# Model of Drawdown in Well Fields Influenced by Boundaries: Technical Description and User's Manual 

Najwan T. Shareef<br>David S. Bowles

Follow this and additional works at: https://digitalcommons.usu.edu/water_rep
Part of the Civil and Environmental Engineering Commons, and the Water Resource Management Commons

## Recommended Citation

Shareef, Najwan T. and Bowles, David S., "Model of Drawdown in Well Fields Influenced by Boundaries:
Technical Description and User's Manual" (1980). Reports. Paper 108.
https://digitalcommons.usu.edu/water_rep/108

This Report is brought to you for free and open access by the Utah Water Research Laboratory at DigitalCommons@USU. It has been accepted for inclusion in Reports by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.

# MODEL OF DRAWDOWN IN WELL <br> FIELDS INFLUENCED BY BOUNDARIES: <br> Technical Description and User's Manual 

by
Najwan T. Shareef and
David S. Bowles
Utah Water Research Laboratory
College of Engineering
Utah State University
Logan, Utah 84322

December 1980
MODEL OF DRAWDOWN IN WELL FIELDS INELUENCED BY BOUNDARTES:
Technical Description and User's Manual
by
Najwan T. Shareef and
David S. Bowles

# Utah Water Research Laboratory <br> College of Engineering <br> Utah State University Logan, Utah 84322 

December 1980

## ACKNOWLEDGMENTS

The model described in this report is based on earlier work by Bowles and Rogers (1973). The text of this report is also the M.S. Thesis for Mr. Najwan T. Shareef. Mr. Shareef was supported in his graduate studies through a scholarship from the Government of Iraq. The authors wish to acknowledge the helpful comments of Drs. Calvin $G$. Clyde and Richard H. Hawkins, Utah State University, and the general guidance and encouragement provided by Mr. T. A. Prickett, Illinois State Water Survey, Urbana, Illinois. Computer funds for both the original work and the work described herein were provided by the College of Engineering, Utah State University, Logan, Utah. The Utah Water Research Laboratory provided funds for printing this report. The careful typing of Ms. Leslie C. Johns on and drafting of Mr. Arthur L. Rivers are gratefully acknowledged.

Najwan T. Shareef David S. Bowles

## TABLE OF CONTENTS

Page
ACKNOWLEDGMENTS ..... ii
LIST OF TABLES ..... vi
LIST OF FIGURES ..... vii
ABSTRACT ..... viii
Chapter
I. INTRODUCTION ..... 1
Background ..... 1
Objective ..... 2
Outline ..... 5
II. LITERATURE REVIEW ..... 6
Groundwater Models ..... 6
Introduction ..... 6
Physical models ..... 6
Analog techniques ..... 6
Mathematical models ..... 7
Analytical solutions ..... 7
Type curve techniques ..... 8
Graphical solutions . ..... 8
Differential analyser solutions ..... 8
Inverse problems ..... 8
Numerical techniques ..... 8
Equations Used for Different Aquifer Hydraulic Characteristics ..... 11
Background ..... 11
Hydraulic cases used in the program ..... 12
Variable discharge ..... 12
Well losses ..... 17
Image Well Theory ..... 18
Definition ..... 18
Barrier boundary ..... 18
Recharge boundary ..... 19

## TABLE OF CONTENTS (Continued)

Page
Multiple boundary system ..... 20
III. MODEL DESCRIPTION, ADVANTAGES, AND LIMITATIONS ..... 22
Background ..... 22
Model Description ..... 22
Input ..... 22
Analysis of geometry ..... 24
Image well generation ..... 25
Drawdown computation ..... 27
Well losses ..... 27
Several real wells ..... 28
Several points of interest ..... 28
Variable pumping rate ..... 28
Analysis of drawdown components ..... 29
Program options ..... 31
Output ..... 31
Model Advantages ..... 32
Time saving ..... 32
Education tool ..... 33
Groundwater contour map ..... 33
Model contribution to agriculture ..... 33
Model contribution to industry ..... 34
Groundwater recharge ..... 34
Construction dewatering ..... 35
Model Limitations ..... 35
IV. MODEL VERIFICATION AND APPLICATION ..... 37
Model Verification ..... 37
No boundaries and drawdown case 11 ..... 37
Single boundary and drawdown case 7 ..... 37
Parallel pair of boundaries and drawdown case 9 ..... 38
$90^{\circ}$ Intersection of two boundaries and drawdown case 5 ..... 38
Seminfinite aquifer and drawdown case 4 ..... 38
Two perpendicular parallel pairs of boundaries and drawdown case 1 ..... 39
Application to Well Spacing ..... 39
Purpose ..... 39
Problem statement ..... 40

## TABLE OF CONTENTS (Continued)

Page
Solution ..... 42
V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS ..... 45
Summary ..... 45
Conclusions ..... 47
Recommendations ..... 48
REFERENCES ..... 50
APPENDICES ..... 52
Appendix A. Model Components ..... 53
Model flow chart ..... 54
Program listing ..... 55
Variable Dictionary ..... 69
Appendix B. Tables and Notes ..... 76
Table B-1. Description of the input data and their formats ..... 77
Table B-2. Data cards used in the verified examples ..... 80
Table B-3. Interpretation of error codes ..... 81
Notes on the use of input data ..... 82
Appendix C. Computer Output for Examples in Chapter IV ..... 83

## LIST OF TABLES

Table Page

1. Definitions of well drawdown cases ..... 13
2. Well drawdown equations ..... 14
3. Definitions of symbols used in Table 2 ..... 15
4. Conversion factors ..... 24
5. Cost and permissible discharge for various well diameters ..... 42
6. Economical analysis of alternative well field designs ..... 44
$B-1$. Description of the input data and their formats ..... 77
B-2. Data cards used in the verified examples ..... 80
B-3. Interpretation of error codes ..... 81

## LIST OF FIGURES

Figure Page1 The interrelationship between the main program andthe subroutine . . . . . . . . . . . . 232 Limitations on the configuration of system ofboundaries . . . . . . . . . . . . . 26
3 Discharge-drawdown curve for varfable pumping case ..... 30
4 Plan and section of selected well field design ..... 41
A-1 Model flow chart ..... - 54

ABSTRACT

# A General Model of Well Fields Influenced by Barrier and Recharge Boundaries 

by
Najwan Tawfeek Shareef, Master of Science
Utah State University, 1981

Major Professor: Dr. David S. Bowles<br>Department: Civil and Environmental Engineering

This model is designed to calculate the drawdown of a piezometric surface or water table at a given point of interest in a well field. Many different well and aquifer hydraulic conditions can be considered. The effect of field boundaries is solved by the image well theory. Many points of interest, wells (recharge or injection), boundaries (barrier or recharge), and time increments as well as constant and variable pumping rate case have been included in the model. The drawdown at a point of interest can be decomposed into the following components for each real well due to the effect of: the existence of boundaries, partial penetration of the well in the aquifer, and the well loss at the walls of the well due to turbulent flow. Both the International Systems of Units (SI) and the English system are available. The option of using graph of drawdown vs distance or drawdown vs time is also available. Also included in this thesis is a well-spacing design problem as an application of the model.

## CHAPTER I

INTRODUCTION

## Background

Groundwater is the body of water which occurs in the saturated zone beneath the ground surface and whose motion is exclusively determined by gravity and by the frictional forces provoked by the motion itself. This body of water in the saturation zone and that in the aeration zone is bounded at the top and bottom by either impervious or semimpervious strata.

Wells generally serve as devices for extracting groundwater from aquifers, which are geologic formations which contain and transmit water. Wells serve such purposes as for subsurface exploration and observation, disposal of sewage, industrial or radioactive wastes, draining agricultural lands, controlling salt water encroachment, relieving pressures under dams or levees, and artificial recharge of aquifers.

Design features of the well depend on the quantity of water extracted, economic factors, the well's objective and hydrologic conditions. Spacing of wells is largely affected by the drawdown within the region of interest. The greater the spacing of the wells, the less their interference, but the greater the cost of connecting pipelines and power installation. The drawdown itself is affected physically by:
a) Well boundary conditions such as well percentage of penetration through the aquifer, well diameter and gravel envelopes, screens, perforation, amount and duration of pumped (or injected) water, and schedule of pumping.
b) Aquifer characteristics and aquifer boundary conditions such as hydraulic characteristics of the aquifer, its thickness, and the existence of barrier and recharge boundaries which delimit the continuity of the aquifer.

The assumption of infinite areal extent of the aquifer is made for both the equilibrium and nonequilibrium formulas in well hydraulics. Practically, in many instances the existence of boundaries serve to limit the continuity of the aquifer, in one or more directions, to distances ranging from a few hundred feet to as much as tens of miles. The boundaries may be either of the barrier type, such as an impermeable outcrop, also termed a "negative boundary," or of the recharge type such as a stream in a hydraulic contact with the aquifer, sometimes called a "positive boundary." The influence on groundwater movement and storage of hydrologic boundaries, which often exist in the real world, may be determined by means of the image well theory as described by Ferris et al. (1962).

## Objective

The objective of this study is to build a computer program to calculate the drawdown of a piezometric surface (for confined aquifer), or the drawdown of a water table (for unconfined aquifer) at any point of interest within a well field for a variety of wells, vertical aquifer, and horizontal aquifer boundary conditions. The program will be designed to be used by engineers for designing well fields influenced by recharge and barrier boundaries. The program will be based on an earlier model by Bowles and Rogers (1973). The specific tasks necessary to fulfill this objective are listed below:

1. To review the literature in well hydraulics, boundary conditions, image wells, and design of well fields.
2. To understand the earlier model by Bowles and Rogers (1973).
3. To modify this model so that it will include the following modifications:
a) The program will handle the following aquifer and well conditions:
i) Steady-state radial flow in isotropic, nonleaky artesian aquifer with fully penetrating well and constant discharge conditions.
ii) Steady-state radial flow in isotropic, watertable aquifer with fully penetrating well and constant discharge conditions.
iii) Unsteady-state radial flow in isotropic-nonleaky artesian aquifer with fully penetrating well and constant discharge conditions.
iv) Unsteady-state radial flow in anisotropic-nonleaky artesian aquifer with fully penetrating well and constant discharge conditions.
v) Unsteady-state radial flow in isotropic-nonleaky artesian aquifer with partially penetrating wells and constant discharge conditions.
vi) Unsteady-state radial flow in isotropic leaky artesian aquifer with fully penetrating wells without water released from storage in aquitard and constant discharge conditions.
vii) Steady-state radial flow in isotropic leaky artesian aquifer with fully penetrating wells without water released from storage in aquitard and constant discharge conditions.
viii) Unsteady-state radial flow in isotropic leaky artesian aquifer with fully penetrating wells with water released from storage in aquitard.
ix) Steady-state radial flow in isotropic leaky artesian aquifer with fully penetrating wells with water released from storage in aquitard.
x) Unsteady-state radial flow in isotropic watertable aquifer with fully penetrating wells and constant discharge conditions.
b) The boundary geometry will be restricted to the following cases:
i) Infinite aquifer
ii) Semiinfinite aquifer
iii) Wedge-shaped aquifer
iv) Infinite-strip aquifer
v) Semiinfinite-strip aquifer
vi) Rectangular aquifer
c) The program will calculate and analyze the following components of the drawdowns:
i) Total interference from other wells
ii) Portion due to partial penetration
iii) Magnitude of well loss (in real well only)
iv) Total drawdown effects from all boundaries
d) The program will be suitable for use as a tool for solving problems with:
i) Constant pumping rate
ii) Variable pumping rate

Finally, the program will use both S.I. and U.S. systems of units.
4. To verify the model by hand calculation and debugging the program.
5. To write thesis in form of a user manual.

## Outline

Chapter II reviews the various types of groundwater models, explains the well drawdown equations and image well theory. Chapter III contains a description of the various components of the computer model, and describes the advantages and the limitations of the model. In Chapter IV, model verification and the application to an optimal well spacing problem, are described. Finally, Chapter $V$ gives the summary, conclusions and recommendations for further work.

## CHAPTER II

LITERATURE REVIEW

## Groundwater Models

## Introduction

Groundwater plays an important role in the hydrologic cycle. The amount of water beneath the ground surface is much greater than surface water. So groundwater has great contribution in water resource planning, especially when surface water is rate. Until the advent of groundwater computer models it was difficult to evaluate groundwater availability and quality, the cost of pumping, or effect of groundwater development. The techniques used in groundwater modeling are as follows (FAO 1978).

## Physical models

In physical models the groundwater prototype is scaled down to a model of similar materials and has the same basic physical properties such as sand models. San models are useful in demonstration and are powerful to represent unsaturated and multiple fluid flow problems.

Analog techniques
Darcy's law in fluid mechanics, Ohm's law in electricity and Fourier's law in heat transfer are similar in principle and application. Analog techniques are based on the similarities in equations for groundwater flow and the flow of electricity. These analogs are devices with similar input-output or cause and effect relations as the true systems.

Electrolytic rank analogs, resistence network analogs, resistence capacitance analogs, viscous fluid parallel plate model, membrane models, etc. are examples of these techniques (Bouwer 1978, FAO 1978).

## Mathematical models

A mathematical groundwater model is a mathematical expression, or group of expressions, that describes the hydraulic relations within the system. It is usually in the form of differential equations together with the auxiliary conditions (the system geometry, the hydraulic characteristics of the system parameters, and the initial and boundary conditions) (Remson, Hornberger, and Molz 1971).

Mathematical models may be grouped into 6 types, each of which is briefly reviewed.

## Analytical solutions

These solutions for groundwater problems, which are in the form of partial differential equations, were dominated in the 1950s. In the 1960 s and 1970 s, there was more attention to numerical modeling because of the invention of high speed computers. But analytical solution is still the ideal way to solve these problems. Many groundwater problems were solved in this method like land subsidence due to artesian pressure, single and multiple boundary aquifer system (Stallman 1963, Vandenberg 1977), fresh water injection in a nonleaky artesian salaquifer (Esmael and Kimbler 1967), and estimating the rate and volume of stream depletion by near by production wells (Theis 1941, Jenkins 1968). Books are available to describe the solution of partial differential equations like deWiest (1965), Verruijt (1970), Kruseman and de Ridder (1970), Walton (1970), Todd (1959), and Bear (1972).

## Type curve techniques

In this method the unknown parameters could be solved by superposing the fitting data curve on a "type curve." The various factors can then be computed. These solutions were summarized by Kruseman and de Ridder (1970), and others.

## Graphical solutions

These solutions have a limited extent. They are based on the application of numerical techniques in heat flow to groundwater such as the graphical solutions for both linear and radial flow conditions (Thomas 1961).

Differential analyser solutions

This analyser is a device to solve differential equations. It is constructed by fluid, electronic or mechanical means to solve the differential equations (Tyson and Weber 1963).

## Inverse problems

This is a trial and error technique to solve the formation factors from water levels and input data, and then adjust the model until historical data reproduced by the model through calibration procedure.

