8-1-2011

Structural and Reduced-Form Models: An Evaluation of Current Modeling Criteria in Econometric Methods

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STRUCTURAL AND REDUCED-FORM MODELS: AN EVALUATION OF CURRENT MODELING CRITERIA IN ECONOMETRIC METHODS

by

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A research project submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Economics

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UTAH STATE UNIVERSITY
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2011

ABSTRACT

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This paper evaluates the structural form model of John Rust's 1987 paper, *Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher,* by using reduced-form models to evaluate the same data and interpret the results. The question is whether reduced-form modeling such as probit and logit models can be as useful as structural models for prediction.

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Introduction

In terms of types of models, there are two that will be discussed. Reduced-form models evaluate endogenous variables in terms of observable exogenous variables and serve to identify relationships between the variables. Structural models are derived from theory and often include unobservable parameters that help describe behavior at a deep level.

In economic research papers authors use structural and reduced-form models to describe and help define and describe the data that has been collected from the world around them. These models contain variables that can be put into two categories: dependent and independent. Dependent variables are functions of the independent variables and receive their value depending on the value of independent variables; they are derived within the model. Furthermore, these variables may be governed by parameters such as utility or cost parameters that give supplemental information about the model and can be used to forecast behavior. Sometimes these parameters are not observable and can only by estimated using structural form models.

The models containing these variables and parameters are simply tools to explain past behavior and forecast future behavior. Econometrics is used to analyze data using different estimators, and a challenge lies in determining which model most effectively describes the data. In an attempt to depict the data correctly, however, models may become too formal, extravagant, and, hence, complicated to estimate. The parameters, which are often left to the econometrician to evaluate, can be abstract and nonreplicable to other researchers but may be needed to estimate parameters that are left out of reduced-form models. Should these two models be used simultaneously as tools in estimation?

In 1976 Economist Robert Lucas wrote a paper criticizing the way macroeconomists built and interpreted models. Later labeled the Lucas Critique, the paper argues that it is unwise to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data. Lucas argued modelers should only include "deep parameters" that govern individual behavior that predict what individuals will do and aggregate the decisions to calculate the macroeconomic effects of a policy change (Lucas, 1976). In other words, Lucas believed microeconomic techniques should be used in estimating models to account for changes in policy through people's reactions to the changes. He believed the best way to see the effect of any policy change is through aggregating all individuals' behavioral changes. This critique changed the way econometricians and others created their models. More and more models included these deep parameters. A good example of one of these parameters is risk aversion or utility. Many macroeconomic models contain such a parameter that changes the output of the model when adjusted. Structural models with varying parameters became the standard and reduced-form models were used sparingly and became substandard.

This paper attempts to parallel previous structural and reduced-form testing using the same data set introduced in John Rust's 1987 paper, *Optimal Replacement of GMC Bus Engines: An Empirical model of Harold Zurcher*. In his paper, Rust creates a structural model with deep parameters to estimate an optimal stopping rule. For the current research, a reduced-form model will be used to test the same data.

The hypothesis to be tested and evaluated in this paper is the following: reduced-form modeling can do as well as structural models and can add value to econometric testing. The results show that it is hard to prove the reduced-form model can do as well as the Rust's structural model and further research is needed.

This paper will continue with a description of the econometric model that will contain a description of the data to be used, data sources, the models, and a description of the variables. Furthermore, an estimation of the reduced-form models and predictions tables will be included. A conclusion reiterating the results will close this paper.

Literature Review on Structural vs. Reduced-Form Models

The previous literature on this topic consists mostly of research papers that briefly mention reduced-form models before passing on to a structural model for estimation. However, a 1997 *Econometrica* paper by Bill Provencher compares and contrasts structural and reduced-form models entitled "*Structural versus Reduced-Form Estimation of Optimal Stopping Problems"*. The paper examines "several statistical, interpretive, and policy implications of reduced-form estimation of optimal stopping problems" (Provencher, 1997). He concludes the failure of reduced-form modeling lies in failing to properly interpret the relationship between the model and underlying optimal stopping problem. Provencher argues an econometrician should be acutely aware of the data-generating process prior to choosing which type of model to use and continues by saying, "failure to understand the process generating the data may lead to incorrect econometric analysis and misinterpretation of coefficients" (Provencher).

