Improved Formulas for Synchrotron Radiation

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Acknowledgments: Mitchell Furst, Tom Lucatorto, Ping Shaw, Uwe Arp

Outline

- Background on synchrotron radiation
  - 1st & 2nd generation only
  - Radiometric utility
  - Work at NIST (very cursory)

- Calculation of SR (other work)

- Calculation approach (this work)
  - Coordinate systems
  - Ultimate formula & conclusions

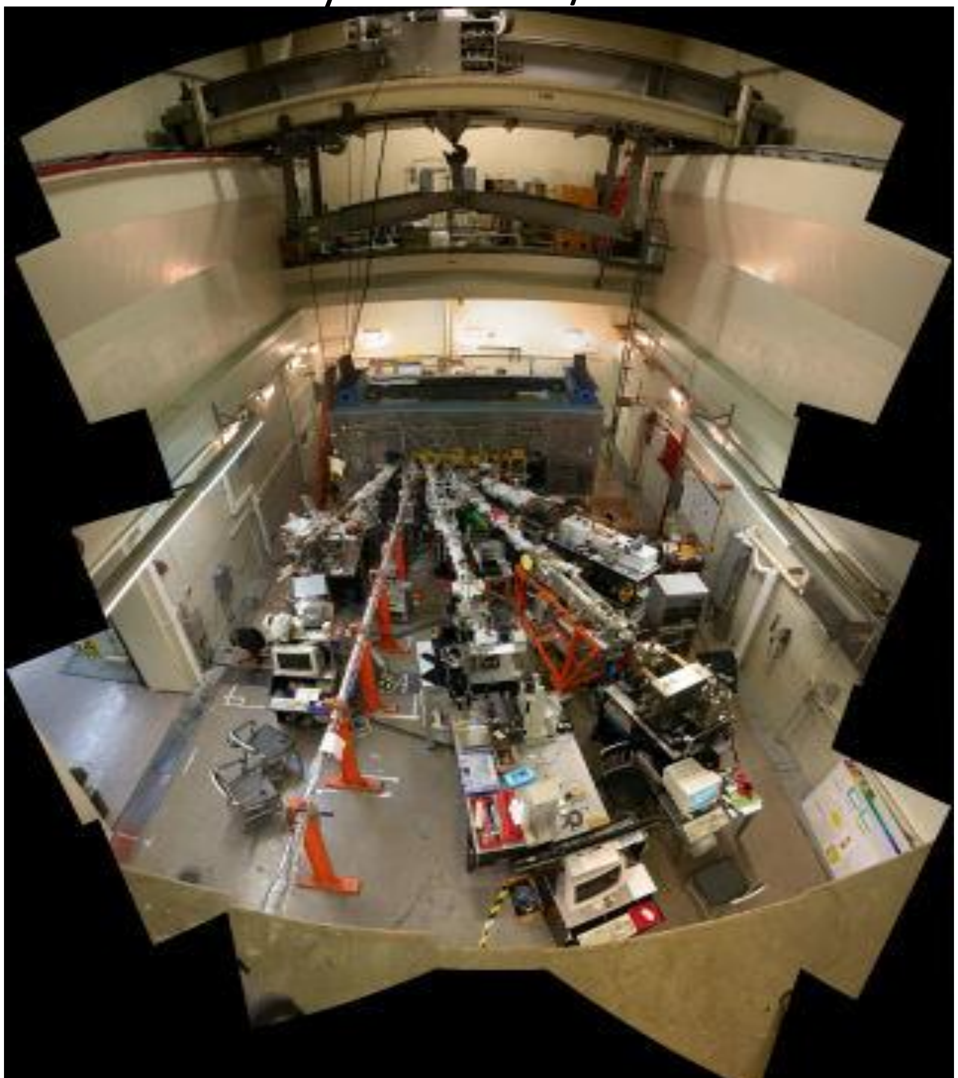
- “Fuzzing” effects & diffraction effects

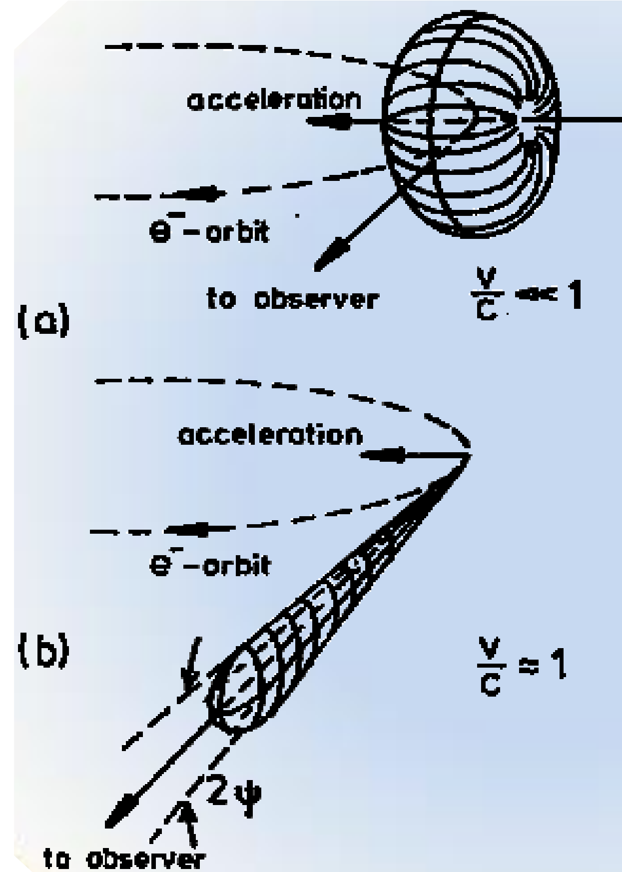
- Conclusions

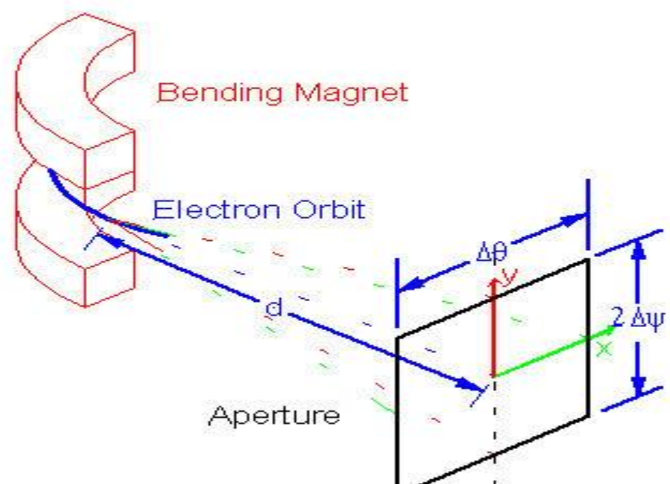
Synchrotron radiation—

Emitted by relativistic charged particles orbiting (accelerated) in magnetic fields

NIST SURF III Synchrotron / Measurement Hall







Schwinger formula (1949):

$$P(\lambda,\gamma,\psi_0,\rho,\Delta\lambda,I_B,\Delta\theta,\Delta\psi)=$$

$$\int_{\psi_0-\Delta\psi}^{\psi_0+\Delta\psi}\frac{2}{3}\frac{e_0\,\Delta\lambda\,\Delta\theta\,I_B\,\rho^2}{\varepsilon_0\,\beta\,\lambda^4\,\gamma^4}\left[1+(\gamma\psi)^2\right]^2$$

$$\times\left[K_{2/3}\left(\xi(\lambda,\gamma)\right)^2+\frac{(\gamma\psi)^2}{1+(\gamma\psi)^2}K_{1/3}\left(\xi(\lambda,\gamma)\right)^2\right]d\psi$$

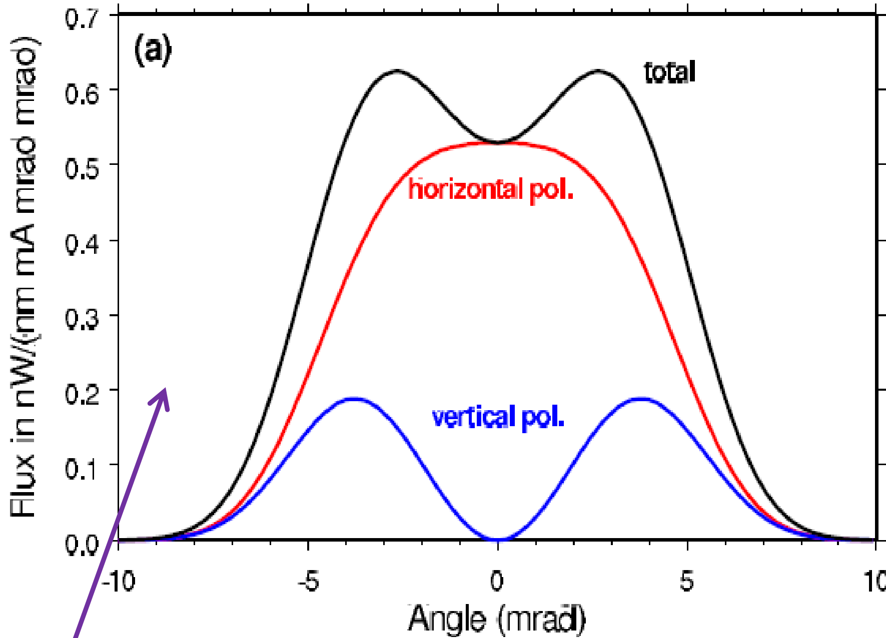
Legend:

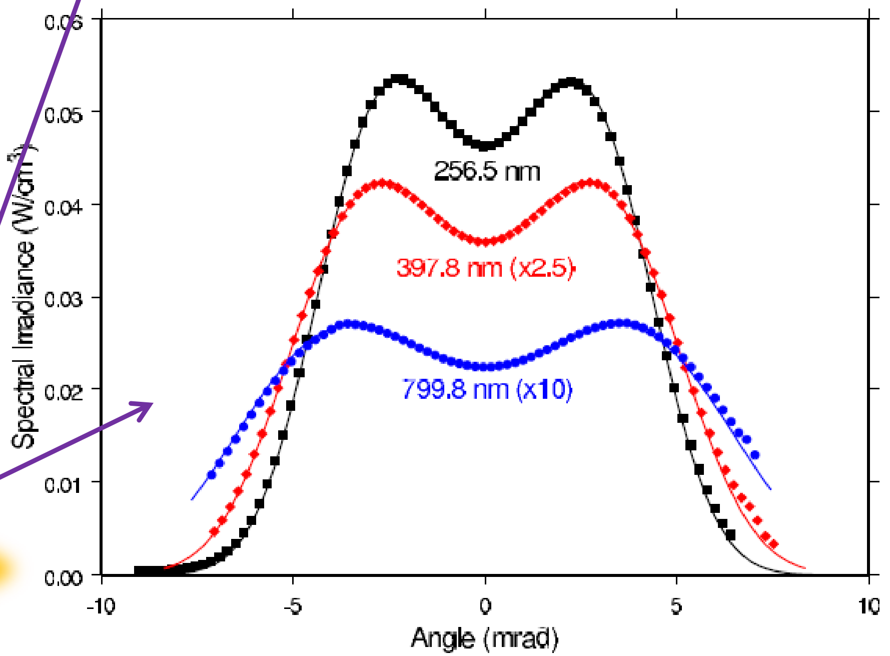
P	power
λ	wavelength
$\gamma=E/(m_e c^2)$	Electron energy
ψ_0, $\Delta\psi$ & θ	collection angle info.
I_B	Beam current
ρ	Orbit radius (also R)
$\xi=\lambda(1+\gamma^2\psi^2)^{\frac{3}{2}}/(2\lambda_c)$	Schwinger param.
$\lambda_c=4\pi\rho/(3\gamma^3)$	Critical wavelength
K_ν	Bessel fcn. (mod.)

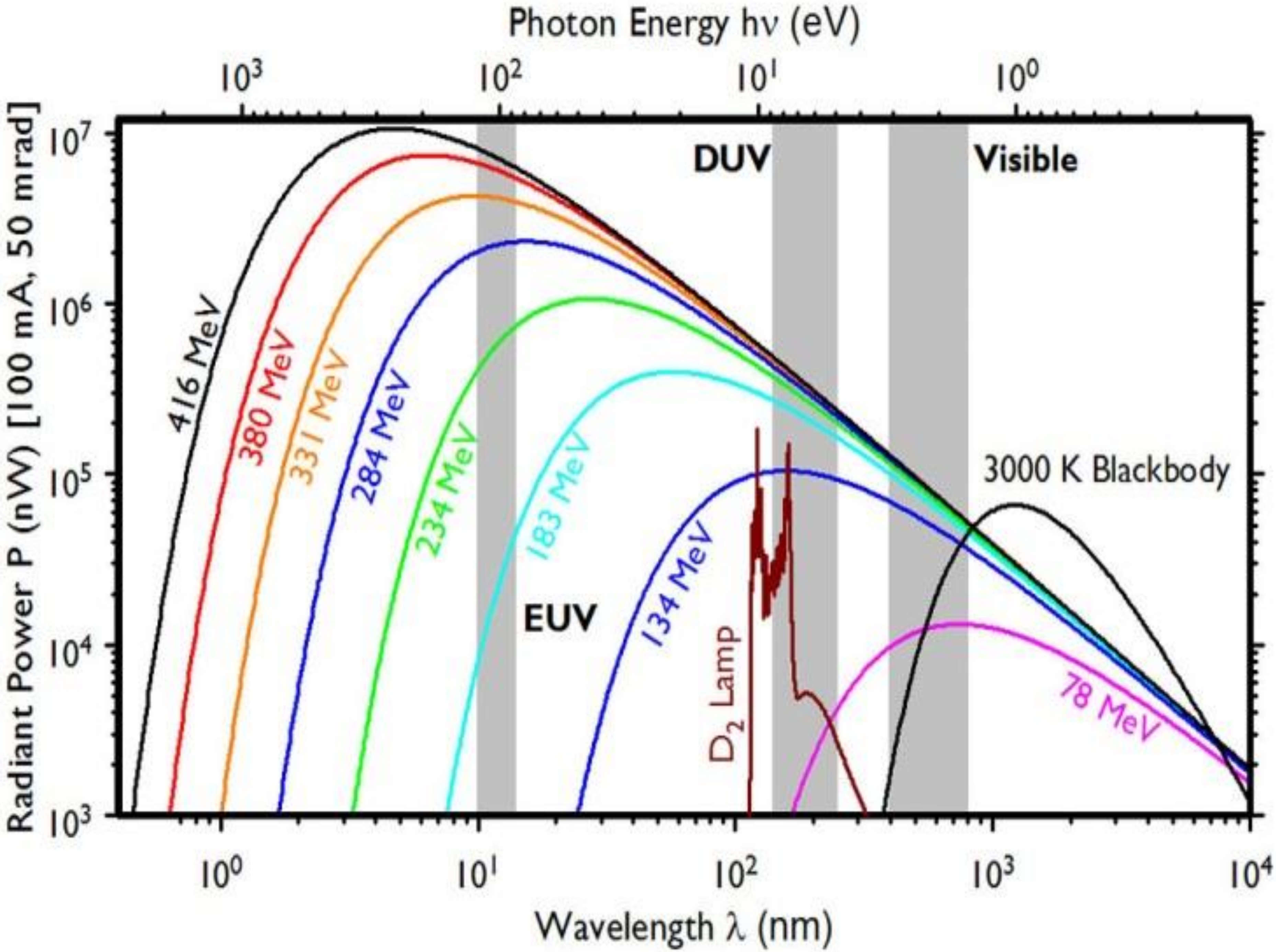
Like a blackbody,
a **calculable** primary source!

Why this work: assess diffraction effects and Schwinger formula

Angular output vs. ψ and λ:







Photon Energy hν (eV)
10^3
10^2
10^1
10^0
Radiant Power P (nW) [100 mA, 50 mrad]
10^7
10^6
10^5
10^4
10^3
DUV
Visible
EUV
416 MeV
380 MeV
331 MeV
284 MeV
234 MeV
183 MeV
134 MeV
78 MeV
D2 Lamp
3000 K Blackbody
10^0
10^1
10^2
10^3
10^4
Wavelength λ (nm)

Application example: Calibration of deuterium lamps (source-based scale)

Synchrotron radiation-based irradiance calibration from 200 to 400 nm at the Synchrotron Ultraviolet Radiation Facility III

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Ping-Shine Shaw, Uwe Arp, Robert D. Saunders, Dong-Joo Shin, Howard W. Yoon, Charles E. Gibson, Zhigang Li, Albert C. Parr, and Keith R. Lykke

A new facility for measuring irradiance in the UV was commissioned recently at the National Institute of Standards and Technology (NIST). The facility uses the calculable radiation from the Synchrotron Ultraviolet Radiation Facility as the primary standard. To measure the irradiance from a source under

