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PROGRESS REPORT ON  
STUDIES OF  
HYDRAULIC GEOMETRY OF LARGE  
BED-ELEMENT STREAMS

by

Dean F. Peterson<sup>1/</sup>

June 30, 1970

Foreword

The purpose of this report is to summarize studies and correlations made by its author principally during 1967 and early 1968. The author has been interested in the hydraulics and channel-forming processes of mountain streams since 1959; and in the years mentioned above, attempted to examine data collected and theory to see if some useful rationale for how streams of an apparent hydraulic class -- Large Bed Element streams -- are formed could be drawn. No conclusions were reached, but some interesting possibilities were postulated and some evidence collected. Further effort since early 1968 has not been possible, nor does the prospect for future detailed attention by the author seem promising. This report is written as a sort of comptes rendus in case it might be useful to someone who may wish to pursue this matter further.

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*Not for distribution  
nor  
discussion*

*Internal  
Use Only*

### Introduction

Leopold and his colleagues<sup>2/</sup> identify three types of flow resistance in open channels. Besides skin resistance, which depends on boundary roughness and the velocity squared, internal distortion resistance caused by boundary features which set up eddies and secondary circulation, and spill resistance occurring as the result of obstructions which cause overflows, occur in open channels. Many streams, especially in mountainous regions, are characterized by large bed elements which individually contribute resistance of the second and third types. These are referred to as large bed element (LBE) streams, Fig. 1<sup>3/</sup> Tentatively, streams in which  $d/D_{50}$  exceeds about 30 have been included in this class. Besides acting individually, large-bed elements may form bars or aggregations to act in combination.<sup>4/5/</sup>

Langbein<sup>6/</sup> has stated that hydraulic considerations alone are insufficient to determine the channel geometry of a stream and a river channel tends toward an equal adjustment of velocity, depth, breadth, and slope in

<sup>2/</sup> Leopold, Luna B., M. Gordon Wolman, and John P. Miller. Fluvial Processes in Geomorphology. W. H. Freeman and Company. San Francisco. 1964. p. 162.

<sup>3/</sup> Ibid. pp. 203-209.

<sup>4/</sup> Judd, Harl E. A Study of Bed Characteristics in Relation to Flow in Rough, High-gradient, Natural Channels. A dissertation presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy. Utah State University. Logan, Utah. 1963. p. 87.

<sup>5/</sup> This class of streams has been called "steep, rough streams," and "rough, high-gradient streams." Neither is particularly appropriate since, relatively speaking, the gradient may not be particularly high nor the hydraulic roughness unusually great.



Logan River



Providence Creek

Figure 1. Typical LBE streams.

accommodating a change in stream power. He and others<sup>6/7/8/</sup> have applied thermodynamic and least action concepts to the description of hydraulic geometry of natural streams. Leopold and Maddock<sup>9/</sup> have noted broad generalizations of the discharge  $Q$  by the relations

$$V = C_m Q^m \quad (1a)$$

$$d = C_f Q^f \quad (1b)$$

$$w = C_b Q^b \quad (1c)$$

$$s = C_z Q^z \quad (1d)$$

where  $V$ ,  $d$ ,  $w$  and  $s$  are the velocity, depth, width and slope respectively,  $C_m$ ,  $C_f$ ,  $C_b$  and  $C_z$  are coefficients and  $m$ ,  $f$ ,  $b$  and  $z$  are exponents. From continuity,  $m + f + b = 1$  and  $C_m C_f C_b = 1$ . These generalizations may be applied to variations among  $V$ ,  $d$ ,  $w$  and  $s$  with  $Q$  at a particular section or reach, within a river system or to broad general classes of rivers.

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<sup>6/</sup> Langbein, W. B., "Geometry of River Channels." Proceedings, Am. Soc. Civ. Engrs, Vol. 90. HY 2. March, 1964.

<sup>7/</sup> Leopold, L. B. and W. B. Langbein. "The Concept of Entropy in Landscape Evolution," Professional Paper 500-A, Geol. Survey. U. S. Department of the Interior, Washington, D. C., 1962.

<sup>8/</sup> Leopold, Luna B., M. G. Wolman, and John P. Miller. Fluvial Processes in Geomorphology. W. H. Freeman and Company. San Francisco, 1964. pp. 268, 307, 431.

