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A Collection and Analysis of Local Middle Grade Math Projects for the Common Core

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A COLLECTION AND ANALYSIS OF LOCAL MIDDLE GRADE MATH PROJECTS FOR THE COMMON CORE

by

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ABSTRACT

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The State of Utah has changed the mathematics core curriculum several times over the past decade. The latest change was the adoption of the Common Core State Standards introducing both grade level content standards and standards of mathematical practice that emphasize how students are to study and reason with mathematics at all grade levels. According to state officials, these standards require more mathematical reasoning, problem solving, and deeper understanding than previous core curriculum documents. One way to address this change is for teachers to educate using projects during their instruction as unit starters, daily lessons, and for evaluation purposes. Yet finding instructional materials takes considerable time and effort; hence, I have gathered, analyzed, and classified several projects teachers currently use in their classrooms according to the Common Core State Standards and presented them here in this report.

This report contains five projects that cover different mathematical components of the new core. The Invention Project is a statistics lesson comprised of a student survey along with several types of statistical follow-up questions that allow students to represent, display and interpret data. The Dilation project is an application based on dilating a greeting card with a scale factor by using proportions and similar figures. The Interior and Exterior Angles Project leads students to find the sum of interior and exterior angles of convex polygons after students have learned the Triangle Sum Theorem. The Celebrity Project uses an applet to show lines of best fit, and then engages the students in creating their own data of ordered pairs in a scatterplot, once the data is obtained students learn the procedure for generating a regression line. The Kite Flying Project allows for multiple learning levels as students pursue the Pythagorean Theorem with a hands-on approach. Materials for project implementation such as student task sheets, scoring rubrics, and the alignment with Common Core State Standards are included in this report.

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Lastly, I would like to thank all the teachers that submitted projects. I believe you are great teachers and want to make a difference in your students’ lives. Thank you for being willing to share your ideas with others.

Lynette Checketts

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CHAPTER 1

INTRODUCTION

During my experience as a secondary mathematics teacher for the past ten years, the Utah Mathematics Core Curriculum has changed three times. From the original core I knew in 2002 to the adjusted version of 2007, curriculum differences seemed only minor. I remember looking at the modified core and realizing that I would have to add in some areas to my current curriculum, while other components were no longer a part of the program. Nonetheless, I found the time in my courses and topic sequencing to make additions to the content without completely omitting topics I valued that were no longer required. Overall the way I taught did not change based on the 2007 core.

The 2011-2012 school year brought about a different change for the State of Utah. The Common Core State Standards (CCSS) were adopted and Utah decided to additionally adopt the International or Integrated model as an organizational plan for the high school grade levels. This core curriculum seemed radically different from the previous two. The ninth (Secondary Math I) and tenth grade (Secondary Math II) courses now blended content from Algebra I, Geometry, and parts of Algebra II split over the two grade levels. Topics, such as factoring, that had been previously taught in Algebra, no matter the age level of the student, would now be part of the 10th grade course. My first cursory glance at the CCSS gave me the impression that the authors threw all the old objectives in a blender and let them spit out wherever they may. But a second more thorough examination shows that there was effort put into the outline and organization of the objectives.

These state standards recommend increased rigor in classroom instruction and application of higher-order thinking skills that are aligned with college and career expectations so that all students are prepared for success upon graduating from high school (CCSS, 2011). According to state education officials, a deeper, more meaningful level of conceptual learning is emphasized compared to the previous Utah State Core Standards of 2007. With the new common core both conceptual understanding and procedural skills are stressed equally which counters the "mile wide, inch deep" (CCSS, 2011) criticism leveled at most current U.S. standards. As Webb (1997) states, reforms are partly based on the idea that student learning will be improved by creating consistent systems of expectations and assessments.

Along with the change to the curriculum there are eight Standards for Mathematical Practice that also have been implemented as a companion piece to the mathematical standards.

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

These practices describe the standard mathematical practices expected of students in classrooms at all grade levels. To realize such practices, teachers should place a larger priority on how students investigate, evaluate and solve problems, rather than merely procuring correct solutions. These adjustments have forced teachers to not only look at what they teach but how they teach.