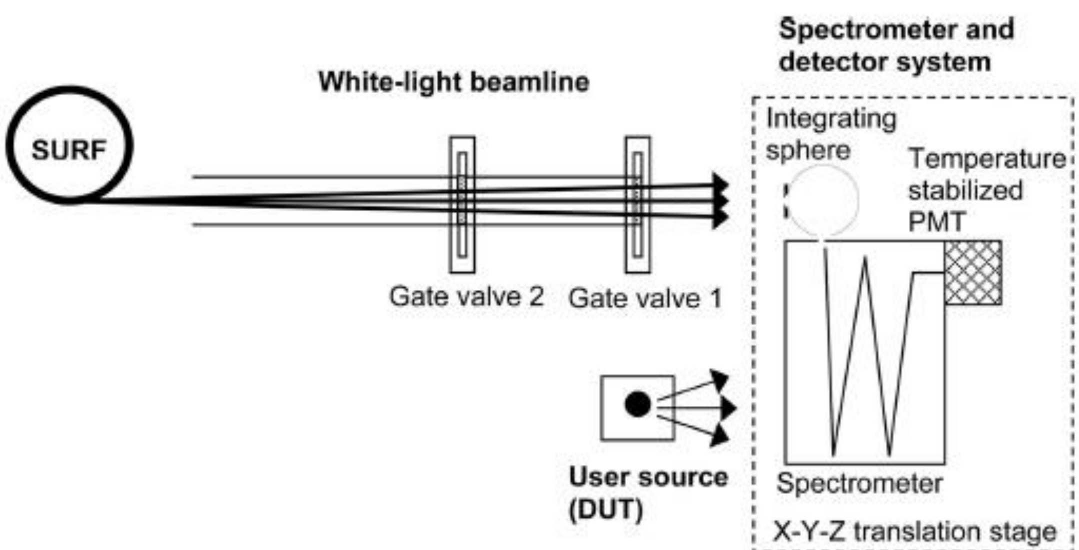
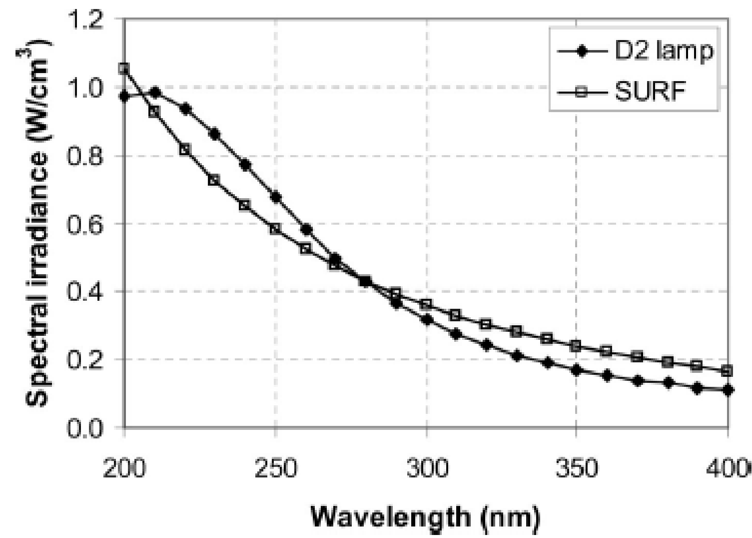
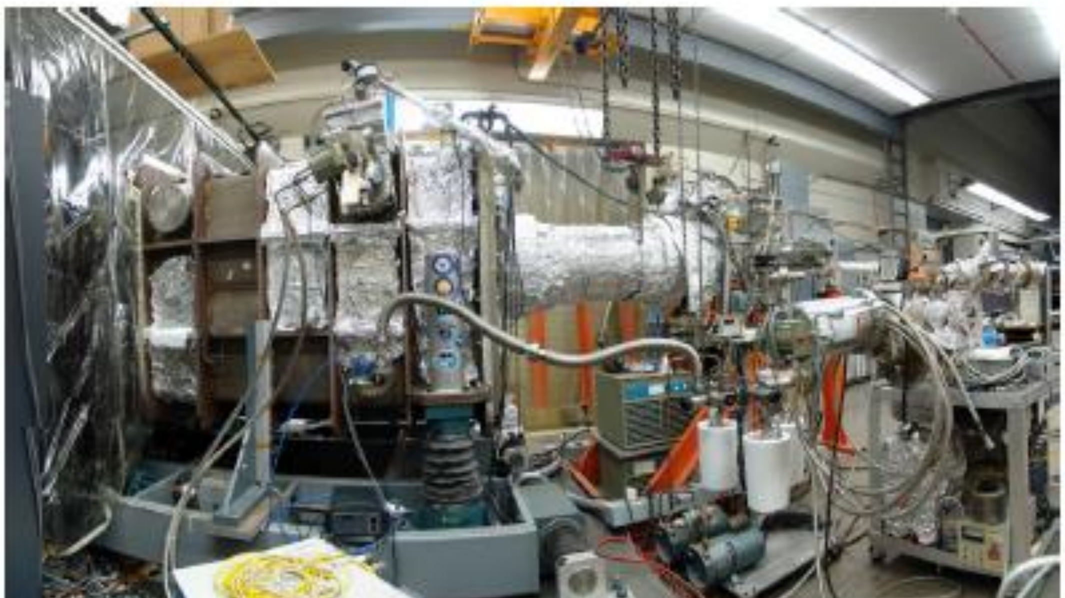


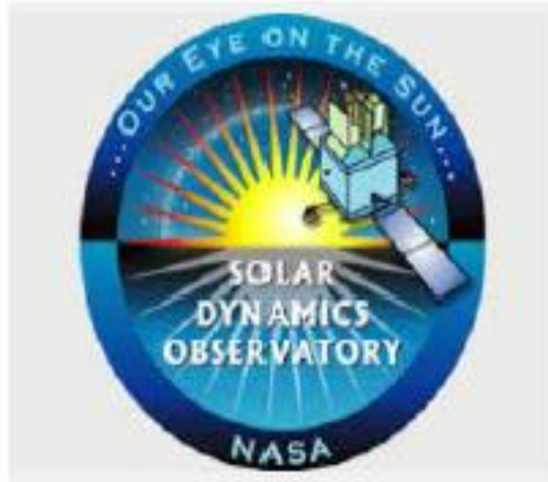
Fig. 1. Schematic of the FICUS for synchrotron-radiation-based irradiance calibration.



BL-2: Large chamber and clean room

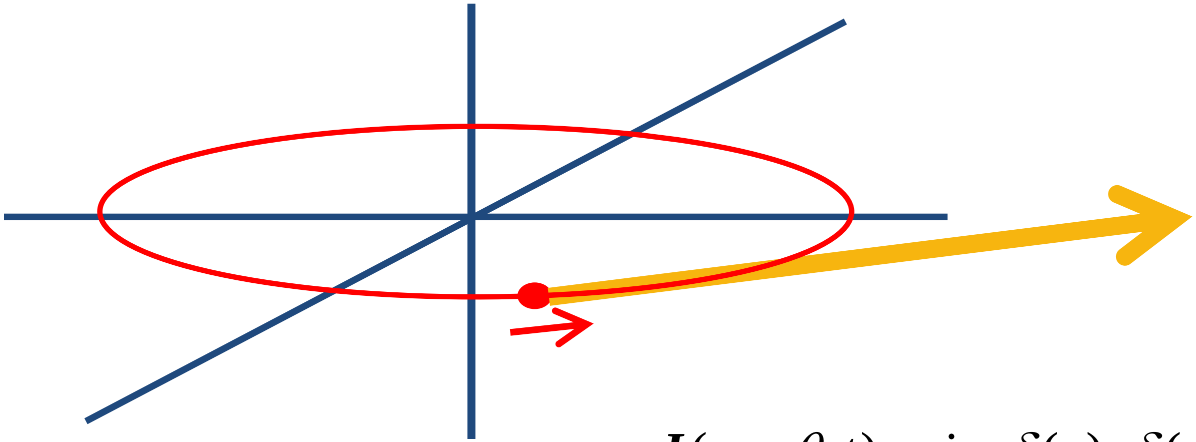


Recent UV/EUV Calibrations at SURF III: Missions and Collaborators



Collaborators: NOAA; NASA Goddard Space Flight Center; Laboratory for Atmospheric & Space Physics; Naval Research Lab; USC Space Flight Center; Jet Propulsion Laboratory.

Calculation of synchrotron radiation



(near-tangential) radiation direction

Time-dependent current:

$$\boldsymbol{J}(r,z,\theta,t) = j_0 \cdot \delta(z)\cdot\delta(r-R)\cdot\hat{\theta}\cdot\sum_{m'}\exp[im'(\theta-\omega_0 t)]$$

Take m^{th} frequency

$$k = m\omega_0 / c; \quad \omega = m\omega_0$$

$$\mathbf{J}(\mathbf{r},t) = \mathbf{J}(\mathbf{r})e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{r},t) = -e^{-i\omega t}\left(\frac{\mu_0 c k^2}{6\pi}\right)\times \int d^3\mathbf{s}\left\{\begin{array}{l}\mathbf{J}(\mathbf{r}-\mathbf{s})\, h_0^{(1)}(ks) \\ -[s^2\mathbf{J}(\mathbf{r}-\mathbf{s}) - 3\mathbf{s}(\mathbf{s}\cdot\mathbf{J}(\mathbf{r}-\mathbf{s}))]\left(\dfrac{h_2^{(1)}(ks)}{s^2}\right)\end{array}\right\}$$

$$\mathbf{B}(\mathbf{r},t) = -ie^{-i\omega t}\left(\frac{\mu_0 k}{4\pi}\right)\int d^3\mathbf{s}\left\{\mathbf{s}\times\mathbf{J}(\mathbf{r}-\mathbf{s})\left(\frac{h_1^{(1)}(ks)}{s}\right)\right\}$$

Note: **r**=point where field is found; **r–s**=point(s) of current density.

Early development in synchrotron radiation theory

Early calculation:

- G. A. Schott, Ann. Phys. (Leipzig) **24**, 635 (1907).

Early calculations motivated by radiative energy loss:

- J. Schwinger, Phys. Rev. **75**, 1912 (1949).
- D. Ivanenko and A.A. Sokolov, Dokl. Akad. Nauk SSSR [Sov. Phys. Dokl.] **59**, 151 (1948).