Christopher Sims has been a leader of research in the application of vector autoregression models (VAR) and takes this argument to the area of forecasting. In his paper, *Are Forecasting Models Usable for Policy Analysis?,* Sims states the following when discussing the interpretation of parameters of structural equations: "There is no unique standard, however, for when a parameter has an economic interpretation. The parameters in a model may have interpretations, yet the model may not be structural, in the sense that we may be unable to use it to predict the consequences of the actions which interest us. Conversely, we may be able to use a model to make such predictions accurately even though some or all of its parameters do not have neat interpretations" (Sims, 1997). Sims goes on to say that many economists have come to think of structural models as models with satisfying interpretations for all parameters. He believes the unarguable assertion that predicting the effects of policy requires identification of a structural model thus becomes "via a semantic confusion", a source of serious misunderstanding (Sims, 1997).

The literature by Provencher and Sims introduces some research completed on reduced-form versus structural models. Because the motivating question to this paper refers to the value of econometric tools when estimating models for *any* data set rather than attempting to explain behavior of a *certain* data set using a specified model (even though only one data set is used in this research), it may not be traditional. In other words, the topics of other existing literature are specific to the data and not the tools used.

The Data

The data and model to be discussed come from John Rust's 1987 paper, *Optimal Replacement of GMC Bus Engines: An Empirical model of Harold Zurcher*. Harold Zurcher was the superintendent of maintenance at the Madison, Wisconsin Metropolitan Bus Company, and Rust's null hypothesis was that "Zurcher's decisions on bus engine replacement coincide with an optimal stopping rule". The data is a panel time series set that includes accumulated odometer readings of buses during the months between December 1974 and May 1985. There is a two-month gap between July and August of 1980 where no data exists because of a strike within the company. Rust estimates the structural parameters to test whether Zurcher's behavior is consistent with the model. Conversely, seeing how a reduced-form model compares using the same data motivates the research in this paper.

The original data given to John Rust included three types of maintenance cost structures; (i) routine maintenance including oil changes, brake improvements, and the like; (ii) major repairs that do not require overhauls; and (iii) engine replacements or major overhauls that are equivalent to engine replacements. We focus on the last type.

The raw data set includes observations for 166 buses that are included within 9 groups. Each group represents a different model bus. For example, T8H203 is a 60x4 matrix for GMC model T8H203. All buses were not in the company's possession at the beginning of the time period. In some cases, the buses were bought and placed on the road midway through the time period. The data accounts for these buses by stating the first month and first odometer reading for each bus. In total, there are 15,964 data points; a significant amount of data by most standards that should give interesting results.

The data contains identifying information for each bus including the bus type and year, and the individual bus number. For the purpose of modeling, a new binary variable was added to the data set called “Replace”. The variable is given a value of 1 if the bus had a replacement in that month and 0 if no replacement occurred.

Econometric Model

Rust describes three models in his paper that ultimately laid the foundation for the structural model that is estimated using a nested fixed-point algorithm. The first model includes some stylized assumptions that restrict the model but allow it to have a closed form solution. The second model is a general maximum likelihood estimation algorithm that does not need the limiting assumptions used in the first model. Finally, he takes the regenerative stopping problem described in the first model and combines it with the algorithm of model the second model to produce a formal structural model.

First, the *regenerative optimal stopping* problem used in the first model of Rust’s paper:

Vθxt=supΠEj=t∞βj-tu(xj, , fj, θ1xt,

where the utility function *u* is given by:

uxt,i,θ1=-cxt,θ1 if it=0,-P-P+c0,θ1 if it=1.

