Use of Curved Slits to Increase the Throughput of a Hadamard Spectrometer

Roy W. Esplin
Utah State University

Use of curved slits to increase throughput of a Hadamard spectrometer

Roy W. Esplin
Stewart Radiance Laboratory
Utah State University
1 DeAngelo Drive
Bedford, Massachusetts 01730

Abstract. The throughput of a Hadamard spectrometer is directly proportional to the length of the slits. However, optical aberrations limit the slit length that can be used. The usable length of curved and straight slits are theoretically and experimentally compared for a typical singly encoded Hadamard spectrometer that has a Czerny-Turner plane-grating mounting. For this Hadamard spectrometer the optical throughput can be increased by a factor of ten by using properly curved slits.

Keywords: spectroscopy; Hadamard spectrometers; throughput.

Optical Engineering 19(4), 623-627 (July/August 1980)

I. INTRODUCTION

The same grating mounting can be used for monochromators, grille spectrometers, and Hadamard spectrometers.[1,2] Fastie[3,4] has shown that for a monochromator that uses the Ebert mounting, astigmatism and curvature of the spectrum lines can be neutralized if the entrance and exit slits are properly curved. Consequently, for this monochromator the slit length can be made much longer for curved slits than for straight slits; and since the optical throughput is directly proportional to the slit length, the optical throughput can be greatly increased by using curved slits. In order to maximize the optical throughput of his grille spectrometer, which used an Ebert mounting, Tinsley[5] used a circularly symmetric grille.

Almost all the Hadamard spectrometers constructed to date have used straight slits. However, Esplin et al.[6] constructed a singly encoded Hadamard spectrometer that used a curved entrance slit and a Hadamard mask with curved slits. Swift et al.[7] constructed a doubly encoded Hadamard spectrometer in which curved slits were used in both the entrance and exit masks. Both these Hadamard spectrometers used the Czerny-Turner mounting.

This paper summarizes the theoretical and experimental results of a study to ascertain how much the optical throughput of a typical Hadamard spectrometer with a Czerny-Turner mounting could be increased by using curved rather than straight slits. The usable length of straight slits was ascertained using both analytic expressions and computer ray tracing. The radius of curvature for curved slits was optimized by ray tracing. The ray tracing was done using the algorithms of Spencer and Murty.[8] The transverse error of the ray-tracing results was less than 0.1 μm. A more extensive report on this study has been given elsewhere.[9]

II. DESCRIPTION OF THE SPECTROMETER

A commercially available Hadamard spectrometer, model 305-HS of Minuteman Laboratories, was used for both the theoretical analysis and for the experimental measurements. This spectrometer is shown schematically in Fig. 1. It is a singly encoded Hadamard spectrometer that uses a Czerny-Turner mounting. The entrance slit is above the plane of symmetry determined by the vertices of the two spherical mirrors and the grating center, while the detector is below this plane. This configuration allows the same optics to be used both for dispersion and dedispersion. The dispersion pass results in the formation of

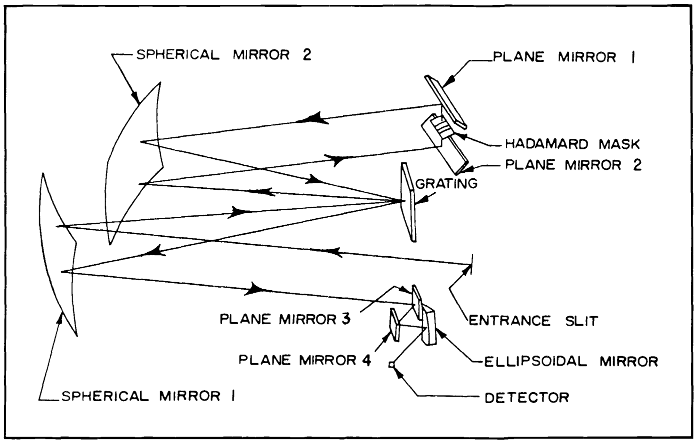

Fig. 1. Model 305-HS Hadamard spectrometer of Minuteman Laboratories, Inc.