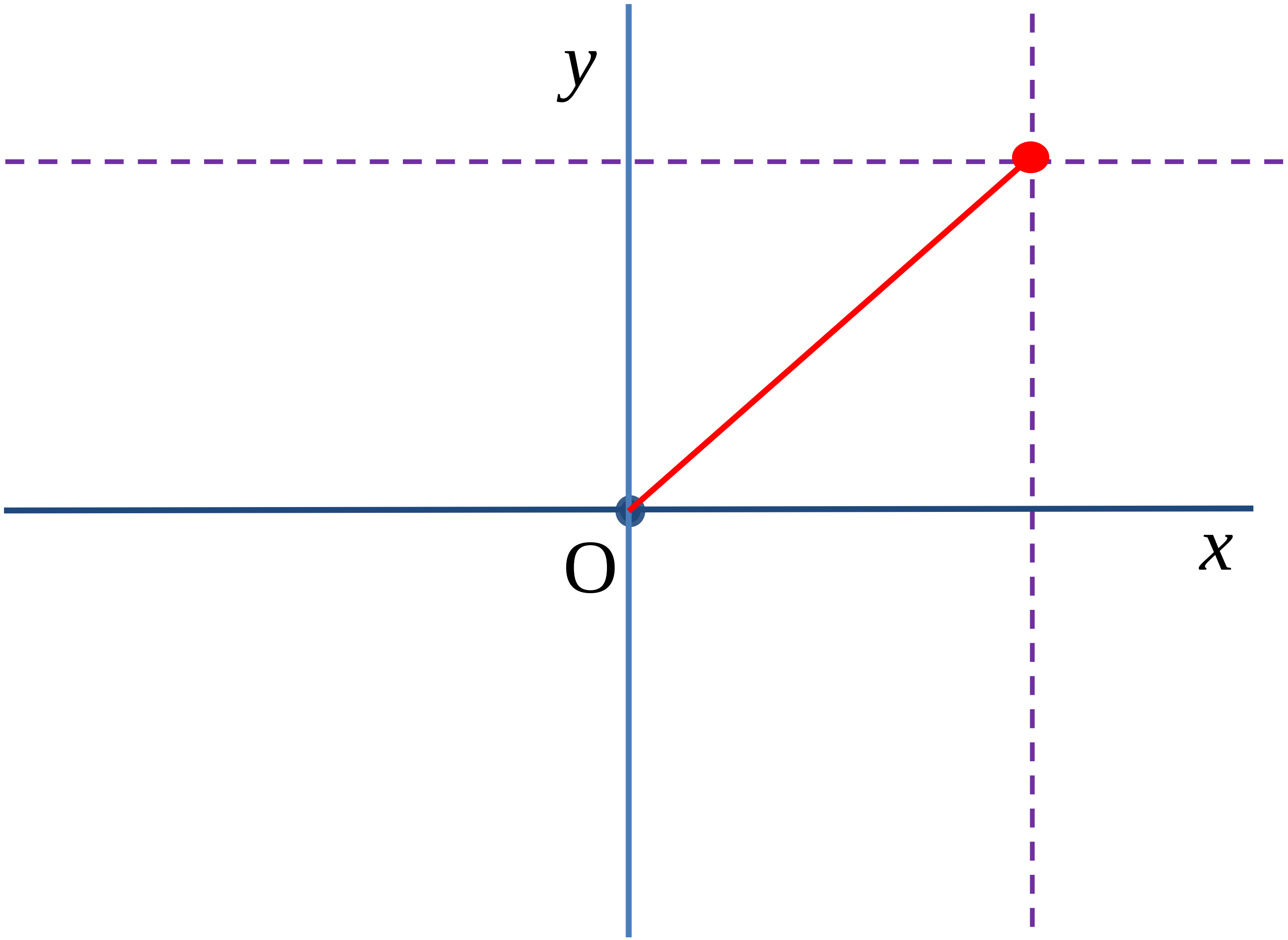
Refinement to Schwinger formula:

- W.B. Westerveld, A. McPherson and J.S. Risley, Atomic Data and Nuclear Data Tables **28**, 21 (1983).

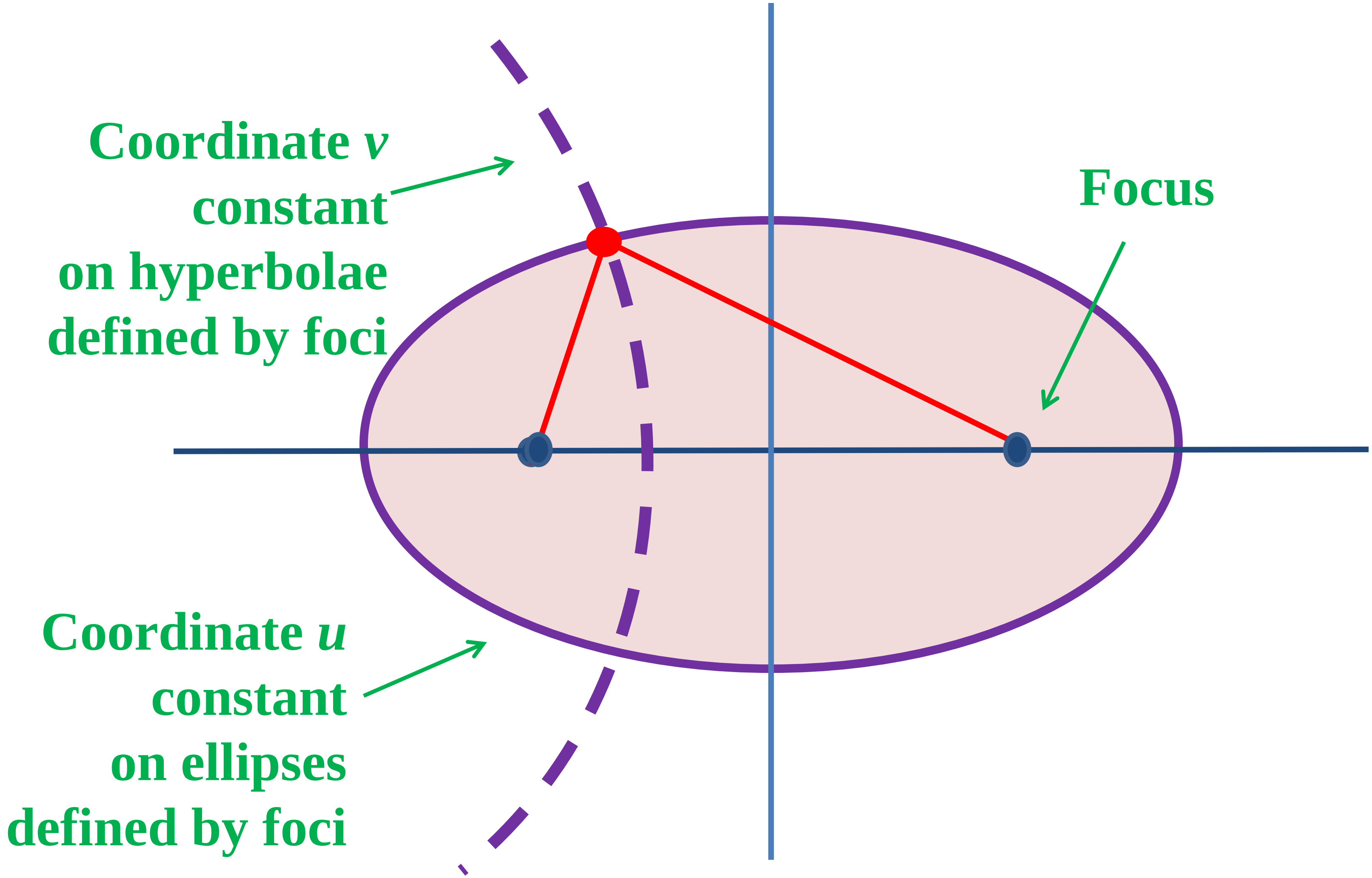
Further general discussion of synchrotron radiation fields:

- J.D. Jackson, *Classical Electrodynamics, 2nd Edition* (Wiley, New York, 1975).
- L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1975).

