Utah State University

DigitalCommons@USU

Calibration of a Parametric-Stochastic Model

Zhida Song

L. Douglas James

Recommended Citation

Utah State University
MERRILL-CAZIER LIBRARY

CALIBRATION OF A PARAMETRIC-STOCHASTIC MODEL

Zhida Song and L. Douglas James
Utah Water Research Laboratory
Utah State University
Logan, UT 84322-8200

ABSTRACT

Conceptually, stochastic parametric modeling offers a powerful tool to select a scale for expressing catchment variability for hydrologic simulation and relating model parameters to catchment characteristics. Practically, success depends on having an efficient method for model calibration. The calibration of a stochastic model is much more difficult than a deterministic one because simulation shifts from using fixed parameters to simulate of flows as deterministic values to taking multiple combinations of parameter values randomly from distributions to simulate flows as stochastic variables. The proposed method calibrates the first two moments of each parameter distribution to represent the average and the variability of catchment characteristics by using two objective functions. One minimizes relative errors between recorded and simulated flows, and the other bounds the range of simulated flows to cover the recorded flows. The method was successfully calibrated for four watersheds, and the results promise new understanding that will contribute to more reliable models.

KEY WORDS: Parametric Modeling, Monte Carlo Simulation, Calibration Techniques

INTRODUCTION

Hydrologists commonly use average values of catchment characteristics to simulate runoff; however, catchment variability is important, because it strongly influences the volume and pattern of runoff, (Wood et al, 1988). Some widely used parametric deterministic models (PDMs) represent variability with assumed distributions; but we can do better. Geographical information systems offer one approach; an alternative is to calibrate a parametric stochastic model (PSM) to quantify catchment variability from measured flow data (Song, 1990). The calibration procedure is much less costly. Any deterministic model can be made stochastic by replacing selected fixed parameters with probability distributions. Sets of values of parameter are selected by Monte Carlo methods from the distributions and used in flow simulation, this process is repeated until the probability distributions of the flows are defined with precision preselected by the user. Flow distributions can be taken for days, months, and the year as a whole.

A PSM is calibrated by estimating moments to define probability distributions for the chosen parameters. Monte Carlo simulation takes many runs. Calibration methodology has not been explored in the literature, and this short paper introduces a method and describes its results.

The process selects two objective functions, develops an efficient calibration strategy for parameter moment searching, and illustrates its application. It provides a working method that others can probably improve.

OBJECTIVE FUNCTIONS

Models are calibrated with objective functions that index how well simulated match recorded flows. A good objective function depends on the purpose of calibration. The purpose of a PSM is to use both the average and the distribution of catchment characteristics to simulate both total flows and their variability. This dual purpose can be pursued with two objective functions.

Objective Function 1

The first objective function expresses goodness of fit as commonly employed in deterministic models with a mathematical expression of simulation error (Sorooshian, 1988). The function uses a relative index to reduce the impact of high flow periods on calibration and is squared to eliminate negative values and permit minimization. It has two parts: TYRME, an index expressing the match of annual total flow, and TRMS, an index expressing the match of the distribution of monthly flows over the year. When the two are weighted equally,

$$Min.\ TYRME + TRMS$$
$$TYRME = \frac{1}{Y} \sum_{i=1}^{Y} \left(\frac{S_i - R_i}{R_i} \right)^2$$
$$TRMS = \frac{1}{Y} \sum_{i=1}^{Y} \left(\frac{1}{M} \sum_{j=1}^{M} \left(\frac{S_{ij} - R_{ij}}{R_{ij}} \right)^2 \right)$$

(1)

Y equals the number of years covered by the simulation. M is 12 for the number of months in a year. TYRME is the relative mean square error in simulating the annual flow. S_i and R_i are, respectively, the simulated and recorded annual total flows in year i. TRMS is the average mean square error in simulating the distribution of monthly flows. S_{ij} and R_{ij} are, respectively, simulated and recorded monthly flows in month j of year i. As the variance in a stochastic model approaches zero; it becomes deterministic and could be calibrated entirely from Eq. 1 to set mean, μ_x, values for the parameters. We need another objective function to guide calibration of parameter variances.

Objective Function 2

The calibration of a stochastic model must also consider how well the band of simulated flows encompasses recorded flows that resulted from a variety of juxtapositions of storms on catchment characteristics. Consequently, this calibration introduced a second objective function, OF2, to measure how well the simulated flows envelop the expected number of recorded flows.