## Numerical techniques

Numerical methods are those methods in which algorithms that use only arithmetic operations and certain logical operations such as algebraic comparison. They give approximated solution but not exact one by using digital computers. These methods are used broadly now because of the invention of high speed computers. There are two methodology in this technique:

1) Finite difference method: Finite differences were introduced by Richardson in 1910 to solve partial differential equations. Application of this method for steady-state seepage problems started by Show and Southwell in 1941. There are many books available to describe this technique like Carnahan, Luther, and Wilkes (1969), Remson, Hornberger, and Molz (1971), etc. The finite difference method depends on useful mathematical properties of square grid networks, in which the nodes are of the same size. The basic idea of these methods is to replace derivatives at a point by ratios of the changes in appropriate variables over a small but finite interval (Remson, Hornberger, and Molz 1971). For example, the derivative $\mathrm{dH} / \mathrm{dX}$ becomes

$$
\begin{equation*}
\frac{\mathrm{dH}}{\mathrm{dX}}=\lim _{\Delta \mathrm{X} \rightarrow 0} \frac{\Delta \mathrm{H}}{\Delta \mathrm{X}}=\frac{\left(\mathrm{H}_{2}-\mathrm{H}_{1}\right)}{\Delta \mathrm{X}} \tag{1}
\end{equation*}
$$

in which
$\dot{H}_{1}$ and $H_{2}$ are the heads at the grid points numbers 1 and 2 The size of the increment $\Delta X$ depends on the problem itself. If $\Delta X$ becomes very small, the functional problem is approximated to a continuous one.
2) Finite element method: This method was started in the early 1950 s in the aeroscope industry, then it was published by Turner et al. in 1956. Melosh (1965) proved that this method was a variation of the Raleigh-Ritz procedure. Basically, in this method, any continuous quantity can be approximated by a discrete model composed of a set of piecewise continuous functions defined over a finite number of subdomines (Segerlind 1976). This technique is established by starting from a formulation of the fundamental problem, not through a differ-
ential equation and boundary condition. Starting with Darcy's equations and the continuity equation:

$$
\begin{align*}
& V_{\mathrm{x}}=-\mathrm{k} \mathrm{~d} \phi / \mathrm{dx} \cdot \text {. . . . . . . . . . . (2) } \\
& V_{y}=-k \mathrm{~d} / \mathrm{dy} \text {. . . . . . . . . . . . (3) } \\
& \frac{d V_{x}}{d x}+\frac{d V_{y}}{d y}=0 \tag{4}
\end{align*}
$$

in which
$V_{x}=$ the velocity in the $x$ direction
$V_{y}=$ the velocity in the $y$ direction
$x$ and $y$ are the cartesian coordinates
$\phi$ is the total head
$k$ is the permeability
Substituting equations 2 and 3 in equation 4 yields the Laplace equation which is:

$$
\begin{equation*}
\frac{\partial}{\partial x}\left(k \frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k \frac{\partial}{\partial y}\right)=0 \tag{5}
\end{equation*}
$$

Now the region is subdivided into a large number of subregions $R_{j}$, the elements, in which each element produces a contribution $V_{j}$ to the value of the function $V$ so that

$$
V=\sum_{j=1}^{n} V_{j}
$$

where
$n$ is the number of elements

Usually the elements are in a triangular shape. At the nodes the basic parameters are taken. The values of these parameters in the interior of the element varies in a linear relationship (Verruijt 1970).

Equations Used for Different Aquifer Hydraulic Characteristics

## Background

Due to hydrologic cycle in nature, earth was formed with different strata or geologic formations. Each stratum had its unique properties like particle size distribution, particle size diameter, chemical components and their chemical behavior, degree of compaction of the soil, capacity to hold and transmit water, which means that the stratum react as a storage for water, etc. These properties might be changed from time to time.

From the groundwater hydrologic point of view, these geologic formations might be classified according to their boundary conditions and amount of water in storage. Bouwer (1978) defined an aquifer as a "groundwater-bearing formation sufficiently permeable to transmit and yield water in usable quantities." Hantush (1964) defined other types of aquifers: "artesian aquifers, also known as confined or pressure aquifers, are those in which groundwater is confined under pressure by impervious or semipervious strata. Water table aquifers, also known as free, phreatic, or unconfined aquifers are those in which the upper surface of the zone of saturation is under atmospheric pressure. Aquifers, whether artesian or water table, that lose or gain water through adjacent semipervious layers are called leaky aquifers. A water table aquifer resting on a semipervious layer that permits slow movement of water is called a leaky water table aquifer. An artesian aquifer that has at least one semipervious confining bed is called a leaky artesian aquifer. If the flow across the confining beds is negligible, the aquifers are called nonleaky aquifers." An aquitard as defined by

Bouwer (1978) is a confined aquifer which is sufficiently permeable to transmit water vertically to or from the confined aquifer, but not permeable enough to laterally transport water like an aquifer.

## Hydraulic cases used in the program

A total of eleven cases with different aquifer and well boundary conditions are treated in this thesis. The definition of each case is presented in Table 1. Table 2 gives the drawdown equations, references to the tabular solution if used, and a reference for the derivation of each equation. In Table 3 the definition of the symbols used in Table 2 are given.

## Variable discharge

Since the equations of groundwater flow are linear for the confined case, and approximately linear for the unconfined case, if the drawdown due to pumping is small compared with the thickness of the saturated zone, the principle of superposition is applicable. That is, the drawdown at any point of interest is additive for any number of wells and this is the basis of the image well technique.

The principle of superposition can also be applied to the case of variable discharge. If $Q_{0}, Q_{1} \ldots, Q_{n}$ are the discharge rates for a certain well at different times, then $\Delta Q_{1}, \Delta Q_{2}, \ldots, \Delta Q_{n}$ are the changes in discharge at each time. Thus the total drawdown at a point of interest can be estimated by the following equation in which each $\Delta Q$ is considered to come from a separate real well located in the same position (Freeze and Cherry 1979).

$$
\begin{equation*}
h_{0}-h=\frac{Q_{0}}{4 \pi T} W\left(u_{o}\right)+\frac{\Delta Q_{1}}{4 \pi T} W\left(u_{1}\right)+\ldots+\frac{\Delta Q_{n}}{4 \pi T} W\left(u_{n}\right) \tag{6}
\end{equation*}
$$

Table 1. Definitions of well drawdown cases.

| Case Number | Stendy | Unsteady | Isotrople | Antsotropic | $\begin{aligned} & \text { Fully } \\ & \text { penetrat lng } \\ & \text { well } \end{aligned}$ | Partially penetrating vell | Confined aquifer | Uncontined aquifer | $\begin{gathered} \text { Leaky } \\ \text { aquifer } \end{gathered}$ | Nonleaky aquifer | WLth vater released from storage in aquitard | Without water released from storage in aquitard |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | x |  | x |  | x |  | x |  |  | x |  |  |
| 2 | x |  | x | . | x |  |  | x |  |  |  |  |
| 3 |  | X | x |  | x |  | X |  |  | x |  |  |
| 4 |  | x |  | x | x |  | x |  |  | x |  |  |
| 5 |  | $x$ | x |  |  | x | $x$ |  |  | x |  |  |
| 6 |  | X | X |  |  | x | x |  |  | x |  |  |
| 7 |  | x | x |  | x |  | X |  | x |  |  | x |
| 8 | x |  | x |  | $x$ |  | x |  | x |  |  | $x$ |
| 9 |  | x | x |  | x |  | $x$ |  | X |  | x |  |
| 10 | x |  | x |  | x |  | x |  | x |  | x |  |
| 11 |  | x | x |  | x |  | . | $\times$ |  |  |  |  |

Table 2. Well drawdown equations.

| Case Number | Equation used | Table no. in the reference | Source reference |
| :---: | :---: | :---: | :---: |
| 1 | $\mathrm{s}=\left\{\mathrm{Q} \ln \left(\frac{\mathrm{R}}{\mathrm{r}}\right) /(2 \pi \mathrm{Km})\right\} 40.783^{*}$ | - | Bouwer (1978) |
| 2 | $s=\left\{\sqrt{Q \ln \left(\frac{R}{r}\right) /(\pi K)}\right\} \quad 40.783^{*}$ | - | Bouwer (1978) |
| 3 | $\mathrm{s}=114.6 \mathrm{Q} \mathrm{W}(\mathrm{u}) / \mathrm{T}$ | - | Walton (1970) |
| 4 | $\mathrm{s}=114.6 \mathrm{Q} \mathrm{W}\left(\mathrm{u}_{\mathrm{xy}}\right) / \sqrt{\mathrm{T}_{\mathrm{xx}} \mathrm{T}_{\mathrm{yy}}-\mathrm{T}_{\mathrm{xy}}{ }^{2}}$ | - | Walton (1970) |
| 5 | $\mathrm{s}=114.6 \mathrm{Q} \mathrm{W}\left(\mathrm{u}, \frac{\mathrm{r}}{\mathrm{m}}, \mathrm{\gamma}\right) / \mathrm{T}$ | T3.3 p. 140 | Walton (1970) |
| 6 | $s_{p}=114.6 \mathrm{Q} \mathrm{W}_{\mathrm{p}}\left(\mathrm{u}, \frac{\mathrm{r}}{\mathrm{m}}, \gamma\right) / \mathrm{T}$ | T3.4 p. 142 | Walton (1970 |
| 7 | $\mathrm{s}=114.6 \mathrm{Q} \mathrm{W}(\mathrm{u}, \mathrm{r} / \mathrm{B}) / \mathrm{T}$ | T2 p. 707 | Hantush (1956) |
| 8 | $\mathrm{s}=229 \mathrm{Q}\left(\mathrm{K} \frac{\mathrm{r}}{\mathrm{OB}}\right) / \mathrm{T}$ | T2 p. 704 | Hantush (1956) |
| 9 | $\mathrm{s}=114.6 \mathrm{Q} \mathrm{W}(\mathrm{u}, \psi) / \mathrm{T}$ | TTIT p. 313 | Hantush (1964) |
| 10 | $\mathrm{s}=229 \mathrm{Q}\left(\mathrm{K} \frac{\mathrm{r}}{\mathrm{O}}\right) / \mathrm{T}$ | Tl p. 704 | Hantush (1956) |
| 11 | $s=114.6 \mathrm{Q} W\left(u_{\text {ay }}, \mathrm{r} / \mathrm{B}\right)$ | Appendix 2 p. 480 | Boulton (1963) |

Table 3. Definitions of symbols used in Table 2.


Table 3. Contimued.

in which
$h_{o}-h=t h e ~ d r a w d o w n$
$u_{i}=\frac{r^{2} S}{4 T t_{i}}$
$t_{i} \quad=$ the time since pumping started for discharge $Q_{i}$
i $\quad=0,1,2, \ldots, n$
$w\left(u_{i}\right)=$ the well function at time $t_{i}$

## Well losses

Because of the small velocity of flow of water in an aquifer, the flow is considered laminar. The formation loss of the aquifer, i.e. the drawdown computed in all well drawdown equations, is directly proportional to the discharging rate of the well. In the vicinity of the well, turbulence occurs due to the well screen, gravel envelope, and the developed zone outside the well casing. This phenomenon causes another head loss called 'well loss.' This loss varies with some power of the discharge Q. The total head loss in the well can be approximated as
$s_{i W}=C_{f} Q+C_{W} Q^{\pi}$.
where
$s_{i w}$ is the total head loss
$Q$ is the pumping rate
$C_{f}$ is the formation constant
$C_{W}$ is the well loss constant
n is the exponent due to turbulence (Bouwer 1978)

## Image Well Theory

## Definition

Image well theory has been described as follows:
"The effect of a barrier boundary on the drawdown in a well, as a result of pumping from another well, is the same as though the aquifer were infinite and a like discharging well were located across the real boundary on a perpendicular thereto and at the same distance from the boundary as the real pumping well. For recharge boundary the principle is the same except that the iamge well is assumed to be discharging the aquifer instead of pumping from it." (Walton 1970)

## Barrier boundary

The barrier boundary is an impermeable barrier. It is assumed that the irregularly slopping boundary can, for practical purposes, be replaced by a vertical boundary, without sensibly changing the nature of the problem. The hydraulic condition imposed by the vertical boundary is that there can be no groundwater flow across it, for the impermeable material cannot contribute water to the pumping well. An imaginary discharging well has been placed at the same distance as the real well from the boundary but on the opposite side, and both wells are on a common line perpendicular to the boundary. At the boundary the drawdown produced by the image well is equal to the drawdown caused by the real well. Therefore, the drawdown cones for the real and image wells will be symmetrical and will produce a groundwater divide at every point along the boundary line. Because there can be no flow across a divide, the image system satisfies the boundary conditions of the real problem.

The resultant drawdown at any point of interest on the cone of depression in the real region is the algebraic sum of the drawdowns produced at that point by the system of real and image wells. The constant profile of the cone of depression is flatter on the side of the real well toward the boundary and steeper on the opposite side away from the boundary than it would be if no boundary were present.

## Recharge boundary

In this case, the well in an aquifer is hydraulically controlled by a perennial stream. For thin aquifers the effect of vertical flow component are small at relatively short distances from the stream, and if the stream stage is not lowered by the flow to the real well there is established the boundary condition that there shall be no drawdown along the stream position. So for most field situations it can be assumed for practical purposes that the stream is fully penetrating and equivalent to a line source at a constant head. An imaginary recharging well is assumed to be placed at the same distance as the real well from the line source but on the opposite side. Both wells are situated on a common line perpendicular to the line source. The imaginary recharging well operates simultaneously with the real well and returns water to the aquifer at the same rate that it is withdrawn by the real well. The resultant drawdown at any point of interest on the cone of depression in the real region is the algebraic sum of the drawdowns produced at that point by the system of real and image wells. The resultant profile of the cone of depression is flatter on the landward side of the well and steeper on the riverward side.

Multiple boundary system
Two or more boundaries are required to delimit a wedge-shaped aquifer; two parallel boundaries form an infinite-strip aquifer; two parallel boundaries intersected at right angles by a third boundary form a semiinfinite-strip aquifer; and four boundaries intersected at right angles form a rectangular aquifer. The image well theory could be applied to such cases by taking into consideration successive reflections on the boundaries.

A number of image wells are associated with a pair of converging boundaries. Each primary image well produces an unbalanced effect at the opposite boundary. The actual well angle "A" is approximated as to one of certain aliquot parts of $360^{\circ}$ as specified by Ferris et al. (1962) as follows:

If the aquifer wedge boundaries are of a like character, "A" must be an aliquot part of $180^{\circ}$. If the boundaries are not of a like character, " $A$ " must be an aliquot part of $90^{\circ}$. Then the number of reflections required to produce abalance image system is given by the equation

$$
\begin{equation*}
N=\frac{360^{\circ}}{A}-1 \tag{8}
\end{equation*}
$$

The locus of image well locations is a circle whose center is at the apex of the wedge and whose radius is equal to the distance from the pumped well to the wedge apex.

If the arrangement of two boundaries is such that they are parallel to each other, the number of image wells are extending to infinity. Practically it is only necessary to add pairs of image wells until the next pair has negligible influence on the point of interest.

Generally, geologic boundaries do not occur as abrupt discontinuities with the geometry allowed for by image well theory. However, for the purpose of analyzing drawdown in well fields it is often possible to approximate them as such.

