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Energy Efficiency in a Delay Constrained Wireless Network

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Abstract—This paper investigates the energy efficiency in a wireless channel with delay requirement. We address the energy efficiency transmission by considering both transmission power and circuit power consumptions based on the effective capacity approach. Effective capacity has been widely used to incorporate QoS requirements in the wireless communication and is an effective approach to modeling the physical layer wireless fading channel with the link layer parameters. We first derive the quasi-convex generalized form of energy efficiency formulation based on effective capacity under delay constraints. The general expression can be applied to various fading scenarios. We further develop a two-step binary search algorithm to find the best energy efficiency and the corresponding optimal transmission signal-to-noise ratio (SNR). The numerical results show that both the circuit power and queuing delay requirement have a big impact on the overall wireless channel energy efficiency.

I. INTRODUCTION

Supporting quality of service (QoS) in the next generation wireless networks such as 4G long term evolution (LTE) [1] is a key performance promise for the multimedia applications. From generation to generation, wireless system spectral efficiency has been substantially improved through technologies ranging from physical layer modulation and coding, Multiple Input Multiple Output (MIMO) schemes to efficient air-interface cross-layer optimization [2]. Due to the ubiquitous applications of real-time services such as video streaming, online gaming, mobile computing, etc, which are all delay sensitive and power consuming, energy efficient transmission under QoS constraints over wireless channels is becoming increasingly important. Conventional analytical results in [3][4][5] based on ergodic Shannon capacity did not address the QoS requirements from the upper layers. Effective capacity [6][7] was proposed as a metric to measure the performance in the presence of statistical QoS needs. It captures the queuing delay in the form of decay rate of buffer occupancy probability and establishes a link-layer model for the wireless physical channel by taking the large deviation theory with the assumption of large queue length. Effective capacity can be considered as the maximum channel capacity under limitations of buffer violation probability or equivalent QoS requirement.

There have been extensive studies on effective capacity to address the QoS requirements in various wireless fading envi-

ronments. [8] discussed effective capacity of MIMO channel in the low-power, wideband, and high signal-to-noise ratio (SNR) region. [9] studied the effective rate of multiple-input and single-output (MISO) transmission in Nakagami-m, Rayleigh, Rician fading channels. [10] investigated the impact of antenna correlation on the effective capacity in a MISO channel. On the other hand, the fast growing data traffic will inevitably trigger tremendous escalation of energy demand and excessive battery consumption. Thus energy saving on both mobile devices and the networking infrastructure equipments are urgent. [11] proposed an optimal power allocation scheme in the Orthogonal frequency division multiple access (OFDMA) system. These works greatly extend the understanding on the two important subjects, namely QoS and energy efficiency, based on which further theoretical research can be conducted to build up a better modeling of energy consumption as well as the relationship between energy efficiency and QoS constrained effective capacity in various wireless environments.

Motivated by above reasons, in this paper, we study the energy efficiency under delay constraint while considering circuit power in different fading environments. As a key performance metric, energy efficiency is defined as the power consumption per information bit for reliable transmission. If there is no circuit power, Shannon capacity shows that the best energy efficiency operation is achieved at low transmission power [3]. When a more practical energy consumption model is considered, the best energy efficiency and optimal transmission SNR will be different. Energy consumption considered in this paper includes both transmission power and circuit power. Furthermore, the best energy efficiency will be dependent on the QoS requirement such as queuing delay from link layer. In this paper, we show that energy efficiency function is quasi-convex with respect to SNR when QoS and circuit power are included in the energy model. We propose a two step binary search algorithm in finding the optimal solution of this quasi-convex function.

The rest of the paper is organized as follows. Section II gives the effective capacity in a general wireless fading channel, followed by the study in effective capacity in a Rayleigh fading channel. Section III presents the optimal energy efficiency given QoS considerations. Section IV derives the numerical search method for the optimal energy efficiency. Section V

presents the simulation results. The paper concludes in section VI.