the spectrum in the Hadamard mask plane, where it is encoded by the translating mask. The dedispersion pass recollects the dispersed energy so that the detector size can be minimized. For many detector types the detector noise is minimized by minimizing the detector size. The slit length for this spectrometer is at most half that of a conventional Czerny-Turner spectrometer because the entrance slit cannot extend below the plane of symmetry. However, because of the obscuration caused by the mounting hardware of plane mirrors 1 and 2 and the Hadamard mask, the bottom of the unobstructed entrance slit is 8.2 mm above the plane of symmetry. The top of the unobstructed entrance slit is 29 mm above the plane of symmetry. Therefore, the unobstructed entrance-slit length of this spectrometer is approximately 20 mm. The unobstructed width of the focal plane is 104 mm. A field stop 100-mm wide was used for this study. The mask translating mechanism is designed for a cyclic code; that is, encoding is accomplished by translating the mask one exit-slit width, the width of the image of the entrance slit formed in the mask plane with a spectral-line source, between each measurement.

The optical aberrations of the dispersion pass determine how much of the unobstructed slit length can be used; that is, the usable portion of the unobstructed slit length is determined by the quality of the spectral images of the entrance slit that are formed in the mask plane. Consequently, the optical system shown in Fig. 2, which is equivalent to the optics of this spectrometer for the dispersion pass, was used to compute

Original manuscript 1613 received Aug. 27, 1979; revised manuscript received and accepted for publication Dec. 5, 1979. This paper is a revision of a paper presented at the SPIE seminar on Multiplex and/or High Throughput Spectroscopy, Aug. 27–28, 1979, San Diego, which appears in SPIE Proceedings Vol. 191.

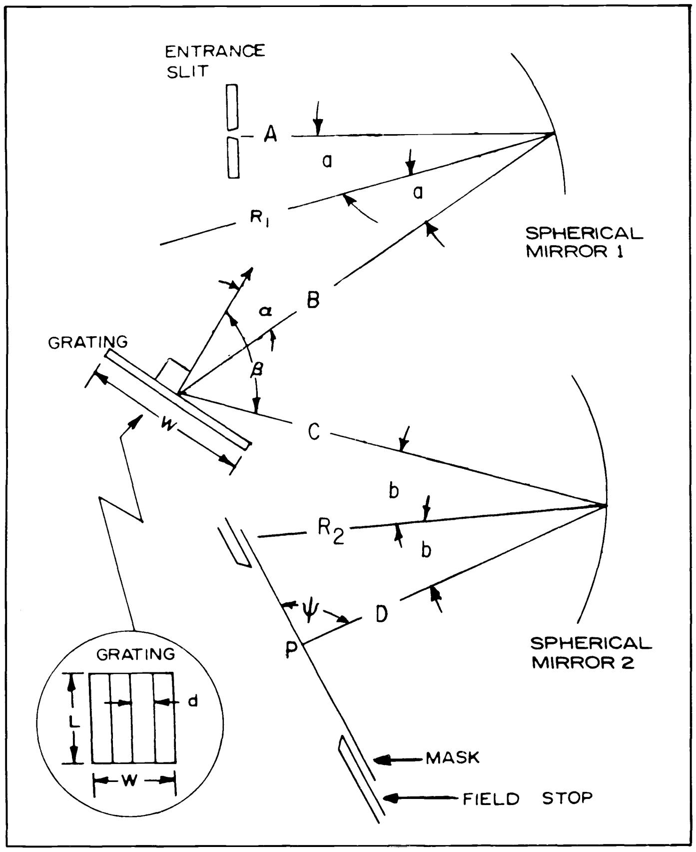

Fig. 2. Optical system used for theoretical analysis.

the theoretical slit-length limits. The basic optical parameters are identified in Fig. 2 and their values tabulated in Table I. The symbols used in Fig. 2 and Table I for these optical parameters are used throughout this paper.

III. STRAIGHT-SLIT THEORY

The usable length of straight slits is limited by the curvature of the spectral images of the entrance slit and by astigmatism.