The image well theory could also be applied in case of multiwell field system. If there are $n$ wells working at $Q_{1}, Q_{2}, \ldots, Q_{n}$ pumping rates at distances of $r_{1}, r_{2}, \ldots r_{n}$ from an observation point, then the drawdown equation is (Freeze 1979):

$$
\begin{equation*}
h_{0}-h=\frac{Q_{1}}{4 \pi T} W\left(u_{1}\right)+\frac{Q_{2}}{4 \pi T} W\left(u_{2}\right)+\ldots+\frac{Q_{n}}{4 \pi T} W\left(u_{n}\right) \tag{9}
\end{equation*}
$$

in which
$h_{0}-h$ is the drawdown at the observation point
$u_{i}=\frac{r_{i}{ }^{2} S}{4 T t_{i}}, i=1,2, \ldots, n$
$t_{i}=$ the time since pumping started for the discharge $Q_{i}$

## CHAPTER III

MODEL DESCRIPTION, ADVANTAGES, AND LIMITATIONS

## Background


#### Abstract

The drawdown model originated with an earlier version written by Bowles and Rogers (1973). That model calculated the drawdown for three aquifer boundary condition cases, namely; steady state, confined aquifer; steady state, unconfined aquifer; and unsteady state, confined aquifer in the English system of units. It included the analysis of boundary conditions, several wells, several points of interests, change of the drawdown as the discharging time continued, and graph. The model described in this chapter is an expanded version of the earlier model and is capable of holding 11 aquifer and well boundary conditions, English or metric system of units, constant or variable pumping rate.


## Model Description

Appendix A contains the program 1 isting and the flow chart illustrating the interrelationship of the main parts of the program. Figure 1 shows the interrelationship between the main program and the subroutines.

## Input

Input to the program is from two sources. A file which contains the tables for well functions, and the data on punched cards which define the scope of the problem to be solved. The data cards define the


Figure 1. The interrelationship between the main program and the subroutine.
number of cases to be solved, the system of units used, the number of boundaries, the number of points of interest, the number of wells, and the type of aquifer and well boundary conditions. Boundary geometry, points of interest, and the cartesian coordinates of real wells must be specified in another set of cards. Aquifer characteristics for the case study are read separately. Time data are read from another card. Finally, if variable discharge problem is to be analyzed, a set of cards to define the discharges of each well must be specified. Appendix $B$ contains examples of the data cards to be used with their format for several example cases.

Errors might arise if wrong data are used or there is a violation in program limitations. Appendix $B$ contains the error messages which may be printed during the program execution.

If the S.I. system of units is used in the input, then the data are converted in the program to the F.P.S. system. Table 4 contains the conversion factors for all the parameters used in the program. Appendix B contains a description of input data and formats.

Table 4. Conversion factors.

| Multiply one | By | To obtain |
| :--- | :--- | :--- |
| Meter | 3.281 | feet |
| cubic meter/day | 0.18345 | gallon/minute |
| square meter/day | 2.28 | gallon/day/foot |
| meter/day | 0.695 | gallon/day/ft |

Analysis of geometry
The boundary configuration is limited to the following six cases which can be analyzed with the image well theory: no boundaries, one
boundary, two intersecting boundaries, two parallel boundaries, three boundaries intersected at right angles, and four boundaries intersected at right angles. Figure 2 illustrates these cases. An error message will be given as an output if the problem to be solved is not one of these cases.

## Image well generation

The purpose of image well generation is to find the number and the coordinates of the image wells due to the existence of boundaries, and the distances from the observation point to the image wells. To do this job, three subroutines are used, REFLEC, PARALL, and INTERS subroutines. Subroutine REFLEC determines the coordinates of the image wells and checks whether the boundary is barrier or recharge one. Subroutine INTERS determines the number of image wells ( $n$ ) in the case of intersecting boundaries with an angle of (A) using equation number (8). Subrountine INTERS calls Subrountine REFLEC to determine the coordinates and type of the image wells (discharge or recharge). Subroutine PARALL determines the number of image wells in case of parallel boundaries, calls Subroutine REFLEC to find the coordinates and type of the image wells, finds the distance from the point of interest to the image well. Subroutine PARALL then calls Subroutine DRAWDO which calculates the drawdown from each well which is explained in the next section. Subroutine PARALL checks the change in drawdown as additional image wells are added to see if it is within a user-specified convergence limit. The maximum number of image wells is 1000 (see the limitation of the program). Subroutine REFLEC is also used by the main program in the case of a single boundary.
Real Well
a. No boundaries

b. Single boundary
d. Infinite strip oquifer
O
O
Reai Well
Reai Well
c. Two boundaries intersecting with an angle
f. Rectangular aquifer
e. Semi-infinite strip aquifer


Figure 2. Limitations on the configuration of system of boundaries.

Real Well
$\qquad$

## Drawdown computation

The coordinates, type, and number of image wells are stored and then used to evaluate the total drawdown at each point of interest as the algebraic sum of drawdown due to each image well and each real well. The computation of drawdown is perfomred by Subroutine DRAWDO. The first four cases (see Table 2) are solved analytically. The other seven cases (see Table 2) use the well function tables to interpolate linearly the value of well function and substitute it in the drawdown equation. This interpolation is done by two subroutines, INTRP1 and INTRP2. If the well function table is one dimensional, Subroutine INTRP1 is used, and if it is two dimensional, Subroutine INTRP2 is used. In the unconfined aquifer unsteady case (case number 11) two alternative equations are used depending on the time since pumping began. Boulton (1963) gave a graph for estimating the time range of each equation from the value of $r / B$. This graph was approximated by a table of times which are interpolated using Subroutine INTRP3 using a given value of $r / B$. An error message is printed by the program if the value of $r / B$ is not within the bounds of the table.

Drawdown may be evaluated for a series of time intervals based on a logarithmic scale. A choice of minutes or days as a time unit and of the number of $\log$ cycles of time (up to 3 ) must be made by the user.

## Well losses

The well loss for each well at each pumping rate, if variable discharge rate is used, is calculated as follows:

$$
s_{W}=C_{W} Q^{n}
$$

in which
$s_{w}=$ the well loss due to turbulence flow near the well
$Q=$ the pumping rate of the well
$C_{W}=$ the well loss constant
$\mathrm{n}=$ an exponent relating the discharge Q to the well loss, $\mathrm{s}_{\mathrm{w}}$.
The two constants $C_{W}$ and $n$ are stored in the main program and then used to calculate the well loss directly.

## Several real wells

Up to fifteen real wells can be analyzed simultaneously using this program. A separate image well system is generated for each real well. At each point of interest, the drawdown is composed of contributions from all real and image wells. The program could be easily changed to accommodate more than 15 real wells.

Several points of interest
The program can calculate drawdown at up to 15 points of interest at the same time. These points could be positioned to give enough drawdown information to enable to draw contour lines of the water table surface or the piezometric surface. If these points are on a straight line a graph option may be used to show the change of drawdown along the line. A relatively simple change for the program would print more than 15 points.

## Variable pumping rate

A variable pumping rate can be justified for any of the unsteady cases (i.e. cases number 3, 4, 5, 6, 7, 9, and 11). Pumping from the real well(s) must be specified for each time interval. The discharge


#### Abstract

in each step or time interval is considered to be constant. For the variable pumping condition the time intervals must be used in days and each interval must be of the same length.

To explain how to treat this case, suppose there are $n$ pumping rates and $m$ wells. Thus the number of time steps is $n+1$. Let $Q_{i j}$ represent the discharge for well number $i$ at stage of pumping $j$. At the initial time of pumping when $j=1$ the discharge $Q_{i 1}=0$ for all wells (i). If the discharge of well $k$ is constant in all time steps, i.e. $Q_{k j}$ is constant for all values of $j$, while the other wells have a variable discharge, then the values of $Q_{k j}$ should be set equal to the same value in each time step.

The number of time increments and the whole pumping time is stored and then used to calculate the drawdown through double matching do-loops (see Figure 3 and the flow chart). The first do-loop is used to calculate the drawdown for the new change in discharge in the recent time interval, and the second do-loop is used to calculate the drawdown continuation from the previous changes in discharges in the previous time intervals.


Analysis of drawdown components
Drawdown is divided to four components for each real well, the drawdown caused by pumping from the real well excluding the effect of partial penetration and boundaries, the effect of partial penetration in the real well only, the effect of boundaries, and the effect of partial penetration on the boundaries. In the program, the drawdown caused by each real well and the effect of partial penetration is stored. As the computation of drawdown for each real well with its


Figure 3. Discharge-drawdown curve for variable pumping case.
image wells is completed the stored value of drawdown is called and subtracted from the total sum of drawdowns caused by real and image wells. This process is repeated twice to find the effect of the system alone, and the effect of partial penetration alone.

## Program options

Many options are used in this program to broaden the utility of the program. A full description of these options is given in the user's manual contained in Appendix A. Any number of runs (NCASES) with different parameters can be run sequentially in the same program execution. S.I. or foot-pound-second system of units can be used (JUNIT). Optional tabular output of the drawdown components (ITABLE) and tabular output of individual image and real well calculations (INDIV) is also available. A time unit option (ITUNT) makes an available choice of using days or minutes. Both cases of constant or variable discharge can be handled using the parameter ITMV. One or more time increments can be used to show the change of drawdown as time proceeds by using NTI option. Finally, an optional graph of drawdown vs. distance or drawdown vs. time is available using IPLOT option.

## Output

All the equations of drawdown computations in this program use the F.P.S. system of units. If the input data is the S.I. system, then it is converted to the F.P.S. system for the calculations and the results are converted back into S.I. units. The conversion factors are given in Table 4.

Three type of output exist:

1. A basic tabular output of input data and results.
2. Two optional tabular outputs. The first one is for the individual processes of the program containing the coordinates of each image well and its contribution to total drawdown at a point of interest. The second one contains a breakdown of the drawdown components.
3. An optional graphical output of drawdown vs. log (time from the beginning of pumping) or drawdown vs. $\log$ (radius from the real well) in case of constant pumping rate. If variable pumping rate is used then a graph of drawdown vs. (time from the beginning of pumping) is drawn.

Examples of the program output are given in Appendix $G$.

## Model Advantages

The purpose of this section is to explain the advantages of model and its practical field application. At this point, the reader should have an adequate understanding of the working of the model. Groundwater models serve as a means to understand the mechanism of groundwater resource in nature and to predict what might happen under various possible future conditions. So groundwater models are a very valuable tool in water resources planning.

Time saving
This model could save a lot of time in hand calculations to predict the drawdown in a well field, especially when the basin is a complex one with many wells, several barrier or recharge boundaries, and there
is a variable discharge rate. The task of placing the input data in the required input format saves days or even weeks of longhand error-prone calculations. This capability for rapid calculation of drawdown under complex conditions will also make possible the examination of many alternative designs which would otherwise be impossible. Convenient tables of input data and results are provided by the program.

## Education tool

This program with its associated tables are stored in a computer disk file which can be readily transfered to other computers. The program would be useful to students of groundwater in helping them develop a rapid feel for drawdown complicated groundwater systems and to enable them to perform sensitivity studies on wuch factors as hydraulic aquifer characteristics and different boundary conditions.

Groundwater contour map
It is possible to predict water table or piezometric surface maps in a simple or complex basin by using this model. The procedure is to make a run with enough points representing the region of interest to find the drawdown at each point as a result of pumping or recharge. Thus, the contour lines of equal heights of water table or piezometric surface can be drawn through these points.

## Model contribution to agriculture

It is important in agriculture to have a permissible range of water table elevations. Maintaining the root zone in the soil with certain percentages of water and air is determined by two factors, soil pro-
perties and water table elevation. Using the model, it is possible to predict the water table elevation and then determine the amount and time for pumping so that the plant will not die neither because of wilting, nor because of extra amount of water in the soil pores. It is also possible to determine the schedule of irrigation for a permissible drawdown in the field in the daily, weekly or monthly basis as needed. This problem can be solved by the variable pumping rate case which is one of the most powerful part of this model.

## Model contribution to industry

Water is as important in industry as in agriculture. Huge amounts of water are used in different industrial production processes. In some cases, the factories have their own wells to produce water. The amounts of water used in industry varies in each day, week, or month according to the period of working hours, days, or according to the rush production. Using the model helps in the drawdown prediction in the well field. A convenient table of pumping schedule can be easily prepared.

Groundwater recharge
Sometimes it is required to recharge a basin with water to compensate the drought periods. Injection well is one of several means to recharge the basin with water. The rise in the water table or piezometric surface can be computed in the model by specifying the well type in the input data as an injection or a discharging well. A combination of discharging and injection well systems in the basin, if exist, can be analyzed normally.

## Construction dewatering

In the construction of building foundations below the water table it is necessary to lower the water table using wells around the periphery of the excavation so that work may be carried out in the dry. Using the model, systems of dewatering wells could be analyzed in an iterative design procedure to find the best layout to achieve the required lowering of the water table.

## Model Limitations

Even when the model covers a variety of situations, it is constrained in its logic to some constraints. Sometimes, when the model is blocked to these constraints, there is an error message as an output.

1. Firstly, the model is constrained to a maximum of 15 points of interest, 15 wells, 4 boundaries, and 15 steps in variable discharge case. These numbers are within practical uses. These limitations can be changed simply by increasing the dimensions of the arrays, but then the program will become more expensive to run.
2. Another limitation is the geometric configuration of the boundaries (six kinds of configurations). These configurations, if violated, will result in an error message. In practical situations, the real world should be approximated to one of the six cases if possible.
3. Convergence limit of the drawdown is one of the most important limitations in the program if a pair of parallel boundaries exists. The existence of parallel boundaries will result in an infinite number of image wells. Practically, this number is determined so that the final image well contribution to the drawdown has a negligible effect. This
negligible effect is the value assigned to the convergence limit in the program logic. If a small convergence limit is assigned in the input data, a large number of image wells will result which causes an expensive run. Error message is given as a warning message if the number of image wells exceeds 1000 and the drawdown in the last image well is still more than the convergence limit.
4. The interpolation of well function from the stored tables in the files in the computer memory is done linearly. So the well function curves are approximated to straight lines between the nodal values of the tables. To have more accurate results, a large number of small increments should be used. Also if the point of interest is too close to the well or far away from it, the interpolating value becomes outside the table limitations.
5. In unsteady, water table variable discharge case, it is assumed that if the drawdown is within 10 percent of the aquifer thickness, then water is assumed to flow horizontally from the aquifer to the well and hence the principle of superposition is applied.
6. Some additional limitations arising from the structure of the computer program are:
a. Only one point of interest can be considered when dealing with more than one time increment in the constant discharge case.
b. Only one point of interest can be considered when dealing with more than one time increment in the variable discharge case.
c. Time must be expressed in fractions of a day and not hours or minutes in the variable discharge case.

CHAPTER IV<br>MODEL VERIFICATION AND APPLICATION

Many examples using different model options have been run and verified by hand calculation. These examples include all boundary types and configurations, aquifer types, equilibrium and nonequilibrium cases, constant and variable pumping cases, and the F.P.S. and S.I. system of units. Several of these cases are described below and example output and output may be found in Appendix $C$.

Model Verification

A set of selected cases from the above mentioned examples is described below. These examples cover the most important components of the model and are made up of various combinations which could be encountered in the field. Refer to Figure 2 for boundary configurations and Table 1 for the definition of drawdown cases used in these examples.

No boundaries and drawdown case 11
The drawdowns were calculated at a point of interest for four days of variable pumping rates from founr fully penetrating wells in an unconfined aquifer, under nonequilibrium conditions. The metric system of units was used. This run cost 11 cents.

Single boundary and drawdown case 7
The drawdown was calculated over two $\log$ cycles of time in days after the start of constant pumping, for a fully penetrating well in a
leaky artesian aquifer without water released from storage in an aquitard under nonequilibrium condition. The graph option of drawdown vs. time was used. This run cost 32 cents.