The state variable xt denotes the accumulated mileage since last replacement on the bus engine at time *t.* Let it denote Zurcher’s replacement decision at time *t*, where it=0 means keep and it=1 means replace the bus engine. Furthermore, the cost function *c* is comprised of the following:

$$cx,\theta1=mx,\theta11+\mu x,\theta12bx,\theta13,$$

where $mx,\theta11$ is the conditional expectation of normal maintenance and operating expenses, $\mu x,\theta12$ is the conditional probability of an unexpected engine failure, and $bx,\theta13$ is the "conditional expectation of towing costs, repair costs, and the perceived dollar cost of lost customer goodwill in the event of an unexpected engine failure" (Rust, 1987). Referring back to the value function$V\theta xt$, Π is an infinite sequence of decision rules $\Pi=ft,ft+1,...$ where each ft refers to Zurcher's replacement decision at time *t*.

A few problems arise when estimating this model that make it unreliable in tests. First, the solution for the likelihood function depends critically on specific choice of functional form: namely, that monthly mileage has an i.i.d. exponential distribution, which the data refutes (Rust). Secondly, and maybe more restrictive, is that the state of the bus is completely described by a single variable, accumulated mileage xt. It is more probable that Zurcher bases his replacement decisions on other information εt also referred to as the unobservable "error term", a state variable which is observed by the agent (Zurcher) but not by the statistician.

The second model described in the Rust paper deviates from the topic to describe structural estimation without closed-form solutions. In a previous paper also written by Rust, a maximum likelihood estimation algorithm for a class of dynamic discrete choice models is developed (Rust, 1987). The model does not require closed-form solutions for the agent's stochastic control problem and treats unobervables εt in a consistent fashion. For a full description, refer to original paper.

In the final structural model, Rust applies the nested fixed-point algorithm to the bus engine replacement data set and the model above is modified to be as follows.

The implied utility function is:

$$uxt,\ i,\ \theta1+\varepsilon ti=-RC-c0,\ \theta1+\varepsilon t1 \quad \text{if } i=1,-cxt,\theta1+\ \varepsilon t0 \qquad \text{if } i=0.$$

Where *RC* denotes the expected cost of a replacement bus engine.

Relaxing the assumptions of the first model, monthly mileage is allowed to have an arbitrary parametric density function *g,* which implies a transition density of the form

$$pxt+1xt,it,\theta3=gxt+1-xt,\ \theta3 \text{ if } it=1,gxt+1-0,\ \theta3 \qquad \text{if } it=0.$$

From this model, Rust's procedure is to "estimate the unknown parameters $\theta=\beta,\theta1,RC,\theta3$ by maximum likelihood using the nested fixed-point algorithm" (Rust). This is where the estimation deviates from the original paper.

Here, a reduced form model is introduced and estimated which has no structural parameters and can be put into the form of probit and logit. In the reduced-form model, $yt$is the dependent variable which will take values $yt=0$ to denote "keep" or $yt=1$ for "replace". The independent variable $\beta1$ denotes bus engine accumulated mileage at time *t.* Another variable, $\beta2$, will be estimated to show the effects of high mileage on a bus engine replacement. Both probit and logistic models are estimated. A linear model will act as a comparison.

$$yt=\alpha+\ \beta1+\ \beta2+\varepsilon t$$

Replace = Intercept + Mileage + Mileage2 + error term.

This simple model is a substitute to the earlier formal model that requires rigorous calculations. For this model, pooled effects are assumed. There are three types of binary choice models for panel data. First, a *random effects* model works under the condition that there are no individual effects within groups but there are unique individual characteristics that are the result of random variation. In other words, the fact that the data is separated by bus type has no effect on the model. Conversely, a *fixed effect* model allows for the correlation of data within a group and, hence, a different intercept for each group. In this data set, that would mean that each bus within a group may be correlated but not with buses in other groups. The model used here is a *pooled effects* model that has the key assumption that there are no unique attributes of the buses within groups or across time. This will lead ultimately to one intercept for the model.

Estimation

Before entering into the estimation results, an introduction of the statistical software is needed. A programming language and statistical software environment called R is used in the following estimation. R is influenced by the language S and is becoming standard software among statisticians partly because of its free source code and graphing capabilities.