Curvature of spectral images

The angular deviation from a straight line of the spectral image of a straight entrance slit is given by[10]

$$\phi_c = \frac{k\lambda}{2d \cos \beta} \theta^2 , \quad (1)$$

where θ is the angular distance from the plane of symmetry and λ is the wavelength. For this spectrometer

$$\theta \doteq \frac{y}{A} = \frac{2y}{R_1 \cos a} , \quad (2)$$

where y is the distance above the plane of symmetry of a point in the entrance slit. The limit on θ resulting from curvature of the spectral images can be found by equating the angle given by Eq. (1) to ϕ_s, the angular exit-slit width. The result is

$$\theta = \sqrt{\frac{2\phi_s d \cos \beta}{k\lambda}} . \quad (3)$$

For this spectrometer if w is the exit-slit width,

TABLE I. Optical Parameters of the Minuteman Hadamard Spectrometer Model 305-HS.

Parameter	Description	Value
R_1	Radius of spherical mirror 1	1000.0 mm
R_2	Radius of spherical mirror 2	925.0 mm
W	Width of grating	64.0 mm
L	Length of grating	64.0 mm
a	Off-axis angle of spherical mirror 1	6.17 degrees
b	Off-axis angle of spherical mirror 2	10.57 degrees
A	Meridional focal length of spherical mirror 1 [$(R_1/2)\cos a$]	497.107 mm
B	Distance from mirror 1 to grating	412.750 mm
C	Distance from grating to mirror 2	365.252 mm
D	Meridional focal length of spherical mirror 2 [$(R_2/2)\cos b$]	454.652 mm
k	Diffraction order	1
λ_p	Wavelength at the center of the field stop	11.5 μm
d	Groove spacing of the grating	11.11 μm
α	Incidence angle at grating	21.26 degrees
β	Diffraction angle for wavelength λ_p	42.26 degrees
ψ	Mask angle	87.7 degrees

$$\phi_s \doteq \frac{w}{D} = \frac{2w}{R_2 \cos b} \quad (4)$$

For diffraction-limited operation Eq. (3) is equivalent to the expression given by Fastie.[3] The angular half width of the diffraction pattern formed by the rectangular pupil of this spectrometer is given by

$$\phi_d = \frac{\lambda}{W \cos \beta} . \quad (5)$$

If ϕ_s in Eq. (3) is set equal to ϕ_d, then

$$\theta = \sqrt{\frac{2d}{kW}} = \sqrt{\frac{2}{kN}} = \sqrt{\frac{2}{R}} , \quad (6)$$

where N is the number of grooves on the grating and R is the diffraction-limited resolving power. For the parameter values tabulated in Table I, the diffraction-limited, exit-slit width resulting from multiplying Eq. (5) by D is 110 μm.

Equations (3) and (6) were derived by assuming that the bottom of the entrance slit was on the plane of symmetry. However, for this spectrometer the unobstructed bottom of the entrance slit is 8.2 mm above this plane. If the difference between the angular deviation for points at the top and bottom of the usable entrance slit is computed using Eq. (1), the difference set equal to the angular exit-slit width ϕ_s and the resulting equation solved for θ, the angular distance from the plane to the top of the usable entrance slit is given by

$$\theta = \sqrt{\frac{2\phi_s d \cos \beta}{k\lambda} + \theta_B^2} , \quad (7)$$

where θ_B is the angular distance from the plane of symmetry to the bottom of the entrance slit. For this spectrometer θ_B equals 8.2 mm divided by A. It follows from Eqs. (2), (5) and (7) that if ϕ_s equals ϕ_d, then for this spectrometer the top of the usable entrance slit is 12.4 mm above the plane of symmetry. Therefore, for diffraction-limited operation of this spectrometer, curvature of the spectral lines limits the length of a straight entrance slit to 4.2 mm (12.4 – 8.2 = 4.2). Equation (7) was confirmed by ray tracing.

Astigmatism

Astigmatism causes a point to be imaged as a line as illustrated in Fig. 3.

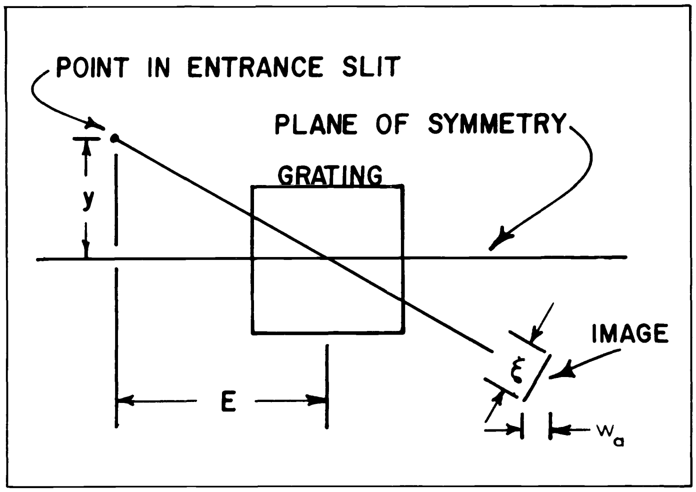

Fig. 3. Effect of astigmatism on the image of a point.