Coordinate systems—Cartesian



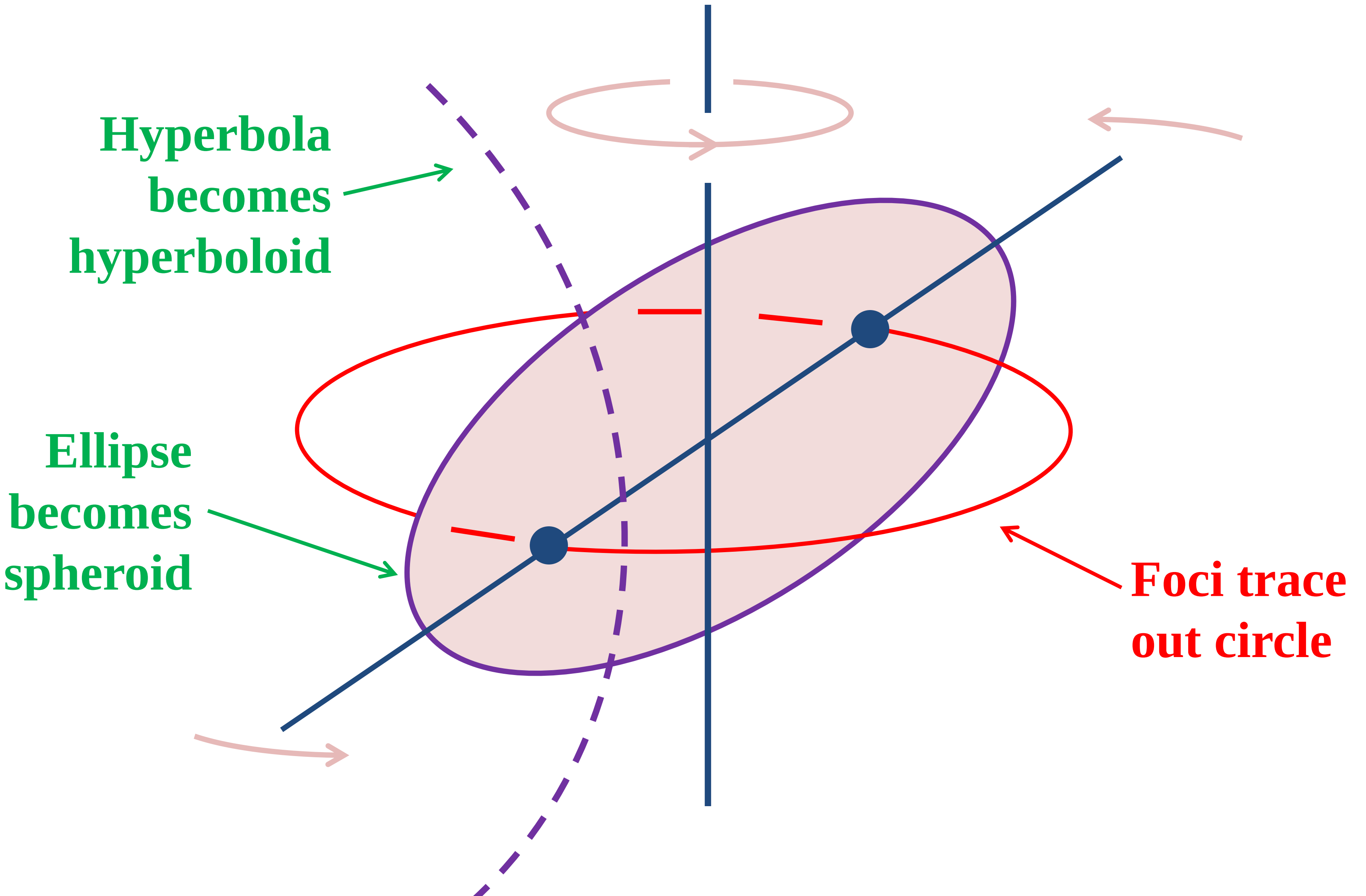
Coordinate systems—Elliptic



Coordinate systems—Oblate spheroidal

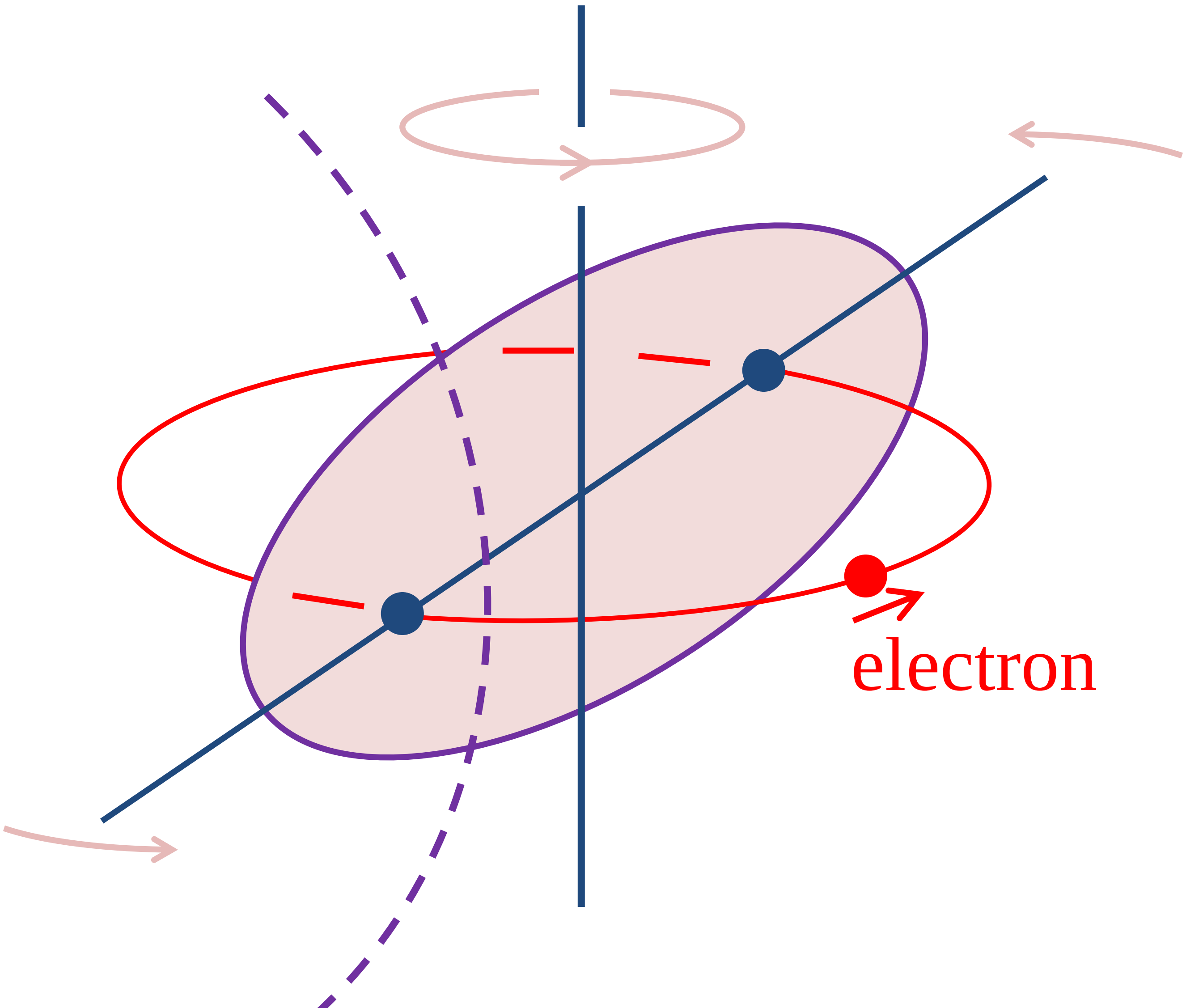
Rotate elliptic system through 360 degrees about minor axis.

Angle θ = 3rd coordinate.