II. EFFECTIVE CAPACITY IN A WIRELESS CHANNEL

A transmission in a wireless channel can usually be formulated as

$$y = \sqrt{p}hx + n \quad (1)$$

Where p is the transmission power, n denotes the additive white Gaussian noise (AWGN), and h is the channel gain, representing channel fading characteristics, including path loss, shadowing and short term fading. For example, Rayleigh fading model is effective to capture the fading in a dense urban environment when there is no line of sight signal between receiver and transmitter [12]. In this paper, we characterize the channel statistics by the square amplitude of coefficient $z = |h|^2$ with probability density function $p(z)$.

A. Shannon capacity

Shannon capacity provides the theoretic upper bound of the information rate that can be supported in a wireless channel at given SNR.

$$C = W \log(1 + \rho E\{|h|^2\}) \quad (2)$$

where W denotes the channel bandwidth, and $\rho = \frac{P_t}{N_0 W}$ is the transmission power P_t normalized by the product of bandwidth W and AWGN variance N_0 . We also call ρ as the transmission SNR. C provides a theoretical upper bound for the achievable data rate in a wireless channel subject to noise and interference. In reality, due to various physical constraints, the achievable data rate is below this value. When considering QoS constraint such as maximum or average queuing delay, the data rate supported is even lower, due to the statistical fluctuations of the wireless channel, which usually cause nontrivial queuing delay at the upper layers.

B. Effective capacity

Shannon capacity provides us with insights on the maximum capacity of a wireless channel subject to noise and interference. But it is not suitable for analytic exploration of a practical system throughput under delay constraints. Effective capacity incorporates the statistical QoS metrics and captures the decay rate of buffer occupancy violation at the large buffer size region. It models the physical layer wireless channel with link-layer parameters, where delay performance can be characterized by triple parameters: source data rate r_s , delay bound D_{max} , delay-violation probability ϵ . D_{max} and ϵ satisfy:

$$P_r\{D(\infty) \geq D_{max}\} \leq \epsilon \quad (3)$$

where $D(\infty)$ is the steady-state queuing delay experienced by the traffic flow. In large deviation theory, the decay rate of queue occupancy probability with a queue of large enough buffer size which is infused with a constant source data rate can be approximated as

$$P_r\{Q \geq q_{max}\} \approx e^{-\theta q_{max}} \quad (4)$$

where Q is the steady state queue length at the transmitter and $q_{max} = r_s D_{max}$. θ is the QoS exponent characterizing the delay constraints. A larger θ denotes a more stringent QoS requirement and delay constraint. When $\theta = 0$, there is no delay constraint and the system can bear unlimited delay. According to theorem proposed in [13], relationship between queue size and delay violation probability is established through $P_r\{D(\infty) \geq D_{max}\} \leq m \sqrt{P_r\{Q \geq q_{max}\}}$ under large queue length assumption, where m is a positive constant. In a block fading channel, where the channel fading process h is constant during time T , effective capacity $R(\theta)$ is defined as a log-moment generation function [7]:

$$R(\theta) = -\frac{1}{\theta T} \log_e \{E\{e^{(-\theta TC)}\}\} \quad (5)$$

where C is the instantaneous Shannon capacity. Substitute (2) into the above formula, and we get the effective capacity formulation as a function of transmission SNR ρ and QoS exponent θ .

$$R(\rho, \theta) = -\frac{1}{A} \log_2 \{E\{(1 + \rho|h|^2)^{-A}\}\} \text{bit/s/Hz} \quad (6)$$

where $A = \theta T W / \ln 2$. According to this definition, ergodic Shannon capacity C can be regarded as the special case of $R(\rho, \theta)$ with $\theta = 0$, which can be verified in the following. Applying L'Hospital Rule to equation (6), we can obtain

$$\begin{aligned} R(\rho, 0) &= \lim_{A \rightarrow 0} -\frac{1}{A} \log_2 \{E\{(1 + \rho|h|^2)^{-A}\}\} \\ &= \lim_{A \rightarrow 0} -\log_2 e \frac{E\{\ln(1 + \rho|h|^2)^{-1} (1 + \rho|h|^2)^{-A}\}}{E(1 + \rho|h|^2)^{-A}} \\ &= E\{\log_2(1 + \rho|h|^2)\} \end{aligned} \quad (7)$$

The ergodic Shannon capacity C can be regarded as the special case when QoS exponent $\theta = 0$, i.e., $C = R(\rho, 0)$. We will use this value as the benchmark comparison in the later sections.

