

## Epicycle Parameter Filter for Long Term Orbital Parameter Estimation

P L Palmer, S A A Gilani  
 Surrey Space Centre, University of Surrey  
 GU2 7XH, Guildford UK; +441483686024  
 P.Palmer@surrey.ac.uk

### ABSTRACT

A parameter estimator for accurate analytic propagation of Epicyclic orbits to be used on board a spacecraft is developed as a replacement of high precision computationally expensive numerical propagators. Long term propagation of orbits using analytical descriptions needs proper choice of orbital parameters. The question arises on how to choose these orbital parameters of an analytical approximation appropriate to a given choice of orbital parameters for the numerically propagated orbit obtained from full nonlinear equations of motion such that the two trajectories remain sufficiently close to each other for long terms usually a week. In this paper we employed statistical data regression technique to accurately determine linear secular growths in argument of latitude and right ascension of the ascending node of an Epicyclic model<sup>1</sup>. This enables precise determination of semi-major axis and inclination of the orbit valid for longer durations with fixed secular variations. Accurately fixing secular quantities minimizes the average drift and improves fidelity over longer periods.

### INTRODUCTION

Satellite orbit dynamical motion is described by second order nonlinear differential equations of motion derived from Newton's law of universal gravitation<sup>2</sup>. Due to Earth's equatorial bulge the accelerations acting on satellite cannot be fully captured without incorporating harmonics of the geo-potential. More significant is zonal harmonic  $J_2$ , which is most considerable geo-potential perturbative force acting on the orbit. The orbital plane under  $J_2$  force does not remain fixed in space but it experiences precession. Also there are other secular and periodic variations to a Keplerian<sup>3,4</sup> orbit due to  $J_2$ . Evolution of satellite orbits are expressed in terms of instantaneous Keplerian orbital elements or osculating elements. These elements under basic *2 body* (Earth-Satellite) accelerations remain constant except for mean anomaly which is a function of time<sup>5</sup>. Analytical satellite theories have been developing since around 50 years incorporating the effect of Earth oblateness. Gauss's planetary equations of motion describe evolution of these orbital elements under perturbing forces<sup>6</sup>. Kozai<sup>7</sup> and Brower<sup>8</sup> found out analytic solutions of perturbed orbital elements of a satellite. Kozai's<sup>7</sup> solution express short periodic variations up to first order and secular variations up to second order in nonspherical Earth Gravitational perturbing forces. Based on the earlier work of King-Hele<sup>9</sup> Hashida and Palmer<sup>1,10</sup> presented analytical formulations of a near circular orbit of a satellite about an axisymmetric potential. In this approach the coordinates of a satellite are expanded in an Earth

Centered Inertial (ECI) frame for small eccentricities. For these type of orbits this description is much simpler than Brouwer's<sup>8</sup> approach. All of these analytical descriptions similes naturally occurring perturbed satellite orbit albeit complex and mathematically rigorous. The analytic solution to Kepler's problem for spherically symmetric Earth and basic *2 body* problem is possible without any linearization<sup>5</sup>. However, for non spherical Earth the equations of motion are linearized about the true trajectory along with other assumptions of circular or low eccentricity orbits. Therefore, the analytic propagation of orbits are not very accurate for long duration of time when compared to true trajectory due to growth of error especially due to linearization process. In this paper Epicycle model around an oblate Earth by Hashida and Palmer<sup>1</sup> is being used. The position of a satellite in an Epicyclic orbit is defined by six osculating coordinates. The geometrical shape of an Epicyclic orbit is described by six constant parameters; semi-major axis ( $a$ ), inclination ( $I_0$ ), right ascension of ascending node ( $\Omega_0$ ), non singular parameters for undefined epicycle phase at perigee passage (needed for equatorial orbits) ( $\zeta_p, \eta_p$ ) and equator crossing time ( $t_E$ ). Extremely precise selection of these parameters is needed to obtain Epicyclic orbital coordinates appropriate to a given numerically propagated orbital coordinates. The most common representation of nonlinear orbital dynamical equations is expressed in Earth Central Inertial (ECI) coordinate frame expressed in equations (1-2) for terms up to  $J_4$  of zonal geo-potential harmonics:

$$\dot{\mathbf{r}} = \mathbf{v} \quad (1)$$

$$\dot{\mathbf{v}} = -\frac{\mu_E}{r^3}\mathbf{r} + \mathbf{a}_G \quad (2)$$

Where the term  $\mathbf{a}_G$  expresses zonal gravitational perturbation terms defined as<sup>6</sup>:

$$J_2(\mathbf{r}) = -\frac{3J_2\mu_ER_E^2}{2r^5} \left\{ \begin{array}{l} \left(1 - \frac{5Z^2}{r^2}\right)X \\ \left(1 - \frac{5Z^2}{r^2}\right)Y \\ \left(3 - \frac{5Z^2}{r^2}\right)Z \end{array} \right\} \quad (3)$$

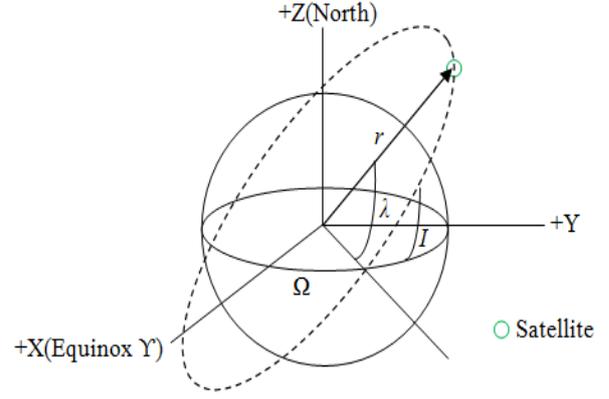
$$J_3(\mathbf{r}) = -\frac{5J_3\mu_ER_E^3}{2r^7} \left\{ \begin{array}{l} \left(3Z - \frac{7Z^3}{r^2}\right)X \\ \left(3Z - \frac{7Z^3}{r^2}\right)Y \\ \left(6Z^2 - \frac{7Z^4}{r^2} - \frac{3}{5}r^2\right) \end{array} \right\} \quad (4)$$