The simulated flows for a given day form a band (Figure 1) whose width can be adjusted by changing the standard deviation, σ_x, of a sensitive parameter. A larger σ_x widens the band of simulated flows and encompasses more recorded flows. If too many flows are encompassed, σ_x should be

reduced. Thus, the match between the number of recorded flows that one would expect to encompass with a band of simulated flows and the number actually encompassed offers a second objective function.

The log normal is the most widely used distribution for catchment characteristics. For this distribution, the μ_x, the σ_x, and the ratios of μ_x to σ_x interact in determining the density curve shapes and the band widths (Figure 2). Thus, the standard deviations cannot be calibrated independently but must be adjusted jointly with the means. PSM using other distributions must be calibrated considering interdependencies among their parameters.
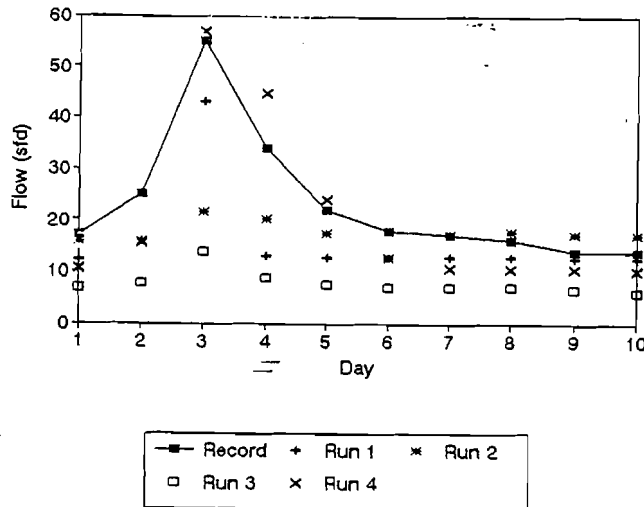

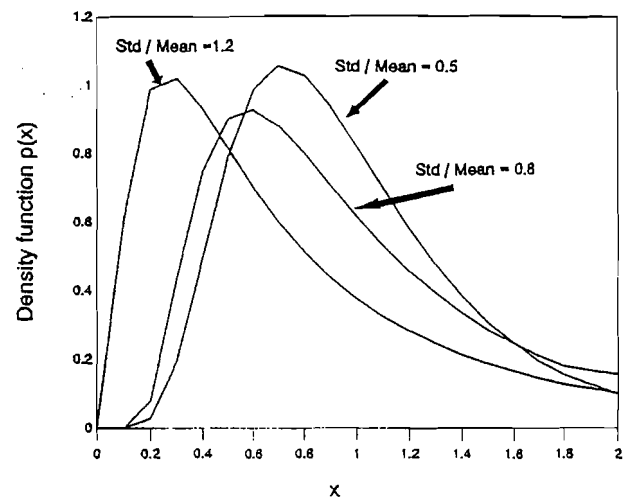FIGURE 1. Recorded flow and Band of Simulated Flows

FIGURE 2. Lognormal Distribution with Different σ/μ Ratio

More recorded flows would be encompassed with more simulations. Extreme flows simulated with rare combinations of random numbers should be discarded. A rule is needed on which ones to discard. Assuming the central limit theorem and thus a normal distribution, a one σ_x variation should encompass the 68.3 percent of the recorded daily flows that fall within $1\sigma_x$ of the μ_x of simulated daily flows. For a year of 365 days, 249 daily flows would be encompassed.

More Monte Carlo simulations would do a better job of defining the distributions but would be more time consuming. A reasonable balance is for the modeler to simulate six flows, discard the highest and lowest, and use the second highest and the second lowest to bound the band; 4/6, or 66.7 percent, of the simulated flows would fall within the band on the average. This is termed the "two-thirds rule." This number is close to 68.3 percent. Even though the six simulations may not represent the distribution well on some days, over the course of a year, the bands will be too wide on some days and too narrow on others; and the isses will balance. OF2 is defined to minimize the departure from the expected one third of the days per year or 122 being outside the band. The mathematical expression is:

$$Min. \quad \sum_{i=1}^{Y} |Day_{out,i} - 122| \tag{2}$$

CALIBRATION STRATEGY

Definitions

In our calibration, zone is a portion of a catchment, and parameter values are varied from zone to zone to represent spatial variability. A run is a simulation for one or more years. Values for each parameter in the set for each zone are taken from the respective distributions and used throughout the run. A run group is a block of six runs used to simulate flows for the same time period and to establish a band. The parameter values taken from the same distribution vary from run to run. A round is a set of run groups used to calibrate the parameter distributions. When a catchment is subdivided into fewer than four zones, a round must have more than one run group to reach the minimum of 24 values to define a trial parameter distribution. The boundaries of the band must be generated numerically since they cannot be derived analytically. Without a mathematical relationship between Day_{out} and the moments of parameter distributions, the calibration process must rely on trends quantified empirically in sensitivity studies.