<sup>9/</sup> Leopold, L. B. and Thomas Maddock, Jr., "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," Professional Paper 252, Geol. Survey. U. S. Dept. of the Interior, Washington, D. C. 1953.

Eqs. 1 (a, b, c, d) are useful in examining the operation of least action principles under different conditions of physical restraint. Using statistical theory, Langbein<sup>6/</sup> postulates that; in the absence of required hydraulic relationships between the terms  $V$ ,  $d$  and  $w$ ;  $m$ ,  $f$  and  $b$  should tend toward equality. The time rate of expenditure of energy per unit length of stream,  $P = Qs = \gamma Vd s$ , should be a minimum within those constraints. The four elements of stream power are reflected by Eqs. 1a, b, c and d.

If large-sized elements are supplied by weathering of canyon walls or because the stream flows through a coarse alluvium deposited by heavier fluvial or glacial action of the past, the river may find the larger bed elements relatively intractable; moving the smaller ones, but leaving the larger ones to form relatively fixed beds, which are modified only by the highest flows. In this case, variation of slope within a particular reach must accommodate to the grand optimization of the total river. Given  $Q_m$ , a bed-forming discharge, a river reach has quite limited freedom to adjust its slope, but has more freedom to adjust width, depth, velocity and bed-element size  $k$ . An optimization among the latter four variables should lead to a relationship among  $Q$ ,  $k$  and  $s$  which could be quite general. Under this postulate, climatic and geophysical factors other than the availability of large, relatively non-transportable sediments would not be involved.

### A Possible Model

During 1967 and 1968, the writer spent considerable time and effort studying a possible model which might be applied to channel formation in the LBE region and looked at some fairly readily-available data as they related to that data. Included in the model were considerations of the hydraulic geometry equations, least action, certain hydraulic constraints as proposed by Leopold and Langbein<sup>7/</sup>, and dimensional homogeneity.

#### Dimensional Analysis

If  $Q(t)[L^3 T^{-1}]$ , the discharge during a bed-forming flood peak is imposed on a sediment characterized by  $k^*[L]^{10/}$  containing relatively large particles (potential LBEs) one could expect that erosion would take place until a stable channel characterized by bed elements of size  $k^*$  is formed at a water surface slope  $s$ . The width  $w$ , depth  $d$ , and velocity  $V$  would be dependent variables; thus

$$F_1(Q(t), k^*, s, \rho_w, \rho_s, \nu, g) = 0 \quad (2)$$

where  $\rho_w$  and  $\rho_s$  are the densities of the sediment and water respectively, and  $g$  and  $\nu$  are the acceleration of gravity and the kinematic viscosity.

Dropping  $\rho_w$  and  $\rho_s$  tentatively because they do not vary appreciably, assuming that  $Q(t)^{10/}$  can be represented by the peak flood flow  $Q_m$  and

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<sup>10/</sup> The star (\*) indicates both mean size and size distribution.  $Q(t)$  indicates a time variable pattern of discharge. Dimensions are given in the brackets [ ].

$k^*$  by  $k$ , the  $D_{50}$  or  $D_{84}$  bed particle size and arranging in non-dimensional combinations gives

$$F_2(Q_m k^{-1} v^{-1}, s, Q_m g^{-1/2} k^{-5/2}) = 0 \quad (3)$$

$Q_m k^{-1} v^{-1}$  and  $Q_m g^{-1/2} k^{-5/2}$  are in Reynolds and Froude number form respectively.

Since drag force and particle weight are both linearly dependent on gravity, one would not expect gravity to influence the bed configuration unless gravity waves were involved. (Conventional Froude number in the form  $V\sqrt{gd} \approx 1.0$ ). For 50 streams, reported by Barnes<sup>11/</sup> conventional Froude number at peak discharge varied from 0.103 to 0.794. On the other hand, stream drag is transferred to the bed through viscosity, so that a Reynolds number form is indicated. With this argument a relationship of the form  $F_2(Q_m k^{-1} v^{-1}, s)$  seems preferred.