$$J_4(\mathbf{r}) = \frac{15J_4\mu_ER_E^4}{8r^7} \left\{ \begin{array}{l} \left(1 - \frac{14Z^2}{r^2} + \frac{21Z^4}{r^4}\right)X \\ \left(1 - \frac{14Z^2}{r^2} + \frac{21Z^4}{r^4}\right)Y \\ \left(5 - \frac{70Z^2}{3r^2} + \frac{21Z^4}{r^4}\right)Z \end{array} \right\} \quad (5)$$

where  $X$ ,  $Y$ , and  $Z$  are the coordinates in Earth Central Inertial (ECI) frame, and  $r = \sqrt{X^2 + Y^2 + Z^2}$ . The inertial frame is defined such that  $X$  axis points to the vernal point in the equatorial plane of the Earth, the  $Z$  axis is the axis of rotation of the Earth in positive direction, and  $Y$  is defined by the right-hand rule.  $\mu_E$  and  $R_E$  are Earth gravitational constant and equatorial radius respectively. Prediction of precise satellite ephemerides are obtained by numerically integrating these equations given some epoch satellite state.

## EPICYCLE ORBIT

An analytic formulation for a near circular orbit of a satellite about an axisymmetric potential was presented by Hashida and Palmer<sup>1</sup>. The model has a simple analytic form, describing all the geo-potential terms arising from the Earth Zonal Harmonics. The description of position of a satellite in ECI frame by Epicyclic coordinates is illustrated in Figure: 1.



**Figure 1: Geometrical Representation of Epicycle Coordinates of  $r$ ,  $\lambda$ ,  $I$  and  $\Omega$  in ECI frame**

The perturbed motion of a satellite about an axisymmetric planet can be described by following osculating Epicyclic coordinates:

$$\begin{aligned} r &= a [1 + \rho - (\xi_p \cos \alpha + \eta_p \sin \alpha) \\ &\quad + \Delta r_2 \cos 2\beta + \chi \sin \beta + \Delta_r] \\ I &= I_0 + \Delta I_2 (1 - \cos 2\beta) + \Delta_I \\ \Omega &= \Omega_0 + \vartheta \alpha + \Delta \Omega_2 \sin 2\beta + \Delta_\Omega \\ \lambda &= \beta + 2[\xi_p \sin \alpha + \eta_p (1 - \cos \alpha)] \\ &\quad - 2\chi (1 - \cos \beta) + \Delta \lambda_2 \sin 2\beta + \Delta_\lambda \\ v_r &= an [(\xi_p \sin \alpha - \eta_p \cos \alpha) - (1 + \kappa) \\ &\quad \times (2\Delta r_2 \sin 2\beta - \chi \cos \beta) + \dot{\Delta}_r] \\ v_\theta &= rn [(1 + \kappa) + 2(\xi_p \cos \alpha + \eta_p \sin \alpha) \\ &\quad + (1 + \kappa)(2\Delta \lambda_2 \cos 2\beta - 2\chi \sin \beta) + \dot{\Delta}_\lambda] \end{aligned} \quad (6)$$

Where  $r$  = radius,  $I$  = inclination,  $\Omega$  = right ascension of the ascending node,  $\lambda$  = argument of latitude,  $v_r$  = radial velocity,  $v_\theta$  = azimuthal velocity,  $\beta = (1 + \kappa)\alpha$ ,  $\Delta x_2$  = short periodic coefficient due to  $J_2$ ,  $\Delta_x$  = short periodic coefficients due to zonal harmonic terms up to  $J_4$ , long periodic variations in the orbit are described by  $\chi$ . Terms defining the geometrical shape of an Epicyclic orbit are described by six constant parameters; semi-major axis ( $a$ ), inclination ( $I_0$ ), right ascension of ascending node ( $\Omega_0$ ),  $\alpha = n(t - t_E)$ , where  $t_E$  = Equator passage time,  $(\xi_p, \eta_p)$  = non singular parameters for undefined epicycle phase at perigee passage where,  $\xi_p = \frac{A}{a} \cos \alpha_p$ ,  $\eta_p = \frac{A}{a} \sin \alpha_p$ ,  $\alpha_p = n(t_p - t_E)$ ,  $t_p$  = perigee passage time,  $A$  = Epicycle amplitude and  $n$  = mean motion.  $\rho, \kappa, \vartheta$  are secular variations in the orbit. The coefficients for these variations for  $J_2$  and  $J_4$  are given by<sup>1</sup>:

$$\begin{aligned}
\rho_2 &= -\frac{1}{4}J_2 \left(\frac{R_E}{a}\right)^2 (2 - 3\sin^2 I_0) \\
\kappa_2 &= \frac{3}{4}J_2 \left(\frac{R_E}{a}\right)^2 (4 - 5\sin^2 I_0) \\
\vartheta_2 &= -\frac{3}{2}J_2 \left(\frac{R_E}{a}\right)^2 \cos I_0 \\
\rho_4 &= \frac{9}{64}J_4 \left(\frac{R_E}{a}\right)^4 (8 - 40\sin^2 I_0 + 35\sin^4 I_0) \\
\kappa_4 &= -\frac{3}{64}J_4 \left(\frac{R_E}{a}\right)^4 (136 - 500\sin^2 I_0 \\
&\quad + 385\sin^4 I_0) \\
\vartheta_4 &= \frac{15}{16}J_4 \left(\frac{R_E}{a}\right)^4 \cos I_0 (4 - 7\sin^2 I_0)
\end{aligned} \tag{7}$$

The above analytical descriptions due to linearization and other orbital assumptions when compared with true nonlinear trajectory (Equation: 1-2) would need proper choice of Epicycle orbital parameters to avoid divergence. Therefore, in order to make these descriptions suitable for real time on board applications for longer durations i.e., week carefully finding out of orbital parameters is highly desirable.