Adjustment Principles

The parameter values for a round are selected by Monte Carlo simulation from the parameter distributions. For a catchment subdivided into four zones, four values are generated for each parameter for each of six runs for a total of 24. Each run simulates a flow sequence and a value for OF1. Each run group simulates a band and a value for OF2. To minimize OF1, the modeler can find parameter moments giving smaller values by examining the 12 parameter values in the three runs with the smallest values of OF1 to estimate the mean and standard deviation for the parameter distributions to use in the next round. Since there are only 12 parameter values, the estimated moments vary around the moments of the underlying population. Different samples of 12 give different values for both the μ_x and the σ_x. This property facilitates calibration by spreading the trial values over a range so that one can find the best match to the recorded flows and minimize OF1.

Search Procedure

This rule for adjusting the parameter moments was applied in five steps:

1. Estimate initial values for the mean and variance for each parameter. The geometric mean and standard deviation of values calibrated for different years with the deterministic model can be used because different years have different spatial juxtapositions of storms and catchment characteristics.

2. Decide whether to take the catchment as a whole or subdivide it into zones. Generate values for each parameter from these distributions and use them to simulate stream flows. Record the generated parameter values and associated values of the two objective functions. Records of when and how far the recorded flows fall outside the band can be used to guide parameter moment adjustments.

3. Check OF2 for each run group. If the value is acceptable, the calibration can concentrate on reducing OF1 when selecting new values of μ_x and σ_x for each parameter distribution for the next round.

4. If OF2 is too large, Widen the simulated flow band during the times of the year when the results are worst by adjusting appropriate parameter moments identified with the sensitivity studies. With six points, one is working in the middle range of the log normal distribution where the band width is not widened by increasing the second moment (Figure 2). The μ_x and σ_x must be increased simultaneously to make the band wider. Then, repeat step 2 using the modified parameter moments.

5. If OF1 cannot be reduced and OF2 is acceptable, the calibration is completed for the selected number of zones. Record the values of μ_x and σ_x for the parameter moments and also the values of OF1 and OF2 to use in determining the optimal number of zones to use in simulation.

CALIBRATION EXAMPLE

The above procedure has been applied four times: to three real catchments (Song, 1990) and to a simulated flow series (James and Song, 1991). Satisfactory results were obtained each time as illustrated in Table 1 where the catchment was taken as one zone. Seven rounds were simulated, and the optimal result was obtained in round 5. The parameter moments for four more sensitive parameters are tabulated. The objective functions for the seven rounds are plotted in Figure 3, and the stepwise adjustment in BIR is illustrated in Figure 4.

DISCUSSION

Figure 3 shows a steady improvement that is not monotonic because of noise in the random component. In multiple-objective optimization, an unambiguous optimal solution is rarely found. For example, Round 5 is not mathematically superior to Round 6 (Figure 4), but we judged the improvement in OF2 from Round 5 to Round 6 to be less important than the loss in OF1 and selected Round 5* as optimal.

TABLE 1.
Parameter and Objective-Function Values for Seven Runs

RUN	LZC μ	LZC σ	BIR MEAN	BIR STD	SUZC MEAN	SUZC STD	DPR MEAN	DPR STD	OF1	OF2
R1	3.50	2.70	7.00	6.60	9.66	6.50	2.45	2.29	.863	62
R2	2.28	2.07	8.22	7.77	8.12	5.00	2.46	1.73	.896	41
R3	4.28	3.50	9.43	7.20	4.50	4.13	1.86	1.47	.640	22
R4	3.78	2.00	12.02	7.21	3.65	1.97	1.28	0.72	.554	54
R5*	4.03	2.00	12.02	7.21	3.75	2.19	1.10	0.66	.380	28
R6	3.33	1.35	15.67	9.70	3.50	2.36	0.92	0.60	.491	23
R7	3.63	1.28	15.01	7.23	4.04	2.89	0.81	0.48	.476	28