#### Least Action Implications

In a complex river system, the particular action minimized at any point in space and time is not clear. Realistically, no single action, but a combination of actions may be involved.<sup>6/</sup> The optimal balance for a river system as a whole will doubtless be different than that for a particular section or reach. The overall process proceeds indomitably but must accommodate to continual perturbations as new potentials are added by geological and climatic changes. Langbein<sup>6/</sup> and others<sup>8/</sup> have

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<sup>11/</sup> Barnes, Harry H., Jr. Roughness Characteristics of Natural Channels. Water Supply Paper 1849. Geol. Survey. U. S. Department of the Interior. Washington, D. C. 1967.



discussed some of these actions and their consequences. Two of these concepts - equal stream power distribution per unit length and equal stream power distribution per unit area are summarized for background following which some other possibilities are considered.

Equal Stream Power Distribution per Unit Length or per Unit Area--

The postulate is that the river system will transport the largest possible discharge at each section with the least expenditure of power. Stream power per unit length  $P = \gamma w d V s$  and

$$\gamma^{-1} \partial P / \partial Q = d V s \partial w / \partial Q + w V s \partial d / \partial Q + w d s \partial V / \partial Q + w V d \partial s / \partial Q = 0 \quad (4)$$

In Eq. 3, the relevant  $Q$  is  $Q_m$ , the maximum or bed-forming  $Q$ .  $Q_m$  implicitly varies as  $x$ , the distance along the stream; thus

$$\partial P / \partial x = \partial P / \partial Q \cdot \partial Q / \partial x = 0$$

since  $\partial P / \partial Q = 0$ . In this case, minimization of power per unit of  $Q$  leads to the conclusion of equal power throughout the length of stream.

The argument for equal power per unit of stream area is more direct. The stream does the same amount of work on each unit area of its bed. This makes sense if each unit of area is equally erodible, otherwise one would expect more work to be done in the more difficultly erodible places. As  $Q$  changes, along the stream length, then the condition that  $\partial(P/w) / \partial Q = 0$  is implied. For convenience this may be written in the form  $\partial(P/w) / \partial Q = \partial P / \partial Q / w - P \partial w / \partial Q / w^2$ .

Consideration of LBE size--One might argue that an LBE stream would expend power in relation to the size and amount of sediments eroded and transported from the alluvium in which it is formed. A measure of such a quantity is  $k$ . For equal power related to bed size per

unit length of stream  $\partial(P/k)/\partial Q$  is needed; if equal power per unit bed area applies, then the needed expression is  $\partial(P/kw)/\partial Q$ .

Hydraulic Geometry Equations--Eqs. 1 express  $V$ ,  $d$ ,  $w$  and  $s$  as power functions of  $Q$ . If these apply, they may be used to make the optimization equations more explicit. Following the Leopold and Langbein approach from these equations  $\partial V/\partial Q = mC_m Q^{m-1}$ ; similar expressions can be obtained for  $\partial d/\partial Q$ ,  $\partial w/\partial Q$  and  $\partial s/\partial Q$ . Substituting in Eq. 3 yields for the case of equal power per unit length

$$m + f + b + z = 0, \text{ or since } m + f + b = 1$$

$$z + 1 = 0 \quad (5)$$

For the case of equal power per unit area

$$1 + z - b = 0 \quad (6)$$

This same approach may be used to relate power expenditure to bed size if one can make the case that  $k = C_p Q^p$ . (1e)

The case for distribution of power along the stream length yields

$$1 + z - p = 0 \quad (7)$$

and for equal power on a bed area basis

$$1 + z - b - p = 0 \quad (8)$$

Hydraulic Constraints--Hydraulic equations relate velocity to channel characteristics. The basic concept is expressed by the Chezy equation

$$V = (C/g^{1/2}) \sqrt{Rs} \quad (9)$$

where  $R$  is the hydraulic radius and  $C$  is a coefficient; which, by tradition, has the dimension  $L^{1/2}/T$ . Much has been written about the nature of the

non-dimensional combination  $C/g^{1/2}$  and its many variants. There is justification<sup>4/</sup> that in the LBE range  $C/g^{1/2}$  varies as a power function of the relative roughness  $d/k$ , i. e.

$$C/g^{1/2} \sim (d/k)^a \quad (10)$$

From Eqs. 9 and 10, if one can assume for a wide stream that  $d$  approximates  $R$ , using the hydraulic geometry equations,

$$C_m Q^m \sim (C_f Q^f / C_p Q^p)^a (C_f Q^f / C_z Q^z)^{1/2}$$

which implies

$$m = (a + 1/2) f + z/2 - ap \quad (11)$$

This approach differs from that of Leopold and Langbein in that Chezy's equation, rather than Manning's, is used.

