

Development of a Drift-Free Stellar Gyroscope

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ABSTRACT

A Stellar Gyroscope is a star based attitude propagator that is capable of propagating a spacecraft's attitude between camera frames by tracking the motion of the stars in the field of view. An algorithm to calculate the attitude maneuver in three degrees of freedom from successive star field images is described. Unlike traditional gyroscopes, the Stellar Gyroscope is capable of drift free attitude propagation when at least three stars are in the field of view. A larger number of stars increases the accuracy of the rotation matrix estimate in a least squares sense. Simulated images based on the SKY2000 Master Star Catalog are used to evaluate the performance of the algorithm. Finally, the operation constraints are discussed and a camera components selection is offered.

INTRODUCTION

The concept of an Image-based Stellar Gyroscope is to infer a spacecraft's rotation and attitude changes from the apparent motion in the camera's field of view. This is a novel scheme in attitude propagation (unlike Star Trackers), where by tracking stars in the field of view between camera frames, the change in orientation between frames can be uniquely resolved in all three degrees of freedom.

The stellar gyroscope can be used to propagate the attitude from a known initial condition without drift. Normally, in the absence of an absolute attitude measurement, attitude is propagated by integrating gyroscope angular rate data (typically MEMS based for small satellites). This results in a drift in the attitude estimate (at best 0.5 degrees per minute), which is essentially a loss of attitude knowledge after a sufficient amount of time. The image based approach can propagate attitude without drift because absolute attitude changes are measured and not angular rates, eliminating the need to perform integration of a noisy signal which causes the drift.

The sequence of images in Figure 1 illustrates this concept. One can intuitively realize that the camera's change in orientation between every frame and the next indicate a unique change in orientation.

Research by Liebe et al. from the NASA Jet Propulsion Laboratory (JPL) studies the feasibility of using a Stellar Gyroscope to estimate high rotation rates.^[1] The basis of operation depends on a single long-exposure

image of the star field and then analyzing the circular arcs caused by the stars' motion. The concept is optimized for high rotation rates outside regular gyro operation ranges, and with the single-exposure method results in a noisy image even for a high quality sensor. The research by JPL, despite the difference in scope, presents the concept of inferring the rotation rate and spin rate from the star streaks. The approach in this paper adopts the idea while eliminating the requirement of taking long exposure images (with very low SNR) by taking a sequence of snap shots and effectively processing a "video" instead of a single long-exposure image.

Star Trackers are traditionally attitude determination systems carrying star catalogs that are used to identify star constellations in order to calculate the spacecraft's attitude in inertial space. Recent research efforts have been focused on improving the hardware and search algorithms of star trackers to increase the update rates to a level where the angular rates can be approximated. Such a high update rate star tracker is sometimes referred to as a Stellar Gyroscope.^[2] Another effort to estimate the angular rate of a satellite using star sensors implements a Kalman filter that models the environmental torques as a random process and depends on the absolute attitude measurements of a high update rate star tracker.^[3] The work in this paper aims to eliminate the need of absolute attitude measurement and a complicated star identification algorithm for attitude propagation.