Table 1 shows estimated coefficients and marginal effects (slope) of the variables. Table 2 gives the standard errors, test statistics, and p-values. Table 3 gives the predictions of the model. Tables 4 through 6 show the same information as the previous models but for a specific bus type. Because the model is binary, meaning the dependent variable takes a value of 0 or 1, marginal effects are computed because estimated coefficients with these models do not convey much information. Marginal effects are essentially the slopes of the curves in question. For a linear model, the slope *is* the coefficient and does not vary in value. Note that the tables include the variables (Intercept, Mileage, Mileage2) and the estimates produced within R using different packages within the software to test the data. Particularly, and function called *glm()* is used. The *general linear model* function allows regressions with binary dependent variables with different "family links" such as probit, logit, and many others that describe the distribution of the model.

Table 1 Regression Coefficients

LM			**Probit**		**Logit**	
Variable	**Coefficient**	**Slope**	**Coefficient**	**Slope**	**Coefficient**	**Slope**
Intercept	-3.69e^-03	-3.69e^-03	-3.49		-7.88	
Mileage	1.20e^-07	1.20e^-07	1.00e^-05	1.66e^-07	2.84e^-05	1.96e^-11
Mileage2	-2.45e^-13	-2.45e^-13	-2.00e^-11	-2.00e^-11	-5.72e^-11	-3.96e^17

Note that when marginal effects are computed, the slopes move closer in value to each other with all three estimators. The above models are shown to be significant at the 99 percent confidence level and have the following test statistics shown in Table 2 Regression Standard Errors. Again, linear, probit , and a logit model are represented. A linear model requires a t value for the test statistic whereas probit and logit models require z values.

Table 2 Regression Standard Errors

LM				**Probit**			**Logit**		
Variable	**Std. Error**	**t value**	**P(>|t|)**	**Std. Error**	**z value**	**P(>|z|)**	**Std. Error**	**z value**	**P(>|z|)**
Intercept	2.12e-13	-1.73	0.082	2.08e-01	-16.76	< 2e-16	6.24e-01	-12.62	<2e-16

Mileage	2.44e-08	4.92	8.37e-07	1.92e-06	5.200	1.99e-07	5.69e-06	4.99	5.86e-07
Mileage2	6.22e-14	-3.93	8.22e-05	4.30e-12	-4.65	3.27e-06	1.26e-11	-4.54	5.58e-06

For the Linear Model, the R-squared, Adjusted R-squared, and p-value are 0.002068, 0.001943, and 6.69e-08 respectively. The R-squared value is extremely low and is not a desired result. Typically, an econometrician likes to see high R-squared values because this means there is not much information being left out of the model. The p-value, however, is also very low and *is* a desired result.

To test the model, prediction tables for the variable "Replace" are created to show the effectiveness of the reduced-form models. At the basic level, a prediction table shows how many times (and at which times) the model correctly predicted the same value as the original data. Specifically, this information is useful in determining the properties of our reduced-form model and how well it fits the data. Both the probit and logit models have the same prediction tables and only one is represented in Table 3 Prediction Chart. The predicted values are those that the reduced-form model predicts, and, conversely, the actual values are those that match the data.

Table 3 Prediction Chart

		Actual		
		0	**1**	**Totals**
Predict	**0**	15,840	124	15,964
	1	0	0	0

	Totals	15,840	124	15,964

The above table shows that the reduced-form model correctly predicts that most of the time no replacement will be made, but it fails to predict that some replacements will be made. In fact, the model did not predict any replacements given the data. This is an interesting result and will be explored in the Interpretations section.

To give further information about the data, a regression on just one bus group was conducted. Bus group T8H230 had a significant proportion of replacements with 27 in total. Most replacements were conducted in the years 1982 to 1985, and each bus that had a replacement, only had one. Other bus groups had no replacements and may not give much information about the data so it is reasonable not to include them in the tests. In this process, we are focusing on data that has the characteristic we are interested in. The following table, Table 4 T8H203 Regression Coefficients, shows the test statistics for the probit regression of bus type T8H230 similar to the full regression of all data points.