The changes in both mathematical and practice standards have opened up a greater need for investigative learning projects to be used in the classroom. Many teachers have created different learning activities for mathematical topics that are no longer a part of their grade level, but these high quality projects which are teacher built and time tested are needed with appropriate alignment to the new state curriculum.

The literature review shows that teachers need the opportunity to create projects so they can relate to the learning process. It was also found that teachers will use projects created by other teachers if there are rubrics associated with objectives they can relate back to their teaching. Teachers need a place to upload and download current projects that they are working on for other teachers to use.

Collecting, revising and aligning teacher created projects was my main goal for this report. I found five teacher created projects that have been used in the classroom, aligned them to the new core standards, and made revisions to share with other colleagues. The projects cover content appropriate for grades 6^{th} through Secondary Math II and cover a variety of mathematical topics. I have standardized the project format so each contains mathematical and learning level objectives, lesson outlines, rubrics and handouts where applicable.

CHAPTER 2

LITERATURE REVIEW

Classroom practices greatly influence not only what students learn but how they will learn. Fennema (1999) describes five different ways mathematical understanding is developed:

a) constructing relationships,

b) extending and applying mathematical knowledge,

c) reflecting about experiences,

d) articulating what one knows, and

e) making mathematical knowledge one's own.

Classroom projects are designed and used often to incorporate most if not all of these five pathways to understanding throughout the course of the task. For the purpose of this report, projects are considered to be any learning activity or extended lesson that teaches one or more standard from the CCSS and involves the students as active participants in the learning process. Often a project includes the use of manipulatives or technology, but this is not required. Furthermore, projects may be used for different purposes as some teachers may use them to enhance a lesson about material already taught, introduce new concepts, review background material, or as a final assessment.

Some general principles of mathematics education support the use of projects in instruction. Mathematics classrooms should reflect the field of mathematics and engage students in activities that involve mathematical work. Since mathematics can be thought of as a discipline of reasoning and logic, communication is another key component to classroom mathematics. Also, learning with understanding is accomplished by engaging students in high-order thinking and opportunities to make the mathematical content of the lesson one's own.

Remillard (2004) states school mathematics should be thought of as a human activity that reflects the work of mathematicians. Students should find out why given techniques work, invent new techniques and justify the conclusions that they deduced. Mathematical work in classrooms should also reflect how mathematicians go through investigations, calculations and justify the final solutions to problems they encounter and invent, in other words, students should be allowed to reason through a problem.

As Beckman (2012) states it is the heart of mathematical reasoning to ask the right questions. To teach using reasoning requires a classroom atmosphere where learners are active participants in the development of mathematics and have time to reflect on the reasoning behind their judgments. One form of reflection is articulation. Articulation requires reflection in that it involves pulling out the critical elements of an activity so the core concepts and ideas can be communicated. As Fennema (1999) states, in order to articulate ideas, one must reflect on them in order to identify and describe critical elements. The ability to communicate one's ideas is an important goal of education and is also a benchmark of understanding. This can be done in a variety of ways: verbal, textual, and visual such as pictures, diagrams, or models.

Students who are equipped with high-order thinking strategies which build upon background knowledge find success as they understand mathematics and begin to reason and think in multifaceted ways. Fennema and Romberg (1999) emphasize that for students to learn with understanding they have to relate what they are learning to their existing knowledge in ways that extend and support the application of that knowledge.

For example, this would be accomplished through a lesson that requires students to find a connection between the Pythagorean Theorem, finding the distance between two points on the coordinate plane, and then how that relates to lengths of authentic objects. Improper implementation of complex thinking tasks can lead to out-of-control classrooms, but findings by Stein et al. (1996) suggest that students can work on high-level tasks without becoming disruptive and nonproductive.

