Utah State University

DigitalCommons@USU

All Graduate Theses and Dissertations

Graduate Studies

5-2008

A Laboratory Study of Streambed Stability in Bottomless Culverts

Brian Mark Crookston Utah State University

Follow this and additional works at: https://digitalcommons.usu.edu/etd



Part of the Civil Engineering Commons

Recommended Citation

Crookston, Brian Mark, "A Laboratory Study of Streambed Stability in Bottomless Culverts" (2008). All Graduate Theses and Dissertations. 93.

https://digitalcommons.usu.edu/etd/93

This Thesis is brought to you for free and open access by the Graduate Studies at DigitalCommons@USU. It has been accepted for inclusion in All Graduate Theses and Dissertations by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.



A LABORATORY STUDY OF STREAMBED STABILITY IN BOTTOMLESS CULVERTS

by

Brian Mark Crookston

A thesis submitted in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Civil and Environmental Engineering

Approved:	
Blake P. Tullis Major Professor	William J. Rahmeyer Committee Member
Jacoba A. Oalianda	Duran D. Durah ara
Joseph A. Caliendo	Byron R. Burnham
Committee Member	Dean of Graduate Studies

UTAH STATE UNIVERSITY Logan, Utah

2008

ii

ABSTRACT

A Laboratory Study of Streambed Stability in Bottomless Culverts

by

Brian Mark Crookston, Master of Science

Utah State University, 2008

Major Professor: Dr. Blake P. Tullis

Department: Civil and Environmental Engineering

Traditional culvert designs, in many cases, have become habitat barriers

to aquatic animal species. In response, environmentally sensitive culvert designs

have been developed to function as ecological bridges. Bottomless and buried

invert culverts are examples of such designs and are commonly used for fish

passage. Additional design guidance specific to streambed stability in buried-

invert or bottomless culverts under high flow events is needed. This study

investigated incipient motion conditions for four substrate materials in a 2-ft

(0.61-m) diameter circular bottomless arch culvert and in a 1-ft (0.30-m) wide

rectangular flume in a laboratory setting. General scour of the streambed within

the bottomless arch culvert was also investigated under partially pressurized and

non-pressurized flow conditions.

This thesis discusses the experimental methods used to determine

incipient motion conditions and analyses of incipient motion prediction methods.

This thesis also presents the experimental results obtained from both test

facilities with the results of other published incipient motion studies on gravel streambeds. Finally, the prediction efficiency of eight stone sizing methods (open channel and culvert application) applied to the experimental results was analyzed, which may be useful for determining stable stone diameters to be used as riprap in simulated streambeds through bottomless culverts.

(237 pages)

ACKNOWLEDGMENTS

Funding for this study was provided by the Alaskan Department of Transportation and Public Facilities and the State of Utah (Utah Water Research Laboratory).

I express sincere appreciation to my major professor, Dr. Blake Tullis, for his assistance, counsel, patience, and encouragement, which have been both generous and invaluable. I would also like to thank Dr. William Rahmeyer and Dr. Joseph Caliendo for their time and advice.

I wish to thank the faculty and staff of the Civil and Environmental Engineering Department at Utah State University for their outstanding efforts to educate, assist, and inspire students. In addition, I would like to thank Alan Taylor and the UWRL shop employees for their assistance.

Finally, I am most grateful for the support and friendship of my gracious wife, Kelsi; your efforts have done so much.

Brian Mark Crookston

CONTENTS

	Page
ABSTRACT	ii
ACKNOWLEDGMENTS	iv
LIST OF TABLES	vii
LIST OF FIGURES	ix
NOMENCLATURE	xvi
INTRODUCTION	1
Research Objectives	2
LITERATURE REVIEW	4
Fish Passage in Culverts Initiation of Motion of Substrate Particle Incipient Motion of Bed Previous Bottomless Culvert Scour Studies Related Sediment Transport Studies	5 14 17
EXPERIMENTAL METHOD	28
Facilities Substrate Materials Bottomless Culvert Testing Procedure Bottomless Culvert Data Collection Rectangular Flume Testing Procedure Rectangular Flume Data Collection Data Analyses Programs	33 38 42 43
BOTTOMLESS CULVERT INCIPIENT MOTION RESULTS	51
General Velocity ObservationsIncipient Motion VelocitiesShields Relation for Beginning of Motion in a Bottomless Culvert	52
DECTANCIA AD ELLIME INCIDIENT MOTION DESLILTS	68

	vi
General Velocity Observations	68
Incipient Motion Velocities	
Shields Relation for Beginning of Motion in a Rectangular Flume	72
SCOUR IN A BOTTOMLESS ARCH CULVERT	78
Bottomless Culvert General Scour Observations	78
Extent of Scour	
STONE SIZING ANALYSES	99
CONCLUSIONS	106
REFERENCES	111
APPENDICES	117
Appendix A: Bottomless Arch Culvert Velocity Data	118
Appendix B: Rectangular Flume Velocity Data	
Appendix C: Shields Relation for Bottomless Arch Culvert	127
Appendix D: Shields Relation for Rectangular Flume	
Appendix E: Streambed Response in Bottomless Arch Culvert	
Appendix F: Riprap Analyses Results	
Appendix G: Substrate Properties	1/6
Appendix H: Visual Basic Code Used for Calculations for	407
Bottomless Arch Culvert in Microsoft Excel	187
Appendix I: Visual Basic Code Used for Calculations for Rectangular Flume in Microsoft Excel	215
Appendix J: Channel Cross-section formulas	
Appendix 6. Chamile 01000 occion formalas	44 1

LIST OF TABLES

Tab	le F	age
1	Substrate sieve analyses results	33
2	Substrate density analyses results	34
3	Bottomless culvert test matrix	35
4	Bottomless culvert H _w /D test matrix	39
5	Curve fit equations for physical properties of water [†]	47
A1	Pea gravel substrate velocity results	119
A2	0.75-inch angular gravel substrate velocity results	119
А3	2-inch cobble substrate velocity results	120
A4	2-inch angular rock substrate velocity results	121
В1	Pea gravel substrate velocity results	123
B2	0.75-inch angular gravel substrate velocity results	124
ВЗ	2-inch cobble substrate velocity results	125
В4	2-inch angular rock substrate velocity results	126
E1	Pea gravel substrate measured depths of scour in inches	148
E2	0.75-inch angular gravel substrate measured depths of scour in inches.	153
E3	2-inch cobble substrate measured depths of scour in inches	158
E4	2-inch angular rock substrate measured depths of scour in inches	164
F1	0.75-inch angular gravel substrate in bottomless arch culvert riprap results	171
F2	2-inch cobble substrate in bottomless arch culvert riprap results	171
F3	2-inch angular rock substrate in bottomless arch culvert riprap results	171

F4	Pea gravel substrate in rectangular flume riprap results	172
F5	0.75-inch angular gravel substrate in rectangular flume riprap results	173
F6	2-inch cobble substrate in rectangular flume riprap results	174
G1	Specific weight analyses results	185
G2	Substrate properties in metric units	185

LIST OF FIGURES

Figu	ire	Page
1	Example of a buried-invert culvert	5
2	Shields relation for beginning of motion	11
3	Critical velocity as a function of stone size (Richardson, Simons, and Julien, 1990)	
4	Overview of supply piping to bottomless culvert test facility	30
5	Overview of bottomless culvert test facility	31
6	Overview of rectangular flume test facility	32
7	Sieve size vs. % finer for tested substrate materials	35
8	0% Contraction with headwall	37
9	33% Contraction, with headwall (A), projecting inlet (B)	37
10	75% Contraction, with headwall (A), projecting inlet (B)	37
11	Headwater depth dimensionless parameter, H _w /D	38
12	0.5 H _w /D with headwall (A), projecting inlet (B)	40
13	1.0 H _w /D with headwall (A), projecting inlet (B)	40
14	1.5 H _w /D with headwall (A), projecting inlet (B)	40
15	Culvert cross-section looking downstream	42
16	Example of scour geometry measurements at entrance	43
17	Approximation of physical properties of water	48
18	Shields diagram (Chien, Wan, and McKnown 1999)	49
19	Reproduction of Chien, Wan, and McNown (1999) shields diagram, based on curve fit	50

20	Velocity vs. H _w /D ratio for pea gravel substrate	53
21	Velocity vs. H _w /D ratio for 0.75-inch angular gravel	54
22	Velocity vs. H _w /D ratio for 2-inch cobbles	56
23	Velocity vs. H _w /D ratio for 2-inch angular rock	57
24	Critical average bed velocity as a function of stone size‡	59
25	Critical average culvert velocity as a function of stone size‡	59
26	Four tested substrates in bottomless culvert plotted on Shields relation for incipient motion.	61
27	Method correlation to shields relation for incipient motion for 0.75-in angular gravel at the culvert entrance	63
28	Method correlation to shields relation for incipient motion for 0.75-in angular gravel at the culvert exit	63
29	Method correlation to shields relation for incipient motion for 2-in cobble substrate at the culvert entrance	64
30	Method correlation to shields relation for incipient motion for 2-in cobble substrate at the culvert exit	64
31	Method correlation to shields relation for incipient motion for 2-in angular rock substrate at the culvert entrance	65
32	Method correlation to shields relation for incipient motion for 2-in angular rock substrate at the culvert exit	65
33	Method correlation to shields relation for incipient motion for pea gravel substrate at the culvert exit	71
34	Method correlation to shields relation for incipient motion for 0.75-in angular gravel substrate at the culvert exit	71
35	Method correlation to Shields relation for incipient motion for 2-in Cobble Substrate at the culvert exit	72
36	Method correlation to Shields relation for incipient motion for 2-in Angular Rock Substrate at the culvert exit	73

37	Four tested substrates in rectangular flume plotted on Shields relation for incipient motion	. 81
38	Method correlation to shields relation for incipient motion for pea gravel substrate in the rectangular flume	. 76
39	Method correlation to shields relation for incipient motion for 2-in cobble substrate in the rectangular flume	. 76
40	Method correlation to shields relation for incipient motion for 2-in angular rock substrate in the rectangular flume	. 77
41	Examples of scour at entrance of culvert for pea gravel (A), 0.75-inch angular gravel (B), 2-inch cobbles (C), and 2-inch angular rock (D)	. 80
42	Example of scoured channel at culvert exit with bank deposition	. 81
43	Example of oval scour hole at culvert exit	. 81
44	Example of antidunes in pea gravel substrate	. 83
45	Average scour vs. H _w /D ratio for pea gravel substrate	. 84
46	Average inlet/outlet scour vs. corresponding average culvert velocities for pea gravel substrate	
47	Max depth of scour at inlet and outlet for pea gravel	. 87
48	Scour of 0.75-inch angular gravel looking upstream from exit, 1 H_w/D	. 87
49	Average scour vs. H _w /D ratio for 0.75-inch angular gravel substrate	. 87
50	Average inlet/outlet scour vs. corresponding average culvert velocities for 0.75-inch angular gravel substrate	. 88
51	Max depth of scour at inlet and outlet for 0.75-inch angular gravel substrate	. 89
52	Average scour vs. H _w /D ratio for 2-inch cobble substrate	. 90
53	Scour of 2-inch cobbles looking upstream from exit	. 91
54	Average inlet/outlet scour vs. corresponding average culvert velocities for 2-inch cobble substrate	. 91

55	Comparison of extent of average cross-sectional scour in 2-inch angular rock and 2-inch cobbles	94
56	Max depth of scour at inlet and outlet for 2-inch cobble substrate	93
57	Average scour vs. H _w /D ratio for 2-inch angular rock substrate	95
58	Scour of 2-inch angular rock looking upstream from exit	96
59	Average inlet/outlet scour vs. corresponding average culvert velocities for 2-inch angular rock substrate	96
60	Max depth of scour at inlet and outlet for 2-inch angular substrate	97
61	Comparison of riprap safety factors for 0.75-inch angular gravel in a bottomless arch culvert	100
62	Comparison of riprap safety factors for 2-inch cobbles in a bottomless arch culvert	101
63	Comparison of riprap safety factors for 2-inch angular rock in a bottomless arch culvert	101
64	Comparison of riprap safety factors for pea gravel in a rectangular flume	102
65	Comparison of riprap safety factors for 0.75-inch angular gravel in a rectangular flume	102
66	Comparison of riprap safety factors for 2-inch angular cobbles in a rectangular flume	103
C1	Pea gravel substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion	128
C2	Pea gravel substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion	129
C3	0.75-inch angular gravel substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion	130
C4	0.75-inch angular gravel substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion	131

C5	2-inch cobble substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion	132
C6	2-inch cobble substrate at station 4 in bottomless arch culvert plotted on shields relation for incipient motion	133
C7	2-inch cobble substrate at station 8 in bottomless arch culvert plotted on shields relation for incipient motion	134
C8	2-inch cobble substrate at station 12 in bottomless arch culvert plotted on shields relation for incipient motion	135
C9	2-inch cobble substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion	136
C10	2-inch angular rock substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion	137
C11	2-inch angular rock substrate at station 4 in bottomless arch culvert plotted on shields relation for incipient motion	138
C12	2-inch angular rock substrate at station 8 in bottomless arch culvert plotted on shields relation for incipient motion	139
C13	2-inch angular rock substrate at station 12 in bottomless arch culvert plotted on shields relation for incipient motion	140
C14	2-inch angular rock substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion	141
D1	Pea gravel substrate in rectangular flume plotted on shields relation for incipient motion	143
D2	0.75-inch angular gravel substrate in rectangular flume plotted on shields relation for incipient motion	144
D3	2-inch cobble substrate in rectangular flume plotted on shields relation for incipient motion	145
D4	2-inch angular rock substrate in rectangular flume plotted on shields relation for incipient motion	146
E1	Pea gravel substrate average streambed profile (a)	149
F2	Pea gravel substrate average streambed profile (b)	149

E3	Pea gravel substrate average streambed profile (c)	150
E4	Pea gravel substrate average streambed profile (d)	150
E5	Pea gravel substrate average streambed profile (e)	151
E6	Pea gravel substrate average streambed profile (f)	151
E7	Pea gravel substrate average streambed profile (g)	152
E8	0.75-inch angular gravel substrate average streambed profile (a)	154
E9	0.75-inch angular gravel substrate average streambed profile (b)	154
E10	0.75-inch angular gravel substrate average streambed profile (c)	155
E11	0.75-inch angular gravel substrate average streambed profile (d)	155
E12	0.75-inch angular gravel substrate average streambed profile (e)	156
E13	0.75-inch angular gravel substrate average streambed profile (f)	156
E14	0.75-inch angular gravel substrate average streambed profile (g)	157
E15	2-inch cobble substrate average streambed profile (a)	159
E16	2-inch cobble substrate average streambed profile (b)	159
E17	2-inch cobble substrate average streambed profile (c)	160
E18	2-inch cobble substrate average streambed profile (d)	160
E19	2-inch cobble substrate average streambed profile (e)	161
E20	2-inch cobble substrate average streambed profile (f)	161
E21	2-inch cobble substrate average streambed profile (g)	162
E22	2-inch cobble substrate average streambed profile (h)	162
E23	2-inch cobble substrate average streambed profile (i)	163
E24	2-inch cobble substrate average streambed profile (j)	163

F2 <i>E</i>	2 inch angular rock autotrata average atroomhad profile (a)	^V
E25	2-inch angular rock substrate average streambed profile (a)	100
E26	2-inch angular rock substrate average streambed profile (b)	165
E27	2-inch angular rock substrate average streambed profile (c)	166
E28	2-inch angular rock substrate average streambed profile (d)	166
E29	2-inch angular rock substrate average streambed profile (e)	167
E30	2-inch angular rock substrate average streambed profile (f)	167
E31	2-inch angular rock substrate average streambed profile (g)	168
E32	2-inch angular rock substrate average streambed profile (h)	168
E33	2-inch angular rock substrate average streambed profile (i)	169
E34	2-inch angular rock substrate average streambed profile (j)	169
G1	Sieve distribution of pea gravel substrate	176
G2	Sieve distribution of 0.75-inch angular gravel substrate	177
G3	Sieve distribution of 2-inch cobble substrate	178
G4	Sieve distribution of 2-inch angular rock substrate	179
G5	Sieve distribution of the four tested substrate materials	180
G6	Sieve analysis of pea gravel substrate	181
G7	Sieve analysis of 0.75-inch angular gravel substrate	182
G8	Sieve analysis of 2-inch cobble substrate	183
G9	Sieve analysis of 2-inch angular rock substrate	184

NOMENCLATURE

A_r	Ranneyer curve in coemicient
а	Shields parameter coefficient originally proposed by Andrews
B_r	Rahmeyer curve fit coefficient
b	Shields parameter coefficient originally proposed by Andrews
C	Safety factor with published values specific to channel geometry and severity of attack by current
C_r	Rahmeyer curve fit coefficient
C_s	Stability coefficient for incipient failure
C_T	Thickness coefficient
C_u	Uniformity coefficient
C_V	Vertical velocity distribution coefficient
C_z	Coefficient of curvature
D	Distance between the pre-scour culvert streambed invert and the culvert crown
D_p	Pipe diameter
D_r	Rahmeyer curve fit coefficient
d_s	A representative particle size diameter where <i>s</i> represents the percent of material in a sample that is smaller than the representative particle size (i.e., s=30, 40, 50)
δ '	Thickness of the viscous sublayer
E_r	Rahmeyer curve fit coefficient
F_d	Drag force
F_n	External forces acting upon the submerged particle

F_r	Rahmeyer curve fit coefficient
F_v	Viscous shear force
f	Overall Darcy-Weisbach roughness coefficient
f_b	Darcy-Weisbach roughness coefficient for bed region
f_w	Darcy-Weisbach roughness coefficient for wall region
φ	Angle of side slope to horizontal, in degrees
g	Acceleration constant of gravity
γ	specific weight of water
γ_s	average specific weight of substrate material
H_w	Total upstream headwater measures relative to the inlet pre-scour culvert invert
H_w/D	Headwater to culvert height ratio
K	Side slope correction factor
k_s	Height of roughness element
L	Characteristic length
μ	Absolute or dynamic viscosity
v	Kinematic viscosity
P_{w_bed}	Wetted perimeter of bed region
P_{w_wall}	Wetted perimeter of wall region
π	3.141592
R_e	Reynolds number
R_e^*	Grain Reynolds number
R_h	Hydraulic radius of the channel

R_{h_bed}	Hydraulic radius of bed region
ho	Mass density of fluid
$ ho_c$	Coefficient for material placement
S	The slope of the channel
SF	Safety factor, ratio of $d_{s_predicted}$ to d_{s_actual}
SF'	Safety factor (USACE)
SG	Specific gravity of riprap material
S_f	The slope of the energy gradeline
Ψ	Side slope angle in degrees
$ au_{\scriptscriptstyle C}$	Critical viscous shear stress
$ au_o$	Average viscous shear stress
θ	Shields parameter
V	Average cross-sectional culvert velocity at location of interest
V^*	Shear velocity
V_{a_ent}	Average velocity at the culvert entrance
$V_{\it eff}$	Effective local bed velocity
χ	Einstein multiplication factor
x	$Log(R_e^*)$
Y	Local flow depth
Y_{ent}	Depth of flow at the culvert entrance
y	Distance measurement from a rigid boundary

INTRODUCTION

Background and Motivation

In general, traditional culvert designs have focused on passing the maximum design discharge while maintaining sufficient freeboard. Increased concern for and interest in better facilitating fish migration and sediment transport through culverts have fostered alternative culvert designs. These alternative designs feature culvert flow velocities favorable to local fish species and continuity between the existing natural channel and the culvert. Culverts designed for fish passage are commonly sized such that the width of the culvert spans the natural channel under base flow conditions. This is done to prevent contraction of the flow entering the culvert, which typically causes local turbulence, scour, and higher culvert flow velocities. Such culvert designs commonly use "Buried-Invert" or "Bottomless" culverts, sometimes referred to as "D" shaped culverts. Buried-Invert culverts consist of culverts with traditional cross-sections that are installed with the culvert barrel invert below the natural grade of the channel and the barrel is partially filled with substrate material. Bottomless culverts are typically arch culverts with no bottom section, usually placed on strip footings. The goal of buried-invert and bottomless culverts is to simulate, to the extent possible, the naturally occurring streambed adjacent to the culvert in order to facilitate fish passage, prevent debris barriers, and facilitate naturally occurring sediment transport.

There is currently limited information available in published literature regarding the required substrate characteristics necessary to prevent or limit (within acceptable levels) the amount of scour that occurs in buried-invert and bottomless culverts. The artificial streambed must be resilient to scour and channel degradation to ensure structural stability and stream integrity. Many traditional culverts have become fish barriers due to excessive channel degradation at the exit, resulting in a perched culvert (the invert of the culvert exit is above the tailwater). A practical requirement for buried-invert or bottomless culverts should be that the bed material within the culvert barrel remains stable up to a specific return period storm event. This should include the stability of the streambed material in the culvert as well as in the vicinity of the culvert inlet and outlet. Existing riprap design methods and criteria do not specifically address streambed material stability in a buried-invert or bottomless culvert applications.

In this study, general scour behavior in a 2-ft diameter circular shaped bottomless arch culvert was evaluated. In addition, incipient motion of the tested substrate materials were also evaluated in a 1-ft wide rectangular flume. The parameters of interest to this study included: incipient motion velocities, scour depth variations, influence of culvert entrance configurations, upstream driving heads, and substrate material characteristics.

Research Objectives

In an effort to develop a better understanding of the scour potential in 2-ft

diameter bottomless arch culverts, this study with the following objectives was undertaken.

- Determine the incipient motion velocities (in the culvert facility and the laboratory flume) associated with four substrate materials, namely pea gravel, 0.75-inch angular gravel, 2-inch angular rock, and 2-inch rounded cobbles in an effort to identify geometric influences on incipient motion.
- 2. Determine the maximum depth of scour and the extent of scour within the culvert and near the culvert inlet and outlet for all substrate materials.
- Examine the effects of various culvert entrance conditions (i.e., channel to culvert contraction ratios) on culvert scour.
- 4. Compare experiment results with previous scour studies and rip-rap design guidelines for channels in an effort to extend these experimental results to larger bottomless culverts.

This study was conducted at the Utah Water Research Laboratory (UWRL) located on the Utah State University Campus in Logan, Utah. A bottomless arch culvert test facility was constructed at the UWRL specifically for this research project.

LITERATURE REVIEW

Fish Passage in Culverts

Design criteria for fish passage bottomless culverts, buried-invert culverts, arch or 'D' shaped culverts with simulated streambeds have been developed or adopted by various state and federal agencies, such as the state of California Department of Fish and Game (2002) and the Washington Department of Fish and Wildlife (Bates et al., 2003). Some of the general guidelines include matching the culvert slope as well as the entrance and exit elevations with the adjacent stream channel. Culvert skew should be minimized to prevent headcutting and accelerated flow paths and the culvert width or diameter should be equal to or greater than the base flow channel width. The bottomless culvert geometries provide wide, low velocity channels at base-flow conditions (see Figure 1). Substrate materials similar to those in the adjacent stream reaches should be incorporated into the simulated culvert streambed to maintain bed continuity. An ecological culvert design should consider species and life stages of fish present and associated flow depth and velocity requirements, etc. Finally, though the culvert is designed for base flow conditions, it should have an adequate flood discharge capacity; the streambed substrate material should be sized to minimize scour at high flow rates to maintain structural integrity and decrease maintenance.



Figure 1. Example of a buried-invert culvert.

Initiation of Motion of Substrate Particle

When the flow-induced bed shear stresses reaches or exceeds a critical value, particles within the flow will begin to move; this critical limit is defined as incipient motion. Though Brahms suggested in 1753 that the velocity for incipient motion is proportional to the grain weight raised to the sixth power, this condition is more appropriately analyzed from the concept of a balance of forces acting upon a particle. Incipient motion, however, has further complexities due to the fluctuating characteristics (turbulence) of the flow over a bed of sediment, composed of innumerable particles of various shapes, sizes, orientation, specific weight, protrusion, roughness, and location (Kirchner et al., 1990). Also, incipient motion is influenced by material gradation, friction angle, sorting, and armoring of

the bed (Forchheimer, 1914; Buffington, Dietrick, and Kirchner, 1992). Therefore, the forces acting upon a particle must be approximated and will vary spatially and temporally, resulting in the incipient motion phenomenon that is probabilistic (stochastic) in nature (Chien, Wan, and McKnown, 2003).

Water flowing over a bed of sediment exerts forces on the particles of sediment that may result in movement or entrainment (Vanoni, 2004). Such forces acting on a particle in a bed of relatively uniform, non-cohesive sediment are: the fluid forces acting on the surface of the particle (form drag, viscous shear, and buoyant), gravity, and external forces acting at the points of contact between neighboring particles.

The form drag force relationship, written in terms of shear velocity is presented as Equation (1).

$$F_d = d_s^2 \gamma \left(\frac{V^{*2}}{g}\right) \tag{1}$$

In Equation (1), F_d is defined as the drag force, d_s is the diameter of the particle, γ is the specific weight of water, and g is the acceleration of gravity. V_* is the shear velocity and is presented as Equation (2).

$$V^* = \sqrt{\frac{\tau_o}{\rho}} \tag{2}$$

In Equation (2), τ_{θ} is the average viscous shear stress and ρ is the mass density of the fluid. The equation generally accepted for average viscous shear stress for laminar flow is presented as Equation (3).

$$\tau_o = \mu \frac{dV}{dy} \tag{3}$$

In Equation (3), μ is the dynamic or absolute viscosity of the fluid and dV/dy is the rate of strain where V refers to the fluid velocity and y refers to a distance measured normal from a rigid boundary, such as a wall. Equation (3) is an idealized representation when grain protrusions destroy the viscous sublayer. Simplifying Equations (2) and (3) for rectangular channels and applied to turbulent flow conditions results in Equation (4), as defined by von Karman (1930).

$$V^* = \sqrt{gR_hS_f} \tag{4}$$

In Equation (4), R_h is the hydraulic radius of the channel, and S_f is the slope of the energy grade line, which is equal to the slope of the channel bed (S) for uniform flow conditions. Prandtl (1925) studied time-averaged mean velocities in two dimensions in turbulent flow and proposed to characterize turbulence with a "mixing length". As a result of his work and work by Von Karman, the velocity distribution, mean velocity, and resistance to flow are incorporated into a logarithmic relationship for shear velocity. This relationship, rearranged in terms of shear velocity and incorporating a multiplication factor developed by Einstein (1950) is presented as Equation (5).

$$V^* = \frac{V}{5.75 \log_{10} \left(\frac{12.27 \chi R_h}{d_s}\right)}$$
 (5)

In Equation (5), χ is the Einstein multiplication factor for logarithmic velocity equations and was presented graphically as a function of the ratio of the roughness height to the viscous sublayer thickness. However, a curve-fit by Rahmeyer (1989) closely approximates this curve for computer use and is presented as Equation (6).

$$\chi \approx A_r + B_r \log\left(\frac{k_s}{\delta'}\right) + C_r \left(\log\left(\frac{k_s}{\delta'}\right)\right)^2 + D_r \left(\log\left(\frac{k_s}{\delta'}\right)\right)^3 + E_r \left(\log\left(\frac{k_s}{\delta'}\right)\right)^4 + F_r \left(\log\left(\frac{k_s}{\delta'}\right)\right)^5$$
(6)

In Equation (6), A_r , B_r , C_r , D_r , E_r and F_r are all empirical coefficients defined as A_r =1.622653, B_r =0.099472, C_r =-2.83296, D_r =1.189237, E_r =2.566298, F_r =-1.64. Equation (6) is appropriate for $0 \le k_s/\delta$ ' ≤ 8 . For k_s/δ ' ≥ 8 , χ =1.00. k_s is the height of the roughness element and δ ' is the thickness of the viscous sublayer and can be calculated using Equation (7).

$$\delta' = \frac{11.6\nu}{V^*} \tag{7}$$

In Equation (7), v is the kinematic viscosity of the fluid. Equation (7) is for hydraulically smooth boundary turbulent flow where the velocity distribution, mean velocity, and flow resistance are dependent on fluid viscosity and not boundary roughness of the bed. Shear velocity now appears on both sides of Equation (5), requiring iterations until the solution converges.

The second fluid force acting upon a particle is the viscous shear force described by Equation (8).

$$F_{v} = d_{s}^{2} \tau_{0} \tag{8}$$

In Equation (8), F_{ν} is defined as the viscous shear force.

The external forces acting upon the submerged particle (gravity force - buoyant force) on a spherical particle are presented in Equation (9).

$$F_n = d_s^3 (\gamma_s - \gamma) \tag{9}$$

In Equation (9), F_n is defined as the external forces acting upon the submerged particle and γ_s is the specific weight of the particle.

Shields (1936) applied dimensional analysis to the forces acting upon a particle to determine incipient motion. The ratio of forces tending to move the particle to the forces resisting movement, developed by Shields, is presented as Equation (10).

$$\frac{F_d}{F_n} = \frac{d_s^2 \gamma \left(\frac{V_*^2}{g}\right)}{d_s^3 (\gamma_s - \gamma)} = \frac{\tau_0}{(\gamma_s - \gamma) d_s} = \theta \tag{10}$$

In Equation (10), θ is defined as the Shields Parameter. When motion is impending, the viscous shear stress (τ_{θ}) acting on the particle of sediment reaches a critical value (τ_{c}) which is of sufficient magnitude to rotate the particle about its point of support or lift it from its position. This critical value of shear stress is also termed the critical tractive force. A generally accepted expression of the average bottom shear stress is presented in Equation (11).

$$\tau_0 = \gamma R_h S \tag{11}$$

In equation (11), S is defined as the slope of the channel, though the energy gradeline would be preferred. Shields parameter is dependent on the shape of the grain and the Reynolds number of the flow. However, if the grains

are spherical in shape then Shields parameter only depends on what is referred to as the Grain Reynolds Number. The formula Shields used to represent the forces acting on a particle is presented in Equation (12).

$$\frac{\tau_0}{(\gamma_s - \gamma)d_s} = f\left(\frac{V * d_s}{\nu}\right) = f(R_e *)$$
 (12)

In Equation (12), $V*d_s/v$ is the Grain Reynolds number, and is often represented by R_e* . The form of the function f in Equation (12) must be determined experimentally.

Shields conducted experiments on fine grain sediments with four specific weights. He measured sediment transport at decreasing levels of bed shear stress and extrapolated to zero (incipient motion). The representative particle diameter selected was the median grain diameter, or d_{50} . The results he obtained are presented in Figure 2, which are similar to results obtained by Nikuradse (1933), indicating regions of hydraulically smooth, transitional, and turbulent flow (Colebrook and White, 1937). Shields originally reported that for turbulent flow, the value of the critical dimensionless shear stress for the median particle size in a streambed (θ) is approximately 0.06. He reported a single value to eliminate curve fitting to obtain a solution.

Nevertheless, Shields did not account for bed forms that developed with sediment transport nor non-uniform bed materials. Also, Shields' work is based upon the average transport of material and not sporadic movement, which is the common movement behavior of gravels and cobbles.

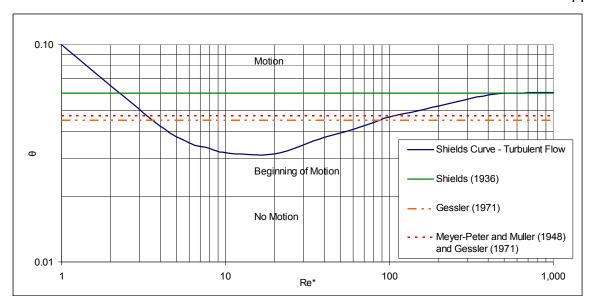


Figure 2. Shields relation for beginning of motion.

There have been various revisions to Shields Curve since its original publication. Gessler (1971) reanalyzed Shields relationship to account for bed forms (correction for sidewall effects and form drag) and found θ to vary with Reynolds number. However, he also suggested one value, θ =0.045 (lower than 0.06), which is commonly used in practice, as shown in Figure 2. Meyer-Peter and Müller (1948) and Gessler (1971) determined from their data sets that the critical Shields parameter for sediment mixtures was 0.047 (USACE, 1989). Buffington and Montgomery (1997), however, analyzed over 600 studies spanning eight decades and reported that the Shields parameter was not limited to 0.045 but had a range from 0.030 to 0.086.

The curve in Figure 2 shows Shields Curve for turbulent flow. Shields parameter is plotted on the ordinate and the Grain Reynolds number on the abscissa. Incipient motion is shown as a line, but in reality incipient motion

would be better represented by a band or range due to the inherent variability of results. Scatter can be caused by approximations made in the balance of forces acting upon a particle, the random or statistical nature of incipient motion, inconsistencies in the definition of incipient motion, the experimental methods utilized by researchers, site conditions or the experiment facility used, and the type and gradation of the bed material (Sturm, 2004). This relationship is more accurate under clearwater conditions, in uniform sands and small gravels (cohesionless) of uniform gradation, and the specific gravity of the material of approximately 2.65. As previously mentioned, incipient motion is inherently a statistical problem and therefore there is a frequency distribution of dimensionless critical shear stresses for each particle size of interest (Buffington and Montgomery, 1997).

Equation (12) can be calculated without directly measuring shear stress at the location and time period of interest. Equations (2) and (11) calculate an average shear stress of the entire channel boundary; therefore a method for separating the shear stress on the bed from the shear stresses on other physical boundaries should be implemented. A flow resistance sidewall correction method developed by Vanoni and Brooks (1957), which makes use of the Darcy-Weisbach formulation to estimate flow resistance, is presented as Equation (13).

$$f = \frac{8gR_hS_f}{V^2} \tag{13}$$

In Equation (12), f is the overall Darcy-Weisbach roughness coefficient, which is dependent on the Reynolds number presented as Equation (14).

$$R_e = \frac{VL}{V} \tag{14}$$

In Equation (13), R_e is the Reynolds number and L is a characteristic length. For closed conduits or full pipe flow, the pipe diameter, D_p , is often used. For open channel flow, $4R_h$ is often used as the characteristic length. Vanoni and Brooks used a procedure which consists of partitioning the cross-section of the flow into two non-interacting parts, referred to as the wall and bed regions. Therefore, with this estimation, Equation (13) translates into Equation (15), which can be used to iteratively calculate the roughness coefficient for the wall region with a smooth hydraulic boundary (Wong and Parker, 2006).

$$\frac{R_e}{f} = \frac{10^{\left(\frac{1}{2\sqrt{f_w}} + 0.40\right)}}{f_w^{\frac{3}{2}}} \tag{15}$$

In Equation (15), f_w is the friction factor for the wall region. By calculating f_w and using a water continuity equation, the friction factor for the bed can be calculated, as presented in Equation (16).

$$f_b = f + \frac{P_{w_- wall}}{P_{w_- hed}} (f - f_w)$$
 (16)

In Equation (16), f_b is the friction factor for the bed region, P_{w_wall} is the wetted perimeter of the wall region and P_{w_bed} is the wetted perimeter of the bed region. Combining Equations (13) and (16) results in Equation (17).

$$f_b = \frac{8gR_{h_bed}S_f}{V^2}$$
 (17)

Equation (17) is used to calculate R_{h_bed} , which is the hydraulic radius of the bed region, based upon the bed friction factor. R_{h_bed} can be substituted for R_h in Equations (4) and (5) to calculate shear velocity, which can be used to calculated the average shear stress of the bed to be used in Equation (10).

Incipient Motion of Bed

Great care is needed when defining incipient motion of the bed. The magnitude of turbulent fluctuations and frequency of fluctuations can cause relatively uncommon particle motion events (dependent upon the hydraulic conditions) and as a result increase subjectivity among investigators. Neill and Yalin (1969) attributed the wide range of scatter of experimental data points on Shields diagram, in part, to subjective definitions of incipient motion and proposed a quantitative definition for the beginning of motion. Buffington and Montgomery (1997) outlined four common methods for defining incipient motion to aid in categorizing studies of incipient motion and understanding data point stratification.

When the shear stress is of sufficient magnitude to cause general incipient movement of the sediment or substrate material in a channel, individual sediment particles should be observed to move at random locations and at random time intervals. It is common for groups of particles to move simultaneously. Such movement of particles is effected by particle size, shape, orientation, and turbulence. Cluster movement caused by the removal of intergranular forces present between larger particles can be referred to as the locking effect. When

one particle is dislodged, its associates are exposed to an increase in external forces and there are fewer points of contact to resist movement (the particles are no longer locked together), thus several more particles are removed, leading to the mobilization of pockets of material.

Turbulent flow at the bed is much more likely than laminar flow. Fluctuations or pulses cause fluctuations in shear stresses exerted upon the bed material and increases in movement. Even with a relatively smooth bed and the presence of a laminar sub-layer, vorticity or eddies from the main region of flow will enter the boundary layer and cause movement fluctuations (Chien, Wan, and McKown 2003).

The resisting forces or the particles ability to resist movement varies according to particle size, density, shape, friction angles, packing, and material gradation (Miller and Byrne, 1966; Li and Komar, 1986). Larger particle sizes generally require more energy to move, but mass density or specific weight is also an important factor. A large, low mass density material such as pumice requires less energy for motion to occur than a smaller particle of higher mass density, such as granite. Particle shape can make a significant contribution to movement resistance. Rounded particles such as river cobbles roll more readily compared to a very angular or non-spherical particle. Particle shape is also related to the amount of drag force experienced by the particle. A smooth, rounded particle will experience a drag force of a lesser magnitude than a rough or angular particle, for a given flow condition. A correlation between the physical characteristics of a particle on a plane bed and Shields parameter were

investigated by White (1970) but multiple regression analyses showed no relationship nor with the depth of flow or the water surface slope.

Due to the difficulty of quantifying shear stress in the field, relationships based upon velocity for incipient motion and particle size have been developed as an alternative to Shields relationship. Various research studies on non-cohesive materials have been collected by the Federal Highway Administration (HIRE) and are presented in Figure 3 (Richardson, Simons, and Julien, 1990).

Figure 3 is a graphical method for predicting the required velocity (critical velocity) to begin motion of a stone of a specific diameter or weight. Other investigators have explored the relationship between critical velocity and stone size (Li, 1959; Zeng and Wang, 1963; Fortier and Scobey; 1926, Keown, 1983). However, the depth of flow is an important factor for predicting motion if the average column velocity is selected.

The observed velocities corresponding to the initiation of substrate motion (incipient motion velocity) and corresponding substrate particle diameters from this study were compared with the relationships presented in Figure 3. This comparison was made in an effort to identify a conservative relationship for predicting incipient motion for the various bottomless arch culvert substrates tested and may assist the selection of riprap materials in larger bottomless culvert applications.

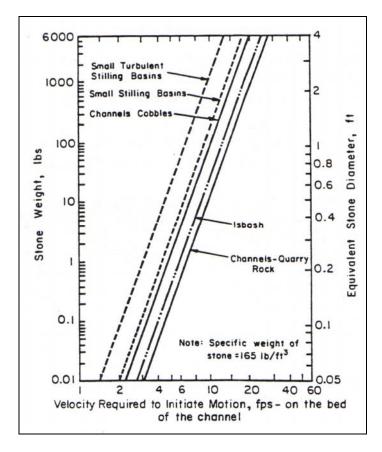


Figure 3. Critical velocity as a function of stone size (Richardson, Simons, and Julien, 1990).

Previous Bottomless Culvert Scour Studies

Scour associated with traditional circular culverts has been of interest for decades. Investigations regarding impinging jets (Abt, Kloberdanz, and Mendoza, 1984); cohesive (Abt, 1980) and cohesionless streambed materials (Chien, Wan, and McNown, 2003); tailwater, discharge, and culvert cross-section influences (Bohan, 1970; Abida and Townsend, 1991); culvert slope (Abt, Ruff, and Doehring, 1985b); and headwall and wingwall effects (Mendoza, Abt, and Ruff, 1983) have been undertaken in an effort to predict and control the amount

of scour occurring at non-embedded culvert inlets, outlets, and adjacent regions (Abt and Thompson, 1996). Riprap protection (FHWA, 1983) and incipient motion relationships for impinging jets (Shafai-Bajestan and Albertson, 1993) at culvert outlets have been developed to protect against perched outlets and possible failure of the structure. Many established riprap design methods, however, have been developed for subcritical flow in trapezoidal channels. The applicability of such methods may be limited for other channel geometries or flow conditions, (i.e., supercritical or highly turbulent flow).

Several recent culvert studies have focused on bottomless culvert scour. The Federal Highway Administration (FHWA) conducted a two-phase study exploring bottomless culvert inlet scour. Phase 1 (FHWA, 2003) tested 0.6-m wide scale models of four commercially available bottomless culvert shapes using four different uniformly graded sands ($d_{50} = 0.05$ -in to 0.1-in) as the substrate material. Various methods were used to calculate representative velocities in the flow contraction area near the culvert inlet, which were correlated with measured scour depths. After developing a maximum scour depth vs. velocity relationship for the test sand, the results were extrapolated for larger substrate materials using the following riprap sizing formula based on the Ishbash method (Ishbash, 1936).

$$d_{50} = \frac{0.69 V_{eff}^{2}}{2g(SG - 1)} \tag{18}$$

In Equation (18), V_{eff} is an effective local bed velocity (ft/s) at the upstream corner of a culvert inlet, d_{50} is the minimum stone diameter (ft) that resist incipient

motion, g is the acceleration of gravity (ft/s²), and SG is the specific gravity of the riprap material.

Phase 2 (FHWA-HRT-07-026) explored several scour countermeasures in a rectangular bottomless culvert (2-ft wide and 0.5-ft high) with vertical wing walls using two uniformly graded gravels (d_{50} =0.5-in and 0.6-in). Clear-water, fixed-bed flow conditions were evaluated for submerged and unsubmerged entrance conditions. The experimental results of Phase 1 and 2 were used to develop Equation (19).

$$d_{50} = \frac{0.38Y_{ent}}{SG - 1} \left(\frac{V_{a-ent}^{2}}{gY_{ent}}\right)^{0.33}$$
 (19)

In Equation (19), Y_{ent} is the depth of flow (ft) at the culvert entrance and V_{a_ent} is the average velocity (ft/s) at the culvert entrance. The unsubmerged inlet conditions produced more scour at the culvert outlet; the submerged inlet produced greater scour at the inlet. Recommendations for additional research from Phase 2 included developing riprap sizing relationship safety factors for bottomless culvert applications; testing non-rectangular, bottomless culverts with various wall roughness (smooth to corrugated); and testing longer culvert lengths with larger substrate mobile-beds modeled. Size-scaling issues should also be addressed.

An evaluation of several well-established riprap design methods (FHWA, 1989; Blodgett and McConaughy, 1986; Maynord, 1979, 1987) developed for traditional culvert design applications was conducted, using data collected at field sites in Minnesota featuring a concrete arch on spread footings (Halvorson and

Laumann, 1996). The following riprap sizing relationship for protecting footings for a 100-year flood event was developed and tentatively proposed.

$$d_{50} = 0.01V^{2.44} \tag{20}$$

In Equation (20), V is the average cross-sectional culvert velocity at the location of interest in the channel. The fourth riprap stone sizing method (FHWA, 1989) is as follows.

$$d_{50} = \frac{CV^{3.95}}{Y^{1.06}} \tag{21}$$

In Equation (21), *Y* is the local flow depth (ft) and *C* is a safety factor with published values specific to channel geometry and the severity of attack by the current. For use in the current study, a *C* value of 0.00117 was used, which corresponds to the most aggressive scour scenario (i.e., a curved channel with a 2:1 side slope, and a total depth of flow less than 10 feet).

The fifth method [Equation (22)] is from the California Department of Public Works, Division of Highways (1970).

$$d_{50} = \left(\frac{0.00002V^6SG}{(SG-1)^3\sin(\rho_c - \psi)} * \frac{6}{\pi\gamma_s}\right)^{\frac{1}{3}}$$
 (22)

In Equation (22), γ_s is the average specific weight (lb/ft³) of the substrate material, ρ_c is 70° for randomly placed material, and ψ is the side slope in degrees. It should be noted that this equation has been modified from predicting stone weight to a spherical stone size. The sixth method [Equation. (23)] comes from the U.S. Army Corp of Engineers (USACE, 1994).

$$d_{30} = SF'C_sC_VC_TY \left[\left(\frac{\gamma}{\gamma_s - \gamma} \right)^{\frac{1}{2}} \frac{V}{\sqrt{KgY}} \right]^{2.5}$$
 (23)

In Equation (23), SF' is defined as a safety factor, assumed to be 1.0 for this study; C_s is a stability coefficient for incipient failure equal to the ratio of d_{84}/d_{16} ; C_V is a vertical velocity distribution coefficient, and C_T is a thickness coefficient equal to d_{100} or $1.5d_{50}$, which ever is greater; γ is the specific weight (lb/ft³) of water; and K (assumed to be 1.0 for this study), is a side slope correction factor. User-defined coefficients have a significant impact on the accuracy of this method and should be based on site-specific field observations and good engineering judgment.

The seventh stone sizing method is from ASCE Manual 54 (Vanoni, 2004) and is presented as

$$d_{50} = \left(\frac{0.000041SGV^6}{(SG-1)^3 \cos^3 \varphi} * \frac{6}{\pi \gamma_s}\right)^{\frac{1}{3}}$$
 (24)

In Equation (24), φ is the angle of the side slope of the streambed to horizontal, in degrees. For this study, the streambed was horizontal and therefore an angle of 0° was used. This equation has been modified to calculate a spherical size of stone instead of a stone weight. The eighth method (Equation (25)) was developed by the United States Bureau of Reclamation (USBR, 1962) by Blodgett and McConaughy specifically for scour protection downstream of stilling basins.

$$d_{40} = 0.0105(V)^{2.6} (25)$$

The objectives of this study were to observe the response of various substrate materials inside a 2-ft (0.61-m) diameter bottomless arch culvert to pressurized and non-pressurized conditions and various inlet geometries; observations included particle transport, bedforms, and scour depths. In addition, incipient motion of the same substrate materials and inlet geometries were evaluated inside the bottomless arch culvert and in a 1-ft (0.30-m) wide rectangular laboratory flume, for comparison. Using the incipient velocity, stone size, and other necessary system parameters, the required stable substrate material size was estimated using each riprap stone size predictive method [Equations (18)-(25)] and compared with the corresponding experimental substrate size parameter. The ratio of the predicted to actual stone size values for each method was calculated for each method and represents a stone-size prediction factor of safety.