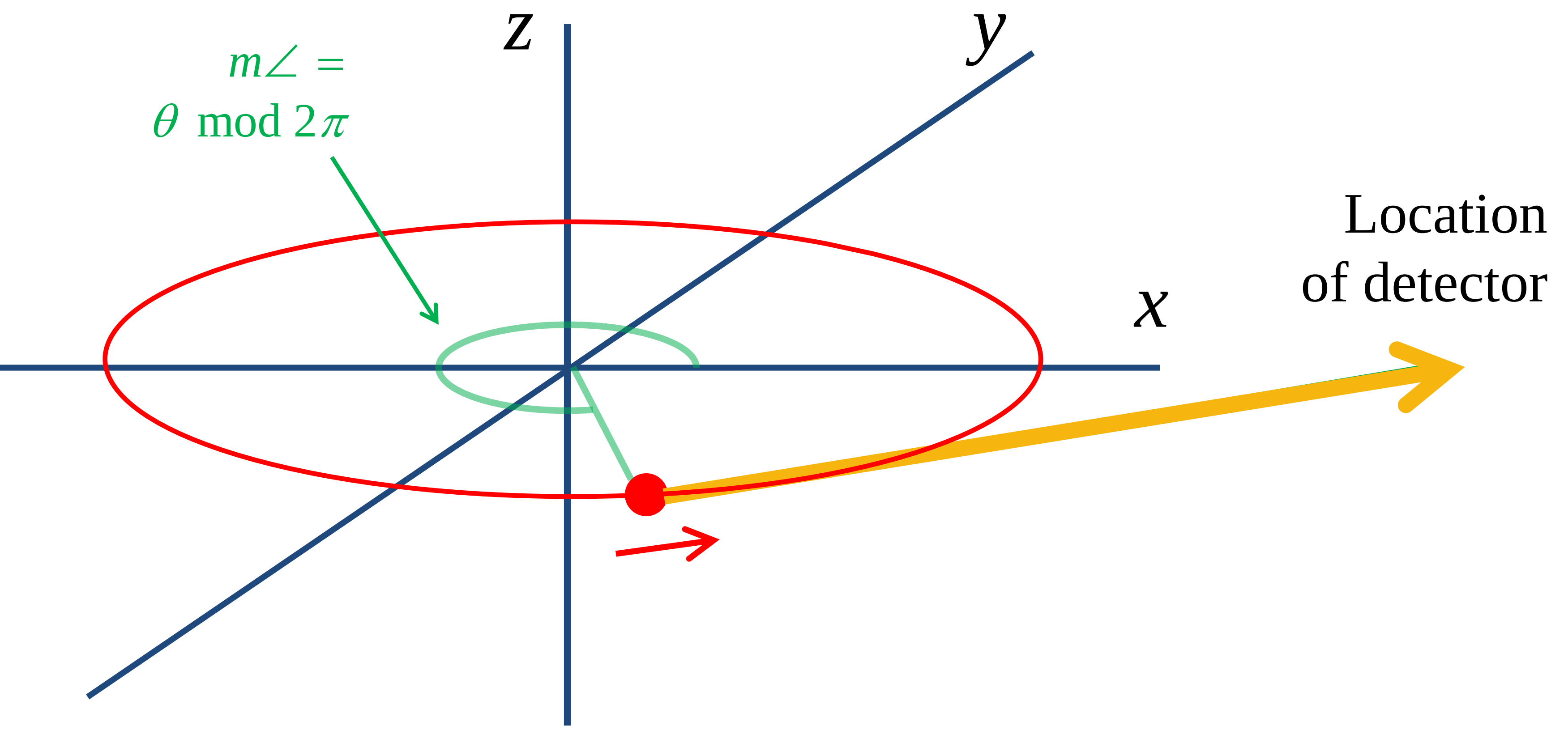


Coordinate systems—Oblate spheroidal

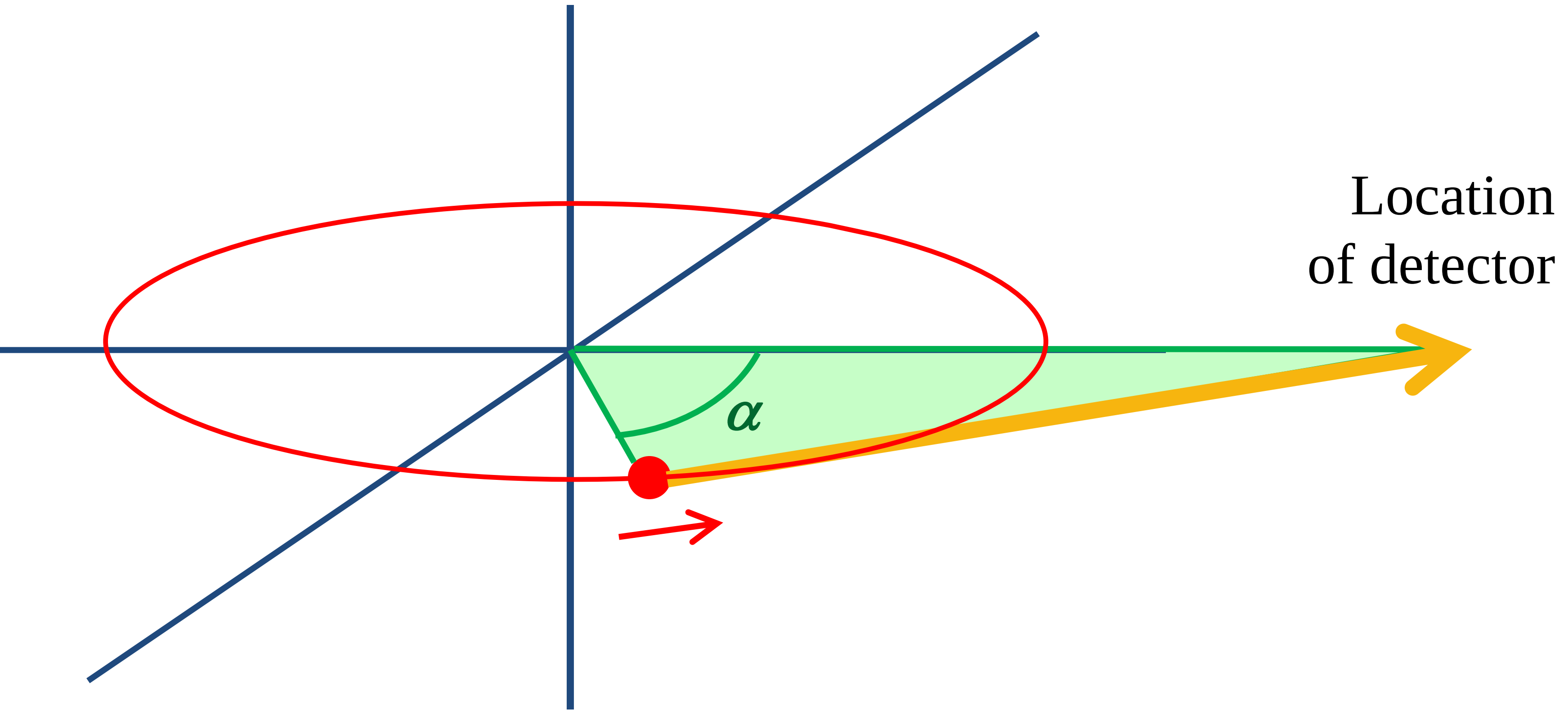
For synchrotron radiation, natural to have electron orbit circle traced by foci.



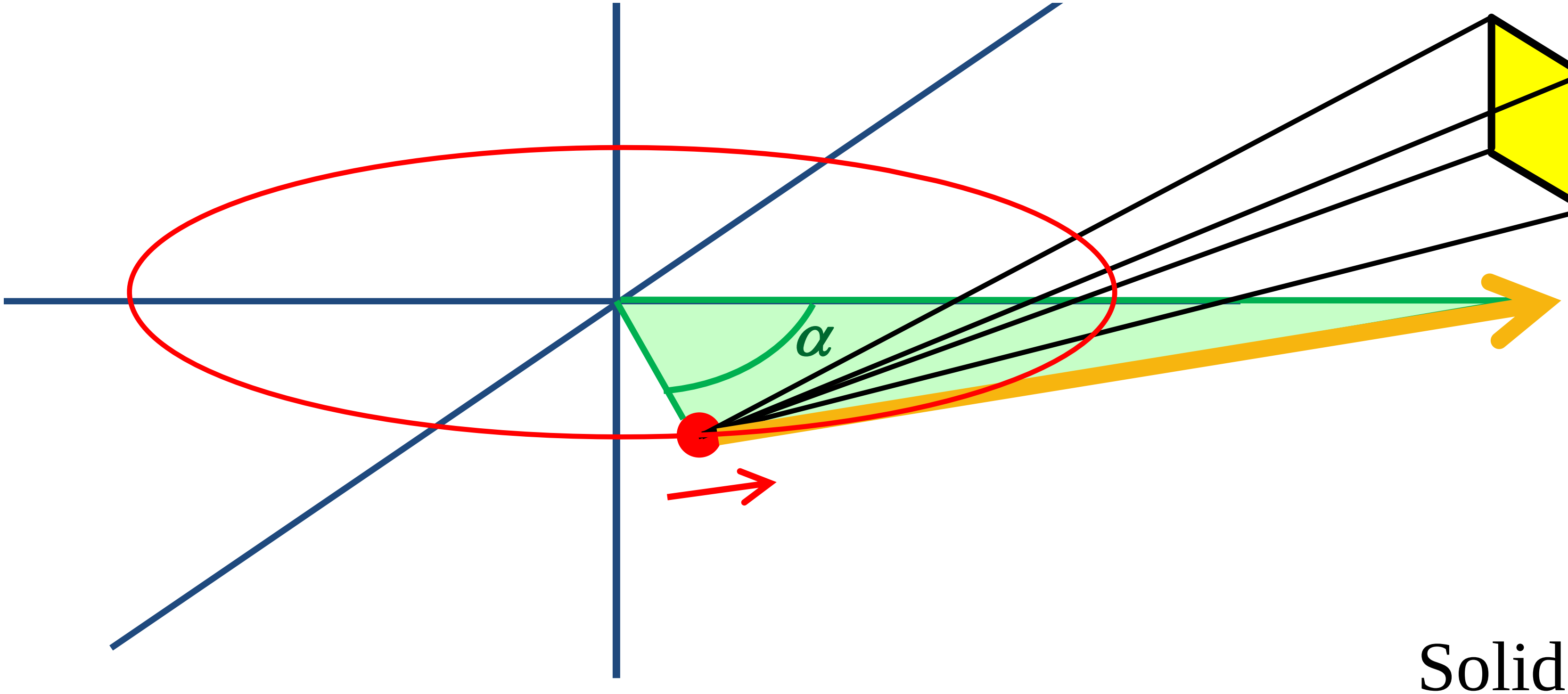
Convenient to use mix of coordinates, depending:



At finite range, azimuthal angle difference between detector vs. relevant tangent point ≠ right angle.

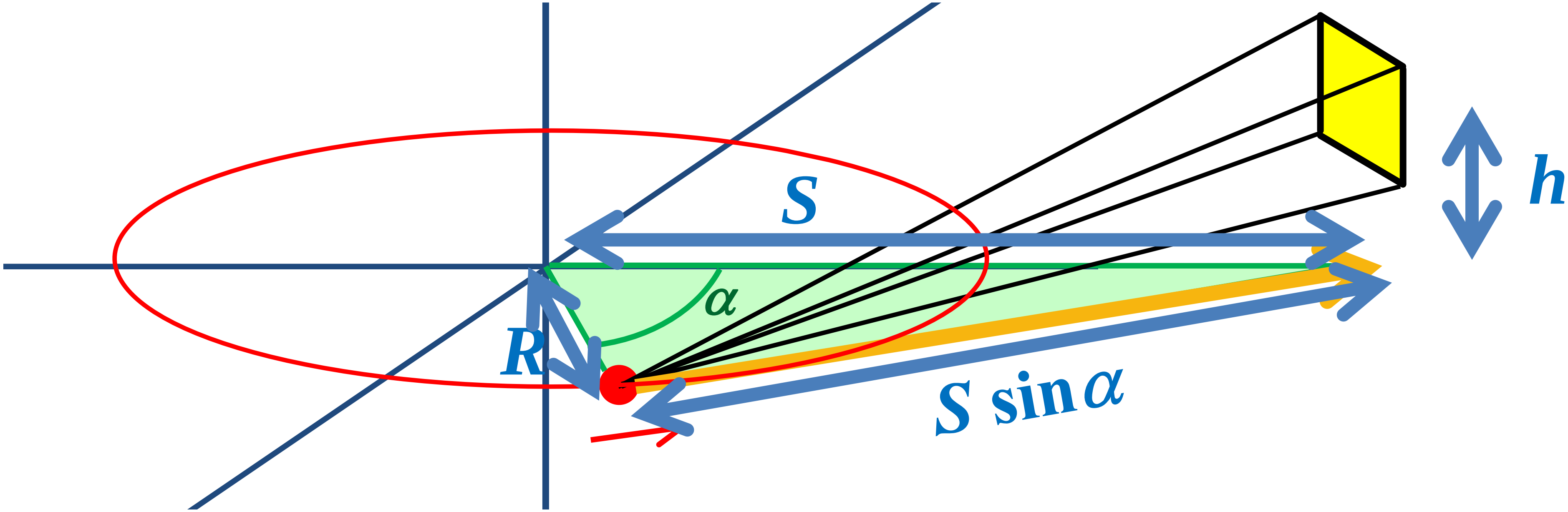


Complete specification of coordinates:



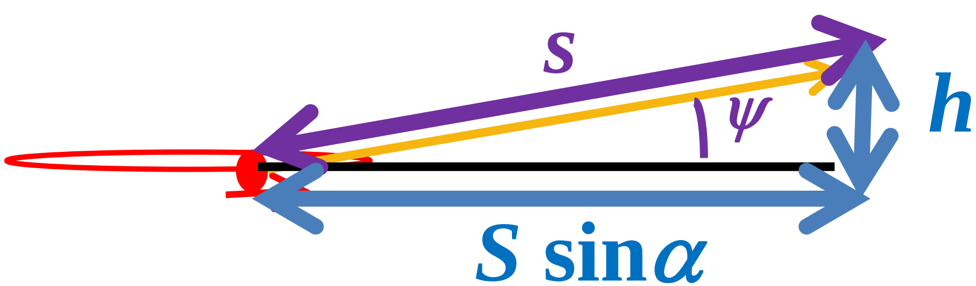
Solid angle effect:

Complete specification of coordinates:



"source detector distance"

$$s^2 = (S\sin\alpha)^2 + h^2$$



"elevation angle"

$$\tan\psi = \frac{h}{S\sin\alpha}$$

Integrating field over electron path, phase of integrand from J and h_l. Near tangent in relativistic case, phase is nearly stationary (along path) at $\theta = -\theta_0 = -\cos^{-1}(v/u)$:

$$ks + m\theta = u\sin\theta_0 + m\theta_0 + (m-v)(\theta-\theta_0) + \frac{v}{6}(\theta-\theta_0)^3 + \ldots$$

From distance to source

From current

Schwinger trick

- Keep 3 nonzero Taylor terms in phase, 2 Taylor terms in all else
- Assume infinite distance to detector (optional)

$$F = \int d\theta\, c(\theta)\exp[i\Psi(\theta)]$$

$$\approx \int_{-\infty}^{+\infty} d\phi\,(\underline{c_1} + \underline{c_2}\phi)\exp[i(\underline{\Psi_0} + \underline{\Psi_1}\phi + \underline{\Psi_3}\phi^3)]$$

$\rightarrow$ Fields =combinations of $K_{1/3}$ & $K_{2/3}$ (or Ai & Ai′)

Real-time approach (more general) (P.-S. Shaw)

$$\mathbf{E}(\mathbf{x},t) \sim \left[\frac{\hat{\mathbf{s}}\times[(\hat{\mathbf{s}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{s}})^3 s}\right]_{ret}$$

For simple circular orbit, "natural" oblate spheroidal partners to θ:

$$u = \frac{k}{2}\left[\sqrt{(S+R)^2+h^2}+\sqrt{(S-R)^2+h^2}\right]$$

$$v = \frac{k}{2}\left[\sqrt{(S+R)^2+h^2}-\sqrt{(S-R)^2+h^2}\right]$$

$$\boxed{\begin{array}{l}\text{Case } h=0: \\ \quad\Rightarrow \quad u=kS;\, v=kR\end{array}}$$

Note reduction of 3D to fictitious 2D problem:

$$ks = k\sqrt{\underline{R^2+S^2-2RS\cos\theta}+h^2} = \sqrt{u^2+v^2-2uv\cos\theta} = \sqrt{(u-ve^{+i\theta})(u-ve^{-i\theta})}$$

Graf's addition theorem gives the following expansion:

$$h_l^{(1)}(ks) = h_l^{(1)}((u^2+v^2-2uv\cos\theta)^{1/2})$$

$$= \left(\frac{2}{\pi ks}\right)^{1/2} H_{l+1/2}^{(1)}((u^2+v^2-2uv\cos\theta)^{1/2})$$

$$= \left(\frac{2}{\pi ks}\right)^{1/2}\left(\frac{u-ve^{-i\theta}}{u-ve^{+i\theta}}\right)^{l/2+1/4} \sum_{k=-\infty}^{+\infty} H_{l+1/2+k}^{(1)}(u) J_k(v) e^{-ik\theta}$$

→ Allows integration over θ of each term, v/u-type geometric progression of terms converges the sum quickly.

Outcomes of new approach of calculation:

- Graf's formula facilitates convenient "exact" calculation
  - Debye's asymptotic formula for *H* is helpful (very large argument & large order)
  - Olver's asymptotic formula for *J* is helpful (large order, argument *very near* order)
  - Asymptotic expansion using Ai and Ai′, but with a slightly different argument
  - Calculations can be done *extremely* quickly

- Analysis of expansion suggests Schwinger results deviates at higher orders in $m^{-1/3}$

Role of fuzzing of beam (horizontal, vertical and orbital tilt spread):

$$\left.\begin{aligned}
\sigma(x_s) &= 0.173\,\text{cm}\\
\sigma(z_s) &= 0.085\,\text{cm}\\
\sigma(s_x) &= 0.000653\,\text{rad}\\
\sigma(s_z) &= 0.000782\,\text{rad}
\end{aligned}\right\}$$

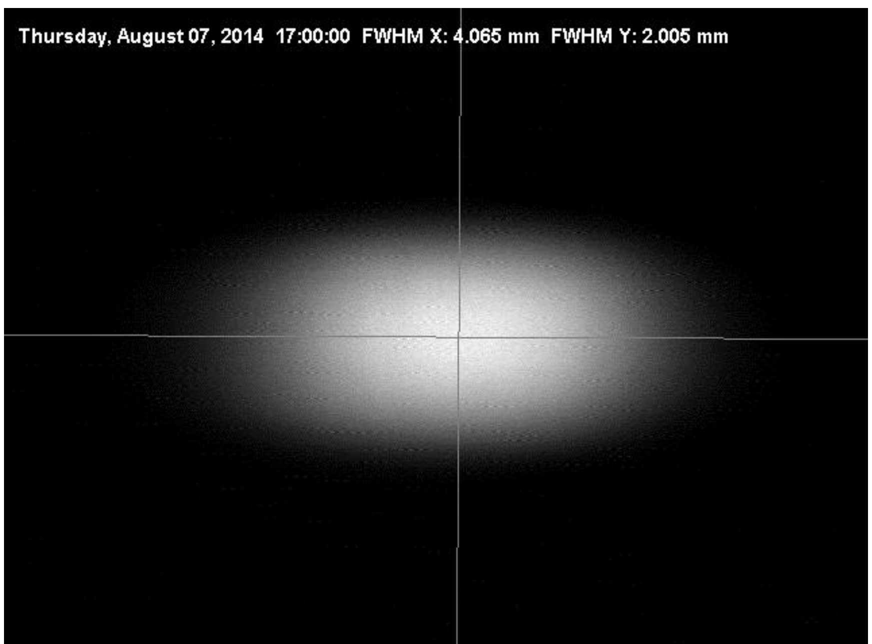
Apparent elevation angle along tangent:

Detector height   Tangent height

$$\Psi_{\mathrm{el}} = \frac{z - z_s}{d_s} - s_z$$

Tangent length

Image of beam:



Credit: Uwe Arp

Preliminary calculation of radiation (for diffraction effects):

Approximate radiation fields (main beam):

$$\begin{Bmatrix} E_x \\ E_z \end{Bmatrix} \cong \frac{1}{d_s} \begin{Bmatrix} \psi \; K_{2/3}(n\psi^3/3) \\ i\Psi_{\mathrm{el}} \; K_{1/3}(n\psi^3/3) \end{Bmatrix} \exp\left[ik\left(\frac{(x-x_s)^2 + (z-z_s)^2}{2d_s} \right) \right] e^{-i\omega t}$$

$$\psi = \sqrt{1/\gamma^2 + \Psi_{\mathrm{el}}^2}$$

Apertures along SURF III Beamline 2:

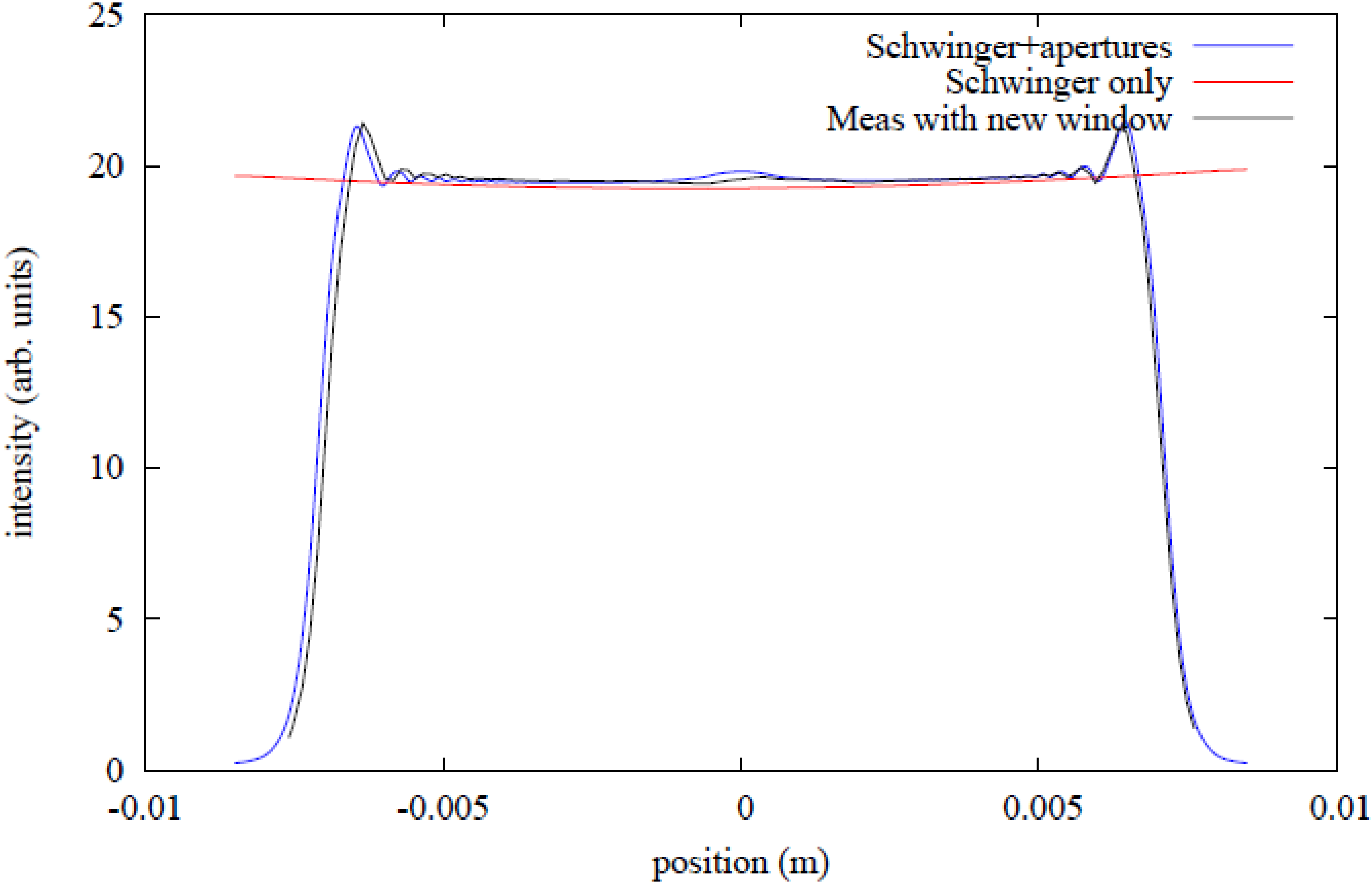
Diameter (mm)	Dist. from tangent (m)	Effect
27.71	0.414	Flux in central region
22.86	2.11	“
22.86	4.82	“
$\boxed{\text{13.00 (main aperture)}}$	$\boxed{10.42}$	“ **+ fringed beam waist**

