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APPLICATION OF
AN ELECTRONIC ANALOG COMPUTER
FOR THE SIMULATION OF HYDROLOGIC EVENTS
ON A SOUTHWEST WATERSHED

by

J. Paul Riley
Duane G. Chadwick
Eugene K. Israelsen

The work reported by this project progress report was supported primarily with funds provided by the United States Department of Agriculture, Agricultural Research Service, Soil and Water Conservation Research Division

Utah Water Research Laboratory
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Logan, Utah

February 1967

ABSTRACT

APPLICATION OF AN ELECTRONIC ANALOG COMPUTER FOR THE SIMULATION OF HYDROLOGIC EVENTS ON A SOUTHWEST WATERSHED

The hydrologic characteristics of watersheds in semiarid regions are dependent upon many variable and often interrelated factors. A quantitative knowledge of these factors and of their relative influence upon the system as a whole is needed in order to improve the efficiency of watershed management in these areas. In an attempt to develop a comprehensive simulation model of a semiarid watershed, research workers in the Agricultural Research Service considered the electronic analog computer, and a cooperative research agreement was subsequently signed with Utah State University. Analog modeling concepts are based upon the development of basic relationships which describe the various processes which occur within the surface hydrologic system of a semiarid watershed. Once established, the model is applicable to any particular geographic unit by determining the appropriate constants of the hydrologic equations. The analog computer is ideally suited to the many time-dependent differential equations which are encountered in hydrologic systems. To test individual equations and to verify the model, a subbasin of Walnut Gulch watershed in southern Arizona was simulated. In preliminary tests, close agreement was achieved between the observed and computed runoff hydrographs for a single storm. Some progress is also reported in the development of an analog technique to plot isohyetal lines corresponding to selected time intervals during the course of a storm.

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J. Paul Riley
Duane G. Chadwick
Eugene K. Israelsen

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PARTIAL LIST OF SYMBOLS[1]

Symbol	Definition
C_s	volume of water stored within a particular length of surface channel
E_r	evaporation rate
F_c	constant infiltration capacity rate under conditions of a saturated soil profile
E_{cr}	potential evaporation rate or evaporation capacity
F_r	infiltration rate
F_{cr}	infiltration capacity or maximum infiltration rate
F_o	soil infiltration capacity rate at the beginning of a storm
i_r	net inflow rate to surface channels within a watershed zone
P	precipitation rate
q_{rb}	rate of flow from the groundwater basin into an effluent channel within a watershed zone
q_{rc}	rate of channel runoff from a particular watershed zone
q_{rg}	rate of seepage loss from a surface channel within a watershed zone

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[1] Notes:
1) All parameters are functions of time
2) The subscript "r" denotes a rate of change with respect to time
3) The subscript "s" denotes a stored quantity
4) Values of all parameters are greater than or equal to zero
5) Symbols not included in this list are defined within the text of the report

Symbol	Definition
q_{rn}	rate of discharge from interflow storage within a watershed zone
Q_{rs}	rate of total surface discharge from a watershed, including channel base flow
q_{rs}	rate of discharge from surface detention storage within a watershed zone
R_{cr}	retention rate capacity or the maximum rate at which precipitation is able to enter retention storage
R_{cs}	retention storage capacity of the vegetation and land surface within a particular area
R_r	rate at which precipitation is entering retention storage
R_s	quantity of precipitation in retention storage
S_r	rate at which water is available for surface runoff within a zone (inflow to surface detention storage)
S_s	quantity of water stored as surface detention

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CHAPTER I

INTRODUCTION

The hydrologic characteristics of the semiarid watersheds of the southwestern part of the United States are dependent upon many variable factors. The high-intensity, short-duration convective storms that cause most of the runoff down the ephemeral channels of these watersheds rarely cover an area of more than a few square miles. The outflow hydrograph is, therefore, greatly affected by the storm pattern. Infiltration and channel transmission loss rates are generally high. Vegetative cover has a significant effect on runoff characteristics, and in areas where the vegetative density is low, the surface runoff frequently carries high sediment loads.

Because of the very limited precipitation in the Southwest , efficient management of available water supplies is becoming increasingly more important. For this reason, the Soil and Water Conservation Research Division of the Agricultrual Research Service, United States Department of Agriculture, recently established four experimental watersheds in southern Arizona and New Mexico. To provide basic data for an evaluation of the various factors which influence the hydrologic processes within these areas, dense instrumentation networks were established.

A hydrologic system is relatively easy to describe from a qualitative standpoint. However, the extension of this qualitative knowledge to obtain specific quantitative results is a difficult problem. The complex interrelation and variable nature of the many different processes occurring simultaneously within a hydrologic system make this so. In addition, compared to many other fields of science, few basic quantitative concepts exist as yet in the area of hydrology. Thus, there is need both to describe the various hydrologic processes in mathematical terms and to develop a practical method of combining these expressions into models which will facilitate a quick and easy examination of hydrologic parameters as they are affected by management and other changes within a prototype basin.

In an attempt to develop a comprehensive simulation model of a semiarid watershed, research workers in the Agricultural Research Service considered the electronic analog computer. In February 1966 an agreement was signed setting out the terms of a cooperative study between the Soil and Water Conservation Division of the Agricultural Research Service and the Utah Water Research Laboratory. Under this agreement the Utah Water Research Laboratory was charged with the responsibility of developing a general electronic analog model which would be applicable to complex semiarid watersheds, considering such factors as the small aerial extent of runoff-producing thunderstorms, channel transmission losses, and momentum waves in the channel systems.

This report outlines the research activities by personnel of the Utah Water Research Laboratory during the first year of this cooperative study. During this time there has been a considerable interchange of information with the staff of the Southwest Watershed Research Center, Soil and Water Conservation Research Division, Agricultural Research Service, at Tucson, Arizona. An initial mathematical model of the surface hydrologic system has been proposed and tested. Progress also has been made in the development of an analog device for automatically plotting isohyetal lines for a particular watershed.

Chapter II of this report deals with the basic concepts that are incorporated into the development of an electronic analog model of a hydrologic system. Chapter III contains the development of preliminary mathematical descriptions of the various processes which occur within the surface hydrologic system of a semiarid watershed. Verification of the model for a particular drainage area is briefly described in Chapter IV. Chapter V discusses the development and present status of the isohyetal plotting technique. Finally, Chapter VI briefly summarizes the results of the past year, and reviews plans for the continuation of the study.

CHAPTER II

ANALOG COMPUTER SIMULATION OF HYDROLOGIC SYSTEMS

Characteristics of the analog computer

Simulation is a technique for investigating the behavior or response of a dynamic system subject to particular constraints and input functions. This technique is usually performed by means of both physical and electronic models. Physical models and also those consisting of electrical resistor-capacitor networks have been used to investigate hydraulic and hydrologic phenomena for many years. However, simulation by means of high-speed electronic computers is a relatively new technique.

Considerable progress has been achieved in digital computer simulation of hydrologic phenomena (1, 4, 5). However, for applications of this nature the electronic analog computer has several important advantages. This type of computer solves problems by behaving electronically in a manner analogous with the problem solution, and is therefore a much faster computing machine than the digital computer. Moreover, the analog computer is a parallel device in that all computations proceed simultaneously. If the size of a problem is doubled, the amount of analog equipment required is also approximately doubled, but the time for solution remains the same. On the other hand, the digital computer, which is a sequential machine, takes twice as long when the problem size is doubled.

Many of the processes which occur in nature are time dependent and as such are differential in form. It is in the solution of differential equations that the great speed of the analog computer is particularly apparent because it can integrate the problem variables continuously instead of using numerical approximations. Frequently, design optimization problems or those involving stochastic variables require differential equations to be solved repeatedly, each with slightly different parameters or functions. Because of its tremendous speed, problems of this nature can be undertaken feasibly by the analog computer when all other methods would require unacceptable lengths of time.

Output on an analog computer is presented in graphical form as a continuous plot of the variable quantities involved. The operator can visualize results as being the actual dynamic responses of the physical system under investigation. Also, the results of possible alternative ways of combining the various components of the entire system can be quickly defined as an aid to determining the changes in specific processes that might be necessary to meet prototype conditions. Thus, the analog is very helpful during the exploratory phases of developing both component relationships and a composite model of a hydrologic system.

In recent years research in the development of electronic analog models of dynamic flow systems has been undertaken at several centers. In the area of flood runoff, Shen (15) discusses the applicability of analog models for analyzing flood flows. The Hydraulic Laboratory at the University of California has built an analog model for the purpose of routing floods in a particular river system (6). In addition, an analog computer program has been developed for simulating flood conditions on the Kitakami River of Japan (13).

Research in electronic analog simulation of hydrologic systems began at Utah State University in 1963 (3). Logically, emphasis was first placed on the formulation of basic and fundamental models of the physical system. Professors Bagley and Chadwick envisioned the simulation of the entire watershed and recommended the design and formulation of a pilot model. These recommendations were accepted, and the Agricultural Research Service, the Soil Conservation Service, and the Utah Water and Power Board provided funding to proceed with the construction of a test model. An electronic analog computing device was subsequently designed and built at Utah State University and completed in November 1964 (2). The success of the project encouraged further work and led to a study supported in part with funds provided by the Department of the Interior, Office of Water Resources Research (14). Under this project important improvements were made both in the mathematical relations for describing the various hydrologic processes and in the capability of the analog computing equipment. The computer developed under these two projects is shown by Fig. 2. 1.