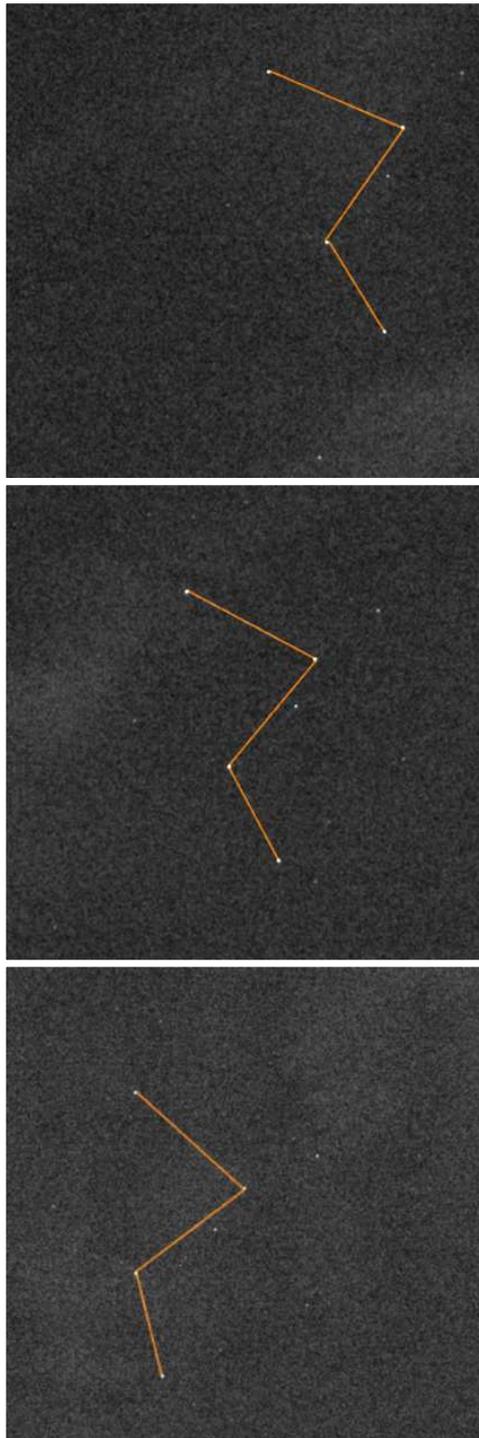


Figure 1: A set of images of the Cassiopeia constellation. The stars appear to be panning and rotating in a way that indicates a unique attitude maneuver of the camera in 3 degrees of freedom.

The field of Machine Vision, namely Egomotion Estimation, while usually based on land systems and images of objects in close proximity, offers valuable insight to the Stellar Gyroscope concept. ^{[4] [5]} A star based attitude propagator has the advantage of eliminating the need to track translation because stars are considered to be infinitely far away, and only rotation is tracked. Also, optical flow techniques don't directly apply to stellar images since such images lack features. The Stellar Gyroscope discussed in this paper is an innovative application of machine vision to spacecraft attitude determination as a unique special case in Egomotion Estimation.

This paper describes an algorithm to identify the change in attitude between two star field images. This will serve as the basis for an autonomous attitude propagation system for small satellites that is capable of drift-free attitude propagation and angular rate estimation.

Using the pinhole camera model, stars are resolved into vectors in body-fixed coordinates. The mathematical model is based on backing out the 3 by 3 rotation matrix (that defines attitude changes) from the "before and after" vectors, which are associated with the stars being tracked in the field of view. Three stars are required as a minimum to find a solution. More stars that are tracked would improve the estimate of the rotation matrix in a least square error sense.

To verify the validity and accuracy of this approach, star field images of known camera orientation are necessary to compare the Stellar Gyroscope results with the actual change in attitude. A mechanical setup would introduce uncertainties, therefore the SKY2000 Star Catalog ^[6] is used to generate simulated star field images that are fed to the Stellar Gyroscope algorithm to perform a one-to-one comparison of the actual attitude change with the estimated attitude change that is found based on the images.

The camera setup and suggested hardware are described in the final section. The Stellar Gyroscope is intended as an experiment on board a future CubeSat to be built at the Space Systems Laboratory at the University of Kentucky, leveraging the heritage of the KySat-1 CubeSat developed by the Kentucky Space Consortium, which failed to reach orbit in March 2011 due to a rocket contingency.

STAR DETECTION

The Stellar Gyroscope operation begins by detecting stars and calculating the unit vectors associated with the stars originating from the spacecraft pointing towards the stars, defined in body-fixed coordinates. The changes of these vectors are tracked and used to infer the rotation changes between frames.

An ideal pinhole camera model is used where a shell of the celestial sphere is mapped onto the camera's sensor as shown in Figure 2. The field of view (FOV) is a function of the focal length and the imaging sensor's physical dimensions.

The vectors associated with each star can be obtained by modeling the camera as in Figure 3. The mapping from pixel coordinates to vector components is done by identifying the origin (the focal point) and the star coordinates on the image plane. One should note that the values of the star indices on the image plane must undergo a transformation to be in units of distance. Finally, the unit vector is found by simply dividing by the vector magnitude.