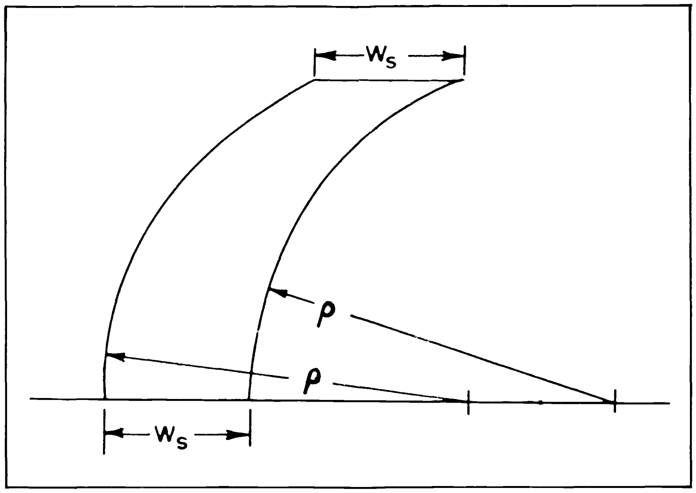

Fig. 4. Slit geometry.

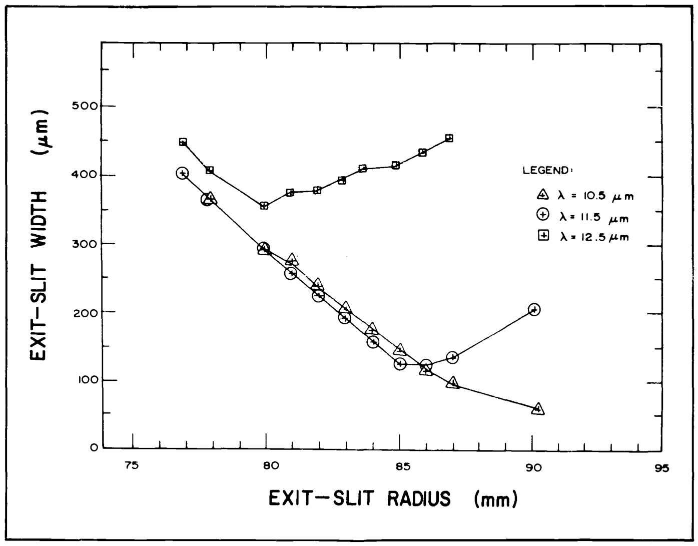

Fig. 5. Relationship between the required exit-slit width and the exit-slit radius for an entrance-slit radius of 91 mm when the top of the entrance slit is 15 mm above the spectrometer's plane of symmetry. The exit-slit width is that required to collect 85 percent of the spot diagram of a point at the top of the entrance slit.

For a straight entrance slit the required exit-slit width is

$$w_a \doteq \frac{\xi y}{E}, \tag{8}$$

where ξ is the length of the astigmatic image, y is the distance from the plane of symmetry to the top of the entrance slit, and E is the distance from the grating center to the entrance slit.

Lindblom[11] has derived a mathematical expression from which the total-path-length variation caused by astigmatism in a Czerny-Turner mounting can be computed. His expression can be converted to a transverse aberration expression.[9] The length ξ is twice this transverse aberration expression; therefore,

$$\xi = \left\{ \frac{\sin^2 a}{R_1 \cos a} + \frac{\sin^2 b}{R_2 \cos b} \right\} R_2 L \cos b \, . \tag{9}$$

Equation (9) was verified to within 2 percent by ray tracing. For the parameters tabulated in Table I, ξ equals 2.8 mm. If this value of ξ, E equal to 91 mm, and y equal to 8.2 mm are substituted in Eq. (8), w_a equals 252 μm. This is more than twice the diffraction-limited width of 110 μm. For the top of the unobstructed entrance slit, y equal to 29 mm, w_a is 892 μm.