Related Sediment Transport Studies

A broader search of scour and sediment transport of gravels and cobbles in channels was conducted for additional information, background, and guidance for experiments in bottomless culverts with larger substrates. Each of following publications has directly or indirectly influenced this study.

One such publication is entitled "Highways in the River Environment" (HIRE), which was published by the Federal Highway Administration (Richardson, Simons, and Julian, 1990). This document contains a comprehensive academic background on alluvial channel flow, scour, and

sediment transport. Of particular interest to this study are the recommendations regarding incipient motion velocities versus equivalent stone diameters (see Figure 3, Critical velocity as a function of stone size (Richardson, Simons, and Julian, 1990).

Wilcock (1988) explored methods for estimating the critical shear stress for different size fractions in non-uniform or mixed-size sediments. He considered two established methods for estimating critical shear stress for a particular size fraction: [1] low transport rate and [2] largest grain displacement. He stated that "the practical considerations involved in determining initial motion for mixed-size sediment suggest strongly that the low transport method is preferable to the largest-grain method." Unfortunately, he also stated that scaling is still difficult and no universal method exists for predicting incipient motion for a large range of practical particle sizes.

A study entitled "Effects of hydraulic roughness on surface textures of gravel-bed rivers" (Buffington, 1995) investigated gravel-bed rivers in Alaska and Washington. Field studies found that for a given shear stress, reaches with relatively high hydraulic roughness experience a textural fining. Rough bed surfaces transform into finer bed surfaces due to channel irregularities such as bars, vegetation, bank boundary aberrations. These irregularities decrease bed shear stresses and sediment transport capacities, therefore allowing finer material to be deposited. The biological implications for salmonid species are that forested river reaches with coarse, unsuitable gravel beds may transform into finer, suitable areas for spawning. Buffington found that observed d₅₀ values

were as much as 90% smaller than those predicted by Shields equation. This phenomenon may prove advantageous for bottomless culvert installations that may require riprap stone sizes that are ecologically unsuitable (too large).

Investigations by Lisle et al. (2000) on six gravel-bed channels with varying sediment yield also found a poor correlation between values of shear stress at bank-full discharge and measured surface d₅₀ stone sizes. Local variations in boundary shear stress acting on the particles were noted to control sediment transport patterns. Buffington and Montgomery conclude that shear stresses obtained from Shields equation to predict particle mobility should be used as a first-order assessment tool; this method showed reasonable agreement if placed in context by means of sediment budgeting.

A publication entitled "Concept of Critical Shear Stress in Loose Boundary Open Channels" studied existing concepts of incipient motion of particles in a 3-ft (0.91-m) wide rectangular laboratory flume (Paintal, 1971). Three uniform sand and gravel materials (d₅₀=0.098-in or 2.49-mm, 0.313-in or 7.95-mm, 0.874-in or 22.2-mm) and two mixtures of the materials were analyzed, with adjustments made for sidewall effects using the Johnson method (Johnson, 1942). Ten previous sediment transport studies were compared with results from the experimental data and the following two conclusions were made: [1] A distinct condition for incipient motion does not exist. [2] Experimental results of critical shear stress and mean stone size correlated very well with the ten bed load transport rates evaluated.

Critical shear stress equations were first developed for sand in laboratory flumes. However, discrepancies arose as these equations were applied to gravel beds due to several factors that are insignificant for fine material: particle shape, imbrication, cluster movement, and grain protrusion. A recent investigation of shields method for predicting motion on gravels was conducted by Petit (1994). Dimensionless critical shear stress was evaluated for four different gravel beds (d₅₀=0.47-in,0.77-in, 0.95-in, 1.54-in) placed in a 1.65-ft (0.50-m) wide rectangular flume. Incipient motion or critical shear stress was defined as the condition when 20% of the particles were set in motion. Shields curve was found to correlate well with the smaller particles, with shields parameters ranging from 0.056 to 0.079, illustrating the range of results frequently collected from laboratory experiments. Petit states that this method did not correlate with experimental results for larger particles and therefore ceases to be applicable (Petit, 1994). An alternative equation for Shields parameter proposed previously by Andrews (1983) was found to be applicable and the form Andrews suggested is presented as Equation (25).

$$\theta = a \left(\frac{d_s}{d_{50}} \right)^b \tag{25}$$

In Equation (24), a and b are coefficients found through experiment for a stone size (d_s) of interest. Still, Petit found values of a and b that were significantly lower than those originally proposed by Andrews but were similar in magnitude to those proposed by Li and Komar (1986) whose results are based upon natural rivers.

Bathurst, Graf, and Cao (1987) investigated the influence of channel slope to predicting incipient motion using Shields diagram. He stated that several recent studies concluded the invalidity of Shields diagram for slopes greater than 5% that were composed of coarse bed materials (gravels and boulders). He investigated an alternative method proposed by Schoklitsch (1962) via a combination of field and flume research. Bathurst found that for increasing slopes, a trend of separate but approximately parallel lines roughly bisected Shields curve. This trend may not be readily noticeable at mild bed slopes and could be mistaken for data scatter. Also, flow velocity instead of shear stress seemed to better represent incipient motion conditions for this study.

Fenton and Abbott (1977) explored grain protrusion effects on incipient motion and state that Shields diagram "implicitly contains variation with protrusion between two extremes; [1] large grains, large Reynolds numbers, low protrusion and [2] small grains, low Reynolds numbers and large protrusions." Fenton and Abbott state that sediment characteristics and bed preparation can greatly influence the degree of protrusion of individual particles and therefore influence incipient motion of the particles and cause discrepancies when using Shields curve, due to the fact that no protrusion parameter is included. For beds with protruding particles (such as natural streambeds with varying particle sizes) Shields diagram becomes less and less applicable with increasing grain Reynolds numbers.

Experiments conducted by White (1940) demonstrated that uncertainties with Shields diagram were as much as ±25% and that the shear stress

experienced by a particle increases as the particle diameter increases, other conditions being held constant. He also concluded that from the various forces acting upon a particle, form drag enables a streambed to withstand a greater drag whereas nonuniform flow conditions and turbulence facilitate grain movement.

A mobile gravel bed without bedforms (flat) experiences not only bed movement effects but permeability effects, which presents a different boundary condition when compared to fixed-bed laboratory studies (Nikora and Goring, 2000). This difference in boundary conditions may alter the flow structure and therefore directly affect particle mobility and incipient motion results.

Incipient motion methods developed in flumes and natural channels and the experimental findings of other research studies on sediment transport of gravel beds, such as those discussed above, should be applied to scour in bottomless culverts to validate which methods are suitable for bottomless culverts and which needs to be adapted for this application. Open channel flow and full pressurized flow conditions should be explored in addition to investigating size scaling techniques for riprap sizing for a range of culvert sizes.

EXPERIMENTAL METHOD

Facilities

All research was conducted at the Utah Water Research Laboratory (UWRL) located on the Utah State University Campus in Logan, Utah. Two facilities at the UWRL were utilized for this study—a bottomless culvert facility and a rectangular flume.

The bottomless arch culvert test facility constituted a 2-ft (0.61-m) diameter bottomless circular arch culvert, a head box, and a tail box, all of which were fabricated specifically for this study. The head box measured 8-ft (2.44-m) wide, 3.5-ft (1.07-m) deep and 17.5-ft (5.33-m) long. The tail box measured 8-ft (2.44-m) wide, 12-ft (3.66-m) long and 3.5-ft (1.07-m) deep. The culvert included only the top half (i.e., half a cylinder) of a circular culvert, fabricated of clear acrylic for flow visualization. The 16-ft (4.88-m) long culvert was installed on top of a 2-ft wide by 13-in deep (0.61-m by 0.33-m) steel box with a rectangular cross-section, which housed the streambed material. An overview of the test facility is shown in Figures 4 and 5. The 8-ft (2.44-m) wide approach channel (the width of the head box) allowed for a reasonable amount of flow contraction entering the culvert. The approach flow contraction was adjustable using movable guide walls. Some specific details pertaining to the bottomless culvert test facility design and operation are presented in the following list.

 The acrylic culvert was mechanically fastened to the steel box for access to the streambed in the culvert for inspection.

- 2. The culvert had a uniform cross-section along the entire length and projected 2-ft (0.61-m) into both the head and tail box.
- 3. The culvert slope was approximately 0.01 ft/ft.
- 4. A 3.5-ft (1.07-m) by 8-ft (2.44-m) sheet of acrylic was placed in a side wall in both the head and tail boxes for visual inspection throughout the system.
- 5. A stop log assembly was installed in the tailbox to permit tailwater control.

The rectangular laboratory flume facility used for this study is 3-ft (0.91-m) wide, 2-ft (0.61-m) deep, and 30-ft (9.14-m) in overall length. It consists of clear acrylic panels for the walls and floor, and a reinforcing steel frame. Flow enters the facility from the floor of the headbox, passes through a perforated aluminum plate to straighten flow lines and decrease turbulence, and exits the flume by spilling over a hinge gate into the tailbox, which conveys the flow through an opening in the floor into a drain channel the laboratory. This rectangular flume facility was modified from the above description for incipient motion investigations. An overview of the modified flume is presented in Figure 6; the modifications are listed as follows:

- 1. The flume width was decreased from 3-ft (0.91-m) to 1-ft (0.30-m) via a contraction and a partition wall, all constructed from lumber.
- Two false floors, spanning the entire width of the flume and 4 inches (101.6-mm) deep, were place at the upstream and downstream ends of the flume to create a 17-ft (5.18-m) long recess for the placement of the substrate materials.

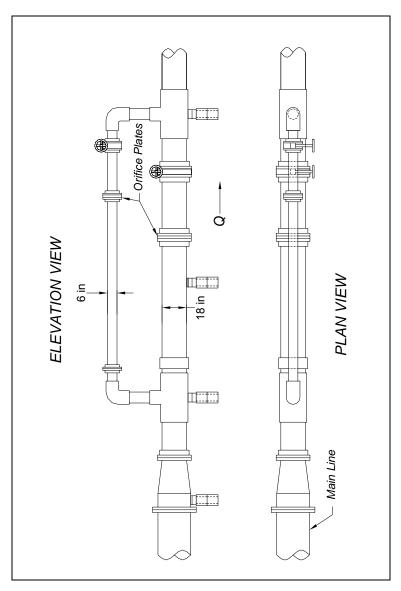


Figure 4. Overview of supply piping to bottomless culvert test facility.

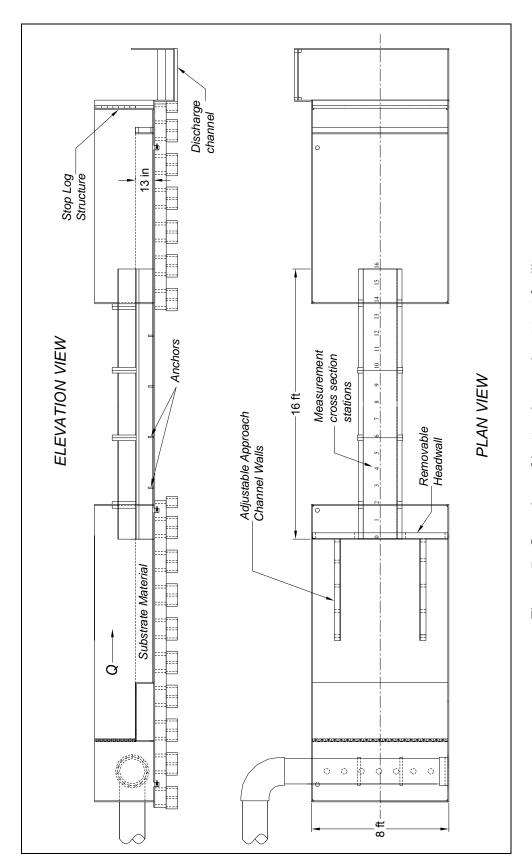


Figure 5. Overview of bottomless culvert test facility.

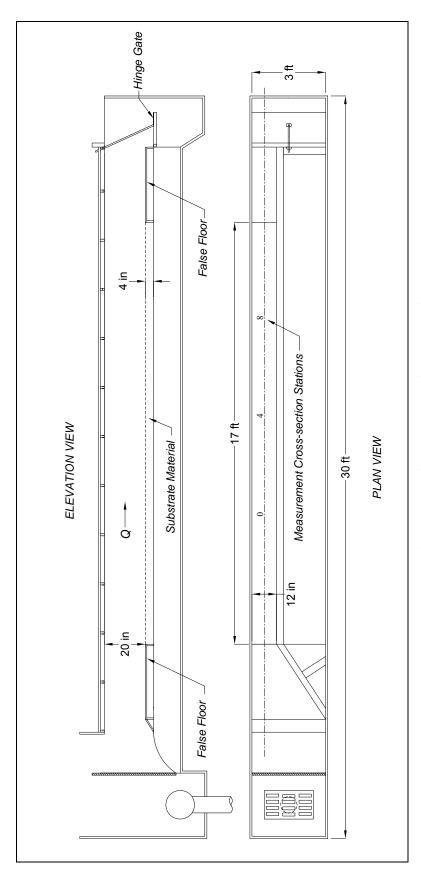


Figure 6. Overview of rectangular flume test facility.

3. The false floors and prepared substrate bed decreased the overall depth of the flume from 24 inches to 20 inches (609.6-mm to 508-mm).

Substrate Materials

The four substrate materials tested were pea gravel, 0.75-inch angular gravel, 2-inch cobbles (rounded) and 2-inch angular rock. These materials were selected to represent armored, non-cohesive streambeds. Sieve analyses were conducted for each substrate material; the sieve analyses results are summarized in Table 1 and Figure 7. Detailed results, including material gradation and standard deviation, are presented in Appendix G Substrate Properties. The average density of each material was also determined; the results are presented in Table 2.

Bottomless Culvert Testing Procedure

For the bottomless culvert test facility, each substrate material was placed upstream, inside, and downstream of the culvert. The elevation of the top of the material was meticulously graded to correspond with the elevation of the top of the steel box or (the interface between the steel box and the acrylic culvert) or the springline of the culvert. Four 4-inch high cross-flow anchors attached to the bottom of the steel box were used to keep the gravel substrate in place and prevented the formation of an artificial shear failure plane at the interface between the substrate material and the bottom of the culvert. Each tested

Table 1. Substrate sieve analyses results

	Pea Gravel		0.75-inch Angular Gravel		2-inch Cobbles		2-inch Angular Rock	
	(in)	(ft)	(in)	(ft)	(in)	(ft)	(in)	(ft)
D ₁₆	0.169	0.014	0.502	0.042	1.055	0.088	0.951	0.079
D ₃₅	0.228	0.019	0.579	0.048	1.201	0.100	1.252	0.104
D ₅₀	0.266	0.022	0.636	0.053	1.299	0.108	1.471	0.123
D ₆₅	0.307	0.026	0.685	0.057	1.417	0.118	1.684	0.140
D_m	0.260	0.022	0.632	0.053	1.371	0.114	1.463	0.122
D ₈₄	0.370	0.031	0.807	0.067	1.713	0.143	1.958	0.163
D ₉₀	0.396	0.033	0.866	0.072	1.969	0.164	2.090	0.174
D ₉₅	0.441	0.037	0.921	0.077	2.205	0.184	2.264	0.189

Table 2. Substrate density analyses results

Substrate	Rock Weight (lb)	Rock Volume (ft^3)	γ	γ_{s}	SG
Pea Gravel	35.25	0.23	62.4	154.80	2.48
0.75-inch	37.20	0.24	62.4	153.44	2.46
Angular Gravel					
2-inch Cobbles	43.20	0.27	62.4	160.90	2.58
2-inch Angular Rock	33.95	0.23	62.4	150.78	2.42

material was placed on top of a 1-inch (25.4-mm) thick layer of pea gravel substrate. This substrate helped keep the streambed materials in place during testing. The incipient motion velocity of 2-inch cobbles, for example, placed on a smooth surface, such as steel or acrylic, would likely be smaller than for the same material resting on a rough surface, such as gravel.

For each substrate material, five entrance configurations were tested. The entrance configurations consisted of three different channel to culvert contraction ratios of 0%, 33% and 75% for both projecting and non projecting (headwall)

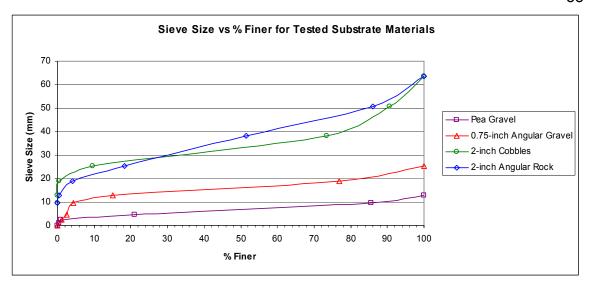


Figure 7. Sieve size vs. % finer for tested substrate materials.

entrance conditions (see Table 3 and Figures 8-10). The 0% contraction configuration was limited to the non-projecting (headwall) condition.

Each of the five entrance configurations were to be tested at two headwater depths. Headwater depths were measured as a dimensionless parameter expressed as the headwater depth relative to the pre-scour invert at invert at the culvert entrance over the distance from the culvert invert to the crown dimension, H_w/D (see Figure 11).

The location of measurement (in the headbox) for the headwater depth was a sufficient distance from the culvert entrance where the measured velocity from the velocity probe was less than 0.5 ft/s. Therefore, the velocity head was a relatively insignificant portion of the total energy head; the headwater depth, H_w, also represents the total energy head at the culvert entrance.

Table 3. Bottomless culvert test matrix

Culvert Inlet Type	Contraction Ratio (approach channel to culvert diameter)			
Square-edged with headwall	0%	33%	75%	
Projecting	Х	33%	75%	

^{*}All tests included incipient motion velocities, scour depths, and extent of scour.

The two H_w/D values tested for 2-inch cobbles and 2-inch angular rock were 1.0 and 1.5 (see Figures 13 and 14). According to Mark Miles of the Alaskan Department of Transportation and Public Facilities (ADOT&PF), a typical design H_w/D for bottomless culverts in Alaska is approximately 1.0 at maximum discharge conditions. The 1.5 H_w/D test provides additional information for culverts that may be undersized. Preliminary testing of the pea gravel at 1.0 & 1.5 H_w/D and 0.75-inch angular gravel at 1.5 H_w/D resulted in rapid and excessive bed degradation; therefore the test H_w/D ratios were modified. The pea gravel was primarily tested at 0.5 H_w/D (see Figure 12); the 0.75-inch angular gravel was primarily tested at 1.0 H_w/D (see Table 4 and Figure 13). There was no controlled tailwater at the exit of the culvert and all tests consisted of a 2-ft (0.61-m) projecting end treatment with a 400% expansion from the culvert into the tailbox. Thirty-five separate tests were conducted from the combination of contraction ratios, tested headwater depths, and streambed materials.



Figure 8. 0% Contraction with headwall.



Figure 9. 33% Contraction, with headwall (A), projecting inlet (B).



Figure 10. 75% Contraction, with headwall (A), projecting inlet (B).

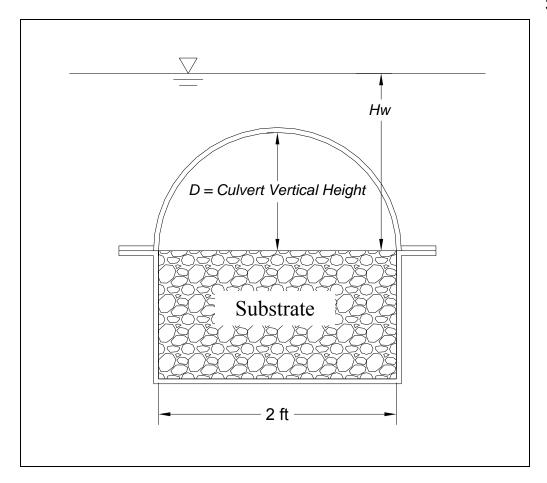


Figure 11. Headwater depth dimensionless parameter, H_w/D.

Bottomless Culvert Data Collection

Flow to the headbox and culvert was supplied by either an 8-inch (20.3-cm) or an 18-inch (45.7-cm) poly vinyl chloride (PVC) pipeline (see Figure 4). Each pipeline contained a calibrated orifice plate and was connected to differential manometry to monitor flow rates. Water entered the headbox through a diffuser and a baffle (synthetic mesh), which created a relatively uniform flow pattern approaching the culvert.

The period of duration for a specific headwater or H_w/D test was

Table 4. Bottomless culvert H_w/D test matrix

Hw/D	Pea Gravel	0.75-inch Angular Gravel	2-inch Cobbles	2-inch Angular Rock
0.5	х			
1.0		х	х	х
1.5			х	х

approximately 2 hours, which is referred to as a duration test. This was based upon test periods used for other relatively similar scour studies and preliminary test investigations for this study, which observed scour rates, scour equilibrium, and any formation and migration of bedforms.

Each test began by slowly increasing the flow rate to the system. This slow increase in flow rate prevented any premature scouring and permitted monitoring of the bed for incipient motion. When incipient motion was reached, the flow rate was held constant and measurements were taken. Generally, from the start of a test, 30 minutes would elapse before incipient motion conditions were reached and measurements were obtained. Subsequently, incremental increases of the flow rate were resumed until the head water depth reached the desired H_w/D ratio for the particular duration test. The flow rate was held constant and measurements were taken at 30-minute intervals during a two-hour period. The measurements taken and data collected during each test (incipient motion and duration) included:

1. Culvert flow rate



Figure 12. $0.5 H_w/D$ with headwall (A), projecting inlet (B).



Figure 13. 1.0 H_w/D with headwall (A), projecting inlet (B).



Figure 14. 1.5 H_w/D with headwall (A), projecting inlet (B).

- 2. Head water elevation (total energy)
- 3. Tail water elevation
- 4. Approximate depth of flow or water surface profile in the culvert
- 5. Local velocities at the entrance and exit of the culvert
- 6. Video and photographic documentation
- 7. General scour observations such as localized scour & bed formations

Velocity measurements were made using a Global Water Velocity Probe (model FP 201) with precision of ± 0.005 ft/s. The probe was placed on top of the substrate at five locations or points (A, B, C, D, E, see Figure 15) at measurement cross section stations 0 and 16 (see Figure 5) and an average local velocity was measured and recorded. The duration of time for determining the average velocity at each measurement location ranged between 15 and 120 seconds, depending upon the time required for the average velocity measurement to become stable. Depth measurements (flow and scour) were taken in the headbox, tailbox, and at stations 0, 4, 8, 12, and 16 using a staff gauge with precision of ± 0.0026 ft (see Figure 4). The station numbers represent the distance in feet from the culvert inlet to the exit.

After each test was completed, the system was slowly drained of water for final scour measurements to be taken. Scour depths were taken within the culvert at stations 0, 1, 2, 4, 6, 8, 10, 12, 14, and 16; at five locations (A, B, C, D, E) across the cross-section of the culvert (see Figures 5 & 15). Geometries (shape, widths, and depths) were also taken at other locations where scour occurred (see Figure 16).

Rectangular Flume Testing Procedure

Each tested substrate material was placed in a 4-in (101.6-mm) deep recess in the laboratory flume. The elevation of the top of the material was meticulously graded to correspond to the height of the false floors and the slope of the laboratory floor.

For each substrate material, three bed slopes were tested; 0.005 ft/ft, 0.010 ft/ft, and 0.015 ft/ft. The second tested slope corresponded with the fixed slope of the bottomless culvert facility, for comparison of incipient motion experimental results. Each individual test was duplicated to investigate the

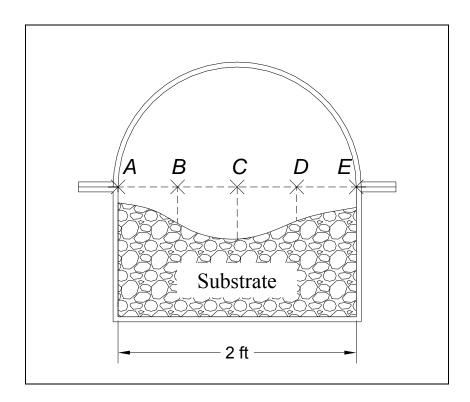


Figure 15. Culvert cross-section looking downstream.

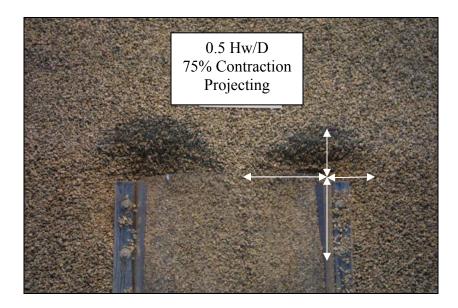


Figure 16. Example of scour geometry measurements at entrance.

variability and repeatability of incipient motion conditions; there was no tailwater control.

Rectangular Flume Data Collection

Incipient motion of each substrate material was investigated for comparison with the bottomless culvert experimental results of this study and for future comparisons with other incipient motion studies on gravel beds.

Flow to the headbox of the flume was supplied by a 12-inch poly vinyl chloride (PVC) pipeline. The pipeline contained a calibrated orifice plate and was connected to differential manometry to monitor flow rates. Water entered the headbox through a diffuser and passed through a baffle (perforated aluminum plate) before entering the flume. Prior to beginning a set of tests, the hinge gate located at the exit of the flume was fully raised and the flume slowly filled with

water. The water supplied to the UWRL from the Logan River is much colder than the air temperature of the laboratory; filling the flume with water and allowing it to cool for 30 to 45 minutes ensured that the flume was cool and not experiencing thermal contraction while data was being collected. After the flume was cooled and ready for testing, the supply flow was turned off and the hinge gate slowly lowered to drain the flume. As soon as the flume was sufficiently drained, testing would commence.

Each test began by slowly increasing the flow rate to the system. As with the bottomless culvert facility, this slow increase in flow rate was necessary to ensure that incipient motion conditions were correctly obtained and not surpassed or falsified by flow surges. When incipient motion was reached, the flow rate was held constant and measurements were taken. Generally, from the start of a test, approximately 20 minutes would elapse before incipient motion conditions were created and measurements were obtained. The measurements taken and data collected during each incipient motion test included:

- 1. Flume flow rate
- 2. Approximate depth of flow or water surface profile of the flume
- 3. Local velocities at locations of incipient motion
- 4. Video and photographic documentation.

Velocity measurements were made using the same velocity probe used during testing of the bottomless culvert (model FP 201) with precision of ± 0.005 ft/s. The probe was placed as close to the substrate particles as possible (less than 0.75-in) at measurement cross section stations 0, 4, and 8 (see Figure 6)

and an average velocity was recorded (3 stations total). The duration of time for determining the average velocity at each measurement location ranged between 15 and 30 seconds, depending upon the time required for the average velocity measurement to become stable. Depth measurements were taken using a point gage (precision of ± 0.0005 -ft) on a rolling carriage, placed upon guide rails that were fastened to the top of the flume walls.

It should be noted that at high flow rates there were prominent surface waves due to the contracting approach flow section. Also, the flow near the exit of the flume exhibited a pronounced S2-type gradually varied flow surface profile. Therefore, the first 5-ft (1.52-m) and last 4-ft (1.22-m) of linear bed length were discounted for incipient motion (see Figure 6). The station numbers represent the distance in feet from the beginning of the investigated bed area to the end of the flume, in 1-ft (0.30-m) increments. After each test was completed, the system was allowed to drain and the substrate material was prepared for the next test.

Data Analyses Programs

Many short functions were programmed in Visual Basic to be used in calculations and analyses conducted in Microsoft Excel. The actual code used for the bottomless culvert and the rectangular flume is presented in Appendix H Visual Basic Code Used for Calculations for Bottomless Arch Culvert in Microsoft Excel and Appendix I Visual Basic Code Used for Calculations for Rectangular Flume in Microsoft Excel, respectively. The reference formulas utilized to

calculate hydraulic radii, wetted perimeters are presented in Appendix J. To facilitate the use of the computer, several curve fit approximations were developed.

A curve fitting was conducted for three physical water properties that change with temperature. These physical properties assume water as a Newtonian fluid and exposed to atmospheric pressure. The data points used as a basis for the curve fitting came from R. E. Bolz and G. L. Tuve (1973) Handbook of Tables for Applied Engineering Science. Curve fitting was conducted using XLfit by IDBS (www.idbs.com) and the results are presented in Table 5 and Figure 17.

No curve fit was conducted for the mass density of water due to the very small incremental change of the physical property over the range of water temperatures (~32°F to ~50°F) measured at the UWRL during testing; it was assumed to be constant at 1.94 slugs/ft³.

Shields diagram for turbulent flow presented by Chien, Wan, and McNown (2003) (see Figure 18) was curve fit to facilitate plotting of experimental data on Shields curve. A sixth order polynomial, was developed, with the log of the Grain Reynolds number ($logRe^*$) as the independent variable (see Figure 19). The polynomial equation used to approximate Shields curve for turbulent flow is presented as Equation (26).

$$\theta = -0.0009x^{6} + 0.0083x^{5} - 0.027x^{4} + 0.0211x^{3} + 0.0703x^{2} - 0.1403x + 0.0999$$
(26)

In Equation (26), x represents the log of the grain Reynolds number (log Re^*).

Table 5. Curve fit equations for physical properties of water[†]

Physical	Equation Type	Equation	Curve Fit
Property	Equation Type	Equation	Constants
	Reciprocal Quadratic	1	a = 10865.95
Dynamic Viscosity		$\mu = \frac{1}{a + b(^{\circ}F) + c(^{\circ}F)^2}$	b = 441.7072
Viscosity			c = 1.467279
	Vapor Pressure Model	$\nu = \exp^{\left(a + \frac{b}{({}^{\circ}F)} + c\ln({}^{\circ}F)\right)}$	a = -5.511624
Kinematic Viscosity			b = -26.35435
Viscosity			c = -1.310849
	Sinusoidal Fit	$\gamma = a + b\cos(c(^{\circ}F) + d))$	a = 59.36498
Specific			b = 3.075081
Weight			c = 0.007833
			d = -0.243022

^{†°}F is a variable; it is the measured temperature of water in degrees Fahrenheit

Equation (26) is limited to grain Reynolds numbers between 1 and 1,000. Equations (26) had correlation coefficient values (R²) exceeding 0.999. An example of Shields diagram utilizing this equation is presented as Figure 20. Though the traditional Shields diagram is limited to grain Reynolds numbers less than 1,000, as illustrated in Figure 20, he theorized that the curve would continue at a constant value of 0.06. His theory has been used as guidance for extrapolating Shields curve for higher grain Reynolds numbers and therefore was adopted for this research study.

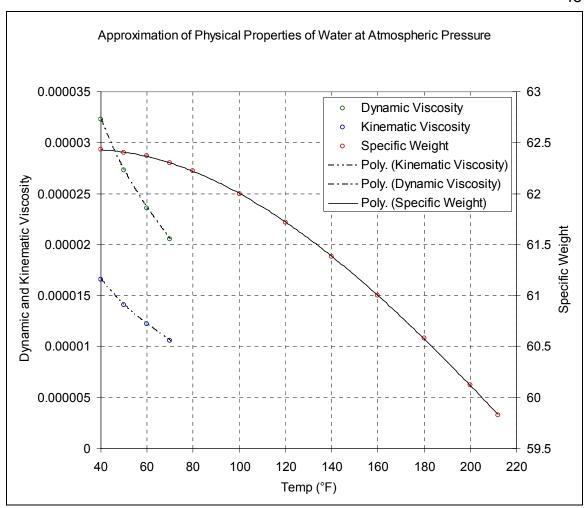


Figure 17. Approximation of physical properties of water.

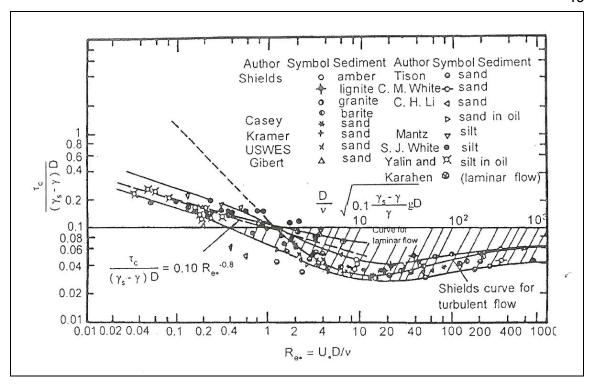


Figure 18. Shields diagram (Chien, Wan, McNown, 2003).

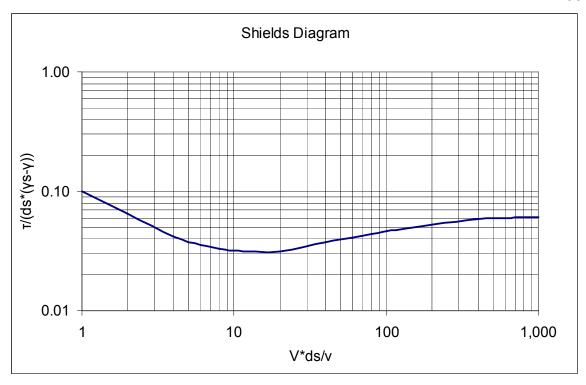


Figure 19. Reproduction of Chien, Wan, and McNown (2003) shields diagram, based on curve fit.

BOTTOMLESS CULVERT INCIPIENT MOTION RESULTS

General Velocity Observations

During testing, several general behavior characteristics were observed concerning local velocities upstream, within, and downstream of the 2-ft (0.61-m) diameter circular bottomless culvert for all four tested substrate materials. First, measured velocities varied in magnitude both spatially and temporally. Measured velocities were lowest upstream of the culvert entrance where the depth of flow was greatest. Velocities increased in the downstream direction, with the highest measured values at the exit of the culvert. After exiting the culvert, the flow was free to expand (400% channel width expansion) in the tailbox and depths rapidly decreased, resulting in a further increase of velocities and in the formation of a channel with deposition along the edges (see Figure 22). In the more resilient materials, an oval scour hole was observed to form instead of a channel (see Figure 23).

Generally, entrance configurations with a 33% or 75% contraction ratio and H_w/D ratios of 1.0 or less resulted in lower bed velocities at points B, C, and D and higher bed velocities at points A and E (at station 0) where the flow contracted to enter the culvert (see figure 12). For H_w/D ratios greater than 1.0, lower bed velocities remained at points B and D, with higher bed velocities occurring at A, C, and E (at station 0). However, as the substrate material was eroded, local velocities would decrease, as expected. Where the eroded material was deposited inside the culvert, local bed velocities would increase.

Also, measured velocities at individual locations were observed to fluctuate with time. Generally, these fluctuations were ± 0.5 ft/s (0.152 m/s) or less, and may be a result of general flow turbulence.

Incipient Motion Velocities

Incipient motion was interpreted as the commencement of sporadic movement of particles of substrate, occurring at an average rate of several incidents per minute. When incipient conditions were reached, bed velocity measurements were taken. In addition, flow conditions, such as flow rate and flow depths, were recorded to calculate an average cross-sectional velocity at each location within the culvert (see Figure 4). To verify that incipient motion was occurring and not a prelude seating effect, the headwater was slightly increased after measurements were taken. If this increase resulted in an increased rate of sporadic movement, the incipient motion conditions and corresponding data were confirmed to be accurate. The headwater was then increased to the desired depth for duration testing.

Incipient motion occurred very suddenly during testing of the pea gravel substrate material. Incipient motion was observed to occur at the entrance of the culvert and adjacent areas, along the entire length of the culvert, and in the tailbox. Incipient motion velocities were not identified for the pea gravel substrate due to the ease with which it moved, resulting in difficulties in differentiating between incipient motion and general movement by observation. The results of

average culvert velocities at tested H_w/D ratios for pea gravel are summarized in Figure 20; Figure 20 only presents motion data at tested H_w/D ratios, and are average velocities for the entire culvert length. Measured and calculated velocities are summarized in Appendix A, Bottomless Arch Culvert Velocity Results.

Incipient motion for the 0.75-inch angular gravel occurred at higher velocities than the approximate values identified for the pea gravel substrate. Initial movement was observed to commence at the exit of the culvert where velocities were greatest. However, prior to movement, individual substrate particles were observed to vibrate or shudder at various locations where small gaps between particles were present but the forces acting upon the particle were not large enough to cause rotation. This vibration or shudder shall be referred to

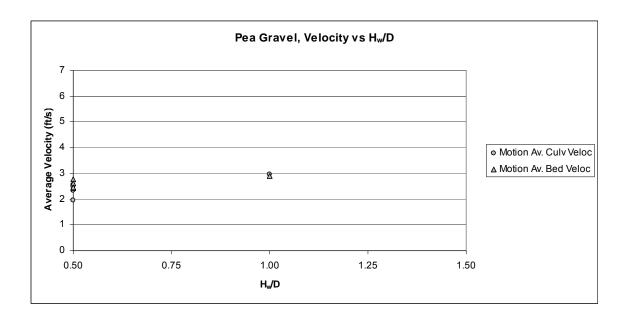


Figure 20. Velocity vs. H_w/D ratio for pea gravel substrate.

as the "shudder effect" and its observed intensity proved an excellent indicator of proximity to incipient motion. After data were collected for incipient motion, the H_w/D ratio was increased as conducted with the pea gravel substrate. The relationship between calculated and observed velocities and H_w/D ratios is summarized in Figure 21. As Figure 21 illustrates, incipient motion frequently occurred at approximately 0.75 H_w/D. Measured bed velocities during incipient motion conditions ranged from 2.7 ft/s to 3.9 ft/s; average culvert velocities ranged from 1.7 ft/s to 2.8 ft/s. Measured and calculated velocities are summarized in Appendix A.

The previously defined shudder effect was observed during testing of incipient motion for the 2-inch cobbles. However, an additional event also occurred prior to incipient motion, a seating effect. A few substrate particles were observed to roll a short distance prior to finding a stable position in an

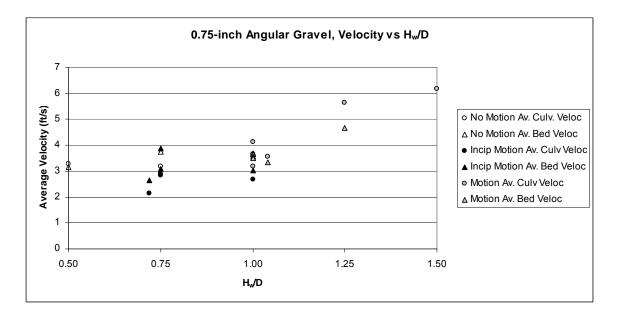


Figure 21. Velocity vs. H_w/D ratio for 0.75-inch angular gravel.

indentation or gap. This seating effect was differentiable from incipient motion since after the seating effect concluded, an increase in the flow rate did not result in an increase in the rate of sporadic movement of particles. A quiescent period was observed to occur between the conclusion of the seating event and the beginning of incipient motion.

Incipient motion for the 2-inch cobbles also began at the exit of the culvert where velocities were greatest. This substrate material was approximately twice as large as the 0.75-inch angular gravel substrate; the d_{50} of the 0.75-inch angular gravel was 0.64 inches and the d_{50} of the 2-inch cobbles was 1.30 inches. However, the average culvert velocities were approximately 3.0 ft/s to 4.7 ft/s, only 1.1 to 1.2 times larger. In contrast, the average velocities measured at the bed were approximately 1.7 to 2.5 times larger, ranging from 4.2 ft/s to 4.9 ft/s. It can be concluded that the measured bed velocity under incipient motion conditions will increase in relation to the size of the material and is a better indicator than an average water column velocity. The relationship between averaged measured velocities and calculated velocities for the entire culvert barrel and $H_{\rm w}/D$ ratios is summarized in Figure 22. Incipient motion occurred over a range of 1.25 to 1.42 $H_{\rm w}/D$. Measured and calculated velocities are summarized in 2-inch Cobbles Velocity Results, Appendix A.

The shudder effect and seating effect were also observed during testing of the 2-inch angular rock. This angular substrate proved more resilient to motion and scour than the three previously tested materials, resulting in higher critical velocities and higher corresponding $H_{\rm w}/D$ ratios, confirming that angular riprap is

preferred for protection. As previously mentioned, the shudder effect and seating effect were both observed; however, differences between the 2-inch cobbles were noted. The shuddering of particles seemed more pronounced and less seating occurred relative to the 2-inch cobbles. These observed differences may be attributed to the differences in material shape and gradation.

Incipient motion velocity results for the 2-inch angular rock substrate were observed to occur at the entrance of the culvert when the flow was contracted from a contraction ratio and at the exit of the culvert where velocities were greatest. The H_w/D ratio present when motion began was slightly higher than the 2-inch cobbles (see Appendix B). A comparison of the 0.75-angular gravel and the 2-inch angular rock present two points of interest: the d_{50} of the 2-inch angular material is approximately twice as large as the d_{50} of the 0.75-inch

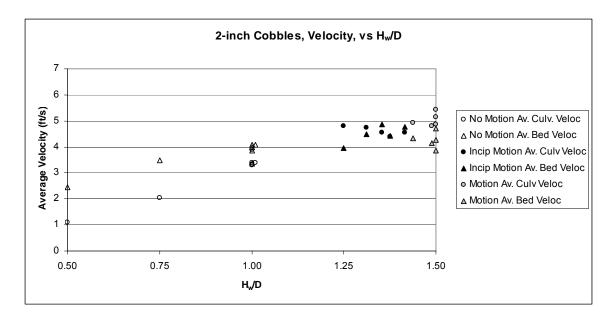


Figure 22. Velocity vs. H_w/D ratio for 2-inch cobbles.

angular gravel; the average H_w/D ratio when incipient motion occurred is approximately twice as large as the average H_w/D ratio when incipient motion occurred in the 0.75-inch angular gravel. The relationship between calculated and observed velocities and H_w/D ratios for this substrate material is summarized in Figure 23.

The critical velocity results for the 0.75-inch angular gravel, the 2-inch cobbles, and the 2-inch angular rock have been compared to the results of previous research studies on non-cohesive materials, collected by the Federal Highway Administration (Richarson, Simons, and Julien, 1990). This comparison is presented in Figures 24 and 25; Figure 24 uses the average measured bed velocity as a function of stone size and Figure 25 uses the average calculated culvert velocity as a function of stone size. Each figure has a series of parallel, diagonal lines which

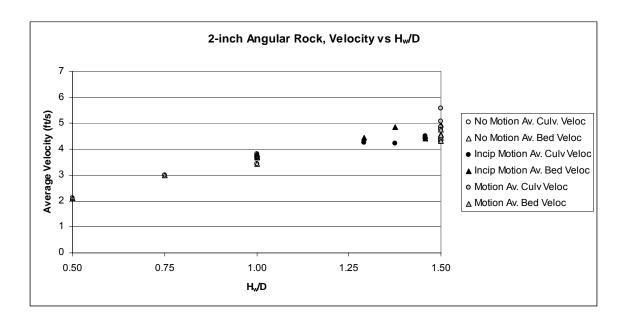


Figure 23. Velocity vs. H_w/D ratio for 2-inch angular rock.

are identified or named to represent the type of substrate tested or a particular application, or the author of the study (such as channel cobbles, small stilling basins, or Ishbash).

The results of average measured bed velocities for the 0.75-inch angular gravel substrate correspond to critical velocities for channel cobbles (see Figure 24). However, using the results of average culvert velocities results in lower values for incipient motion and correspond to critical velocities for small turbulent stilling basins (see Figure 25). The results of average measured bed velocities for 2-inch cobble substrate and 2-inch angular rock substrate correspond to critical velocities found by Ishbash. However, using the average culvert velocity results in a less conservative correlation with channel cobbles for the 2-inch angular rock, whereas the 2-inch cobbles continued to correspond with Ishbash.

Shields Relation for Beginning of Motion in a Bottomless Culvert

Due to the impracticality of quantifying shear stress in the field, shear stress was not measured during testing of the four substrate materials. Four methods were used to calculate shear velocities for each test at stations 0, 4, 8, 12, and 16; Equations (4) (Von Karman) and (5) (Prandtl with Einstein correction factor) are generally accepted as acceptable shear velocity approximations for one-dimensional and two-dimensional flows, respectively. Equations (4) and (5) were modified by utilizing a sidewall correction developed in rectangular laboratory flumes, Equation (17), resulting in two standard and two corrected shear

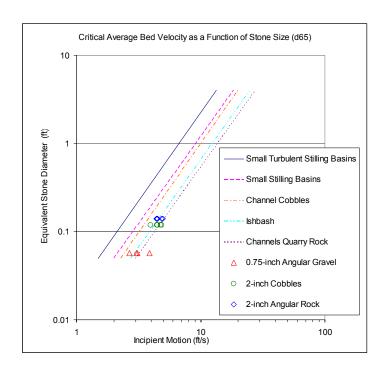


Figure 24. Critical average bed velocity as a function of stone size ‡.

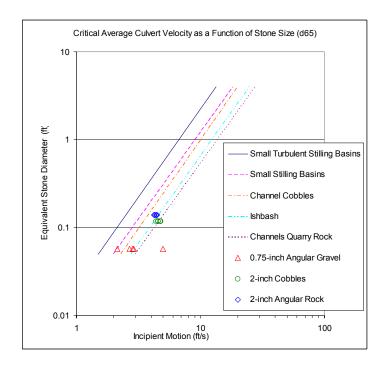


Figure 25. Critical average culvert velocity as a function of stone size‡.
‡adapted from Richarson, Julien, and Simons, 1990

velocities at a particular station and time during testing, which are presented in the following list:

- 1. Shields No Correction
- 2. Shields Correction
- Prandtl Einstein No Correction
- 4. Prandtl Einstein Correction.