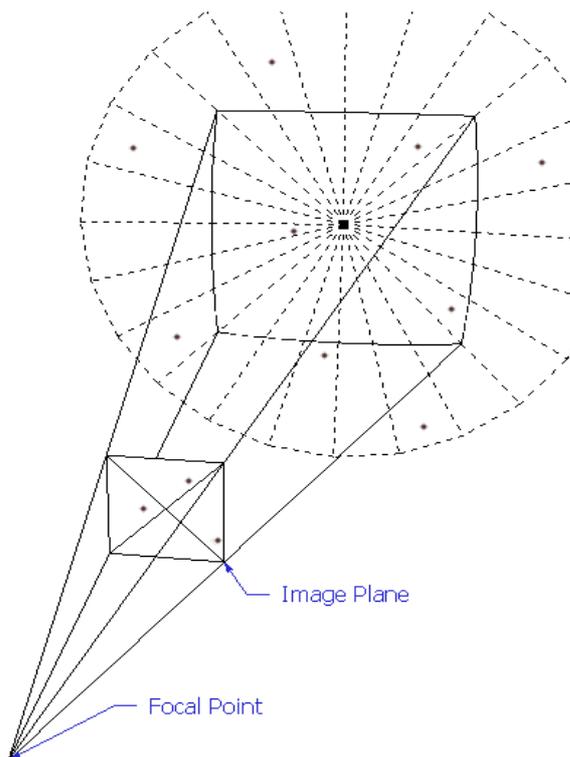


Figure 2: Camera Model. A celestial shell is mapped onto the image sensor

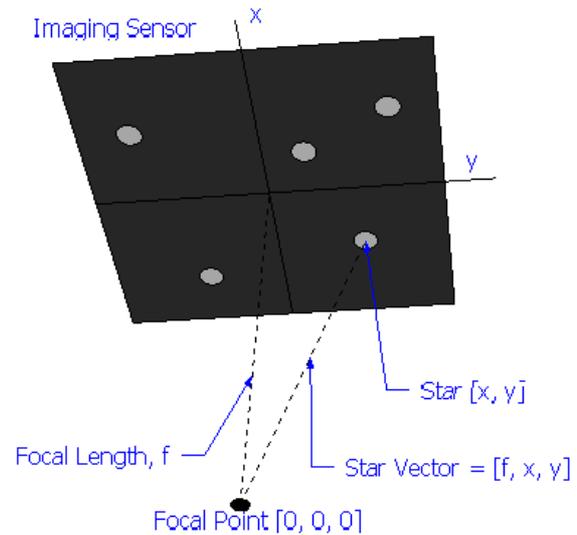


Figure 3: Resolving star vector from pixel coordinates.

The unit vector for a star u using the notation in Figure 3 is then:

$$\vec{u} = \frac{1}{\sqrt{x^2 + y^2 + f^2}} \cdot \begin{bmatrix} x \\ y \\ f \end{bmatrix}$$

where f is the camera focal length, and x and y are the pixel locations in space in units of distance.

A single star affects multiple pixels on the sensor as shown for a 14.7 megapixel image in Figure 4. This is mainly due camera imperfections. A simple algorithm would calculate the star vector by choosing the pixel coordinates of the maximum value in the star vicinity. Using the maximum value results in a reasonable estimate of the star location, and the error would be minimized proportional to image resolution. Sub-pixel resolution can be achieved when a normal distribution curve is fit over the data.^[7] Calculating the expected value as the star location utilizes information in several pixels and results in a more accurate estimate of the star location. This approach is often referred to as centroiding. The expected value can be found by first thresholding the noise to ensure that the noise values will not contribute to the expectation. Also, scaling the star region to sum up to a total probability of unity is necessary. Figure 4 shows a star cross section to illustrate this process, the expected value is found as:

$$E(x) = \sum x \cdot f_x(x)$$

where $f_x(x) = \sum_y f_{xy}(x, y)$

and where x and y represent the pixel coordinates and $f_{xy}(x, y)$ is the thresholded and scaled star distribution. $f_x(x)$ is the marginal probability along the x image axis. The same approach applies to the y image axis.