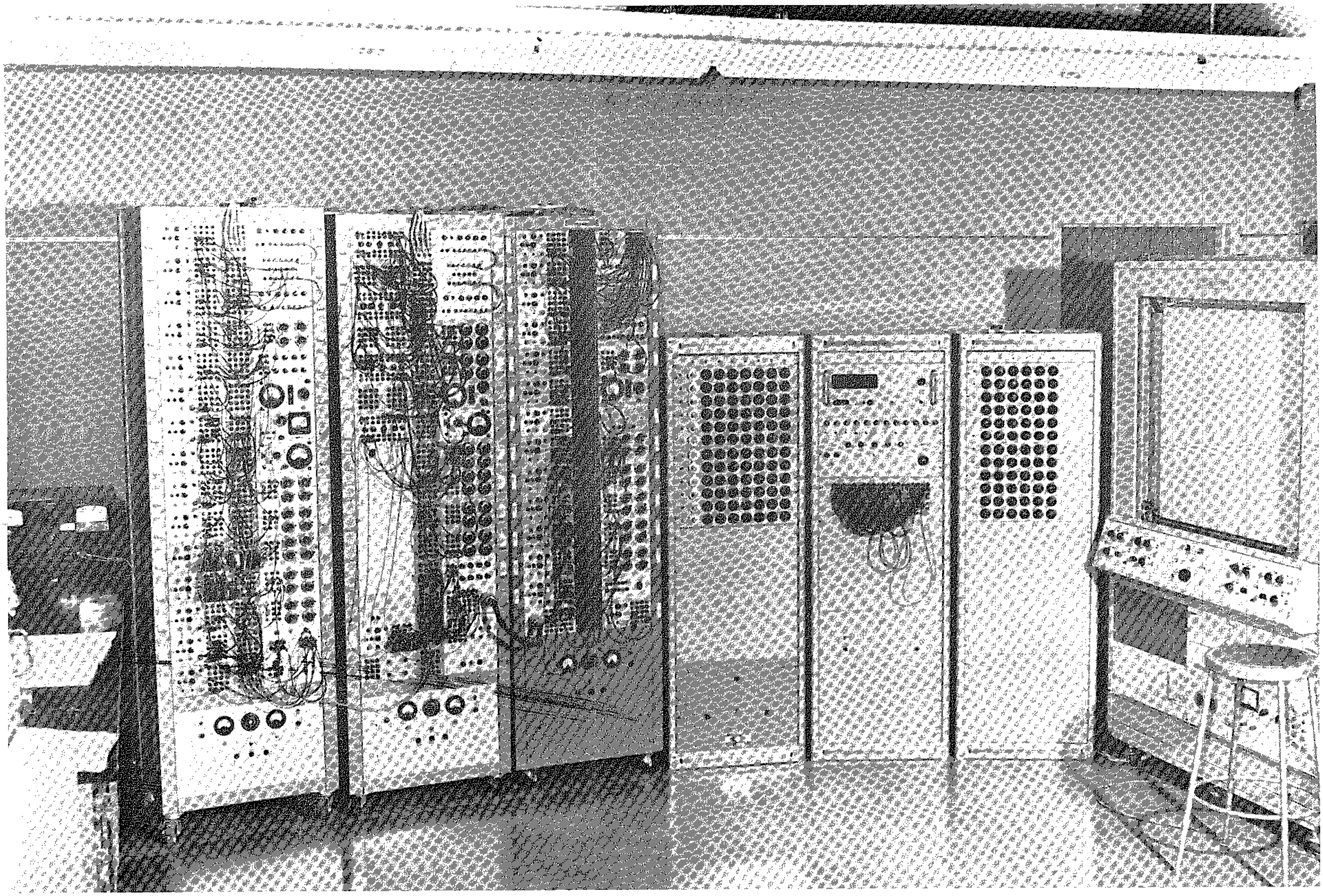

Fig. 2.1. Analog computing facility for hydrologic simulation at Utah State University.

The only available independent variable on an analog computer is time, and computations are performed continuously throughout the integration period. It is for this reason that differential equations with respect to time are very applicable to this type of computer. For example, in the case of precipitation, intensity or rate is given by the following differential expression:

$$P_r = \frac{dP}{dt} \qquad 2.1$$

From Eq. 2.1 the total precipitation occurring during a period of time, n, is given as follows:

$$P(n) = \int_{t_o}^{t_n} P_r \, dt \qquad 2.2$$

It is recognized that meteorological data are frequently recorded in digital format over finite time intervals. For example, precipitation information might be available as inches per day or per month, which is expressed in finite form as follows:

$$\bar{P}_r = \frac{\Delta P}{\Delta t} \qquad 2.3$$

in which $\bar{P}_r$ designates the average rate of precipitation during the time period Δt, and ΔP is the precipitation which occurred during this period. In many cases it is very convenient to input to the analog computer the data in its digital format, and this is possible if the period of integration coincides with the finite period over which the record was obtained. In this case equation 2.2 becomes:

$$P\ (n) = \bar{P}_r \int_0^1 dt = \bar{P}_r\ (1) \quad \ldots\ldots\ldots\ldots \quad 2.4$$

This function is, of course, continuous only within a particular finite period. A solution over a longer period of time is achieved, however, because conditions existing at the end of period n then become initial conditions for integration over period (n + 1), and so on. As a second alternative for the input of digital information, continuous functions of time can be developed from digital data by interpolation techniques. These functions are then input to the computer by means of electronic function generating devices.

Throughout this report the subscript r applied to any parameter is used to designate the variation of that parameter with respect to time, whether it be an instantaneous rate or an average value occurring over a finite time interval, such as a day or an hour.

Concepts of a hydrologic model

The fundamental requirements of a computer model of a dynamic hydrologic system are that:

1. It simulates on a continuous basis all important processes and relationships within the system that it represents.
2. It is nonunique with respect to space. This implies that it can be easily applied to different geographic areas with existing hydrologic data.

3. The computing equipment posses a high degree of capacity and capability.

Requirements one and two are approached by developing a preliminary model from an analysis of published information and established concepts. Through the operation of the model, quantitative relationships and hydrologic concepts are further defined and improved. At the same time, the third requirement is met through improvements in equipment design and modeling techniques. When the model is properly verified so that it accurately simulates a particular system, input and individual model parameters can be varied, and the effects of these changes can be observed at any point in the system.

A dynamic system consists of three basic components namely the medium or media acted upon, a set of constraints, and an energy supply or driving forces. In a hydrologic system water in any one of its three physical states is the medium of interest. The constraints are applied by the physical nature of the hydrologic basin, and the driving forces are supplied by both direct solar energy and gravity and capillary potential fields. The various functions and operations of the different parts of the system are interrelated by the concepts of continuity of mass and momentum. Unless relatively high velocities are encountered, such as in channel flow, the effects of momentum are negligible, so that for many hydrologic models continuity of mass is the only link between the various processes within the system.

Continuity of mass is expressed by the general equation:

Input = Output + Change in Storage 2.5

A hydrologic balance is the application of this equation in order to achieve an accounting of physical hydrologic measurements within a particular unit. Through this means and the application of appropriate translation or routing functions, it is possible to predict the movement of water within a system in terms of its occurrence in space and time.

The concept of the hydrologic balance is pictured by the block diagram of Fig. 2.2. The inputs to the system are precipitation and surface and groundwater inflow, while the output quantity is divided among surface outflow, groundwater outflow, and evapotranspiration. As water passes through this system, storage changes occur on the land surface, in the soil moisture zone, in the groundwater zone, and in the stream channels. These changes occur rapidly in surface locations and more slowly in the subsurface zones.

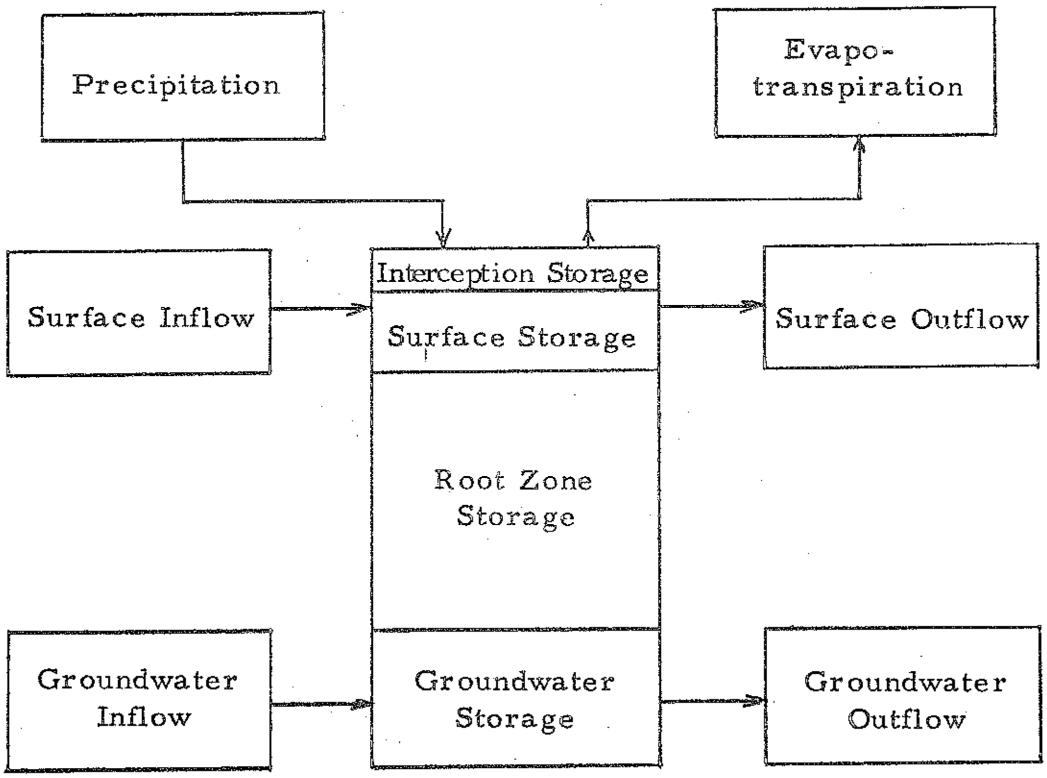

Fig. 2.2. A simplified diagram of the hydrologic balance.

CHAPTER III

THE HYDROLOGIC MODEL

The hydrologic flow chart which was followed in the development of a preliminary mathematical model for a semiarid watershed is shown by Fig. 3.1. It will be noted that this chart includes certain modifications resulting, in part, from the fact that precipitation rarely occurs in the form of snow on the rangeland watersheds of the Southwest. In addition, evaporation was assumed to be negligible during the period of a typical runoff-producing event. For the initial model discussed here only the surface hydrologic system was simulated, so that as water entered the soil surface it was removed from any further consideration in the model. This modification led to a further simplification. Precipitation which enters interception storage is lost through evaporation. The component of precipitation which enters depression storage eventually enters the soil profile, and in this model was also lost because only the surface system was considered. Thus, both interception and depression storage were combined and treated as a single entity termed retention storage.