#### DEVELOPMENT OF EPICYCLE PARAMETER FILTER

The ensuing description develops orbital parameter estimation algorithm called Epicycle Parameter Filter (EPF) using statistical data regression technique of linear least squares<sup>11</sup>. Data for time history of osculating Epicycle coordinates is generated by firstly numerically integrating equations of motion given in Equations: 1-2 for a week. A transformation is applied to convert ECI into Epicycle coordinates expressed in Equation: 9. Given this time history of osculating epicycle coordinates, determine the set of orbital parameters for an Epicyclic model defined as:

$$\mathbf{x}_0 = (a, \xi_p, \eta_p, I_0, \Omega_0, \alpha_0)^T \tag{8}$$

Where, the term  $\alpha_0$  = initial Epicycle phase.

$$\begin{aligned}
r &= \sqrt{X^2 + Y^2 + Z^2} \\
v &= \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}
\end{aligned} \tag{9}$$

$$\begin{aligned}
\lambda &= \cos^{-1} \left( \frac{\dot{Z} - \frac{v_r Z}{r}}{v_\theta \sin I} \right), \\
&\text{if } \left( \frac{Z}{r \sin I} < 0 \right), \lambda = 2\pi - \lambda \\
&\text{if } (\lambda > \pi), \lambda = \lambda - 2\pi \\
I &= \cos^{-1} \left( \frac{X\dot{Y} - Y\dot{X}}{r v_\theta} \right) \\
\Omega &= \cos^{-1} \left( \frac{X\dot{Z} - Z\dot{X}}{r v_\theta \sin I} \right) \\
&\text{if } ((Y\dot{Z} - Z\dot{Y}) < 0), \Omega = 2\pi - \Omega \\
v_r &= \frac{\mathbf{r} \cdot \mathbf{v}}{r} \\
v_\theta &= \sqrt{v^2 - v_r^2}
\end{aligned}$$

#### Least Squares Formulation

Coordinates of argument of latitude and right ascension of the ascending node are angular descriptions repeating itself after an orbital period therefore; data for these coordinates are unravelled to obtain time evolution of continuously increasing angular quantities. First stage is to unravel  $\lambda$  and  $\Omega$  so that these grow linearly instead of the usual  $-\pi \leq \lambda < \pi$  and  $0 \leq \Omega < 2\pi$  respectively. The equation for these two coordinates may be separated into linear and oscillating terms therefore one may rewrite from Equation: 6.

$$\begin{aligned}
\tilde{\lambda} &= \lambda - d\lambda \triangleq \beta \\
\tilde{\Omega} &= \Omega - d\Omega \triangleq \Omega_0 + \vartheta\alpha
\end{aligned} \tag{10}$$

Where,

$$\begin{aligned}
d\lambda &= 2\xi_p \sin \alpha + 2\eta_p(1 - \cos \alpha) \\
&\quad - 2\chi(1 - \cos \beta) + \Delta\lambda_2 \sin \beta \\
&\quad + \Delta\lambda
\end{aligned} \tag{11}$$

$$d\Omega = \Delta\Omega_2 \sin \beta + \Delta\Omega$$

Therefore osculating coordinates of  $\lambda$  and  $\Omega$  from Equation: 6 can be rewritten as:

$$\tilde{\lambda} = (1 + \kappa)n(t - t_E) = p_1(t - t_E) \quad (12)$$

$$\tilde{\Omega} = \Omega_0 + \vartheta n(t - t_E) = \Omega_0 + p_2(t - t_E) \quad (13)$$

In fact  $\tilde{\lambda}$  and  $\tilde{\Omega}$  are linearly increasing coordinates therefore linear least squares can be used to estimate  $p_1, p_2, \Omega_0$  and  $t_E$ . The reason for this assumption is quite valid as linear secular growth in angular quantities of  $\lambda$  and  $\Omega$  are more dominant than periodic terms in  $d\lambda$  and  $d\Omega$ . Therefore, these do not make much impact on a linear least square fit. The cost function for linear least squares problem can now be conveniently written as:

$$\mathcal{J} = \sum_i [\tilde{\lambda}_i - p_1(t_i - t_E)]^2 + \sum_i [\tilde{\Omega}_i - \Omega_0 - p_2(t_i - t_E)]^2 \quad (14)$$

The cost function is now differentiated with respect to four variables i.e.,  $p_1, p_2, \Omega_0$  and  $t_E$  and equated to zero. This provides following set of four simultaneous equations:

$$\overline{\lambda t} - \bar{\lambda} t_E - p_1(\overline{t^2} - 2t_E \bar{t} + t_E^2) = 0 \quad (15)$$

$$\overline{\Omega t} - \bar{\Omega} t_E - p_2(\overline{t^2} - 2t_E \bar{t} + t_E^2) = \Omega_0(\bar{t} - t_E)$$

$$\bar{\Omega} - \Omega_0 - p_2(\bar{t} - t_E) = 0$$

$$p_1 \bar{\lambda} + p_2 \bar{\Omega} = p_2 \Omega_0 + (p_1^2 + p_2^2)(\bar{t} - t_E)$$