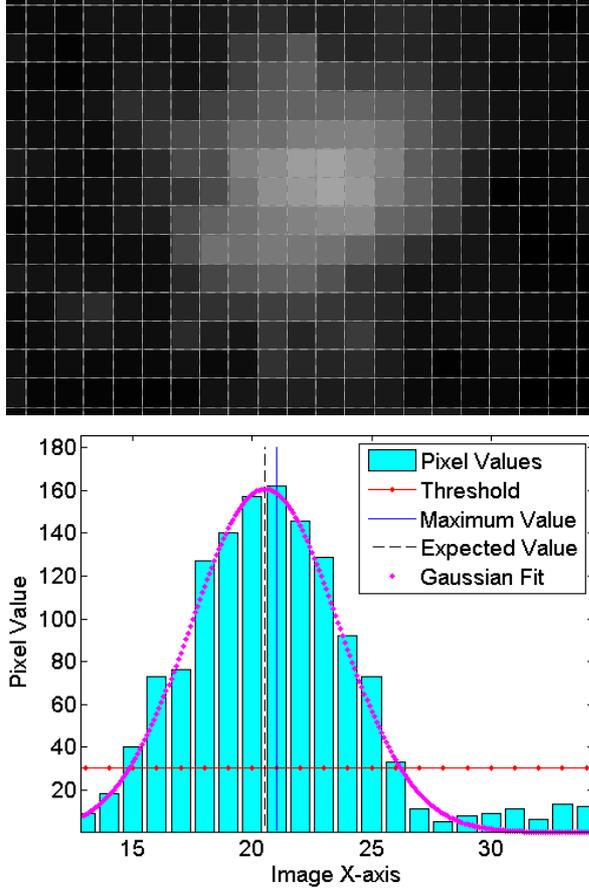


Figure 4: Zoomed in photograph of a star (upper) and an illustration of a normal distribution fit over the star region to obtain sub-pixel resolution (lower).

As it can be seen in Figure 4, the maximum value and the calculated centroid (expected value) deviate from each other. It is noted that there is no need to calculate the covariance matrix of the distribution in the scope of this paper, calculating the expected value is sufficient to identify a star's location to a high accuracy.

MATHEMATICAL MODEL

The mathematical model to find the attitude change between two image frames is based on identifying the stars' motion between frames. A single star moving in the field of view would imply a panning motion of the camera, however a single star cannot imply anything regarding the camera's rotation about its boresight. Therefore, intuitively, multiple stars are needed to estimate the camera's motion in 3 degrees-of-freedom.

The following approach requires at least 3 stars to obtain an estimate of the attitude maneuver. The Direction Cosine Matrix is used for attitude representation. For example, the rotation between frame a and frame b for a star u is given by:

$$\begin{aligned} \vec{u}^b &= \mathbf{C}_{ba} \vec{u}^a \\ \vec{u}^b &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \vec{u}^a \end{aligned} \quad (1)$$

where \vec{u}^a and \vec{u}^b are the unit vectors pointing at the same star in two image frames in body-fixed coordinates. The matrix \mathbf{C}_{ba} hence defines the camera attitude change between those two frames.

It is noted that the star correspondence between the two frames is done by evaluating the proximity of the stars to in each frame to the previous frame, while dropping stars near the edges that may be entering or leaving the camera field of view. For missions with high expected angular rates where star locations are expected to vary significantly between frames, if the frame rate cannot be increased enough to compensate, a more sophisticated algorithm for star association can be developed that performs predictions of where the star will be in the next frame before it measures the proximities.

Using multiple pairs of vectors between the two frames it is possible to solve for \mathbf{C}_{ba} . The goal is to estimate \mathbf{C}_{ba} such that it satisfies the collective changes of all these vectors. For the case of 3 stars u , v , and w , equation (1) can be expanded to be:

$$\begin{aligned} \begin{bmatrix} \vec{u}^b & \vec{v}^b & \vec{w}^b \end{bmatrix} &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \vec{u}^a & \vec{v}^a & \vec{w}^a \end{bmatrix} \\ \begin{bmatrix} u_1^b & v_1^b & w_1^b \\ u_2^b & v_2^b & w_2^b \\ u_3^b & v_3^b & w_3^b \end{bmatrix} &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} u_1^a & v_1^a & w_1^a \\ u_2^a & v_2^a & w_2^a \\ u_3^a & v_3^a & w_3^a \end{bmatrix} \end{aligned}$$

A solution can now be obtained for \mathbf{C}_{ba} by finding the inverse matrix of $[\overline{\mathbf{u}^a} \ \overline{\mathbf{v}^a} \ \overline{\mathbf{w}^a}]$, i.e.