C. Effective capacity in a Rayleigh Fading Channel

Our study in this paper focuses on the energy efficiency in a Rayleigh fading channel. We start with the effective capacity study in such a channel. For a Rayleigh fading channel, square of channel amplitude $z = |h|^2$ obeys an exponential distribution with a probability density function $p(z) = \frac{1}{\Omega} e^{-\frac{z}{\Omega}}$, with parameter Ω . Substitute $p(z)$ into formula (6), the Rayleigh channel's effective capacity can be calculated as

$$\begin{aligned} R_{EC}(\rho, \theta) &= -\frac{1}{A} \log_2 \{E\{(1 + \rho|h|^2)^{-A}\}\} \\ &= -\frac{1}{A} \log_2 \int_0^\infty (1 + \rho z)^{-A} \frac{1}{\Omega} e^{-\frac{z}{\Omega}} dz \\ &= -\frac{1}{A} \log_2 \{(\rho \Omega)^{-A} e^{\frac{1}{\rho \Omega}} \int_{\frac{1}{\rho \Omega}}^\infty \frac{1}{z^A e^z} dz\} \\ &= \log_2(\rho \Omega) - \frac{\log_2 e}{A \rho \Omega} - \frac{1}{A} \log_2 \Gamma(1 - A, \frac{1}{\rho \Omega}) \end{aligned} \quad (8)$$

where $\Gamma(p, q) = \int_q^\infty \frac{1}{z^{1-p}e^z} dz$ is the upper incomplete gamma function. We consider the special case of no QoS requirement, i.e., $\theta = 0$. Then the effective capacity can be written as

$$\begin{aligned} R_{EC}(\rho, 0) &= E\{\log_2(1 + \rho|h|^2)\} \\ &= \int_0^\infty \log_2(1 + \rho z) \frac{1}{\Omega} \exp^{-\frac{z}{\Omega}} dz \\ &= \log_2 e (e^{\frac{1}{\rho\Omega}}) \int_{\frac{1}{\rho\Omega}}^\infty \frac{1}{ze^z} dz \\ &= \log_2 e (e^{\frac{1}{\rho\Omega}}) \Gamma(0, \frac{1}{\rho\Omega}) \end{aligned} \quad (9)$$

We will show later the close form expression for the optimal energy efficiency is hard to derive. However, we are always interested in the asymptotic system behaviors such as QoS and energy efficiency at high and low SNR regions.

1) *Effective capacity at High SNR*: High-SNR approximation was initially used to study system performance in code division multiple access system with random spreading, and later on was employed to study ergodic Shannon capacity in MIMO system [17]. The same approach is used in this paper to exploit the effective capacity under the delay constraint in a Rayleigh fading channel. The impact of QoS constraints on performance in the high SNR region can be captured by two measurements, high-SNR slope S_∞ and power offset L_∞ , which are defined in [17] as

$$S_\infty = \lim_{\rho \rightarrow \infty} \frac{R_{EC}(\rho, \theta)}{\log_2 \rho}, \quad L_\infty = \lim_{\rho \rightarrow \infty} (\log_2 \rho - \frac{R_{EC}(\rho, \theta)}{S_\infty}) \quad (10)$$

Thus at high SNR, effective capacity can be approximated as

$$R_{EC}(\rho, \theta) = S_\infty (\log_2 \rho - L_\infty) + o(1) \quad (11)$$

where $o(1)$ denotes a finite constant value when SNR approaches infinity. In a single transmitter and single receiver antenna case, we substitute $n_T = 1$ and $n_R = 1$ into formula (10). Thus $S_\infty = \frac{1}{A}$ and $L_\infty = \frac{1}{A} \log_2 E\{|h|^2\}^{-A} = \log_2 \frac{\Gamma(1-A, 0)^{\frac{1}{A}}}{\Omega}$. Using Theorem 6 in [8], the approximated effective capacity at high-SNR can be derived as

$$R_{EC}(\rho, \theta) = \frac{1}{A} \log_2 \frac{\rho\Omega}{\Gamma(1-A, 0)^{\frac{1}{A}}} + o(1). \quad (12)$$