For each method listed above, equation (2), Lotter Equation, was used to calculate shear stresses from corresponding shear velocities, which in turn were used in Equation (10) to calculate Shields parameters. Equation (12) was utilized to calculate corresponding Grain Reynolds' numbers to be plotted with Shields relation for the beginning of motion (previously presented in Figure 2). Equation (12) required the previously obtained shear velocities and the d₅₀ from the sieve analyses conducted on the substrate materials (see Figure 5 and Appendix D, Substrate Properties). The results of station 16, utilizing the Shields No Correction method for each test conducted on the four substrate materials are presented in Figure 26. For individual plots of each substrate material, see Appendix C, Shields Relation for Bottomless Arch Culverts. Figure 26 illustrates the calculated values of the Grain Reynolds' number and Shields parameter and is a compilation of the incipient motion tests and the duration tests, presenting the substrate behavior regarding no movement, incipient motion, and movement. Though it may be argued that the d_m or d₆₅ may be a more representative particle diameter, the d₅₀ was selected to correlate with Shields work. The results of each shear velocity method for all four substrate materials are presented in

Appendix C Shields Relation for Bottomless Arch Culverts. Figures 27 to 32 illustrate how well each method corresponded to Shields Relation for Incipient Motion at the culvert entrance and exit. The Grain Reynolds' number is placed on the abscissa, and the ratio of a methods' predicted Shields parameter to the Shields parameter predicted from Shields Relation (actual) pertains to the ordinate. Figure 27 to 28 corresponds to the 0.75-inch Angular Gravel, Figure 29 to 30 corresponds to the 2-inch Cobble Substrate, and Figure 31 to 32 corresponds to the 2-inch Angular Rock Substrate. There is no figure for the smallest material, due to the fact that incipient motion conditions were not quantified for the Pea Gravel Substrate. The results for stations 0, 4, 8, 12, and 16 are presented in Appendix D.

Despite the fact that Shields relation was developed using materials classified as sands, Shields Relation for Motion appears to apply to the pea gravel substrate (d_{50} =0.27 inches) due to the fact that all motion points exceeded the beginning of motion curve; calculated Shields parameters predicted motion, which was in agreement with observations during testing. Shields relation under predicted incipient motion of the 0.75-inch angular gravel substrate (d_{50} =0.64 inches), notwithstanding some of the calculated Reynolds' numbers exceeded the known range for this relation (R_e * \leq 1,000). As shown in Figures 27 and 28, there is scatter to the data and though there appears to be no clear separation between no movement, incipient motion, and movement, one must remember

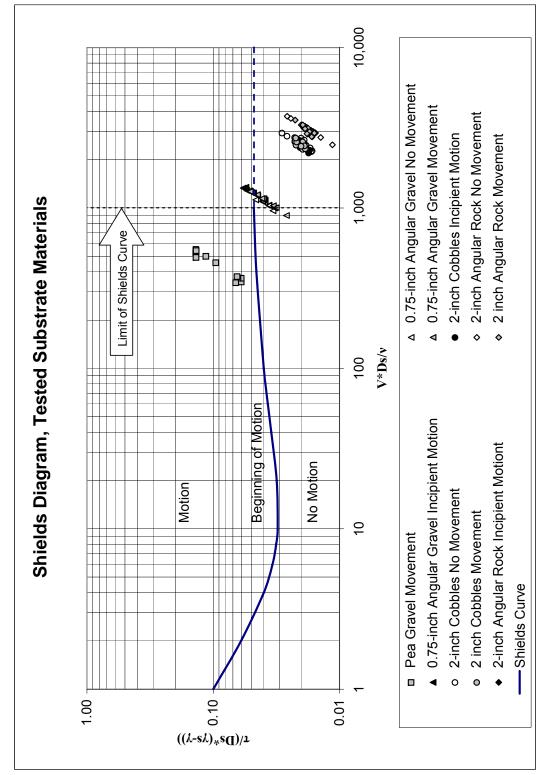


Figure 26. Four tested substrates in bottomless culvert plotted on Shields relation for incipient motion.

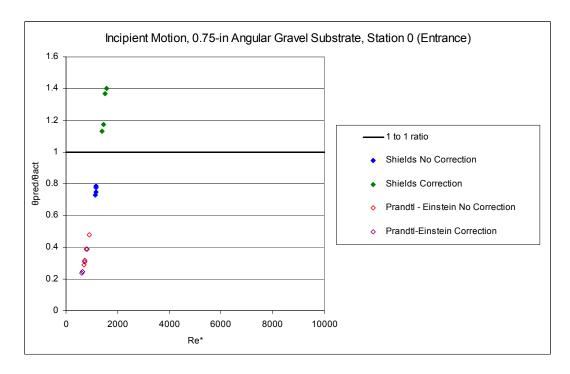


Figure 27. Method correlation to shields relation for incipient motion for 0.75-in angular gravel at the culvert entrance.

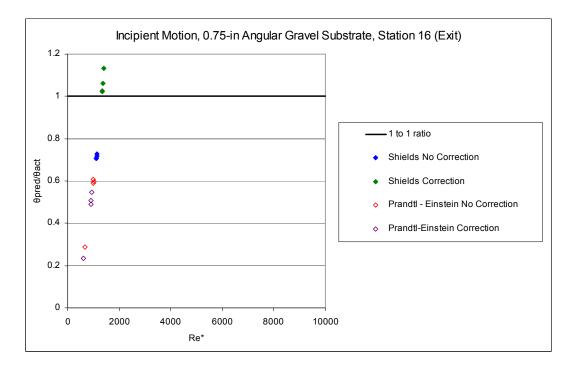


Figure 28. Method correlation to shields relation for incipient motion for 0.75-in angular gravel at the culvert exit.

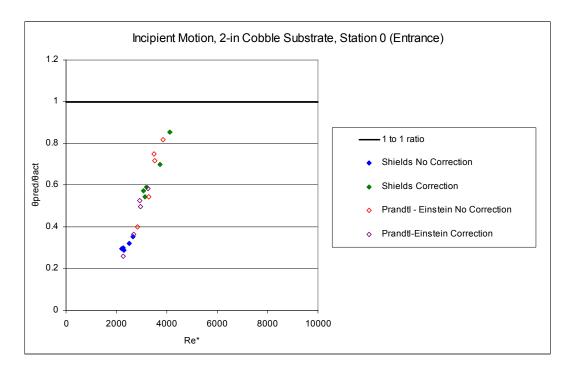


Figure 29. Method correlation to shields relation for incipient motion for 2-in cobble substrate at the culvert entrance.

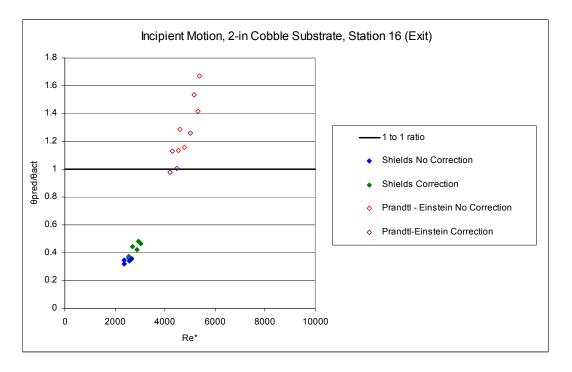


Figure 30. Method correlation to shields relation for incipient motion for 2-in cobble substrate at the culvert exit.

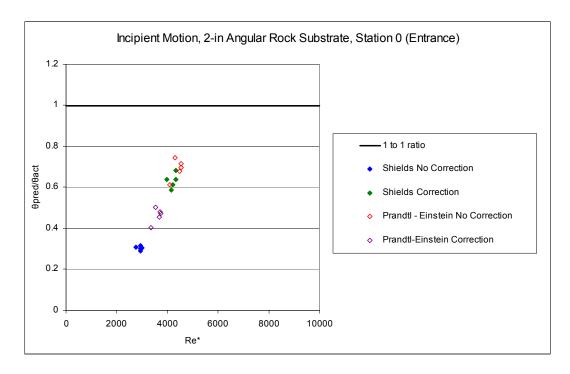


Figure 31. Method correlation to shields relation for incipient motion for 2-in angular rock substrate at the culvert entrance.

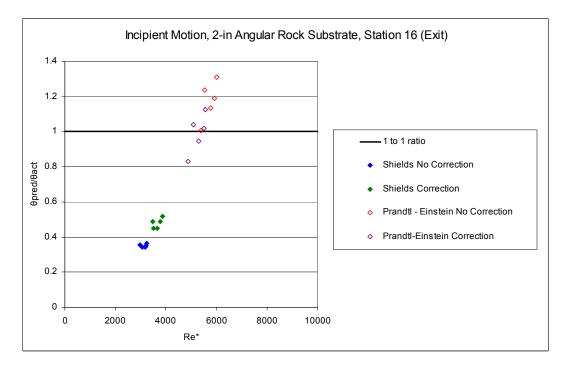


Figure 32. Method correlation to shields relation for incipient motion for 2-in angular rock substrate at the culvert exit.

The calculated Grain Reynolds' numbers for the 2-inch cobbles ranged from approximately 2,000 to 3,000, which exceeds the upper limit of published If Shields curve, data in the traditional Shields diagram (Re*\leq1,000). representing beginning of motion, is extrapolated to approach a Shields parameter of 0.06 as suggested by Shields (1936), Meyer-Peter and Müller (1948), and Gessler (1971) proposed from their data sets (0.047), then it appears that a one-dimensional method for calculating Shields parameter does not apply to this larger substrate material (d_{50} =1.30 inches) for this diameter of bottomless culvert. All methods under predicted incipient motion at the culvert entrance and inside the culvert barrel, both for pressurized and non-pressurized conditions. However, incipient motion was approximated at the culvert exit by means of the Prandtl-Einstein Correction method (two-dimensional flows, sidewall correction). It should be noted that the culvert exit was never submerged and that the point where the water surface detached from the culvert crown fluctuated between stations 4 and 14 during testing.

The calculated Reynolds numbers for the 2-inch angular rock resulted in even higher values than the 2-inch cobbles, ranging from 2,000 to 4,000. As shown in Figures 29 and 30, calculated prediction ratios range from 0.2 to 0.8 at the culvert entrance and inside the barrel, slightly larger than the values calculated for the 2-inch cobbles. Extrapolating the curve for beginning of motion as done previously, Shields relation does not apply to this substrate material $(d_{50} = 1.47 \text{ inches})$. However, the Prandtl-Einstein Correction method also approximated incipient motion at the culvert exit for this material.

A comparison of the incipient motion shear stresses of the 2-inch cobbles (0.21 lb/ft^2) and the 2-inch angular rock (0.20 lb/ft^2) revealed a minor difference in magnitude. The differences in the materials' shapes (angular vs. round) and gradation appear to have a small influences on critical shear stress values calculated during this study, which are not parameters included in the shear velocity equations utilized in this study.

Though it is generally understood that incipient motion is an event which is best represented by enveloping curves (a range instead of a thin line) on Shields relation for incipient motion, this method appears to best apply to smaller substrate materials, such as sands and fine gravels.

RECTANGULAR FLUME INCIPIENT MOTION RESULTS

General Velocity Observations

During data collection, several general observations were made regarding hydraulic conditions in the rectangular laboratory flume. First, the flow depth was not identical but very similar at each of the measurement stations, attributed to the gradually varied water surface profile and surface turbulence. Secondly, due to the width of the flume and the relative size of the impeller located at the sampling end of the velocity probe, no variation in bed velocities across a cross section were discernable, therefore the bed velocity at a station was sampled at the midpoint and assumed to be representative of the entire cross-section.

During testing in the rectangular flume, the tested substrate materials behaved similarly to testing conducted in the bottomless culvert; the same seating, vibrations, and shuddering effects noted prior to and at incipient motion conditions were observed. Generally, incipient motion was observed to occur simultaneously at stations 0, 4, and 8 (see Figure 6). Finally, measured velocities were observed to fluctuate in magnitude. Generally, these fluctuations were ± 0.5 ft/s (0.152 m/s) or less, and may be a result of general flow turbulence.

Incipient Motion Velocities

The standard established for incipient motion in the bottomless culvert was also applied to the laboratory flume. Incipient motion was interpreted as the commencement of sporadic movement of particles of substrate, occurring at an

average rate of several times per minute. When incipient movement was believed to occur, bed velocity measurements were taken. Also, flow conditions, such as flow rate and flow depths, were recorded to calculate an average crosssectional velocity at each location along the length of the rectangular flume (see Figure 6). The same procedure to verify incipient motion conditions in the bottomless culvert was also followed during testing in the rectangular flume. To verify that incipient motion was occurring and not a prelude seating effect, the upstream flow depth was slightly increased after measurements were taken by means of increasing the flow rate to the flume. If this small incremental increase resulted in an increased rate of sporadic movement, measured critical velocities were confirmed to be accurate. No duration tests were conducted within the rectangular flume, facilitating the duplication of test runs. Once an incipient motion test was concluded, the flume was prepared and the test repeated to observe variability associated with incipient motion. Also, as described previously, the slope of the flume (bed slope) was not a constant parameter, as was implemented during testing of the bottomless culvert. The three slopes that were tested resulted in three general groupings for incipient motion.

An example of the influence of slope for the pea gravel substrate is presented in 33; as shown, small differences existed between computed average water column velocities and measured velocities at the stream bed. Also, there is not a linear relationship between flow depth and velocity; as velocities became larger and larger, the incremental increase in flow depth became less and less. For a bed slope of 0.005 ft/ft, incipient motion velocities occurred within the range

of 2.5 and 3.0 ft/s, with the flow depths not exceeding 6 inches. For a slope of 0.01 ft/ft, the flow depth decreased significantly with incipient motion velocities occurring at approximately 2.5 ft/s. An increase of bed slope to 0.015 ft/ft also resulted in incipient motion velocities of approximately 2.5 ft/s and measured flow depths of approximately 3 inches.

Testing of the 0.75-inch angular gravel substrate found minor differences between average water column velocities and velocities measured at the streambed. However, the influence of bed slope affected incipient motion velocities differently than observed in the pea gravel substrate. There is a smaller distinction or data cluster separation for bed slopes of 0.005 ft/ft and 0.01 ft/ft. For a bed slope of 0.005 ft/ft and 0.1 ft/ft, incipient motion velocities occurred approximately at 5 ft/s, with the flow depth being the largest measured, at approximately 1.1 inches (see Figure 34). A bed slope to 0.015 ft/ft resulted in incipient motion velocities of approximately 4.5 ft/s and measured flow depths of approximately 9-inches.

Incipient motion for the 2-inch cobble substrate did not begin until the depth of flow exceeded 1 ft, with velocities ranging from 5 ft/s to 6 ft/s. The influence of bed slope resulted in three vertical bands or clusters of data points, the steepest slope oriented left or with lower flow depths, and the mildest slope oriented to the right (largest flow depths). Velocities and flow depths prior to and during incipient motion are presented in Figure 35.

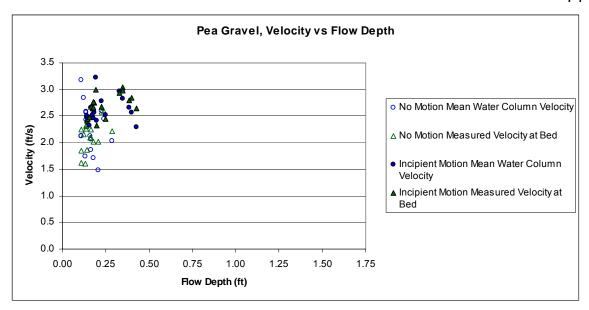


Figure 33. Method correlation to shields relation for incipient motion for pea gravel substrate at the culvert exit.

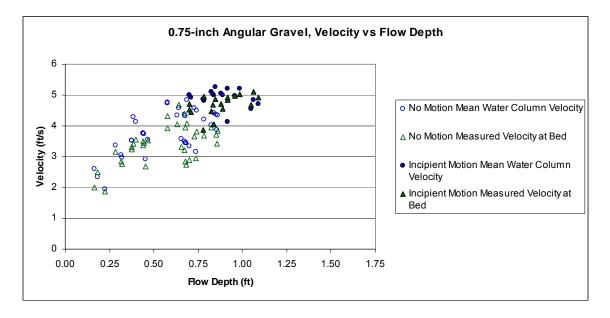


Figure 34. Method correlation to shields relation for incipient motion for 0.75-in angular gravel substrate at the culvert exit.

Incipient motion conditions were not obtained for the 2-inch angular rock substrate, see Figure 36. The maximum flow rate available to the flume is

approximately 8 cfs, limited by the size of the orifice plate utilized for flow measurements. In addition, flow depths exceeding 1.75-ft would overtop the flume walls. However, testing did determine that flow velocities must exceed 1.5 ft/s for each bed slope tested. When juxtaposed with the 2-inch cobble data, the differences in particle shape and material gradation appear to provide a difference of at least 0.5 ft/s for incipient motion velocities, underscoring the influence of the physical characteristics of a substrate material on incipient motion velocities and scour.

Shields Relation for Beginning of Motion in a Rectangular Flume

The methods used to calculate Shields parameter and the Grain

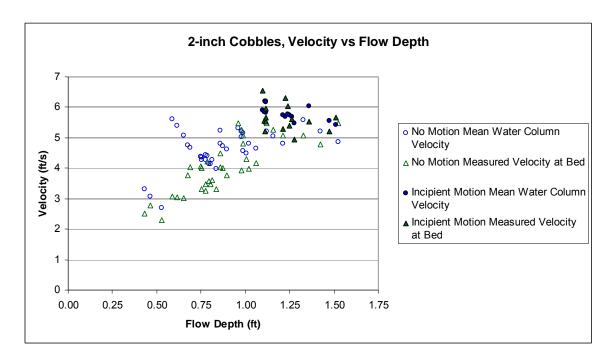


Figure 34. Method correlation to shields relation for incipient motion for 2-in cobble substrate at the culvert exit.

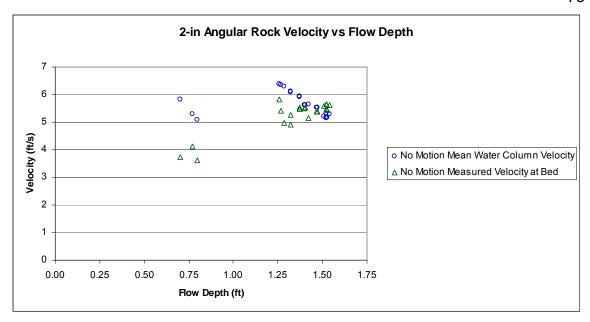


Figure 35. Method correlation to shields relation for incipient motion for 2-in angular rock substrate at the culvert exit.

Reynolds number in the bottomless culvert were also used to evaluate incipient motion in the rectangular laboratory flume. The four methods used to calculate shear velocities for each test at stations 0, 8, and 12 are presented below.

- 1. Shields No Correction
- 2. Shields Correction
- 3. Prandtl Einstein No Correction
- 4. Prandtl Einstein Correction

Equation (2), Lotter Equation, was used to calculate shear stresses from corresponding shear velocities, which in turn were used in Equation (10) to calculate Shields parameters. Equation (12) was utilized to calculate corresponding Grain Reynolds' numbers to be plotted with Shields relation for the beginning of motion. Equation 12 required the previously obtained shear

velocities and the d₅₀ from the sieve analyses conducted on the substrate materials (see Figure 5 and Appendix D, Substrate Properties). The results of all three stations, utilizing the Prandtl-Einstein (PE) Correction Method for each test conducted on the four substrate materials are presented in Figure 37. For individual plots of each substrate material, see Rectangular Flume Shields Relation, Appendix D. Figure 37 illustrates the calculated values of the Grain Reynolds' number and Shields parameter presenting the substrate behavior regarding no movement and incipient motion.

Figures 38 to 40 illustrate how well each method corresponded to Shields Relation for Beginning of Motion in the rectangular flume. The Grain Reynolds' number is placed on the abscissa, and the ratio of a methods' predicted Shields parameter to the Shields parameter predicted from Shields Relation (actual) pertains to the ordinate. Figure 38 corresponds to the pea gravel substrate, Figure 39 corresponds to the 0.75-inch angular gravel substrate, and Figure 40 corresponds to the 2-inch cobble substrate. There is no figure for the 2-inch Angular Rock substrate, due to the fact that the capacity of the flume was reached before the development of incipient motion.

The Shields Correction method most poorly predicted Shields curve for the pea gravel substrate. Grain Reynolds numbers were approximately between 200 and 400. The Shields method and the corrected and non-corrected Prandtl-Einstein methods approximated Shields curve with approximately ±20%. The Prandtl-Einstein Corrected method best approximated Shields curve for the 0.75-in angular gravel substrate, but consistently over predicted Shields parameter by

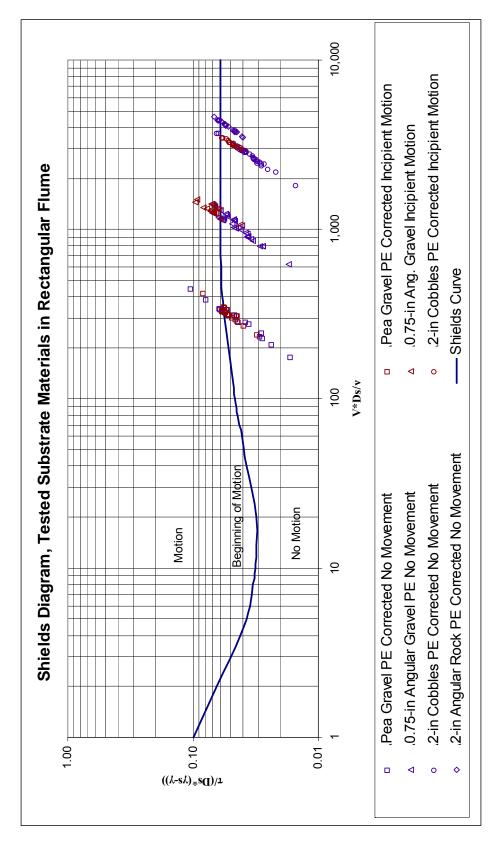


Figure 37. Four tested substrates in rectangular flume plotted on Shields relation for incipient motion.

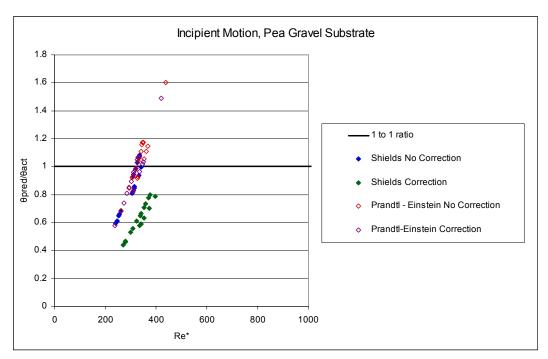


Figure 38. Method correlation to shields relation for incipient motion for pea gravel substrate in the rectangular flume.

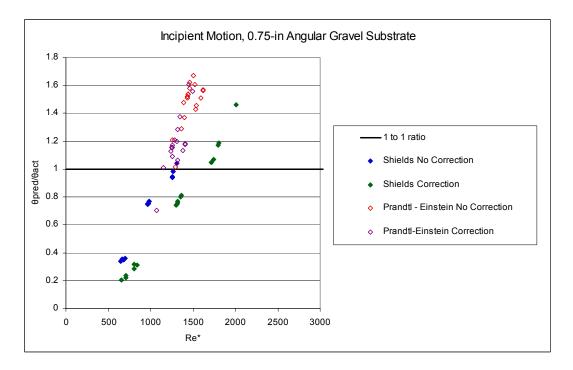


Figure 39. Method correlation to shields relation for incipient motion for 2-in cobble substrate in the rectangular flume.

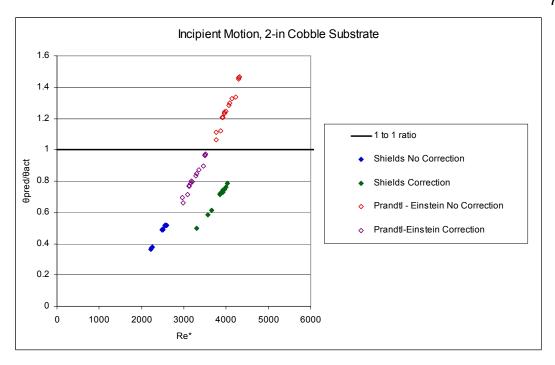


Figure 40. Method correlation to shields relation for incipient motion for 2-in angular rock substrate in the rectangular flume.

20%, on average. Finally, both Prandtl-Einstein methods best approximated Shields curve. Applying the sidewall correcting resulted in under-predicting Shields parameters, without the Prandtl-Einstein method over-predicted; the amount of error is shown in Figure 40.

SCOUR IN A BOTTOMLESS ARCH CULVERT

Bottomless Culvert General Scour Observations

For engineering purposes, there are two sources of sediment transported by a natural stream: bed material that composes the streambed and the fine material from the banks and watershed (Rahmeyer, 1989). Material moving along the streambed (sliding, rolling, saltating) shall be referred to as bed load. Suspended materials e.g., (dissolved material, clay, silt, and fine sands) shall be referred to as washload. In this test facility, there was no appreciable washload present except during testing of the 0.75-inch angular gravel and the 2-inch cobbles (testing occurred during spring runoff). During spring runoff, water was an opaque brown and visibility was low. Testing of the other two substrate materials was conducted in clear water conditions with high visibility. Yang and Simões (2005) suggests that suspended or washload materials can influence scour, however, the effects of washload were assumed negligible for this study and were not a controlled parameter.

The following discussion summarizes the general scour or bed response observations of the four tested substrate materials inside the bottomless arch culvert.

Substrate particles for all tested materials were observed to move by means of rolling and saltating along the streambed. At higher flow rates, the pea gravel particles did not exclusively experience brief suspension at the culvert entrance where flow contraction and vortices were observed, but also at random

locations inside the culvert barrel. No suspension was observed for the other three substrate materials.

For all tested substrate materials, scour was most severe at the entrance and exit of the culvert, however, for certain test scenarios particle movement was observed along the entire length of the culvert. Local scour holes were observed to occur at the edges of the culvert entrance, and generally were conical in shape (see Figure 41). Material removed from the culvert entrance was generally deposited between stations 1 and 6 (see Figure 4) and at cross-section locations B, C, and D (see Figure 16). Particles transported inside the culvert barrel for the 0.75-inch angular gravel, 2-inch cobbles, and 2-inch angular rock often resulted in the deepest scour depths near the middle of the cross section (see Figure 16) and the maximum overall depth of scour at or near the culvert exit. As the flow exited the culvert, material in the tailbox was removed and a channel was created with material being deposited along the edges (see Figure 42), or in the more resilient substrate materials, an oval scour hole was formed (see Figure 43).

Finally, test scenarios that included pressurized inlet conditions resulted in an unstable or fluctuating point where the water surface detached from the culvert crown. As previously mentioned, the detachment location would fluctuate between stations 4 and 14 with residence times ranging approximately 2 to 5 minutes before shifting.

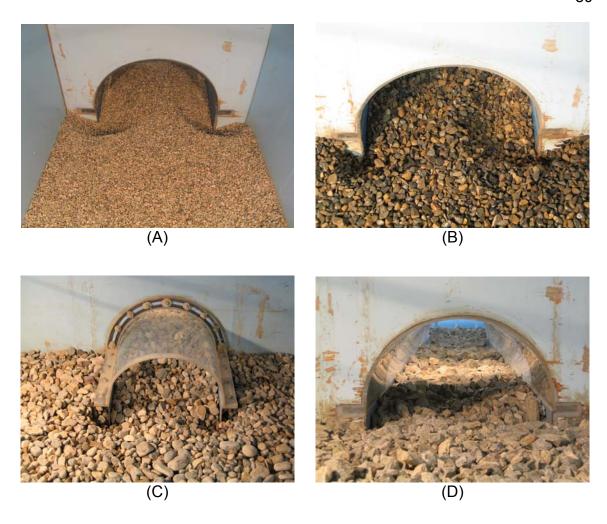


Figure 41. Examples of scour at entrance of culvert for pea gravel (A), 0.75-inch angular gravel (B), 2-inch cobbles (C), and 2-inch angular rock (D).

Extent of Scour

The pea gravel, the 0.75-inch angular gravel, the 2-inch cobbles, and the 2-inch angular rock substrate material are classified as armored beds due to the absence of finer materials, such as sands and silts. The three larger substrate materials are also classified as plane or flat bedforms; the flat graded bed



Figure 42. Example of scoured channel at culvert exit with bank deposition.

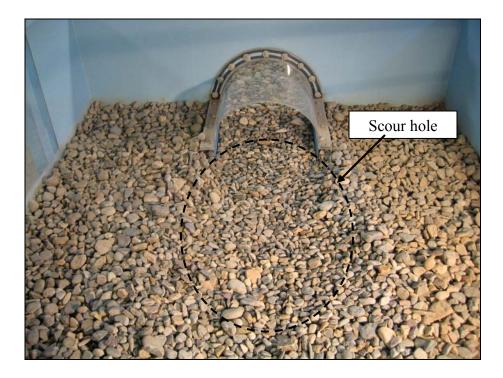


Figure 43. Example of oval scour hole at culvert exit.

created prior to the commencement of each test did not metamorphose into other bedform types. However, bedforms were observed to form in the pea gravel; therefore it is classified as a moveable bed, constantly changing in response to the hydraulic conditions.

For entrance configurations with a 33% or 75% contraction, the pea gravel substrate particles were briefly suspended in vortices at the edges of the culvert entrance (locations A and E), resulting from flow contraction. Bedforms developed parallel to the flow (longitudinal) at H_w/D = 0.5. Testing at Hw/D = 1 (stopped after 30-min) resulted in the formation of antidunes, as shown in Figure 44. Antidunes form as a series or train of in-phase (coupled) symmetrical sediment and water waves (Rahmeyer, 1989). They gradually build up from a plane bed and a plane water surface. Although antidunes can remain stationary or migrate up or downstream, in this study antidunes were observed to form at the exit of the culvert and migrate upstream (see Figure 44). The average extent of scour for each individual test is presented in General Scour Data, Appendix C.

As expected, the depth of scour was observed to increase as the H_w/D ratio was increased. Due to the accelerated rate of scour for H_w/D ratios larger than 0.5 for the pea gravel substrate, duration tests were confined to 0.5 H_w/D . The rate of scour decreased with time during the 2-hour duration tests, resulting in little to no movement of material at the conclusion of the tests (approximate equilibrium). The average depth of scour at stations 0, 4, 8, 12, and 16 of the culvert (average depth of points A through E) for each test and the corresponding H_w/D ratio are presented in Figure 45. The average culvert scour depths that



Figure 44. Example of antidunes in pea gravel substrate.

occurred at $0.5~H_w/D$ ranged from 0 to 4 inches with a large fraction less than 2 inches. However, actual measured depths of scour at each location ranged from 0 to 5.25 inches. Apart from general bed degradation, local pier-type inlet scour was observed to occur for the 33% and 75% contraction ratios. Measured depths of scour are presented in Streambed Response in Bottomless Arch Culvert, Appendix E.

The results of the average amount of scour that occurred at the culvert inlet and outlet (stations 0 and 16) and the corresponding average culvert velocities for testing of the pea gravel substrate are presented in Figure 46. As scour occurred, the local velocities would slowly decrease until the bed material became stable and reached equilibrium. For each test, the maximum depths of

scour and corresponding velocities at the inlet and outlet of the culvert are presented in Figure 47.

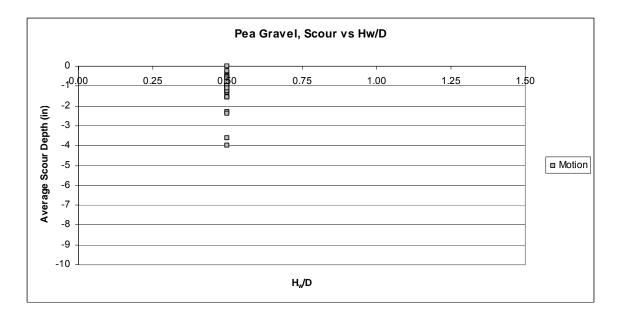


Figure 45. Average scour vs. H_w/D ratio for pea gravel substrate.

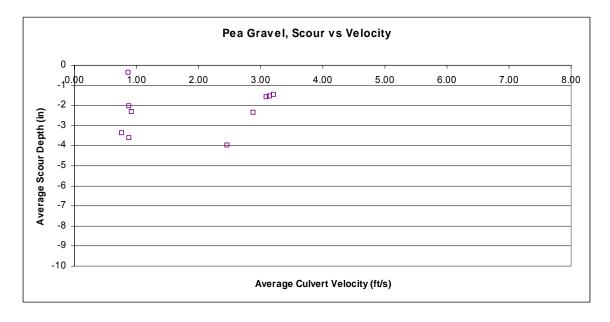
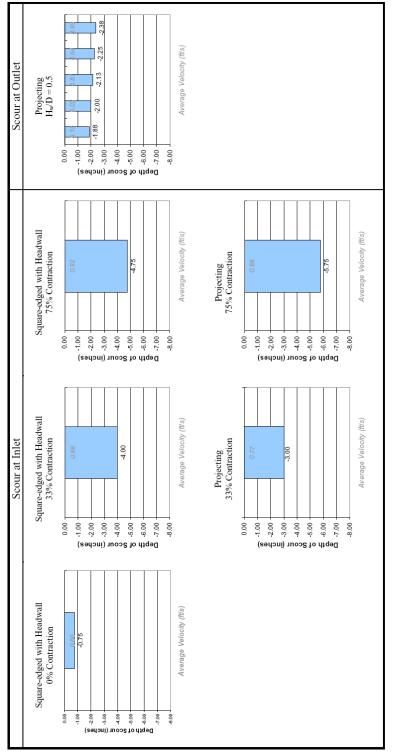


Figure 46. Average inlet/outlet scour vs. corresponding average culvert velocities for pea gravel substrate.



0.0 0.0 0.5 0.0 0.0

Figure 47. Max depth of scour at inlet and outlet for pea gravel.

The 0.75-inch angular gravel substrate, as expected, was more resistant to scour than the pea gravel substrate. Substrate particles moved by sliding or saltating; no particles were observed to become suspended. Local scour holes, though smaller than those occurring during testing of the pea gravel substrate, were formed at the entrance of the culvert in response to the entrance configurations with a 33% or 75% inlet contraction. In addition, maximum depths of scour occurred at the exit region of the culvert and flow exiting the culvert scoured a shallow channel in the tailbox (see Figure 48). The bedform inside the culvert remained a flat plane bedform during testing. Average scour depths at each station were observed to increase as the H_w/D ratio was increased; average scour at stations 0, 4, 8, 12, and 16 are presented in Figure 49.

The time duration for exploratory duration tests of larger H_w/D ratios was truncated due to the intensity and rate of scour; testing at 1.25 H_w/D ratio was 70 minutes, resulting in an average depth of scour of 6 inches at the exit. The time duration for the 1.5 H_w/D ratio was 11 minutes, resulting in average depths of scour exceeding 8 inches at the exit. Therefore, testing of this substrate material focused on 1.0 H_w/D with a 2-hour testing period; average depths of scour ranged 1.0 to 3.25 inches at the entrance and 3.0 to 4.0 inches at the exit of the culvert. Measured depths of scour are summarized in Depths of Scour Measurements, 0.75-inch Angular Gravel, Appendix C. The average culvert velocities associated with depths of scour for this material at stations 0 and 16 are presented in Figure 50. For each test, the maximum depths of scour and

corresponding velocities at the inlet and outlet of the culvert are presented in Figure 51.



Figure 48. Scour of 0.75-inch angular gravel looking upstream from exit, 1 H_w/D.

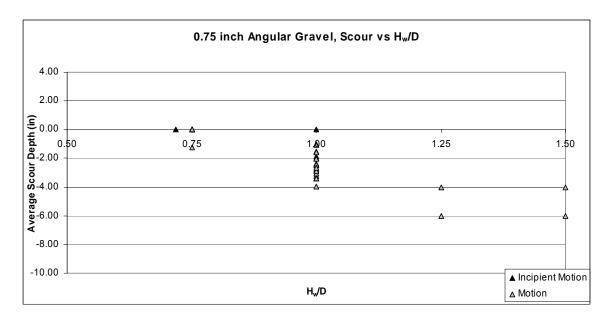


Figure 49. Average Scour vs. H_w/D ratio for 0.75-inch angular gravel substrate.

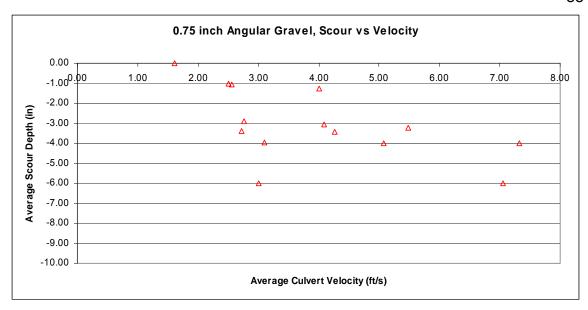
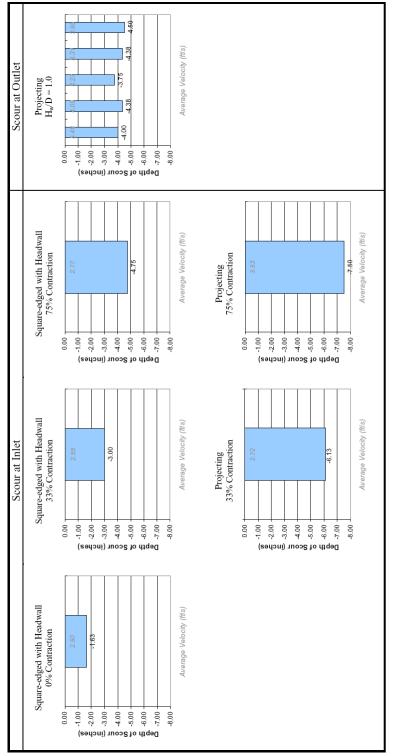


Figure 50. Average inlet/outlet scour vs. corresponding average culvert velocities for 0.75-inch angular gravel substrate.

The 2-inch cobble substrate particles were observed to roll in clusters or groups and not individually, which is attributed to the rounded shape of the substrate and interlocking of the particles. Incipient motion was observed to occur between 1.25≤H_w/D≤1.42. No appreciable amount of scour was observed to occur below 1.4 H_w/D. Measured depths of scour are summarized in Depths of Scour Measurements, 2-inch Cobbles, Appendix C. A summary of the average depths of scour that occurred at tested H_w/D ratios is presented in Figure 52.

Small localized scour holes at the entrance of the culvert began to form at an H_w/D ratio of 1.0, generally of a depth equal to one particle diameter. Larger scour holes formed at 1.5 H_w/D , depositing material downstream between



Contraction 0% 33%

Figure 51. Max depth of scour at inlet and outlet for 0.75-inch angular gravel substrate.

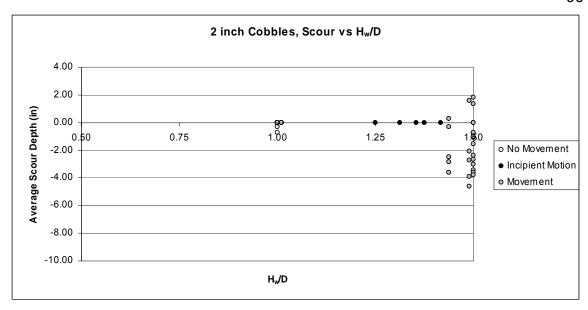


Figure 52. Average Scour vs. H_w/D ratio for 2-inch cobble substrate.

stations 1 and 6. The most considerable scour occurred at the exit of the culvert, generally beginning at station 16 and migrating upstream (see Figure 52). This scour was observed to be general bed scour and not localized scour since degradation was observed across the entire width of the bed. However, during pressurized test scenarios the streambed profile had a slight wave due to the water surface profile and the deposition of substrate material from the culvert inlet. During scour, bed velocities measured at station 16 ranged from 4.6 ft/s to 6.0 ft/s, with average culvert flow velocities ranging from 5.1 ft/s to 6.8 ft/s. For a summary of velocities during testing of 2-inch cobbles, see 2-inch Cobble Velocity Results, Appendix A. The relationship between average culvert velocities and the average amount of scour that occurred at the culvert entrance and exit (stations 0 and 16) is presented in Figure 53.

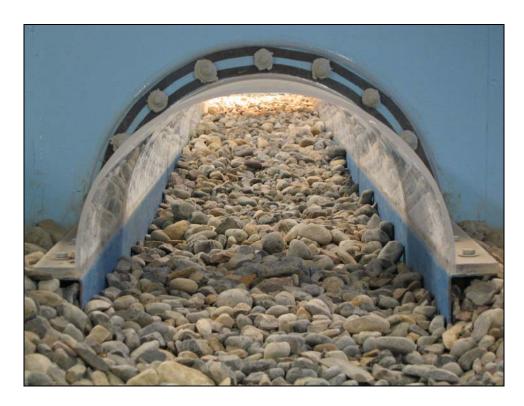


Figure 53. Scour of 2-inch cobbles looking upstream from exit.

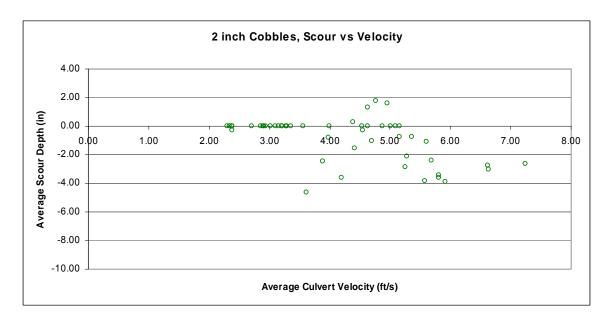
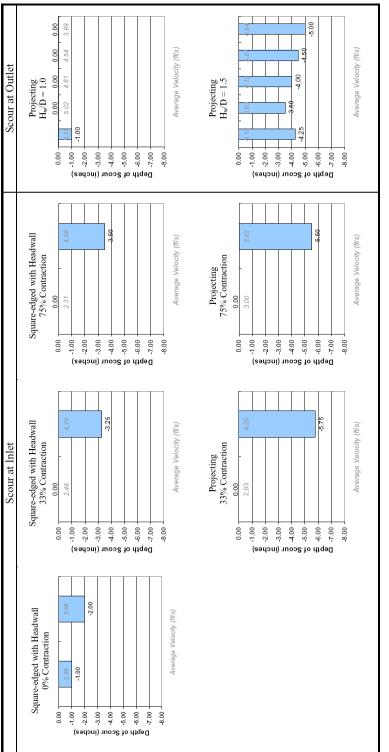


Figure 54. Average inlet/outlet scour vs. corresponding average culvert velocities for 2-inch cobble substrate.

As shown in Figure 54, appreciable scour depths began to occur when the average culvert velocity approached or exceeded 3.75 ft/s. These velocities were observed to occur at the exit of the culvert. Corresponding average culvert velocities at the entrance of the culvert ranged from 2.5 ft/s to 3.5 ft/s. These velocities were insufficient to cause appreciable scour except for two entrance conditions, 33% contraction projecting and 75% contraction projecting. These two entrance conditions resulted in flow vortices at cross-section locations A and E (see Figure 12 and 13), resulting in measured bed velocities at each point ranging from 3.5 ft/s to 4.2 ft/s. For each test, the maximum depths of scour and corresponding velocities at the inlet and outlet of the culvert are presented in Figure 36. These results illustrate that bed velocities may be larger at locations of potential scour (such as at entrance contractions or exiting jets) than average flow velocities and should be taken into consideration. Areas of potential scour are larger if pressurized flow conditions are likely to occur and it may be beneficial to use larger materials at such locations and smaller material where acceptable.

The substrate material that was the most resistant to scour was the 2-inch angular rock. Movement was observed to commence at slightly higher H_w/D ratios (1.3 to 1.5) than the 2-inch cobble substrate. Though incipient motion conditions were similar for both substrates, the extent of scour that occurred was much less for the angular bed material. A comparison of the most pronounced scour that occurred during testing of the 2-inch cobbles and 2-inch angular rock is given in Figure 56. The results show that for the 2-inch angular substrate, 50%



		Inlet			Projecti	Projecting Outlet
	% Contraction	Q/ ^M H	Velocity (ft/s)	Scour Depth (in)	Velocity (ft/s)	Scour Depth (in)
	/60	1.0	2.38	-1.00	5.15	-1.00
	0%0	1.5	3.98	-2.00	6.12	-4.25
Same adread with Handwall	338/	1.0	2.46	0.00	5.02	00.00
square-eugeu with Headwan	3370	1.5	4.76	-3.25	6.63	-3.50
	/052	1.0	2.39	0.00	4.81	0.00
	13%	1.5	4.68	-3.50	6.12	-4.00
	330/	1.0	2.93	0.00	4.54	00.00
Decipation	3370	1.5	4.20	-5.75	5.61	-4.50
Fiolecing	/052	1.0	3.00	0.00	3.99	00:00
	1370	1.5	3.62	-5.50	6.64	-5.00

Figure 55. Max depth of scour at inlet and outlet for 2-inch cobble substrate.

of the bed inside the culvert remained unchanged whereas 100% of the bed composed of the 2-inch cobbles was altered. Measured depths of scour are summarized in Depths of Scour Measurements, 2-inch Angular Rock, Appendix C. The range of average depths of scour was from 0 to 4 inches, with a maximum deposition depth of approximately 1.75 inches, see Figure 57. The area where scour was most severe was at the culvert exit, at locations B, C, and D, see Figure 58.

The bed velocities, which were measured when incipient motion occurred, ranged from 3.5 ft/s to 4.2 ft/s at the entrance and 4.5 ft/s to 6.0 ft/s at the exit.

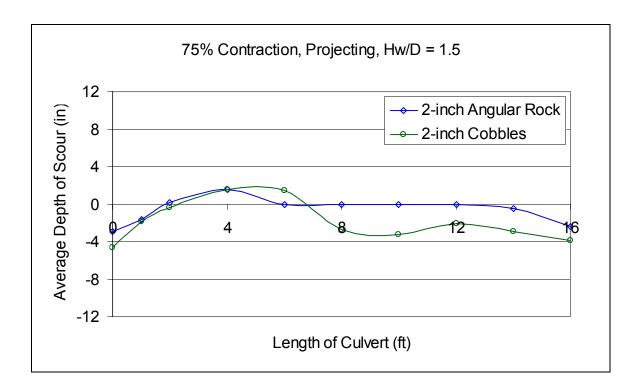


Figure 56. Comparison of extent of average cross-sectional scour in 2-inch angular rock and 2-inch cobbles.