Precipitation

Spatial integration of precipitation was performed by means of the Thiessen weighting technique as follows:

$$P_z = \frac{1}{A_z} \sum_{i=1}^{n} a_i P_i \quad \ldots \ldots \quad 3.1$$

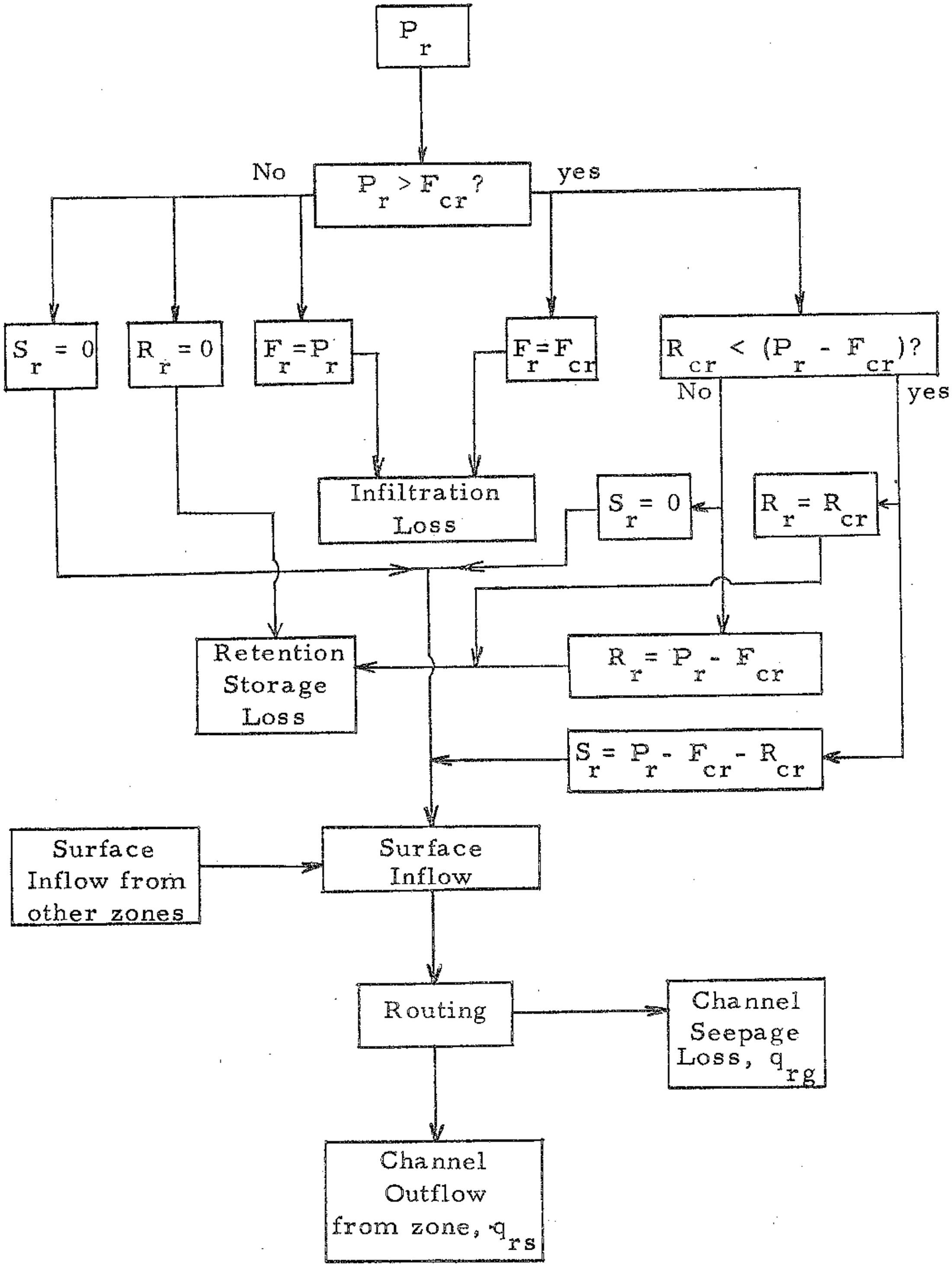

Fig. 3.1. Hydrologic chart for the surface flow system of a semiarid watershed.

in which

n = total number of Thiessen weighting areas included within the zone

P_z = mean precipitation in a zone during a particular time period

A_z = area of the zone

a_i = the Thiessen area within the zone over which precipitation occurs

P_i = the precipitation recorded at station i

Studies conducted by personnel of the Agricultural Research Service for the Walnut Gulch watershed indicated no significant difference between the results of the Thiessen and isohyetal methods for computing average precipitation over an area within the watershed for a particular period of time.

Infiltration

The first abstractive process considered by the model was infiltration. For a particular soil the capacity infiltration rate at any time is usually primarily a function of the soil moisture content. However, the soil moisture system was not included in this initial model, and the infiltration capacity equation as proposed by Horton (7) was therefore applied. This equation is entirely time dependent, and is of the form:

$$F_{cr} = F_c + (F_o - F_c) \exp(-kt) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.2$$

in which

k is an empirical constant depending upon the soil type,

t is the time from the beginning of the rainfall, and the remaining terms are defined at the beginning of the report. It is obvious that infiltration actually occurs at the capacity rate, F_{cr}, only when the precipitation rate equals or exceeds F_{cr}. Thus,

$$F_r = F_{cr} \quad , \quad P_r \geq F_{cr} \quad . . . . \quad 3.3$$

$$F_r = P_r \quad , \quad P_r < F_c \quad . . . . \quad 3.4$$

Retention

Retention was the next abstractive process applied in the model. Inflow to this form of storage was assumed to occur only when the precipitation rate exceeded the capacity infiltration rate of the soil. Losses from retention occur as evaporation and infiltration.

The capacity rate of inflow to retention storage decreases throughout the period of a storm. As the more shallow depression of the vegetative and ground surfaces within the precipitation area become filled, the rate of retention storage increase decreases, and consequently, for a particular supply rate, the rate of surface runoff increases. In general terms, the rate of retention storage increase becomes less with time and is a function of the quantity in storage. Thus, the maximum or limiting rate at which precipitation enters retention storage can be given by an expression of the form:

$$R_{cr} = k_r \left[R_{cs} - R_s(t) \right] \quad . . . . \quad 3.5$$

in which

k_r = a constant depending upon vegetation and soil surface characteristics

R_{cs} = retention storage capacity of the vegetation and land surface within a particular area

The value of $R_s(t)$ in Eq. 3.5 is given by the following expression:

$$R_s(t) = \int_0^t (R_r)\, dt \quad . . . . \quad 3.6$$

in which the actual retention rate, R_r, is given by the three limiting equations:

$$R_r = 0 \quad , \quad 0 \geq (P_r - F_{cr}) \quad . . . . \quad 3.7$$

$$R_r = (P_r - F_{cr}) \quad , \quad 0 < (P_r - F_{cr}) < R_{cr} \quad . . . . . \quad 3.8$$

$$R_r = R_{cr} \quad , \quad (P_r - F_{cr}) \geq R_{cr} \quad . . . . \quad 3.9$$

As shown by Fig. 3.1, surface runoff occurs when the rate of precipitation exceeds the sum of the capacity retention and infiltration rates at any particular time. Thus,

$$S_r = P_r - F_{cr} - R_{cr} \quad , \quad S_r \geq 0 \quad . . . . \quad 3.10$$

in which the quantity S_r represents the rate of inflow to surface detention storage.

Surface flow

Surface runoff from a watershed occurs first as overland flow, sometimes called the land phase (9), and secondly as channel flow. The time distribution of outflow from surface detention storage (land phase) within a watershed zone might be approximated by the following general expression:

$$q_{rs} = k_s\, S_s\, (t) \quad . . . . \quad 3.11$$

in which k_s is a constant characteristic of surface conditions and slope within the watershed zone, and surface detention storage is given by

$$S_s(t) = \int (S_r - q_{rs})\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.12$$

in which the value of S_r is given by Eq. 3.10. Using the differential form of Eq. 3.12 and substituting into it Eq. 3.11 yields the equation

$$q_{rs} + \frac{1}{k_s} \frac{d}{dt} (q_{rs}) = S_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.13$$

The solution of this differential equation provides an estimate of the required quantity, q_{rs}.

Several techniques for the routing of surface runoff in the channel phase are described in the literature. Many of these techniques are based upon the familiar continuity of mass principle in which the stream channel is considered as a series of reaches with the length of a given reach being established by the channel distance between the upstream and downstream boundaries of the particular watershed zone in which the reach is situated. Thus, for a particular reach, channel inflow and outflow are related by the following differential equation:

$$i_r\,dt - q_{rc}\,dt = d\,[\,C_s(t)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.14$$

Expressed as an indefinite integral, Eq. 3.14 appears as

$$C_s(t) = \int (i_r - q_{rc})\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.15$$

The form of Eq. 3.15 is identical to that of Eq. 3.12. The channel outflow, q_{rc}, is expressed as a function of the channel storage, thus:

$$q_{rc} = k\,C_s\,(t) \qquad 3.16$$

Laurenson (10) expressed Eq. 3.16 in the form

$$C_s\,(t) = k'\,q_{rc} \qquad 3.17$$

in which C_s is expressed in cubic feet, q_{rc} in cubic feet per second, and k' in seconds. In tests conducted on a watershed near Sidney, Australia, he found the value of k' in Eq. 3.17 to be a function of the channel discharge rate in accordance with the following relationship

$$k'\,(q_{rc}) = k_r q_{rc}^{-0.27} \qquad 3.18$$

in which q_{rc} is again expressed in cubic feet per second. Laurenson established the value of k_r by a particular application of Eq. 3.18 as follows:

$$t_m = k_r q_{rcm}^{-0.27} \qquad 3.19$$

in which

t_m = mean lag time in hours for the watershed zone

q_{rcm} = the mean discharge rate in cfs, including base flow, caused by a runoff producing event

Thus, if values of t_m and q_{rcm} are known, k_r is estimated from Eq. 3.19. For the particular watershed on which he tested his theories (South Creek), Laurenson found the mean value of k_r to be 64.

From Eq. 3.18, Eq. 3.17 can be written as follows:

$$C_s(t) = k_r q_{rc}^{0.73} \qquad 3.20$$

in which C_s is expressed in terms of cubic feet. Now, substituting

Eq. 3.20 into Eq. 3.14 yields the expression

$$q_{rc} + 0.73\, k_r q_{rc}^{-0.27} \frac{d(q_{rc})}{dt} = i_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .3.21$$

The solution of this differential equation provides an estimate of the channel discharge rate, q_{rc}. Initial values of q_{rc} are established from streamflow records. In the case of ephemeral streams, $q_{rc}(0) = 0$.

In the general case, the term i_r in Eq. 3.21 consists not only of surface runoff and interflow from within a particular zone, n, but also of channel flow from a higher zone, m. In addition, i_r is increased by groundwater inflow originating within the zone in the event of an effluent reach, and decreased by seepage losses in the case of an influent channel. Also, channel evaporation losses represent a loss from the zone and should be considered. A general expression i_r is, therefore, given by

$$i_r(n) = q_{rc}(m) + q_{rs}(n) + q_{rn}(n) + q_{rb}(n) - q_{rg}(n) - E_{cr}(n) \quad . \quad . \quad 3.22$$

In the preliminary model developed during the initial phase of this study some of the terms on the right side of Eq. 3.22 were neglected. Thus, the rate of entry of water to the channel by interflow, q_{rn}, the groundwater or base flow inflow rate, $q_{rb}(n)$, and evaporation losses, $E_{cr}(n)$, were each assumed equal to zero. $q_{rc}(m)$ represents the computed channel runoff from zone m which is situated immediately above zone n. $q_{rs}(n)$ is the rate of surface runoff from the land phase (detention storage) within zone n as given by Eq. 3.13.