2) *Effective capacity at low SNR*: At low-SNR, we can approximate effective capacity with second order Taylor expansion as

$$R_{EC}(\rho, \theta) = \dot{R}_{EC}(0, \theta)\rho + \ddot{R}_{EC}(0, \theta)\frac{\rho^2}{2} + o(\rho^2) \quad (13)$$

where

$$\begin{aligned} \dot{R}_{EC}(0, \theta) &= \log_2 e \frac{E\{(1 + \rho|h|^2)^{-(A-1)}|h|^2\}}{E\{(1 + \rho|h|^2)^{-A}\}} \Big|_{\rho=0} \\ &= \log_2 e E\{|h|^2\} \\ &= \log_2 e \Omega \end{aligned} \quad (14)$$

and

$$\begin{aligned} \ddot{R}_{EC}(0, \theta) &= \log_2 e \left\{ A \left(\frac{E\{(1 + \rho|h|^2)^{-A-1}|h|^2\}}{E^2\{(1 + \rho|h|^2)^{-A}\}} \right)^2 \right. \\ &\quad \left. - (A+1) \frac{E\{(1 + \rho|h|^2)^{-A-2}|h|^4\}}{E\{(1 + \rho|h|^2)^{-A}\}} \right\} \Big|_{\rho=0} \\ &= \log_2 e (A E^2\{|h|^2\} - (A+1) E\{|h|^4\}) \\ &= -\log_2 e (A+2)\Omega^2 \end{aligned} \quad (15)$$

III. OPTIMAL ENERGY EFFICIENCY BASED ON EFFECTIVE CAPACITY

In this section we study the wireless channel energy efficiency based on its effective capacity. The demand for high data rate real-time services such as video streaming in a wireless network has been soaring over the last decade, and this trend would continue due to the increasing popularity of multimedia applications [14]. However, the energy consumption also goes up and the battery technology has not kept up with this trend, which makes energy efficiency and QoS in wireless network design more and more important. In this paper, we define the energy efficiency function E_b as power consumption per information bit transmission under a delay constraint, which is captured in the effective capacity. In our model, energy consumption includes both transmission power P_t and the circuit power consumption P_c . We assume circuit power is a constant value as long as the system is powered on. The transmission power can be adapted in order to achieve a desirable SNR and thus a target transmission rate. Without loss of generality, we normalize the energy efficiency function E_b with noise power spectral density N_0 for simplification. Hence, the normalized energy efficiency function can be denoted by

$$\frac{E_b}{N_0} = \frac{P_t + P_c}{R(\rho, \theta) N_0 W} = \frac{\rho + \frac{P_c}{N_0 W}}{-\frac{1}{A} \log_2 (E\{(1 + \rho|h|^2)^{-A}\})} \quad (16)$$

where A has been defined earlier as $A = \theta T W / \ln 2$.

Proposition 3.1: Energy efficiency function defined in (16) is a strictly quasi-convex function with respect to transmission SNR ρ .

Proof: We first prove the denominator in (16), which is effective capacity R , is a concave function of ρ . Define function $g(\rho) = (1 + \rho z)^{-A}$.

$$\dot{g} = -A(1 + \rho z)^{-A-1} z, \quad \ddot{g} = A(A+1)(1 + \rho z)^{-A-2} z^2 > 0$$

where \dot{g} and \ddot{g} denote the first and second order derivation of $g(\rho)$ with respect to ρ . Since $z = |h|^2 > 0$, then $\ddot{g} > 0$. So $g(\rho)$ is a strictly convex function of ρ . $E\{(1 + \rho z)^{-A}\} = \int (1 + \rho z)^{-A} p(z) dz$ is also a convex function of ρ because $\int f(z, v) dv$ is convex if $f(z, v)$ is convex with respect to z for each v in the domain [15]. As function $-\frac{1}{A} \log_2(z)$ is a strictly decreasing function of z , $R(\rho, \theta) = -\frac{1}{A} \log_2 (E\{(1 + \rho|h|^2)^{-A}\})$ is a concave function of ρ .