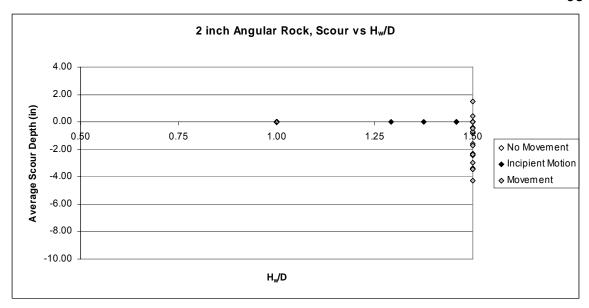


Figure 57. Average Scour vs. H_w/D ratio for 2-inch angular rock substrate.

Average culvert velocities and the depth of scour that occurred are presented in Figure 59. Generally, scour would become appreciable when average culvert velocities exceeded 4.5 ft/s.

Though the 2-inch cobble substrate and 2-inch angular rock had similar d_{50} sizes, reached incipient motion under similar H_w/D ratios, attained similar maximum depths of scour inside the culvert barrel, and similar velocities were present during movement, two influential characteristics created a relatively large difference in the extent of scour inside the culvert, angularity and gradation. A much larger percentage of the 2-inch angular substrate inside the culvert remained unchanged compared to the 2-inch cobbles. For each test, the maximum depths of scour and corresponding velocities at the inlet and outlet of the culvert are presented in Figure 60. The scour results from these two

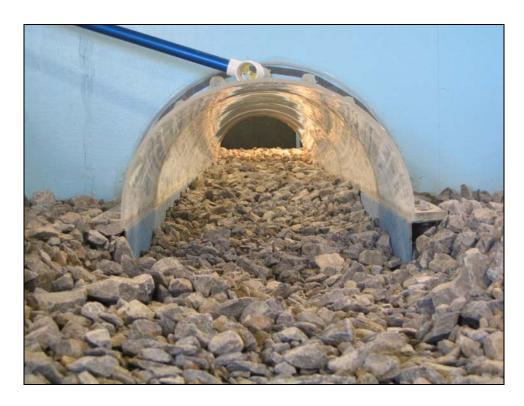


Figure 58. Scour of 2-inch angular rock looking upstream from exit.

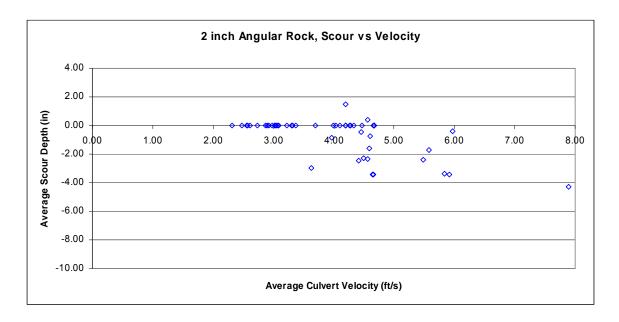
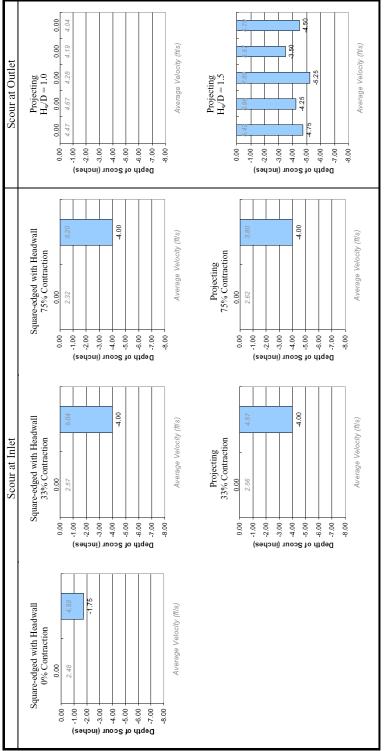


Figure 59. Average inlet/outlet scour vs. corresponding average culvert velocities for 2-inch angular rock substrate.



		Inlet			Projecti	Projecting Outlet
	% Contraction	H _w /D	Velocity (ft/s)	Scour Depth (in)	Velocity (ft/s)	Scour Depth (in)
	/00	1.0	2.48	0.00	4.47	0.00
	0%0	1.5	4.50	-1.75	5.42	-4.75
Compact advantage Hooder	7000	1.0	2.57	0.00	4.67	0.00
oduaie-eugeu wiiii Heauwaii	33%	1.5	5.04	-4.00	96.9	-4.25
	/63F	1.0	2.32	0.00	4.28	0.00
	1370	1.5	5.20	-4.00	7.85	-5.25
	7900	1.0	2.56	0.00	4.19	00.00
Designation	33%	1.5	4.57	-4.00	5.57	-3.50
giingafaia	/852	1.0	2.62	0.00	4.04	00.00
	1370	1.5	3.80	-4.00	5.70	-4.50

Figure 60. Max depth of scour at inlet and outlet for 2-inch angular substrate.

substrates suggest that the most resilient material to scour would be a relatively large, angular, well-graded material. However, angular material may not be ecologically acceptable, less practical, or more costly for streambed simulation as naturally occurring streambed material is generally rounded.

STONE SIZING ANALYSES

A means for scaling the experimental incipient motion results from this study to larger bottomless culvert applications would be beneficial, as the use of a 2-ft (0.61-m) diameter bottomless culverts is unlikely for fish passage; larger bottomless culverts would be more effective and practical. For example, the State of California Department of Fish and Game specify a minimum culvert diameter of 6-ft for their streambed simulation design method. Consequently, a literature review produced eight riprap sizing techniques. The first two techniques listed below were developed for bottomless culverts; the other six methods were developed for open channel applications.

- 1. FHWA Phase 1
- 2. FHWA Phase 2
- 3. Halvorson
- 4. HEC-11
- 5. Cal B&SP
- 6. USACE EM-1601
- 7. ASCE Manual 54
- 8. USBR EM-25

Two recently published FHWA methods (Equations (18) and (19)) and six well-established stone or riprap sizing methods (Equations (20)-(25)) were evaluated based on their ability to predict the minimum particle or stone size required for a non-erodible substrate, using the experimentally determined

average cross-sectional bed velocity at incipient motion as the independent variable. The objective was to evaluate the applicability of Equations (18) though (25) as potential size-scaling relationships for bottomless culvert substrate stone sizing. The results of the predicted and experimental stable d_s values are presented in terms of a factor of safety, which is defined as the ratio of d_s predicted over d_s actual. Factors of safety (SF) for each substrate material tested in both the bottomless culvert and the rectangular flume were calculated. For the bottomless culvert, Figures 61 – 63 present the calculated factors of safety (SF) and the corresponding H_w/D ratios. Figures 64 – 66 present the calculated factors of safety and corresponding flow depths for the rectangular flume. More detailed information is presented in Appendix F.

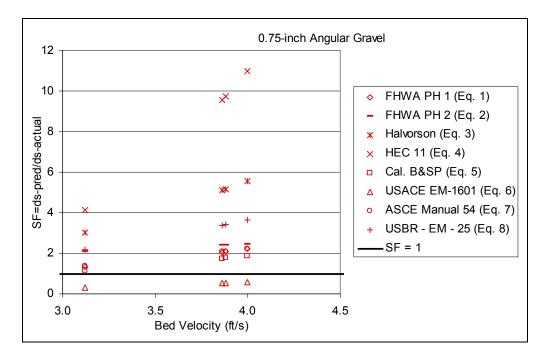


Figure 61. Comparison of riprap safety factors for 0.75-inch angular gravel in a bottomless arch culvert.

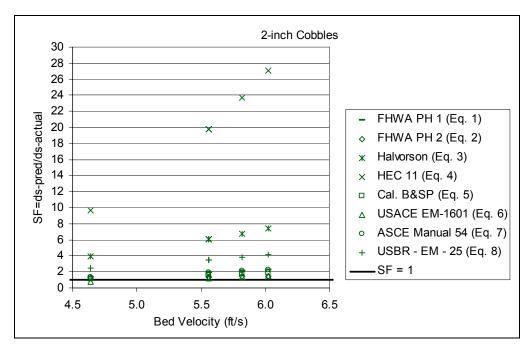


Figure 62. Comparison of riprap safety factors for 2-inch cobbles in a bottomless arch culvert.

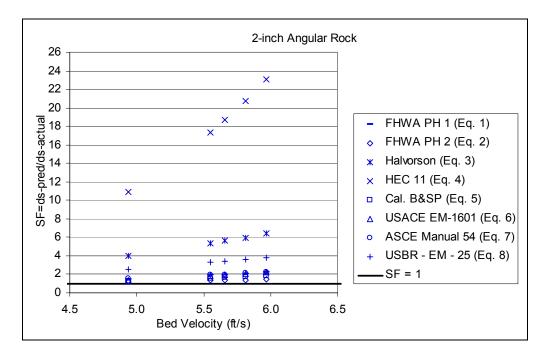


Figure 63. Comparison of riprap safety factors for 2-inch angular rock in a bottomless arch culvert.

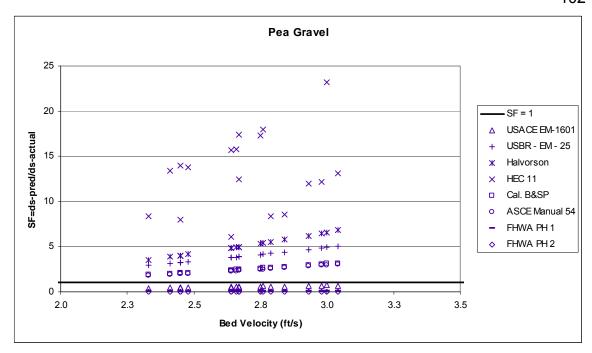


Figure 64. Comparison of riprap safety factors for pea gravel in a rectangular flume.

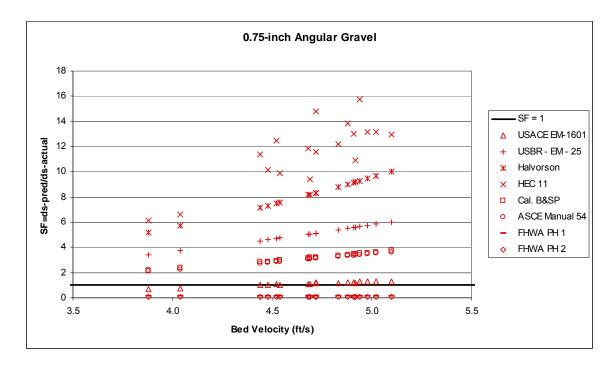


Figure 65. Comparison of riprap safety factors for 0.75-inch angular gravel in a rectangular flume.

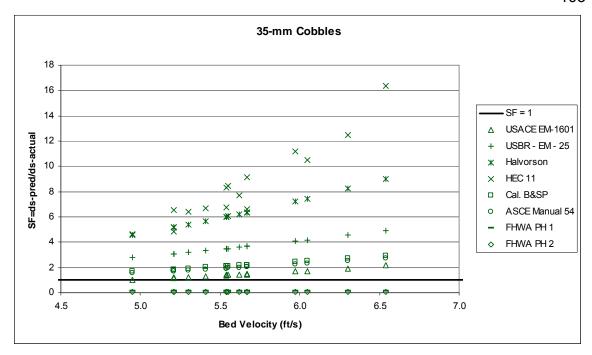


Figure 66. Comparison of riprap safety factors for 2-inch cobbles in a rectangular flume.

As can be seen from comparing Figures. 64 to 66, the performance of some of the riprap stone sizing methods are application specific (i.e., bottomless culvert vs. the rectangular flume). In general, SF increased with increasing velocity for most methods in both the bottomless culvert and the rectangular flume.

The USACE-EM-1601 riprap sizing method (Equation 23) under predicted stone sizes for the smaller substrate materials (Pea gravel and 0.75-inch angular gravel), with SF ranging from 0.8 to 1 for the culvert and 0.5 to 1.5 for the rectangular flume data. For the larger substrate materials (2-inch cobbles and 2-inch angular rock), SF ranged from 1 to 2 for the culvert and flume data. It is important to note that in order to determine a d_{30} riprap stone size distribution

using this method, flow velocity and flow depth data are required, along with five condition-specific empirical coefficients, which must be determined or estimated.

The USBR-EM-25 riprap sizing method (Equation 25) produced *SF* values ranging from approximately 3 to 6 in the laboratory flume and 2 to 4 for the 2-inch cobbles and the 2-inch angular rock. The Halvorson riprap sizing method (Equation 20) produced SF values ranging from approximately 4 to 10 in the laboratory flume and approximately 4 to 6 in the 2-inch cobbles and 2-inch angular rock substrate materials in the bottomless culvert. The large SF values suggest that this method may be overly conservative.

HEC 11 (Equation 21) was the most conservative method (SF often exceeding 10) and produced the most scatter, with the stone size increasing sharply with increasing velocity. This method may be too conservative for bottomless culvert applications. The Cal B&SP riprap sizing method (Equation 22) predicted SF values ranging from approximately 2 to 4 in the laboratory flume, but only 1 to 2 in the bottomless culvert. The ASCE Manual 54 riprap sizing method (Equation 24) produced very similar results to the CAL B&SP stone prediction method.

The results of each FHWA method were very similar and predicted factors of safety comparable to ASCE Manual 54 and Cal B&SP methods in the bottomless culvert. The FHWA methods, however, under predicted stone sizes in the laboratory flume and therefore appear to be specific to culvert applications. When selecting a stone sizing method for an application (e.g., bottomless culvert, trapezoidal channel, etc.) it is important to have a general sense of performance

as some appear to be best suited for culverts and others for trapezoidal crosssections.

When implementing a riprap design method, it is important to refer to the original publication for any additional guidelines, limitations, or additional information. Site-specific information should also be obtained, where possible, to identify specific hydraulic conditions the riprap will experience. The accuracy of each riprap design method hinges on the correct estimation of input parameters.

CONCLUSIONS

An evaluation of the scour potential of four substrate materials in a 2-ft (0.61-m) diameter bottomless circular culvert and a 1-ft (0.30-m) wide rectangular flume was conducted in a research laboratory. The substrate materials were pea gravel (0.25-in), 0.75-inch angular gravel, 2-inch cobbles, and 2-inch angular rock. For the bottomless culvert, each substrate material was tested for incipient motion conditions and streambed response at various H_w/D values (partially pressurized and non-pressurized culvert flow) for both projecting and nonprojecting entrance conditions and three inlet contraction ratios. The bed slope was fixed at 0.01 ft/ft, there was no controlled tailwater at the exit of the culvert. and all tests consisted of a 2-ft (0.61-m) projecting end treatment with a 400% channel width expansion from the culvert to the tailbox; the culvert exit was never submerged. Testing in the rectangular flume investigated incipient motion conditions and three bed slopes were evaluated, 0.005 ft/ft, 0.01 ft/ft, and 0.015 Incipient motion results of the four tested substrates were compared to ft/ft. predicted results of eight published riprap stone sizing methods in an effort to scale the experimental results of this study to larger bottomless culvert diameters.

Incipient motion velocities could not be quantified for the pea gravel substrate in the bottomless culvert test facility due to the highly erodible nature of the material; there was no observed differentiation between the commencement of motion and considerable movement. In addition, incipient motion velocities

were never attained for the 2-inch angular rock due to flow and depth limitations of the rectangular flume. Incipient motion velocities for the 2-inch cobbles and the 2-inch angular substrate were approximately the same magnitude and exceeded those of the 0.75-inch angular gravel, which exceeded incipient motion velocities of the pea gravel. The incipient motion velocities for the two larger materials occurred between 1.25<H_w/D<1.5 in the bottomless culvert. The Incipient motion velocity corresponds to the condition where materials first began to move and, as such, are not necessarily a good indicator of the extent of scour. Similar flow velocities were observed for the 0.75-inch angular gravel and 2-inch cobbles in the rectangular flume.

Four methods were used to calculated Shields parameters and Grain Reynolds' numbers for comparison to Shields relation for the beginning of motion for the substrate materials tested. The Von Karman equation was utilized with and without a sidewall correction (one-dimensional flow) and Prandtl equation with the Einstein modification was utilized with and without a sidewall correction. The 1-dimensional methodologies appeared to apply reasonably well to the pea gravel substrate; however, they under predicted incipient motion conditions for the 0.75-inch angular gravel. If Shield's curve is assumed to remain constant for Re* values exceeding 1,000, these two methods appear to under-predict incipient motion for the 2-inch cobbles and the 2-inch angular rock. The two-dimensional methodologies appear to be better predictors of incipient motion for the three larger substrates when using Shields relation. The data had varying degrees of

scatter, reiterating that Shields curve would be more accurate as an upper and lower bound pair of curves instead of a line.

Scour evaluations in the bottomless arch culvert observed scour occurring along the entire length of the culvert for certain test scenarios. The most severe scour occurred primarily at the entrance and exit regions of the culvert. Flow contraction caused local pier-type scour holes at the culvert entrance. For pressurized tests bed degradation also occurred at the culvert entrance. Due to the fact that there was no controlled tailwater (unsubmerged outlet) the highest velocities and deepest scour depths were recorded at the outlet.

The pea gravel proved to erode more readily than the other tested substrate materials as expected. The 2-inch cobble and 2-inch angular rock substrates scoured the least; the depth of scour and incipient motion velocities were similar. However, the extent of scour (the area over which scour occurred) was greater in the 2-inch cobble substrate; angularity and gradation decrease the extent of scour inside the culvert barrel, compared to rounded cobbles.

In an effort to scale the results of this study to larger bottomless culverts and substrate materials, eight riprap sizing methods were applied to the experimental results of the 0.75-inch angular gravel substrate, the 2-inch cobble substrate and the 2-inch angular rock substrate. The two FHWA published stone sizing methods produced safety factors ranging from 1 to 3 in the culvert and 0 to 0.3 in the rectangular flume. The Halvorson method was more conservative, producing SF values ranging from 3 to 8 in the culvert and 3 to 10 in the rectangular flume. The stone sizing method of HEC 11 produced the most

conservative and scattered SF values in the bottomless culvert, and slightly less conservative values in the rectangular flume, at 4 to 28 and 6 to 18, respectively. Methods by Cal. B&SP, ASCE Manual 54, and USBR-EM-25 produced similar SF values ranging from 1 to 4 in the culvert and 2 to 6 in the flume. Finally, the USACE EM-1601 stone sizing method produced SF values of 0.5 to 2 in the culvert and rectangular flume.

Lack of conservatism for HEC-11 may be attributed to the application of a 1-dimensional modeling tool to a 2 and 3-dimensional flow conditions. Also, the two FHWA published methods were less conservative in the rectangular flume as each was developed for culverts. Methods by USACE, Cal B&SP, ASCE Manual 54, and FHWA all produced factors of safety approximately between 1 and 2 in the larger tested substrate materials in the bottomless culvert, with factors of safety slightly increasing with increasing bed velocities. These methods appear to be the best for designers as they would be able to choose their own factors of safety for larger installations. Methods by the USBR and Halvorson appear to contain factors of safety in the larger materials of 2 to 4 and 4 to 8, respectively.

Additional research in this area should included: testing larger bottomless culvert diameters; determining the influence of sidewall roughness, such as corrugations, on bottomless culverts scour; evaluate larger stone sizes; and streambed materials with fine material present (non-armored beds), and submerged culvert outlets. This additional research would help to identify the most appropriate scaling methods as well as limitations for determining stable bottomless culvert streambed material sizes for field applications.

By creating favorable hydraulic conditions for aquatic life and simulating the natural streambed, bottomless culverts can be acceptable ecological passageways. Bottomless culverts should be designed not only for base flow conditions, but also for high flow events. Knowledge of the scour behavior of bottomless culverts under pressurized and non-pressurized conditions should aid in bottomless culvert design. Riprap is a viable option for protecting streambeds through bottomless culverts during high flow events; relative conservativeness or factors of safety of available stone sizing methods will assist a designer in selecting a stone sizing method.

REFERENCES

- Abida H., and R. D. Townsend. 1991. Local scour downstream of box-culvert outlets. Journal of Irrigation and Drainage, ASCE, 117, June, 25–40.
- Abt, S. R. 1980. Scour at culvert outlets in cohesive bed material. Unpublished Ph.D. dissertation. Colorado State University, Fort Collins, Colorado. 157 p.
- Abt, S. R., R. L. Kloberdanz, and C. Mendoza. 1984. Unified culvert scour determination. Journal of Hydraulic Engineering 11010: 1475-1479.
- Abt, S. R., J. F. Ruff, and F. K. Doehring. 1985b. Culvert slope effects on outlet scour. Journal of Hydraulic Engineering, ASCE 11110: 1363-1367.
- Abt, S. R., and P. L. Thompson. 1996. Scour at culvert outlets: considerations present and future. Streambed Stability and Scour at Highway Bridges, 3927-3931.
- Andrews, E. D. 1983. Entrainment of gravel from naturally sorted riverbed material. Geological Society of America Bulletin 94: 1225-1231.
- Bates, K., B. Barnard, B. Heiner, J. P. Klavas, and P.D. Powers. (WDF&D). 2003. Fish passage design at road culverts. Washington Department of Fish and Wildlife, Olympia.
- Bathurst, J. C., W. H. Graf, and H. H. Cao. 1987. Bed load discharge equations for steep mountain rivers. Presented in Sediment transport in gravel-bed rivers, edited by C.R. Thorne, J.C. Bathurst, and R.D. Hey, 453-491. John Wiley, New York.
- Blodgett, J.C., and C. E. McConaughy. 1986. Rock riprap design for protection of stream channels near highway structures: volume 2 evaluation of riprap design procedures. Water Resources Investigations Report 86-4128, Sacramento, California.
- Bohan, J. P. 1970. Erosion and riprap requirements at culvert and storm-drain outlets, Research Report H-70-2, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi.
- Bolz, R. E., and G. L. Tuve. 1973. Handbook of tables for applied engineering science. CRC Press, Cleveland. 1184 p.

- Buffington, J. M. 1995. Effects of hydraulic roughness and sediment supply on surface textures of gravel-bedded rivers, Unpublished M.S. thesis. University of Washington, Seattle. 184 p.
- Buffingtion, J. M., W. E. Dietrich, and J. W. Kirchner. 1992. Friction angle measurements on a naturally formed gravel streambed: implications for critical boundary shear stress, Water Resources Research 28: 411-425.
- Buffington J.M., and D. R. Montgomery. 1997. A systematic analysis of eight decades of incipient motion studies, with spatial reference to gravel bedded rivers. Water Resources Research 33: 1993–2029.
- California Department of Fish and Game (CDFG). 2002. Culvert criteria for fish passage. Fisheries Branch Publications, http://www.dfg.ca.gov/nafwb/pubs/2002/culvert_criteria.pdf July 9, 2007.
- California Department of Public Works, Division of Highways (Cal B&SP). 1970.

 Bank and shore protection in california highway practice. Sacramento,
 California.
- Chien, N., Z. Wan, J. S. McNown (Translator). 1998. Mechanics of sediment transport. American Society of Civil Engineers, Reston, Virginia.
- Colebrook, C. F., and C. M. White. 1937. Experiments with fluid friction in roughened pipes. Proceedings of the Royal Society of London Series A 161: 367–81.
- Einstein, H. A. 1950. The bed-load function for sediment transportation in open channel flows, U.S. Department of Agriculture Soil Conservation Service Technology Bulletin, 1026, 73.
- Federal Highway Administration (FHWA). 1989. Use of riprap for bank protection. Hydraulic Engineering Circular #11 (HEC 11). FHWA-IP-89-016, Federal Highway Administration, Washington, D.C.
- Federal Highway Administration (FHWA). 2003. Bottomless culvert scour study: phase 1 laboratory report. FHWA-RD-O2-078, Federal Highway Administration, Washington, D.C.
- Federal Highway Administration (FHWA). 2006. Hydraulic design of energy dissipators for culverts and channels. Hydraulic engineering circular #14 (HEC 14). FHWA-NHI-06-086, Federal Highway Administration, Washington D.C.

- Federal Highway Administration (FHWA). 2007. Bottomless culvert scour study: phase 2 laboratory report. FHWA-HRT-07-026. Federal Highway Administration, Washington, D.C.
- Fenton, J. D., J. E. Abbott. 1977. Initial movement of grains on a stream bed: the effect of relative protrusion, Proceedings of the Royal Society of London Series A 352: 523-537.
- Forchheimer, P. 1914 Hydraulik, Ist ed. (2d ed., 1924, and 3d ed., 1930), Teubner, Leipzig-Berlin. 566 p.
- Fortier, S., and F. C. Scobey. 1926. Permissible canal velocities. Transaction of ASAE 89: 940–948.
- Gessler J. 1971. Beginning and ceasing of sediment motion. Presented in River Mechanics, edited by Shen, H. W.: Fort Collins, Colorado: 7:1–7:22.
- Halvorson, D. V., and F. J. Laumann. 1996. Scour protection in bottomless culverts. North American Water and Environment Congress & Destructive Water: 3932-3941.
- Ishbash, S. V. 1936. Construction of dams by depositing rock in running water. Second Congress on Large Dams, Communication No. 3, Washington, D.C.
- Johnson, J. W. 1942. The importance of considering side-wall friction in bed-load investigations, Civil Engineering 12: 329-331.
- Karman, Th. von, 1930. Mechanische anlichkeit und turbulenz, nach. ges. wiss. gottingen, Math. Phys. Klasse, Vol. 58, and Proceedings of 3rd International Congress of Applied Mechanics, Stockholm, Sweeden. Pt. I, Vol. 85.
- Keown, M. P. 1983. Streambank protection guidelines. U.S. Army Engineer, Environmental Laboratory, Vicksburg, Mississippi.
- Kirchner, J. W., W. E. Dietrich, F. Iseya, and H. Ikeda. 1990. The variability of critical shear stress, friction angle, and grain protrusion in water worked sediments, Sedimentology 37: 647-672.
- Li, Baoru. 1959. Calculations of threshold velocity of sediment particles. Journal of Sediment Research 4(1): 71-77.

- Li, M. S., and P D. Komar. 1986. Laboratory measurements of pivoting angles for applications to selective entrainment of gravel in a current. Sedimentology 33: 413-423.
- Lisle, T.E., J. M. Nelson, J. Pitlick, M. A. Madej, and B. L. Barkett. 2000. Variability of bed mobility in natural, gravel-bed channels and adjustments to sediment load at local and reach scales. Water Resources Research 36(12): 3743-3755.
- Maynord, S. T. 1979. Practical riprap design, U.S. Army Waterways Experiment Station, Misc. Paper H-78-7, Vicksburg, Mississippi.
- Maynord, S. T. 1987. Stable riprap size for open channel flows. Unpublished Ph.D. dissertation, Colorado State University, Fort Collins, Colorado. 122 p.
- Mendoza, C., S. R. Abt, and J. F. Ruff. 1983. Headwall influence on scour at culvert outlets. Journal of Hydraulic Engineering 109(7): 1056-1060.
- Meyer-Peter, and E., R. Müller. 1948. Formulas for bed-load transport, in proceedings of the 2nd meeting of the international association for hydraulic structures research, pp. 39–64, International Association for Hydraulic Research, Delft, Netherlands.
- Miller, R. T., and R. J. Byrne. 1966. The angle of repose for a single grain on a fixed rough bed, Sedimentology 6: 303-314.
- Neill, C. R., and M. S. Yalin. 1969. Quantitative definition of beginning of bed movement, Journal of Hydraulics Division of American Society of Civil Engineers 95: 585-588.
- Nikora, V. I., and D. G. Goring. 2000. Flow turbulence over fixed and weakly mobile gravel beds. Journal of Hydraulic Engineering 126(9): 679–690.
- Nikuradse, J. 1933. Stromungsgesetze in rauhen rohren. In Forschungsheft 361, Volume B, VDI Verlag, Berlin.
- Paintal, A. S., 1971. Concepts of critical shear stress in loose boundary open channels, Journal of Hydraulic Research 9: 91-113.
- Petit, F. 1994. Dimensionless critical shear stress evaluation from flume experiments using different gravel beds, Earth Surf. Processes Landforms 19: 565-576.
- Prandtl, L. 1925, Uber die ausgegildete turbulenz, ZAMM, 5: 136-139.

- Rahmeyer, W. J. 1989. Sedimentation engineering Vol. 1 & 2. CEE 6470 Course Notes. Logan, Utah.
- Richardson, E. V., D. B. Simons, and P. Y. Julien. 1990. Highways in the river environment. (HIRE) U.S. Dept. of Transportation, Federal Highway Administration, Washington, D.C.
- Schoklitsch, A. 1962. 3rd ed. Handbuch des wasserbues. Vienna, Austria: Springer-Verlag.
- Shafai-Bajestan, M., and M. L. Albertson. 1993. Riprap criteria below pipe outlet. Journal of Hydraulic Engineering 119(2): 181-200.
- Shields, A. 1936. Anwendung der aenlichkeitshmechanik und der turbulenzforschung auf die geschiebebewegung. Mitteilungen det Preussischen Versuchsanstalt fur Wasserbau und Schiffbau: Berlin, Germany.
- Sturm, T. W. 2004. Enhanced abutment scour studies for compound channels, FHWA-RD-99-156. Federal Highway Administration.
- U.S. Army Corps of Engineers (USACE). 1989 Sedimentation investigations of rivers and reservoirs: engineering manual (EM 1110-2-4000). Washington, D.C.
- U.S. Army Corps of Engineers (USACE). 1994. Hydraulic design of flood control channels: engineering manual. (EM 1110-2-1601). Washington, D.C.
- U.S. Bureau of Reclamation (USBR). 1962. Studies of tractive forces of cohesive soils in earth canals: division of engineering laboratories., Hydraulics Division, Reports of Engineering Practice No. 504, New York.
- Vanoni, V. A. 2004. Sedimentation engineering, ASCE manual No. 54. American Society of Civil Engineers, Sedimentation Committee of Hydraulics Division, New York.
- Vanoni, V. A., and N. H. Brooks. 1957. Laboratory studies of the roughness and suspended load of alluvial streams, Sediment. Lab. Rep. E68, 121, California Institute of Technology, Pasadena.
- White, C. M. 1940. The equilibrium of grains on the bed of a stream.

 Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences 174(958):322-338.

- White, S. J. 1970. Plane bed thresholds of fine grained sediments, Nature 228: 152-153.
- Wilcock, P. R. 1988. Methods for estimating the critical shear stress of individual fractions in mixed-size sediment, Water Resources Research 24: 1127-1135.
- Wong, M., G. and Parker. 2006. Re-analysis and correction of bed load relation of Meyer-Peter and Müller using their own database. Journal of Hydraulic Engineering, 132: 1159-1168.
- Yang, C. T., J. M. Simões. 2005. Wash load and bed-material load transport in the yellow river. Journal of Hydraulic Engineering 131(5): 413-418.
- Zeng, Z., and S. Wang. 1963. Study on threshold criteria for granular sediment. Journal of Sediment of Tianjin University, 19-40.

APPENDICES

Appendix A

Bottomless Arch Culvert Velocity Data

Table A1. Pea gravel substrate velocity results

Run	Hw/D	Control	Flow	Depth	Measured Avg. Velocity @ Bed	Mean Velocity	Froude #	Reynolds #
(#)	()	(Inlet,Outlet)	(cfs)	(ft)	(ft/s)	(ft/s)	()	()
1a incipient	-		-	-	-	-	-	-
1a incipient	0.50	Inlet	1.689	0.36 / 0.13	2.77 / 0.11	2.52 / 0.87	0.49 / 0.1	148452 / 16305
1b incipient	-		-	-	-	-	-	-
1b duration	1.00		4.407		Excessiv	e degradatio	n - discard	
2 incipient	-	-	-	-	-	-	-	-
2 duration	0.50	Inlet	1.721	0.37 / 0.14	2.42 / 0.36	2.54 / 0.94	0.57 / 0	158463 / 19114
3 incipient	-	-	-	-	-	-	-	-
3 duration	0.50	Outlet	1.789	0.39 / 0.14	2.43 / 0.43	2.48 / 0.86	0.57 / 0	157815 / 19030
4 incipient	-	-	-	-	-	-	-	-
4 duration	0.50	Outlet	1.516	0.43 / 0.17	2.62 / 0.32	1.94 / 0.74	0.46 / 0	118660 / 15377
5 incipient	-	-	-	-	-	-	-	-
5 duration	1.00		4.174		Excessiv	e degradatio	n - discard	
6 incipient	-	-	-	-	-	-	-	-
6 duration	0.50	Outlet	1.712	0.4 / 0.13	2.47 / 0.51	2.33 / 0.76	0.54 / 0	151197 / 16372
7 incipient	-	-	-	-	-	-	-	-
7 duration	1.00	Outlet	4.345	0.84 / 0.19	2.9 / 0.14	2.95 / 0.25	0.87 / 0	273582 / 62084

Table A2. 0.75-inch angular gravel substrate velocity results

					Measured Avg.	Mean		
Run	Hw/D	Control	Flow	Depth	Velocity @ Bed	Velocity	Froude #	Reynolds #
(#)	()	(Inlet,Outlet)	(cfs)	(ft)	(ft/s)	(ft/s)	()	()
8 incipient	-		-	-	-	-	-	-
8 duration	1.00	Inlet	4.432	0.68 / 0.38	3.7 / 0.42	4.11 / 1.92	0.86 / 0.43	307028 / 91360
9 incipient	-		-	-	-	-	-	-
9 duration	1.50	Inlet	7.911	0.78 / 0.3	-	6.17 / 1.61	1.76 / 0	503749 / 170783
10 incipient	-	-	-	-	-	-	-	-
10 duration	1.25	Inlet	7.977	0.85 / 0.2	4.66 / 0.16	5.63 / 0.79	1.47 / 0	473971 / 120535
11 a	0.50	Outlet	2.360	0.39 / 0.14	3.14 / 0.88	3.28 / 1.16	0.66 / 0.13	214004 / 23101
11 b	0.75	Outlet	3.090	0.55 / 0.22	3.74 / 1.3	3.17 / 1.16	0.7 / 0.19	249488 / 42961
12 incipient	0.75		3.257	0.62 / 0.11	3.88 / 1.41	2.84 / 0.45	0.66 / 0.09	252120 / 22787
12 duration	1.00	Outlet	4.489	0.71 / 0.19	3.65 / 0.34	3.57 / 0.73	0.81 / 0.21	322749 / 50204
13 incipient	0.75		3.224	0.59 / 0.04	3.1 / 1.1	2.9 / 0.18	0.68 / 0.03	258192 / 8672
13 duration	1.00	Inlet	4.679	0.75 / 0.23	3.54 / 0.57	3.66 / 0.84	0.8 / 0.28	330708 / 66377
14 incipient	0.72		2.355	0.58 / 0	2.66 / 0.65	2.14 / 0	0.5 / 0	186552 / 0
14 duration	1.00	Outlet	4.774	0.83 / 0.02	3.5 / 0.33	3.17 / 0.11	0.77 / 0.11	317780 / 20480
15 incipient	1.00	•	3.149	0.65 / 0.16	3.04 / 1.15	2.68 / 0.54	0.62 / 0.12	237566 / 29945
15 duration	1.04	Inlet	4.594	0.77 / 0.2	3.34 / 0.64	3.56 / 0.41	0.96 / 0	299715 / 85968

Table A3. 2-inch cobble substrate velocity results

					Measured Avg.	Mean		
Run	Hw/D	Control	Flow	Depth	Velocity @ Bed	Velocity	Froude #	Reynolds #
(#)	()	(Inlet,Outlet)	(cfs)	(ft)	(ft/s)	(ft/s)	()	()
16 a	0.50	Outlet	1.154	0.54 / 0	2.45 / 0.91	1.11 / 0	0.26 / 0	100844 / 0
16 b	0.75	Outlet	2.925	0.79 / 0	3.5 / 1.13	2.05 / 0	0.49 / 0	211563 / 0
16 incipient	-		-	-	-	-	-	-
16 duration	1.00	Outlet	4.220	0.85 / 0.24	3.36 / 1.37	2.58 / 1.91	0.64 / 0.27	290662 / 64457
17 incipient	1.31		6.634	0.84 / 0.16	4.48 / 1.52	4.73 / 0.64	1.17 / 0.26	419625 / 86025
17 duration	1.50	Inlet	7.246	0.84 / 0.19	4.71 / 0.64	5.16 / 0.8	1.37 / 0.21	466090 / 96053
18 incipient	-		-	-	-	-	-	-
18 duration	1.00	Inlet	4.059	0.72 / 0.17	3.86 / 0.8	3.3 / 0.87	0.74 / 0.18	314162 / 46128
19 incipient	1.25		6.594	0.82 / 0.21	3.96 / 0.95	4.79 / 1.14	1.31 / 0.22	447065 / 104973
19 duration	1.50	Inlet	7.480	0.8 / 0.18	4.28 / 0.61	5.41 / 0.83	1.53 / 0.12	513265 / 107659
20 incipient	-		-	-	-	-	-	-
20 duration	1.00	Inlet	4.159	0.72 / 0.17	4.07 / 0.94	3.31 / 0.74	0.76 / 0.18	314244 / 43237
21 incipient	1.35		6.714	0.87 / 0.16	4.87 / 1.34	4.56 / 0.59	1.16 / 0.26	439890 / 88832
21 duration	1.44	Inlet	7.060	0.84 / 0.19	4.32 / 0.47	4.93 / 0.68	1.4 / 0.16	477961 / 103142
22 incipient	-		-	-	-	-	-	-
22 duration	1.01	Inlet	4.364	0.19 / 0.35	4.07 / 1.11	2.11 / 1.82	0.8 / 0.17	326883 / 61710
23 incipient	1.38		6.473	0.88 / 0.2	4.44 / 1.58	4.39 / 0.74	1.15 / 0.29	446298 / 99094
23 duration	1.50	Inlet	7.282	0.88 / 0.21	3.87 / 0.91	4.85 / 0.73	1.4 / 0.13	505799 / 108209
24 incipient	-		-	-	-	-	-	-
24 duration	1.00	Outlet	4.411	0.18 / 0.32	3.98 / 0.98	2.12 / 1.77	0.77 / 0.17	366069 / 42022
25 incipient	1.42		6.674	0.89 / 0.23	4.76 / 1.78	4.55 / 1.01	1.19 / 0.27	459283 / 104657
25 duration	1.49	Inlet	7.480	0.86 / 0.23	4.15 / 0.73	4.8 / 1.62	1.45 / 0.19	531214 / 113925

Table A4. 2-inch angular rock substrate velocity results

					Measured Avg.	Mean		
Run	Hw/D	Control	Flow	Depth	Velocity @ Bed	Velocity	Froude #	Reynolds #
(#)	()	(Inlet,Outlet)	(cfs)	(ft)	(ft/s)	(ft/s)	()	()
26 a	0.50	Outlet	1.468	0.39 / 0.08	2.11 / 1.07	1.95 / 0.39	0.4 / 0.04	158440 / 9190
26 b	0.75	Outlet	2.780	0.55 / 0.11	3.01 / 1.57	1.98 / 0.95	0.38 / 0.08	163988 / 43337
26 incipient	-		-	-	-	-	-	-
26 duration	1.00	Inlet	4.227	0.72 / 0.15	3.67 / 1.81	3.34 / 0.65	0.77 / 0.16	356427 / 45912
27 incipient	1.29		6.136	0.86 / 0.18	4.45 / 1.54	4.26 / 0.67	1.06 / 0.28	451024 / 95858
27 duration	1.50	Inlet	7.533	0.85 / 0.18	4.44 / 0.55	5.06 / 0.68	1.49 / 0.06	576117 / 115881
28 incipient	-		-	-	-	-	-	-
28 duration	1.00	Inlet	4.634	0.77 / 0.19	3.43 / 0.91	3.48 / 0.75	0.78 / 0.22	367174 / 56573
29 incipient	1.46		7.060	0.97 / 0.17	4.41 / 0.73	4.49 / 0.61	1.41 / 0	420235 / 91646
29 duration	1.50	Inlet	7.707	0.82 / 0.21	4.32 / 0.77	5.56 / 0.94	1.63 / 0.14	538436 / 126826
30 incipient	-		-	-	-	-	-	-
30 duration	1.00	Outlet	4.339	0.8 / 0.18	3.76 / 0.92	3.12 / 0.7	0.72 / 0.18	345974 / 49392
31 incipient	1.38		6.513	0.91 / 0.17	4.84 / 1.36	4.23 / 0.59	1.07 / 0.25	459984 / 91318
31 duration	1.50	Inlet	7.300	0.89 / 0.19	4.56 / 0.62	4.86 / 0.97	1.34 / 0.26	525627 / 110684
32 incipient	-		-	-	-	-	-	-
32 duration	1.00	Outlet	4.308	0.76 / 0.15	3.68 / 1.02	2.04 / 1.74	0.74 / 0.17	354888 / 46264
33 incipient	1.46		6.554	0.88 / 0.16	4.5 / 1.64	4.4 / 0.55	1.09 / 0.23	474010 / 91473
33 duration	1.50	Inlet	7.172	0.9 / 0.15	4.53 / 0.79	4.76 / 0.47	1.34 / 0.16	502244 / 100560
34 incipient	-		-	-	-	-	-	-
34 duration	1.00	Outlet	4.152	0.22 / 0.37	3.81 / 1.26	1.94 / 1.65	0.68 / 0.2	334919 / 48435
35 incipient	1.50		6.594	0.94 / 0.19	4.96 / 1.42	4.37 / 0.75	1.41 / 0	420653 / 101088
35 duration	1.50	Inlet	6.594	0.92 / 0.18	4.75 / 1.12	4.3 / 0.57	1.24 / 0.12	443019 / 91481

Appendix B

Rectangular Flume Velocity Data

Table B1. Pea gravel substrate velocity results

				Point Gage	Point Gage			Measured Avg.			
Run	Motion?	Station	So	False Floor	W.S.	Depth	D ₅₀ /Depth	Velocity @ Bed	Mean Velocity	Froude #	Reynolds #
(#)			(ft/ft)	(ft)	(ft)	(ft)		(ft/s)	(ft/s)	()	()
106	yes	8	0.0108	0.003	0.227	0.22	0.10	2.67	2.78	1.035	110699
107	yes	4	0.0108	0.043	0.236	0.19	0.11	3.00	3.22	1.294	115651
108	yes	0	0.0108	0.026	0.273	0.25	0.09	2.45	2.52	0.894	107291
109	no	8	0.0108	0.005	0.167	0.16	0.14	2.25	2.06	0.904	65071
110	no	4	0.0108	0.030	0.168	0.14	0.16	2.32	2.42	1.150	67519
111	no	0	0.0108	0.025	0.183	0.16	0.14	2.10	2.12	0.939	65467
112	no	8	0.0108	0.022	0.152	0.13	0.17	1.61	1.75	0.855	46453
113	no	4	0.0108	0.044	0.151	0.11	0.21	1.62	2.12	1.145	48213
114	no	0	0.0108	0.023	0.130	0.11	0.21	1.85	2.12	1.145	48213
115	yes	8	0.0108	0.017	0.213	0.20	0.11	2.33	2.41	0.961	87529
116	yes	4	0.0108	0.037	0.221	0.18	0.12	2.75	2.57	1.057	89065
117	yes	0	0.0108	0.016	0.196	0.18	0.12	2.76	2.63	1.092	89589
118	no	8	0.0148	0.010	0.149	0.14	0.16	1.87	2.50	1.181	65983
119	no	4	0.0148	0.038	0.160	0.12	0.18	2.15	2.85	1.436	67786
120	no	0	0.0148	0.017	0.126	0.11	0.20	2.24	3.18	1.701	69233
121	yes	8	0.0148	0.018	0.161	0.14	0.15	2.41	2.51	1.171	67874
122	yes	4	0.0148	0.028	0.174	0.15	0.15	2.45	2.46	1.136	67559
123	yes	0	0.0148	0.005	0.160	0.16	0.14	2.48	2.32	1.038	66630
124	no	8	0.0148	0.017	0.152	0.14	0.16	2.26	2.57	1.234	66398
125	no	4	0.0148	0.031	0.165	0.13	0.17	2.31	2.59	1.248	66503
126	no	0	0.0148	0.011	0.151	0.14	0.16	2.45	2.48	1.168	65880
127	yes	8	0.0148	0.015	0.188	0.17	0.13	2.64	2.52	1.066	78535
128	yes	4	0.0148	0.027	0.191	0.16	0.14	2.67	2.65	1.155	79599
129	yes	0	0.0148	0.002	0.179	0.18	0.13	2.66	2.46	1.030	78071
130	no	8	0.0052	0.015	0.180	0.17	0.13	2.09	1.86	0.809	55748
131	no	4	0.0052	0.024	0.204	0.18	0.12	2.01	1.71	0.710	54518
132	no	0	0.0052	0.000	0.207	0.21	0.11	2.02	1.49	0.576	52436
133	yes	8	0.0052	0.008	0.355	0.35	0.06	3.04	2.97	0.887	146360
134	yes	4	0.0052	0.017	0.403	0.39	0.06	2.79	2.67	0.756	139918
135	yes	0	0.0052	0.005	0.406	0.40	0.06	2.84	2.57	0.714	137588
136	no	8	0.0052	0.013	0.250	0.24	0.09	2.59	2.44	0.885	94715
137	no	4	0.0052	0.038	0.261	0.22	0.10	2.58	2.60	0.970	96549
138	no	0	0.0052	0.016	0.300	0.28	0.08	2.22	2.04	0.675	89037
139	yes	8	0.0052	0.009	0.354	0.35	0.06	2.98	2.83	0.850	139366
140	yes	4	0.0052	0.040	0.370	0.33	0.07	2.93	2.96	0.909	141885
141	ves	0	0.0052	0.007	0.432	0.43	0.05	2.64	2.30	0.622	127313