Kirchhoff diffraction integral (Gaussian optics):

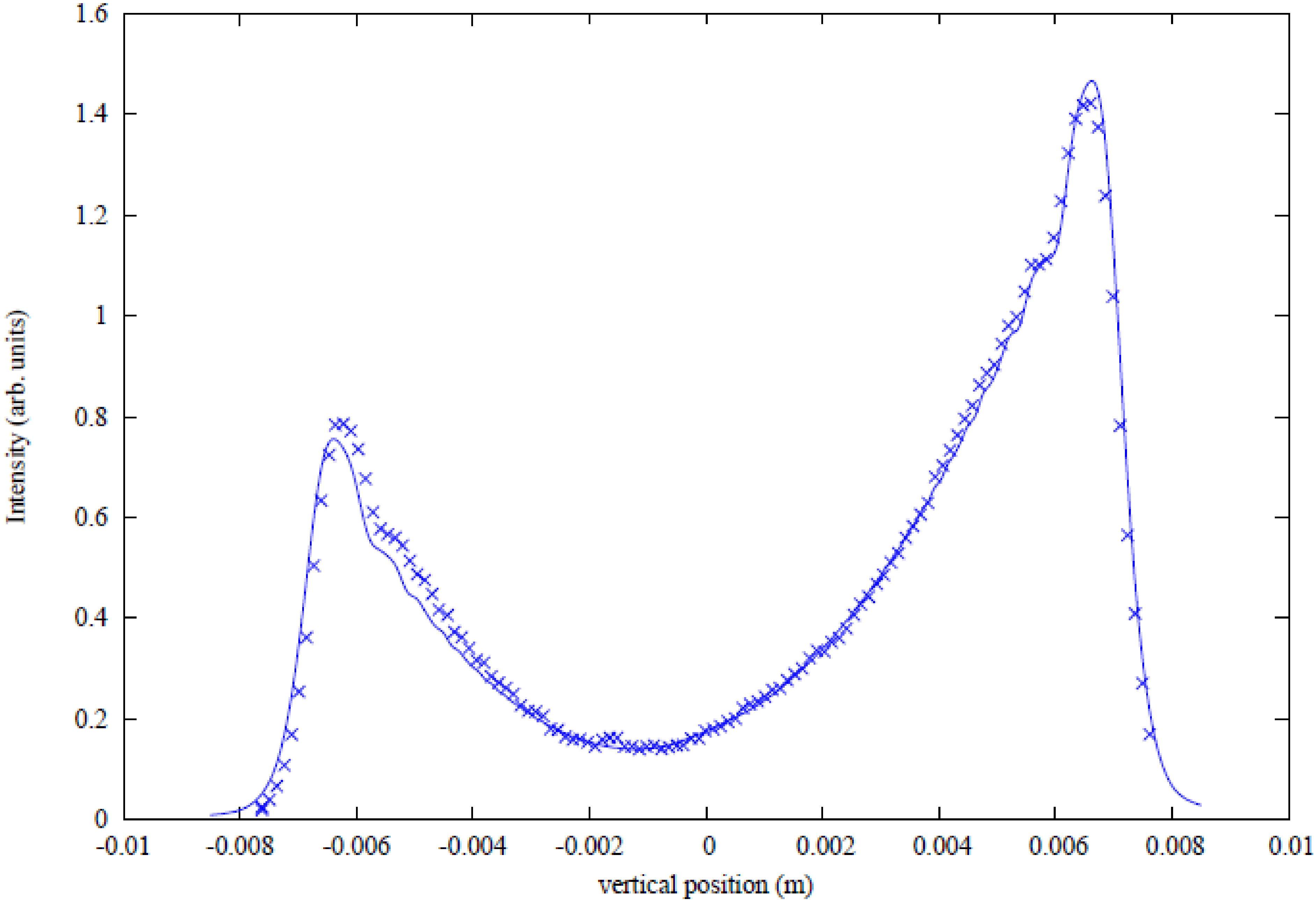
$$E_i(x_d, d_s + d_d, z_d) \cong \frac{1}{i\lambda d_d} \iint_A dx\, dz\, E_i(x, d_s, z) \exp\left[ik\left(\frac{(x-x_d)^2 + (z-z_d)^2}{2d_d} \right) \right]$$

Appropriately treated, aperture effects can be chiefly additive.

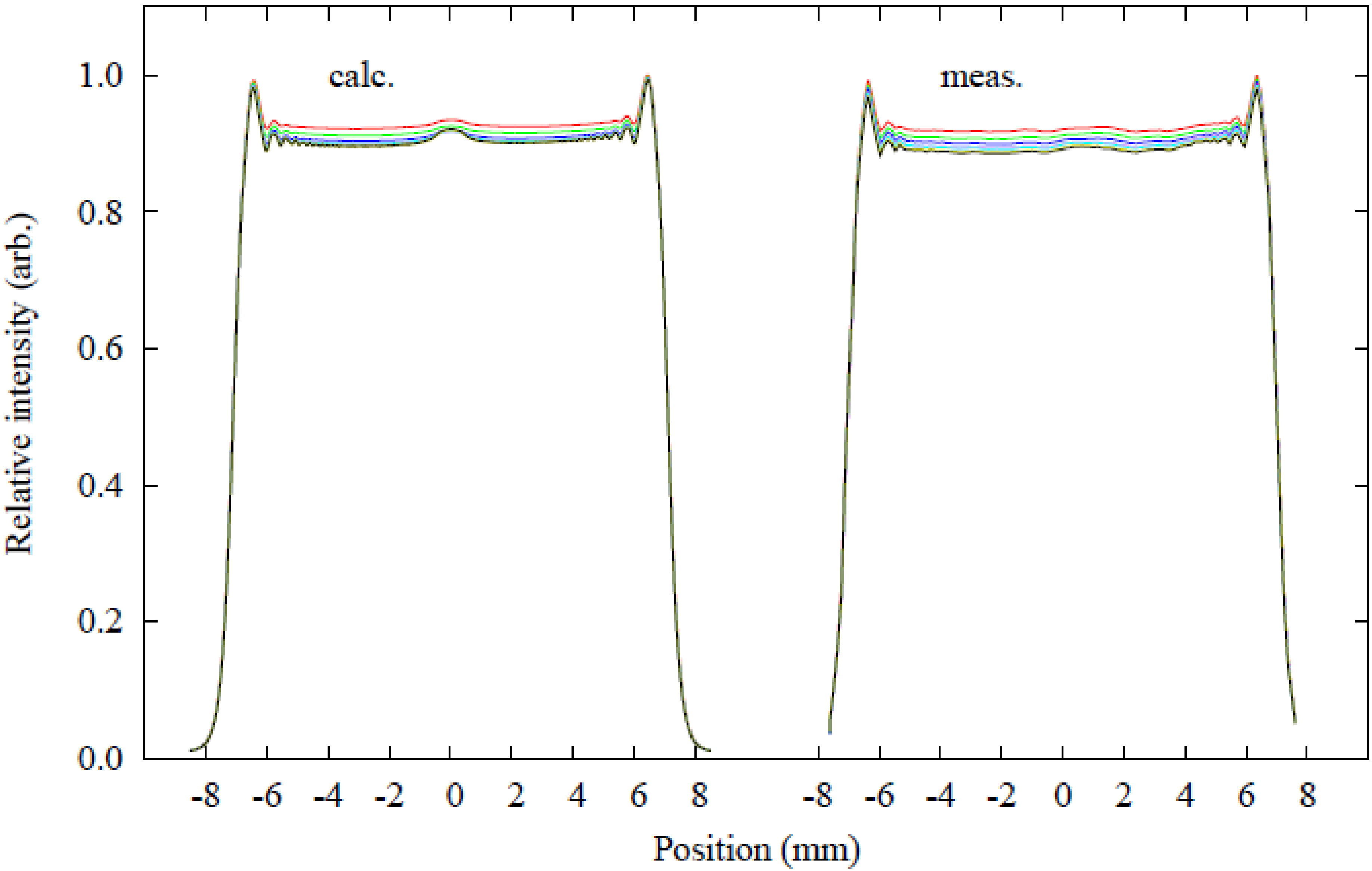
Irradiance profile (λ=334 nm):



Profile with vertical polarizer (locates orbit plane):



Varying fuzz (extrapolating orbital tilt variation) changes total flux (shown for 334 nm):



Conclusion

- SURF III is available for calibrations

- We are improving on the Schwinger formula
  - Important at longer λ

- We are correcting for diffraction
  - To matter in the future, also longer λ

Thank you!