We further prove in the following $\frac{E_b}{N_0}$ is a strictly quasi-convex function. If sub-level set $\frac{E_b}{N_0} = \left\{ \rho \mid \frac{\rho + \frac{P_c}{N_0 W}}{-\frac{1}{A} \log_2 (E\{(1 + \rho|h|^2)^{-A}\})} \leq \alpha, \rho \geq 0 \right\}$ is a

convex set for any α , then $\frac{E_b}{N_0}(\rho)$ is a quasi-convex function with respect to ρ .

- when $\alpha \leq 0$, there is no feasible ρ .
- when $\alpha > 0$, according to the conclusion derived above, $\frac{\alpha}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}) + \rho + \frac{P_c}{N_0 W}$ is a convex function of ρ , so set $\{\frac{\alpha}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}) + \rho + \frac{P_c}{N_0 W} \leq 0, \rho \geq 0\}$ is also convex. Therefore, $\{\rho | \frac{\rho + \frac{P_c}{N_0 W}}{-\frac{1}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\})} \leq \alpha, \rho \geq 0\}$ is a convex set.

So the energy efficiency function is a strictly quasi-convex function. ■

Proposition 3.2: There exists a unique minimum value for the energy efficiency function. The optimal SNR is achieved at $\rho_0 > 0$, with $\frac{E_b}{N_0}|_{\rho=\rho_0} = 0$. $\frac{E_b}{N_0}$ denotes the first order derivation of $\frac{E_b}{N_0}$ with respect to ρ .

Proof: As a first step, we check the asymptotic bounds for $\frac{E_b}{N_0}$.

- when $\rho \rightarrow 0$, $R(\rho, \theta) \rightarrow 0$, so $\frac{E_b}{N_0}|_{\rho=0} \rightarrow +\infty$
- when $\rho \rightarrow +\infty$, as

$$\begin{aligned} \frac{E_b}{N_0} &= \frac{\rho + \frac{P_c}{N_0 W}}{-\frac{1}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\})} \\ &\geq \frac{\rho + \frac{P_c}{N_0 W}}{\frac{1}{A} \mathbb{E}\{-\log_2((1 + \rho|h|^2)^{-A})\}} \\ &= \frac{\rho + \frac{P_c}{N_0 W}}{\mathbb{E}\{\log_2((1 + \rho|h|^2))\}} \\ &\geq \frac{\rho + \frac{P_c}{N_0 W}}{\log_2(1 + \rho \mathbb{E}\{|h|^2\})} \end{aligned} \quad (17)$$

The two inequalities in the above formula are derived from Jensen's inequality by applying it to the concave function $\log_2(\cdot)$ twice. Obviously, the last expression in (17) approaches $+\infty$ when $\rho \rightarrow +\infty$, thus energy efficiency function $\frac{E_b}{N_0}$ also approaches $+\infty$. Therefore, the optimal SNR for minimum energy efficiency function must be achieved between 0 and $+\infty$ if it exists.

Next, we prove that the optimal SNR exists and is uniquely achieved at $\frac{E_b}{N_0}|_{\rho=\rho_0} = 0$. The first order derivation for energy efficiency function can be expressed as

$$\frac{\dot{E}_b}{N_0} = \frac{R(\rho, \theta) - (\rho + \frac{P_c}{N_0 W})\dot{R}(\rho, \theta)}{R^2(\rho, \theta)} = \frac{f(\rho)}{R^2(\rho, \theta)} \quad (18)$$

where function $f(\rho)$ is defined as $f(\rho) = R(\rho, \theta) - (\rho + \frac{P_c}{N_0 W})\dot{R}(\rho, \theta)$ and

$$\dot{R}(\rho, \theta) = \log_2 e \frac{\mathbb{E}\{(1 + \rho|h|^2)^{-(A-1)}|h|^2\}}{\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}} \quad (19)$$

Further calculation shows:

- when $\rho = 0$, $f(\rho) = -\log_2 e \frac{P_c}{N_0 W} \mathbb{E}\{|h|^2\} < 0$
- when $\rho = +\infty$, $f(\rho) = -\frac{1}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}) - \log_2 e \rightarrow +\infty$