Table B2. 0.75-inch angular gravel substrate velocity results

				Point Gage	Point Gage			Measured Avg.			
Run (#)	Motion?	Station	S _o (ft/ft)	False Floor (ft)	W.S. (ft)	Depth (ft)	D ₅₀ /Depth	Velocity @ Bed (ft/s)	Mean Velocity (ft/s)	Froude #	Reynolds #
52	no	8	0.0103	0.050	0.217	0.17	0.32	1.99	2.61	1.124	74434
53	no	4	0.0103	0.000	0.223	0.22	0.24	1.88	1.95	0.729	68669
54	no	0	0.0103	0.008	0.193	0.19	0.29	2.51	2.35	0.964	72478
55	no	8	0.0103	0.050	0.427	0.38	0.14	3.23	3.52	1.012	172809
56	no	4	0.0103	0.000	0.455	0.46	0.12	2.69	2.92	0.763	158694
57	no	8	0.0103	0.017	0.394	0.38	0.14	3.32	3.52	1.012	172809
58	yes	8	0.0103	0.045	0.925	0.88	0.06	4.72	5.05	0.949	367302
59	yes	4	0.0103	0.030	0.917	0.89	0.06	4.54	5.01	0.938	365448
60	yes	0	0.0103	-0.010	0.836	0.85	0.06	4.88	5.25	1.007	376580
61	no	8	0.0103	0.000	0.286	0.29	0.19	3.15	3.36	1.106	138965
62	no	4	0.0103	0.025	0.340	0.32	0.17	2.85	3.05	0.957	134020
63	no	0	0.0103	0.000	0.322	0.32	0.16	2.77	2.98	0.926	132879
64	no	8	0.0103	0.000	0.728	0.73	0.07	3.66	4.58	0.945	308700
65	no	4	0.0103	0.028	0.770	0.74	0.07	3.82	4.49	0.919	305221
66	no	8	0.0103	0.005	0.693	0.69	0.08	4.07	4.84	1.029	319094
67	yes	8	0.0103	0.015	0.853	0.84	0.06	4.04	5.00	0.963	356627
68	yes	4	0.0103	0.006	0.840	0.83	0.06	4.68	5.03	0.970	357696
69	yes	0	0.0103	0.000	0.823	0.82	0.06	4.48	5.09	0.990	360670
70	no	8	0.0046	0.000	0.657	0.66	80.0	3.31	3.59	0.781	239014
71	no	4	0.0046	0.008	0.692	0.68	0.08	2.75	3.45	0.735	233563
72	no	0 8	0.0046	0.029 0.000	0.703 0.823	0.67	0.08 0.06	3.22 3.94	3.50 4.02	0.752 0.781	235553 293092
73 74	no	4	0.0046	0.000	0.874	0.82 0.86				0.781	293092 285959
74 75	no	8	0.0046 0.0046	0.018	0.814	0.86	0.06 0.07	3.43 3.68	3.87 4.22	0.737	301995
75 76	no	8	0.0046	0.030	1.096	1.09	0.07	4.92	4.71	0.840	301995 378192
77	yes yes	4	0.0046	0.003	1.090	1.09	0.05	5.10	4.84	0.794	385461
78	yes	0	0.0046	0.015	0.989	0.98	0.05	5.02	5.22	0.020	405461
79	no	8	0.0046	0.000	0.680	0.68	0.03	2.84	3.44	0.736	222085
80	no	4	0.0046	0.003	0.705	0.70	0.08	2.89	3.33	0.701	218020
81	no	0	0.0046	0.001	0.740	0.74	0.07	2.96	3.17	0.649	211509
82	no	8	0.0046	0.000	0.850	0.85	0.06	3.70	4.38	0.837	308539
83	no	4	0.0046	0.000	0.859	0.86	0.06	3.83	4.33	0.824	306496
84	no	8	0.0046	0.002	0.841	0.84	0.06	4.06	4.43	0.853	311074
85	yes	8	0.0046	0.000	1.050	1.05	0.05	4.69	4.54	0.782	344751
86	yes	4	0.0046	0.004	0.961	0.96	0.06	4.98	4.99	0.898	366756
87	yes	0	0.0046	0.035	0.951	0.92	0.06	4.83	5.21	0.959	377376
88	no	8	0.0149	0.010	0.451	0.44	0.12	3.36	3.75	0.996	222421
89	no	4	0.0149	0.020	0.420	0.40	0.13	3.56	4.14	1.153	232554
90	no	0	0.0149	0.010	0.395	0.39	0.14	3.42	4.30	1.221	236496
91	no	8	0.0149	0.000	0.631	0.63	0.08	4.04	4.34	0.963	306146
92	no	4	0.0149	0.014	0.593	0.58	0.09	3.93	4.73	1.095	320900
93	no	8	0.0149	0.006	0.582	0.58	0.09	4.31	4.75	1.104	321795
94	yes	8	0.0149	0.000	0.701	0.70	0.08	4.52	4.99	1.051	368601
95	yes	4	0.0149	0.020	0.732	0.71	0.07	4.44	4.91	1.027	365255
96	yes	0	0.0149	0.008	0.708	0.70	0.08	4.72	5.00	1.053	368908
97	no	8	0.0149	0.000	0.466	0.47	0.11	3.53	3.55	0.917	214215
98	no	4	0.0149	0.000	0.444	0.44	0.12	3.46	3.73	0.986	219207
99	no	0	0.0149	0.001	0.440	0.44	0.12	3.49	3.77	1.003	220375
100	no	8	0.0149	0.000	0.680	0.68	0.08	3.96	4.32	0.924	311633
101 102	no	4 8	0.0149 0.0149	0.005 0.000	0.678	0.67 0.64	0.08 0.08	4.40 4.68	4.37 4.58	0.939 1.008	313493 322003
102	no	8	0.0149	0.000	0.642 0.783	0.64	0.08	4.68 4.94	4.58 4.82	0.961	322003 368164
103	yes yes	4	0.0149	0.000	0.763	0.78	0.07	3.88	4.86	0.972	369894
104	yes	0	0.0149	-0.200	0.716	0.78	0.07	4.91	4.12	0.759	333584
100	yes	U	0.0149	-0.200	0.710	0.92	0.00	4.91	4.12	0.759	JJJJ04

Table B3. 2-inch cobble substrate velocity results

				Point Gage	Point Gage			Measured Avg.			
Run	Motion?	Station	So	False Floor	W.S.	Depth	D ₅₀ /Depth	Velocity @ Bed	Mean Velocity	Froude #	Reynolds #
(#)			(ft/ft)	(ft)	(ft)	(ft)		(ft/s)	(ft/s)	()	()
1	no	8	0.0141	0.000	0.778	0.78	0.14	3.25	4.28	0.856	292361
2	no	4	0.0141	0.000	0.835	0.84	0.13	3.30	3.99	0.770	279878
3	no	0	0.0141	0.000	0.802	0.80	0.13	3.47	4.15	0.818	286971
4	no	8	0.0141	0.000	0.754	0.75	0.14	3.30	4.36	0.885	300282
5	no	4	0.0141	0.000	0.793	0.79	0.14	3.59	4.15	0.821	291225
6	no	0	0.0141	0.000	0.749	0.75	0.14	4.06	4.39	0.894	301484
7	yes	8	0.0141	0.000	1.228	1.23	0.09	6.30	5.68	0.904	462605
8	yes	4	0.0141	0.000	1.212	1.21	0.09	5.30	5.76	0.922	466929
9	yes	0	0.0141	0.000	1.275	1.28	0.08	4.95	5.47	0.855	450356
10	no	8	0.0141	0.000	0.614	0.61	0.18	3.04	5.39	1.213	341921
11	no	4	0.0141	0.000	0.652	0.65	0.17	3.01	5.08	1.108	330642
12	no	0	0.0141	0.000	0.589	0.59	0.18	3.06	5.62	1.291	349770
13	no	8	0.0141	0.000	0.861	0.86	0.13	4.49	5.23	0.995	381202
14	no	4	0.0141	0.000	1.004	1.00	0.11	4.30	4.49	0.790	344958
15 16	no	8 8	0.0141	0.000	0.987 1.250	0.99 1.25	0.11	4.81 5.41	4.57 5.75	0.810 0.906	348901
17	yes yes	o 4	0.0141 0.0141	0.000 0.000	1.261	1.25	0.09 0.09	5.62	5.75	0.894	472398 469447
18	yes	0	0.0141	0.000	1.242	1.24	0.09	6.05	5.78	0.094	474567
19	no	8	0.0154	0.000	0.785	0.79	0.14	4.23	4.41	0.877	310776
20	no	4	0.0154	0.000	0.778	0.78	0.14	3.48	4.44	0.888	312478
21	no	0	0.0154	0.000	0.811	0.81	0.13	3.62	4.26	0.835	304612
22	yes	8	0.0154	0.000	1.096	1.10	0.10	6.54	5.91	0.995	468517
23	yes	4	0.0154	0.000	1.114	1.11	0.10	5.97	5.81	0.971	463291
24	yes	0	0.0154	0.000	1.108	1.11	0.10	5.55	5.84	0.979	465020
25 26	no no	8 4	0.0154 0.0154	0.000 0.000	0.688 0.676	0.69 0.68	0.16 0.16	4.03 3.78	4.68 4.76	0.995 1.022	310560 313729
26	no	0	0.0154	0.000	0.676	0.00	0.16	4.02	4.76	0.869	294450
28	no	8	0.0154	0.000	0.979	0.73	0.14	5.26	5.22	0.929	395519
29	no	4	0.0154	0.000	0.960	0.96	0.11	5.48	5.32	0.957	400666
30	no	8	0.0154	0.000	0.989	0.99	0.11	5.07	5.16	0.915	392862
31	yes	8	0.0154	0.000	1.112	1.11	0.10	5.21	6.21	1.038	490732
32	yes	4	0.0154	0.000	1.114	1.11	0.10	5.54	6.20	1.035	490124
33	yes	0	0.0154	0.000	1.117	1.12	0.10	5.67	6.18	1.031	489215
34 35	no no	8 4	0.0049 0.0049	0.050 0.093	0.514 0.523	0.46 0.43	0.23 0.25	2.78 2.50	3.06 3.30	0.792 0.888	172641 178953
36	no	0	0.0049	0.093	0.525	0.43	0.25	2.31	2.69	0.654	162051
37	no	8	0.0049	0.030	1.063	1.02	0.21	3.98	4.82	0.841	379038
38	no	4	0.0049	0.093	1.070	0.98	0.11	3.93	5.04	0.898	390330
39	no	8	0.0049	0.030	1.091	1.06	0.10	4.16	4.64	0.794	369325
40	no	8	0.0049	0.033	1.361	1.33	0.08	5.07	5.59	0.855	475568
41	no	4	0.0049	0.065	1.490	1.43	0.08	4.77	5.21	0.769	451605
42	no	0	0.0049	0.002	1.527	1.53	0.07	5.49	4.86	0.694	429303
43 44	no no	8 4	0.0103 0.0103	0.055 0.010	0.915 0.906	0.86 0.90	0.13 0.12	4.03 3.78	4.81 4.62	0.915 0.861	358518 349273
45	no	0	0.0103	0.010	0.888	0.90	0.12	4.02	4.74	0.893	354865
46	no	8	0.0103	0.020	1.177	1.16	0.09	5.26	5.04	0.826	414391
47	no	4	0.0103	0.061	1.180	1.12	0.10	5.48	5.21	0.868	424117
48	no	8	0.0103	0.027	1.240	1.21	0.09	5.07	4.81	0.769	400844
49	yes	8	0.0103	0.035	1.508	1.47	0.07	5.21	5.56	0.807	488513
50	yes	4	0.0103	0.024	1.382	1.36	0.08	5.54	6.03	0.912	518749
51	yes	0	0.0103	0.033	1.545	1.51	0.07	5.67	5.41	0.776	479044

Table B4. 2-inch angular rock substrate velocity results

				Point Gage	Point Gage			Measured Avg.			
Run	Motion?	Station	So	False Floor	W.S.	Depth	D ₅₀ /Depth	Velocity @ Bed	Mean Velocity	Froude #	Reynolds #
(#)			(ft/ft)	(ft)	(ft)	(ft)		(ft/s)	(ft/s)	()	()
142	no	8	0.0052	0.010	1.408	1.40	0.09	5.50	5.62	0.838	500876
143	no	4	0.0052	0.020	1.543	1.52	0.08	5.47	5.16	0.737	469927
144	no	0	0.0052	0.003	1.510	1.51	0.08	5.58	5.21	0.748	473674
145	no	8	0.0052	0.008	1.413	1.41	0.09	5.52	5.62	0.835	501362
146	no	4	0.0052	0.022	1.550	1.53	0.08	5.45	5.16	0.736	470954
147	no	8	0.0052	0.000	1.523	1.52	0.08	5.64	5.18	0.740	472118
148	no	8	0.0102	0.010	1.382	1.37	0.09	5.46	5.92	0.891	527045
149	no	4	0.0102	0.023	1.489	1.47	0.08	5.37	5.54	0.807	501845
150	no	0	0.0102	0.021	1.543	1.52	0.08	5.66	5.34	0.763	487946
151	no	8	0.0102	0.010	1.380	1.37	0.09	5.52	5.95	0.896	529333
152	no	4	0.0102	0.025	1.495	1.47	0.08	5.39	5.54	0.806	502464
153	no	8	0.0102	0.005	1.545	1.54	0.08	5.61	5.29	0.752	485222
154	no	8	0.0151	0.008	0.807	0.80	0.15	3.62	5.09	1.005	384332
155	no	4	0.0151	0.044	0.744	0.70	0.18	3.74	5.81	1.225	416040
156	no	0	0.0151	0.002	0.770	0.77	0.16	4.11	5.30	1.066	393728
157	no	8	0.0151	0.046	1.329	1.28	0.10	4.98	6.28	0.978	554503
158	no	4	0.0151	0.039	1.359	1.32	0.09	4.91	6.11	0.937	543230
159	no	8	0.0151	0.060	1.328	1.27	0.10	5.42	6.36	0.995	559207
160	no	8	0.0151	0.000	1.422	1.42	0.09	5.14	5.65	0.835	511019
161	no	4	0.0151	0.016	1.337	1.32	0.09	5.27	6.08	0.933	539362
162	no	0	0.0151	0.040	1.300	1.26	0.10	5.81	6.38	1.001	558056

Appendix C

Shields Relation for Bottomless Arch Culvert

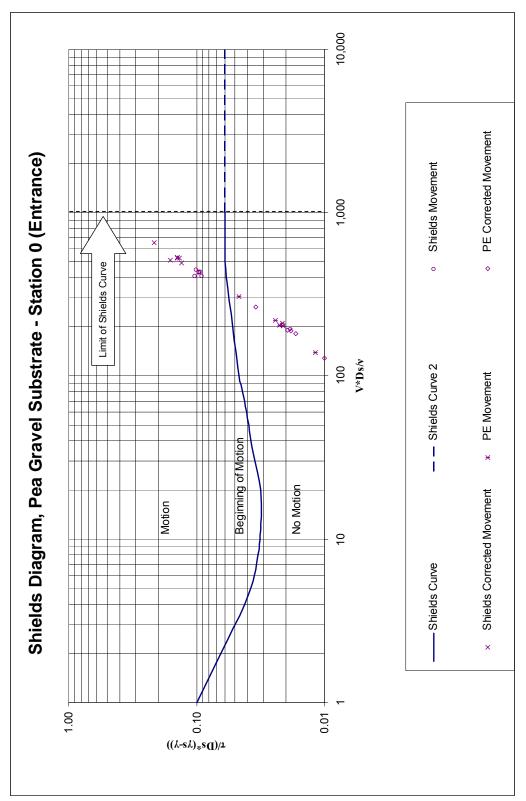


Figure C1. Pea gravel substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion

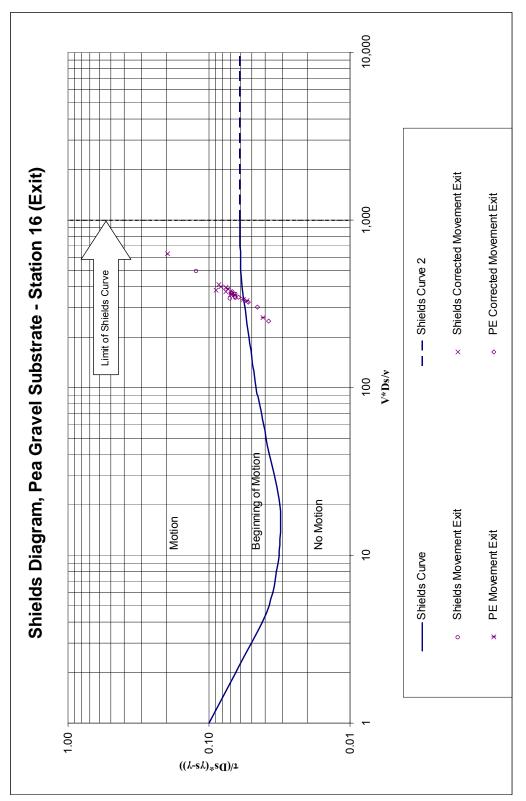


Figure C2. Pea gravel substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion

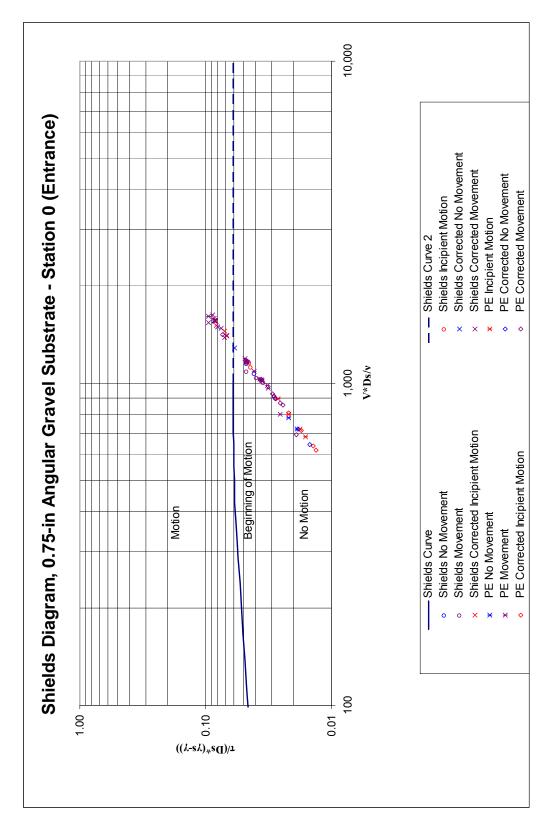


Figure C3. 0.75-inch angular gravel substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion

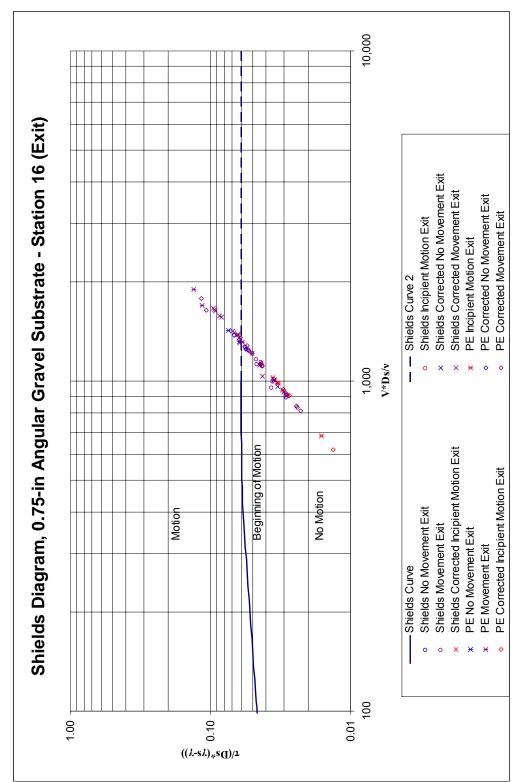


Figure C4. 0.75-inch angular gravel substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion

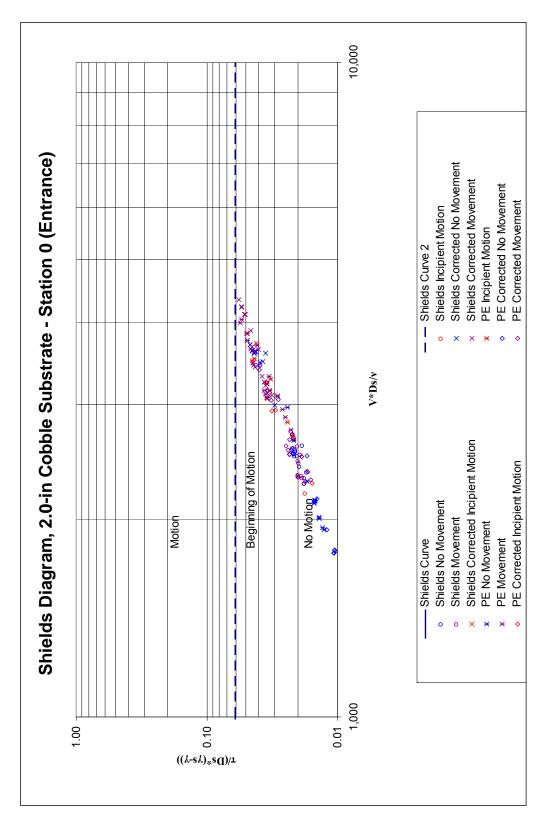


Figure C5. 2-inch cobble substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion

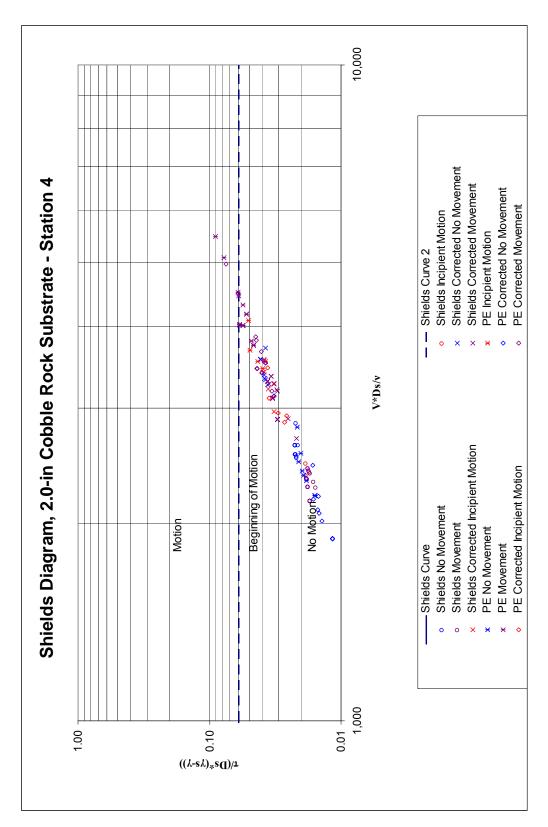


Figure C6. 2-inch cobble substrate at station 4 in bottomless arch culvert plotted on shields relation for incipient motion

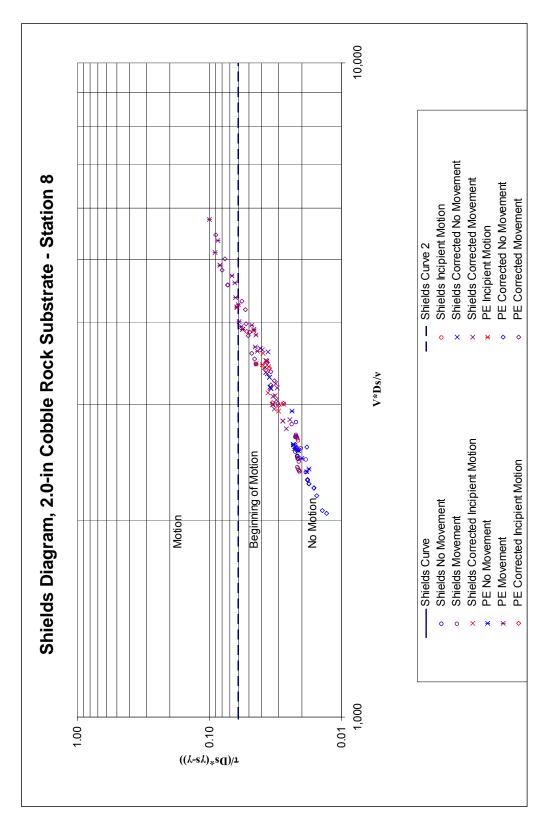


Figure C7. 2-inch cobble substrate at station 8 in bottomless arch culvert plotted on shields relation for incipient motion

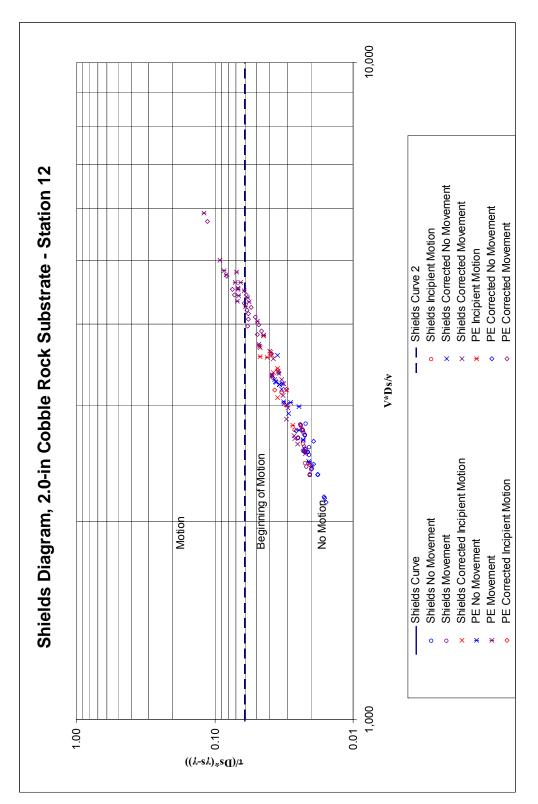


Figure C8. 2-inch cobble substrate at station 12 in bottomless arch culvert plotted on shields relation for incipient motion

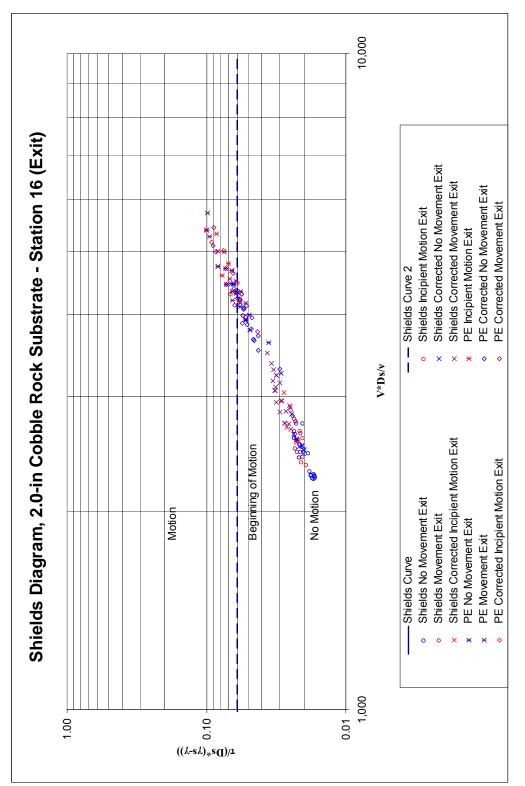


Figure C9. 2-inch cobble substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion

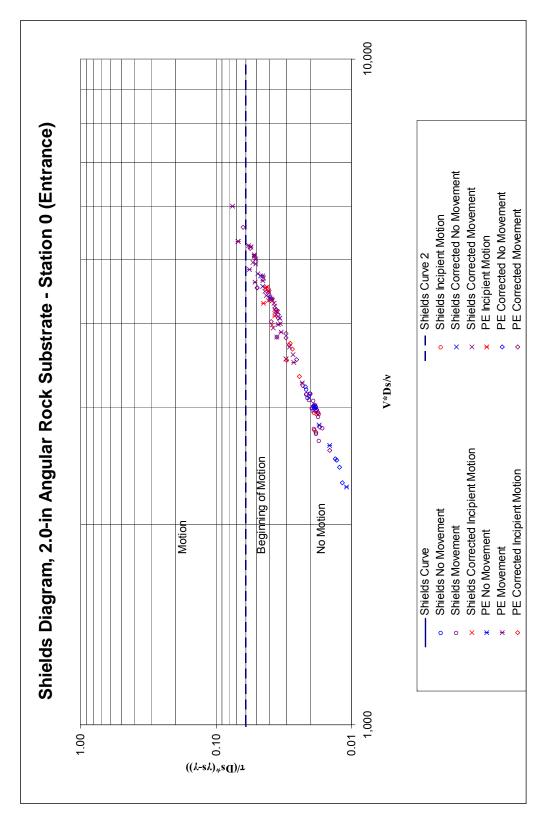


Figure C10. 2-inch angular rock substrate at station 0 in bottomless arch culvert plotted on shields relation for incipient motion

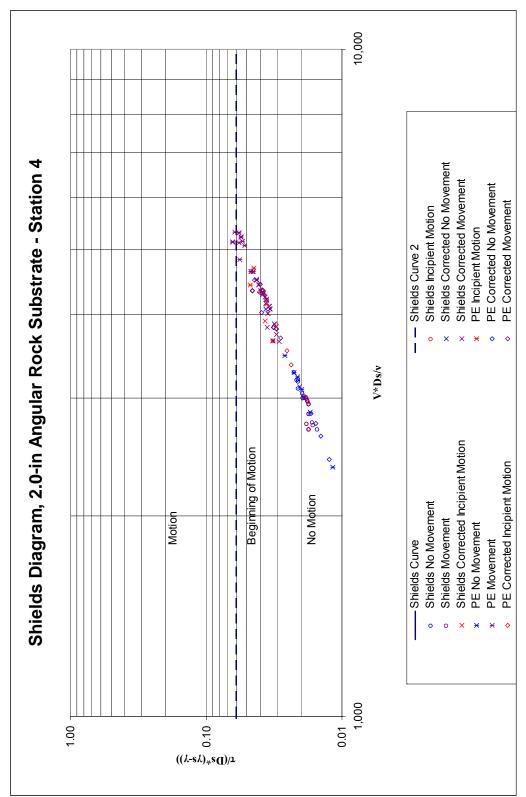


Figure C11. 2-inch angular rock substrate at station 4 in bottomless arch culvert plotted on shields relation for incipient motion

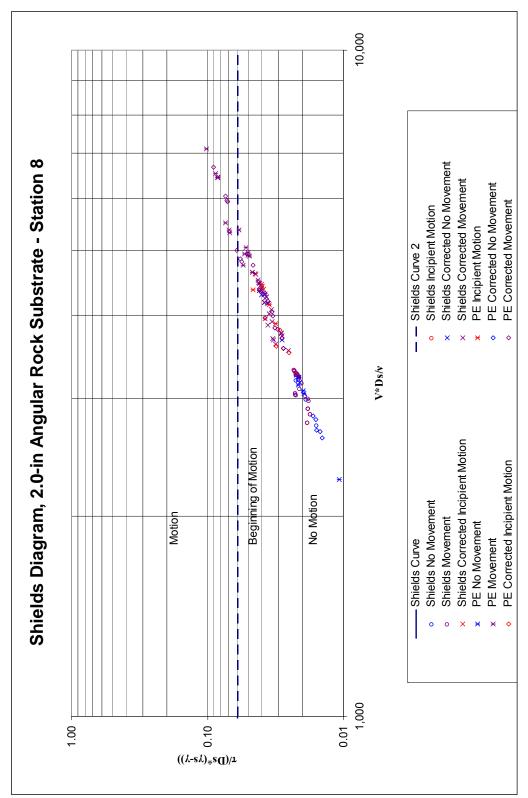


Figure C12. 2-inch angular rock substrate at station 8 in bottomless arch culvert plotted on shields relation for incipient motion

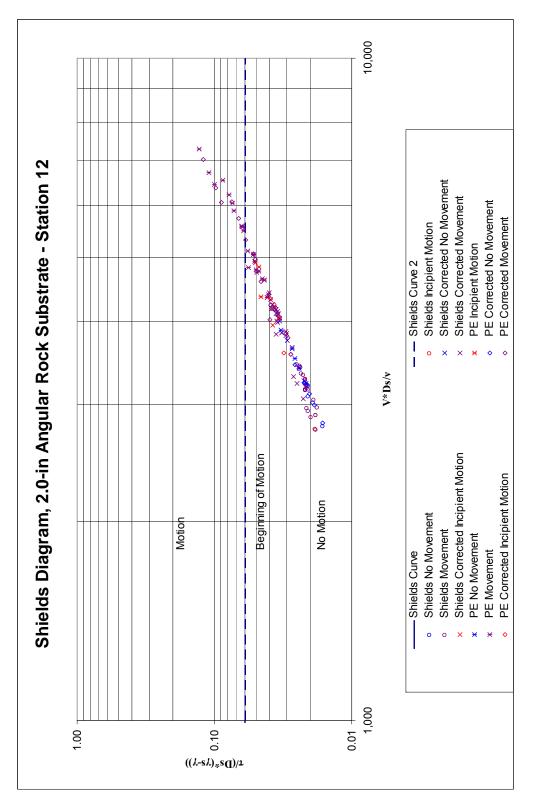


Figure C13. 2-inch angular rock substrate at station 12 in bottomless arch culvert plotted on shields relation for incipient motion

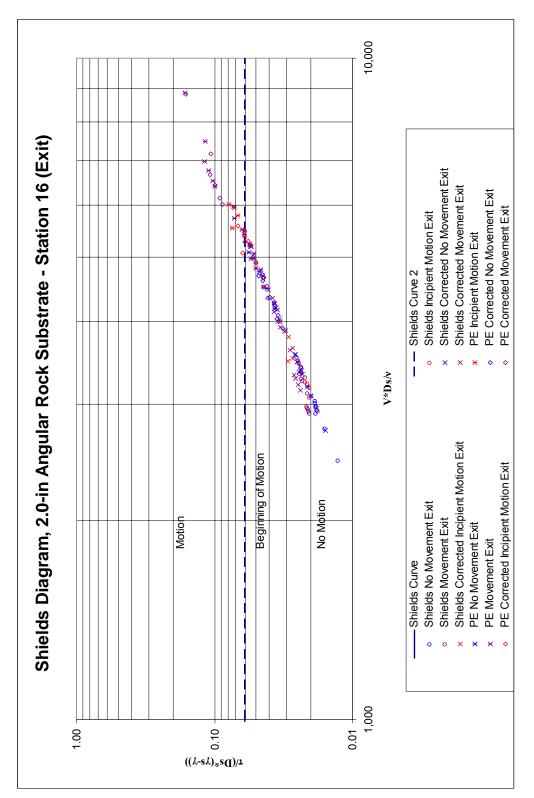


Figure C14. 2-inch angular rock substrate at station 16 in bottomless arch culvert plotted on shields relation for incipient motion

Appendix D

Shields Relation for Rectangular Flume

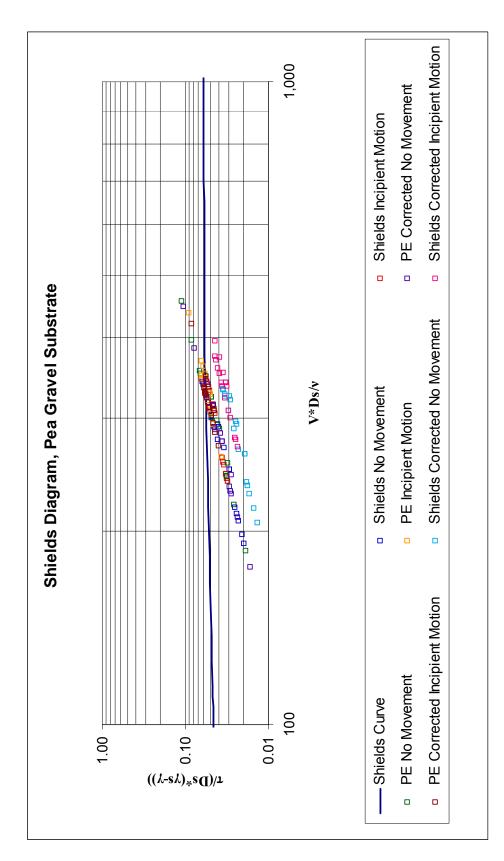


Figure D1. Pea gravel substrate in rectangular flume plotted on shields relation for incipient motion

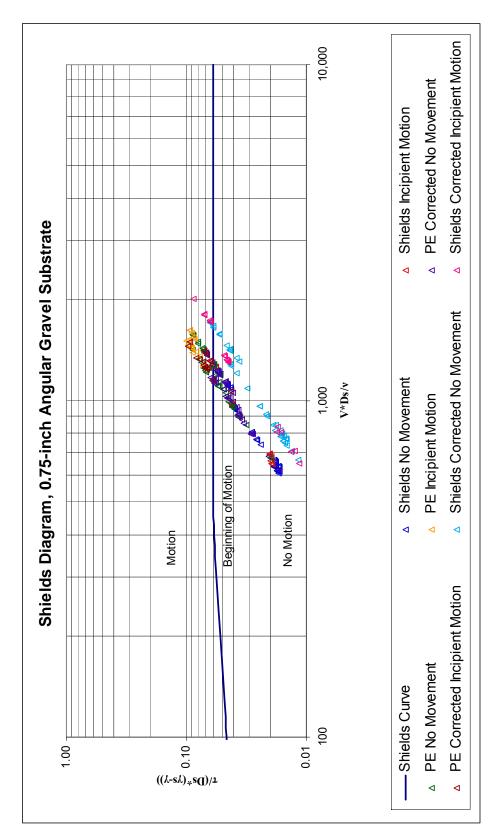


Figure D2. 0.75-inch angular gravel substrate in rectangular flume plotted on shields relation for incipient motion

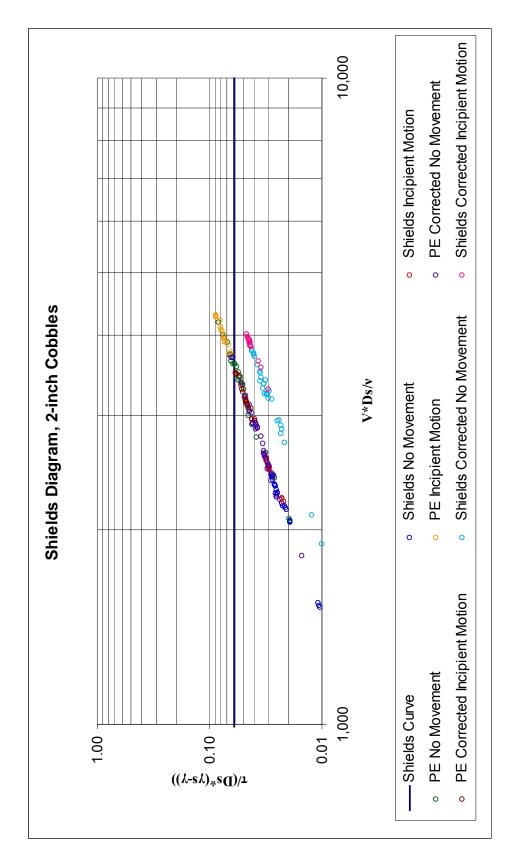


Figure D3. 2-inch cobble substrate in rectangular flume plotted on shields relation for incipient motion

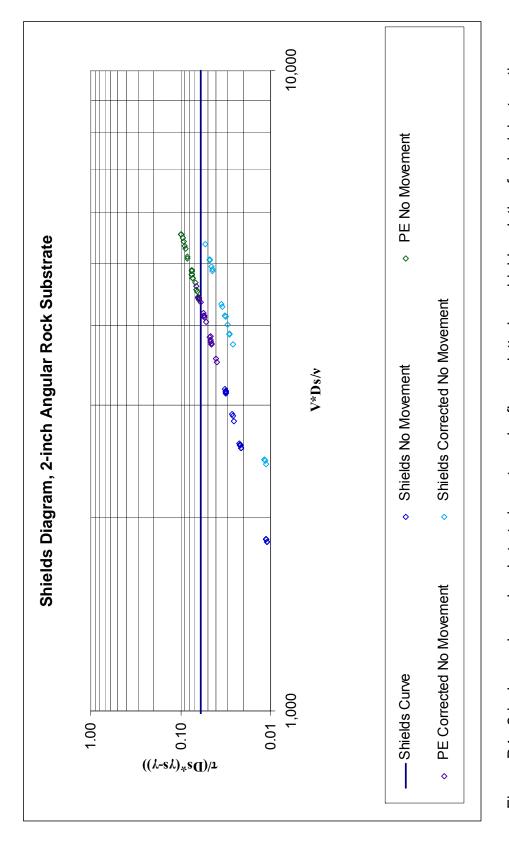


Figure D4. 2-inch angular rock substrate in rectangular flume plotted on shields relation for incipient motion

Appendix E

Streambed Response in Bottomless Arch Culvert

Table E1. Pea gravel substrate measured depths of scour in inches

	Entrance (Configuration				Location										
Run	Projecting	Contraction	Hw/D	Run Duration	Station	0	1	2	4	6	8	10	12	14	16	
(#)	(yes or no)	(0%,33%,75%)	()	(min)												
1	no	0%	0.50	123.00	Α	-0.25	0.00	0.00	0.00	0.13	0.00	0.00	-0.25	-0.50	-1.50	
					В	-0.75	-0.75	-1.00	-1.00	-0.75	-1.00	-1.38	-1.50	-1.63	-1.75	
					С	0.00	-0.50	-1.00	-1.38	-1.75	-0.50	-1.75	-1.88	-1.88	-1.88	
					D	-0.63	-0.50	-0.88	-1.00	-1.50	-1.63	-1.50 0.00	-1.50	-1.75	-1.88	
					E	-0.13 -0.35	0.00 -0.35	0.00 -0.58	-0.38 -0.75	0.00 -0.78	0.00 -0.63	-0.93	0.00 -1.03	0.00 -1.15	-0.75 -1.55	
					Average Min	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00	-0.75	
					Max	-0.75	-0.75	-1.00	-1.38	-1.75	-1.63	-1.75	-1.88	-1.88	-1.88	
2	no	33%	0.50	125.00	A	-3.50	-1.50	-0.25	-1.25	-0.13	0.00	0.00	0.00	-0.25	-1.00	
					В	-0.75	-0.63	-0.50	-0.88	-1.25	-1.75	-2.00	-1.88	-2.00	-1.75	
					С	0.00	-0.50	-0.50	-1.13	-1.50	-1.88	-1.88	-1.75	-1.88	-1.75	
					D	-2.00	-1.13	-0.50	-0.75	-0.63	-0.88	-1.25	-1.50	-1.75	-1.75	
					E	-4.00	-2.25	-1.50	-0.50	-0.13	0.00	0.00	-0.25	-0.75	-1.00	
					Average	-2.05	-1.20	-0.65	-0.90	-0.73	-0.90	-1.03	-1.08	-1.33	-1.45	
					Min	0.00	-0.50	-0.25	-0.50	-0.13	0.00	0.00	0.00	-0.25	-1.00	
3		75%	0.50	137.00	Max	-4.00 -4.25	-2.25 -1.63	-1.50 -1.00	-1.25 0.00	-1.50 -0.50	-1.88 0.00	-2.00 0.00	-1.88 0.00	-2.00 -0.50	-1.75 -1.00	
3	no	75%	0.50	137.00	A B	-4.25 -1.50	0.00	0.00	-0.25	-0.50	-1.13	-1.50	-1.38	-0.50 -1.75	-1.88	
					Č	0.00	0.00	0.00	-0.25	-0.63	-1.13	-1.50	-1.75	-2.13	-2.00	
					Ď	-1.75	-1.00	-0.25	-1.00	-1.50	-1.38	-1.50	-1.38	-1.50	-1.75	
					Ē	-4.13	-2.13	-2.00	-1.25	-0.75	-0.88	-0.50	-0.50	-0.75	-1.25	
					Average	-2.33	-0.95	-0.65	-0.55	-0.85	-0.93	-1.00	-1.00	-1.33	-1.58	
					Min	0.00	0.00	0.00	0.00	-0.50	0.00	0.00	0.00	-0.50	-1.00	
					Max	-4.25	-2.13	-2.00	-1.25	-1.50	-1.38	-1.50	-1.75	-2.13	-2.00	
4	yes	33%	1.00	10.00	Α	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
	*Excessive scour	, approximate depth	of -8 enter	red	В	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					С	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					D	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					E	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					Average Min	-8.00 -8.00										
					Max	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
5	yes	33%	0.50	120.00	A	-1.00	0.00	0.00	-0.13	-0.25	0.00	0.00	-0.75	-0.25	-1.13	
J	ycs	0070	0.50	120.00	В	-0.25	-0.44	-0.75	-0.75	-0.75	-1.38	-1.63	-2.00	-2.25	-2.25	
					C	-0.75	-0.25	-0.44	-0.63	-0.75	-1.13	-1.44	-1.75	-1.88	-2.00	
					D	-1.00	-0.44	-0.63	-1.00	-0.88	-1.00	-1.25	-1.38	-1.38	-1.88	
					E	-3.00	-1.25	-0.75	-0.50	-0.50	-0.44	-0.75	-0.63	-0.88	-0.63	
					Average	-1.20	-0.48	-0.51	-0.60	-0.63	-0.79	-1.01	-1.30	-1.33	-1.58	
					Min	-0.25	0.00	0.00	-0.13	-0.25	0.00	0.00	-0.63	-0.25	-0.63	
					Max	-3.00	-1.25	-0.75	-1.00	-0.88	-1.38	-1.63	-2.00	-2.25	-2.25	
6	yes	75%	0.50	120.00	A	-5.13	0.00	0.00	-0.25	0.00	-0.25	-0.25	-0.50	-1.00	-2.38	
					В	-3.00	0.25	0.00	-1.00	-1.13	-1.75	-2.25	-2.25	-2.13	-2.38	
					C D	-0.75 -3.50	0.75 -1.00	-0.75 -1.38	-1.25 -2.25	-1.50 -2.00	-2.25 -1.50	-2.13 -1.38	-1.50 -1.38	-1.63 -0.50	-2.38 -2.38	
					E	-5.75	-2.00	-2.00	-1.75	-0.88	-0.50	0.00	0.00	-0.13	-2.38	
					Average	-3.63	-0.40	-2.00	-1.75	-1.10	-1.25	-1.20	-1.13	-0.13	-2.38	
					Min	-0.75	0.75	0.00	-0.25	0.00	-0.25	0.00	0.00	-0.13	-2.38	
					Max	-5.75	-2.00	-2.00	-2.25	-2.00	-2.25	-2.25	-2.25	-2.13	-2.38	
7	yes	75%	1.00	15.00	A	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
		, approximate depth		red	В	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					С	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					D	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.0	
					E	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					Average	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					Min	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	
					Max	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	