With reference to the channel seepage losses, $q_{rg}(n)$, an average value was established for each watershed zone on the basis of the dimensions of the surface channel and the vertical permeability of the bed materials within the zone. From Darcy's equation the rate of channel seepage loss can be written as

$$q_{rg} = k\, i\, a \qquad 3.23$$

in which

k = the permeability of the bed material

i = the gradient

a = the area of surface over which seepage is occurring

If seepage is assumed to occur mainly in a vertical direction, Eq. 3.23 can be written in the form

$$q_{rg}(t) = k_g\, w(t) l \qquad 3.24$$

in which

k_g = the vertical permeability of the bed material

w = the average width of the surface of the water within the channel at any time, t

l = the length of channel within the zone under consideration

For a given volume of water storage per unit length of channel at a particular point, the value of w depends upon the cross-sectional area of flow, A_s, within the channel at this point. Thus,

$$w(t) = f[A_s(t)] \qquad 3.25$$

in which the function f is dependent upon the shape of the channel cross-section. Now, if the distribution of storage within a zone is assumed to be uniform, the average cross-sectional area of flow at any time, t, is given by

$$A_s(t) = \frac{C_s(t)}{l} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.26$$

Substituting Eq. 3.25 and 3.26 into Eq. 3.24 yields the general expression

$$q_{rg}(t) = k_g f \frac{C_s(t)}{l} \, l \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.27$$

The application of Eq. 3.27 is demonstrated by assuming a trapezoidal channel cross-section with an average side slope equal to m horizontal to one vertical and a base width, b. In this case the average cross-sectional area of flow is given by

$$A_s(t) = \frac{w^2 - b^2}{4m} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.28$$

from which

$$w(t) = [\, b^2 + 4mA_z(t)]^{1/2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.29$$

From Eq. 3.26, w now can be expressed in terms of channel storage as follows:

$$w(t) = [\, b^2 + \frac{4m}{l} \, C_s(t) \,]^{1/2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.30$$

Substituting Eq. 3.30 into Eq. 3.24 gives

$$q_{rg}(t) = k_g l \, [\, b^2 + \frac{4m}{l} \, C_s(t) \,]^{1/2} \quad . \; . \; . \; . \; . \; . \quad 3.31$$

Special cases of Eq. 3.31 occur for rectangular shaped channels ($m = 0$), and triangular channels ($b = 0$). It is also noted that this equation establishes a limiting minimum rate for channel seepage losses, namely:

$$q_{rg}(\text{min}) = k_g l b \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.32$$

Thus, in the absence of channel storage, C_s, if the gross inflow rate to a particular reach of channel does not exceed the rate established by Eq. 3.32, there is no outflow, q_{rc}, from the reach. It, therefore, follows that the actual total rate of channel loss is given by the sum of the seepage and evaporation terms contained on the right side of Eq. 3.22 only when the positive or gross inflow terms on the right side of this same equation jointly equal or exceed this sum. When this is not the case, the rate of channel loss is limited by the supply rate.

As an example of the application of the preceding channel routing analysis to a particular zone, consider the Walnut Gulch experimental watershed which is situated in southern Arizona. With reference to an estimate of k_r by Eq. 3.19, the mean lag time of many watersheds is closely approximated by the more easily obtained hydrograph rise time. In the case of the Walnut Gulch watershed the average rise time is given by the expression (12):

$$t_r = 25.3\, A_z^{-0.14} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 3.33$$

in which

t_r = rise time in minutes

A_z = area of the runoff surface or zone in square miles

For this particular watershed rise time is apparently independent of discharge rate so that the value of k' in Eq. 3.17 is also independent of q_{rc}, and in fact is equal to the rise time estimated by Eq. 3.33. Thus, Eq. 3.17 can be written in the form:

$$C_s(t) = 25.3\, A_z^{-0.14}\, q_{rc} \quad \ldots\ldots\ldots \tag{3.34}$$

in which C_s and q_{rc} are expressed in terms of cubic feet and cubic feet per minute respectively. Substituting Eq. 3.34 into Eq. 3.14 yields the result

$$q_{rc} + 25.3\, A_z^{-0.14} \frac{d}{dt}(q_{rc}) = i_r \quad \ldots\ldots\ldots \tag{3.35}$$

This equation is a particular form of the more general routing Eq. 3.21.

If for short duration storms channel evaporation losses are neglected and the inflow to channels of interflow, or quick seepage, and base flow is assumed to be negligible, Eq. 3.22 can be written as

$$i_r(n) = q_{rc}(m) + q_{rs}(n) - q_{rg}(n) \quad \ldots\ldots\ldots \tag{3.36}$$

If a trapezoidal channel cross-section is assumed, q_{rg} is given by Eqs. 3.31 and 3.34 as follows:

$$q_{rg} = k_g l \left(b^2 + \frac{4m}{l}\, 25.3\, A_z^{-0.14}\, q_{rc}\right)^{1/2} \quad \ldots\ldots\ldots \tag{3.37}$$

Eqs. 3.36 and 3.37 are now substituted into Eq. 3.35 to yield a channel routing expression for Walnut Gulch watershed as follows:

$$q_{rc}(n) + 25.3\, A_z(n)^{-0.14} \frac{d}{dt} [\, q_{rc}(n)] + k_g l [\, b^2 + \frac{4m}{l} 25.3\, A_z{}^{-0.14} q_{rc}]^{1/2}$$

$$= q_{rc}(m) + q_{rs}(n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 3.38$$

CHAPTER IV

TESTING AND VERIFICATION OF THE MODEL

Preliminary tests of the initial hydrologic model developed in Chapter IV were made by simulating a subbasin of the Walnut Gulch experimental watershed. This watershed contains approximately 37,200 acres and is situated at Tombstone in southeastern Arizona. It is an ephemeral tributary of the San Pedro River. Runoff characteristics of the area are being intensively studied by the Agricultural Research Service of the U. S. Department of Agriculture, and hydrologic instrumentation is therefore very complete (8, 12, 14). Fig. 4.1 shows the general outline of the basin and the approximate locations of the precipitation and runoff measuring stations. Annual precipitation on the watershed is approximately 14 inches, with between 50 percent and 70 percent of the precipitation and essentially all of the runoff, occurring during the June to September period as the result of intense, small-diameter, convective storms. Precipitation is measured with a network of 91 recording rain gages. Runoff is measured at the watershed outlet and from seven subwatersheds.

Elevations vary from 4,200 feet above mean sea level at the watershed outlet to over 6,000 feet at the upper portions of the drainage area. Stream channels in the watershed are typical of the semiarid Southwest. Gradients are steep (approximately one percent), and consequently flow velocities are high. Throughout most of the channel system several feet

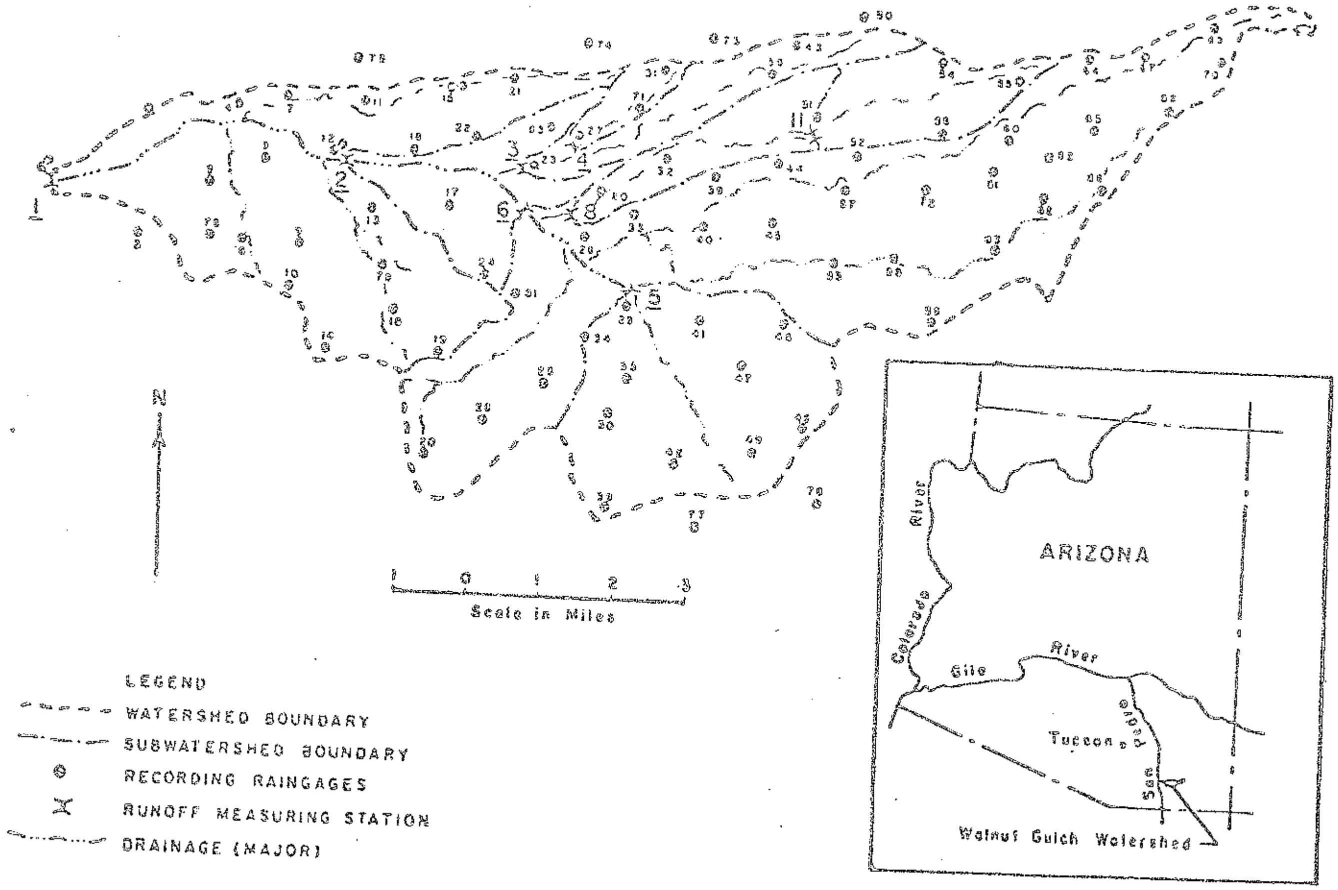

Fig. 4.1. Walnut Gulch experimental watershed.

of sand and gravel overlie a more consolidated material of fine-textured sediment or rock. The vegetative cover is the grama grass and creosote bush mixture typical of many of the rangelands of the Southwest. The regional groundwater table is situated at a depth of approximately 400 feet beneath the land surface.

The subunit of the Walnut Gulch watershed which was selected for initial simulation was subwatershed 11. Good precipitation and runoff data are available for four storms which occurred within this particular catchment area during July 1966. This report will present simulation results corresponding to the single event of July 20, 1966. An enlarged plan of subwatershed 11, which contains a total area of 2,035 acres, is shown by Fig. 4.2.