$$\begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{C}_{13} \\ \mathbf{C}_{21} & \mathbf{C}_{22} & \mathbf{C}_{23} \\ \mathbf{C}_{31} & \mathbf{C}_{32} & \mathbf{C}_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1^b & \mathbf{v}_1^b & \mathbf{w}_1^b \\ \mathbf{u}_2^b & \mathbf{v}_2^b & \mathbf{w}_2^b \\ \mathbf{u}_3^b & \mathbf{v}_3^b & \mathbf{w}_3^b \end{bmatrix} \begin{bmatrix} \mathbf{u}_1^a & \mathbf{v}_1^a & \mathbf{w}_1^a \\ \mathbf{u}_2^a & \mathbf{v}_2^a & \mathbf{w}_2^a \\ \mathbf{u}_3^a & \mathbf{v}_3^a & \mathbf{w}_3^a \end{bmatrix}^{-1}$$

This solution is based on inverting a square 3×3 matrix. Therefore, it only works in the case of 3 stars, and it is contingent on the existence of the matrix inverse. The inverse exists if the three vectors are linearly independent. Linear dependence of 3 vectors occurs only when they all lie in one plane. Therefore, the inverse does not exist if the three stars lie on a straight line in the image.

Even though this may rarely occur, the discussion will be resumed to expand the solution for a larger amount of stars which will practically eliminate the chances of all stars in the field of view lining up in a straight line, and will provide a more accurate estimate of \mathbf{C}_{ba} . For stars $s1, s2 \dots sN$, equation (1) can be expanded again to be:

$$\begin{aligned} \begin{bmatrix} \overline{s1^b} & \overline{s2^b} & \dots & \overline{sN^b} \end{bmatrix}_{3 \times N} \\ = \mathbf{C}_{ba3 \times 3} \cdot \begin{bmatrix} \overline{s1^a} & \overline{s2^a} & \dots & \overline{sN^a} \end{bmatrix}_{3 \times N} \\ \mathbf{S}_{3 \times N}^b = \mathbf{C}_{ba3 \times 3} \cdot \mathbf{S}_{3 \times N}^a \end{aligned}$$

This is the case of an over-determined system with more equations than unknowns. The least squares solution is then^[8]:

$$\widetilde{\mathbf{C}}_{ba} = \mathbf{S}^b \mathbf{S}^{aT} (\mathbf{S}^a \mathbf{S}^{aT})^{-1} \quad (2)$$

EXPERIMENT

In order to evaluate the accuracy of the star detection approach and the rotation matrix derivation described thus far, a pair of simulated images is created based on the SKY2000 Master Star Catalog.^[6] Because the attitude difference between the two images is known, the estimate found using the Stellar Gyroscope can be directly compared to the actual attitude maneuver. Figure 5 shows two images of Polaris in the northern sky. The images are created with the hardware implementation described in the following section in mind. The parameters of the simulated images are summarized in Table 1.

The attitude change is extreme in this experiment and star association between frames seems challenging. This is done for illustrative purposes, in the actual operation of the Stellar Gyroscope images would be taken at a rate where the rotation is no more than a few degrees between frames and the proximity based star correspondence is feasible.



Figure 5: Simulated star fields for known orientations using the SKY2000 Master Catalog. The brightest star in the images is the north star, Polaris.

Table 1: Parameters of Simulated Images

Parameter	Value
Image Resolution	5 megapixels
Focal length	16 mm
Field of View	15 ° × 15°
Star Magnitude Threshold	5.75
Star Size	1°

Using all 8 stars in the two images, the direction cosine matrix C_{ba} was estimated using equation (2). To display the results in a more readable and intuitive format, the rotation matrix is converted to the 1-2-3 Euler Angle representation which describes the orientation maneuver as a sequence of rotations around the body X, Y, and Z axes respectively.^[9] In this experiment, the camera is pointing along the body X-axis. The actual attitude change between the two images is described by the angle set:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 16.067487148167718^\circ \\ 0.162200887147300^\circ \\ 0.989417931361931^\circ \end{bmatrix}$$

The estimated attitude change between the two images, using the Stellar Gyroscope algorithm is:

$$\begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \\ \widetilde{\theta}_3 \end{bmatrix} = \begin{bmatrix} 16.018860946764185^\circ \\ 0.158859418630225^\circ \\ 0.990416316507994^\circ \end{bmatrix}$$

The error between the actual and the estimated orientations for the worst case (θ_1) is 0.0486°, or 0° 2' 54.9594". The error originates mainly from inaccuracies in calculating the star vectors from the images.