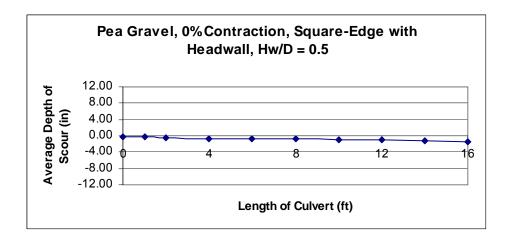


Figure E1. Pea gravel substrate average streambed profile (a)

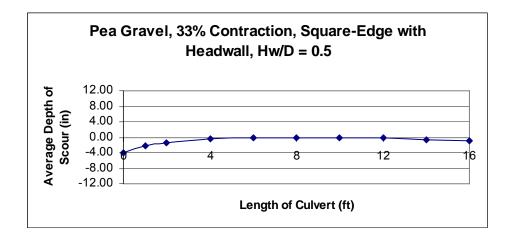


Figure E2. Pea gravel substrate average streambed profile (b)

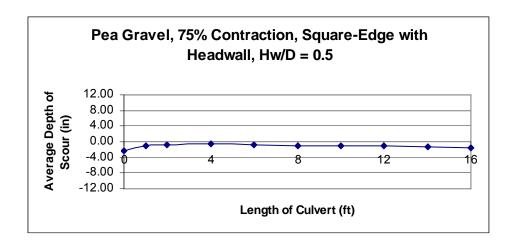


Figure E3. Pea gravel substrate average streambed profile (c)

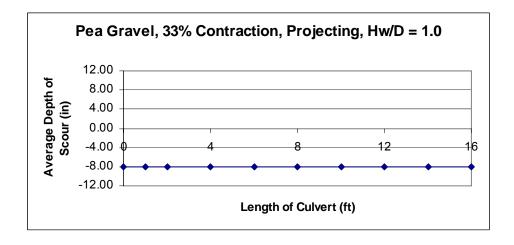


Figure E4. Pea gravel substrate average streambed profile (d)

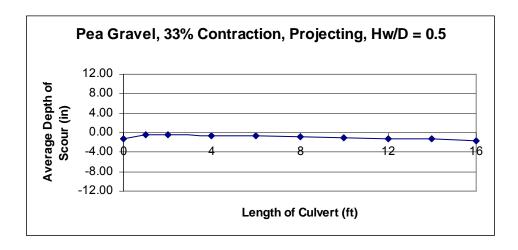


Figure E5. Pea gravel substrate average streambed profile (e)

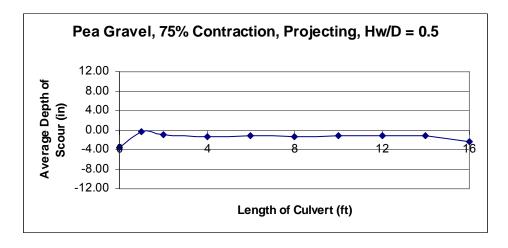


Figure E6. Pea gravel substrate average streambed profile (f)

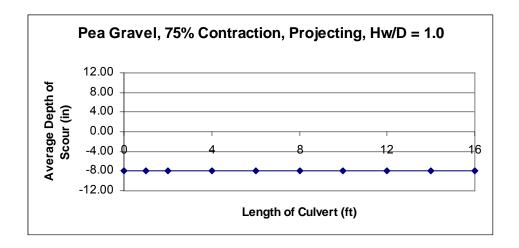


Figure E7. Pea gravel substrate average streambed profile (g)

Table E2. 0.75-inch angular gravel substrate measured depths of scour in inches

	Entrance Co						Location								
Run (#)	Projecting (yes or no)	Contraction (0%,33%,75%)	Hw/D	Run Duration (min)	Station	0	1	2	4	6	8	10	12	14	16
8	no	0%	1.00	128.00	Α	-0.88	-1.00	-1.00	-1.00	-1.13	-1.50	-1.75	-2.00	-2.75	-2.50
					В	-0.75	-1.13	-1.88	-1.75	-2.25	-2.38	-3.00	-3.25	-3.63	-4.00
					C D	-1.63 -1.00	-2.63	-2.50	-2.75	-3.25	-3.25	-3.38 -3.13	-3.50	-3.75	-4.00
					E	-0.88	-2.13 -1.25	-2.38 -1.00	-2.38 -1.50	-2.75 -2.00	-2.88 -2.38	-3.13	-3.00 -2.00	-3.25 -2.50	-3.38 -2.25
					Average	-1.03	-1.63	-1.75	-1.88	-2.28	-2.48	-2.55	-2.75	-3.18	-3.23
					Min	-0.75	-1.00	-1.73	-1.00	-1.13	-1.50	-1.50	-2.73	-2.50	-2.25
					Max	-1.63	-2.63	-2.50	-2.75	-3.25	-3.25	-3.38	-3.50	-3.75	-4.00
9 & 10	no	0%	1.25 & 1.5	70 & 11	A	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
	*bad runs - little data				В	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					С	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					D	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					E	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					Average	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					Min	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
					Max	-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.00
11	no	33%	0.75	90.00	Α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25
					В	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25
					С	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25
					D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25
					Ε	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25 -1.25
					Average Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25 -1.25
					Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.25 -1.25
12	no	33%	1.00	78.00	A	-3.00	0.50	0.00	-1.00	-1.13	-2.00	-1.50	-2.00	-3.00	-2.50
12	110	3370	1.00	70.00	В	0.00	-0.50	0.13	-0.63	-1.31	-2.13	-2.25	-2.25	-3.63	-3.50
					Č	0.25	-1.50	-1.75	-2.50	-2.38	-2.63	-2.38	-3.25	-4.38	-4.00
					Ď	0.00	-1.50	-1.13	-2.00	-1.50	-2.00	-2.50	-2.38	-2.75	-3.00
					E	-2.50	-1.13	-0.63	-1.50	-1.25	-1.00	-1.50	-2.00	-1.50	-2.25
					Average	-1.05	-0.83	-0.68	-1.53	-1.51	-1.95	-2.03	-2.38	-3.05	-3.05
					Min	0.25	0.50	0.13	-0.63	-1.13	-1.00	-1.50	-2.00	-1.50	-2.25
					Max	-3.00	-1.50	-1.75	-2.50	-2.38	-2.63	-2.50	-3.25	-4.38	-4.00
13	no	75%	1.00	130.00	Α	-4.75	-2.38	-1.75	-2.25	-1.88	-2.00	-3.38	-3.00	-3.25	-3.13
					В	-3.50	-2.75	-1.13	-2.00	-1.63	-2.25	-2.75	-3.00	-3.00	-3.00
					С	-0.25	-2.00	-1.50	-2.00	-2.00	-2.25	-2.75	-2.75	-3.63	-3.75
					D	-3.00	-1.88	-1.00	-0.88	-1.38	-2.13	-2.50	-2.75	-3.50	-3.50
					E	-3.00	-1.25	-0.38	-0.63	-0.50	-1.63	-2.63	-3.00	-3.50	-3.75
					Average Min	-2.90 -0.25	-2.05 -1.25	-1.15 -0.38	-1.55 -0.63	-1.48 -0.50	-2.05 -1.63	-2.80 -2.50	-2.90 -2.75	-3.38 -3.00	-3.43 -3.00
					Max	-0.25 -4.75	-1.25	-0.36	-0.03	-2.00	-2.25	-3.38	-3.00	-3.63	-3.75
14	yes	33%	1.00	128.00	A	-6.13	-3.00	-2.50	-2.25	-1.50	-2.00	-2.13	-2.50	-2.75	-3.88
	,00	0070	1.00	120.00	В	-4.38	-3.50	-2.75	-2.38	-2.25	-2.25	-2.63	-3.50	-3.75	-4.13
					c	-1.75	-1.00	-2.00	-2.63	-3.13	-4.00	-3.75	-4.50	-4.13	-4.38
					D	-1.13	-1.50	-1.50	-2.50	-2.38	-3.00	-3.13	-3.75	-3.88	-4.00
					E	-3.50	-2.75	-2.63	-2.50	-2.75	-2.00	-1.88	-2.75	-3.00	-3.50
					Average	-3.38	-2.35	-2.28	-2.45	-2.40	-2.65	-2.70	-3.40	-3.50	-3.98
					Min	-1.13	-1.00	-1.50	-2.25	-1.50	-2.00	-1.88	-2.50	-2.75	-3.50
L					Max	-6.13	-3.50	-2.75	-2.63	-3.13	-4.00	-3.75	-4.50	-4.13	-4.38
15	yes	75%	1.00	1047.00	A	-7.50	-5.25	-3.25	0.00	0.50	0.00	0.50	0.00	-0.25	-1.75
					В	-8.75	-6.00	-4.00	-1.25	-0.25	-1.00	-0.25	-0.88	-1.63	-2.13
					C D	-9.50 -8.00	-6.88 -5.75	-4.25 -3.00	-2.38 -2.75	-3.00 -2.63	-3.00 -4.13	-3.25 -4.38	3.50 -4.00	-3.75 -4.25	-4.13 -4.50
					E	-8.00	-5.75 -3.75	-3.00 -1.25	-2.75 -2.38	-2.63	-4.13 -4.50	-4.38 -3.88	-3.75	-4.25 -3.63	-4.50 -4.00
					Average	-9.15	-5.53	-3.15	-1.75	-1.68	-2.53	-2.25	-1.03	-2.70	-3.30
					Min	-7.50	-3.75	-1.25	0.00	0.50	0.00	0.50	3.50	-0.25	-3.30
					Max	-12.00	-6.88	-4.25	-2.75	-3.00	-4.50	-4.38	-4.00	-4.25	-4.50

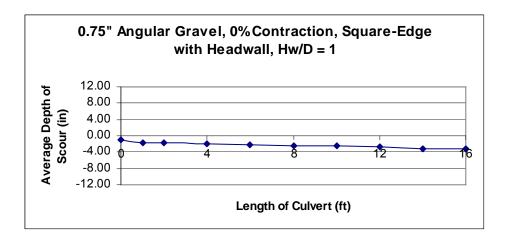


Figure E8. 0.75-inch angular gravel substrate average streambed profile (a)

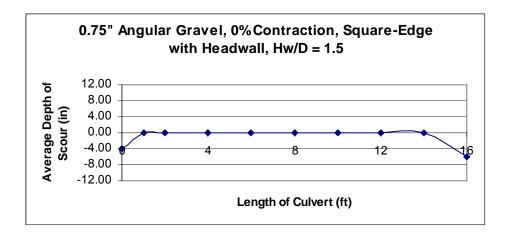


Figure E9. 0.75-inch angular gravel substrate average streambed profile (b)

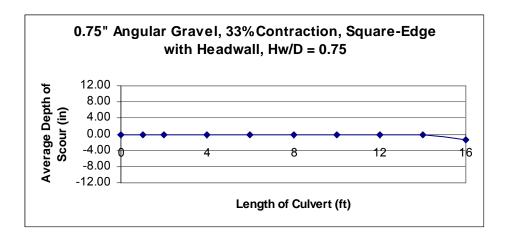


Figure E10. 0.75-inch angular gravel substrate average streambed profile ©

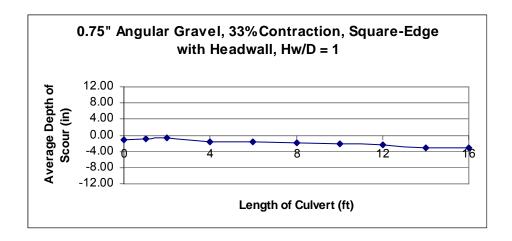


Figure E11. 0.75-inch angular gravel substrate average streambed profile (d)

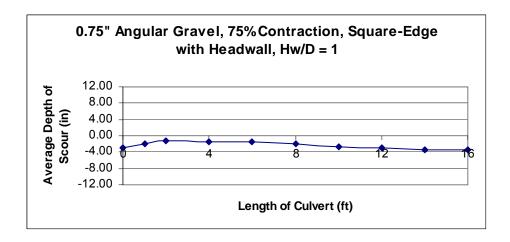


Figure E12. 0.75-inch angular gravel substrate average streambed profile (e)

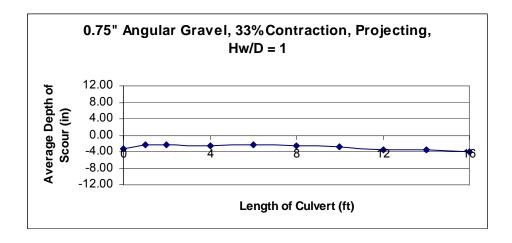


Figure E13. 0.75-inch angular gravel substrate average streambed profile (f)

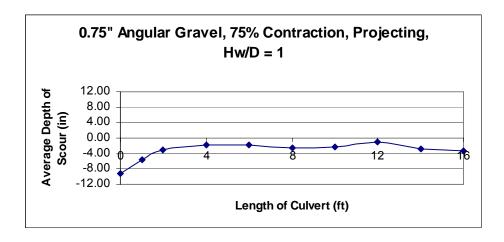


Figure E14. 0.75-inch angular gravel substrate average streambed profile (g)

Table E3. 2-inch cobble substrate measured depths of scour in inches

	Entrance (Configuration								Loca	ition				
Run (#)	Projecting (yes or no)	Contraction (0%.33%.75%)	Hw/D	Run Duration (min)	Station	0	1	2	4	6	8	10	12	14	16
16	no	0%	1.00	125.00	А	-0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.50
					B C	0.00	0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	-0.50 -1.00
					D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.75
					E Average	-1.00 -0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00 -0.75
					Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.50
17	no	0%	1.50	130.00	Max A	-1.00 -1.00	0.00	0.00	0.00	0.00	0.00	-0.50	0.00 -1.44	-3.13	-1.00 -3.50
"	110	078	1.50	130.00	В	-0.75	0.00	0.00	0.00	0.00	0.00	-0.50	-1.75	-3.50	-4.25
					C D	0.00 -0.25	0.00	0.00	0.00	0.00 0.00	0.00	-0.38 0.00	-1.13 -0.75	-3.25 -2.63	-4.00 -2.50
					E	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.75	-0.88	-3.00
					Average Min	-0.80 0.00	0.00	0.00	0.00	0.00	0.00	-0.28 0.00	-1.06 -0.25	-2.68 -0.88	-3.45 -2.50
					Max	-2.00	0.00	0.00	0.00	0.00	0.00	-0.50	-1.75	-3.50	-2.50 -4.25
18	no	33%	1.00	120.00	A B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	no	33%	1.50	120.00	Max A	0.00 -2.50	0.00 -0.50	0.00 1.25	1.00	0.00 -0.50	0.00	0.00 -4.00	0.00 -2.25	0.00 -2.25	0.00 -3.38
					B C	-1.00 0.75	-0.75	1.00 2.00	1.50 3.00	0.00 -0.63	-2.00 -1.00	-3.50 -3.75	-2.25 -2.50	-2.25 -2.50	-3.50 -3.13
					D	-1.63	-1.13 0.50	1.50	2.00	-0.63 1.13	-1.00 -0.88	-3.75 -3.00	-2.50 -2.75	-2.50 -2.75	-3.13 -3.00
					E	-3.25	-0.75	1.75	1.50	2.00	-0.50	-1.75	-3.50	-3.50	-2.00
					Average Min	-1.53 0.75	-0.53 0.50	1.50 2.00	1.80 3.00	0.40 2.00	-0.73 0.75	-3.20 -1.75	-2.65 -2.25	-2.65 -2.25	-3.00 -2.00
					Max	-3.25	-1.13	1.00	1.00	-0.63	-2.00	-4.00	-3.50	-3.50	-3.50
20	no	75%	1.00	125.00	A B	0.00	0.00 0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00 0.00
					С	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D E	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00
					Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Min Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	no	75%	1.50	130.00	Α	-3.50	1.50	1.25	0.00	-1.00	0.00	-1.00	-2.00	-2.13	-3.13
					B C	-2.50 -1.75	0.75 1.13	1.50 0.00	0.50 0.13	0.00 -1.00	-1.25 0.00	-1.13 -2.13	-3.13 -4.00	-4.00 -4.00	-3.63 -3.50
					D	-1.13	0.00	0.75	0.00	-0.75	-0.75	-0.75	-3.00	-3.75	-4.00
					E Average	-3.50 -2.48	1.25 0.93	2.00 1.10	0.75 0.28	0.00 -0.55	0.50	1.00 -0.80	-2.25 -2.88	-3.75 -3.53	-3.75 -3.60
					Min	-1.13	1.50	2.00	0.75	0.00	0.50	1.00	-2.00	-2.13	-3.13
22	yes	33%	1.00	35.00	Max A	-3.50 0.00	0.00	0.00	0.00	-1.00 0.00	-1.25 0.00	-2.13 0.00	-4.00 0.00	-4.00 0.00	-4.00 0.00
	,				В	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					C D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					E	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average Min	0.00	0.00 0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00 0.00
					Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23	yes	33%	1.50	135.00	A B	-3.88 -1.75	1.00 1.75	1.75 2.25	1.00 2.00	1.25	-0.75 -1.50	-0.75 -1.50	-2.00 -3.00	-3.25 -2.25	-3.25 -3.25
					С	-2.25	-0.50	0.00	1.13	0.25	-1.00	-1.00	-1.75	-2.00	-4.25
					D F	-4.50 -5.75	-2.00 -1.25	-0.25 0.50	1.00 1.50	0.00 1.00	-1.25 -0.75	-2.00 -2.25	-2.75 -2.50	-3.13 -3.50	-4.50 -3.75
					Average	-3.63	-0.20	0.85	1.33	0.90	-1.05	-1.50	-2.40	-2.83	-3.80
					Min Max	-1.75 -5.75	1.75 -2.00	2.25 -0.25	2.00 1.00	2.00 0.00	-0.75 -1.50	-0.75 -2.25	-1.75 -3.00	-2.00 -3.50	-3.25 -4.50
24	yes	75%	1.00	55.00	A	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					B C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					E Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ne.	1/00	750/	1.50	120.00	Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	yes	75%	1.50	129.00	A B	-5.00 -4.50	0.00 -3.25	0.00 -0.25	2.00 1.75	1.50 2.00	-2.50 -3.75	-3.25 -3.00	-1.75 -1.50	-3.50 -2.75	-3.25 -3.25
					С	-3.75	-4.00	-1.50	2.00	2.25	-3.50	-4.00	-2.25	-3.88	-4.50
					D E	-4.25 -5.50	-2.00 -0.25	-0.75 0.75	1.50 0.75	2.00 -0.50	-2.00 -2.00	-2.75 -3.00	-2.00 -3.00	-2.75 -1.75	-5.00 -3.50
					Average	-4.60	-1.90	-0.35	1.60	1.45	-2.75	-3.20	-2.10	-2.93	-3.90
					Min Max	-3.75 -5.50	0.00 -4.00	0.75 -1.50	2.00 0.75	2.25 -0.50	-2.00 -3.75	-2.75 -4.00	-1.50 -3.00	-1.75 -3.88	-3.25 -5.00
					- nux	3.00	1.00			2.00	20	1.00	3.00	3.00	2.00

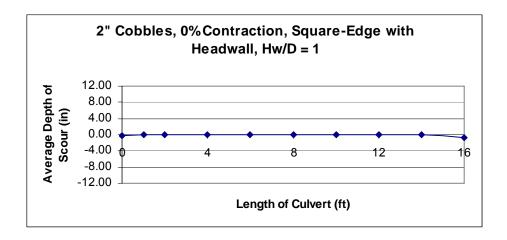


Figure E15. 2-inch cobble substrate average streambed profile (a)

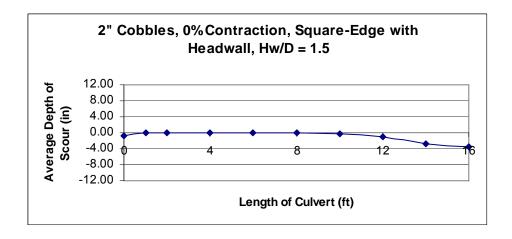


Figure E16. 2-inch cobble substrate average streambed profile (b)

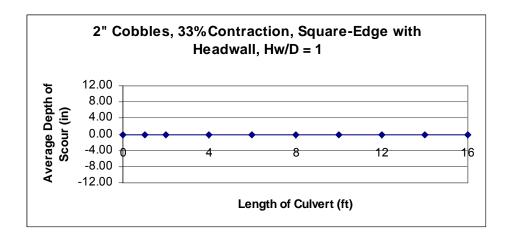


Figure E17. 2-inch cobble substrate average streambed profile (c)

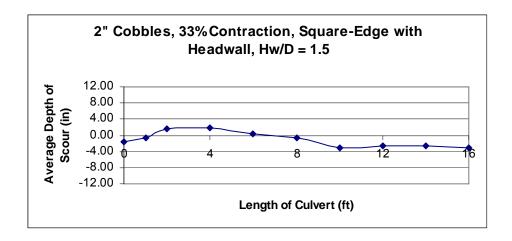


Figure E18. 2-inch cobble substrate average streambed profile (d)

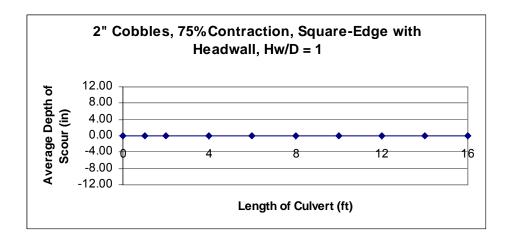


Figure E19. 2-inch cobble substrate average streambed profile (e)

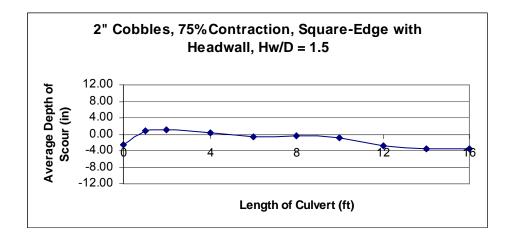


Figure E20. 2-inch cobble substrate average streambed profile (f)

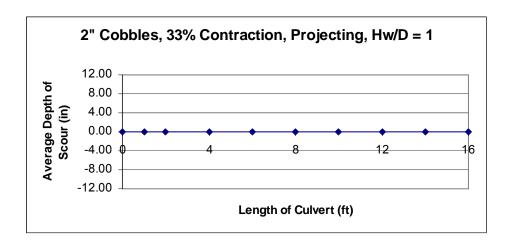


Figure E21. 2-inch cobble substrate average streambed profile (g)

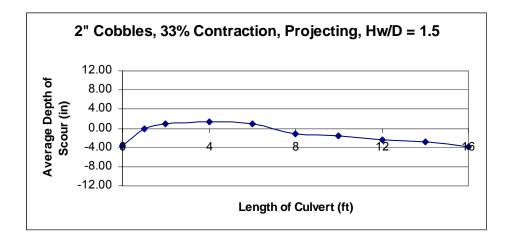


Figure E22. 2-inch cobble substrate average streambed profile (h)

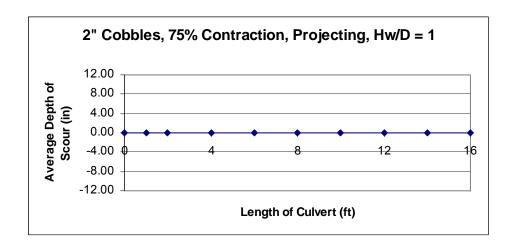


Figure E23. 2-inch cobble substrate average streambed profile (i)

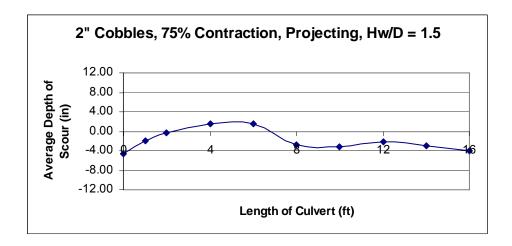


Figure E24. 2-inch cobble substrate average streambed profile (j)

Table E4. 2-inch angular gravel substrate measured depths of scour in inches

	Entrance C	Configuration								Loca	ation				
Run (#)	Projecting (yes or no)	Contraction (0%, 33%, 75%)	Hw/D	Run Duration (min)	Station	0	1	2	4	6	8	10	12	14	16
26	no	0%	1.00	107.00	Α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					B C	0.00	0.00	0.00 0.00	0.00	0.00 0.00	0.00	0.00 0.00	0.00	0.00 0.00	0.00 0.00
					D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					E Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00
					Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	no	0%	1.50	122.00	Max A	0.00 -1.75	0.00	0.00	0.00	0.00	0.00 -0.50	0.00 -2.00	0.00 -3.25	0.00 -3.25	0.00 -2.50
		0,0	1.00	122.00	В	0.00	0.00	0.00	0.00	0.00	0.00	-2.75	-4.25	-4.75	-4.25
					C D	-0.50 -1.00	-0.75 -0.50	0.00	0.00	0.00	-0.75 -0.68	-3.38 -1.50	-4.38 -3.00	-4.38 -3.50	-4.25 -3.13
					E	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	-2.00	-3.50	-3.00
					Average Min	-0.85 0.00	-0.25 0.00	0.00	0.00	0.00	-0.39 0.00	-1.93 0.00	-3.38 -2.00	-3.88 -3.25	-3.43 -2.50
					Max	-1.75	-0.75	0.00	0.00	0.00	-0.75	-3.38	-4.38	-4.75	-4.25
28	no	33%	1.00	115.00	A B	0.00	0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00 0.00
					С	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D E	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00
					Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Min Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	no	33%	1.50	120.00	A	-4.00	0.50	0.75	0.00	0.00	-0.50	-2.38	-2.75	-3.00	-4.25
					В	-2.50	-2.00	0.75	0.00	0.50	-1.00	-2.75	-3.50	-3.50	-4.00
					C D	-1.50 -1.25	-2.13 -1.50	-0.50 0.00	0.00	0.75 0.00	-0.88 -0.75	-1.50 -0.75	-4.00 -4.00	-3.50 -2.25	-2.75 -2.75
					E	-2.25	-1.00	0.00	0.00	-0.25	-0.50	-0.50	-2.75	-2.25	-3.50
					Average Min	-2.30 -1.25	-1.23 0.50	0.20 0.75	0.00	0.20 0.75	-0.73 -0.50	-1.58 -0.50	-3.40 -2.75	-2.90 -2.25	-3.45 -2.75
		750/	4.00	44.00	Max	-4.00	-2.13	-0.50	0.00	-0.25	-1.00	-2.75	-4.00	-3.50	-4.25
30	no	75%	1.00	41.00	A B	0.00	0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00 0.00
					С	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D E	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Min Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00
31	no	75%	1.50	140.00	Α	-3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-2.75	-4.00
					B C	-1.50 -1.25	0.00 -2.75	1.50 -1.50	0.00	0.00	0.00	0.00	-2.00 -2.25	-3.50 -4.50	-5.25 -3.38
					D	-2.50	-1.88	-0.50	0.00	0.00	0.00	-1.50	-2.00	-4.00	-4.50
					E Average	-4.00 -2.45	0.00 -0.93	0.50 0.00	0.00	0.00	0.00	0.00 -0.30	-1.75 -1.60	-2.75 -3.50	-4.25 -4.28
					Min	-1.25	0.00	1.50	0.00	0.00	0.00	0.00	0.00	-2.75	-3.38
32	yes	33%	1.00	30.00	Max A	-4.00 0.00	-2.75 0.00	-1.50 0.00	0.00	0.00	0.00	-1.50 0.00	-2.25 0.00	-4.50 0.00	-5.25 0.00
02	,00	0070	1.00	00.00	В	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					C D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00
					Е	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
33	yes	33%	1.50	120.00	A B	-2.75 -2.00	-1.75 -2.25	1.75 0.00	0.00 1.00	-1.00 0.00	0.00	0.00	0.00 -1.00	0.00 -1.00	0.00 -3.00
					С	0.00	-2.00	0.50	0.00	0.00	0.00	0.00	-1.00	-1.00	-3.50
					D F	-3.00 -4.00	-0.50 0.75	1.50 0.50	0.50	0.50 1.25	0.00	0.00	0.50 -0.75	-0.75 0.00	-2.00 0.00
					Average	-2.35	-1.15	0.85	0.40	0.15	0.00	0.00	-0.75	-0.55	-1.70
					Min Max	0.00 -4.00	0.75 -2.25	1.75	1.00	1.25 -1.00	0.00	0.00	0.50	0.00 -1.00	0.00 -3.50
34	yes	75%	1.00	30.00	Α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	-				B C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					D	0.00	0.00	0.00	0.00	0.00 0.00	0.00	0.00	0.00	0.00	0.00 0.00
					E	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					Average Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 0.00
0.5		750/	4.50	400.00	Max	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
35	yes	75%	1.50	120.00	A B	-3.50 -2.00	-1.50 -1.75	1.50 1.50	2.00 2.25	0.00	0.00	0.00	0.00	0.00 -1.25	0.00 -3.50
					С	-2.38	-2.50	-1.50	1.75	0.00	0.00	0.00	0.00	-1.00	-4.50
					D E	-3.00 -4.00	-2.25 0.00	-2.25 1.75	1.50 0.00	0.00 0.00	0.00	0.00	0.00	0.00	-3.00 -1.00
					Average	-2.98	-1.60	0.20	1.50	0.00	0.00	0.00	0.00	-0.45	-2.40
					Min Max	-2.00 -4.00	0.00 -2.50	1.75 -2.25	2.25 0.00	0.00	0.00	0.00	0.00	0.00 -1.25	0.00 -4.50
					IVIGA	7.00	-2.00	-2.20	0.00	0.00	0.00	0.00	0.00	-1.20	7.00

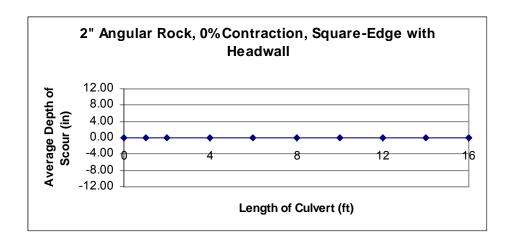


Figure E25. 2-inch angular rock substrate average streambed profile (a)

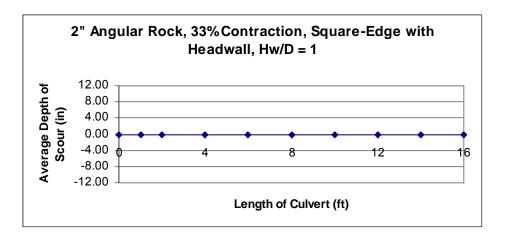


Figure E26. 2-inch angular rock substrate average streambed profile (b)

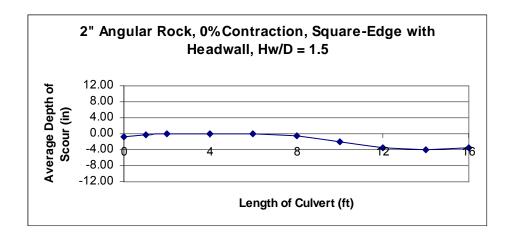


Figure E27. 2-inch angular rock substrate average streambed profile (c)

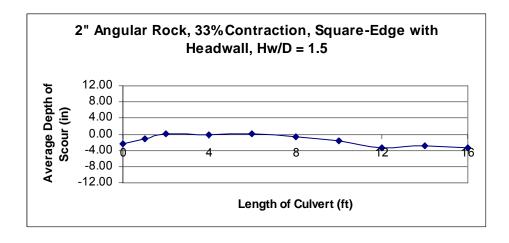


Figure E28. 2-inch angular rock substrate average streambed profile (d)

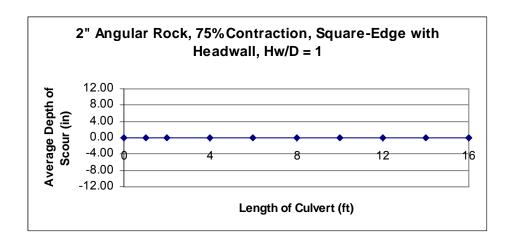


Figure E29. 2-inch angular rock substrate average streambed profile (e)

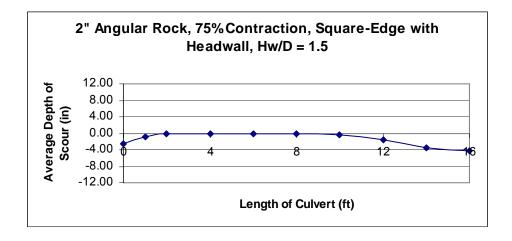


Figure E30. 2-inch angular rock substrate average streambed profile (f)

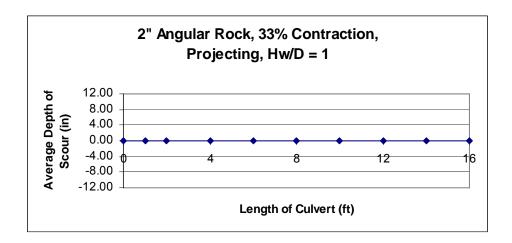


Figure E31. 2-inch angular rock substrate average streambed profile (g)

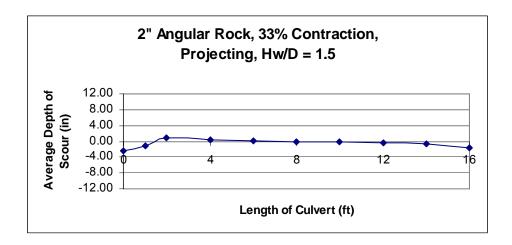


Figure E32. 2-inch angular rock substrate average streambed profile (g)

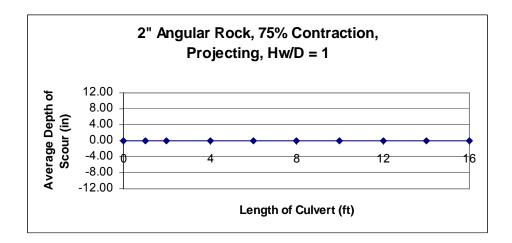


Figure E33. 2-inch angular rock substrate average streambed profile (i)

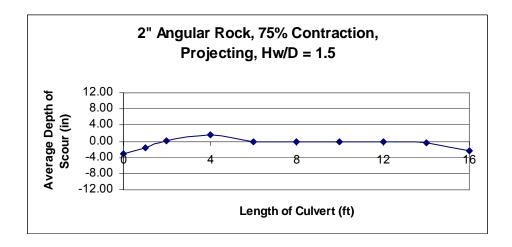


Figure E34. 2-inch angular rock substrate average streambed profile (j)

Appendix F

Riprap Analyses Results

Table F1. 0.75-inch angular gravel substrate in bottomless arch culvert riprap results

				Exit				USACE EM-1	601 (Eq. 6)	USBR - EM - 25 (Eq. 8	- 25 (Eq. 8)	Halvorson (Eq. 3)	(Eq. 3)	HEC 11 (Eq. 4)	Cal. B&SP	(Eq. 5) A	ASCE Manual 54	154 (Eq. 7)	FHWA PH	1 (Eq. 1)	FHWA PH 2	2 (Eq. 2)
				Mean				Predicted		Predicted		Predicted		Predicted		Predicted		Predicted		Predicted	Ī	Predicted	
		Inlet	ontraction	Bed	Actual	Actual	Actual	Design	Average	Design	Average	Design	Average	Design	Average	Design /	Average	Design	Average	Design	Average	Design	Average
Run	Hw/D P.	rojecting	Ratio	Velocity	q ₃₀	d ₄₀	q ₅₀	q ³⁰	SF	d ₄₀	ŞF	q ₂₀	SF	q ₅₀	SH **	d ₅₀	SF.	q ₂₀	SF.	d ₅₀	SF.	d ₅₀	SF.
#)	Š	(es or no)	(%)	(ft/s)	£	ŧ	Œ	Œ	0	Œ	0	Œ	С	Œ	0	Œ	С	Œ	0	Œ	0	æ	С
12 incipient	0.75	ou	33%	4.00	0.046	0.050	0.053	0.0262	0.57	0.183	3.65	0.294	5.56	0.583	11.00	660'0	1.87	0.118	2.23	0.118	2.22	0.130	2.45
13 incipient	0.75	00	75%	3.88	0.046	0.050	0.053	0.0243	0.53	0.171	3.43	0.273	5.16	0.517	9.75	0.093	1.76	0.111	2.10	0.111	5.09	0.127	2.40
14 incipient	0.72	yes	33%	3.12	0.046	0.050	0.053	0.0141	0.31	0.109	2.19	0.161	3.03	0.218	4.12	0.060	1.14	0.072	1.36	0.071	1.35	0.110	2.08
15 incipient	1.00	yes	75%	3.86	0.046	0.050	0.053	0.0240	0.52	0.170	3.39	0.270	5.09	0.506	9.55	0.092	1.74	0.110	2.08	0.109	2.07	0.127	2.40

Table F2. 2-inch cobble substrate in bottomless arch culvert riprap results

								USACE EM-1	601 (Eq. 6)	USBR - EM -	· 25 (Eq. 8)	Halvorson	(Eq. 3)	HEC 11 (E	≦q. 4)	Cal. B&SP	(Eq. 5) A	4SCE Manua	al 54 (Eq. 7)	FHWAPH	1 (Eq. 1)	FHWA PH	2 (Eq. 2)
				exit				Predicted		Predicted		Predicted		Predicted	Ē	Predicted		Predicted		Predicted		Predicted	
		Inlet	Contraction	Mean	Actual	Actual	Actual	Design	Average	Design	Average	Design	Average	Design	Average	Design	Average	Design	Average	Design	Average	Design	Average
	Hw/D	Projecting	Ratio	Velocity	q ₃₀	d ₄₀	q ₅₀	q ₃₀	SF	d ₄₀	₽₽	d ₅₀	SF **	d ₅₀	SF.	d ₅₀	S **	d ₅₀	SF	d ₅₀	SF **	d ₅₀	SF.
(#)	0	(yes or no)	(%)	(ft/s)	æ	£	Œ	Œ	0	Œ	0	£	С	ŧ	С	£	\Box	£	С	(£)	Э	Œ	0
incipient	1.31	ou	%0	5.56	960'0	0.103	0.108	0.1184	1.24	0.360	3.51	0.658	6.07	2.140	19.76	0.177	1.63	0.211	1.95	0.210	1.94	0.149	1.38
incipient	1.25	91	33%	4.64	960.0	0.103	0.108	0.0754	0.79	0.248	2.42	0.423	3.91	1.047	29.6	0.123	1.14	0.147	1.36	0.146	1.35	0.132	1.22
incipient	1.35	91	75%	5.82	960.0	0.103	0.108	0.1328	1.39	0.395	3.86	0.735	6.79	2.563	23.67	0.194	1.79	0.231	2.14	0.230	2.12	0.154	1.42
3 incipient	1.38	yes	33%	5.56	960.0	0.103	0.108	0.1184	1.24	0.360	3.51	0.658	6.07	2.140	19.76	0.177	1.63	0.211	1.95	0.210	1.94	0.149	1.38
25 incipient	1.42	yes	75%	6.02	960.0	0.103	0.108	0.1445	1.51	0.424	4.13	0.798	7.37	2.929	27.05	0.207	1.92	0.248	2.29	0.246	2.27	0.157	1.45

Table F3. 2-inch angular rock substrate in bottomless arch culvert riprap results

								USACE EM-16	301 (Eq. 6)	USBR - EM - 25	25 (Eq. 8)	Halvorson (Eq	1.3) H	HEC 11 (Eq. 4	4)	al. B&SP (Ed	d. 5) A	SCE Manual	54 (Eq. 7) F	FHWA PH 1	(Eq. 1)	HWA PH 2	Eq. 2)
				exit				Predicted		Predicted		Predicted		Predicted		Predicted		Predicted		Predicted		Predicted	
		Inlet	Contraction	Mean	Actual	Actual	Actual	Design	Average	Design	Average	Design /	Average	Design	Average	Design	Average	Design	Average	Design	Average	Design	Average
Run	D/WH	Projecting	Ratio	Velocity	о [©] р	d ₄₀	d ₅₀	q ₃₀	-R	d ₄₀	SF÷	d ₅₀	S **	d ₅₀	S **	d ₅₀	SF	d ₅₀	SF	d ₅₀	SF.	d ₅₀	S ++
(#)	\Box	(yes or no)	(%)	(ft/s)	(ft)	(tt)	(#)	(tt)	0	(ft)	С	(ft)	С	(t)	\Box	(£)	0	(tt)	0	(ft)	\Box	(H)	\odot
27 incipient	1.29	ou	%0	5.55	860.0	0.109	0.123	0.1827	1.86	0.358	3.28	0.655	5.34	2.124	17.33	0.196	1.60	0.235	1.91	0.233	1.90	0.166	1.36
29 incipient	1.46	01	33%	4.94	0.098	0.109	0.123	0.1365	1.39	0.282	2.58	0.493	4.02	1.341	10.94	0.156	1.27	0.186	1.52	0.185	1.51	0.154	1.26
31 incipient	1.38	01	75%	5.81	0.098	0.109	0.123	0.2048	2.08	0.394	3.60	0.732	5.97	2.546	20.76	0.215	1.76	0.257	2.10	0.255	2.08	0.171	1.40
33 incipient	1.46	yes	33%	99.9	0.098	0.109	0.123	0.1919	1.95	0.373	3.41	0.687	2.60	2.296	18.72	0.204	1.67	0.244	1.99	0.242	1.98	0.168	1.37
35 incipient	1.50	yes	75%	2.97	0.098	0.109	0.123	0.2192	2.23	0.417	3.81	0.782	6.38	2.834	23.11	0.227	1.85	0.271	2.21	0.270	2.20	0.174	1.42

Table F4. Pea gravel substrate in rectangular flume riprap results

FHWA PH 2		Average	SF.	С	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0
FHW/	Predicted	Design	q ₂₀	(#)	0.057	0.056	0.058	0.048	0.051	0.051	0.040	0.041	0.043	0.048	0.046	0.049	0.084	0.085	0.088	0.082	0.079	000
PH 1		Average	SF *÷	\Box	60.0	0.09	0.10	0.10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.11	0.11	0.10	0.10	7
FHWA PH 1	Predicted	Design	d ₅₀	(ft)	0.052	0.065	0.043	0.039	0.055	0.055	0.042	0.043	0.045	0.050	0.052	0.051	0.067	0.056	0.058	0.064	0.062	010
nual 54		Average	SF +÷	С	2.34	2.96	1.97	1.79	2.49	2.51	1.91	1.97	2.02	2.29	2.34	2.33	3.04	2.56	2.65	2.92	2.82	000
ASCE Manual 54	Predicted	Design	d ₅₀	(ft)	0.052	990.0	0.044	0.040	0.055	0.055	0.042	0.044	0.045	0.051	0.052	0.052	0.067	0.057	0.059	0.065	0.063	0.054
SP		Average	SF **	\Box	2.45	3.09	5.06	1.86	2.60	2.62	1.99	2.06	2.11	2.39	2.45	2.43	3.17	2.67	2.77	3.05	2.95	000
Cal. B&SP	Predicted	Design	d ₅₀	(#)	0.054	0.068	0.046	0.041	0.058	0.058	0.044	0.046	0.047	0.053	0.054	0.054	0.070	0.059	0.061	0.068	0.065	0 0 0
1		Average	S +÷	\Box	12.48	23.17	8.01	8.40	17.28	17.94	13.40	13.99	13.78	15.70	17.37	15.79	13.11	8.34	8.59	12.19	11.95	90
HEC 11	Predicted	Design	d ₅₀	(#)	0.276	0.513	0.177	0.186	0.383	0.397	0.297	0.310	0.305	0.348	0.385	0.350	0.290	0.185	0.190	0.270	0.265	7010
son		Average	SF.	\odot	4.96	6.59	4.02	3.56	5.33	5.38	3.86	4.02	4.14	4.82	4.96	4.91	6.81	5.52	5.77	6.48	6.22	007
Halvorson	Predicted	Design	d ₅₀	(#)	0.110	0.146	0.089	0.079	0.118	0.119	0.086	0.089	0.092	0.107	0.110	0.109	0.151	0.122	0.128	0.144	0.138	7010
M - 25		Average	SF	\Box	3.87	4.92	3.24	2.92	4.11	4.15	3.14	3.24	3.33	3.78	3.87	3.84	5.06	4.24	4.40	4.86	4.69	2 70
USBR - EM	Predicted	Design	d ₄₀	(ft)	0.079	0.101	0.067	0.060	0.084	0.085	0.064	0.067	0.068	0.078	0.079	0.079	0.104	0.087	0.090	0.100	960.0	0 0 0 0
CE EM-1601		Average	SF	\Box	0.54	0.76	0.43	0.40	0.62	0.63	0.47	0.49	0.50	0.56	0.59	0.57	0.68	0.53	0.55	0.64	0.62	37.0
	Predicted	Design	q ₃₀	(#)	8600.0	0.0136	0.0077	0.0072	0.0111	0.0113	0.0085	0.0088	0.0089	0.0102	0.0106	0.0103	0.0122	9600.0	0.0099	0.0116	0.0112	0000
		Actual	d ₅₀	(tt)	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	000
		Actual	d ₄₀	(tf)	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	60.0
		Actual	q ₃₀	(tt)	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	010
		Bed	Velocity	(ft/s)	2.67	3.00	2.45	2.33	2.75	2.76	2.41	2.45	2.48	2.64	2.67	2.66	3.04	2.79	2.84	2.98	2.93	200
			Depth	(tt)	0.22	0.19	0.25	0.20	0.18	0.18	0.14	0.15	0.16	0.17	0.16	0.18	0.35	0.39	0.40	0.35	0.33	07.0
			Run	#)	106	107	108	115	116	117	121	122	123	127	128	129	133	134	135	139	140	177