Projects are beneficial to both teachers and students as they help teachers better prepare their students to apply reasoning for addressing complicated problems with multiple solution possibilities. Remillard (2004) believes that creating student projects as part of the curriculum provides “teachers with opportunities to learn about the teacher’s role in orchestrating student learning.” When students can see that a problem can be solved in more than one way it empowers them to think differently than those around them. Students learn to devise problems and develop and apply strategies to find solutions in a broad range of contexts. As they explore problems on their own, they can interpret results and verify solutions, while learning to apply mathematical modeling and address real-world problem situations. Teachers can use projects with open-ended prompts to ensure that students stay focused on their mathematical process, “How did you do that?” not, “What is your answer?” As Stein et al. (1996) states, “multiple-solution strategies would be seen as one way of helping students to develop the view that mathematics involves making decisions about how to go about solving problems, not simply employing teacher-supplied procedures.” (p. 472)

Since project tasks can be solved in multiple ways, investigations which focus on expanding understanding of mathematical ideas and emphasize connections among topics can help the students attain a deeper understanding of the curriculum that they are learning. "The main goal is to prepare students to be college and career ready" (Krupa, 2011 p. 15). Hence, students need to apply the knowledge they have learned in their math classes in many different ways. Life is not full of nice tidy answers where there is a teacher's edition with all the right answers ready and waiting to be opened. Sometimes it is messy and one answer may look different than another's, but if the justification is accurate then it is still correct. Active learning through projects helps prepare students for broader applications, and allows teachers to focus on the classroom discourse.

While projects may be used at any time to enhance learning, some authors advocate using projects as an introductory part of a unit. Fennema (1999) states that every instructional unit should begin with an empirical situation that is engaging to the students and presents a problem that can be referred to throughout the unit. It should be engaging and worthy of investigation. Laumakis (2012) talks about how he does this in his Statistics class:

> One way is to introduce the data early in the course, when students first study single-variable descriptive statistics topics. Later they will be given a detailed description of the data, along with how and why they were collected. Then students can perform the simple summary statistical procedures, including constructing histograms and box plots and generating basic summary statistics. Later in the course, when students are studying statistical inference procedures of constructing confidence intervals and performing hypothesis tests, they can revisit the already familiar data sets and complete their inferential procedures. Finally, whenever two-variable statistical topics are covered, students can perform scatter plot and correlation analysis. Using the posed problem and data sets in this manner provides a course-long themed investigation to which students respond very well. (p. 358)

Hence, introductions to mathematical topics can be motivated by students' genuine curiosity about observed phenomena or real-life data. Others use projects as reviews or final assessments because they allow students to demonstrate their comprehension of a multitude of topics through one activity or written report.

Not only can projects be used at different times, but considerable variation exists with how teachers utilize project materials. There is a multilayered and complex relationship between teachers and their curriculum materials as most teachers have a set style of teaching (Remillard & Bryans, 2004). Some teachers are the go-by-the-book, never-deviate-from-the-plan, straight-arrow type, while others look at a lesson, plan for the idea, and then redo the whole layout so they can make it their own. If given a project to teach, some teachers will take it and automatically tweak it, while others will teach it exactly the way it is given to them (Remillard & Bryans, 2004). For some, a classroom is a sacred learning space where the teacher gives information, and the students are there to absorb each morsel with reverenced awe. Other classrooms can more aptly be described as organized chaos where knowledge is passed back and forth between teachers and students at varied amounts of speed and insight. “Two classrooms in which the same curriculum is supposedly being implemented may look very different; the activities of the teacher and students in each room may be quite dissimilar, with different learning opportunities available, different mathematical ideas under consideration, and different outcomes achieved” (Tarr et al., 2008 p. 249). Different philosophies lead teachers to adopt project materials in different ways according to how they conduct their classes.

Indeed, a teacher’s job is immensely complex, and deciding on instructional materials to use is one piece of that complexity. “The textbook adopted by a school or district is often not perfectly aligned with the intended curriculum (the local or state mathematics curriculum framework); consequently, teachers must make decisions, often on a daily basis, about what to use from the textbook, what to skip and what to supplement from other resources" (Tarr et al., 2008 p. 250). Kober (2011) explains when talking about the CCSS that much work will need to be done at the school district level, as the final responsibility for ensuring that students master the knowledge and skills in the standards resides with the districts, schools, administrators and teachers. Many districts will need to change their curriculum, instruction, assessments, and teacher development to align them with the new standards. She also states that districts have cited inadequate curriculum materials to support integration of the CCSS in classroom instruction as a challenge.