Where, over bar indicates average over all data points. These equations are solved algebraically to determine  $p_1, p_2, \Omega_0$  and  $t_E$ :

$$p_1 = \frac{1}{(\overline{t^2} - \bar{t}^2)} (\overline{\lambda t} - \bar{\lambda} \bar{t}) \quad (16)$$

$$p_2 = \frac{1}{(\overline{t^2} - \bar{t}^2)} (\overline{\Omega t} - \bar{\Omega} \bar{t})$$

$$t_E = \bar{t} - \frac{\bar{\lambda}}{p_1}$$

$$\Omega_0 = \bar{\Omega} - \frac{p_2 \bar{\lambda}}{p_1}$$

### **Determination of Semi Major Axis “a” and Inclination “I<sub>0</sub>”**

The estimates of  $p_1$  and  $p_2$  are basically slopes of linearly increasing  $\lambda$  and  $\Omega$ . Their implicit dependence upon  $a$  and  $I_0$  is as expressed in Equation: 7. Therefore, we need to determine  $a$  and  $I_0$  to keep secular terms in Equation: 7 accurate. The estimate of  $I_0$  is found out by estimate of  $p_2$  (Equation: 16) in an iterative scheme wherein this estimate and analytical expression of slope “ $\vartheta n$ ” obtained from analytical equation of  $\Omega$  (Equation: 10) are equated fixing semi major axis “ $a$ ”. We start by assuming a value of “ $a$ ” and it is convenient to choose the first value of the radial coordinate.

$$\therefore \vartheta = \frac{p_2}{n} \equiv \vartheta_2 + (\vartheta - \vartheta_2) \quad (17)$$

where,  $\vartheta_2$  is the value of  $\vartheta$  for just  $J_2$ . This is much larger than  $(\vartheta - \vartheta_2)$ . Rewriting equation for  $\vartheta_2$  as a function of  $x$ :

$$\vartheta_2 = -\frac{3}{2} J_2 \left(\frac{R}{a}\right)^2 x \quad (18)$$

Where,  $x = \cos I_0$ , one can get first estimate of  $x$  from:

$$\vartheta_2(x_1) = \frac{p_2}{n} \quad (19)$$

The iterative scheme from Equation: 17 for  $i^{th}$  estimate of  $\vartheta_2(x_i)$  would be:

$$\vartheta_2(x_i) = \frac{p_2}{n} - [\vartheta(x_{i-1}) - \vartheta_2(x_{i-1})] \quad (20)$$

Then iteration of equation for  $\vartheta_2$  be carried out until the change in result decreases below selected tolerance. We may now use the value of  $\vartheta_2$  to find value of  $I_0$  from Equation: 18. The value of  $p_1$  Equation: 16 would now be used to estimate semi-major axis  $a$ . This can be done by using Newton-Raphson<sup>12</sup> method. From Equation: 16 Let,

$$f = 1 + \kappa - p_1 a^{3/2} \mu^{-1/2} \quad (21)$$

So when  $a$  satisfies  $(1 + \kappa)n = p_1$ , then  $f = 0$ . Equation: 21 can be written for Newton-Raphson formulation as:

$$a_i = a_{i-1} - \frac{f(a_{i-1})}{\dot{f}_a(a_{i-1})} \quad (22)$$

Where,  $\dot{f}_a(\cdot)$  indicates derivative of function with respect to  $a$ . After some derivations the  $i^{th}$  estimate of  $a$  may be written as:

$$a_i = a_{i-1} + a_{i-1} \frac{1 + \kappa - p_1 a^{3/2}}{\kappa' + \frac{3}{2} p_1 a^{3/2}} \quad (23)$$

Where,  $\kappa' = \sum_{l=1}^l 2l\kappa_{2l} + 4\kappa_{22}$  and  $\kappa_{22}$  is  $J_2^2$  coefficient in post epicycle equation<sup>1</sup> expressed as:

$$\kappa_{22} = \frac{3}{16} J_2^2 \left( \frac{R}{a} \right)^4 (14 + 17\sin^2 I_0 - 35\sin^4 I_0) \quad (24)$$

Again Equation: 23 will be iterated until the change in semi-major axis “ $a$ ” is less than selected tolerance. Now with newly found out value of “ $a$ ” we go back to Equation: 20 for  $I_0$  and repeat until both the values  $a$  and  $I_0$  converge.

### Determination of “ $\xi_P$ ” and “ $\eta_P$ ”

The two quantities of  $\xi_P$  and  $\eta_P$  are now being estimated using the equations of epicycle coordinates of  $r$  and  $v_r$  from Equation: 6. By separating out oscillating terms the equations may be rewritten as:

$$\frac{r}{a} = dr - \xi_P \cos \alpha - \eta_P \sin \alpha \quad (25)$$

$$\frac{v_r}{an} = dv_r + \xi_P \sin \alpha - \eta_P \cos \alpha$$

Where,

$$dr = 1 + \rho + \chi \sin \beta + \Delta r_2 \cos 2\beta + \Delta_r \quad (26)$$

$$dv_r = -(1 + \kappa)(2\Delta r_2 \sin 2\beta - \chi \cos \beta) + \dot{\Delta}_r$$

The above may again be simplified on the lines of Equation: 10 as:

$$\therefore \frac{r}{a} - dr \equiv \tilde{r} = -\xi_P \cos \alpha - \eta_P \sin \alpha \quad (27)$$

$$\frac{\dot{r}}{an} - dv_r \equiv \tilde{v} = \xi_P \sin \alpha - \eta_P \cos \alpha \quad (28)$$

We may now conveniently define least square cost function as:

$$\begin{aligned} \mathfrak{J} = \sum_i [\tilde{r}_i + \xi_P \cos \alpha_i + \eta_P \sin \alpha_i]^2 \\ + \sum_i [\tilde{v}_i - \xi_P \sin \alpha_i + \eta_P \cos \alpha_i]^2 \end{aligned} \quad (29)$$