The derivation of function $f(\rho)$ can be expressed as

$$\dot{f}(\rho) = -(\rho + \frac{P_c}{N_0 W})\ddot{R}(\rho, \theta) \geq 0. \quad (20)$$

$\dot{f}(\rho)$ is always positive since we have proved that $R(\rho, \theta)$ is a concave function of ρ and thus $\ddot{R}(\rho, \theta) \leq 0$. So $f(\rho)$ is an increasing function of ρ . $f(\rho) < 0$ when $\rho < \rho_0$ and $f(\rho) > 0$ when $\rho > \rho_0$, and the zero-crossing point is uniquely achieved at ρ_0 . From (18), we can conclude that $\frac{E_b}{N_0} < 0$ when $\rho < \rho_0$ and $\frac{E_b}{N_0} > 0$ when $\rho > \rho_0$. That means $\frac{E_b}{N_0}$ first monotonically decreases when $\rho < \rho_0$ and then monotonically increases when $\rho > \rho_0$. Therefore, minimum value of energy efficiency function is unique and is uniquely achieved at ρ_0 , where $\frac{E_b}{N_0}|_{\rho_0} = 0$. ■

IV. BINARY SEARCH FOR OPTIMAL ρ_0

Setting $\frac{E_b}{N_0}|_{\rho=\rho_0} = 0$, we get:

$$(\rho + \frac{P_c}{N_0 W}) \frac{\mathbb{E}\{(1 + \rho|h|^2)^{-(A-1)}|h|^2\}}{\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}} = \frac{\ln\{(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\})\}}{-A}$$

The closed-form expression for optimal ρ_0 is very difficult to derive from above formulation. We propose a two-step binary search algorithm to find the optimal solution for the quasi-convex function $\frac{E_b}{N_0}$. For the optimal energy efficiency problem, we introduce a new variable t and convert the quasi-convex optimization problem into a convex problem, whose optimal solution can be searched using the step binary search algorithm. The primary quasi-convex optimization problem is

$$\min \frac{E_b}{N_0} = \frac{\rho + \frac{P_c}{N_0 W}}{-\frac{1}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\})} \quad (21)$$

subject to

$$\rho \geq 0$$

By letting $t = \frac{E_b}{N_0}$, the primary optimization problem can be reformulated as

min t subject to

$$\rho + \frac{t}{A} \log_2(\mathbb{E}\{(1 + \rho|h|^2)^{-A}\}) + \frac{P_c}{N_0 W} \leq 0, \rho \geq 0 \quad (22)$$

The binary search algorithm is detailed in Table I.

V. NUMERICAL RESULTS

In the section, we numerically evaluate the energy efficiency under various delay constraints and circuit power values. Without loss of generality, Rayleigh fading channel is assumed in the effective capacity calculation. Channel block length $T = 1$ ms and system bandwidth $W = 1$ MHz in the simulation. Figure 1 shows the effective capacity under different delay constraints, i.e., different θ values. Curves with QoS parameter $A = 0, 1, 3, 5, 7, 9$ are plotted from top to bottom, $A = \theta T W / \ln 2$. Note that when $A = 0$ or equivalently $\theta = 0$, there is no delay constraint and the effective capacity is equal to the ergodic Shannon capacity. As expected, a strict queuing delay constraint from upper layers would decrease the wireless channel capacity, which leads to a lower data rate supported at the upper layers. At low-SNR, all the curves have the same slope at zero SNR $\dot{R}_{EC} = \log_2 e \mathbb{E}\{|h|^2\}$ regardless of the delay requirements or A values. This could be explained in the

TABLE I
TWO-STEP-BINARY-SEARCH-ALGORITHM FLOW

```

1: Step 1: Initialize  $t = t1 = t2 = t0 > 0$ ,
2: if the solution for constraint (22) is feasible then
3:   repeat
4:      $t = t1 = t1/2$ ;
5:     Using gradient descent method to search the minimum value of
(22)
6:   until solution for  $\rho$  becomes infeasible
7:    $t2=t1*2$ ;
8: else
9:   repeat
10:    Using gradient descent method to search the minimum value of
(22)
11:  until solution for  $\rho$  becomes feasible
12:   $t1=t2/2$ ;
13: end if
14: Step2: Initialize  $\epsilon = 1e - 4$ ;
15: while  $|t1 - t2| \geq \epsilon$  do
16:    $t = (t1 + t2)/2$ ;
17:   gradient descent search the minimum value of (22)
18:   if solution for  $\rho$  is feasible then
19:      $t1=t$ ;
20:   else
21:      $t2=t$ ;
22:   end if
23: end while