Fig. 4.3 is the analog flow diagram for the simulation model of subwatershed 11. The time scale adopted for the model was one second equal to five minutes of real time. Precipitation input values to the computer were therefore those quantities which occurred at each rain gage during succeeding five-minute intervals throughout the storm. Precipitation input data for the event of July 20, 1966, are shown by Table A 1. Although these data were received in digital form, the time increment varied considerably. A digital computer program was therefore developed to convert this data into the desired format of Table A 1. This program together with sample output information are included within the appendix of this report. It will be noted that the data have been output for three particular time intervals. Any desired interval within

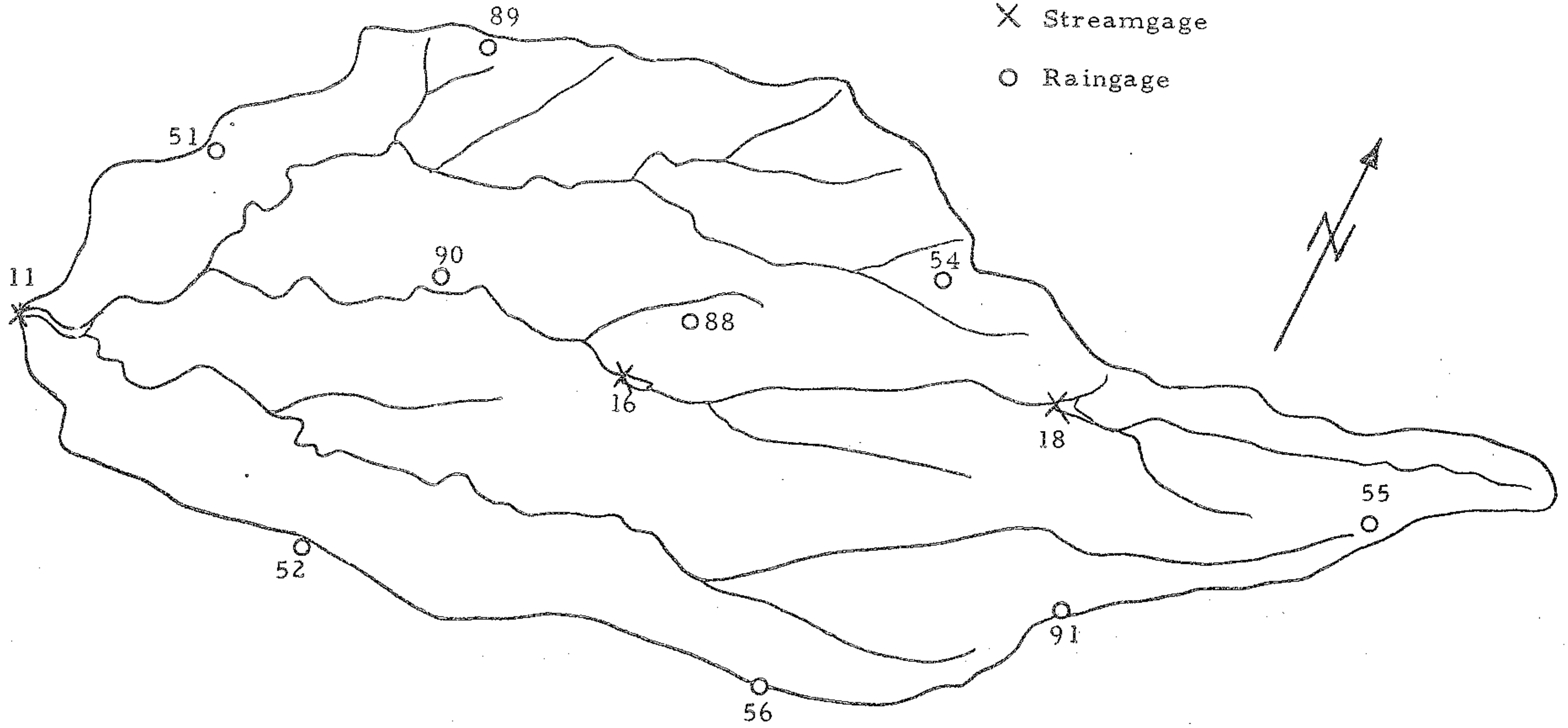

Fig. 4.2. Outline of subwatershed 11, Walnut Gulch, Arizona

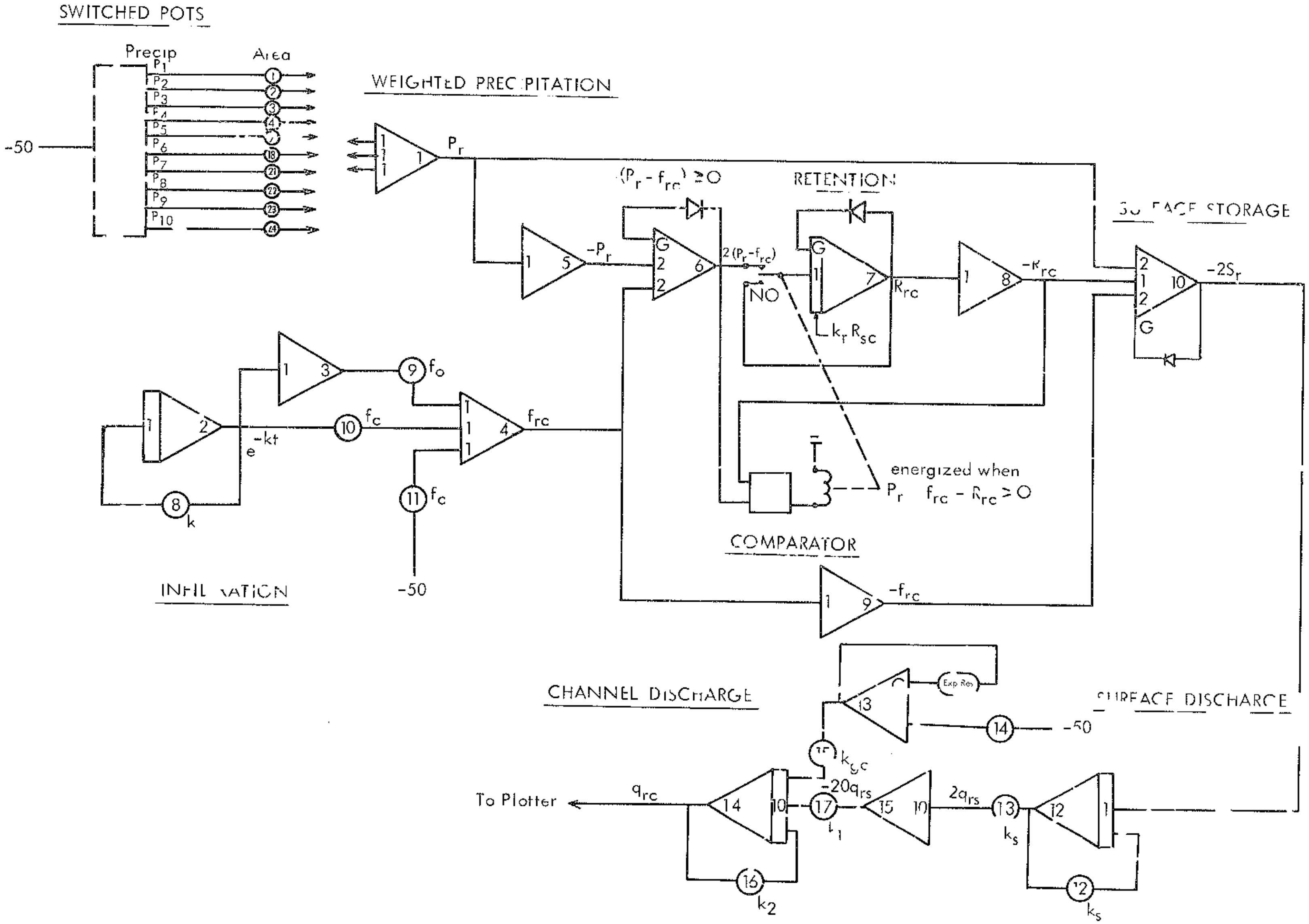

Fig. 4.3. Analog flow diagram for subwatershed 11, Walnut Gulch, Arizona

the range of the data may be specified in the digital program. Input values characteristic of the prototype area are included within Table A2. Most of the information in this table was obtained from the results of field studies carried out by personnel of the Agricultural Research Service.

Analog computer output functions plotted directly on the x-y variplotter are shown by Fig. 4.4. It will be noted that the vertical scale of the curve representing the rate of inflow to surface channels, q_{rs}, is ten times larger than that of the other accompanying curves. The area beneath this curve is, therefore, ten times greater than the cross-hatched area beneath the plot which represents the rate of input to surface detention storage. Except for the discharge hydrograph, all values are expressed in terms of inches over the area of the subwatershed per time interval. The outflow hydrogrpahs are expressed in terms of acre-feet per time interval. Reasonable agreement was achieved between the computed and observed hydrographs. The peak discharge rates of 0.146 acre-feet per five minutes (approximately 20 cfs) coincided exactly.

The time delay, in this case of about 50 minutes, between the outflow from detention storage and the appearance of an appreciable flow at the stream gaging station, is dependent largely upon the location of the storm on the subwatershed. This variable delay problem can be approached by subdiving the area into a number of small zones and

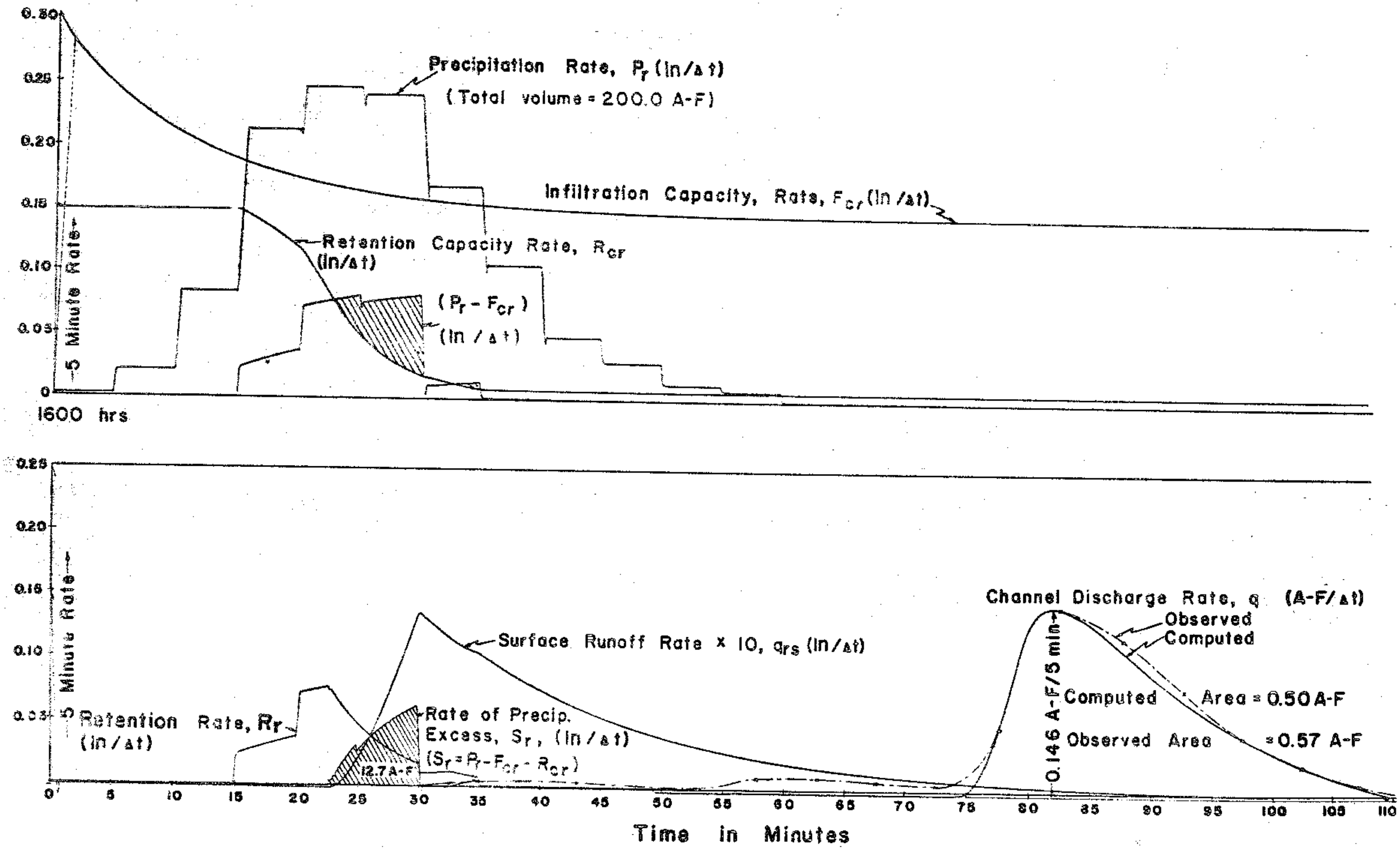

Figure 4.4. Analog computer plots for event of July 20, 1966, on subwatershed 11, Walnut Gulch, Arizona.