The above function be differentiated with respect to  $\xi_P$  and  $\eta_P$  and equated to zero. The resultant simultaneous equation can be solved to provide estimates of  $\xi_P$ ,  $\eta_P$  expressed as:

$$\xi_P = \overline{v \sin \alpha} - \overline{r \cos \alpha} \quad (30)$$

$$\eta_P = -\overline{v \cos \alpha} - \overline{r \sin \alpha}$$

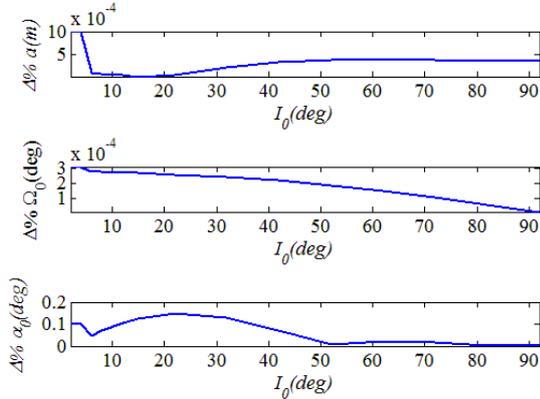
Now these parameters can be conveniently used in Equation: 11 to compute  $d\lambda$  and  $d\Omega$  and estimates of secular terms would be repeated as in Equations 20 & 23. Repetition of the algorithm is carried out until the estimates are converged for the orbital parameters.

## RESULTS

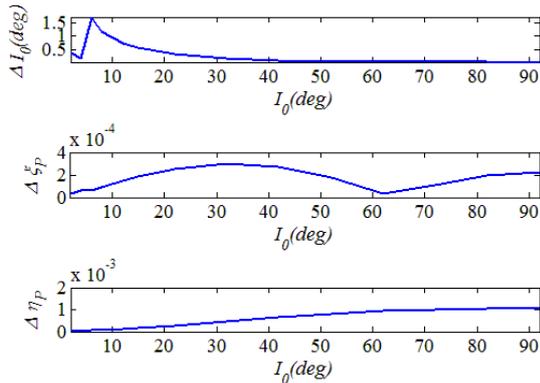
### Parameter Estimation Accuracy

In order to test the usefulness of Epicycle Parameter Filter (EPF), its accuracy of determination of Epicycle parameters is carried out over all orbital inclinations at  $a = 7000$  km orbit. Estimation of these parameters is compared with true values used for numerical propagation of truth orbit (Equation: 1-2). The batch of numerically propagated data used for estimation is for a week. The time scale of 1 week corresponds to approximately 100 orbital periods for LEO micro or nano satellite. Figure: 2 indicates percentage errors in estimating the parameters of  $a$ ,  $\Omega_0$ , and  $a_0$  whereas Figure: 3 illustrates errors for  $I_0$ ,  $\xi_P$ , and  $\eta_P$ . Results reveal that errors cannot be fully eliminated as the two trajectories are being propagated differently i.e., numerical and analytical. In order to keep the two trajectories sufficiently close to each other for long duration slightly perturbed parameters are found out in

order to compensate for process noise<sup>11</sup>. In general process noise is a time varying quantity and is inherent in all analytically derived models due to linearization and approximations to full dynamics of the nonlinear problem.



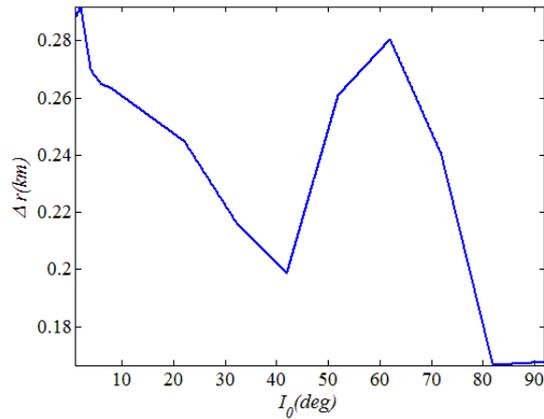
**Figure 2: Percentage Estimation Errors for Semi-Major Axis "a"(Top), Right Ascension of the Ascending Node "Ω<sub>0</sub>"(Middle), and Initial Epicycle Phase "α<sub>0</sub>"(Bottom), as a Function of Inclination of the Orbital Plane.**



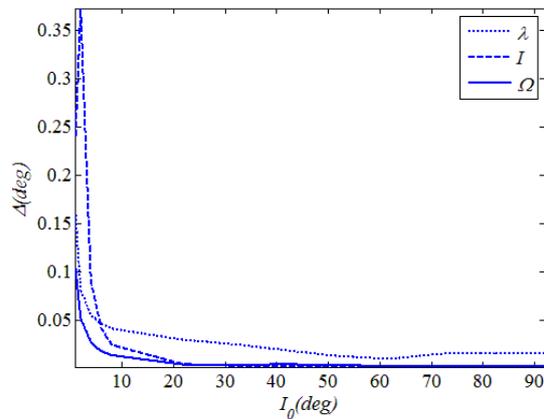
**Figure 3: Estimation Errors for Inclination "I<sub>0</sub>"(Top), "ξ<sub>P</sub>" (Middle) and "η<sub>P</sub>" (Bottom) as a Function of Inclination of the Orbital Plane.**

**Error Statistics**

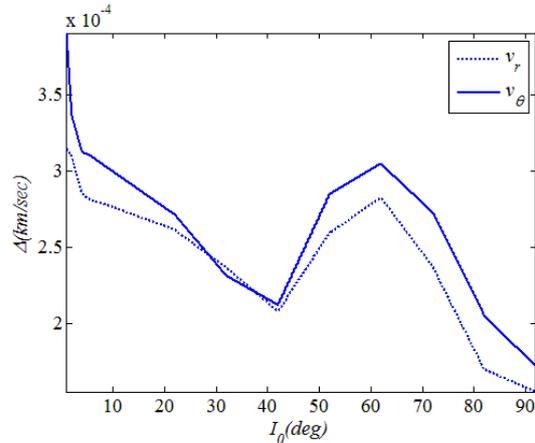
After having been chosen optimal orbital parameters the error statistics in terms of Epicycle coordinates for the numerical and analytically propagated orbits is carried out. The maximum errors for the period of one week are observed only at lower inclinations. The errors decrease almost exponentially at higher inclinations. Figure: 4-6 illustrates maximum errors in Epicycle coordinates over the period of one week.