Table 4 T8H203 Regression Coefficients

	Estimate	**Std. Error**	**z value**	**Pr (>|z|)**
Intercept	-5.955	1.42	-4.182	2.89 e-05
Mileage	3.66 e-05	1.53 e-05	3.287	0.0170
Mileage2	-8.566 e-11	4.041 e-11	-2.120	0.0340

A prediction table for bus group T8H230 follows that is similar to the previous prediction table. For probit and logit regressions, predictions of the data are used to show the effectiveness of the model. Similar to using r-squared and adjusted r-squared values in the linear model to see how well the model fits the data. Table 5 T8H203 Prediction Chart shows the replacements the probit model predicted.

Table 5 T8H203 Prediction Chart

		Actual		
		0	**1**	**Totals**
Predict	**0**	3,333	27	3,360
	1	0	0	0
	Totals	3,333	27	3,360

This model has a correct prediction rate of 99.12 percent. Again, the model correctly predicts that most of the time no replacement will be made, but still fails to predict that some replacements will be made.

Interpretation

The original regression that included the full data set showed that a reduced-form model, whether it is probit or logit, is highly significant at any level but because of the low R-squared value, more data should be included. Other data could include the cost structure of replacements. For example, replacements are not only a function of odometer readings but how much a replacement will cost or how the drivers take care of the bus through care while driving. Or, maybe the month of the year affects replacements. Unfortunately, this information is not immediately available and could be included in future research.

Next, the data was narrowed to just one bus group that had a high proportion of total replacements. A similar process was used in Rust's paper. He put the buses into four categories and left out two or three bus types depending on the test. In every test at least one type was left out of the testing. For this test just one bus type was used. Bus Type T8H230 had the most replacements at 24. Again, the model was highly significant and predicted replacements almost *100 percent* of the time but failed to predict *any* replacements.

The results are interesting and can be interpreted different ways. One way is to push the effectiveness of the reduced-form model by stating that with a miniscule p-value and high percentage of correct predictions, we failed to reject the null hypothesis. Statistics are a powerful tool and information presented in the right light carries much persuasion with readers. The interesting result, though, is that the reduced-form models never predicted a true replacement. An unbiased interpretation should not leave that information untouched; the model did indeed fail to match the data. However, there are only 127 replacements in the entire data set. That is, out of 15,964 data points only 127 have the qualifying characteristic we are interested in: engine replacement. Proportionally, that results in less than one percent of the data points that had a bus engine replacement (true value is 0.77 percent). This leads to the question; can a structural model predict replacement? If not, then a richer data set is needed. Rust's paper does not include predictions of the structural model and leaves an opening for future research and a few unanswered questions for this paper.

Further analysis of the data could be conducted using Monte Carol simulation to test data derived from the structural model. A Monte Carlo simulation takes the moments (mean, variance, skewness, and kertosis) of the original data set, use that information as the natural data generating process (DGP) and generates new data for testing. Probit and Logit models could again be used to test the data. After prediction tables are made, a comparison of new and old prediction rates could give more answers.

Conclusion

Given estimations and significant values, it can be concluded that, indeed, a reduce-form model can perform well given the data set provided from *Optimal Replacement of GMC Bus Engines,* but another step is needed: evaluating the performance of the structural model. In deriving the results it was shown that there are failures of the reduced-form model. Maybe it is that the data set is not rich enough for an optimal stopping problem, or maybe it is that more independent variables are needed in the regression. Either way the reader chooses interpret these results is not wrong. For the purpose is to show that reduced-form models are a just another tool and should be used supplementary to structural models. The ease of computation alone should motivate an econometrician to expand her tool set to include reduced-form.

Further Research

An extension of this paper could include an estimation of John Rust's structural model as described before. Prediction tables of the artificial data derived from the structural model are the next step in this process. Using those estimations as the data generating process, a Monte Carlo study could be conducted to "compete" the two types of models to see which outperforms. The assumptions are that, again, the probit and logit will do just as well as the structural model.

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