```

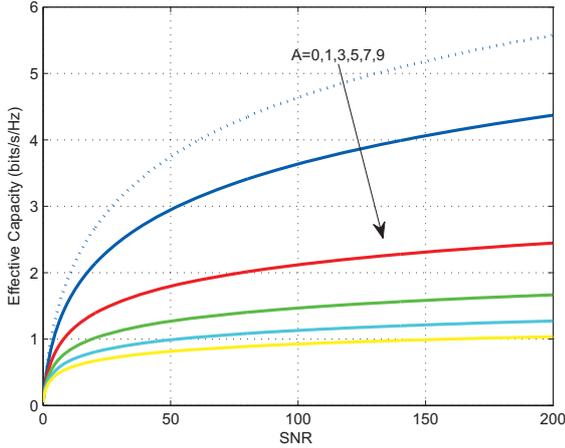


Fig. 1. Effective Capacity under different QoS Constraint

way that a close-to-zero transmission SNR supports a close-to-zero data rate, hence decreases buffer violation probability. However, a more stringent QoS requirement or a higher A always has a smaller increase slope of effective capacity curve especially at low SNR as illustrated in Figure 1 and formula (15). A similar behavior can be observed at high-SNR, i.e., curve slope S_∞ inversely increases with QoS parameter A . A strict QoS requirement decreases the rate of effective capacity.

Figure 2 plots the energy efficiency function with respect to transmission SNR under different QoS requirements. Energy efficiency with and without circuit power consumption are plotted in dotted and solid lines respectively. Curves with QoS parameter $A = 9, 7, 5, 0$ are ranked from top to bottom in both cases. As expected, energy efficiency with circuit

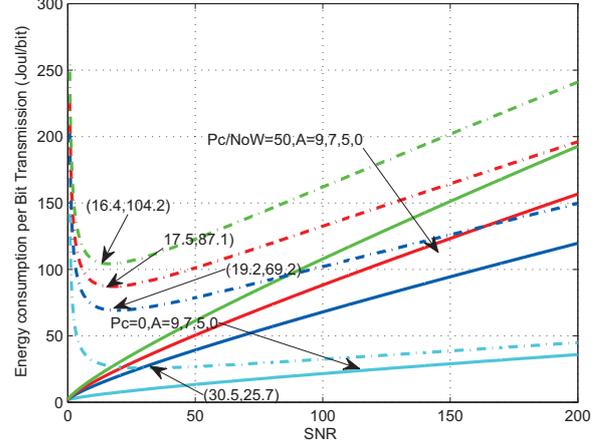


Fig. 2. Impact of QoS parameter on energy efficiency

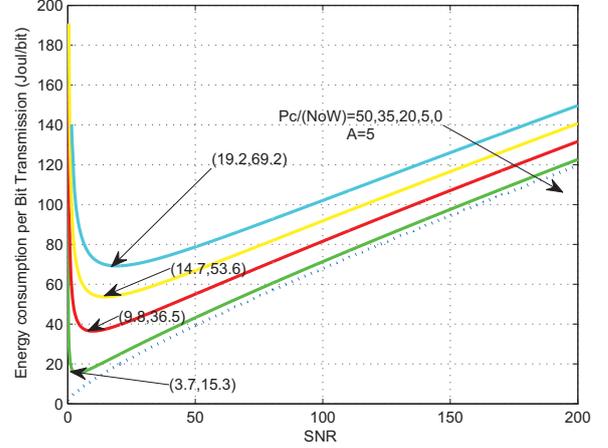


Fig. 3. Impact of circuit power on energy efficiency

power is a quasi-convex function, which first decreases and then increases with transmit SNR. In the no circuit power case, the function monotonically increases with SNR and minimum energy efficiency function is always achieved at $SNR = 0$ with $\frac{E_b}{N_0}|_{min} = \frac{1}{\log_2 e E\{|h\|^2\}}$ irrespective of the QoS requirements. With the increase of QoS requirements or parameter A value, a lower energy efficiency is observed in both cases. So a higher transmission power is always required for a faster transmission if all other system parameters such as W are kept the same. In Figure 3, we illustrate the impact of circuit power on energy efficiency. Obviously, adding circuit power will lower energy efficiency. Furthermore, we observe that a lower circuit power will need a lower transmission power to achieve the maximum energy efficiency. And the corresponding maximum energy efficiency is higher for a lower circuit power than that for a higher circuit power.