IV. CURVED-SLIT THEORY

Fastie[3,4] showed that for an Ebert monochromator the effects of both astigmatism and curvature of the spectral lines could be neutralized by properly curving both the entrance and exit slits. Kudo[12] has shown that the optimum slit shape for the Czerny-Turner monochromator is elliptical. Since the field-stop width of a Hadamard spectrometer is large, the optimum slit shape varies significantly across the field stop. However, all slits were assumed to have the same curvature because for a cyclic-code Hadamard spectrometer a typical slit is used across a significant portion of the field stop.

The curvatures for the entrance slit and the mask slits were optimized using ray-tracing data. Spot diagrams were computed using rays spaced across the entrance pupil such that each ray was representative of an equal amount of energy. The slot widths required to collect the rays were computed as the slot curvature was varied. The optimization was done for the slit geometry illustrated in Fig. 4, where w_s is the width and ρ is the radius of the slit.

As shown by Fastie,[3,4] the optimum slit radius for an Ebert monochromator equals the distance between the entrance slit and the grating normal when the grating is in the zero-order diffraction position. For this spectrometer this distance is 91 mm. Plots of the required exit-slit width for the central wavelength and for wavelengths at approximately both extremes of the field stop with an entrance slit radius of 91 mm are shown in Fig. 5. These plots show the slot width required to collect 85 percent of the spot diagram for a point located in the entrance slit 15 mm above the plane of symmetry. It can be seen from these plots that the minimum slot width is approximately 360 μm, and the optimum exit-slit radius is 80 mm. Since the required slit width to encode 255 spectral elements within the 100 mm field stop is 392 μm, a 255 code can be used with an entrance slit 6.8 mm long (15 – 8.2 = 6.8). As can be seen from Fig. 5, the long wavelength end of the spectrum, 12.5 μm, determines the minimum exit-slit width. If the spectral interval were 10.5 to 11.5 μm, the minimum slit width would reduce to 125 μm, and the optimum exit-slit radius would be 86 mm.

As can be seen from Fig. 6, the required exit-slit width increases significantly if the entrance slit length is increased to 16.8 mm (25 – 8.2 = 16.8). The minimum exit-slit width for this entrance-slit length is 725 μm, and the optimum exit-slit radius is 81 mm. Since the required slit width to encode 127 spectral elements is 787 μm, a 127 code could be used with an entrance-slit length of 16.8 mm. It can also be seen from Fig. 6 that if the spectral interval were 10.5 to 11.5 μm, the minimum exit-slit width would be reduced to 275 μm, and the optimum exit-slit radius would be 86.6 mm.

It was found that the length of the entrance slit could be increased by

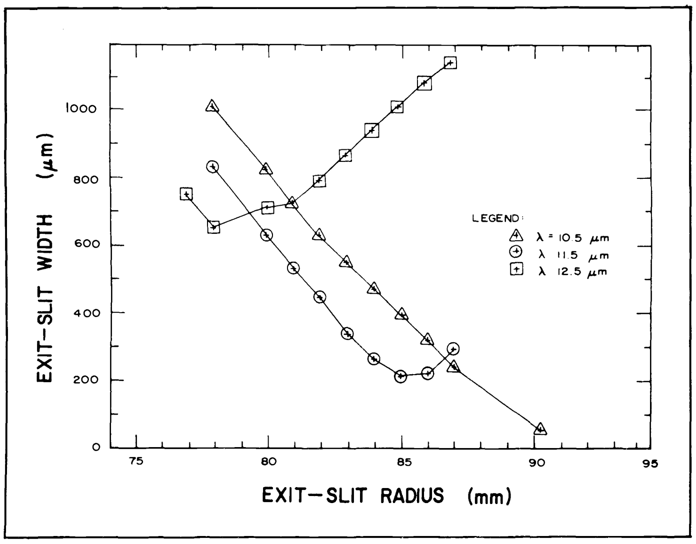

Fig. 6. Relationship between the required exit-slit width and the exit-slit radius for an entrance slit radius of 91 mm when the top of the entrance slit is 25 mm above the spectrometer's plane of symmetry. The exit-slit width is that required to collect 85 percent of the spot diagram for a point at the top of the entrance slit.