routing down the channels from one zone to the next. In subsequent simulation studies of subwatershed 11 it is proposed to model this area as eight individual zones.

Unfortunately, the only point at which quantitative verification of the model was possible was at the stream gaging station. However, channel transmission losses for portions of the Walnut Gulch watershed have been estimated as a function of average peak discharge (14). On the basis of this relationship the computed transmission loss of about 12 acre-feet for the storm of July 20, 1966, on subwatershed 11 is a reasonable value.

CHAPTER V

PLOTTING OF ISOHYETAL LINES USING AN ANALOG COMPUTER

As indicated in Chapter I, the scope of this study includes an investigation of computer techniques which might be applicable to hydrologic problems. Personnel of the Soil and Water Conservation Research Division of the Agricultural Research Service at Tuscon suggested that consideration be given to the possibility of using the analog computer for plotting isohyetal lines within a watershed area. Accordingly, during the phase of the project reported herein, an investigation of this problem was begun at the Utah Water Research Laboratory. It is noted here that staff members of the Agricultural Research Service indicated the isohyetal study as being of secondary importance to other areas of research in the Walnut Gulch simulation project. For this reason, and because considerable expenditures of both time and money are needed to develop and test the technique, approximately two-thirds of the cost of the isohyetal study to date has been provided from other funding sources.

The plotting technique being investigated is an adaption of the procedure proposed by Lym in 1965 (11). A diagram of the analog computer program being tested is shown by Fig. 5.1. Basically, this program solves the two LaPlacian equations

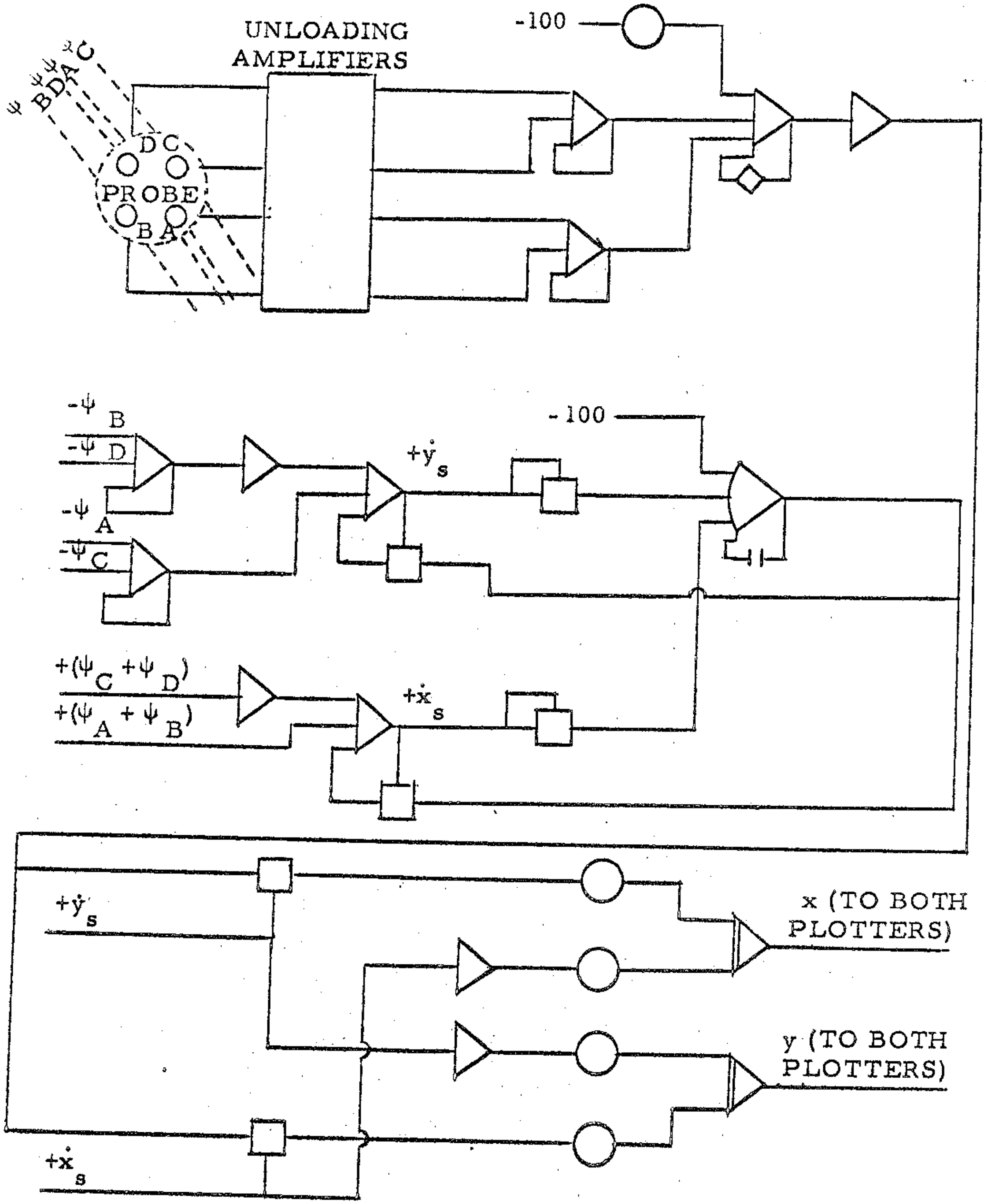

Fig. 5. . Circuit for plotting equipotential lines in a potential field.

$$\dot{x}_s = \frac{\partial \psi}{\partial y} \tag{5.1}$$

and

$$\dot{y}_s = \frac{\partial \psi}{\partial x} \tag{5.2}$$

in which

$\dot{x}_s$ = the x component of velocity along an isohyetal line

$\dot{y}_s$ = the y component of velocity along an isohyetal line

ψ = the potential of the isohyetal line

The plotting technique requires the use of electrical resistance paper in conjunction with a large 30 inch by 30 inch x-y plotter and a four-point probe. A close-up view of this probe is shown by Fig. 5.2. The probe is mounted on one of the arms of the plotter in place of the pen. In addition to this equipment, other components required by the technique are 23 operational amplifiers, six x-y multipliers, a limiter, about 100 potentiometers, and a second x-y plotter for plotting the isohyetal lines.

The boundaries of a particular watershed area are established on the resistance paper according to an appropriate scale factor. Provision is then made for applying voltages at points on the resistance paper corresponding to the locations of the raingages on the watershed. The voltages applied at these points can be varied by means of potentiometers and can be set at values which are proportional to the quantities of precipitation recorded at the raingages within the area.

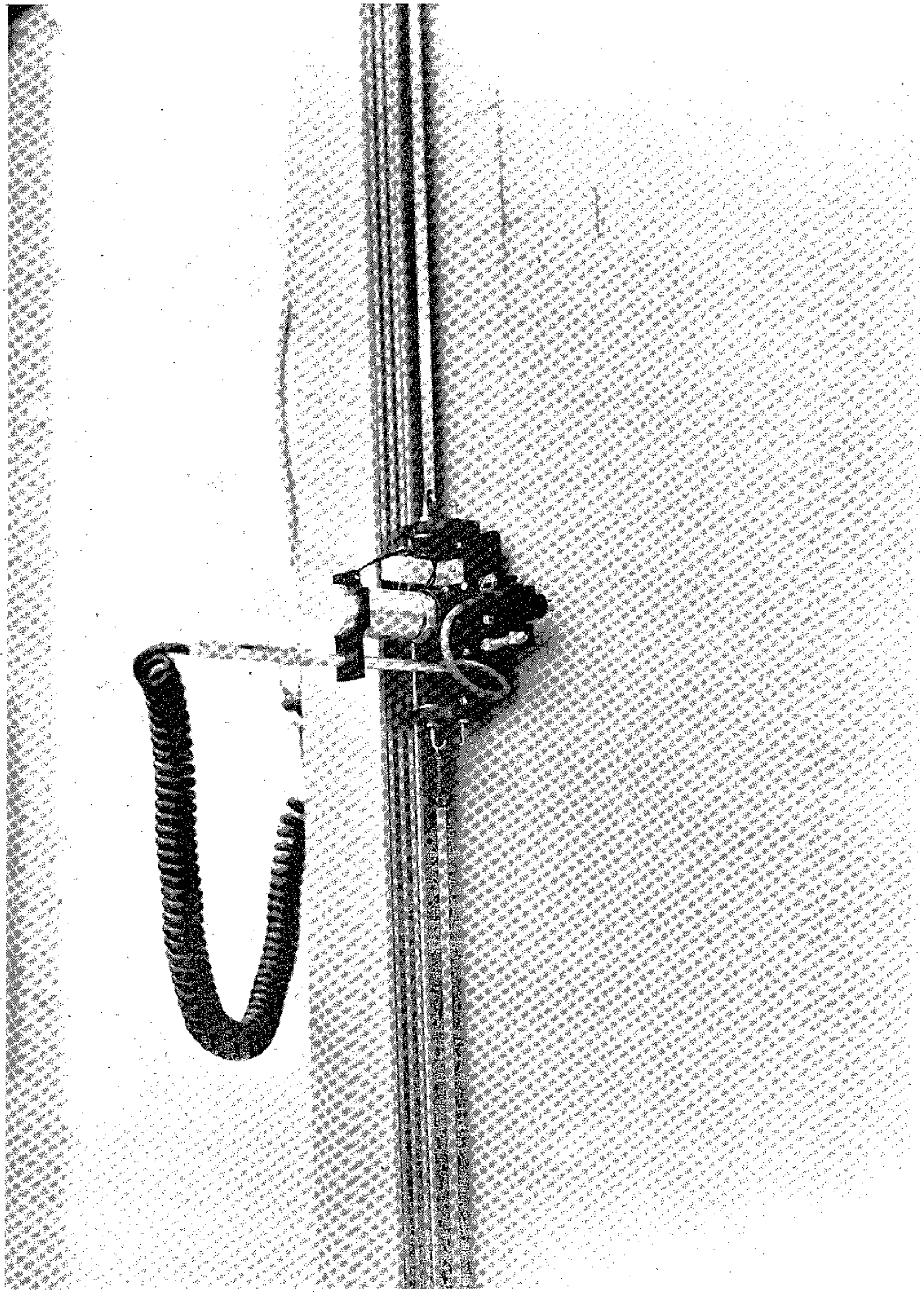

Fig. 5.2. View of the four-point potential sensing probe.