Parallel pair of boundaries and drawdown case 9
Drawdown was calculated at the middle point between two parallel boundaries due to pumping at a constant rate from a fully penetrating well in a leaky artesian aquifer, with water released from storage in aquitard under nonequilibrium condition. Time is in minutes and the S.I. system of units were used. In this run, the tabulation of the individual contributions to drawdown from each real and image well was included. This run cost 19 cents.
$90^{\circ}$ intersection of two boundaries and drawdown case 5

The drawdown was calculated at a point of interest due to four partially penetrating wells in an unconfined aquifer with various percentages of penetration at each well. The drawdown was calculated at four time increments under variable pumping conditions for the nonequilibrium case. The S.I. system of units was used. This run cost 15 cents.

Semiinfinite aquifer and drawdown case 4
Drawdown was calculated at 15 points of interest laying on a straight line. A fully penetrating well is located in a confined semiinfinite strip anisotropic aquifer. One of the boundaries is a recharge one while the parallel pair are of the barrier type. This run cost 15 cents. The S.I. system of units was used. The computer output in Appendix $C$ includes a graph of drawdown vs. distance.

Two perpendicular parallel pairs of boundaries and drawdown case 1

The rise in the piezometric surface due to the injection of water via a fully penetrating well was calculated in a rectangular, confined aquifer under equilibrium conditions. This rectangular aquifer might represent a water storage basin. The S.I. system of units was used. This run cost 13 cents. The printout, including the tabulation of the individual contributions to drawdown from each real and image wells, is given in Appendix C.

## Application to Well Spacing

## Purpose

The problem of well spacing is a practical problem facing groundwater engineers. Two objectives should be satisfied in this problem, engineering feasibility and cost minimization. The amount of drawdown at any point of interest depends on the spacing of the wells and the distances between the observation point and the production wells. For a given aquifer and boundary configuration, there are many alternative well field designs, ranging from a small number of wells spaced far apart, and hence less well interference, to a larger number of wells with closer spacing. In the first case, the pipeline cost is more and the well drilling cost plus casing and maintenance cost is less than the second case. A design requirement is that the drawdown be limited and the design pumping rate be obtained. The criterion for selecting an alternative design is that it will have the minimum cost.

## Problem statement

It is required to obtain a design discharge of 750 gpm in a well field to be located in a confined aquifer type (drawdown case 3) such that the drawdown in the most affected well is not more than $10.00 \pm$ 0.15 feet to ensure artesian flow conditions and to minimize the installation and operation cost of the pumping unit. The wells should be located on a circumference of a circle, and the discharging water from these wells will be collected, by means of pipelines radiating from the center of the circle. There is a barrier boundary located 15,000 feet from the center of the circle. The following data are available (see Figure 4).

Permeability of the aquifer, $k=400 \mathrm{gpd} / \mathrm{ft}^{2}$
Thickness of the aquifer, $m=120 \mathrm{ft}$
Coefficient of storage of the aquifer, $S=0.0005$
We11 loss constant, $C_{W}=0.00001$
Well loss exponent, $n=1.75$
Depth of each well $=150 \mathrm{ft}$
Maximum period of well system operation $=5$ days
Cost of cementing each well $=\$ 200$
Annual maintenance of each well $=\$ 200$
Discount rate $=10 \%$
Estimated life span for the wells and pipelines $=30$ years
Annual maintenance for the pipelines $=\$ 0$.
Cost of drilling wells with casing, the pipeline cost, and the permissible discharge for each diameter are given in Table 5 (personal communication with the well drilling companies in Logan, Utah 1980).


Figure 4. Plan and section of selected well field design.

Table 5. Cost and permissible discharge for various well diameters.

| Well or pipe <br> diameter <br> $(i n)$. | Drilling cost <br> of well <br> $(\$ / f t)$ | Cost of pipeline | Permissible <br> discharge <br> $(\$ / f t)$ |
| :---: | :---: | :---: | :---: |
| 6 | 21.00 | 3.00 | 100 |
| 8 | 24.00 | 4.30 | 200 |
| 10 | 32.00 | 5.25 | 400 |
| 12 | 39.00 | 6.25 | 600 |

The object of the problem is to design the most economical spacing of the well system.

## Solution

The procedure to solve this problem begins by assuming a certain number of wells located on a circle circumference with an assumed radius. The discharge of each well is equal to the design discharge divided by the number of wells. The coordinates of each well are computed and a model run is performed to compute the drawdown at the well nearest to the barrier boundary. Well loss at the well is added to the total drawdown and the resultant drawdown is compared with the maximum permitted drawdown in the aquifer system. By varying the radius of the circular well field the radius at which the selected number of wells results in the maximum permissible drawdown is obtained. For each radius a new model run is made to compute drawdown at the well nearest the barrier boundary. As the radius is increased the interwell interference decreases, but the boundary interference increases in the wells located nearer to the boundary.

The next step is to assume some other numbers of wells and repeat the above procedure to obtain a maximum radius such that the constraint on maximum drawdown is still met.

The result is a set of alternative well field designs (see Table 6), each of which has approximately the same maximum amount of drawdown in the well nearest to the boundary and each of which has the same design discharge. Each alternative differes from the others by the number of wells and their spacing and therefore the cost of installation and maintenance. The total cost of each design is expressed as an equivalent annual amount. It is composed of the equivalent annual amount of all the wells and the connecting pipelines between the wells and the center of the well system circle, and the annual maintenance of the well system. No operation cost is required since no pumping is needed under artesian flow condition. Cost figures are also included in Table 6. The selected design is the one with minimum total cost. From Table 6 it can be seen that this design comprises 7 wells located around a circumference of a circle of 1600 feet diameter. The total annual cost is $\$ 4,669$. The computer model output for some runs performed in this well spacing case study is included in Appendix $C$. The role of the computer model in this well field design problem is to define the feasible set of designs that satisfy the maximum drawdown condition.

Table 6. Economical analysis of alternative well field designs.

| Number of wells | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total drawdown at the well (ft) | 9.98 | 10.01 | 10.00 | 10.10 | 10.09 | 10.00 | 9.99 |
| Discharge from each well (gpm) | 187.50 | 150.00 | 125.00 | 107.10 | 93.70 | 83.30 | 74.00 |
| Diameter of well (in) | 8.00 | 8.00 | 8.00 | 6.00 | 6.00 | 6.00 | 6.00 |
| Radius of center of wells (ft) | 5,000 | 2,750 | 2,000 | 1,600 | 1,400 | 1,300 | 1,200 |
| Total pipeline length (ft) | 20,000 | 13,750 | 12,000 | 11,200 | 11,200 | 11,700 | 12,000 |
| Total pipeline cost (\$) | 86,000 | 59,125 | 51,600 | 33,600 | 33,600 | 35,100 | 36,000 |
| Cost of cementing all the wells (\$) | 800 | 1,000 | 1,200 | 1,400 | 1,600 | 1,800 | 2,000 |
| Cost of drilling all the wells (\$) | 14,400 | 18,000 | 21,600 | 22,050 | 25,200 | 28,350 | 31,500 |
| Capital investment cost (\$) | 101,200 | 78,125 | 74,400 | 57,050 | 60,400 | 65,250 | 69,500 |
| Equivalent annual capital cost (\$) (Based on 30 yr life, $10 \%$ ) | 5,800 | 4,477 | 4,264 | 3,269 | 3,461 | 3,740 | 3,983 |
| Yearly maintenance (\$) | 800 | 1,000 | 1,200 | 1,400 | 1,600 | 1,800 | 2,000 |
| Total annual cost | 6,600 | 5,477 | 5,464 | 4,669 | 5,061 | 5,540 | 5,983 |

CHAPTER V<br>SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this chapter is to give a brief summary of the work, to list the conclusions from the study, and to give recomendations for further work which might continue the process of model improvement.

## Summary

This study involves the computation of the drawdown in a well field as a result of removal or recharge of water. A total of eleven different aquifer hydraulic cases which are described in Walton (1970) and Bouwer (1978) have been included in the model.

The problem of a finite aquifer caused by the existence of hydraulic boundaries has been solved using image well theory as described by Ferris et al. (1962). Six cases of boundary configurations, barrier or recharge boundaries, or combinations of both, can be analyzed with this program.

Water table or piezometric surface profiles can be calculated with the model, and can be displayed as a graph of drawdown vs. distance by using many points of interest (up to 15 points) arranged on a straight line from the real well. Another graph option is available to show the change of the drawdown with time as pumping continues.

Frequently, many wells working simultaneously may exist in a well field. Up to 15 real wells (discharging or injection wells) with the well loss computation for each one can be treated by the model.

In addition to constant pumping rate case, a stepped pumping rate case can be analyzed in any of the aquifer, well, or boundary situations. A maximum of 15 time increments for the stepped pumping case can be used. These time increments should be equal in length. A fraction of a day can be used as a time increment. The change of drawdown in each time increment at one point of interest can be displayed on a graphical output but only days as a time unit is permitted in this case.

The drawdown components can be analyzed for each well at each point of interest or at each time increment. Four components have been considered: the effect of pumping from the well only, the effect of partial penetration of the well alone, the effect of the boundaries (if they exist) alone, and the effect of partial penetration by all the boundaries. The total effect of pumping on the drawdown is the sum of these four components.

To make the model more flexible, there is an option to use the S.I. or F.P.S. system of units. Another option is to use a time unit in minutes or days except in the variable pumping case when days must be used.

The output is in a tabular form with a graphical option. The drawdown components table is option. There is another optional output which shows the calculation of drawdown for each image well. Care should be taken in using this option especially if a pair of parallel boundaries exist in the problem because it can become quite voluminous.

A number of error messages are included in the program to inform the user of incorrect input data or violations of the program limitatations.

The following conclusions were reached based on the program performance.

1. Convergence limit (CONV). Theoretically, the number of image wells in the case of a pair of parallel boundaries should extend to infinity. A convergence limit (CONV) is assigned by the user to truncate the series of image wells when the drawdown contribution from the latest image well is less than the convergence limit. The affect of changing the convergence limit to the accuracy of results and cost of running depends on the nature of the problem to be solved. In some cases, changing the convergence limit neither changed the accuracy of results nor the cost of running. In other cases, reducing CoNV from 0.05 to 0.01 feet, for example, increase the accuracy to $\pm 0.025$ feet while the cost of running increased by 20 percent. Sometimes in the cases of interpolating the well function from their tables, a little change in CONV will cause the interpolating values to be outside the table bounds.
2. Interpolation error, The effect of linear interpolation from the well function tables cause a small negligible error. This interpolation error in most cases is less than 5 percent than the interpolation from the well function curves.
3. Time until steady state condition achieved. The time at which steady state flow conditions are reduced can be determined using the model by running the unsteady state case with many time increments until the increase in the pumping period has no effect on the amount of drawdown.
4. Factors affecting well spacing. From running the well spacing example, it was found that the cost of the connecting pipeline has more affect on well spacing than the cost of drilling the wells. The cost of yearly maintenance of the well system is very sensitive to the well spacing.
5. Cost of running and compilation of the model. The cost of compiling the program is about $\$ 4.50$ on the Burroughs $B 6800$ computer. The cost of storing the program in a computer disk plus the cost of storing the well function tables in the files is about $\$ 5.50$ per month. The cost of running different cases depends on the nature of the problem itself. The cost of running the case study increases with decreases in the convergence limit, and with increases in the number of boundaries, well boundaries, points of interest, and stages in the varible pumping case. The cost of each run is given in each example and ranges from 11¢ to 32 ¢. These costs are the night computer costs which are 10 percent of the usual daytime costs.

## Recommendations

Nothing is complete in the real world because we are human beings. The recent knowledge is built on the previous knowledge and so for the future knowledge. This study needs to be continued, and there are several recommendations for future work.

1) The boundary geometry is limited to six regular cases of boundaries in this model. In the real world the boundaries are located in irregular shapes and curves. Introducing the capability for analyzing well fields defined by such boundary configurations would be a useful addition.
2) Another recommendation is to solve cases 5 through 11 (see Table 1) analytically without using the well function tables and an interpolation procedure.
3) It would also be useful to add the sloping water table situation to the program.
4) A capability for obtaining the drawdown at many points of interest during many time intervals should be included in the model.
5) The model can be adapted to utilize any pumping schedule which might be proposed or used in the field.
6) A.graph plotting subroutine can be added to plot drawdown vs time and distance or vs two distances axes.
7) The well spacing analysis described in Chapter IV could be included as a separate subroutine to avoid the current trial and error procedure which involves many separate computer runs.
8) Convert the program to be interactive.
9) A groundwater contour map subroutine should be added to the model.

## REFERENCES

Bear, J. 1972. Dynamics of fluids in porous media. American Elsevier, Pub. Co., New York.

Boulton, N. S. 1963. Analysis of data from nonequilibrium pumping tests allowing for delayed yield from storage. Proc. Inst. Civil Engrs. (London) 26(6693).

Bouwer, H. 1978. Groundwater hydrology. McGraw-Hill, New York.
Bowles, D. S., and V. J. Rogers. 1973. A computer program to calculate drawdown using the image well theory. Utah State University, Logan, Utah.

Carnahan, B., H. A. Luther, and J. O. Wilkes. 1969. Applied numerical methods. John Wiley and Sons, New York.
de Wiest, R. J. M. 1965. Geohydrology. John Wiley and Sons, Inc., New York.

Esmael, O. J., and 0. K. Kimbler. 1967. Investigation of the technical feasibility of storing freshwater in saline aquifers. Water Resour. Res. 3(3):683-695.

Ferris, J. G., D. B. Knowles, B. M. Brown, and R. W. Stallman. 1962. Theory of aquifer test. U.S. Geological Survey Water Supply Paper 1536-E.

Food and Agricultural Organization of the United Nations. 1978. Groundwater models. FAO Irrigation and Drainage Paper, Rome.

Freeze, R. A., and John A. Cherry. 1979. Groundwater. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Hantush, M. S. 1956. Analysis of data from pumping tests in leaky artesian aquifer. Trans. Am. Geophys. Union 37 (6).

Hantush, M. S. 1964. Hydraulics of wells. In: Advances in Hydroscience. Academic Press, Inc., New York.

Jenkins, C. T. 1968. Techniques for computing rate and volume of stream depletion by wells. Ground water 6(2):37-46.

Kruseman, G. P., and N. A. de Ridder. 1970. Analysis and evaluation of pumping test data. Bulletin 11. International Institute for Land Reclamation and Improvement, Washington.

Melosh, R. J. 1965. Basis for derivation of matrices for the direct stiffness method. Journal American Institute for Aeronautics and Astronautics 1:1631-37.

Personal communication with the well drilling companies in Logan, Utah. 1980.

Remson, I., G. N. Hornberger, and F. J. Molz. 1971. Numerical methods in subsurface hydrology with an introduction to the finite element methods. John Wiley and Sons, New York.

Richardson, L. G. 1910. The approximate arithmetical solution by finite differences of physical problems involving differential equations with an application to the stress in a masonary dam. Phil. Trans. Royal Soc. A210:307-357.

Segerlind, L. J. 1976. Applied finite element analysis. John Wiley and Sons, Inc., New York.

Show, F. S., and R. V. Southwell. 1941. Relaxation methods applied to engineering problems; VII. Problems relating to the percolation of fluids through porous materials. Proc. Roy. Soc. Al78:1-17.

Stallman, R. W. 1963. Type curves for the solution of single-boundary problems. In: Shortcuts and Special Problems in Aquifer Tests. U.S. Geol. Surv., Water Supply Paper 1545-C:45-47.