**Figure 4: Maximum Absolute Errors in "r" as Function of Inclination of the Orbital Plane.**



**Figure 5: Maximum Absolute Errors in λ, I, and Ω as a Function of Inclination of the Orbital Plane.**



**Figure 6: Maximum Absolute Errors in v<sub>r</sub> and v<sub>θ</sub> as a Function of Inclination of the Orbital Plane.**

**NORAD TLE to Epicycle Parameters - UK Disaster Monitoring Constellation**

SSTL conceived the innovative and unique Disaster Monitoring Constellation (DMC), the first Earth observation constellation of low cost small satellites providing daily images for applications including global disaster monitoring<sup>13</sup>. The DMC currently comprises:

- Beijing-1
- NigeriaSat-1
- UK-DMC-1
- UK-DMC-2
- Deimos-1

We obtained classical orbital elements<sup>6</sup> from Two Line Element (TLE) data from NORAD<sup>14</sup> for UK-DMC-1 satellite and converted these into Epicycle parameters and carried out estimation process for a period of one week. The maximum absolute errors over this period are  $\Delta r = 314\text{ m}$ ,  $\Delta \lambda = 13.3 \times 10^{-3}\text{ deg}$ ,  $\Delta I = 2.8 \times 10^{-3}\text{ deg}$ ,  $\Delta \Omega = 1.97 \times 10^{-3}\text{ deg}$ ,  $\Delta v_r = 5.82 \times 10^{-4}\text{ km/s}$ ,  $\Delta v_\theta = 7.09 \times 10^{-4}\text{ km/s}$ .

**Time History of Errors in Epicycle Coordinates**

In order to observe time history of errors in Epicycle coordinates we selected a sun synchronous LEO with following initial conditions in ECI coordinates expressed as  $\mathbf{z}_0 = [\mathbf{r}_0 \ \mathbf{v}_0]^T$ :

$$\mathbf{r}_0 = [7003.02657 \ 0.0 \ 0.0]^T \text{km} \tag{31}$$

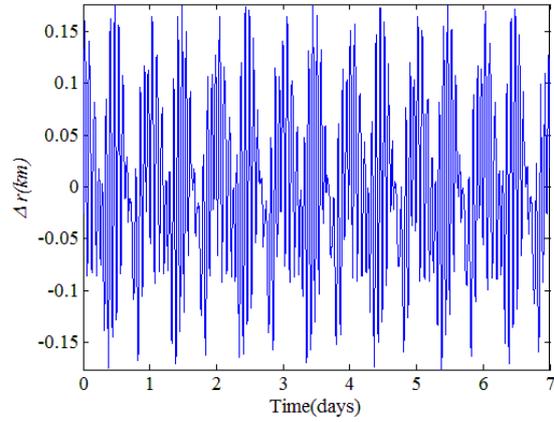
$$\mathbf{v}_0 = [-0.00795 \ -1.05022 \ 7.47274]^T \text{km/s}$$

The time history of errors for the period is shown in Figures: 7-12. The estimates are approximately zero mean and converged which shows consistency in estimates. The maximum absolute errors over this period are  $\Delta r = 176\text{ m}$ ,  $\Delta \lambda = 9.6 \times 10^{-3}\text{ deg}$ ,  $\Delta I = 2.3 \times 10^{-3}\text{ deg}$ ,  $\Delta \Omega = 2.09 \times 10^{-3}\text{ deg}$ ,  $\Delta v_r = 2.07 \times 10^{-4}\text{ km/s}$ ,  $\Delta v_\theta = 2.03 \times 10^{-4}\text{ km/s}$ .

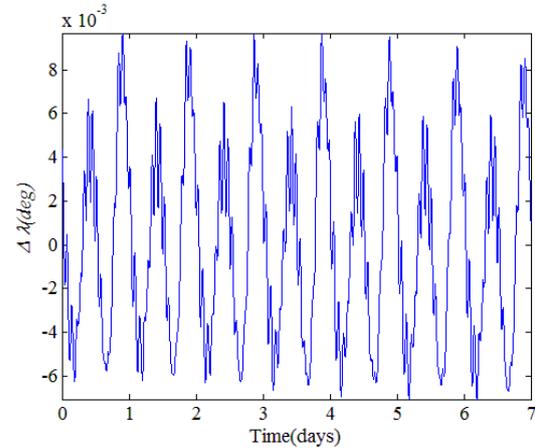
**Time History of Errors in Epicycle Coordinates Without Estimation**

The time history of errors will now be observed for initial conditions as expressed in Equation: 31 without properly choosing the Epicycle orbital parameters. One may clearly observe increase and drift in errors for all the coordinates if we do not carry out proper selection of orbital parameters. The errors in the coordinates are presented in Figures: 13-18. Divergence and increased errors are quite evident from the plots. Especially in argument of latitude and right ascension of the ascending node which amounts to significant in-track

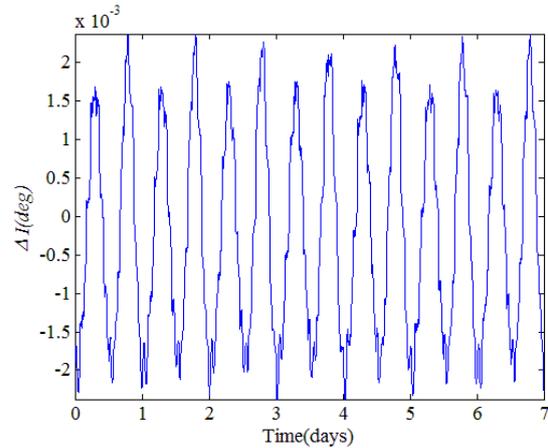
and cross track errors in Local Vertical Local Horizontal Coordinate (LVLH) system<sup>6</sup>.