The minimum map scale is limited by the physical size of the four-point probe. Ideally, the probe would contact the resistance paper at only one point. From a practical consideration, however, this is not possible. The probe is equipped with four contacts arranged in a square pattern and spaced at a distance of 0.2 inches apart (Fig. 5.2). This is considered to be about the minimum practical spacing (11).

Initial tests of the isohyetal plotting technique were made by simulating the lower portion of the Walnut Gulch watershed. Because of the minimum scale limitation, the entire watershed could not be included within the area of the 30 inch by 30 inch plotting board. Consequently, only that part of the watershed which extends eastward as far as raingages 44 and 46 was included in the simulation (Fig. 5.3). The locations of the 60 raingages within this area are shown by Fig. 4.1. Fig. 5.4 is a view of the probe and resistance board in position on the plotter. The white dots on the black resistance paper indicate raingage locations (or voltage source points).

Preliminary tests of the isohyetal plotting technique indicate that the system has good possibilities. However, problems are still being encountered in establishing boundary conditions on the resistance paper, and much additional work will be needed to establish the practicality of this technique.

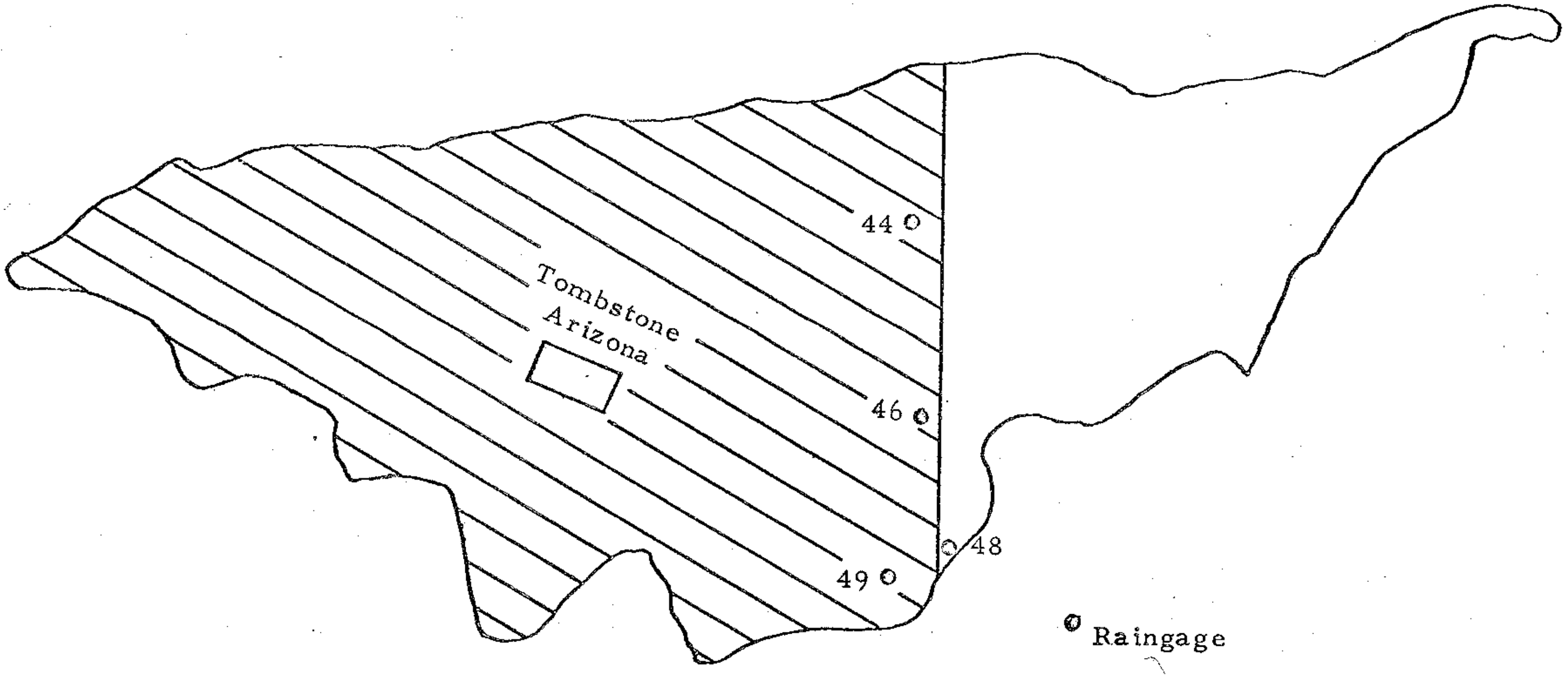

Fig. 5.3. An area of Walnut Gulch simulated by a conductive sheet for analog plotting of isohyetal lines.

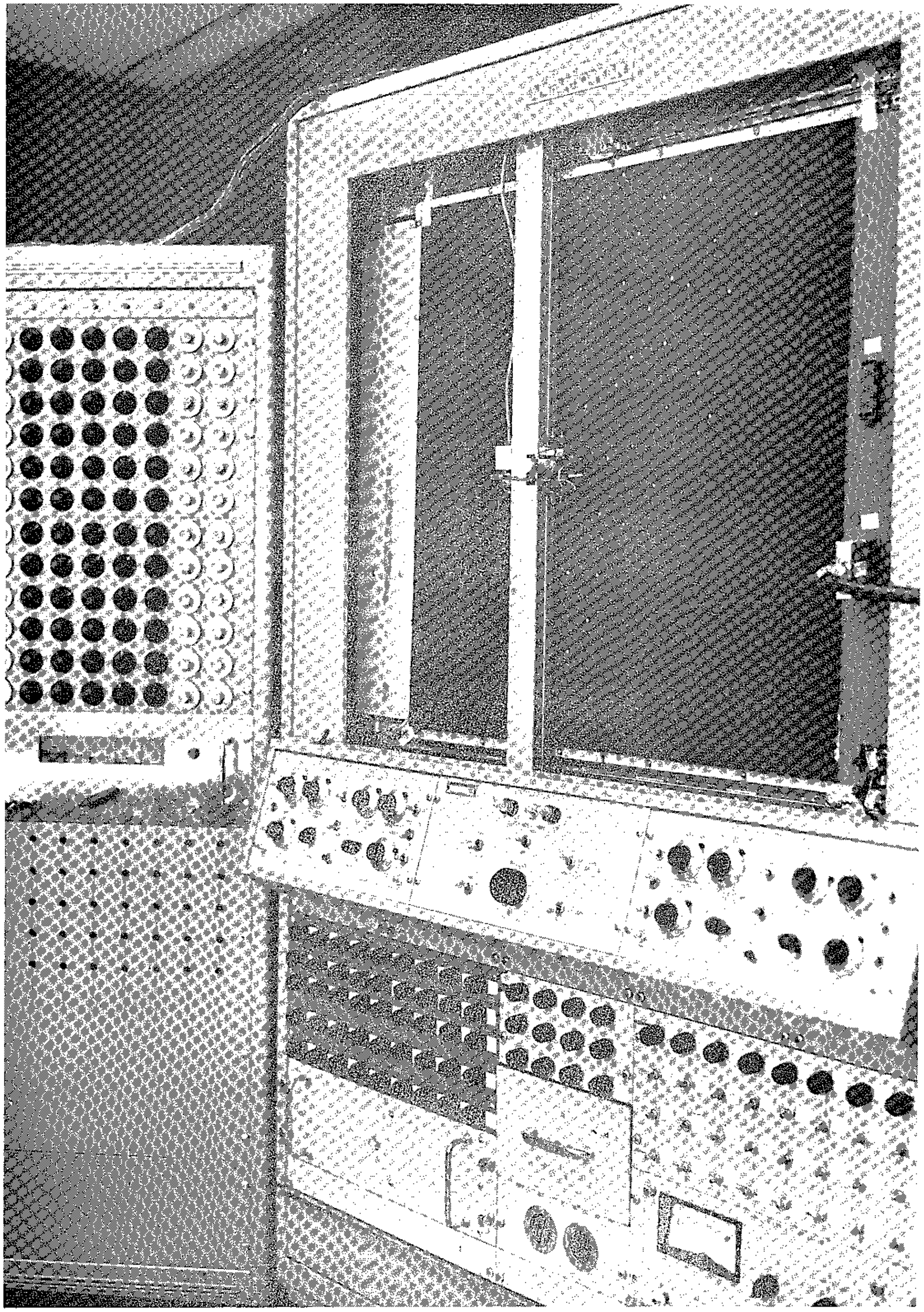

Fig. 5.4. Analog test equipment for plotting isohyetal lines.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This report proposes a preliminary analog model of the surface hydrologic system within a semiarid watershed. The basic components of the model are:

1. Fundamental and logical mathematical representations of the hydrologic processes.
2. An electronic analog computer which is capable of representing the individual hydrologic processes and of synthesizing them into a complex system.

Electronic analog simulation of hydrologic systems has many practical applications in the areas of both research and project planning and management. As a research tool the computer is valuable in the process of investigating and improving mathematical relationships. In this respect, the computer is applied not only for its calculating potential, but also for its ability to yield optimum solutions. Simulation is also ideal for investigations of hydrologic sensitivity. Problems range from the influence of a single factor upon a particular process to the effects of an entire process, such as evapotranspiration, upon the system as a whole.

In many ways analog simulation can assist in planning and development work. Models can provide the designer with runoff estimates from the input of recorded precipitation data. In addition, simulated streamflow records from statistically generated input information enable the establishment of synthetic flow frequency distribution patterns.

In the area of water resource management, analog computer simulation will permit the rapid evaluation of the effects of various management alternatives upon the entire system. These alternatives might involve such variables as watershed treatment, including urbanization, the construction of a storage reservoir, or changes in irrigation practices within a basin.

To test individual equations and to verify the model, a particular hydrologic unit was simulated. Flow records at the outlet of the subbasin provided data for quantitative verification of the model. For the storm of July 20, 1966, close agreement was achieved between the computed and observed outflow hydrographs from subwatershed 11 of the Walnut Gulch catchment area.