Theis, C. V. 1941. The effect of a well on the flow of a nearby stream. Trans. Am. Geophys. Union, Part 3:734-738.

Thomas, R. G. 1961. Graphical solutions of groundwater flow problems. Bulletin of the International Association of Scientific Hydrology, VI(4):50-69. December.

Todd, D. K. 1959. Groundwater hydrology. John Wiley and Sons, Incs., New York.

Turner, M. J., R. W. Clough, H. C. Martin, and L. J. Topp. 1956. Stiffness and deflection analysis of complex structures. Journal Aeronatical Science 23:805-824.

Tyson, H. N., and E. M. Weber. 1963. Use of electronic computers in the simulation of the dynamic behavior of groundwater basins. Water Resources Conference, Milwaukee, Wisconsin. May.

Vandenberg, A. 1977. Type curves for analysis of pump tests in leaky strip aquifers. J. Hydro1. 33(1/2):15-26.

Verruijt, A. 1970. Theory of groundwater flow. MacMillan, New York.
Walton, C. W. 1970. Groundwater resource evaluation. McGraw-Hill Book Company, New York.

APPENDICES

APPENDIX A

Mode1 Component

1. Model flow chart
2. Program listing
3. Variable Dictionary


Figure A.1. Model flow chart.

Program listing.

| Fil |  |
| :---: | :---: |
| $\begin{aligned} & \text { FTLE } \\ & \text { FHLE } \end{aligned}$ |  12(KIMO=DISK, HITLE"TABLE2";FILETYPE-1) |
|  |  |
|  |  |

## COMMON/BDSDH (4), C(9), BiYPE(9), TH(4), NANDS

1/PIE/XP(45),Yp(45)




T/CONI/KPROB, PI, SCONY(2), 1 ( 301 , CONX, JUMII, IVY, IM, IRK
\%OUT/T1TLE(20), $\times(100), Y(100), 2(100), 0(25,11)$, GAMA(i5), 1U1

1/TMA1/H(12), N(12), $1(300,15), V(15,15), H(65,65,121, W F(300,15)$
$2 / W L 08 / C W(15)$

DIALNSION YH1 (15,45),YP1(15,45),YW2(15,45),YP2(15,45),YTOT3(15,45) dimension xheliodol,ym(1000)


```
501 l',',
```




509 FOMMT(z0As)
550 format (1His













FORHAT IH





c ** mitialise
NNPKO
P1:22,
1isi:
liumio

TOELT:
NINTAO
NPAMAMO
NSINO
NSINO=0
IME
I
NIMOST1
DO 3 ispen,
y(13p)"O,

KPARA $($ PARARA,IL)=0

s


* InPu
c HEAOING

c * control card

C *** read hell function thbles fron the stored files if needed

```
IF(KPROB,LE,*) 60 10 %1
EAO(1,1)(M(J),J=1,10),(N(1),1=1,11)
aEA0(1,1,)((1(1,k),1:1,N(k)),k=1,11)
QE{O(13,\)((CH(1,J,K),I=1,M(K)),J=1,N(X)),K*1,9)
RE{O(i3,刀)((ch(1,J,k),i=1,M(k)),j=1,N(x)),k=1,q)
```

    Finembs, 01.4)60 10100
    
MERRF1
$60100_{0}$

If（ABHOS，ED．0） 6010 is
OO 10 16：fi，NAHDS
EAD（5，502）TH（18），C（18），日TYPE（10）
IF（JUNIT：EO，1）C（18）MC（IB）＊1， $2 B$ i
nahenati
ififh（i日）

ATH：16］＊P1／1AO
aュ0
OM（ID） $\mathrm{COHAN(TA})$
continue
${ }_{13}$
point of inteatst coordinates
READ
IF（SUNIT．EG．1）XP（IP） 1 XP $(1 P)=3,20$
15 comijnue
c real mell data

c＊aguirer characterisilcs and chanaing or units


o conisinue
IF（KPROD．LT，a）READ（5，503）PERM，1HCK，COT，HO



st，MoAR，Pgap



if（Junit，Ea．1）Conveconv＊3，2＊1

IT（JUNIt．E日，i）HAAREMERRA； 261


IF（JUNIT，E日，1）YXY＝1XY＊z．28
ir（JUNIT，EO，1）PGARABAR $+0,695$

IFIIUNT，RE，I，OR IYMY EO．0）G0 1023
nearne
RIIE（6．509）HERA
$0010 \$ 00$


00 21 KJ M 1, NHELLS


IF（ITUNT，EO，1）11＝T1／1440
IIISII
If（IDELT，EQ．0）IDELT：




MERRE 3
OOPO
Soo
c＊＊ANALYAIS op gounday ceohetry and check for compliance with
$\epsilon_{25}$＊SEPARATE NO BOUNOARY CASE
c－Binde goundsay clase


ksingel
to 0 ro
c＊fimo parallel paias of soundaries
0010 Igimb，（HaHosel）

MPARARMPARA＋！

Co $10(45,27,30),(N 8 N B E=1)$
CONTMUE
If TNPARA．EQ，210010 43
00 10（31，13；19），（NBNDS－1）
C＊one parallel pait and aingle goundary cabe
NSING：
ifiB．E日，KPARA（1，1）100070 29

KSING：18
coto 34
29 Conithue
 KINT（1，1）＊

KINI(1.2)me
601040
C 3 M mon-pratlel bounortics - 3 intersections
Mintaj
$K 1 N(1,1)=1$
$K(N)$
$\mathrm{KINT}(2,1)=1$
$\mathrm{KINT}(2,2) n\}$
KINT(2,2)*
$K 1 N T(3: 1) * 2$
KIM1(3.2)
$60 \$ 1040$
$t$ * ChECK single goundary perpendiculak to parallet patr


MERRMA
6010400
C * a boundiries must at a parallel paiss
NERRES 5
OD TO 490
$t$ * check intersecilion angles comply with fermis limitaitions NT=1,NINT


THOENN*90;
ITHDEITIX



| 42 |
| :--- |
| continue |
| 60 to |

Ca3* chick 2 pairs or papallel lines pempendiculan


NERRAT
6010000
c * gengration of image nells
45 DO os jumf,NHELLS

- meth losses in constant punping case

c** computation of orawonin in case or vanianle oischange







siot:o.
splofer.
IVY:ITYM1
- hell losses in yariable purpina case


00 99 11141.17vat

y12011
JJ1 1
JJ2*0
NIMOS:


YYFI(JJ)MHTYPE(IK)

IFININT, Ea, j)00 TO 59

CALL INTE
CONTINUE
c parallel patas of goundarieg IFENPARA, EG, Oigo to 65


37 CALL PARALL(IPARA, J)
54
CALL PARALLI(IPARA,
34
GO CALL PARA
confinue
C as alte goundary If (NSING, E0,0) 1001079 if (JJ1, 最, oing 1073

c * compute drahoown dor nphti points of interest

1f(1fHY,NE.0) 00 1079

YHi $(1 H, I R)=0$.
YFi(1H, IR)

16 YTorifinilitio.
7 IFITHV, EQ.0) siot:o.
ifitnotvere inoto






 WRIIE (6,551)
If (ITUNT,NE, AND, JINIT, NE, I) HRTIE $(6,552)$
If (ITUNT,EO.1. ANO, JMNIY, NE,
If (ITUNT,EO.1, ANO.JUNIY, NE, I) WRTIE(6,553)

c - compute deahoowh contriaution ay well jJ
$\begin{aligned} & X x A B S(X P(I P)-X I(J J)) \\ & Y Y=A B S(Y P(I P)-Y I(J J))\end{aligned}$



c o** compuration of omahoonn as time varies
IT DO 90 INDE1:(NLOG\&)

HE1T**(to**(1n0-1)
spoy=0
sproted
00 fa line 1,15
YHI (1H THE1:45

Y $42(1 \mu, I R)=0$.

101 IF(TI.HE.0.) 60 10 84
NERRAG
GOTO 400
84 CALL ORAWDA(R,TI, 3 )
IF (KPROB,NE.5) 60 10 81
крRoemb
CALL DRANDO (R,T1, SP
KP20日*S






srotesiotis

Y(18) $\left.\mathrm{Y}(1)^{3}\right)+8$
YSP(1s)
SPTOT:SPTOItSP
c ** ordhoonn component





Tr(JJ.NE, NRMGB) DO 10 os





If (JUNIT,EG, I) RH=R/3.281


IF (INDIV,NE, liodoto ob




 golo 06




66


so continue



96

Conthut cu.0) 60 lo es
of conithue
If(ITUNT, Eg.1) 111:T1*14a0.
4. *. outpur
call output
c * plotied output H(IPLOT, NE, 1)0070 120

IF (IINVEQ,O) $X$ HAXPALOGIO $X(1)$




135
conimue
130
YuIN=Y(1)
Maxey
$001 \% 0$ 18:2. shay
If (Y(1s) , LT:YMIM)YMIHEY(IB)
140

00 105 $13 \times 1,19 \mathrm{Hax}$
z (is)
105

120 continue
ouro 100
c ** ERRor hessages
300 EOHIINUE
subaoutine intersainis
COHMON/BOS/OH(0), C(4), 日TYPE(5),TH(4), NBHOS
/PT3/XP(45),YP(45)

4/HYOR/PERM, THCK, COT, HO, SY, HBAR, SEAR, PGAR,ALPA, ALFTHT, TIMT









jusit (lsco./THOLFi-1.)
Do 10 In Elina

 1日TYPEKINI(IINT,JL)),JJJ)
10 continue
return
Eno
subrauyine parall (Ipara,j)
COHMON/OOS/OH(4),C(A), DTYPE(4), TH(A), NOMOS /PIB/XP(45),1p(43)


$5 / 14 W L / X I(1000), Y 1(1000)$ YYPI(1000), $001 F(15)$

/CONT/RPROA, PI, SCONV (2), (130), CONY, JUMIT, IVY, 1H, IRK



c ** fhis suaroutine conthole the reflection process in the cage of
504 SORHATIHH: TERROR NO: 12
OO 5 IPRAI: 2

IR:O
IR
IR
IR

IL=( $1+(-1) * * 1 P P) / 2)+1$


c : calculation of phin
DO 20 \{PMINPNTS


IFR.GT,0.000001)6010 is

WRITE $(6,504)$ NERA
BTOP
15 IV (IP, EQ, I) AMINRA
20
CONITNUE

If (ABS(s).LT, CONVSFO 10
5 Coninus

aEturn
END

SUBRDUTINE REFLEC(DHM,ITH,CC, BETYPE,J)




5/1HML XI (1000), Yi (1000), TYP I (1000),001F(15)






C *** THIS BusROUTiNE REFLECTE a HELL about a gountary
504 FORHAY(IMI,'ERROR NO:IIZ
3J~JJ3:
MERAJ, LE 1000 SaIO
MERPE1O
HPIIE $(6,504)$ NERR
C. * SEPARATE CABL WhEN GOUNDATIES ARE PARALLLL TO COORDIMATE AXE DO 1 NAET, 1
IF(TTH.EO, (NAHAOD))00 10 10
1 continue
c e evaluation of codroinates of thage hell



00 10 25
$c^{10} 0010(15,20,15,20), \mathrm{NA}$
$c^{10}$ © ANO 180 DES, CASE

901025
C * 90 ano 270 Deg. case
$x!(J)=2+c(=x$
$y$
C ** Evaluilion of IYPE of juIge hell
IYPI(JJ) WBATYPATYPI(J)
NIHGBJJ
aE furn
EnO

## SUBROUTINE DRAHDO\{R,TI; 3


1/P13/XP(45), Yp(45)
1/GEDHPKINT(S, 2), KPARA (2,2), KSINQ, NSING, MIHT, NPARA
5/MML/XI(1000), YI(1000), TYPI(1000), 80 IF (1S)


-ANISAXXYY
Q/ANISAXX,YY, TXY, IXX, TYY, NLOG, ITHY,TINC, ITV, ITME (15), OVAR (15, 15)




504 Format(ini, 'erron no titis)
00 to $(1,2,3,3,6,6,7,8,9,3,11), \mathrm{KPRO}$
c * ateady staye, confined abuifer
$1 \quad$ OD*OO, $\quad 183 * O C O N S T(1 H) /\left(2, * P 1 * P_{E} R H * T H C K\right)$

60104
c ** beady stale . hater table aquifer
$2 \quad \mathrm{RF}=\mathrm{RAD}_{\mathrm{A}} \mathrm{I}\left(\mathrm{I}^{H}\right) / \mathrm{R}$

F(1HO*\#2-XX).GT.0.15010 5
HRIIE 6,504 )NERR
\$10p
5 Hasght HO**2, -xx

c ** Unsieady state , confineo hquifer



on yocomsy


c oo for expansion of exponemital iniegal $0010 \mathrm{kal}, 10$
oo 14 int.
14 FAciapActí
 EERERASERIE1 +00 TTER

ERIED.LT. CONveoto 20
10 CONTITUE
MRIIE $(6,504)$ NERA MRIIE
910 P

20 SAERRIE2*TYPI(JJ) Co 104
c. ** Unsteady state confined aduifer pparilally pene traling mells * total effect and effect of penetrailon alone





 $5=114.6 *$
60704
c** unsieady gtate , leaky confimed aguifer hithouy hater released

* proh storage in aguitiro
 atal
 60104

- RBAR/GQRY(PERH*THCKAMBAR/P8AR

CALL INTRPI(RB,1,10)

** Unstexoy state geaky confimed adutfer hith hater meleased fabm
c storage in aduitabo

(1)

RGER SUR (PERHATHCK*HAAR,PEAR)





## WHIMM CALL IN <br> 



unsteady shate, water table aduifea

CALL MTRP3(RG.S. 11$)$
THT*AFTHY/ALFA

 1F (11.LE.TH1)

 AATHOAS/THCK
 NERREI
WAITE
S. 50a) NERR
WAITE
sTop
4 geturn
END

SUBROUTINE INTRPI(UT,I,K)

 1/GEOHKKINITS,2);RPRA(2,2),KSING,NSINGNINT,NPARA







c.** This subroutine interpolales the value of well function
sot pormat(ihi,ierfor no l,iz)

00 5 ( $=2, N(k) \quad$ (K)
If fuT, EG, (If,k)) oo $101^{5}$
IF(UT, (T, I(1,K)) $60 ~ 1020$
5 CONTHUE
MNEWF
00 10
30
15 Wwanf (1,k)
00 to 30

MERA15

fiop
30 $\begin{gathered}\text { AETURN } \\ \text { END }\end{gathered}$
subroutine intrez (ut, RD, fi, j,k]
COHMOL/BOS/OM(4), C(4), BTYPE(4),TH(4), NBNOS

## /PYF/XP(45),YP(45) <br> 

3/HYOF

S/INOX/O, IF, IW,JJ,JJI,JJR,NIMGS,NERR,TIZ, TII,NIL,ISMAX,IIUNT
T/CONT/KPRO日, PI, SCONY (z), 1 (10); CONY, SUNIT, IVY,IM,IRY




G **" inis subroutine interroletes the value of hell function




 1001 CONTANUE

001002 (a2, H(K)





10
102
CONTHUE
MERGDIA
NRIIE (6, 5a4) HERR


$\$ 10^{\circ}$
6010111
Wh* $H(1,1, k)$
 1) 0010111
ion $N=H(1, j, k$

t) 10 111

106 MH2H(1, $1, K)$
107



11
100

WH:H(1,j, K)
 1! 0 ra 111
110
111 RETUAN
RETUR
END
subroutine intarstutal.ks
COMMON/BOS/DH(4), C(4), 日TYPE(4),TH(4), NBNDB /PT:/XP(45): PP(45)
THELC/XM(15), YW(15), WTYPE(19), OCONST(15), RAOI(15), NHELL 3 , WK GMYOR/PEAH, HMCK, CST, HO,SY, HBAR, SBAR, PBAR, ALFA,ALFYNT, YIMT






FORHAT(AHI, IERRUR HO ',12)

oo 5 Iol:N(K)

If 010.10
CONTINUS
5

1K)
ALFTMTME (I, K)


GOTO 10
MERR 10
Whill (6,50a) HERR

stop
so $\begin{gathered}\text { Relurn } \\ \text { eno }\end{gathered}$
suenouline output
COMMON/GDS/OM(4), C(4), BYYPE(4), TH(4), NBNO:

## /P18 PP(aS),YP(0y)

/GEDH/KINT(3,2),KPMRA(2, 2FIS), OCONST(1S), RAD1(15),




G/ANIS SXX,YY, IXY, TXX, TYY, NLOG, ITHV, INC, ITY, TIME (15); GYAR (15, 15)


DIMENATON THU(2), TYPM(2), TYPA(?)












 qEST:
FOPH
 201710ns





 6 - ronkaftix.