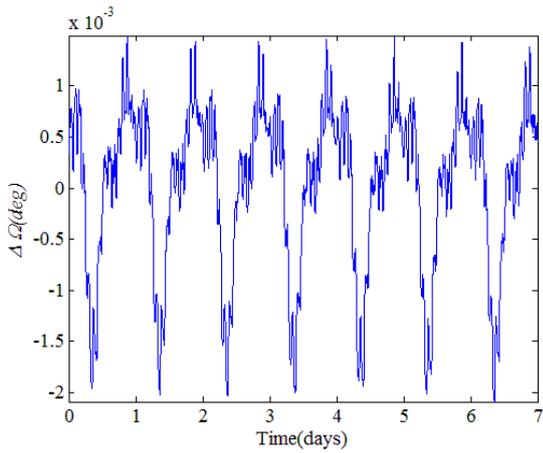
**Figure 7: Time History of Errors in “r”.**



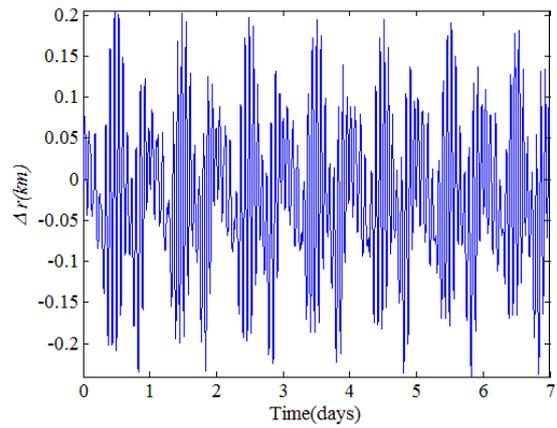
**Figure 8: Time History of Errors in “λ”.**



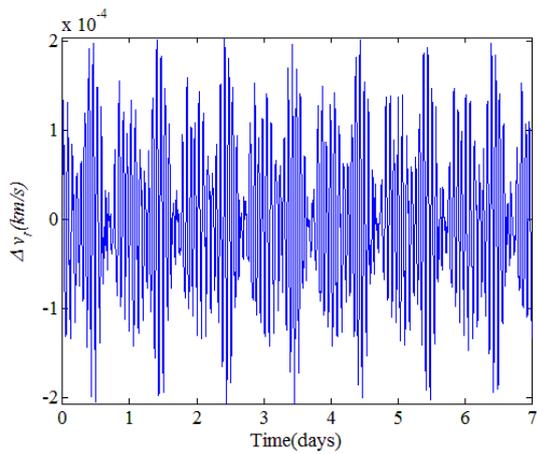
**Figure 9: Time History of Errors “I”.**



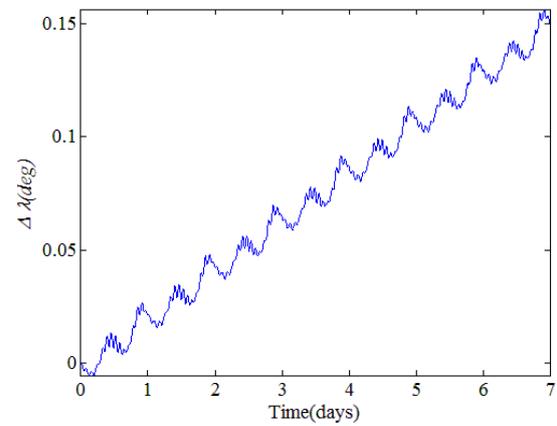
**Figure 10: Time History of Errors “Q”.**



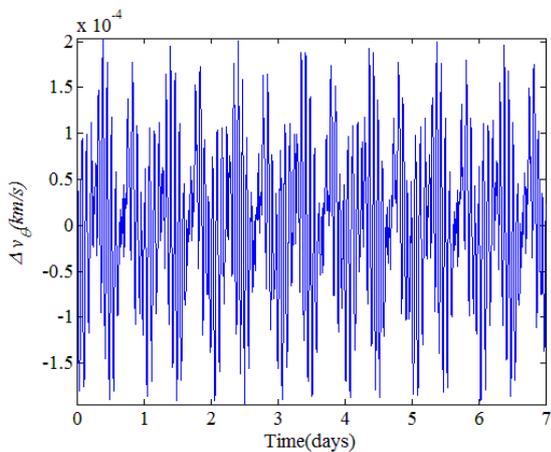
**Figure 13: Time History of Errors in “r” Without Estimation.**



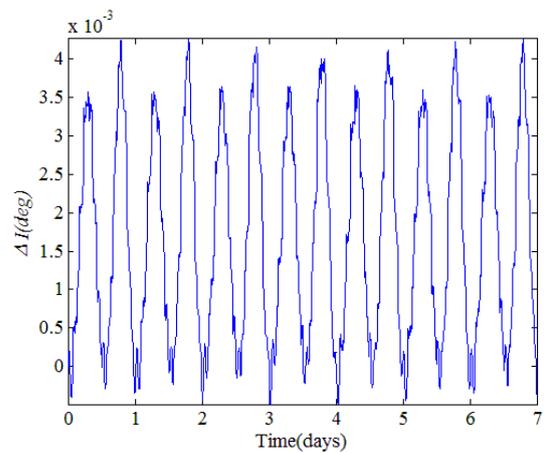
**Figure 11: Time History of Errors  $v_r$ .**



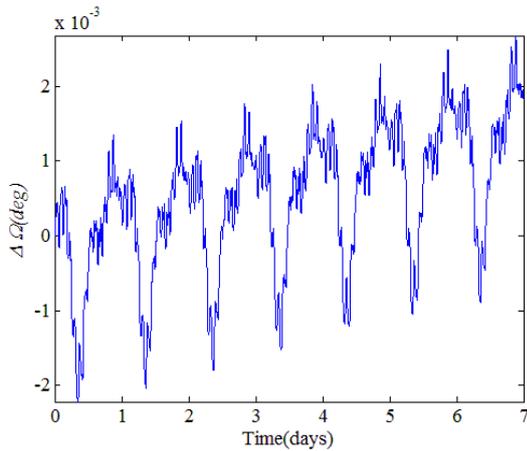
**Figure 14: Time History of Errors in “λ” Without Estimation.**