DRIFT-FREE PROPAGATION

Because this approach does not involve an integration process of a noisy signal, there is no drift. Typically, when the attitude is propagated from angular rate measurements, the measurement noise compounds and causes drift in the attitude estimate.

Resolving attitude changes from star motion is not free of error as shown in the previous section. However, subsequent estimates of attitude would refer back to the first set of vectors from the first image, which is how the error does not compound.

The main contrast between the Stellar Gyroscope and standard angular rate measurement gyroscopes is in the inherent preference to operate at low angular rates, which is typical for spacecraft. The main challenge in gyroscope technology is in sensing minute angular rates near the noise level. This is even a greater challenge in small satellites at low mass and volume. The Stellar Gyroscope has the advantage that it prefers to operate at low angular rates rather than high rates like typical gyroscopes and is therefore more suitable for the small satellite domain.

No method of absolute attitude measurement has been discussed thus far, and it is left as future work. However, once an absolute measurement is obtained, using a Sun sensor, Earth horizon sensor, or an occasional star constellation matching algorithm, the attitude can be propagated from that known state by observing the motion of the stars in the focal plane.

ON-ORBIT EXPERIMENT

The Stellar Gyroscope is being developed at the Space Systems Laboratory (SSL) at the University of Kentucky and is intended to be a payload on one of the lab upcoming CubeSat missions. The mission will entail image collection of star fields to analyze the work that has already been done, and the collection of Earth surface, Earth horizon, and Moon images, in order to enable further development to use non-star elements as features in the object tracking algorithm, and extending the system to perform absolute attitude estimation given known objects. The on-orbit data will be vital to the SSL's research towards a camera-only attitude determination system.

Hardware

The Stellar Gyroscope system being developed consists of a low-cost camera assembly and processing hardware, it is designed to occupy mass and volume comparable to a set of three orthogonal gyroscopes. This is a convenient solution for small satellite missions where Laser Ring Gyroscopes are not feasible and (noisy) MEMs technology is the only alternative.

Table 2: Summary of Stellar Gyroscope CubeSat Hardware

Margin	Letter
Sensor	Aptina MT9P031 Monochrome 5 megapixel CMOS detector
Optics	16 mm focal length
Field of View	15° by 20.2°
Pixel Dynamic Range	70.1 dB

The camera is based on the Aptina MT9P031 Monochrome 5 megapixel CMOS sensor and a lens with a focal length of 16mm. This configuration results in a 15° by 20.2° field of view. Table 2 summarizes the camera specifications.

This configuration is similar to commercially available Star Trackers; it has been shown that with the selected sensor, field of view and an exposure time of 100 ms, star magnitudes brighter than 5.75 can be captured, and at least 3 stars will be visible in over 99.99% of the sky.^[10] This is a relatively short exposure and can tolerate slew rates up to 1 °/second for star magnitudes 5.75 and brighter.

Slew rates beyond 1 °/second can be tolerated using the same configuration with loss of dim stars. The energy from Stars with magnitudes near the 5.75 threshold in this case would be spread across multiple pixels and become indistinguishable from the noise floor. To ensure that three stars or more remain in the field of view, a wider field of view can be used. Another approach to tolerate higher slew rates would be to increase the exposure time to collect more star energy and increase the signal-to-noise ratio. This comes at the cost of motion blur which is tolerable using the described centroiding algorithm where the center of the blur is identified as the star position. Because the Stellar Gyroscope is based on relative measurements and sudden changes in rotation rates are not expected, slew rates well-beyond 1°/second should be tolerable. Future work will address this design space.

CONCLUSION

An algorithm to resolve attitude changes from the motion of stars in a camera field of view has been presented. The algorithm is capable of finding an estimate at a minimum of 3 stars as long as they do not lie on a straight line in the image. More stars in the focal plane improve the estimate in a least squares sense.

Using the described approach, attitude can be propagated without an integration process which is the main cause of drift in typical attitude determination systems.

At a time where gyroscope technologies are facing physical limits in attempts to measure low rotation rates, the Stellar Gyroscope operates well in that domain and is therefore more suitable for spacecraft. It is intended as a low cost alternative for small satellites.

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