 $1 / 1$
CORMAT Fокнит $15,11,3 x, 5,2,3 x, 56,1,13 x, 55,2)$








 209




HRITE(6,10)(TITLE(J),J*1,19)
60 10 (101,102,103,109,105,105,107,108,109,110,111), KPROB
101 HR1TE(6,11)
6011053
WRTE 56,72$)$

- 60 10 53,72
- 001053

104 WR1TE $(6,74)$


lot white ( 6,71 )

log WAITE $(6,19)$
60.1053


C ** COHyERSION OF UNIT

TXX=1XX,
$T X Y=T X Y / 2: 2 \mathrm{Z}$
PERNEPERH 0.695
PEAR
H0 $\mathrm{HO} / 3.2 \mathrm{Bl}$

## CONv=CONV/3.201


THCK=THCK/3.Es
$X P(1 \mathrm{RK})=X P(\{\mathrm{RK}) / 3,281$
YP(IRK) $x P(1 A K) / 3,281$
$\begin{array}{ll}X H(1 R K) \\ Y & X H(1 R K) \\ Y\end{array}$
YH(IRK) $=Y H(1 R K) / 3,2$
$Y(I R K)=Y(1 R K)$
SEE(IAK)=SEE (IRK)/B.28
GADI(IRK)*RADI (IRK)/3.28
(ODS (1RK) 0 .
oconsi(1RK)wacanst(IRK)/0.103a5

QUAR(1C,10)= GVAR(16.10)/0.10343


YPN(ic:IO)EYPN(ic, iD)/3.20


C 57 ** Mrite out he avifer characteristics
57 IF (JUNII,EO, I.AND.KPROB.EQ:9) KRITE $(6,66)$ TXX, TYY, IXY


If (JUMIT, NE, 1 ) WRIEE(o,13) PERM

IF (JUNII.NE, 1) MRITE $(6,14)$ Ho
IF (JUNI $, E a, 1)$ HRITE $(6,203)$ Ho
$1 F$ Jumil
GO TO
If (JUNIT. NE, I) HAITE (6,15) THCK
IF (JUNITEG: 1 ) $H$ HIEE 6,204 ) IHCK




IF (manos.to.0jeo 10 a
c ** YRIPE OUT BOUSDARY DATA
IF $(J U N I T, H E, 1)$ MRITE( 6,17$)$
IF $(J U N I T, E Q, 1)$

J8:1


35

00103
 CONIDNE WRIE $(6,20)$ nsing,mint, HPAAA


OO 4: IMES, HWELLS
IF(NIYPE(IH),EQ,-t.)KWMx
 iH)
MRIE(6,22)
IF (JUNIT,NE, 1) WRIIE (6,06) TIMU(ITUNT +1 )
IF (IIHV,EO, O) so 10 al
21 DO 47 ITVE2,1 TMy +
al Do as iHEI, NNELL
c** priniout * hore than one time interyal or more ihan one point of

e * atngle point of inteaget case - lime interyal valics WRIE(6, 24)×P(1),YP(1) IF(JUMII, NE:1) MRIE(6,25) IIMU(IIUNT+1)

co to 6
\& ** single time interval cast - point of inieresp varit


ao if (JUMITEEO;1) Mrite (b,207)
WRIE (6, $\frac{1}{2}$ )

1 CONTEStute

- If(AFARA. NE . 0)co 1031

31 If TJUNIT.NE, 1) WRITE (6,11) CONV
C an orandomn cohponents option
If (ITABLE,NE, 1) 00 10 30

Do 3915 \#1. 15 max


 11s)
50 retuan
EHD

SUBROUTINE GARPH(HEADNO, XX,YY, M, VARY, XHEN, XMAX,YHIN,YHAX, HPAGES,
 REAL MGY/I*I,

WRIYE(b,9) HEADNG
OYMXMN-YMAX YMIN
Yaxis(I):YMIN
00 10 $(12,14)$, MPAGES
$\times \$ P A C E=5510$
XSPACEE50.
HSPACEESI
601016
XSPACE $=100.0$
HSPACEE101


IF (OYMXMN-1000,0)90,100,100
IF YYAX $-1000,0550,100,100$


If (AES(yHAX)-(1.0E-02)) 100:90:110
IF (YHAx) $100,110,100$
NRITE( 6,1$)($ Yaxis $(k), k=2,11)$

DO $130 \mathrm{j}=1.112$
LNE JYロ
WRITE $(6,3)$ LINE
KF $(x+1 \neq) 140,170,170$
IF $(x+1 \times x)$
$170,170,150$
Do $160 \mathrm{~J}=10.112$
1 IME (J) 28 BL
6010200
$D 0$ 180 Jx10,110
INE (J)=Dor

$c^{190}$
200
LINE(11!):blank

$x_{A} \times 18(1)=x H_{1}$

KKR1
XINVL $\leq 0 \times \mathrm{HX}$
VARXPXHIN
230
230



11Nを(JY+1)rdot

```
soo Lime(10)=Dar
    JY*9
    270 IF(L-1)280,130,200
    280 IF(L-1i) 290,5002900
    290 IF(L-21) 300,340,300
    10 IF(t-41) 320,340,320
    \20 17(L-51) 321,340,121
    221 00 10 (430,322),4P40E|
    23 IF(L-61) 323,340,323
    323 IF(L-11) 324,340,324
    25 if(L-91) 326,340,325
    36 IF(C-101) 430,340,430
    CINE(JY+1)
    30 LINE(JY+1)=PLU*
    KK"G
    350 17(XHAX=1000,0) 360,910,410
    360 IF (XHIN+100,0) a10,910,370
    310 if (ABS (XMINJ-(1,0E-02)) 380, 380,300
    380 IF(XHIN) 40;190,410
    380 IT(XHIN) 90, 190,410
    410 WRITE(B,a) XAXIA(Kk)
    GD 10430 xaxiatkk
    420 Wa!!E(0,5) xax18(kK)
910 IF(CVARx+xIHYL/Z,0)-ABS(YARN) 480,900,040
c APPENDIX N
    MOMHMMOUNT+1
    90 Do 480 j=10,110
    LINE(J)=DOT
    10 LINE(J):P!ug
        LINE(IIJ):ULANK
    LNE(:1:2)= %
4 8 0
    MHAX=O
    TRY#Kx(1)-VARX
    \MRYTRY;(XINYL/2,0)
490 IT(TIRY4XINVG 500,510,510
510 K=(YY(I)=YMIN):100%010YMXKN+9.5
\12 1F(K-111) 512,525;525
512
520
LIFE(x+1)*0
20
GO FO 530
525
CaNTHNU(L)-YAIN).100,010YHXHN+9.5
\ IF(J-111) 540,500,580
550 if(linE(J:I).0) 570,560,510
    LINE(J+1)=0
```

570 LINE (J+1)=AS
so $J=0$

jYisjY+1
1F(LTME (112)-n) $000,120,600$
$600 \mathrm{IF}(J Y-J)$
610 IF $(J Y=K)$
$660,810,610$



$t^{650}$
$60 \begin{aligned} & 6010750 \\ & \text { If (LLOKK) }\end{aligned}$
660


$0.00 \quad 10750$

white (6,6)(Line (JJ),JJa10,112)

10 K月I位 (6,6)(LIM2 (JJ), JJ=10,112)

150 DO $160 J=10,112$
LINE (J) $\operatorname{sBLANK}$
VARXVYARXXINYL
170 conilnue

i00 if (DYMXMH-1000,03 $800,870,810$
000 IF $(Y M A X=1000,0810,970,670$
010 if $(Y H 1 N+100,0)$





aso 0010900
0 WRIPE(B, B) XHIN, XHAX,YAIN, YHAX


3 Format (1H, 9x, iosal
formay (i, fipeg.z)
FORMAT(1+t $+9 \times 9$ ?



$900 \underset{\substack{\text { RETUNO } \\ \text { ENO }}}{ }$

VARIABLE DICTIONARY

| Variable | Definition |
| :---: | :---: |
| AITHD | AITHD $=$ FLOAT (ITHD) |
| ALFA | ALFA $=1 /$ (delay index) |
| ALFTWT | The interpolated well function |
| B | $\mathrm{B}=\sqrt{\mathrm{PERM} . \mathrm{THCR} /(7.48 \mathrm{ALFA} \cdot \mathrm{SY})}$ |
| BBTYPE | $\mathrm{BBTYPE}=\mathrm{BTYPE}(\mathrm{I})$ |
| BTYPE (I) | The type of boundary |
| $C$ (I) | The boundary intercept on the $X$-axis (or $Y$-axis if the boundary is parallel to the X -axis) |
| CC | $\mathrm{CC}=\mathrm{C}(\mathrm{I})$ |
| CONV | The convergence limit |
| CST | The coefficient of storage |
| $\mathrm{CW}(\mathrm{I})$ | Well loss constant |
| DD | $\mathrm{DD}=40.783$ QCONST (I) $/ 2 \pi$ PERM.THCK) |
|  | $\mathrm{DD}=114.6$ QCONST $(\mathrm{I}) /(\mathrm{PERM} . \mathrm{THCR})$ |
|  | DD - 114.6 QCONST (I) //TXX.TYY-TXY ${ }^{2}$ |
| DK | $\mathrm{DK}=\mathrm{YI}(\mathrm{I})+\mathrm{XI}(\mathrm{I}) / \mathrm{DMM}$ |
| DM (I) | The tangent of the angle between two intersecting boundaries |
| DMM | $\mathrm{DMM}=\mathrm{DM}(\mathrm{I})$ |
| DRAWDO ( $A, B, C)$ | The subroutine which computes the drawdown |
| FACT | Factorial term |
| GAMA (J) | The percent of penetration of well in the aquifer |
| GRADF | $\mathrm{GRADF}=\mathrm{DMM}^{2}+1$ |
| $\begin{aligned} & \operatorname{GRAPH}(A, B, C \\ & I, D, E, F, G, H, J) \end{aligned}$ | The subroutine which draws the graph |
| H | $\mathrm{H}=\sqrt{\mathrm{HO}^{2}-\mathrm{XX}}$ |


| Variable | Definition |
| :---: | :---: |
| H0 | The depth to water table |
| IDELT | The time increment within a cycle |
| INDIV | The printout option of individual image well coordinates and contribution to total drawdown |
| INTERS (I) | The subroutine which controls the reflection process in the case of intersecting boundaries |
| $\operatorname{INTRP1}(\mathrm{A}, \mathrm{I}, \mathrm{J})$ | The subroutine which interpolates the value of well function in one dimensional array tables |
| $\operatorname{INTRP} 2(A, B, I, J, K)$ | The subroutine which interpolates the value of well function in two dimensional array tables |
| $\operatorname{INTRP} 3(A, I, J)$ | The subroutine which interpolates the time for a given value of $R / B$ |
| IPLOT | Graphical output option |
| ITABLE | Drawdown components table option |
| ITHD | ITHD $=$ IFIX (THD) |
| ITMV | The number of stages in variable pumping rate case |
| ITUNT | The index to the time unit used |
| JUNIT | The option for the system of units |
| $\operatorname{KINT}(\mathrm{I}, \mathrm{J})$ | The number of intersections |
| $\operatorname{KPARA}(\mathrm{I}, \mathrm{J})$ | The number of parallels |
| KPROB | The problem type |
| $\operatorname{KSING}(I, J)$ | The number of singles |
| M (I) | The number of rows in the well function table |
| MBAR | The thickness of aquitard |
| N (I) | The number of columns in the well function table |
| NBNDS | The number of boundaries |
| NCASES | The number of cases to be analyzed |
| NERR | The error number |


| Variable | Definition |
| :---: | :---: |
| NIMGS | The number of image wells |
| NINT | The number of intersections |
| NLOG | The number of time increment cycles for which drawdown is to be evaluated |
| NN | $\mathrm{NN}=1$ or 2 |
| NPARA | The number of parallels |
| NPNTS | The number of points of interest |
| NR | The number of reflections required in the case of intersecting boundaries |
| NS ING | The number of singles |
| NTI | The option for the number of time increments |
| NWELLS | The number of the real wells |
| OUTPUT | The subroutine which produces tabular output of problem details |
| PARALL (I, J) | The subroutine which controls the reflections process in the case of parallel boundaries |
| PBAR | The permeability of aquitard |
| PERM | The aquifer permeability |
| QCONST (I) | The constant well pumping rate |
| QDIF (I) | The difference between two sequent variable pumping rates |
| $\operatorname{QUAR}(I, J)$ | The variable well pumping rate |
| R | The distance between the observation point and the well |
| $\mathrm{RADI}(\mathrm{I})$ | The radius of influence of the real well |
| RB | The vertical coordinate which interpolates the well function |
| $\begin{aligned} & \operatorname{REFLECT}(A, B, \\ & C, D, I) \end{aligned}$ | The subroutine which reflects a well about a boundary |
| RM | $\mathrm{RM}=\mathrm{R} / 3.281$ |


| Variable | Definition |
| :---: | :---: |
| RMIN | The minimum allowable distance between the observation point and the well |
| RR | $R \mathrm{R}=\mathrm{RADI}(\mathrm{I}) / \mathrm{R}$ |
| S | The drawdown at the observation point |
| SBAR | The coefficient of storage of aquitard |
| $S E(I, J)$ | The well loss in variable pumping rate case |
| SEE (I) | The well loss in constant pumping rate case |
| SERIE1 | SERIE1 $=\operatorname{DD}(-0.5772-\operatorname{ALOG}(\mathrm{U}))$ |
| SERIE2 | SERIE2 $=$ SERIE1 + (DD.TERM $)$ |
| SI | The vertical coordinate which interpolates the well function in case number 9 |
| SM | $S M=S / 3.281$ |
| SMTOT | SMTOT $=\mathrm{STOT} / 3.281$ |
| SP | The drawdown due to partial penetration effect only |
| STOT | The total sum of drawdown at the observation point |
| SY | The specific yield |
| $\mathrm{T}(\mathrm{I}, \mathrm{J})$ | The value of the horizontal coordinate of the well function table |
| TA | The angle the boundary makes with the X-axis in radius |
| TH(I) | The angle the boundary makes with the X-axis in degrees |
| THCK | The aquifer thickness |
| THD | $\mathrm{THD}=90 \mathrm{NN} / \mathrm{THDIF}$ |
| THDIF | The angle between two boundaries |
| TI | The time interval after which drawdown is required |