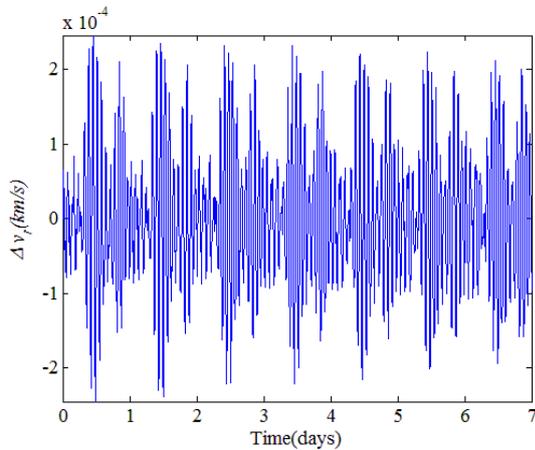
**Figure 12: Time History of Errors in  $v_\theta$ .**



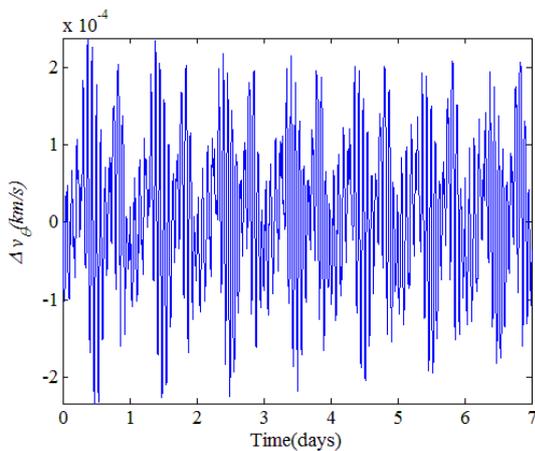
**Figure 15: Time History of Errors in “I” Without Estimation.**



**Figure 16: Time History of Errors in “Ω” Without Estimation.**



**Figure 17: Time History of Errors in  $v_r$  Without Estimation.**



**Figure 18: Time History of Errors in  $v_\theta$  Without Estimation.**

## CONCLUSION

In this paper we have developed an Epicycle Parameter Filter (EPF) for longer propagations of Epicycle orbits including Geo-potential zonal harmonic term  $J_4$ . The high precision accuracy achieved using EPF can easily be extended for higher order zonal perturbative terms. The estimation results show improved Epicycle coordinates compared to the true nonlinear trajectory. The maximum errors were reduced as 26% in  $r$ , 93% in  $\lambda$ , 45% in  $I$ , 20% in  $\Omega$ , 16% in  $v_r$  and 15% in  $v_\theta$ . We have found out that by keeping 10m as the threshold for drift in mean position of satellite in LVLH coordinates fresh estimation of these parameters would be needed after eleven days. The Epicyclic orbit equations can be used on board as a replacement of high precision computationally expensive numerical propagators. It can be conveniently used for computing Epicycle orbital parameters from NORAD TLE fit for long durations. The parameters can be used to update orbital parameters for the space catalogues of commercial and non – commercial spacecrafts. Design constellations based on orbital parameters which is more intuitive rather than using differential equations.

## References

1. Hashida, Y., Palmer, P., “Epicyclic Motion of Satellites About an Oblate Planet,” *Journal of Guidance Control & Dynamics*, Vol 24, No.3, 2001, pp 586-596.
2. Battin, R., “An Introduction to the Mathematics and Methods of Astrodynamics,” AIAA Education Series, Jun 1987.
3. Donahue, W. H., “New Astronomy,” Cambridge University Press, 1992 translation of Kepler, J., “*Astronomia Nova...*”, Heidelberg, 1609.
4. Aiton, E.J., Duncan, A.M., Field, J.V., “The Harmony of the World,” *Memoirs of American Philosophical Society*, Vol.209, Philadelphia, 1997, translation of Kepler, J., “*Harmonices Mundi Libri V.Linz*”, 1619.
5. Montenbruck, O., and Gill, E., *Satellite Orbits Models Methods and Applications*, Springer, Berlin, 2005.
6. Vallado, D, A., *Fundamentals of Astrodynamics and Applications (Third Edition)*, Space Technology Library, Published by Microcosm Press Publisher 2007.
7. Kozai, Y., “The Motion of a Close Earth Satellite,” *Astronomical Journal*, Vol. 64, No.1274, 1959, pp. 367-377.

8. Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," *Astronomical Journal*, Vol. 64, No. 1274, 1959, pp. 378-397.
9. King-Hele, D.G., "The Effect of Earth's Oblateness on the Orbit of a near Satellite," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol.247, 1958, pp.49-72.
10. Hashida, Y., "Analytical Solution for Autonomous Determination of Near Circular Orbits," PhD dissertation Apr 2003, Surrey Space Centre, University of Surrey, UK.
11. Shalom, B., Li, X.R., Kirubarajan, T., "Estimation with Applications to Tracking and Navigation," Wiley-Interscience Publication 2001, John Wiley & Sons, Inc.
12. Thomas, G.B., Finney., R.L., "Calculus of Analytical Geometry," 9<sup>th</sup> Edition, Addison-Wesley Publishing Company, Jun 1998.
13. Surrey Satellite Technology Limited (SSTL), EADS Astrium, <http://www.sstl.co.uk/> [Viewed on 23 May 2011].
14. North American Aerospace Defence Command (NORAD), Peterson Air Force Base, Colorado USA.  
<http://celestrak.com/NORAD/elements/dmc.txt>  
[Viewed on 2 Jun 2011].