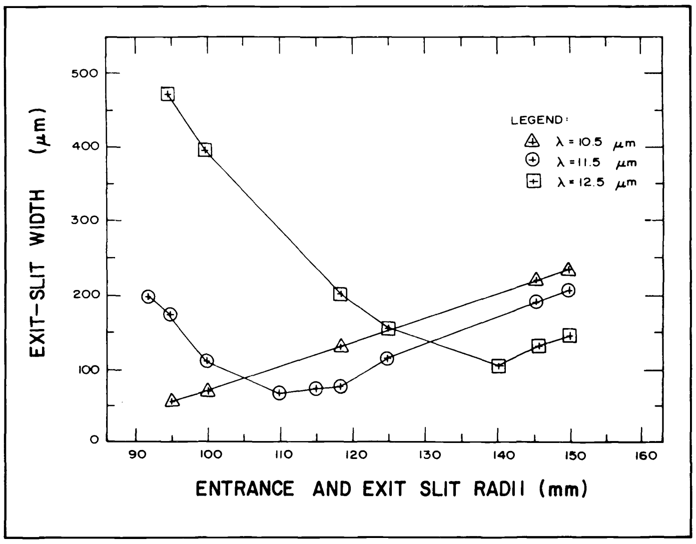

Fig. 7. Exit-slit width required when the radii of the entrance and exit slits are equal and the top of the entrance slit is 15 mm above the plane of symmetry. The exit-slit width is that required to collect 85 percent of the spot diagram of a point at the top of the entrance slit.

changing the 91 mm entrance-slit radius constraint. The required exit-slit width when entrance and exit slit radii are equal is plotted in Fig. 7. As can be seen from this figure, the minimum exit-slit width is 150 μm, and the optimum radius of curvature for the entrance and exit slits is 125 mm.

Since 150 μm is considerably smaller than the 197 μm required to encode 511 spectral elements within the 100 mm field stop, a 511 code can be used with an entrance slit at least 6.8 mm long.

The Hadamard mask used for the experimental measurements was fabricated with a slit radius of 118 mm before the plots in Fig. 7 were made. This radius was selected by ascertaining the radius for which the principal ray from a point in the entrance slit 29 mm above the plane of symmetry at the central wavelength passed through the centerline of the exit slit. As can be seen from Fig. 7, using a radius of 118 mm instead of 125 mm improves the spectrometer preformance for the spectral interval 10.5 to 11.5 μm but degrades it for spectral elements with wavelengths near 12.5 μm. The computed slit-length limitations of the

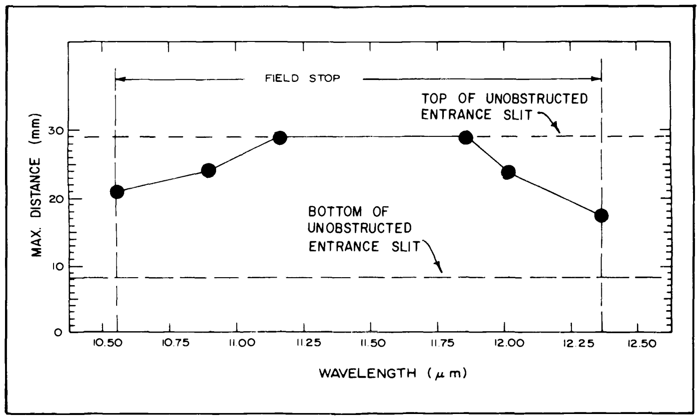

Fig. 8. Maximum distance above the plane of symmetry for which there is no significant deterioration in the resolving power when the entrance and exit-slit radii both equal 118 mm and the exit-slit width is 197 μm.

Hadamard spectrometer with entrance and exit slit radii of 118 mm and an exit-slit width of 197 μm are summarized in the plot shown in Fig. 8. As can be seen from this plot, there is no resolution degradation across the entire spectral interval contained within the field stop for an entrance-slit length of 9.3 mm (17.5 - 8.2 = 9.3). However, terminating the entrance slit at 17.5 mm above the plane of symmetry is not a good solution to the resolution degradation at the ends of the field stop. A much better solution is to use an entrance slit 20.8-mm long (29 - 8.2 = 20.8) and shape the field stop so that the portions of the spectral images corresponding to points above the plot in Fig. 8 are obstructed by the field stop.