| Variable | Definition |
| :---: | :---: |
| TI2 | $\mathrm{TI2}=\mathrm{TI}$ |
| TII | $T I I=T I$ |
| TIME (I) | The time since variable pumping rate starts |
| TIMT1 | TIMT1 $=0.27 \mathrm{MBAR} . \mathrm{SBAR} / \mathrm{PBAR}$ |
| TIMT2 | $\mathrm{TIMT} 2=0.27 \mathrm{MBAR} . \mathrm{SBAR} /(74.8 \mathrm{PBAR})$ |
| TIMU (I) | The double precision for time in days or minutes |
| TINC | The time interval for each variable pumping stage |
| TITLE (I) | The title of the analysis |
| TTH | $T T H=T H(I)$ |
| TWT | TWT $=$ ALFTWT/ALFA |
| TXX | The XX component of the second rank symmetric tensor of transmissibility |
| TXY | The XY component of the second rank symmetric tensor of transmissibility |
| $\operatorname{TYPB}(\mathrm{I})$ | The double precision for boundary type |
| TYPI(I) | The type of the image well |
| TYPW (I) | The double precision for well type |
| TYY | The YY component of the second rank symmetric tensor of transmissibility |
| U | $U=1.87 \mathrm{R}^{2} .(S T /$ (PERM.THICK.TI) $\quad$ in case no. 3 |
|  | $U=1.97 \operatorname{CST} \frac{T X X . Y Y^{2}+T X Y . X X^{2}-2 T X Y . X X . Y Y}{T I\left(T X X . T Y Y-T X Y{ }^{2}\right) \text { in case no. } 4}$ |
| UA | $\mathrm{UA}=0.27 \mathrm{R}^{2} \cdot \operatorname{CST}(7.48 / \mathrm{PERM}$. THCK.TI) |
| UDBAR | UDBAR $=1.87 \mathrm{R}^{2}(1+\mathrm{SBAR}) /(3 \mathrm{PERM} . \mathrm{THCK} . T \mathrm{~T})$ |
| UT | $\mathrm{UT}=1.87 \mathrm{R}^{2}$. CST/(PERM. THCK.TI) |
| UY | $\mathrm{UY}=0.25 \mathrm{R}^{2} \mathrm{SY}(7.48 / \mathrm{PERM}$. THCK.TI) |
| $V(I, J)$ | The value of the vertical coordinate of the well function table |


| Variable | Definition |
| :---: | :---: |
| $\bar{W}(\mathrm{I}, \mathrm{J}, \mathrm{K})$ | The value of the well function in the two dimensional array table |
| W1 | The horizontal interpolation of the well function |
| W2 | The vertical interpolation of the well function |
| WF(I, J) | The value of the well function in the one dimensional array table |
| WN(I) | Exponent due to turbulent flow |
| WTYPE(I) | The type of well |
| WW | The interpolated well function |
| X(I) | The time since pumping started or the distance from the pumped well to the observation point |
| XI(I) | The X -coordinate of the image well |
| XM (I) | $\mathrm{XM}(\mathrm{I})=\mathrm{XI}(\mathrm{I}) 3.281$ |
| XMAX | The maximum distance or time used |
| XMIN | The minimum distance or time used |
| XP (I) | The X -coordinate of the observation point |
| XW(I) | The X -coordinate of the real well |
| XX | $\mathrm{XX}=\mathrm{ABS}(\mathrm{XP}(\mathrm{I})-\mathrm{XW}(\mathrm{I}))$ |
| Y (I) | The accumulated drawdown |
| YI (I) | The Y -coordinate of the image well |
| YM(I) | $\mathrm{YM}(\mathrm{I})=\mathrm{YI}(\mathrm{I}) / 3.281$ |
| YMaX | The maximum accumulated drawdown |
| YMIX | The minimum accumulated drawdown |
| YP(I) | The Y-coordinate of the observation point |
| YPI (I, J) | The total drawdown due to the partial penetration effection in the real well only |
| YP2 (I, J) | The total drawdown due to the effect of partial pentration at the image wells |


| Variable | Definition |
| :---: | :---: |
| $\overline{\mathrm{YPB}}$ (I, J) | YPB (I, J) $=$ YP2 (I, J |
| $\operatorname{YPN}(\mathrm{I}, \mathrm{J})$ | The partial penetration effect due to the real well and its associated image wells |
| YSP(I) | The accumulated drawdown due to the partial penetration effect |
| YTOT (I, J) | $\operatorname{YTOT}(\mathrm{I}, \mathrm{J})=\operatorname{YTOT3}(\mathrm{I}, \mathrm{J})$ |
| YTOT3(I, J) | The total drawdown due to the real well and its associated image well including the partial penetration effect |
| YW(I) | The $Y$-coordinate of the real well |
| YW1(I,J) | The drawdown due to the real well excluding the partial penetration effect |
| YW2 (I, J) | The drawdown due to the image wells excluding the partial penetration effect |
| YWB ( $\mathrm{I}, \mathrm{J}$ ) | YWB (I,J) - YW2 (I, J) |
| YWO (I, J) | $\mathrm{YWO}(\mathrm{I}, \mathrm{J})=\mathrm{YW} 1(\mathrm{I}, \mathrm{J})$ |
| YY | $Y Y=A B S(Y P(I)-Y W(I))$ |
| Z (I) | $Z(I)=$ YMIN - 5 |

## APPENDIX B

Tables and Notes

1. Table B-1. Description of the input data and their formats.
2. Table $B-2$. Data cards used in the verified examples.
3. Table B-3. Interpretation of error codes.
4. Notes on the use of input data.

TableB-1. Description of the input data and their formats.

| Card <br> No. | Identifier | Definition | Col. <br> No. |
| :---: | :---: | :---: | :---: |
| 1 | Basic Data (Free Format) |  |  |
|  | NCASES | The number of cases to be analyzed | 1 |
|  | JUNLT | Unit option (if JUNLT $=1$, S.I. system of units must be used, if JUNIT $\neq 1$, F.P.S. system of units must be used) | 3 |
|  | ITABLE | Drawdown components table option (if ITABLE $=1$, table given) | 5 |
| 2 | Heading (Format 509) |  |  |
|  | TITLE | The title of analysis | 1-80 |
| 3 | Control Card (Format 501) |  |  |
|  | NBNDS | The number of boundaries (NBNDS $\leq 4$ ) | 10-13 |
|  | NPNTS | The number of points of interest (NPDNTS $\leq 15$ | 21-25 |
|  | NWELLS | The number of real wells (NWELLS $\leq 15$ ) | 34-38 |
|  | KPROB | The problem type | 45-46 |
|  | CONV | The convergence limit of drawdown, meters or feet | 53-57 |
|  | INDIV | ```Printout option (if INDIV = 1, tabular output of individual image well coordinates and contribution to total drawdown given), meters or feet``` | 60 |
|  | IPLOT | Graphical output option (if IPLOT $=1$, graphical output given) | 62 |
| 4 | Boundary Data, one card for each boundary (FORMAT 502) |  | 7-13 |
|  | TH (IB) | The angle the boundary makes with the Xaxis in degrees measured counterclock wise |  |
|  | $C(I B)$ | The boundary intercept on the $X$-axis, or on the $Y$-axis if the boundary is parallel to the $X$-axis, meters or feet | 17-23 |
|  | BTYPE (IB) | The boundary type (BTYPE (IB) $=+1$. for barrier boundary, BTYPE(IB) $=-1$. for recharge boundary) | 27-29 |
| 5 | Point of Interest Data, one card for each point (Format 502) |  |  |
|  | $\mathrm{XP}(\mathrm{IP})$ | The X-coordinate of the point, meters or feet | 7-13 |
|  | YP (IP) | The Y-coordinate of the point, meters or feet | 17-23 |
| 6 | $\begin{aligned} & \text { Real } \\ & \mathrm{XW}(\mathrm{IW}) \end{aligned}$ | ell Data, one card for each well (Format 50 ) The $X$-coordinate of the real well, meters feet | 5-12 |

Table B-1. Continued.

| Card <br> No. | Identifier | Definition | Col. <br> No. |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} 6 \\ (\operatorname{con} t) \end{gathered}$ | YW (IW) | The Y-coordinate of the real well, meters feet | 16-23 |
|  | WTYPE (IW) | The type of well (WTYPE (IW) $=+1$, for discharging well, WTYPE(IW) =-1. for recharging well) | 27-29 |
|  | QCONST(IW) | The well pumping rate, not used in variable pumping case, $\mathrm{m}^{3} /$ day or gpm | 37-42 |
|  | RADI (IW) | The radius of influence of the well, meters or feet | 47-53 |
|  | GAMA (IW) | The percent of penetration of the well in the aquifer | 57-60 |
| 7a | The Aquifer Characteristics |  |  |
|  | For KPROB < 4 (Format 503) |  | -16 |
|  | THCK | The aquifer thickness, not used if $K P R O B=2$, meters or feet | 23-28 |
|  | CST | The coefficient of storage of the aquifer, not used if $K P R O B<3$ | 33-40 |
|  | HO | The depth to water table, used only for $K P R O B=2$, meters or feet | 45-50 |
| b | For KPROB $=4$ (Format 559) |  |  |
|  | TXX | The XX component of the second rank symmetric tensor of transmissibility, $\mathrm{m}^{2} /$ day or gpd/ft | 9-16 |
|  | TYY | The YY component of the second rank symmetric tensor of transmissibility, $\mathrm{m}^{2} /$ day or $\mathrm{gpd} / \mathrm{ft}$ | 22-29 |
|  | TXY | The XY component of the second rank symmetric tensor of transmissibility, $\mathrm{m}^{2} /$ day or gpd/ft | 35-42 |
|  | CST | The coefficient of storage of the aquifer | 48-55 |
| c | For $\mathrm{KPROB}=5$, or $\mathrm{KPROB}=6$ (Format 560) |  |  |
|  | PERM | The permeability of the aquifer, meter/day or gpd/ft ${ }^{2}$ | 9-16 |
|  | THCK | The aquifer thickness, meters or feet | 23-27 |
|  | CST | The coefficient of storage of the aquifer | 33-40 |
| d | For $\mathrm{KPROB}=7,8$, or 10 (Format 561) |  |  |
|  | PERM | The permeability of the aquifer, meter/day or gpd/ft ${ }^{2}$ | 7-14 |
|  | THCK | The aquifer thickness, meters or feet | 23-28 |
|  | CST | The coefficient of storage of the aquifer | 35-42 |
|  | MBAR | The aquitard thickness, meters or feet | 49-54 |
|  | PBAR | The permeability of the aquitard, meter/day or $g p d / f t^{2}$ | 60-67 |

Table B-1. Continued.


Table B-2. Data cards used in the verified examples

| Case Study Example | Card Numbers Used |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Example 1 No boundaries | X | X | X |  | X | X | X | X | X | X |
| Example 2 Single boundary | X | X | X | X | X | X | X | X | X |  |
| Example 3 Parallel paix of boundaries | X | X | X | X | X | X | X | X | X |  |
| Example 4 A $90^{\circ}$ intersection of two boundaries | X | X | X | X | X | X | X | X | X | X |
| Example 5 Three boundaries with two of them parallel | X | X | X | X | X | X | X | X | X |  |
| Example 6 Two perpendicular parallel pairs of boundaries | X | X | X | X | X | X | X | X | X |  |

Table B-3. Interpretation of error codes.

| ERROR |  |
| :---: | :---: |
| No. | MEANING |
| 1 | The limits off the following has been exceeded, NBNDS, NPNTS, and NWELLS. |
| 2 | In variable discharge, time unit must be used in days. |
| 3 | Only one point of interest is permitted when more than one time increment is specified (in variable or constant discharge cases). |
| 4 | A single boundary must be perpendicular to a pair of parallel boundaries. |
| 5 | Two nonparallel boundaries are not permitted with a pair of parallel boundaries. |
| 6 | The angle between two intersecting boundaries violates Ferris's limitation. |
| 7 | Two pairs of parallel lines should be mutually perpendicular. |
| 8 | Point of interest should not be coincident with the real well. |
| 9 | The time at which drawdown is to be evaluated is zero. |
| 10 | The program limit of 1000 image wells has been reached. |
| 11 | The undisturbed water depth specified for the equilibrium water table case is too small and thus results in complex drawdown values. |
| 12 | The convergence limit has not been met in computing the drawdown. This is a warning message. |
| 13 | The ratio of drawdown to the aquifer thickness in variable discharge case is more than 10 percent. This results in nonlinearity of flow motion equation which should not be applied in this case. |
| 14 | The interpolating values are not within the two-dimensional table limitation for well function. |
| 15 | The interpolating value is not within the one-dimensional table limitation for well function. |
| 16 | The interpolating value is not within the table limitation which defines the graph of ( $r / B$ ) vs. ( $\alpha$ twt). |

## Notes on the Use of Input Data

Frequently, errors might arise due to incorrect input data. The following notes help in performing successful runs.

There should be at least one of the following card numbers for each case study, $1,2,3,5,6,7,8$, and 9 . The number of cards 4, 5, and 6 are the same as the number of boundaries, points, and wells respectively. If there are no boundaries, card number 4 must not be used.

In some cases, part of the input data must be read and they are not going to be used like the radius of influence, percent of well penetration, constant discharging rates in variable pumping case, etc. In case of variable pumping rate, the pumping rates for all real wells at the first stage should be read as zeros, because the computation is held at the end of each time increment and not at the beginning. The second set of pumping rate values will stand for the first time increment and so on. The time unit in the variable pumping case should be used in days or fraction of days only.

More than one point of interest is not permitted in using more than one time increment in either constant or variable pumping rate.

If there are more than one case study, then the input data sequence from card numbers 2 to 10 is repeated except where indicated above.

## APPENDIX C <br> Computer Output for Examples in Chapter IV

1. No boundary, variable pumping rate.
2. Example on graph of drawdown vs. time.
3. Example on drawdown component table, variable pumping rate.
4. Example on parallel boundaries.
5. Example on graph of drawdown vs. distance.
6. Example on rectangular aquifer, injection well.
7. Example on well spacing design.


USFP RO42bn/Yollsi
7 gun thajan
f data datit
1.1.0
exampigi no boindarteg, vahiarle pumpthg rate








2,0,5,.3,1,4,1.
 1000., 1400..5500..50co..8000..