LZC: Lower zone soil moisture capacity
BIR: Basic infiltration rate
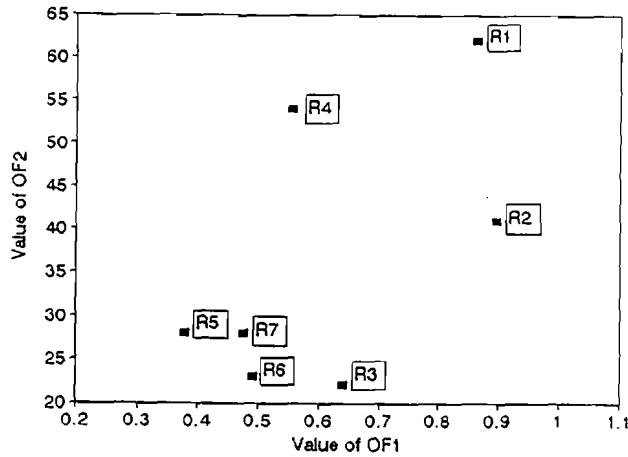SUZC: Seasonal upper zone soil moisture capacity
DPR: Deep percolation rate

FIGURE 3. Values of Objective Functions
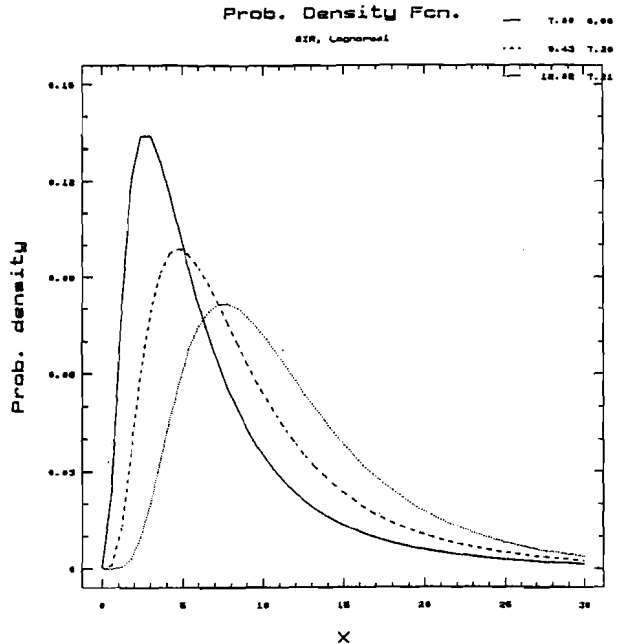
Prob. Density Fcn.

FIGURE 4. Stepwise Adjustment of a Parameter

It is more difficult to adjust the parameter moments to impact OF2 than OF1 and more difficult to adjust when more zones are used because smaller zones contribute less to the total simulated flow. If the μ_x and σ_x of the parameter distributions are not varied when the number of zones increases, the band of simulated flows narrows and shrinks the mathematically feasible region.

CONCLUSIONS

Based on examples tried in this study, parametric models can be expanded to stochastic models and successfully used to assess the spatial variability in the model parameters. The calibration of the resulting parametric-stochastic model (PSM) can be treated as a multiobjective optimization with the objective functions based on the mean and variance (standard deviation) of the parameter distributions. The guided search technique reported in this paper works to calibrate a PSM and can be refined and improved over time.

REFERENCES

1.   James, L.D., and Z. Song, 1991. Parametric Stochastic Modeling for Assessing Hydrologic Scale, Proceedings of International Conference on Computer Application in Water Resources, Tamsui, Taiwan.

2.   Song, Z., 1991. Use of Parametric Stochastic Model to Quantify the Scale Effect in Catchment Characterization, PhD. Dissertation, Department of Civil and Environmental Engineering, Utah State University, Logan, Utah.

3.   Sorooshian, S., 1988. Parameter Estimation, Model Identification, and Model Validation: Case of Conceptual-Type Models, NATO-ASI Recent Advances in the Modeling of Hydrologic Systems, Sintra, Portugal.

4.   Wood, E.F., M. Sivapalan, K. Beven, and L. Band, 1988. Effects of Spatial Variability and Scale with Implications to Hydrologic Modeling, Journal of Hydrology, Vol. 102.