consumption, at least in the short run. So it is very important for the policymakers to consider the effects of price and subsidy policies on these goods. However, the higher expenditure elasticities of commodity group (5) [η_m=1.999] indicate that this situation might improve automatically as income increases. The low expenditure share (or level) of commodity group (4) [Medical care & health expense] is also a very important issue that the policymakers should consider about (This might be the reason that one of the goals of Six-Year National Development Plan is to promote the social welfare and health of Taiwanese). In sum, our study produces important insights into consumer behavior in Taiwan. This analysis also has important implications for government production strategies relating to price and subsidy policies and promotion of reasonable standards of social welfare and health. In general, this study represents our attempt to make a systematic estimation of consumer demand to identify the broad tendencies of individual demand behavior in Taiwan. However, we don't take into account the income-distribution aspects of consumer behavior. A logical and further extension of this study would be to test the linearity assumption of the Engle curve by specifying an appropriate functional form where marginal expenditure shares vary with income, and then to study the responses of prices and expenditure elasticities with respect to the distribution of income in Taiwan.

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