7 (no
*** fxamplfi no houndarieg, vartagle pumpting pate
*... trpe of analrsis ifully penetratina well in mateg table aouifer ---NON-EOUILTARIUM CONDITICNS
*** hourfar characteristics

| pfrufability | 500. $1 / 0$ |
| :---: | :---: |
| thickness of houifer | 10n. 19 |
| coffficient of slorage | . 360E-01 |
| spectific yield | .200E-02 |
| alfa | 50.0 (1/0ar) |

*** real hell data


** Pumping ratts ant mell losses
NIT. THIF Pumaing well losses

| 1 | 1.70 | 100力.0 | 0.00 |
| :---: | :---: | :---: | :---: |
| 1 | 2.00 | 2000.0 | 0.00 |
| 1 | 3.80 | 3000.0 | $0 \times 00$ |
| 1 | n.0n | 1000.0 | 0.00 |
| 2 | 1.00 | 40nn.0 | 0.01 |
| ? | 2.00 | $450 n .0$ | 0.01 |
| 2 | 3.10 | 5000.0 | 0.01 |
| $z$ | 4.00 | 5500.0 | 0.01 |
| 3 | 1,00 | 800.0 | 0.00 |
| 3 | 2.00 | 1000.0 | 0.00 |
| 3 | 3.09 | 150n." | 0.00 |
| 3 | a.n0 | 5000.0 | 0.01 |
| 4 | 1.00 | 500n.0 | 0.01 |
| " | ?.10 | 8000.0 | 0.02 |
| a | 3.10 | 700 n - | 0.02 |
| 4 | 4.0n | 8 80\%\%.0 | 0.03 |

** tamigation of diawdom at each time periog
Tamgarion of diardom at each time perioo


| IIME AFIEN | dramumin at |
| :---: | :---: |
| - InP Jut stants (atrs) | politi nf miterest |
| 1. | 6.80 |
| 2. | 1.00 |
| 3. | 1.26 |
| a. | 2.011 |

* Converafnef lithitz 0.05 ml . *
dif data rapes usut in this exabfle abf
? User biapgucymusta
3 pun najhan
? data dalh
1,0.0
exampla un tifaph of babamone vs. the





0.000005 .1 .75

2,1,5.1.1.1.1.1.
1 FHO



** Anumftrmbacifalstics


Ferbeatility
THICKMESA OF ADUFER
chrficifent of sponabe
thickerss iff anuitard
PEPAAntlity of aquitaro
100n. 600/F1S0
100. Fit
-10nt*00
150. Ft
100.grosari
geundartes

## -


INTEREEP (F1)

TYPE


- anagyts hf gemetry


```
    mimbar of sthgles = ,
    mignatr of intersectuns * "
```

    nembed of pahallfls s \#
    A. reaf well data



** Pumping rates ano wfll legseg

| $n \mathrm{n}$. | $\begin{gathered} \text { YME } \\ (0 \Delta Y 5) \end{gathered}$ | punating Ralf (GPM) | $\text { WFLL }^{\text {WEASFs }}$ |
| :---: | :---: | :---: | :---: |
| 1 | 98.00 | 50\%.n | 0.36 |

4* fabillation of doamonw at each time perino




```
F USER RGaphi/YOHSLA
7 Rum najwan
l bata bigm
1,1,"
    examfle: gagallel goundagies, s.l.systen of umris, thmy in minuts
```



```
    1Hzo0. c8,30.00 07m:%
    N=00. CB=70.n bTx+1
    ***50. VF=50.
```




```
0.00001.1.75
1,1,9479.0.1,0.1..
z (NO
```



## 7 RUM NAJFAN

1.1.n
 By feat well mo tano its asociateo image mell syatem with otacharge of omo. ©0 cm/0






```
** am|fin ratoaciffistirs
\begin{tabular}{|c|c|}
\hline reanemerity & 1000. H/0 \\
\hline mitcomss or a obiter & 1011. Mr \\
\hline cotfficient if shonage & . \(230 \mathrm{fa}-\mathrm{nz}\) \\
\hline minckess or ablitito & 49. Mi \\
\hline Prameamility of agutiahb & 1000.mp \\
\hline
\end{tabular}
** guuncaktes
    ---.--
```



```
- anatugis or gromptay
```


## ---......................

```
numgat of singles a a
nurese of therastions: \(=0\)
numer of rathlefts *
** Pat mille nata
```




```
1051: M0.aphs/winges
T RuM ma.a*ata
? mata ma!a
1,1,1
```




```
    TM=%. ce=0.0 gTall,
    TH=90. CB=0.4 B.12+1.
xPe>.00 vP=2.00
```






```
    Prpm= 5."E0a mek=307. fS1= a.0Emoz
0,00m01,1,56,0.00001,1,75,n,0060u5,1,6,0,00001,1,85
2,n,5,5,1,3,1..
```



```
1fm
```



** sourfrr characimatitics
Truefilillity
50n. No
mitcouss of antifte
387. 11
caffationi me storamat
. 0 Onf-n
** Roundarima

| N11. |  | $\begin{gathered} \text { MifRtept } \\ \text { (MI) } \end{gathered}$ |  | houndiay <br> 17pF |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $1{ }^{1}$ | $y=$ | 0.7 | OAFAIER |

* afalysis of gonhetry
-ave-
buhtife ar shacts: 0
memate ge inifpseftumo a
humbra of vatilitis.


| *** | $\begin{gathered} x-a \times 1 a \\ (H 1) \end{gathered}$ | $\begin{gathered} \text { r-ayis } \\ \text { (HII) } \end{gathered}$ | $\begin{aligned} & \text { no } 1,1 \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \text { panius of } \\ & \text { INFLITEMCF (M1) } \end{aligned}$ | $\begin{aligned} & \text { Pf wifze } \\ & \text { lick } \end{aligned}$ | mell loss CONSTANI | En |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5. | 3. | nischater | 8 2¢00. | 0.25 | .0n00! | 1.560 |
| z | 25. | P. | mischation | - 3 \%na. | 0.15 | .0nomi | 1.758 |
| - | 5. | 25. | olstrapgi | -1500. | 0.50 | - nonos | 1.600 |
| 4 | 25. | 5. | bistramep | F 35f\%. | ". 5 \% | -pront | 1.850 |



THE DATA CAGDS USEO in tuis EXAMPLE aRE

```
1 user amazga/yousra
| bun najuan
p drac daim
1.1,0
Example; three derpendiculap bounorries, ghafll df drawoowil vs. distance
    HBNOS: 3 NPNT:Sx 15 NHELLSS 1 KPROB=4 CONV=.05 0 |
    THmpo, C8=15. BJa+1.
    IH=50. CB=#15. BI*+1.
    MH=00. CB=0.000 Brz-1.
    XP=0. YP=10.
    XP:O. VP=20.
    xP=0. YP=30.
    XP=O. YP=no.
    XP=0, YP=50.
    XP=0. YP=60.
    XI=0. YPs>0.
    KP=0. YP=AO.
    XP*O. YP=go.
    *P=0, YP=100
    xOO. YP=200.
    xp=0. 
    XP=O, YP=A00.
    XP=0. YP=500.
    XP#0. YP=60n,
```



```
    TXXM.704E.05 TYYE.821E+05 TXY=.599E.03 C5TE.360E-02
0.0non06.1.75
1,0,5,.3,1,0,1.
| END
```





(maters


| tix | . 7004045 | 1930,0 |
| :---: | :---: | :---: |
| iyy | .6218+05 | +150,0 |
| Pr | . 5898403 | N19a/0 |
| Coffictant of stopaba | . 360 Ex - 2 |  |

*** puindapils

| H0. | $\begin{aligned} & \text { RGGLE With } \\ & x=A \times I S \text { (bigis } \end{aligned}$ | mitretay (HI) | goundary <br> TYFE |
| :---: | :---: | :---: | :---: |
| 1 | no. | $x=15.0$ | dafmiter |
| ? | 90. | $x=-15.0$ | gamiter |
| 3 | $n$. | $y=0.0$ | bfehafg |

** AnAcyis of feorfipy
-....-.-........................
humber af shifles * $\quad 1$
mampo of thtrasetions $x$
mane of eanatifls =
*** bat watl mata


lhf bata ralpa usfo in inis example abf

```
* usir onazba/yousr
1 num 114,1mam
gala dala
1.1.0
    EXAMPL:G FOUS DOUHOARIFS, INJFCIIOM NFLL
```




```
    TH=O. CE=n. Blu-1.
    IH= on. re=3Mn. Bl=-1.
    H1: 90, C8% 00. BY=-1.
    xP=200. YF=200,
```




```
0.00009.1.235.
1,0,30.,3,1,0.t.
? END
```


 hlime Discuapge of $500.00 \mathrm{Cm} / \mathrm{O}$

| no | 17\% | $\times$ coned My | y COORD <br> MI | badus | lime | ORAmROM | $\begin{gathered} \text { CUHULATIYE } \\ \text { DRAMDOKN } \\ \text { M1 } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | pfat | 100.0 | 100, 0 | 191.0 | equar | -17.091 | -0.091 |
| 2 | imaga | 100.0 | 50 ma | 316.2 | Eour | -0.066 | -0.151 |
| 3 | iHagf | 100.0 | -5no.0 | 107.1 | eourn | -0.040 | -0.191 |
| $\square$ | image | 100.0 | 1100.0 | 905.5 | \%our | -0.032 | - 0.230 |
| 5 | thagf | 1010.0 | -100.0 | 316.2 | cour | 00.066 | -0. 205 |
| 6 | thage | 100.0 | 100.0 | 909.9 | cous | -0.05: | -0.34h |
| 1 | imaga | 100.0 | - -700.0 | 0075.5 | EOur | -0.032 | -0.378 |
| , | thafe | 500.0 | 50n.0 | 424.3 | Equer | -0.056 | -0.435 |
| 9 | thase | -500.0 | 500.0. | 761.6 | EDUH | -0.034 | -0.473 |
| 10 | itagy | -100.0 | 30n.0 | 224.3 | Eouly | -1.056 | -0.529 |
| 11 | thaga | 90n.0 | 590.0 | 503.1 | Eourr | -7,046 | -0.575 |
| 12 | lmata | -100.0 | $50 n .08$ | 948.7 | equir | -0.031 | - 0.606 |
| 13 | image | 500.0 | -500.0 | 761.6 | cour | -n.038 | -9.840 |
| 19 | thagf | -10n.0 | -500.9 | 703.6 | Eaus | -n.03s | -0.682 |
| 15 | thage | 500.0 | 1100.0 | 948.7 | E0ur | *0.011 | -0.113 |
| 16 | 1 HagF | -10n.0 | 110n.0 | 948.7 | cour | -0.031 | -10.73 |
| 17 | 1maga | 500,0 | -100.0 | 424.3 | faur | -0,056 | -0,800 |
| 18 | imaga | -50n.0 | -100.0 | 961.6 | court | -0.038 | -0.036 |
| 19 | 14iga | -100.0 | -10n.0 | 424.3 | EOU: | -0.056 | -0.894 |
| 20 | 1Haga | 106.7 | -100.0 | 583.1 | foutr | -0.046 | -0.941 |
| 21 | tmater | -709.n | -190.0 | 948.1 | foul | -9.031 | - 0.971 |
| 23 | imaga | 500.0 | 10n.0 | 5 59.1 | EDUF | -7.006 | -1.018 |
| 23 | thaga | -50n.0 | 700.0 | $96 n .2$ | four | -0.034 | -1.052 |
| 34 | Imariz | -100.0 | 740.0 | 58.3 | nutir | -7.046 | -1.090 |


| 2 | Inafit | mo.n | ma.n | 707.1 | boutr | -n,nun | -1.138 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | thaga | -Pma." | 100.n | 1027.6 | fouar | -8.02\% | -1.158 |
| 21 | Imaci | 508.0 | -700.0 | 948.7 | Enum | -0.1131 | -1.197 |
| PA | thati | -1nn.n | -70n.0 | 948.7 | Ecuat | -0.031 | -1.228 |
| 29 | thria | 3nn." | 1an. 0 | 118.2 | nour | -14.086 | -1.290 |
| $3{ }^{1}$ | inasat | -509.0 | 10n.0 | 101.1 | fous | -10.040 | -1.338 |
| 31 | thasis | 1104." | 10n.1 | 0n5. 5 | 806) | -11.032 | -1.366 |
| 32 | magr | -1月n." | 197.0 | 316.2 | ruar | -0.040 | -1.432 |
| 15 | leaga | 170.n | 100.0 | 509.9 | fluty | -0.051 | -1.483 |
| 30 | tmagt | $\rightarrow$ P6".0 | $10 n .0$ | 905.5 | Eour | -0.032 | -1.515 |
| mata contatation |  |  |  |  |  |  | -1.515 |

```
** Amalysis of canmpint
    Amaty515 Of Gfnm&1RY
Numato br shugles a 0
        numegr of Interstctons:0
        Numgen of parallels * ?
*** 的AL MFIL mata
    *)
        .00009 1.235
```

        * puhping paifs ano wfll losses
    
$130.00 \quad 500.00 .01$
-4. tabulation af bramoonh at ach poitio of intepebt
FntNis of intifisi oramotm

| (Hi) |
| :--- |
| (nilafsi |

                orahoomm
    (mi)
no. $x=$ counds $y+$ cooros
$1200.0 \quad 200.0 \quad-1.51$


```
7 usim mumatarymuspa
% mum majhan
? 1%T: IM,M
1,0,0
    Exanfle: welt spafmar ofsign
```



```
    1H290. CG=15000. 㫙天+1.
    XP=1251.0 YP=998,?
```



```
MN=-1250.6 998.7 m!=*1. NC=107.1 AY=200#.0 GM=1.00
```






```
xH=0.0 1000.0 Wfat1, 06#107.1 A1=2006,0 GMz1.00
```




```
0.00001,1.75
1.0.5..3.1.0.1..
7 [:%
```


 －－．．．．．．．．．．．．．．．．No－－
＊a＊a duifen eharactieristics

| Perteagility | 4nin．griafisn |
| :---: | :---: |
| whirmess tif aquifer | 120． 51 |
| rotafigient or stomagr | ．584F－n1 |

＊＊＊monnatits

＊＊Lhatrois uf gamilay

NHMER UF StMEIES：$\quad 1$
minafr of intrasectons＝

＊＊UEAL will Bata

| Hit． | $\begin{gathered} x-A \times 13 \\ \text { (Fi) } \end{gathered}$ | $\begin{gathered} \text { r-A×1a } \\ \text { (F: } \end{gathered}$ | $\begin{aligned} & \text { Will } \\ & \text { ivof } \end{aligned}$ |  |  | $\begin{aligned} & \text { GRL loss } \\ & \text { consimhe } \end{aligned}$ | Exponent |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | t251． | 933. | 015chalg\％ | \％2000． | 1.00 | ．00001 | 1．750 |
| $z$ | －1251． | 894. | Hfschabet | $t$ zune． | 1.00 | ，Un001 | 1.750 |
| 3 | － 1560. | － 355. | HISctaplig | E 24n4． | 1.00 | ．00081 | 1.150 |
| 1 | 1540. | －35\％． |  | 5 zame。 | $1.0 \%$ | ．09001 | 1.150 |
| \％ | 695. | －19\％． | niscrampr | F za00． | 1.00 | ．1000 1 | 1.750 |
| \％ | －695． | －144t． | Discrabig | ＋ $210 \%$ ． | 1．00 | ．00001 | 1.150 |
| 7 | $\because$ | 1607. | biscrabise | （19\％， | 1.00 | ．90081 | 1.150 |

*. Tumpm; ants ath whll Lbisis

| 111. |  | рияррия <br> palf (atr) | $\begin{aligned} & \text { WFAL Ln5sFs } \\ & \text { (1) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 1 | 5.00 | 101.1 | 0.04 |
| a | 5,00 | 101.1 | 0.00 |
| 1 | 5.00 | 307.1 | 0.018 |
| 4 | 5.00 | 107.1 | $0 \cdot 00$ |
| 5 | 5.00 | 107.1 | 0.04 |
| 6 | 5.00 | 107.1 | 0.04 |
| 7 | 5.00 | 107.1 | 0.74 |

-     * tagmation fif brahbone at each moint of inifabit