Comments--From the dimensional analysis, one would infer that the least action criteria ought to appear in Reynolds number form. This is consistent for both Eqs. 7 and 8, if one assumes  $Q/kv = C_1 s^{-\beta}$ ; i. e., that the functional relationship of Eq. 3 is a power form<sup>12/</sup>. Substituting from Eq. 1 with  $\beta = 1$ ,  $QC_z Q^z (C_p Q)^{-p} = C_1 v$ , which is satisfied if  $z + 1 - p = 0$ . Otherwise,  $Q(C_z Q^z)^\beta (C_p Q)^{-p} = C_1 v$ . Here  $1 + z \beta - p = 1 + z - p - b = 0$  and  $\beta = (z-b)/z$ . This implies that for power/bed size distributed uniformly along the stream length, Eq. 3 would appear in the form  $Qs/k = C$ ; for an areal distribution the form would be  $Q/k = Cs^{-(b-z)/z}$ .

<sup>4/</sup> p. 40.

<sup>12/</sup>  $C$  signifies only a non-dimensional constant but does not imply a value unless specifically stated otherwise.

Experimental and Field Information

Several years ago an associate<sup>13/</sup> and the writer conducted a series of experiments using a sloping flume. A constant discharge of clear water was allowed to flow over a bed composed of a designed distribution of non-cohesive particles of sand, pebbles and small gravel until a stable, paved channel was formed. A recent review of these experiments supports the postulate that  $Qk^{-1} = C_1 s^{-\beta}$  in which  $k$  is taken as the  $D_{50}$  size of a sample of the paved bed,  $s$  is the slope of the water surface and  $\beta$  is an exponent.  $C_1$  has the dimensions of kinematic viscosity  $\nu$ , so that

$$Q_m k^{-1} \nu^{-1} = C s^{-\beta} \quad (12)$$

$Q_m k^{-1} \nu^{-1}$  is in the form of a Reynolds number.

Recent publication by the U. S. Geological Survey<sup>11/</sup> of information on flood discharges, bed material sizes and channel geometry for some natural streams provides an opportunity to examine the relevance of Eq. 3 and some of the concepts set forth by Langbein as they apply to large-bed element streams. Additional data examined include those published by Blench and Qureshi<sup>14/</sup> and others.

<sup>13/</sup>Hariri, Davoud. Relation Between the Bed Pavement and the Hydraulic Characteristics of High-gradient Channels in Non-cohesive Sediments. Dissertation prepared in partial fulfillment for the degree of Doctor of Philosophy. Utah State University, Logan, Utah. 1964.

<sup>14/</sup>Blench, Thomas and Moh'd Ali Qureshi. Practical Regime Analysis of River Slopes. Journal of Hydraulics Division of the American Society of Civil Engineers. March, 1964. p. 95.

Table 6. Application of Optimizing Criteria to Hariri and USGS Data

<u>Criterion</u>	<u>Hariri</u>	<u>USGS</u>
1. Equal power/length $1 + z = 0$	$1 - 0.57 \neq 0$	$11 - 0.98 \rightarrow 0$
2. Equal power/area $1 + z - b = 0$	$1 - 0.57 - 0.53 \rightarrow 0$ $- 0.10 \rightarrow 0$	$1 - 0.98 - 0.53 \neq 0$
3. Equal power/length x k $1 + z - p = 0$	$1 - 0.57 \neq 0$	$1 - 1.11 + 0.11 = 0$
4. Equal power/area x k $1 + z - p - b = 0$	$1 - 0.57 - 0.38 \rightarrow 0$ $0.05 \rightarrow 0$	$1 - 1.11 - 0.53 + 0.11 \neq 0$ $0.29 \neq 0$
5. Hydraulic constraint, $C/\sqrt{g} \sim (d/k)^a$ $.m = (a + 1/2) f + z/2 - ap$	$0.40 \rightarrow (1.15 + 0.50) 0.33 - 0.57/2$ $0.40 \rightarrow 0.36$	$0.09 \neq (0.33 + 0.50) 0.38$ $- 1.11/2 + (0.33) 0.11$ $0.09 \neq -0.29$