It was ascertained that the slit-length limits were essentially constant for small changes in the grating angle. The optimal curvature changed only 2 percent when the grating was rotated so that the central spectral element had a wavelength of 9.3 μm instead of 11.5 μm.

V. THEORETICAL SUMMARY

For straight slits the image of a point at the bottom of the unobstructed entrance slit is 252–μm wide. Therefore, a 511 code, which requires a slit width of 197 μm, cannot be used with straight slits even a fraction of a millimeter long. In contrast, curved slits over 20–mm long can be used with a 511 code.

VI. EXPERIMENTAL COMPARISON

The instrumental profile of the spectrometer was measured using both straight and curved slits. The following procedure was used. Two Hadamard masks, one with curved slits and the other with straight slits, were fabricated. The exit-slit width of both masks was 197 μm. The radius of curvature of the curved-slit mask was 118 mm. Two entrance slits, one curved with a radius of 118 mm and one straight, were fabricated. The width of both entrance slits was 171 μm. The anamorphic magnification of this spectrometer is 1.15, and 197 μm divided by 1.15 equals 171 μm. In each mask all the slits except one whose width equaled the exit-slit width, 197 μm, were masked off. The instrumental profile was recorded by plotting the detector signal versus mask position as the unmasked slit was translated past an entrance-slit image formed using a source consisting of a mercury lamp with a filter to isolate the 0.5461–μm line. The 18th and 19th orders of the 0.5461–μm line whose locations in the field stop are the same as those of first-order lines at 9.830 and 10.376 μm, respectively, were used. The instrumental profile was measured for spectral elements at various locations across the field stop. The spectral elements were identified by numbering them consecutively from 1 to 511, with spectral element number 1 at the short wavelength end of the field stop. For each instrumental profile measurement, the grating was rotated until either the 18th or 19th order entrance-slit image was located in the desired spectral element. Typical measurements of the instrumental profile are shown in Fig. 9. If the entrance-slit image and the mask slit were

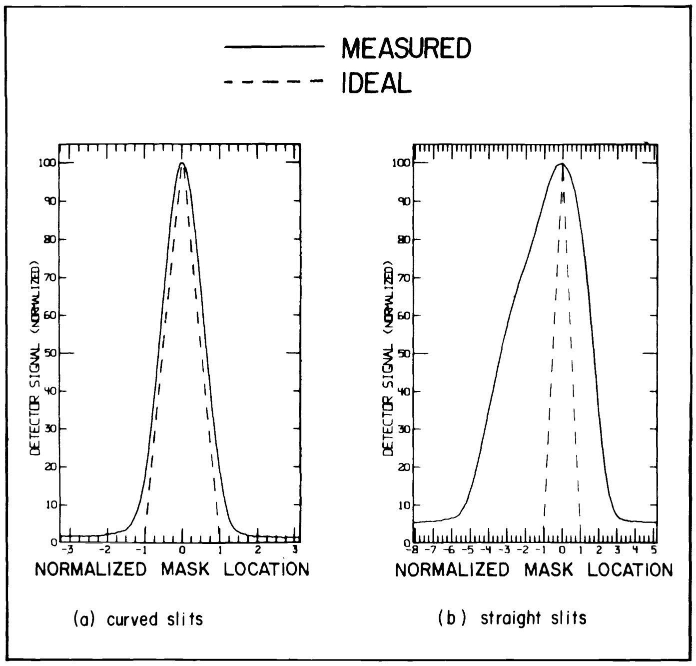

Fig. 9. Instrumental profile at spectral element number 500 for both curved and straight slits 20-mm long.

perfectly matched, the instrumental profile would be the dotted triangles shown in this figure. Instrumental profiles for various entrance-slit lengths were measured by masking down the top of the entrance slit.

The results of the instrumental-profile measurements are summarized in Fig. 10, where the width of the instrument profile at 50 percent of its peak value is plotted versus the location of the spectral element in the field stop. The superiority of curved over straight slits is evident from this figure. The width of the instrumental profile is less for 20–mm tall curved slits than it is for 2–mm tall straight slits. Inasmuch as the optical throughput is directly proportional to the slit length, the throughput of this spectrometer is a factor of ten larger for curved slits than for straight slits. For 20–mm tall curved slits, the normalized instrumental profile width approaches the ideal of 1 for all spectral elements within the field stop.