Due to the complexity of energy efficiency function, it is hard to find the closed-form expression for minimum

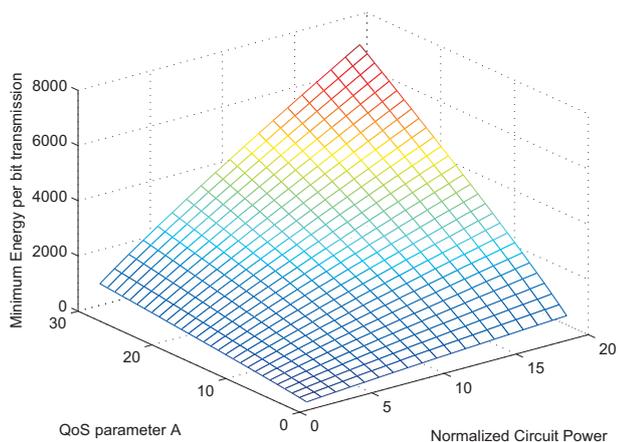


Fig. 4. Best energy efficiency under different QoS parameter and circuit power combination

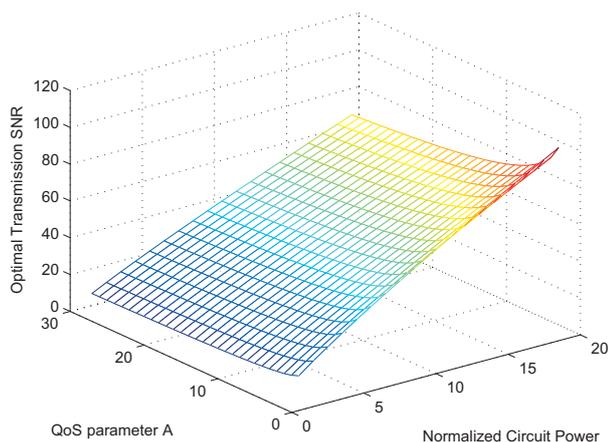


Fig. 5. Optimal SNR under different QoS parameter and circuit power combination

transmission SNR. Thus, the two step binary gradient descent search algorithm is used to find the optimal solution. We use 3D Figure 4 to illustrate the energy efficiency as a function of QoS parameter and circuit power and use 3D Figure 5 to illustrate the optimal transmission SNR as a function of QoS parameter and circuit power. As shown in Figure 4, both QoS and circuit power will increase energy consumption. In Figure 5, the corresponding optimal SNR value decreases when the QoS requirement goes up. The data rate becomes lower as transmission SNR decreases. But a more stringent QoS requirement indicates a lower buffer violation probability, which is only possible when the arrival rate is lower. Therefore, the optimal transmission SNR is always lower with a more strict QoS requirement in order to achieve a lower buffer violation probability when circuit power is considered.

VI. CONCLUSION

In this paper, we have investigated the wireless channel energy efficiency under QoS constraints that are captured through limitations on buffer violation probability in a large queue length region. Effective capacity is employed as the performance metric to measure the maximum system throughput under such constraints. The energy efficiency function is defined as the total energy consumption including both transmission power and circuit power consumption, per information bit transmission. The wireless channel energy efficiency thus depends on transmission power, circuit power as well as QoS requirement. In order to find the best energy efficiency operation and optimal transmission SNR, we have proposed a two-step binary search algorithm to solve the quasi-convex energy efficiency problem. Numerical study showed that the increase of QoS requirements always lower down the energy efficiency. The study also showed that adding circuit power will lower energy efficiency as well.

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