Table F5. 0.75-inch angular gravel substrate in rectangular flume riprap results

_				_																	
	Average	Ŗ	С	0.04	0.04	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.05	200
Predicted	Design	d ₅₀	Œ	0.212	0.207	0.211	0.185	0.203	0.196	0.251	0.253	0.238	0.237	0.232	0.221	0.177	0.176	0.182	0.202	0.171	000
	Average	SF.	С	0.14	0.15	0.14	0.15	0.14	0.15	0.15	0.15	0.14	0.15	0.14	0.14	0.14	0.14	0.14	0.14	0.15	77
Predicted	Design	d ₅₀	Œ	0.164	0.151	0.175	0.120	0.161	0.147	0.178	0.191	0.185	0.162	0.182	0.171	0.150	0.145	0.164	0.179	0.111	777
	Average	SF.	Э	3.11	2.88	3.32	2.28	3.06	2.80	3.38	3.63	3.52	3.07	3.46	3.26	2.85	2.75	3.11	3.41	2.10	000
Predicted	Design	d ₅₀	Œ	0.165	0.152	0.176	0.121	0.162	0.148	0.179	0.192	0.186	0.163	0.183	0.172	0.151	0.146	0.165	0.180	0.111	0 7 7 0
	Average	SF.	\Box	3.21	2.97	3.44	2.35	3.16	2.90	3.49	3.75	3.64	3.17	3.58	3.37	2.95	2.84	3.21	3.52	2.17	07.0
Predicted	Design	d ₅₀	Œ	0.170	0.158	0.182	0.125	0.167	0.153	0.185	0.199	0.193	0.168	0.190	0.178	0.156	0.151	0.170	0.187	0.115	707
	Average	Ŗ,	\Box	11.61	9.88	13.81	6.62	11.89	10.15	10.89	12.93	13.16	9.39	13.13	12.19	12.46	11.42	14.80	15.73	6.11	70
Predicted	Design	q	Œ	0.615	0.523	0.732	0.351	0.630	0.538	0.577	0.685	0.697	0.498	969.0	0.646	0.660	0.605	0.784	0.834	0.324	0000
	Average	Ŗ,	0	8.32	7.57	9.03	5.69	8.15	7.33	9.21	10.05	9.67	8.19	9.49	8.80	7.49	7.17	8.32	9.30	5.16	97.0
Predicted	Design	d ₅₀	Œ	0.441	0.401	0.478	0.302	0.432	0.388	0.488	0.533	0.513	0.434	0.503	0.466	0.397	0.380	0.441	0.493	0.273	907
	Average	Ş₽÷	\Box	5.13	4.74	5.50	3.73	5.05	4.61	5.59	6.02	5.83	5.07	5.73	5.38	4.70	4.53	5.13	5.64	3.43	6.0
Predicted	Design	d ₄₀	Œ	0.257	0.237	0.275	0.186	0.252	0.231	0.280	0.301	0.292	0.253	0.287	0.269	0.235	0.226	0.257	0.282	0.171	0 0 0
	Average	SF	C	1.13	1.02	1.24	0.78	1.12	1.01	1.19	1.31	1.28	1.06	1.27	1.19	1.07	1.02	1.20	1.30	0.71	707
Predicted	Design	q ₃₀	Œ	0.0519	0.0470	0.0570	0.0356	0.0515	0.0463	0.0546	0.0601	0.0589	0.0489	0.0581	0.0545	0.0493	0.0470	0.0550	0.0599	0.0328	0.0507
<u> </u>		d ₅₀	Œ	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	000
	Actual	d	(£)	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	000
	Actual	q ₃₀	Œ	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	010
	Bed	Velocity	(tt/s)	4.72	4.54	4.88	4.04	4.68	4.48	4.92	5.10	5.02	4.69	4.98	4.83	4.52	4.44	4.72	4.94	3.88	5
		Depth	ŧ	0.88	0.89	0.85	0.84	0.83	0.82	1.09	1.06	0.98	1.05	96.0	0.92	0.70	0.71	0.70	0.78	0.78	0
		Run	#	58	29	09	29	89	69	9/	77	78	82	98	87	94	92	96	103	104	101
	1 Predicted Predicted Predicted P	Actual Actual Actual Actual Actual Average Design A	Predicted Predic	Predicted Pred	Predicted Pred	Predicted Pred	Predicted Pred	Predicted Pred	Predicted Actual Ac	Predicted Pred	Predicted Actual Ac	Predicted Pred	Predicted Actual Ac	Predicted Actual Ac	Predicted Actual Ac	Predicted Actual Ac	Predicted Actual Ac	Hard Hard	Predicted Actual Actual Actual Actual Actual Actual Actual Design Average Des	Head	Actual Actual

Table F6. 2-inch cobble substrate in rectangular flume riprap results

Predicted	2.11 0.210 1.94 0.208 0.15 0
Predicted	2.11 0.210 1.94 0.208 0.15
Predicted USBR - EM - 25	2.11 0.210 1.94 0.208
Predicted USBR - EM - LA Predicted	2.11 0.210 1.94
Main Color USBR - FM - 25	2.11 0.210
March Marc	2.11
Marie Mari	
Mariage Design Average Design Desi	0.229
New Year	
Mark	6.76
Marage Design Predicted	0.731
Name	6.02
EM-1601 USBR - EI Predicted Average Design (1) (1) (1) (2) (1) (2) (3) (1) (4) (2) (3) (4) (1) (4) (2) (3) (4) (2) (4) (3) (4) (4) (3) (4) (4) (3) (4) (4) (4) (4) (4) (4) (4) (4) (4) (4	0.652
Average D Pr	3.48
E E	0.357
o o o o o o o o o o o o o o o o o o o	1.36
Design day (f) (f) (7) (7) (7) (7) (7) (7) (7) (7) (7) (7	0.1300
Actual d _{S0} d _{S0} (ft) (7) (7) (7) (8) (7) (8) (9) (9) (9) (9) (9) (9) (9) (9) (9) (9	0.108
Actual d40 (ft) (7) (103 0.103	0.103
Actual d ₃₀ (ft) (7) (0.096 0.006 0	0.096
Bed Velocity (ff/s) (530 5.30 5.52 6.55 5.57 5.57 5.57 5.57 5.57 5.57 5.57	5.54
(#) Depth (#23 123 124 125 126 126 127 1	1.36
Run (#) 7 7 7 18 22 23 23 33 33 33 33 33 33 33 33 33 33	

Appendix G

Substrate Properties

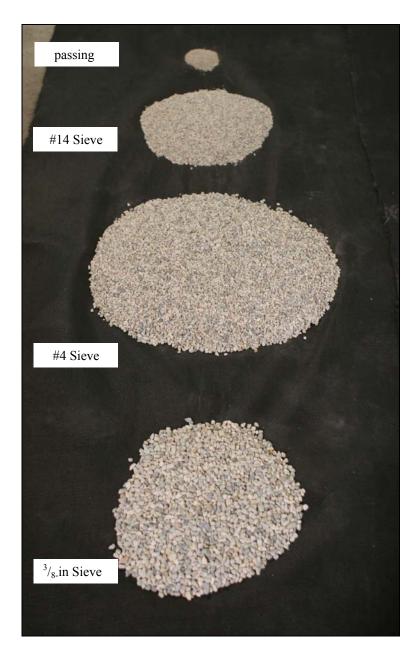


Figure G1. Sieve distribution of pea gravel substrate

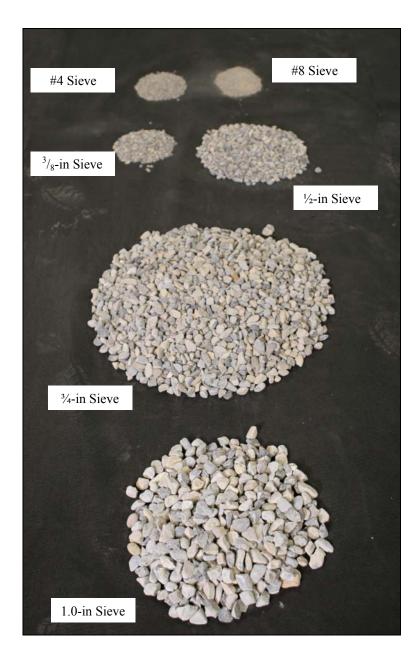


Figure G2. Sieve distribution of 0.75-inch angular gravel substrate

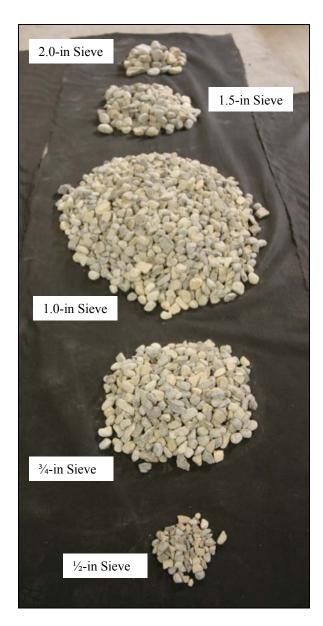


Figure G3. Sieve distribution of 2-inch cobble substrate



Figure G4. Sieve distribution of 2-inch angular rock substrate

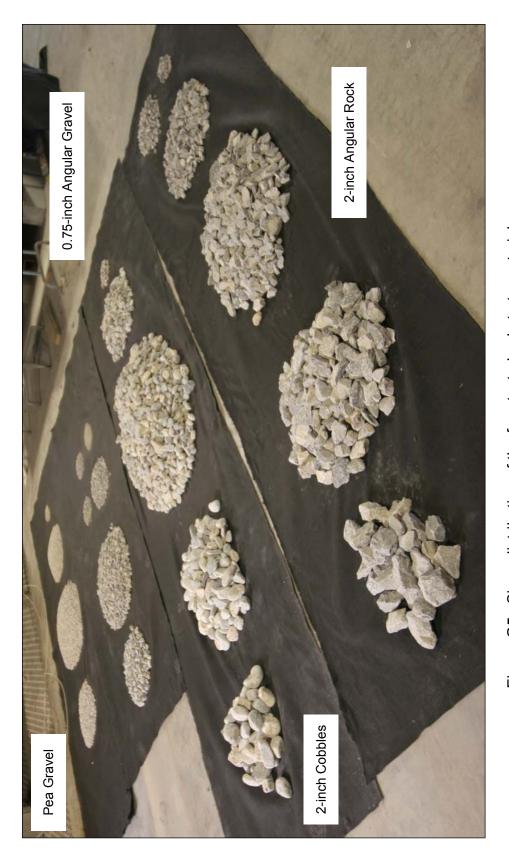


Figure G5. Sieve distribution of the four tested substrate materials

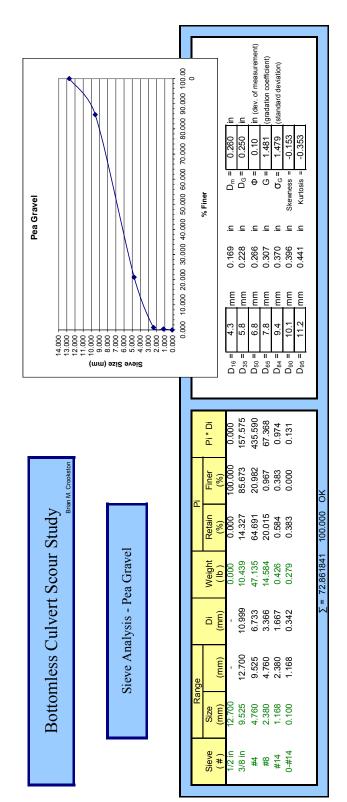


Figure G6. Sieve analysis of pea gravel substrate

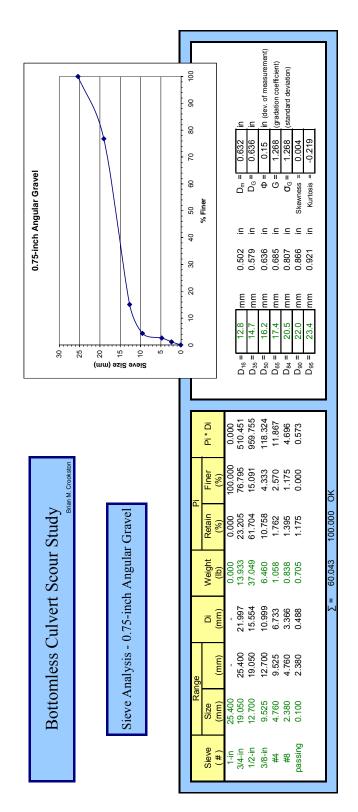


Figure G7. Sieve analysis of 0.75-inch angular gravel substrate

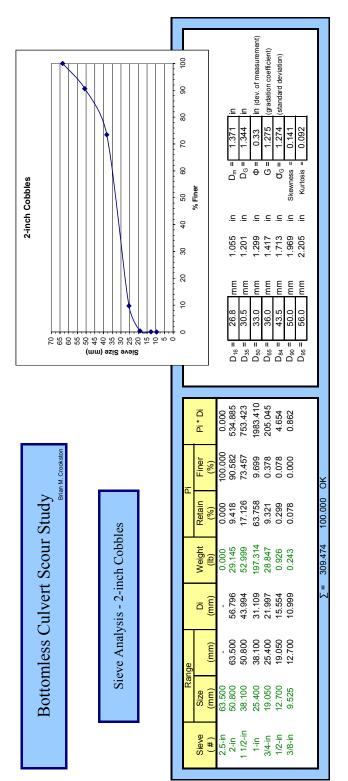


Figure G8. Sieve analysis of 2-inch cobble substrate

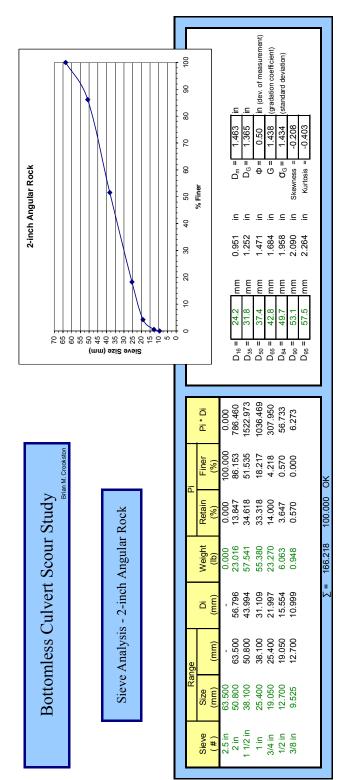


Figure G9. Sieve analysis of 2-inch angular rock substrate

Table G1. Specific weight analyses results

Substrate	Rock Weight (lb)	Rock Volume (ft^3)	γ	$\gamma_{ m s}$	SG
Pea	35.25	0.23	62.4	154.80	2.48
Gravel					
0.75-inch	37.20	0.24	62.4	153.44	2.46
Angular Gravel					
2-inch	43.20	0.27	62.4	160.90	2.58
Cobbles					
2-inch	33.95	0.23	62.4	150.78	2.42
Angular Rock					

Table G2. Substrate properties in metric units

		0.75-in		2-in
	Pea	Angular	2-in	Angular
	Gravel	Gravel	Cobbles	Rock
	(mm)	(mm)	(mm)	(mm)
D ₁₆	4.30	12.75	26.80	24.17
D ₃₅	5.80	14.70	30.50	31.81
D ₅₀	6.75	16.15	33.00	37.37
D ₆₅	7.80	17.40	36.00	42.78
D_m	6.60	16.05	34.82	37.16
D ₈₄	9.40	20.50	43.50	49.72
D_{90}	10.05	22.00	50.00	53.08
D_{95}	11.20	23.40	56.00	57.50
Cu	1.81	1.36	1.34	1.77
Cz	1.00	0.97	0.96	0.98
Υs	154.80	153.44	160.90	150.78
SG	2.48	2.46	2.58	2.42

Appendix H

Visual Basic Code Used for Calculations for

Bottomless Arch Culvert in Microsoft Excel

```
Option Explicit
'Automatic couter of test runs
Function RUNCOUNTER(wii)
  RUNCOUNTER = wii + 1
End Function
'Temperature Conversion & Check
Function TEMPERATURE(wdTemp)
  If wdTemp <= 32 Then
    MsgBox ("Water is either frozen or not entered in Fahrenheit.")
    Exit Function
  Elself wdTemp >= 70 Then
    MsgBox ("Most unlikely, please double check.")
  TEMPERATURE = 5 / 9 * (wdTemp - 32)
End Function
'Orifice Flow Calculations
Function FLOW(strPipe, strFluid, wdManomH)
Dim wdDOrifice As Double
                                                                  'Diameter of Orifice
                                                                 'Pipe Diameter of Inflow 
'Calculate deltaH for Flow Equation
Dim wdDPipe As Double
Dim wdDeltaH As Double
Dim wdPi As Double
Dim wdA1 As Double
                                                             'PI contstant
                                                               'Cross-sectional Area of Pipe of Inflow
Dim wdA2 As Double
                                                              'Cross-sectional Area of Throat of Orifice
Dim wsK As Single
                                                            'Constant from Orifice Calibration
                                                            'Specific weight of Manometer Fluid 
'Acceleration of Gravity Constant
Dim wsGammaM As Single
Dim wsG As Single
wdPi = 3.14159265359
wsG = 32.174
                                                       'Define Constant 
'Define Constant
 'Pipe Check
If strPipe = "8" Then
    wdDOrifice = 6.004 / 12
                                                                   'Data from calibration, separate excel file
    wdDPipe = 7.9183 / 12
wsK = 0.6856
  WSR = 0.0550
Elself strPipe = "18" Then
wdDOrifice = 10.9944 / 12
wdDPipe = 17.9266 / 12
                                                                     'Data from calibration, separate excel file
    wsK = 0.6785
  Else
    MsgBox ("Pipe diameter of Inflow entered incorrectly. Please enter 8 or 18.")
  End If
   'Calculate deltaH from manometer
  If strFluid = "Blue" Then
  wsGammaM = 1.75
Elself strFluid = "Hg" Then
wsGammaM = 13.5536
    MsgBox ("Manometer fluid entered incorrectly, please use 'Blue' or 'Hg'.")
    Exit Function
  End If
  wdManomH = wdManomH * 0.0328083989501 'cm to ft conversion
  wdDeltaH = wdManomH * (wsGammaM - 1)
  'Calculate Cross Sectional Areas
  wdA1 = 0.25 * wdPi * wdDPipe ^ 2
wdA2 = 0.25 * wdPi * wdDOrifice ^ 2
  FLOW = wsK*wdA2*(2*wsG*wdDeltaH) ^ (1 / 2) \\ "Venturi equation wdQ = wsk*wdA2*((2*wsG*wdDeltaH)) ^ 0.5 \\ "Venturi equation wdQ = wsk*wdA2*((2*wsG*wdQeltaH)) ^ 0.5 \\ "Venturi equation wdQ = wsk*wdA2*((2*wsG*wdQeltaH)) ^ 0.5 \\ "Venturi equation wdQ = wsk*wdA2*((2*wsG*wdQeltaH)) ^ 0.5 \\ "Venturi equation wdQeltaH) ^ 0.5 \\ "Ventur
'Calculate Specific Weight of Water as a function of Temperature (Fahrenheit)
Function GAMMAH2O(wdTemp)
  GAMMAH20 = 59.364982 + 3.0750805 * Cos(0.0078331697 * (wdTemp) - 0.24302151)

'Slight adjustment of gamma to match values given in Engineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson
    If wdTemp = 40 Then
GAMMAH2O = 62.43
    Elself wdTemp = 50 Then
       GAMMAH2O = 62.4
    End If
End Function
'Calculate Dynamic Viscosity of Water as a functio of Temperature (fahrenheit)
Function MUH2O(wdTemp)
MUH2O = 1 / (10865.946 + 441.70715 * (wdTemp) + 1.4672793 * (wdTemp) ^ 2)
   'Slight adjustment of mu to match values given in Engineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson
    If wdTemp = 40 Then
MUH2O = 0.0000323
    Elself wdTemp = 50 Then
MUH2O = 0.0000273
    End If
End Function
'Calculate kinematic viscosity of Water as a function of Temperature (Fahrenheit)
Function NUH2O(wdTemp)
```

```
NUH2O = Exp(-5.5116242 - 26.354346 / wdTemp - 1.3108492 * (Log(wdTemp) / Log(2.71828182846)))
'Slight adjustment of nu to match values given in Engineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson
  If wdTemp = 40 Then
NUH2O = 0.0000166
   Elself wdTemp = 50 Then
NUH2O = 0.0000141
   End If
End Function
'Calculate Density of Water as a function of Temperature (Fahrenheit)
Function RHOH2O(wdTemp)
If wdTemp > 32 And wdTemp < 80 Then
RHOH2O = 1.94
   MsgBox ("Water Temperature Invalid")
 End If
End Function
'Calculate Reynolds Number at entrance
Function REA(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw, wdnu)
                                     'Depth of Flow in box
Dim wdYbox As Double
Dim wdYculv As Double
                                     'Depth of Flow in Culvert
Dim wdBoxArea As Double
                                     'Area of flow in box
'Total area of flow in culvert
Dim wdTArea As Double
Dim wdPi As Double
                                  'Pi Constant
                                    'angle in radians
'Cross-sectional area of Culvert
Dim wdbeta As Double
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert Dim wdL As Double 'Length of Culvert
Dim wdWetPerim As Double
                                        Wetted Perimeter
                                    '(1-Y/R) term in beta
Dim wsForbeta As Single
                                 'Acceleration of Gravity
Dim wsG As Single
Dim wdPw As Double
Dim wdPw2 As Double
                                    'wetted perimeter in culvert
'wetted perimeter of riprap above box below culvert
Dim wdPwbox As Double
                                      'wetted perimeter in box
Dim wdTPw As Double
Dim wdRh As Double
                                     'Total wetted perimeter
                                    'Hydraulic Radius
Dim wdTw As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
  wdboxw = wdboxw / 12
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged
   'Calculate Depths
wdYbox = wdDbed - wdr
    If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
      wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
      wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
    wdYbox = 0
End If
     wdYculv = wdr - wdWs
      If wdYculv > wdr Then
       REA = "error"
       Exit Function
      End If
   'box cross-sectional area
   wdBoxArea = wdboxw * wdYbox 'box wetted perimeter
   If wdYbox = 0 Then
   wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
   End If
   'culvert cross-sectional area
    If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
```

```
If wdYculv = wdr Then
        wdbeta = wdPi
       \label{eq:wdculv} \begin{array}{ll} \mbox{wubeta} - \mbox{wur} & \mbox{wdCulvArea} = ((\mbox{wdr} \mbox{ }^2) \mbox{ }^* \mbox{ } (\mbox{wdbeta} - \mbox{Sin}(\mbox{wdbeta}) \mbox{ }^* \mbox{ } Cos(\mbox{wdbeta}))) \mbox{/ } 2 \\ \mbox{wdPw} = \mbox{wdPi} \mbox{ }^* \mbox{ } \mbox{wdr} \end{array}
      Else
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
      End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   If wdHwd >= 1.01 Then 'Submerged
   'Calculate Depths
    wdYbox = wdDbed - (wdr + wdWs)
    If wdYbox < 0.0001 Then
      wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
      wsForbeta = -(wdr Culv / Wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdr * wdbeta - wdPi * wdr
wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
    End If
     wdYculv = wdr
      If wdYculv > wdr Then
       Exit Function
      End If
   'box cross-sectional area
    wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
If wdYbox = 0 Then
   wdPwbox = wdTw
End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
   End If
   'culvert cross-sectional area
If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdPw = 2 * wdbeta * wdr - wdPi * wdr
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdbeta * wdr - wdPi * wdr
      End If
    End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted Perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   End If
 wdRh = wdTArea / wdTPw
REA = 4 * wdFlow * wdRh / (wdTArea * wdnu) 'open channel
End Function
'Calculate Reynolds Number in culvert
Function RE(wdDbed, wdWs, wdFlow, wdr, wdboxw, wdnu)
                                        'Depth of Flow in box
Dim wdYbox As Double
Dim wdYculv As Double
                                        'Depth of Flow in Culvert
Dim wdBoxArea As Double
                                          'Area of flow in box
                                         'Total area of flow in culvert
Dim wdTArea As Double
Dim wdPi As Double
                                      'Pi Constant
Dim wdbeta As Double
                                       'angle in radians
```

```
Dim wdCulvArea As Double
                                       'Cross-sectional area of Culvert
Dim wdCulvArea2 As Double
                                        'Cross-sectional area of Riprap above box below Culvert
                                  'Length of Culvert
le 'Wetted Perimeter
Dim wdl. As Double
Dim wdWetPerim As Double
                                  '(1-Y/R) term in beta
'Acceleration of Gravity
Dim wsForbeta As Single
Dim wsG As Single
Dim wdPw As Double
                                     'wetted perimeter in culvert
Dim wdPw2 As Double
                                      'wetted perimeter of riprap above box below culvert 
'wetted perimeter in box
Dim wdPwbox As Double
Dim wdTPw As Double
Dim wdRh As Double
                                      'Total wetted perimeter
                                     'Hydraulic Radius
Dim wdTw As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
  wdr = wdr / 12
 wdboxw = wdboxw / 12
  'Calculate Depths
wdYbox = wdDbed - wdr
    If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
     ws-roupeta = -(warcun/ wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
    End If
    wdYculv = wdr - wdWs
      If wdYculv > wdr Then
RE = "error"
        'Exit Function
      End If
   'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
    If wdYbox = 0 Then
   wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
End If
   'culvert cross-sectional area
    If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdPw = wdPi * wdr
      Else
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
      End If
    Fnd If
    If wdYculv < 0 Then
wdPw = 0
        wdCulvArea = 0
    End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'total wetted perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
 wdRh = wdTArea / wdTPw
RE = 4 * wdFlow * wdRh / (wdTArea * wdnu) 'closed conduit form
End Function
```

'State Laminar or Turbulent Flow

```
Function LTFLOW(wdRe)
 If wdRe <= 2000 Then
LTFLOW = "Laminar"
Elself wdRe > 2000 Then
LTFLOW = "Turbulent"
 End If
End Function
'Froude # in Bottomless Culvert Model (Closed Conduit and Open Channel Case) Function FROUDEA(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw)
Dim wdYbox As Double
Dim wdYculv As Double
                                      'Depth of Flow in box
'Depth of Flow in Culvert
Dim wdBoxArea As Double
                                        'Area of flow in box
                                      'Total area of flow in culvert
Dim wdTArea As Double
Dim wdPi As Double
                                   'Pi Constant
Dim wdbeta As Double
Dim wdCulvArea As Double
                                     'angle in radians
'Cross-sectional area of Culvert
Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert
Dim wdL As Double Length of Culvert
Dim wdWetPerim As Double Wetted Perimeter
Dim wsForbeta As Single ('(1-Y/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
                                               'Top Width of water surface
Dim wdTwSurface As Double
Dim wdTw As Double
Dim wdPwbox As Double
Dim wdPw2 As Double
Dim wdPw As Double
Dim wdTPw As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
  wdr = wdr / 12
 wdboxw = wdboxw / 12
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged
    'Calculate Depths
     wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
      wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
      wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
     End If
     wdYculv = wdr - wdWs
      If wdYculv > wdr Then
        FROUDEA = "error"
'Exit Function
   'box cross-sectional area
     wdBoxArea = wdboxw * wdYbox
     'box wetted perimeter
   If wdYbox = 0 Then
wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
   End If
   'culvert cross-sectional area
     If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
      End If
   'Total Cross-Sectional Area
```

```
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   Fnd If
   If wdHwd >= 1.01 Then 'Submerged
    'Calculate Depths
     wdYbox = wdDbed - (wdr + wdWs)
     If wdYbox < 0.0001 Then
      wdYculv = -1 * wdYbox
     wsForbeta = -(wdYculv / wdr) \\ wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) \\ wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 \\ wdPw2 = 2 * wdr * wdbeta - wdPi * wdr \\ wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
     End If
     wdYculv = wdr
      If wdYculv > wdr Then
FROUDEA = "error"
        'Exit Function
       End If
   'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
     'box wetted perimeter
     If wdYbox = 0 Then
   wdPwbox = wdTw
   If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
   End If
   'culvert cross-sectional area
     If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
        wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
       End If
     End If
   'Total Cross-Sectional Area
   wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
'Total wetted perimeter
     wdTPw = wdPw - wdPw2 + wdPwbox
 wdTwSurface = 2 * wdr * Sin(wdbeta)
If wdTwSurface < 0.01 Then
FROUDEA = "-"
   Exit Function
FROUDEA = wdFlow / (wdTArea * wsG / wdTwSurface) ^ 0.5
'Froude # in Bottomless Culvert Model (Closed Conduit and Open Channel Case) Function FROUDE(wdDbed, wdWs, wdFlow, wdr, wdboxw)
                                      'Depth of Flow in box
'Depth of Flow in Culvert
Dim wdYbox As Double
Dim wdYculv As Double
Dim wdBoxArea As Double
'Area of flow in box
'Area of flow in culvert
Dim wdYculv As Double
Dim wdPi As Double
                                    'Pi Constant
Dim wdbeta As Double 'angle in radians
Dim wdbeta As Double 'Cross-sectional area of Culvert
Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert
Dim wdCulvArea2 to Dim wdL As Double ULength of Culvent Wetted Perimeter Wetted Perimeter (1-Ty/R) term in beta
Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
Dim wdTw As Double
                                      'Top Width
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
```

```
'Inches to feet conversion
 wdr = wdr / 12
 wdboxw = wdboxw / 12
'check for submergence
   'Calculate Depths
    wdYbox = wdDbed - wdr
    If wdYbox <= 0 Then
     wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
     wdbeta = Atri(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atri(1) wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
    End If
    wdYculv = wdr - wdWs
     If wdYculv > wdr Then
FROUDE = "error"
       'Exit Function
     End If
  'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
   'culvert cross-sectional area
    If wdYculv > 0 Then
     wsForbeta = -(wdYculv / wdr)
     If wdYculv = wdr Then
wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
     wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
End If
    End If
    If wdYculv < 0 Then
       wdCulvArea = 0
   'Total Cross-Sectional Area
    wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
 wdTw = 2 * wdr * Sin(wdbeta)

If wdTw < 0.01 Then

FROUDE = "-"
  Exit Function
FROUDE = wdFlow / (wdTArea * wsG / wdTw) ^ 0.5
End Function
'Calculated mean velocity at entrance of wing walls Function VContraction(wdDepth, wdContraction, wdFlow)
Dim wdWidth As Double
  If wdContraction = 0.33 Then
    wdWidth = 3
  End If
  If wdContraction = 0.75 Then
    wdWidth = 8
  End If
  If wdContraction = 0 Then wdWidth = 2
  VContraction = wdFlow / (wdDepth * wdWidth)
End Function
Function AVelocity(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw)
Dim wdYbox As Double
                                 'Depth of Flow in box
Dim wdYculv As Double
                                 'Depth of Flow in Culvert
```

```
Dim wdBoxArea As Double
                                        'Area of flow in box
                                      'Total area of flow in culvert
Dim wdTArea As Double
                                    'Pi Constant
Dim wdPi As Double
Dim wdbeta As Double
                                     'angle in radians
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double
Cross-sectional area of Culvert
Cross-sectional area of Riprap above box below Culvert
Dim wdL As Double
                                   'Length of Culvert
Dim wdL As Double Lengur of Control.

Dim wdWetPerim As Double 'Wetted Perimeter
Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wdWetPerim As Double
Dim wsForbeta As Single '(1-Y/R) term in Deta
'Acceleration of Gravity
Dim wdTw As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
  wdboxw = wdboxw / 12
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged
   'Calculate Depths
     wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
     wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea2 = (wdr ^2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdTw = 2 * wdr * Sin(wdbeta) wdYbox = 0
     End If
     wdYculv = wdr - wdWs
      If wdYculv > wdr Then
       AVelocity = "error"
'Exit Function
   'box cross-sectional area
     wdBoxArea = wdboxw * wdYbox
   'culvert cross-sectional area
     If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
      End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   End If
   If wdHwd >= 1.01 Then 'Submerged
   'Calculate Depths
     wdYbox = wdDbed - (wdr + wdWs)
     If wdYbox < 0.0001 Then
      wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
      wsForbeta = -(wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
      wdYbox = 0
     End If
     wdYculv = wdr
      If wdYculv > wdr Then
AVelocity = "error" ' AVelocity = "error"
'Exit Function
```

```
'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
    'culvert cross-sectional area
     If wdYculv > 0 Then
       wsForbeta = -(wdYculv / wdr)
       If wdYculv = wdr Then
         wdbeta = wdPi
         wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
       End If
     End If
    'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   End If
  AVelocity = wdFlow / wdTArea
 End Function
Function BCDEVelocity(wdDbed, wdWs, wdFlow, wdr, wdboxw)
Dim wdYbox As Double
Dim wdYculv As Double
                                        'Depth of Flow in box
'Depth of Flow in Culvert
Dim wdYculv As Double
Dim wdBoxArea As Double
'Area of flow in box
'Area of flow in culvert
 Dim wdPi As Double
                                      'Pi Constant
Dim wdbeta As Double 'angle in radians
Dim wdCulvArea As Double 'Cross-sectional area of Culvert
Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert
                                    'Length of Culvert
ble 'Wetted Perimeter
Dim wdL As Double Length of Culvert
Dim wdWetPerim As Double Wetted Perimeter
Dim wsForbeta As Single (1.7/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
wsG = 32.174
wdPi = 3.14159265359
 wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
wdboxw = wdboxw / 12
'check for submergence
    'Calculate Depths
     wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
      wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
       wdYbox = 0
     End If
     wdYculv = wdr - wdWs
       If wdYculv > wdr Then
         BCDEVelocity = "error"
         Exit Function
       End If
    'box cross-sectional area
     wdBoxArea = wdboxw * wdYbox
    'culvert cross-sectional area
     If wdYculv > 0 Then
       wsForbeta = -(wdYculv / wdr)
       If wdYculv = wdr Then
wdbeta = wdPi
         wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
       End If
      If wdYculv < 0 Then
         wdCulvArea = 0
```

```
End If
   'Total Cross-Sectional Area
    wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   BCDEVelocity = wdFlow / wdTArea
End Function 'Calculate V* * Ds/nu at entrance of culvert
Function ShieldsXA(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw, wdnu, wdDs, wds)
                                     'Depth of Flow in box
Dim wdYculv As Double
Dim wdBoxArea As Double
                                    'Depth of Flow in Culvert
'Area of flow in box
Dim wdTArea As Double
                                     'Total area of flow in culvert
Dim wdPi As Double
                                   'Pi Constant
Dim wdbeta As Double
                                    'angle in radians
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double
                                      'Cross-sectional area of Culvert
'Cross-sectional area of Riprap above box below Culvert
Dim wdL As Double
                                  'Length of Culvert
Dim wdWetPerim As Double
                                    'Wetted Perimeter
'(1-Y/R) term in beta
Dim wsForbeta As Single
Dim wsG As Single
Dim wdPw As Double
                                 'Acceleration of Gravity
                                    'wetted perimeter in culvert
Dim wdPw2 As Double
                                     'wetted perimeter of riprap above box below culvert
Dim wdPwbox As Double
Dim wdTPw As Double
                                     'wetted perimeter in box
'Total wetted perimeter
Dim wdRh As Double
                                    'Hydraulic Radius
Dim wdVstar As Double
                                    'Shear Velocity
Dim wdTw As Double
Dim wdPwwalls As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdDeltaRew_fw As Double
Dim wdChange As Double
Dim wdPwb As Double
Dim wdfb As Double
Dim wdAreab As Double
Dim wdRhb As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'inches to feet conversion
 wdr = wdr / 12
wdboxw = wdboxw / 12
  wdFlow = wdFlow
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged 
'Calculate Depths
     wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
      wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
     wsForbeta = -(wd rculv / wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
     End If
    wdYculv = wdr - wdWs
      If wdYculv > wdr Then
ShieldsXA = "error"
       Exit Function
      End If
   'box cross-sectional area
    wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
If wdYbox = 0 Then
   wdPwbox = wdTw
   End If
  If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
End If
   'culvert cross-sectional area
    If wdYculv > 0 Then
```

wsForbeta = -(wdYculv / wdr)

```
If wdYculv = wdr Then
       wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
wdPw = wdPi * wdr
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
     End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   If wdHwd >= 1.01 Then 'Submerged
   'Calculate Depths
    wdYbox = wdDbed - (wdr + wdWs)
    If wdYbox < 0.0001 Then
     wdYculv = -1 * wdYbox
     wsForbeta = -(wdYculv / wdr)
     wsForbeta = -(wd tculv / wd1)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdr * wdbeta - wdPi * wdr
wdTw = 2 * wdr * Sin(wdbeta)
     wdYbox = 0
    End If
    wdYculv = wdr
      If wdYculv > wdr Then
       ShieldsXA = "error"
       Exit Function
      End If
   'box cross-sectional area
    wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
If wdYbox = 0 Then
  wdPwbox = wdTw
End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
   End If
  'culvert cross-sectional area
If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
       wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
       wdPw = 2 * wdbeta * wdr - wdPi * wdr
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdbeta * wdr - wdPi * wdr
      End If
    End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted Perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   End If
 wdRh = wdTArea / wdTPw
 wdVstar = (wsG * wdRh * wds) ^ (1 / 2)
 ShieldsXA = wdVstar * wdDs / wdnu
'Calculate V* * Ds/nu at entrance using Modified Prandtl/Einstein
Function\ Shields PEA (wdDbed,\ wdWs,\ wdFlow,\ wdHwd,\ wdr,\ wdboxw,\ wdnu,\ wdDs,\ wds,\ wdD65,\ wdVmean)
Dim wdYbox As Double
                                    'Depth of Flow in box 
'Depth of Flow in Culvert
Dim wdYculy As Double
Dim wdBoxArea As Double
                                       'Area of flow in box
                                      'Total area of flow in culvert
Dim wdTArea As Double
```

```
Dim wdPi As Double
                                     'Pi Constant
Dim wdbeta As Double
                                       'angle in radians
                                         'Cross-sectional area of Culvert
'Cross-sectional area of Riprap above box below Culvert
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double
Dim wdL As Double 'L
Dim wdWetPerim As Double
                                    'Length of Culvert
e 'Wetted Perimeter
Dim wsForbeta As Single
                                      '(1-Y/R) term in beta
                                   'Acceleration of Gravity
'wetted perimeter in culvert
Dim wsG As Single
Dim wdPw As Double
Dim wdPw2 As Double
Dim wdPwbox As Double
                                       'wetted perimeter of riprap above box below culvert 
'wetted perimeter in box
Dim wdTPw As Double
                                        'Total wetted perimeter
                                      'Hydraulic Radius
'Shear Velocity
Dim wdRh As Double
Dim wdVstar As Double
Dim wdTw As Double
Dim wdPwwalls As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdDeltaRew_fw As Double
Dim wdderRew_fw As Double
Dim wdChange As Double
Dim wsPwb As Double
Dim wdAreab As Double
Dim wdRhb As Double
Dim wdX As Double
Dim wdDeltaX As Double
Dim wdDeltaPrime As Double
Dim wdPhi As Double
Dim wdXnew As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
  wdr = wdr / 12
  wdboxw = wdboxw / 12
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged
   'Calculate Depths
wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
       wsForbeta = -(wdYculv / wdr)
     wsi-ruipeta - -(wu r cuiv / Wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
       wdYbox = 0
     End If
     wdYculv = wdr - wdWs
       If wdYculv > wdr Then
        ShieldsPEA = "error"
Exit Function
       End If
   'box cross-sectional area
   wdBoxArea = wdboxw * wdYbox 'box wetted perimeter
     If wdYbox = 0 Then
   wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
   'culvert cross-sectional area
     If wdYculy > 0 Then
      wsForbeta = -(wdYculv / wdr)
       If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
        wdPw = wdPi * wdr
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
```

```
End If
  Fnd If
 'Total Cross-Sectional Area
  wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
 'Total Wetted perimeter
   wdTPw = wdPw - wdPw2 + wdPwbox
 If wdHwd >= 1.01 Then 'Submerged
 'Calculate Depths
  wdYbox = wdDbed - (wdr + wdWs)
  If wdYbox < 0.0001 Then
    wdYculv = -1 * wdYbox
   wsForbeta = -(wdYculv / wdr) wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw2 = 2 * wdr * wdbeta - wdPi * wdr wdTw = 2 * wdr * Sin(wdbeta)
  wdYbox = 0
End If
  wdYculv = wdr
    If wdYculv > wdr Then
ShieldsPEA = "error"
     Exit Function
    End If
 'box cross-sectional area
  wdBoxArea = wdboxw * wdYbox
 'box wetted perimeter
 If wdYbox = 0 Then
wdPwbox = wdTw
 End If
 If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
 End If
 'culvert cross-sectional area
  If wdYculv > 0 Then
wsForbeta = -(wdYculv / wdr)
    If wdYculv = wdr Then
     wdbeta = wdPi
     wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
wdPw = 2 * wdbeta * wdr - wdPi * wdr
   wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdbeta * wdr - wdPi * wdr End If
  End If
 'Total Cross-Sectional Area
 wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
'Total Wetted Perimeter
  wdTPw = wdPw - wdPw2 + wdPwbox
 End If
wdRh = wdTArea / wdTPw
 wdX = 1
 wdDeltaX = 0.05
 wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10))
 wdDeltaPrime = 11.6 * wdnu / wdVstar
 If wdD65 / wdDeltaPrime <= 8 Then
  Do While wdDeltaX >= 0.0001
wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10))
    wdDeltaPrime = 11.6 * wdnu / wdVstar
    wdPhi = (Log(wdD65 / wdDeltaPrime)) / Log(10)
wdXnew = 1.62265 + 0.09947 * wdPhi - 2.833 * wdPhi ^ 2 + 1.18924 * wdPhi ^ 3 + 2.5663 * wdPhi ^ 4 - 1.64 * wdPhi ^ 5
    wdDeltaX = wdX - wdXnew
    wdX = wdXnew
  Loop
 End If
 If wdD65 / wdDeltaPrime > 8 Then
  wdX = 1
  wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10))
```

ShieldsPEA = wdVstar * wdDs / wdnu

End Function

End If

'box cross-sectional area wdBoxArea = wdboxw * wdYbox 'box wetted perimeter If wdYbox = 0 Then wdPwbox = wdTw End If