VII. CONCLUSIONS

The optical throughput of a Hadamard spectrometer with either a Czerny-Turner or Ebert mounting can be significantly increased by using curved rather than straight slits. For the particular Hadamard spectrometer discussed in this paper, the optical throughput was increased by more than a factor of ten by using curved slits instead of straight slits.

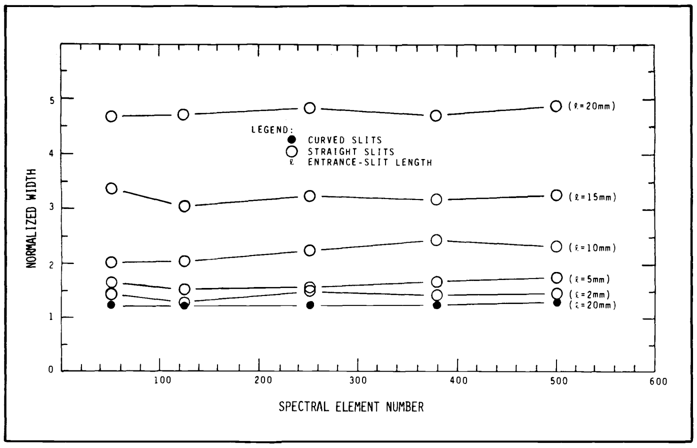

Fig. 10. Measurements of the width of the instrumental profile made with straight and curved slits. The width of the instrumental profile at 50 percent of its peak value, normalized to the exit-slit width of 197 μm, is plotted.

ACKNOWLEDGMENTS

The author wishes to acknowledge the invaluable contributions of George A. Vanasse, who suggested the use of curved slits. The author also wishes to thank Doran J. Baker, Ronald J. Huppi, Clair L. Wyatt, L. Rex Megill, William L. Jones, Chris S. Corey, and Randy B. Shipley for their assistance.

The work reported in the paper was funded by the Air Force Office of Scientific Research (AFOSR) through a contract administered by the Air Force Geophysics Laboratory (AFGL).

REFERENCES

1. Decker, J. A., Jr., "Hadamard Transform Spectroscopy," in *Spectrometric Techniques, Vol. I,* G. A. Vanasse, Ed., Academic Press(1977).
2. Harwit, M., and N. J. A. Sloane, *Hadamard Transform Optics,* Academic Press(1979).
3. Fastie, W. G., "Image Forming Properties of the Ebert Monochromator," J. Opt. Soc. Am. 42, 647-651(1952).
4. Fastie, W. G., "Experimental Performance of Curved Slits," J. Opt. Soc. Am. 43, 1174-1175(1953).
5. Tinsley, B. A., "The Circularly Symmetric Grille Spectrometer," Appl. Opt. 5, 1139-1145(1966).
6. Esplin, R. W., R. B. Shipley and R. J. Huppi, *Far Infrared Atmospheric Instrumentation and Measurements,* AFCRL–TR–75–0482(1975).
7. Swift, R. D., R. B. Wattson, J. A. Decker, Jr., R. Paganetti and M. Harwit, "Hadamard Transform Imager and Imaging Spectrometer," Appl. Opt. 15, 1595-1609(1976).
8. Spencer, G. H., and M. V. R. K. Murty, "General Ray-Tracing Procedure," J. Opt. Soc. Am. 52, 672-678(1962).
9. Esplin, R. W., *"Increasing the Throughput of Hadamard Spectrometers by the Use of Curved Slots,"* Ph.D. dissertation, Utah State Univ.(1978). Also published as AFGL–TR–78–0232(1978).
10. James, J. F., and R. S. Sternberg, *The Design of Optical Spectrometers,* Chapman and Hall(1969).
11. Lindblom, P., "Theory of the Two-Mirror Plane-Grating Spectrograph," J. Opt. Soc. Am. 62, 756-762(1972).
12. Kudo, K., "Plane Grating Monochromators of Ebert, Pfund and Czerny-Turner Types," Science of Light 9, 1-30(1960).