During the phase of the project reported herein efforts were also expended to improve the capability of the analog computing equipment for hydrologic applications. Some progress has been made in the development of an analog technique to plot isohyetal lines corresponding to selected time intervals during the course of a storm. It is anticipated that this technique, if successful, will facilitate the distribution of precipitation within a particular area with respect to the dimensions of both time and space.

In a research program of this nature certain constraints or boundary conditions limit the degree of achievement during any particular phase of the overall program. The most important of these limiting features are the extent to which research information and basic input data are available, the degree of accuracy established by the time and spatial increments adopted for the model, equipment limitations, and the necessary time limit imposed upon the investigation period.

In this study a high level of cooperation has been achieved between personnel of the Agricultural Research Service and the Utah Water Research Laboratory. Through this cooperation extensive technical knowledge and basic data are available to the project. In addition, adequate computing facilities permit detailed modeling in terms of both time and space.

During the next phase of the study, development of the isohyetal plotting technique will be continued. The hydrologic model will be improved by modeling an area, such as subwatershed 11, as a number of subzones. Thus, variable watershed characteristics and storm patterns will be more accurately represented by the model. Channel routing is also an area of possible improvement. Consideration is now being given to representing this flow by the simultaneous solution of the conservation of mass and momentum equations. Subsequent models will also be further generalized by including the soil moisture system and the evapotranspiration process.

The overall objective of this study involves many challenges, not only in the mathematical representations of the hydrologic processes, but also in equipment requirements and modeling techniques. However, achievements during the initial phase of the study reported herein have demonstrated the soundness and validity of the analog simulation approach to the complex hydrologic problems of semiarid regions.

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APPENDIX

Table A1. Precipitation data for event of July 20, 1966, on subwatershed 11, Walnut Gulch, Arizona.

Gage No.	Theissen Area (acres)	Time from 1600 hours (minutes)[1]											
		5	10	15	20	25	30	35	40	45	50	55	60
44	27.1	0.03	0.08	0.10	0.22	0.20	0.21	0.15	0.04	0.04	0.02	0.01	0.00
51	197.2	0.01	0.04	0.08	0.16	0.21	0.18	0.10	0.06	0.04	0.02	0.02	0.00
52	239.4	0.00	0.05	0.09	0.14	0.25	0.15	0.08	0.06	0.05	0.04	0.01	0.01
54	258.8	0.00	0.00	0.06	0.20	0.24	0.30	0.20	0.10	0.04	0.04	0.01	0.00
55	157.4	0.00	0.02	0.14	0.27	0.35	0.20	0.22	0.09	0.03	0.01	0.01	0.00
56	197.5	0.00	0.00	0.06	0.09	0.19	0.20	0.16	0.15	0.04	0.03	0.02	0.02
88	355.4	0.00	0.02	0.06	0.24	0.14	0.20	0.15	0.12	0.06	0.03	0.01	0.01
89	142.9	0.00	0.00	0.03	0.09	0.19	0.22	0.15	0.08	0.05	0.02	0.01	0.01
90	247.6	0.00	0.00	0.04	0.12	0.17	0.21	0.13	0.09	0.04	0.02	0.01	0.01
91	211.7	0.00	0.03	0.13	0.37	0.35	0.29	0.19	0.09	0.03	0.01	0.00	0.00

---

[1] The tabulated values in the main body of the table are the precipitation caught in inches during each five minute time interval.

Table A2. Constant input values for subwatershed 11, Walnut Gulch

Symbol	Description	Value
Δt	time increment or scale factor adopted for the model	5 minutes
b	average base width of the surface channels within a watershed zone	20 feet
l	total length of the surface channels within a watershed zone	52,800 feet
m	average side slope (horizontal to vertical) of the surface channels within a watershed zone	0.6
F_c	constant infiltration capacity rate under conditions of a saturated soil profile	0.15 in/Δt
F_o	soil infiltration capacity rate at the beginning of a storm	0.3 in/Δt
k	a constant in the Horton infiltration equation	0.40
k_g	a constant applied in the computation of channel seepage loss	0.20
k_r	a constant applied in the computation of capacity surface retention rate	1.0
k_s	a constant applied in the computation of outflow rate from surface detention storage	0.3
R_{cs}	retention storage capacity of the vegetation and land surfaces within a particular area (interception plus depression)	0.15 in.

DIGITAL COMPUTER PROGRAM FOR COMPUTING

INCREMENTAL PPECIPITATION

```
      DIMENSION TP(50),T(50),P(50),DELT(10)
    1 READ 99,DT1,DT2,NTO,L,ND,(DELT(I),I=1,ND)
   99 FORMAT(2A3,I9,2I5,5F9.0)
      READ 100
      PUNCH 100
  100 FORMAT(80H
     1                              )
      PUNCH 101,DT1,DT2,NTO
  101 FORMAT(15X,5HDATE 2A3,8X,9HTIME ZEROI6)
      PUNCH 110
  110 FORMAT(7HGAGE NO3X,13HACC TIME(MIN)4X,19HINTERVAL PRECIP(IN)3X,20H
     1ACC STORM PRECIP(IN))
    2 READ102,K,NGA
  102 FORMAT(2I5)
      READ 104,(TP(J),J=1,K)
  104 FORMAT (20X,10F6.2)
      DO324 J=1,K
      I=TP(J)*0.1
      T(J)=I
      B=I*10
  324 P(J)=TP(J)-B
  323 DO 15KK=1,ND
      PUNCH 360,DELT(KK)
  360 FORMAT(20HTIME INCREMENT(MIN)=F4.0)
      CR=1.0
      AP=P(1)
      AT=DELT(KK)
      J=1
      ASP=0
    3 IF(AT-T(J+1))5,7,8
    5 DT=DELT(KK)
      API=AP
      GO TO 10
    8 J=J+1
      IF((J+1)-K)6,6,14
    6 IF(AT-T(J+1))9,7,8
    7 TAP=P(J+1)
      GO TO 11
    9 DT=AT-T(J)
      API=P(J)
      GO TO 10
   14 TAP=P(K)
      GO TO 11
   10 TAP=DT*(P(J+1)-P(J))/(T(J+1)-T(J))+API
```

```
   11 PI=TAP-AP
      PIR=PI+0.005
      ASP=ASP+PI
      ASPR=ASP+0.005
      NAT=AT
      PUNCH 106,NGA,NAT,PIR,ASPR
  106 FORMAT(I5,I12,2F22.2)
      IF(AT-T(K))12,15,15
   12 CR=CR+1.0
      AT=CR*DELT(KK)
      AP=TAP
      GO TO 3
   15 CONTINUE
      L=L-1
      IF(L)20,20,2
   20 PAUSE
      GO TO 1
      END
```

Sample Output

```
PRECIPITATION DATA FOR WALNUT GULCH     SUB WATERSHED 11     PROJECT WG'38
                DATE 072066           TIME ZERO  1600
GAGE NO    ACC TIME(MIN)     INTERVAL PRECIP(IN)    ACC STORM PRECIP(IN)
TIME INCREMENT(MIN)=  5.
   32              5                    .02                      .02
   32             10                    .06                      .08
   32             15                    .06                      .14
   32             20                    .07                      .21
   32             25                    .13                      .34
   32             30                    .15                      .49
   32             35                    .11                      .60
   32             40                    .07                      .67
   32             45                    .03                      .70
   32             50                    .01                      .71
   32             55                    .01                      .71
   32             60                    .01                      .72
   32             65                   0.00                      .72
TIME INCREMENT(MIN)= 10.
   32             10                    .08                      .08
   32             20                    .13                      .21
   32             30                    .28                      .49
   32             40                    .18                      .67
   32             50                    .03                      .71
   32             60                    .01                      .72
   32             70                   0.00                      .72
```

```
TIME INCREMENT(MIN)= 15.
    32          15              .14           .14
    32          30              .35           .49
    32          45              .21           .70
    32          60              .02           .72
    32          75             0.00           .72
TIME INCREMENT(MIN)=  5.
    33           5              .03           .03
    33          10              .04           .07
    33          15              .06           .13
    33          20              .05           .17
    33          25              .09           .26
    33          30              .12           .38
    33          35              .15           .53
    33          40              .10           .63
    33          45              .04           .67
    33          50              .02           .68
    33          55              .01           .69
    33          60              .01           .70
    33          65              .01           .71
TIME INCREMENT(MIN)= 10.
    33          10              .07           .07
    33          20              .10           .17
    33          30              .21           .38
    33          40              .25           .63
    33          50              .06           .68
    33          60              .02           .70
    33          70              .01           .71
TIME INCREMENT(MIN)= 15.
    33          15              .13           .13
    33          30              .26           .38
    33          45              .29           .67
    33          60              .04           .70
    33          75              .01           .71
TIME INCREMENT(MIN)=  5.
    38           5             0.00          0.00
    38          10              .05           .05
    38          15              .11           .16
    38          20              .11           .27
    38          25              .16           .43
    38          30              .21           .64
    38          35              .14           .78
    38          40              .05           .83
    38          45              .04           .87
    38          50              .03           .90
    38          55              .03           .92
TIME INCREMENT(MIN)= 10.
    38          10              .05           .05
    38          20              .22           .27
    38          30              .37           .64
    38          40              .19           .83
    38          50              .07           .90
    38          60              .03           .92
```

```
TIME INCREMENT(MIN)= 15.
   38            15                              .16                      .16
   38            30                              .48                      .64
   38            45                              .23                      .87
   38            60                              .05                      .92
TIME INCREMENT(MIN)=  5.
   39             5                             0.00                     0.00
   39            10                              .03                      .03
   39            15                              .09                      .12
   39            20                              .14                      .26
   39            25                              .13                      .39
   39            30                              .08                      .48
   39            35                              .13                      .60
   39            40                              .10                      .70
   39            45                              .06                      .76
   39            50                              .02                      .78
   39            55                              .01                      .79
   39            60                              .01                      .80
   39            65                             0.00                      .80
TIME INCREMENT(MIN)= 10.
   39            10                              .03                      .03
   39            20                              .23                      .26
   39            30                              .22                      .48
   39            40                              .23                      .70
   39            50                              .08                      .78
   39            60                              .01                      .80
   39            70                             0.00                      .80
TIME INCREMENT(MIN)= 15.
   39            15                              .12                      .12
   39            30                              .36                      .48
   39            45                              .29                      .76
   39            60                              .04                      .80
   39            75                             0.00                      .80
TIME INCREMENT(MIN)=  5.
   44             5                              .03                      .03
   44            10                              .08                      .11
   44            15                              .10                      .21
   44            20                              .22                      .43
   44            25                              .20                      .63
   44            30                              .21                      .84
   44            35                              .15                      .99
   44            40                              .04                     1.03
   44            45                              .04                     1.07
   44            50                              .02                     1.09
   44            55                              .01                     1.10
   44            60                             0.00                     1.10
TIME INCREMENT(MIN)= 10.
   44            10                              .11                      .11
   44            20                              .32                      .43
   44            30                              .41                      .84
   44            40                              .19                     1.03
   44            50                              .07                     1.09
   44            60                              .01                     1.10
```