If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
End If

'Calculate sidewall correction by Vanoni & Brooks (1957) 'Calculate V* * Ds/nu at entrance using Modified Prandtl/Einstein Function ShieldsPEACorr(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw, wdnu, wdDs, wds, wdD65, wdVmean, wdRe) Dim wdYhox As Double 'Depth of Flow in box Dim wdYculv As Double 'Depth of Flow in Culvert Dim wdBoxArea As Double 'Area of flow in box 'Total area of flow in culvert Dim wdTArea As Double Dim wdPi As Double 'Pi Constant Dim wdbeta As Double Dim wdCulvArea As Double 'angle in radians
'Cross-sectional area of Culvert Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert 'Length of Culvert e 'Wetted Perimeter Dim wdL As Double Dim wdWetPerim As Double '(1-Y/R) term in beta
'Acceleration of Gravity Dim wsForbeta As Single Dim wsG As Single Dim wdPw As Double wetted perimeter in culvert 'wetted perimeter of riprap above box below culvert 'wetted perimeter in box Dim wdPw2 As Double Dim wdPwbox As Double Dim wdTPw As Double Dim wdRh As Double 'Total wetted perimeter 'Hydraulic Radius Dim wdVstar As Double 'Shear Velocity Dim wdTw As Double Dim wdPwwalls As Double Dim wdf As Double Dim wdfw As Double Dim wdRe_f As Double Dim wdRew_fw As Double Dim wdDeltaRew_fw As Double Dim wdderRew_fw As Double Dim wdChange As Double Dim wdPwb As Double Dim wdfb As Double Dim wdAreab As Double Dim wdRhb As Double Dim wdX As Double Dim wdDeltaX As Double Dim wdDeltaPrime As Double Dim wdPhi As Double Dim wdXnew As Double wsG = 32.174 wdPi = 3.14159265359 wdCulvArea2 = 0 'Inches to feet conversion wdr = wdr / 12 wdboxw = wdboxw / 12 'check for submergence If wdHwd < 1.01 Then 'Unsubmerged 'Calculate Depths wdYbox = wdDbed - wdr If wdYbox <= 0 Then wdYculv = -1 * wdYbox wsForbeta = -(wdYculv / wdr) wsForbeta = -(wd rculv / wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta) wdYbox = 0End If wdYculv = wdr - wdWs If wdYculv > wdr Then ShieldsPEACorr = "error" Exit Function

```
'culvert cross-sectional area
    If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
       wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
       wdPw = wdPi * wdr
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
       wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw = 2 * wdr * wdbeta - wdPi * wdr
      End If
    End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted perimeter
wdTPw = wdPw - wdPw2 + wdPwbox
   If wdHwd >= 1.01 Then 'Submerged
   'Calculate Depths
    wdYbox = wdDbed - (wdr + wdWs)
    If wdYbox < 0.0001 Then
      wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
      wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdr * wdbeta - wdPi * wdr
     wdTw = 2 * wdr * Sin(wdbeta)
wdYbox = 0
    End If
    wdYculv = wdr
      If wdYculv > wdr Then
       ShieldsPEACorr = "error"
       Exit Function
      End If
   'box cross-sectional area
    wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
    If wdYbox = 0 Then
   wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
   'culvert cross-sectional area
If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
       wdbeta = wdPi
wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
wdPw = 2 * wdbeta * wdr - wdPi * wdr
      Else
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw = 2 * wdbeta * wdr - wdPi * wdr
      End If
    End If
   'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted Perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   End If
 wdRh = wdTArea / wdTPw
'Separate shear to shear on riprap and shear on model boundaries
wdf = 8 * wsG * wdRh * wds / (wdVmean) ^ 2
wdRe_f = wdRe / wdf
wdfw = 0.01 'guess
   wdRew_fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)
wdDeltaRew_fw = wdRe_f - wdRew_fw
```

```
\label{eq:wdderRew_fw} wd der Rew_fw = Exp(1.15129 / wdfw ^ (1 / 2)) * (-3.76783 * wdfw ^ (1 / 2) - 1.44596) / wdfw ^ 3 wdChange = wdDeltaRew_fw / wdderRew_fw wdfw = wdfw + wdChange | f Abs(wdChange) < 0.000001 Then | Exit Delta (Control of the control of the
        Exit Do
     End If
  If wdPwbox >= 0 Then
     wdPwb = wdboxw
  If wdPw2 > 0 Then
      wdPwb = wdTw
  End If
   wdPwwalls = wdTPw - wdPwb
  wdfb = wdf + wdPwwalls / wdPwb * (wdf - wdfw)
wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wsG * wds)
   wdRhb = wdAreab / wdPwb
      wdDeltaX = 0.05
      wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65))) / Log(10)) wdDeltaPrime = 11.6 * wdnu / wdVstar If wdD65 / wdDeltaPrime <= 8 Then
        | Do While wdDeltaX >= 0.0001
| wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))
| wdDeltaPrime = 11.6 * wdnu / wdVstar
| wdPhi = (Log(wdD65 / wdDeltaPrime)) / Log(10)
| wdXnew = 1.62265 + 0.09947 * wdPhi - 2.833 * wdPhi ^ 2 + 1.18924 * wdPhi ^ 3 + 2.5663 * wdPhi ^ 4 - 1.64 * wdPhi ^ 5
            wdDeltaX = wdX - wdXnew
           wdX = wdXnew
        Loop
      End If
      If wdD65 / wdDeltaPrime > 8 Then
        wdX = 1
        wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))
   ShieldsPEACorr = wdVstar * wdDs / wdnu
End Function
'Calculate sidewall correction by Vanoni & Brooks (1957)
'Calculate V* * Ds/nu at entrance of culvert
Function ShieldsXACorr(wdDbed, wdWs, wdFlow, wdHwd, wdr, wdboxw, wdnu, wdDs, wds, wdVmean, wdRe)
                                                                        'Depth of Flow in box
Dim wdYbox As Double
Dim wdYculv As Double
                                                                        'Depth of Flow in Culvert
Dim wdBoxArea As Double
Dim wdTArea As Double
                                                                        'Area of flow in box
'Total area of flow in culvert
Dim wdPi As Double
                                                                   'Pi Constant
                                                                     'angle in radians
'Cross-sectional area of Culvert
Dim wdbeta As Double
Dim wdCulvArea As Double
                                                                  le 'Cross-sectional area of Riprap above box below Culvert 
'Length of Culvert 
e 'Wetted Perimeter
Dim wdCulvArea2 As Double
Dim wdL As Double 'Lo
Dim wdWetPerim As Double Wetted Perimeter
Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
Dim wdPw As Double
                                                                       'wetted perimeter in culvert
                                                                        'wetted perimeter of riprap above box below culvert 
'wetted perimeter in box
Dim wdPw2 As Double
Dim wdPwbox As Double
Dim wdTPw As Double
Dim wdRh As Double
                                                                     'Total wetted perimeter
'Hydraulic Radius
Dim wdVstar As Double
                                                                       'Shear Velocity
Dim wdTw As Double
Dim wdPwwalls As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdDeltaRew_fw As Double
Dim wdderRew_fw As Double
Dim wdChange As Double
Dim wdPwb As Double
Dim wdfb As Double
Dim wdAreab As Double
Dim wdRhb As Double
wsG = 32.174
```

wdPi = 3.14159265359

```
wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
 wdboxw = wdboxw / 12
'check for submergence
   If wdHwd < 1.01 Then 'Unsubmerged
   'Calculate Depths
     wdYbox = wdDbed - wdr
     If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
wsForbeta = -(wdYculv / wdr)
     wsi orocta - -(wti routy / wdi)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
      wdYbox = 0
     End If
     wdYculv = wdr - wdWs
       If wdYculv > wdr Then
        ShieldsXACorr = "error"
        Exit Function
   'box cross-sectional area
   wdBoxArea = wdboxw * wdYbox
'box wetted perimeter
If wdYbox = 0 Then
     wdPwbox = wdTw
End If
     If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
   'culvert cross-sectional area
     If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
        wdbeta = wdPi
        wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2 wdPw = wdPi * wdr
      wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr End If
     End If
   'Total Cross-Sectional Area
   wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
'Total Wetted perimeter
     wdTPw = wdPw - wdPw2 + wdPwbox
   End If
   If wdHwd >= 1.01 Then 'Submerged
   'Calculate Depths
     wdYbox = wdDbed - (wdr + wdWs)
     If wdYbox < 0.0001 Then
      wdYculv = -1 * wdYbox
      wsForbeta = -(wdYculv / wdr)
     wsForDeta = -(wdYculv / wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdr * wdbeta - wdPi * wdr
wdTw = 2 * wdr * Sin(wdbeta)
     wdYbox = 0
End If
     wdYculv = wdr 'sometimes observed true, other times observed differently...this is approximation
      If wdYculv > wdr Then
ShieldsXACorr = "error"
        Exit Function
       End If
   'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
```

```
box wetted perimeter
     If wdYbox = 0 Then
      wdPwbox = wdTw
     End If
    If wdYbox > 0 Then
      wdPwbox = 2 * wdYbox + wdboxw
    End If
   'culvert cross-sectional area
    If wdYculv > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2 wdPw = 2 * wdbeta * wdr - wdPi * wdr
        \begin{tabular}{ll} wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) \\ wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 \\ wdPw = 2 * wdbeta * wdr - wdPi * wdr \\ \end{tabular} 
      End If
    End If
   'Total Cross-Sectional Area
    wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted Perimeter
    wdTPw = wdPw - wdPw2 + wdPwbox
   End If
 wdRh = wdTArea / wdTPw
'Separate shear to shear on riprap and shear on model boundaries
wdf = 8 * wsG * wdRh * wds / (wdVmean) ^ 2
wdRe_f = wdRe / wdf
wdfw = 0.01  'guess
 wdRew_fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)

wdDeltaRew_fw = wdRe_f - wdRew_fw

wdderRew_fw = Exp(1.15129 / wdfw ^ (1 / 2)) * (-3.76783 * wdfw ^ (1 / 2) - 1.44596) / wdfw ^ 3

wdChange = wdDeltaRew_fw / wdderRew_fw
  wdchange = wdbeltartew_nw / wdde
wdfw = wdfw + wdChange
If Abs(wdChange) < 0.000001 Then
   Exit Do
   End If
 Loop
 If wdPwbox >= 0 Then wdPwb = wdboxw
 If wdPw2 > 0 Then
   wdPwb = wdTw
 Fnd If
 wdPwwalls = wdTPw - wdPwb
 wdfb = wdf + wdPwwalls / wdPwb * (wdf - wdfw)
wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wsG * wds)
 wdRhb = wdAreab / wdPwb
 wdVstar = (wsG * wdRhb * wds) ^ (1 / 2)
ShieldsXACorr = wdVstar * wdDs / wdnu
End Function
'Calculate V* * Ds/nu inside barrel of culvert
Function ShieldsX(wdDbed, wdWs, wdFlow, wdr, wdboxw, wdnu, wdDs, wds)
Dim wdYbox As Double
                                      'Depth of Flow in box
Dim wdYculv As Double
                                     'Depth of Flow in Culvert
'Area of flow in box
Dim wdBoxArea As Double
Dim wdTArea As Double
Dim wdPi As Double
                                      'Total area of flow in culvert
                                   'Pi Constant
                                     'angle in radians
'Cross-sectional area of Culvert
'Cross-sectional area of Riprap above box below Culvert
Dim wdbeta As Double
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double
Dim wdL As Double 'Le
Dim wdWetPerim As Double
                                   'Length of Culvert
e 'Wetted Perimeter
                                     '(1-Y/R) term in beta
Dim wsForbeta As Single
                                  'Acceleration of Gravity
'wetted perimeter in culvert
Dim wsG As Single
Dim wdPw As Double
Dim wdPw2 As Double
                                      'wetted perimeter of riprap above box below culvert
Dim wdPwbox As Double
                                      'wetted perimeter in box'
'Total wetted perimeter
Dim wdTPw As Double
Dim wdRh As Double
                                     'Hydraulic Radius
Dim wdVstar As Double
                                      'Shear Velocity
Dim wdTw As Double
```

```
wsG = 32 174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
 wdboxw = wdboxw / 12
'check for submergence
  'Calculate Depths
    wdYbox = wdDbed - wdr
   If wdYbox <= 0 Then
wdYculv = .1 * wdYbox
wsForbeta = -(wdYculv / wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
    wdYbox = 0
End If
    wdYculv = wdr - wdWs
     If wdYculv > wdr Then
ShieldsX = "error"
       Exit Function
     End If
  'box cross-sectional area
    wdBoxArea = wdboxw * wdYbox
  'box wetted perimeter
  If wdYbox = 0 Then
wdPwbox = wdTw
  End If
  If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
  End If
  'culvert cross-sectional area
    If wdYculv > 0 Then
     wsForbeta = -(wdYculv / wdr)
     If wdYculv = wdr Then
       wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
       wdPw = wdPi * wdr
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
      wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw = 2 * wdr * wdbeta - wdPi * wdr
     End If
  If wdYculv < 0 Then
wdPw = 0
    wdCulvArea = 0
  End If
  'Total Cross-Sectional Area
    wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
  'Total Wetted perimeter
wdTPw = wdPw - wdPw2 + wdPwbox
 wdRh = wdTArea / wdTPw
 wdVstar = (wsG * wdRh * wds) ^ (1 / 2)
 ShieldsX = wdVstar * wdDs / wdnu
End Function
'Calculate V* * Ds/nu inside barrel using Modified Prandtl/Einstein
Function\ ShieldsPE(wdDbed,\ wdWs,\ wdFlow,\ wdr,\ wdboxw,\ wdnu,\ wdDs,\ wdS,\ wdD65,\ wdVmean)
Dim wdYbox As Double
                                   'Depth of Flow in box
Dim wdYculv As Double
                                   'Depth of Flow in Culvert
Dim wdBoxArea As Double
                                    'Area of flow in box
Dim wdTArea As Double
Dim wdPi As Double
                                   'Total area of flow in culvert
                                 'Pi Constant
                                  'angle in radians
'Cross-sectional area of Culvert
Dim wdbeta As Double
Dim wdCulvArea As Double
```

```
Dim wdCulvArea2 As Double 'Cross-sectional area of Riprap above box below Culvert
Dim wd. As Double 'Length of Culvert
Dim wd. As Double 'Length of Culvert
Dim wdWetPerim As Double 'Wetted Perimeter
Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wWetPerim As Double

Dim wsForbeta As Single
Dim wsG As Single

"Acceleration of Gravity

"wetted perimeter in culvert
                                           'wetted perimeter of riprap above box below culvert
Dim wdPwbox As Double
Dim wdTPw As Double
                                          'wetted perimeter in box 
'Total wetted perimeter
Dim wdRh As Double
                                         'Hydraulic Radius
Dim wdVstar As Double
                                          'Shear Velocity
Dim wdTw As Double
Dim wdX As Double
Dim wdDeltaX As Double
Dim wdDeltaPrime As Double
Dim wdPhi As Double
Dim wdXnew As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
 wdr = wdr / 12
wdboxw = wdboxw / 12
'check for submergence
   'Calculate Depths
wdYbox = wdDbed - wdr
     If wdYhox \le 0 Then
       wdYculv = -1 * wdYbox
      wdTuur - - 1 wdTuur wdTuur wdTuur wsForbeta = -(wdYculv | wdr)
wsForbeta = -(wdYculv | wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
     wdYbox = 0
End If
     wdYculv = wdr - wdWs
       If wdYculv > wdr Then
ShieldsPE = "error"
         Exit Function
       End If
   'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
    'box wetted perimeter
   If wdYbox = 0 Then
wdPwbox = wdTw
    End If
   If wdYbox > 0 Then
wdPwbox = 2 * wdYbox + wdboxw
    End If
    'culvert cross-sectional area
     If wdYculv > 0 Then
       wsForbeta = -(wdYculv / wdr)
       If wdYculv = wdr Then
        wdbeta = wdPi
wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
         wdPw = wdPi * wdr
        wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw = 2 * wdr * wdbeta - wdPi * wdr
       End If
     End If
     If wdYculv < 0 Then
     wdCulvArea = 0
     End If
    'Total Cross-Sectional Area
    wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
    'Total Wetted perimeter
     wdTPw = wdPw - wdPw2 + wdPwbox
```

```
wdRh = wdTArea / wdTPw
   wdX = 1
   wdDeltaX = 0.05
  wdVstar = wdVmean / (5.75 ^{\star} (Log(12.27 ^{\star} wdX ^{\star} wdRh / wdD65)) / Log(10)) wdDeltaPrime = 11.6 ^{\star} wdnu / wdVstar If wdD65 / wdDeltaPrime <= 8 Then
    wdPhi = (Log(wdD65 / wdDeltaPrime)) / Log(10)
     wdXnew = 1.62265 + 0.09947 * wdPhi - 2.833 * wdPhi ^ 2 + 1.18924 * wdPhi ^ 3 + 2.5663 * wdPhi ^ 4 - 1.64 * wdPhi ^ 5 wdDeltaX = wdX - wdXnew
      wdX = wdXnew
  Loop
End If
   If wdD65 / wdDeltaPrime > 8 Then
    wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10))
   ShieldsPE = wdVstar * wdDs / wdnu
End Function
'Calculate sidewall correction by Vanoni & Brooks (1957)
'Calculate V* * Ds/nu inside barrel of culvert using Modified Prandtl/Einstein
Function ShieldsPECorr(wdDbed, wdWs, wdFlow, wdr, wdboxw, wdnu, wdDs, wds, wdD65, wdVmean, wdRe)
Dim wdYbox As Double
                                   'Depth of Flow in box
Dim wdYculv As Double
Dim wdBoxArea As Double
                                  'Depth of Flow in Culvert
'Area of flow in box
Dim wdTArea As Double
                                   'Total area of flow in culvert
                                'Pi Constant
Dim wdPi As Double
Dim wdbeta As Double
                                  'angle in radians
Dim wdCulvArea As Double
Dim wdCulvArea2 As Double
                                    'Cross-sectional area of Culvert
'Cross-sectional area of Riprap above box below Culvert
                                'Length of Culvert
e 'Wetted Perimeter
'(1-Y/R) term in beta
Dim wdL As Double
Dim wdWetPerim As Double
Dim wsForbeta As Single
Dim wsG As Single
Dim wdPw As Double
                                'Acceleration of Gravity
                                  'wetted perimeter in culvert
Dim wdPw2 As Double
                                   'wetted perimeter of riprap above box below culvert
                                   'wetted perimeter in box 
'Total wetted perimeter
Dim wdPwbox As Double
Dim wdTPw As Double
Dim wdRh As Double
                                  'Hydraulic Radius
Dim wdVstar As Double
                                  'Shear Velocity
Dim wdTw As Double
Dim wdPwwalls As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdRe_f As Double
Dim wdderRew_fw As Double
Dim wdRew fw As Double
Dim wdDeltaRew_fw As Double
Dim wdChange As Double
Dim wdPwb As Double
Dim wdfb As Double
Dim wdAreab As Double
Dim wdRhb As Double
Dim wdX As Double
Dim wdDeltaX As Double
Dim wdDeltaPrime As Double
Dim wdPhi As Double
Dim wdXnew As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
wdr = wdr / 12
wdboxw = wdboxw / 12
   'Calculate Depths
    wdYbox = wdDbed - wdr
    If wdYbox <= 0 Then
wdYculv = -1 * wdYbox
```

```
wsForbeta = -(wdYculv / wdr)
      wsf of leta = 4(wf Cutr' wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
       wdYbox = 0
     End If
     wdYculv = wdr - wdWs
       If wdYculv > wdr Then
         ShieldsPECorr = "error"
         Exit Function
       End If
   'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
   'box wetted perimeter
If wdYbox = 0 Then
    wdPwbox = wdTw
   End If
   If wdYbox > 0 Then
   wdPwbox = 2 * wdYbox + wdboxw
End If
   'culvert cross-sectional area
     If wdYculv > 0 Then
       wsForbeta = -(wdYculv / wdr)
       If wdYculv = wdr Then
wdbeta = wdPi
         wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
wdPw = wdPi * wdr
         wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
       End If
     End If
   'Total Cross-Sectional Area
     wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted perimeter
wdTPw = wdPw - wdPw2 + wdPwbox
  wdRh = wdTArea / wdTPw
'Separate shear to shear on riprap and shear on model boundaries
\label{eq:wdf} \begin{array}{l} wdf = 8 * wsG * wdRh * wds / (wdVmean) ^ 2 \\ wdRe\_f = wdRe / wdf \\ wdfw = 0.01 'guess \\ \end{array}
  Do
   wdRew_fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)
wdDeltaRew_fw = wdRe_f - wdRew_fw
wdderRew_fw = Exp(1.15129 / wdfw ^ (1 / 2)) * (-3.76783 * wdfw ^ (1 / 2) - 1.44596) / wdfw ^ 3
wdChange = wdDeltaRew_fw / wdderRew_fw
   wdfw = wdfw + wdChange

If Abs(wdChange) < 0.000001 Then

Exit Do
   End If
 Loop
 If wdPwbox >= 0 Then
   wdPwb = wdboxw
  Fnd If
 If wdPw2 > 0 Then
 wdPwb = wdTw
End If
 wdPwwalls = wdTPw - wdPwb
 wdf = wdf + wdPwwalls / wdPwb * (wdf - wdfw)
wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wsG * wds)
  wdRhb = wdAreab / wdPwb
   wdX = 1
   wdDeltaX = 0.05
   wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10)) wdDeltaPrime = 11.6 * wdnu / wdVstar
   WdDeltaPrime = 11:0 wdd WdStail

If wdD65 / wdDeltaPrime <= 8 Then

Do While wdDeltaX >= 0.0001

wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))

wdDeltaPrime = 11.6 * wdnu / wdVstar
```

```
wdX = wdXnew
   Loop
  End If
  If wdD65 / wdDeltaPrime > 8 Then
   wdX = 1
    wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))
  End If
 ShieldsPECorr = wdVstar * wdDs / wdnu
End Function
'Calculate V* * Ds/nu at entrance of culvert Function ShieldsXCorr(wdDbed, wdWs, wdFlow, wdr, wdboxw, wdnu, wdDs, wds, wdVmean, wdRe)
Dim wdYbox As Double
                                 'Depth of Flow in box
Dim wdYculv As Double
                                'Depth of Flow in Culvert
Dim wdBoxArea As Double
                                  'Area of flow in box
                                 'Total area of flow in culvert
Dim wdTArea As Double
Dim wdPi As Double
                              'Pi Constant
Dim wdbeta As Double
Dim wdCulvArea As Double
                               'angle in radians
'Cross-sectional area of Culvert
Dim wdCulvArea2 As Double
                                  'Cross-sectional area of Riprap above box below Culvert
                              'Length of Culvert
e 'Wetted Perimeter
Dim wdL As Double
Dim wdWetPerim As Double
Dim wsForbeta As Single
Dim wsG As Single
                             '(1-Y/R) term in beta
'Acceleration of Gravity
Dim wdPw As Double
                                'wetted perimeter in culvert
                                 'wetted perimeter of riprap above box below culvert 
'wetted perimeter in box
Dim wdPw2 As Double
Dim wdPwbox As Double
Dim wdTPw As Double
Dim wdRh As Double
                               'Total wetted perimeter 
'Hydraulic Radius
Dim wdVstar As Double
                                'Shear Velocity
Dim wdTw As Double
Dim wdPwwalls As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdDeltaRew fw As Double
Dim wdderRew_fw As Double
Dim wdChange As Double
Dim wdPwb As Double
Dim wdfb As Double
Dim wdAreab As Double
Dim wdRhb As Double
wsG = 32.174
wdPi = 3.14159265359
wdCulvArea2 = 0
'Inches to feet conversion
wdr = wdr / 12
 wdboxw = wdboxw / 12
  'Calculate Depths
    wdYbox = wdDbed - wdr
    If wdYbox <= 0 Then
     wdYculv = -1 * wdYbox
     wsForbeta = -(wdYculv / wdr)
    wsroneta = -(warcurv wdr)
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea2 = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
wdPw2 = 2 * wdbeta * wdr - wdr * wdPi
wdTw = 2 * wdr * Sin(wdbeta)
     wdYbox = 0
    End If
    wdYculv = wdr - wdWs
     If wdYculv > wdr Then
      ShieldsXCorr = "error"
      Exit Function
     End If
  'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
  'box wetted perimeter
If wdYbox = 0 Then
  wdPwbox = wdTw
  End If
  If wdYbox > 0 Then
  wdPwbox = 2 * wdYbox + wdboxw
```

```
End If
   'culvert cross-sectional area
    If wdYculy > 0 Then
      wsForbeta = -(wdYculv / wdr)
      If wdYculv = wdr Then
       wdbeta = wdPi
       wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2 wdPw = wdPi * wdr
       wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2 wdPw = 2 * wdr * wdbeta - wdPi * wdr
      End If
    End If
   'Total Cross-Sectional Area
     wdTArea = wdBoxArea + wdCulvArea - wdCulvArea2
   'Total Wetted perimeter
wdTPw = wdPw - wdPw2 + wdPwbox
 wdRh = wdTArea / wdTPw
'Separate shear to shear on riprap and shear on model boundaries
\label{eq:wdf} \begin{tabular}{ll} $\sf wdf = 8 * wsG * wdRh * wds / (wdVmean) ^ 2 \\ $\sf wdRe\_f = wdRe / wdf \\ $\sf wdfw = 0.01 \ 'guess \\ \end{tabular}
  wdRew fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)
  wdDeltaRew_fw = wdRe_f - wdRew_fw
wddeltaRew_fw = Exp(1.15129 / wdfw ^ (1 / 2)) * (-3.76783 * wdfw ^ (1 / 2) - 1.44596) / wdfw ^ 3
wdChange = wdDeltaRew_fw / wdderRew_fw
  wdfw = wdfw + wdChange

If Abs(wdChange) < 0.000001 Then
    Exit Do
  End If
 Loop
 If wdPwbox >= 0 Then
   wdPwb = wdboxw
 End If
 If wdPw2 > 0 Then
   wdPwb = wdTw
 End If
 wdPwwalls = wdTPw - wdPwb
 wdfb = wdf + wdPwwalls / wdPwb * (wdf - wdfw)
 wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wsG * wds)
wdRhb = wdAreab / wdPwb
 wdVstar = (wsG * wdRhb * wds) ^ (1 / 2)
ShieldsXCorr = wdVstar * wdDs / wdnu
End Function
'Calculate Specific Head at Critical Depth
'Only used for unsubmerged, inlet control equation form 1
Function SHEADCRIT(wdFlow, wdRip1D, wdRip2D)
 Dim wdYc As Double
 Dim wdboxw As Double
Dim wdr As Double
                                        'Width of box
                                     'culvert radius
 Dim wdYbox As Double
                                        'Depth of Flow in box
                                       'Depth of Flow in Culvert
'Total Cross-Sectional Area
 Dim wdYculv As Double
 Dim wdTArea As Double
 Dim wdYboxmax As Double
Dim wdTw As Double
                                      'max possible depth of box section
'Width of Channel at Water Surface
 Dim wdVc As Double
                                      'Critical Velocity
                                         'Cross-sectional area of Box
 Dim wdBoxArea As Double
 Dim wdPi As Double
                                     'Pi Constant
                                       'angle in radians
'Cross-sectional area of Culvert
 Dim wdbeta As Double
 Dim wdCulvArea As Double
                                   '(1-Y/R) term in beta
'Acceleration of Gravity
 Dim wsForbeta As Single
 Dim wsG As Single
                                        'Riprap to be scoured depth in box 
'Pea Gravel Substrate (1 in ideal)
  'Dim wdRip1D As Double
 'Dim wdRip2D As Double
 wsG = 32.174
 wdPi = 3.14159265359
 wdboxw = Cells(5, 14).Value
```

```
wdr = Cells(8, 14).Value
wdYboxmax = Cells(6, 14).Value
'Calculate Critical Depth
 wdYc = (wdFlow ^ 2 / wsG) ^ (1 / 3)
'Inches to feet conversion
  wdboxw = wdboxw / 12
wdr = wdr / 12
  wdYboxmax = wdYboxmax / 12
wdRip1D = wdRip1D / 12
wdRip2D = wdRip2D / 12
'Calculate Cross-Sectional Area
 'split into two sections, box & culvert
wdYbox = wdYboxmax - wdRip1D - wdRip2D
   If wdYbox < 0.0001 Then
    wdYbox = 0
   End If
  wdYculv = wdYc - wdYbox
If wdYculv > wdr Then
    SHEADCRIT = "-"
    Exit Function
   End If
 'Box cross-sectional area
   wdBoxArea = wdboxw * wdYbox
  'Culvert cross-sectional area
   If wdYculv > 0 Then
    wsForbeta = (wdYculv / wdr) - 1
If wdYculv = wdr Then
      wdbeta = wdPi
      wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
     wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1) wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
    End If
  Fnd If
  'Total Cross-Sectional Area
   wdTArea = wdBoxArea + wdCulvArea
'Calculate Top Width (Width of Channel at Water Surface)
 If wdYculv = 0 Then
wdTw = wdboxw
 Elself wdYculv = wdr Then
  wdTw = 0
 wdw - 0
Elself wdYculv > 0 And wdYculv < wdr Then
wdTw = 2 * wdr * Sin(wdbeta)
wdYculv = wdYc - wdYbox
'Calculate Critical Velocity
 wdVc = (wsG * wdTArea / wdTw) ^ 0.5
SHEADCRIT = wdYc + wdVc ^ 2 / (2 * wsG)
End Function
'Calculate Q/AD^.5 (Unsubmerged Case)
Function QUAD(wdFlow, wdHw, wdRip1D, wdRip2D) 'wdRip are substrate depths
 Dim wdboxw As Double
                                     'Width of box
 Dim wdr As Double
                                  'culvert radius
 Dim wdYbox As Double
                                     'Depth of Flow in box
 Dim wdYculv As Double
Dim wdTArea As Double
                                     'Depth of Flow in Culvert
'Total Cross-Sectional Area
 Dim wdYboxmax As Double
                                        'max possible depth of box section
 Dim wdBoxArea As Double
                                       'Cross-sectional area of Box
 Dim wdPi As Double
                                   'Pi Constant
 Dim wdbeta As Double
Dim wdCulvArea As Double
                                    'angle in radians
'Cross-sectional area of Culvert
 Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
 wsG = 32.174
 wdPi = 3.14159265359
'Cell Input
 wdboxw = Cells(5, 14).Value
wdr = Cells(8, 14).Value
wdYboxmax = Cells(6, 14).Value
 'inches to feet conversion
   wdboxw = wdboxw / 12
   wdr = wdr / 12
   wdHw = wdHw / 12
```

```
wdYboxmax = wdYboxmax / 12
  wdRip1D = wdRip1D / 12
  wdRip2D = wdRip2D / 12
'Calculate Cross-Sectional Area 'split into two sections, box & culvert
   wdYbox = wdYboxmax - wdRip1D - wdRip2D
  If wdYbox < 0.0001 Then
    wdYbox = 0
  End If
  wdYculv = wdHw - wdYbox
  If wdYculv > wdr Then
QUAD = "-"
    Exit Function
  End If
 'box cross-sectional area
wdBoxArea = wdboxw * wdYbox
 'culvert cross-sectional area
  If wdYculv > 0 Then
    wsForbeta = (wdYculv / wdr) - 1
    If wdYculv = wdr Then
     wdbeta = wdPi
     wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
    Else
     wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
     wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
    End If
 'Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea
'Calculate QUAD Function
 QUAD = wdFlow / (wdTArea * wdr ^ 0.5)
End Function
'Calculate (Q/AD^.5)^2
Function QUAD2(wdFlow, wdHw, wdRip1D, wdRip2D)
 Dim wdboxw As Double
                                    'Width of box
 Dim wdr As Double
Dim wdr As Double
Dim wdYbox As Double
Dim wdYculv As Double
                                'culvert radius
                                   'Depth of Flow in box
'Depth of Flow in Culvert
 Dim wdTArea As Double
                                    'Total Cross-Sectional Area
'max possible depth of box section
 Dim wdYboxmax As Double
 Dim wdBoxArea As Double
                                      'Cross-sectional area of Box
 Dim wdPi As Double
Dim wdbeta As Double
                                 'Pi Constant
                                   'angle in radians
 Dim wdCulvArea As Double
                                      'Cross-sectional area of Culvert
 Dim wsForbeta As Single '(1-Y/R) term in beta
Dim wsG As Single 'Acceleration of Gravity
 Dim wsG As Single
 wsG = 32.174
 wdPi = 3.14159265359
'Cell Input
 wdboxw = Cells(5, 14).Value
wdr = Cells(8, 14).Value
wdYboxmax = Cells(6, 14).Value
'inches to feet conversion
  wdboxw = wdboxw / 12
wdr = wdr / 12
  wdHw = wdHw / 12
  wdYboxmax = wdYboxmax / 12
wdRip1D = wdRip1D / 12
  wdRip2D = wdRip2D / 12
'Calculate Cross-Sectional Area
 'split into two sections, box & culvert
wdYbox = wdYboxmax - wdRip1D - wdRip2D
  If wdYbox < 0.0001 Then
    wdYbox = 0
  End If
  wdYculv = wdHw - wdYbox
  If wdYculv > wdr Then
QUAD2 = "-"
    Exit Function
 'box cross-sectional area
  wdBoxArea = wdboxw * wdYbox
 'culvert cross-sectional area
If wdYculv > 0 Then
    wsForbeta = (wdYculv / wdr) - 1
```

```
If wdYculv = wdr Then
wdbeta = wdPi
wdCulvArea = ((wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta))) / 2
Else
wdbeta = Atn(-wsForbeta / Sqr(-wsForbeta * wsForbeta + 1)) + 2 * Atn(1)
wdCulvArea = (wdr ^ 2) * (wdbeta - Sin(wdbeta) * Cos(wdbeta)) - wdPi / 2
End If
Total Cross-Sectional Area
wdTArea = wdBoxArea + wdCulvArea

'Calculate QUAD Function
QUAD2 = (wdFlow / (wdTArea * wdr ^ 0.5)) ^ 2
End Function

'Calculate Hw/D - Hc/D +.5S Ratio
Function RATIO(wdHw, wdHc)

Dim wdr As Double 'Slope

'Cell Input
wdr = Cells(8, 14).Value
wds = Cells(6, 18).Value
'ConConvert from inches to feet
wdr = wdr / 12
If wdHc = "." Then
RATIO = "."
Exit Function

End If

RATIO = wdHw / wdr - wdHc / wdr + 0.5 * wds
End Function
```

Appendix I

Visual Basic Code Used for Calculations for

Rectangular Flume in Microsoft Excel

```
Option Explicit
'Automatic couter of test runs
Function RUNCOUNTER(wii)
 RUNCOUNTER = wii + 1
End Function
'Temperature Conversion & Check
Function TEMPERATURE(wdTemp)
If wdTemp <= 32 Then
  MsgBox ("Water is either frozen or not entered in Fahrenheit.")
  Exit Function
 Elself wdTemp >= 70 Then
  MsgBox ("Most unlikely, please double check.")
 End If
 TEMPERATURE = 5 / 9 * (wdTemp - 32)
End Function
'Orifice Flow Calculations
Function FLOW(strPipe, strFluid, wdManomH)
Dim wdDOrifice As Double 'Diameter of
                                        'Diameter of Orifice
Dim wdDPipe As Double
Dim wdDeltaH As Double
                                       'Pipe Diameter of Inflow 
'Calculate deltaH for Flow Equation
Dim wdPi As Double
                                     'PI contstant
                                      'Cross-sectional Area of Pipe of Inflow 
'Cross-sectional Area of Throat of Orifice
Dim wdA1 As Double
Dim wdA2 As Double
Dim wsK As Single
                                    'Constant from Orifice Calibration
Dim wsGammaM As Single
                                          'Specific weight of Manometer Fluid
Dim wsG As Single
                                    'Acceleration of Gravity Constant
wdPi = 3.14159265359
                                      'Define Constant
                                 'Define Constant
wsG = 32.174
'Pipe Check
If strPipe = "2" Then
wdDOrifice = 1.035 / 12
                                        'Data from calibration, separate excel file
  wdDPipe = 2.042 / 12
wsK = 0.507
 Elself strPipe = "4" Then
  wdDOrifice = 3# / 12
wdDPipe = 4.026 / 12
                                      'Data from calibration, separate excel file
 wsK = 0.7452
Elself strPipe = "12" Then
wdDOrifice = 8.005 / 12
                                       'Data from calibration, separate excel file
  wdDPipe = 12# / 12
wsK = 0.6671
  MsgBox ("Pipe diameter of Inflow entered incorrectly. Please enter 2,4 or 12.")
 End If
 'Calculate deltaH from manometer
 If strFluid = "Blue" Then
 wsGammaM = 1.75
Elself strFluid = "Hg" Then
wsGammaM = 13.5536
Else
  MsgBox ("Manometer fluid entered incorrectly, please use 'Blue' or 'Hg'.")
Exit Function
End If
 wdManomH = wdManomH * 0.0328083989501 'cm to ft conversion
 wdDeltaH = wdManomH * (wsGammaM - 1)
'Calculate Cross Sectional Areas
wdA1 = 0.25 * wdPi * wdDPipe ^ 2
wdA2 = 0.25 * wdPi * wdDOrifice ^ 2
 FLOW = wsK * wdA2 * (2 * wsG * wdDeltaH) ^ (1 / 2) 'Venturi equation wdQ = wsk * wdA2 * ((2 * wsG * wdDeltaH)) ^ 0.5
'Calculate Specific Weight of Water as a function of Temperature (Fahrenheit)
Function GAMMAH2O(wdTemp)
GAMMAH2O = 59.364982 + 3.0750805 * Cos(0.0078331697 * (wdTemp) - 0.24302151)
  Slight adjustment of gamma to match values given in Engineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson
  If wdTemp = 40 Then
GAMMAH2O = 62.43
  Elself wdTemp = 50 Then
GAMMAH2O = 62.4
End Function
'Calculate Dynamic Viscosity of Water as a functio of Temperature (fahrenheit)
Function MUH2O(wdTemp)
MUH2O = 1 / (10865.946 + 441.70715 * (wdTemp) + 1.4672793 * (wdTemp) ^ 2)
 'Slight adjustment of mu to match values given in Éngineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson
  If wdTemp = 40 Then
MUH2O = 0.0000323
  Elself wdTemp = 50 Then
MUH2O = 0.0000273
  End If
End Function
```

'Calculate kinematic viscosity of Water as a function of Temperature (Fahrenheit)

```
Function NUH2O(wdTemp) NUH2O = Exp(-5.5116242 - 26.354346 / wdTemp - 1.3108492 * (Log(wdTemp) / Log(2.71828182846)))
  'Slight adjustment of nu to match values given in Engineering Fluid Mechanics 7th edition by Crowe, Elger, Roberson If wdTemp = 40 Then NUH2O = 0.0000166
   Elself wdTemp = 50 Then
NUH2O = 0.0000141
   End If
End Function
'Calculate Density of Water as a function of Temperature (Fahrenheit) Function RHOH2O(wdTemp)
 If wdTemp > 32 And wdTemp < 80 Then
RHOH2O = 1.94
 MsgBox ("Water Temperature Invalid")
End If
End Function
'Calculate Mean Cross-section water column velocity
Function MVelocity(wdY, wdflumeW, wdflow)
Dim wdArea As Double 'Cross-sectional Area
 wdArea = wdY * wdflumeW
MVelocity = wdflow / wdArea
End Function
'Calculate Reynolds Number Function RE(wdY, wdflumeW, wdflow, wdNu)
Dim wsG As Single
                                 'Acceleration of Gravity
Dim wdArea As Double
Dim wdPw As Double
                                    'Cross-sectional Area 
'wetted perimeter in culvert
Dim wdRh As Double
                                   'Hydraulic Radius
   wsG = 32.174
   wdArea = wdY * wdflumeW
   wdPw = wdY * 2 + wdflumeW
wdRh = wdArea / wdPw
 RE = 4 * wdflow * wdRh / (wdArea * wdNu) 'Closed Conduit form - for consistancy
End Function
'State Laminar or Turbulent Flow Function LTFLOW(wdRe)
 If wdRe <= 2000 Then
LTFLOW = "Laminar"
Elself wdRe > 2000 Then
LTFLOW = "Turbulent"
 Fnd If
End Function
'Froude # in Flume (Open Channel Case)
Function FROUDE(wdY, wdflumeW, wdflow)
Dim wsG As Single
Dim wdArea As Double
                                 'Acceleration of Gravity 
'Cross-sectional Area
Dim wdTw As Double
                                   'Top width of cross-section
   wsG = 32.174
   wdTw = wdflumeW
wdArea = wdY * wdflumeW
 FROUDE = wdflow / (wdArea ^ 3 * wsG / wdTw) ^ 0.5
Function M Velocity(wdY, wdflumeW, wdflow)
                                   'Cross-sectional Area
Dim wdArea As Double
    wdArea = wdY * wdflumeW
   M_Velocity = wdflow / wdArea
End Function
'Calculate V* * Ds / nu in flume
Function ShieldsX(wdY, wdflumeW, wdflow, wdNu, wdDs, wdS)
Dim wdArea As Double
                                    'Cross-sectional area
Dim wsG As Single
Dim wdPw As Double
                                 'Acceleration of Gravity
'Wetted perimeter
Dim wdRh As Double
                                    'Hydraulic Radius
```

Dim wdVstar As Double

'Shear Velocity

```
wsG = 32.174
  wdArea = wdY * wdflumeW
wdPw = wdY * 2 + wdflumeW
wdRh = wdArea / wdPw
  wdVstar = (wsG * wdRh * wdS) ^ (1 / 2)
 ShieldsX = wdVstar * wdDs / wdNu
End Function
'Calculate V* * Ds/nu in Flume using Modified Prandtl/Einstein
Function\ Shields PE(wdY,\ wdflumeW,\ wdflow,\ wdNu,\ wdDs,\ wdD65,\ wdS,\ wdVmean)
Dim wdArea As Double
Dim wsG As Single
Dim wdPw As Double
Dim wdRh As Double
Dim wdVstar As Double
Dim wdX As Double
Dim wdDeltaX As Double
Dim wdDeltaPrime As Double
Dim wdPhi As Double
Dim wdXnew As Double
wsG = 32.174
  wdArea = wdY * wdflumeW
wdPw = wdY * 2 + wdflumeW
  wdRh = wdArea / wdPw
  wdX = 1
  wdDeltaX = 0.05
  wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10)) wdDeltaPrime = 11.6 * wdNu / wdVstar
  If wdD65 / wdDeltaPrime <= 8 Then
   wdDeltaX = wdX - wdXnew
    wdX = wdXnew
  Loop
End If
  If wdD65 / wdDeltaPrime > 8 Then
   wdX = 1
    wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRh / wdD65)) / Log(10))
  Fnd If
  ShieldsPE = wdVstar * wdDs / wdNu
End Function
'Calculate sidewall correction by Vanoni & Brooks (1957)
'Calculate Shields Parameters using Modified Prandtle Éinstein
Function ShieldsPECorr(wdW, wdDs, wdD65, wdY, wdVmean, wdRe, wdS, wdNu)
Dim wdRh As Double
Dim wdArea As Double
Dim wdPw As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdfwnew As Double
Dim wdDeltaRew_fw As Double
Dim wdfb As Double
Dim wdRhb As Double
Dim wdAreab As Double
Dim wdPwb As Double
Dim wdVstar As Double
Dim wdShearb As Double
Dim wdX As Double
Dim wdDeltaX As Double
Dim wdDeltaPrime As Double
Dim wdPhi As Double
Dim wdXnew As Double
Dim wdChange As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdderRew_fw As Double
```

Dim wdG As Single

```
wdG = 32.174
wdArea = wdW * wdY
wdPw = wdW + wdY * 2
wdRh = wdArea / wdPw
wdf = 8 * wdG * wdRh * wdS / (wdVmean) ^ 2
wdRe_f = wdRe / wdf
wdfw = 0.01 'guess
 Do
wdRew_fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)
  Exit Do
   End If
 wdfb = wdf + 2 * wdY / wdW * (wdf - wdfw)
 wdPwb = wdW
 wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wdG * wdS)
 wdRhb = wdAreab / wdPwb
  'wdVstar = (wdG*wdRhb*wdS)^(1/2)'von karmon
   wdX = 1
   wdDeltaX = 0.05
   wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10)) wdDeltaPrime = 11.6 * wdNu / wdVstar If wdD65 / wdDeltaPrime <= 8 Then
    TWODb5 / WdDeltaPrime <= 6 Their
Do While wdDeltaX >= 0.0001
wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))
wdDeltaPrime = 11.6 * wdNu / wdVstar
wdPhi = (Log(wdD65 / wdDeltaPrime)) / Log(10)
wdXnew = 1.62265 + 0.09947 * wdPhi - 2.833 * wdPhi ^ 2 + 1.18924 * wdPhi ^ 3 + 2.5663 * wdPhi ^ 4 - 1.64 * wdPhi ^ 5
      wdDeltaX = wdX - wdXnew
      wdX = wdXnew
   Loop
End If
   If wdD65 / wdDeltaPrime > 8 Then
    wdVstar = wdVmean / (5.75 * (Log(12.27 * wdX * wdRhb / wdD65)) / Log(10))
 ShieldsPECorr = wdVstar * wdDs / wdNu
End Function
'Calculate sidewall correction by Vanoni & Brooks (1957)
'Calculate Shields Parameters using Von Karmon
Function ShieldsVKCorr(wdW, wdDs, wdD65, wdY, wdVmean, wdRe, wdS, wdNu)
Dim wdRh As Double
Dim wdArea As Double
Dim wdPw As Double
Dim wdf As Double
Dim wdfw As Double
Dim wdfwnew As Double
Dim wdDeltaRew_fw As Double
Dim wdfb As Double
Dim wdRhb As Double
Dim wdAreab As Double
Dim wdPwb As Double
Dim wdVstar As Double
Dim wdShearb As Double
Dim wdChange As Double
Dim wdRe_f As Double
Dim wdRew_fw As Double
Dim wdderRew_fw As Double
Dim wdG As Single
wdG = 32.174
wdArea = wdW * wdY
wdPw = wdW + wdY * 2
wdRh = wdArea / wdPw
wdRh = wdArea / wdPw
wdf = 8 * wdG * wdRh * wdS / (wdVmean) ^ 2
wdRe_f = wdRe / wdf
wdfw = 0.01 'guess
```

```
Do
    wdRew_fw = (10 ^ (1 / (2 * wdfw ^ (1 / 2)) + 0.4)) / wdfw ^ (3 / 2)
    wdDeltaRew_fw = wdRe_f - wdRew_fw
    wdderRew_fw = Exp(1.15129 / wdfw ^ (1 / 2)) * (-3.76783 * wdfw ^ (1 / 2) - 1.44596) / wdfw ^ 3
    wdChange = wdDeltaRew_fw / wdderRew_fw
    wdfw = wdfw + wdChange
    If Abs(wdChange) < 0.000001 Then
    Exit Do
    End If
    Loop

wdfb = wdf + 2 * wdY / wdW * (wdf - wdfw)
    wdPwb = wdlW
    wdAreab = wdfb * wdVmean ^ 2 * wdPwb / (8 * wdG * wdS)
    wdRhb = wdAreab / wdPwb

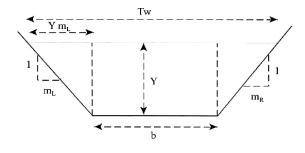
wdVstar = (wdG * wdRhb * wdS) ^ (1 / 2) 'von karmon

ShieldsVKCorr = wdVstar * wdDs / wdNu

End Function
```

Appendix J

Channel Cross-section Formulas



$$A = (Y \cdot b) + \left(\frac{Y^2}{2} m_L\right) + \left(\frac{Y^2}{2} m_R\right)$$

$$P = b + \left(Y \cdot \sqrt{1 + m_L^2}\right) + \left(Y \cdot \sqrt{1 + m_R^2}\right)$$

$$T_W = b + Y \cdot \left(m_L + m_R\right)$$

$$A \cdot h_c = \left(\frac{Y^2 \cdot b}{2}\right) + \left(\frac{Y^3}{6} m_L\right) + \left(\frac{Y^3}{6} m_R\right)$$

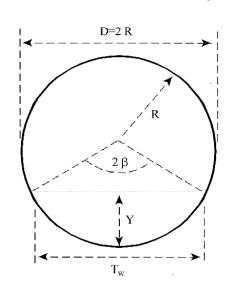
$$A = (Y \cdot b) + (Y^2 \cdot m)$$

$$P = b + \left(2 \cdot Y \cdot \sqrt{1 + m^2}\right)$$

$$T_w = b + (2 \cdot Y \cdot m)$$

$$A \cdot h_c = \left(\frac{Y^2 \cdot b}{2}\right) + \left(\frac{Y^3 \cdot m}{3}\right)$$

$$R_h = \frac{A}{P}$$
 $E = Y + \frac{Q^2}{2g \cdot A^2} + \Delta z$ $M = (A \cdot h_c) + \frac{Q^2}{g \cdot A}$



$$\beta = a\cos\left(1 - \frac{Y}{R}\right) \quad in \ radians$$

$$A = R^{2} \cdot \left(\beta - \cos(\beta) \cdot \sin(\beta)\right)$$

$$P = 2 \cdot R \cdot \beta$$

$$T_{W} = 2 \cdot R \cdot \sin(\beta)$$

$$A \cdot h_{c} = R \cdot \left(\frac{2}{3}R^{2} \cdot \sin(\beta)^{3} - A \cdot \cos(\beta)\right)$$

(Rahmeyer, 2003)