Teachers should actively develop curriculum implemented in their classrooms by creating application problems that relate to specific experiences that are pertinent and therefore interesting to the student while covering the necessary curriculum. "Regardless of how the problem, data, and possible extensions are used, students are always genuinely excited to be applying what they are learning in the classroom to analyze real data and make decisions on the basis of their analysis"(Laumakis, 2012 p. 358). Students should not view mathematics as a set of memorized rote facts, but as an engaging process of "gathering, discovering and creating knowledge in the course of some activity having a purpose" (Stein et al., 1996 p. 456). By adapting relevant instructional activities, the students should be intrinsically motivated to seek the needed information so they can find and make sense of a problem situation. Fennema (1999) also states that at the same time the task should require students to generate inferences, check plausibility and build models of the problem situation. Each student is different, but if some curiosity is fostered, and if the task is not deemed too challenging or meaningless, then the students will want to solve the problem at hand.

Once teachers have had the time, support, and opportunity to implement active learning projects, they tend to continue using them in their instruction. “We observed considerable stability in the way the teachers used the investigations curriculum” (Remillard & Bryans, 2004 p. 385). Teachers have their favorite projects that they do year after year because they enjoy them, and the kids learn and understand the curriculum while participating in them. The teachers have tweaked them and adjusted them; each project is teacher tested and revised. “Curriculum development should be grounded in empirically-based learning progressions that have been researched, tested, and revised over long periods of time” (Krupa, 2011 p. 10). Students should learn an array of concepts and skills as they learn to solve problems and relate information to other areas of study.

The introduction of the CCSS in Utah has posed many challenges for teachers, but also provides an opportunity to develop curriculum and improve classroom instruction. “Teachers need experiences that renew and strengthen their interest in and love for mathematics, help them represent mathematics as a living discipline to their students by exemplifying mathematical practices, figure out how to pose tasks to students that highlight the essential ideas under consideration, to listen to and understand students’ ideas, and to respond to those ideas and point out flaws in students’ arguments” (Beckman et al., 2012 p. 34). The remainder of this literature review will focus on realizing the goals of the CCSS.

The common core puts a greater emphasis on the way the students obtain their solutions, while the previous Utah math core simply required the correct response and left the way the material was taught up to the discretion of the teacher. An example as listed in Table 1, the 2007 Utah Secondary Mathematics Core Curriculum (USOE) required students to solve linear equations both algebraically and graphically. The CCSS, however, defines the process with more depth along with an added piece of explanation for each step. The standard explicitly requires students to justify their solution procedure, and allows the student to choose the way they would like to solve the given problem. This comparison of similar objectives highlights the dramatic differences between the two mathematics core curricula (USOE).

The CCSS is organized by Domains, overall broad topics, which are broken down into Standards under each domain. For example in Table 1, A-REI.1, the Domain is A-REI, Algebra- Reasoning with Equations and Inequalities, Standard 1, which is listed in detail in the table. Each domain and its corresponding standard can be cross-referenced in the Mathematics Appendix of the CCSS to find which grade level the standard belongs. A-REI.1 fits under Secondary Math I, 9th grade.

2007 Utah Mathematics Core Curriculum Algebra 1; Standard III Objective 2a	CCSS Secondary Math I; A-REI.1
Solve single-variable linear equations and inequalities algebraically and graphically.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Table 1: Comparison of an objective from Utah's 2007 Mathematic Core with a standard on the same mathematical topic from the CCSS.

Studies have shown that when students are given innovative, pertinent, teacher created materials they are more motivated in their studies than from doing book work everyday. The goal as teachers is to "demand that students engage in rather sophisticated mathematical thinking and reasoning—either connecting procedures to underlying concepts and meaning or tackling complex mathematical problems in novel ways" (Stein, Grover & Henningsen, 1996 p. 482). This can be accomplished by allowing students to participate in projects that allow for multiple solution strategies, solutions and representations as long as the justification is also present.

"There is a need for better mechanisms for sharing existing information. States should not attempt to create or reinvent their own supporting documents. Instead, they need take advantage of nationally developed materials and resources, modifying them if necessary to meet the needs of their local contexts to share successful materials across state lines" (Krupa, 2011 p. 13). Currently, the Jordan-Granite Consortium website solicits teachers across the state of Utah to upload resources they have developed and has categorized them by the CCSS. Teachers are then able to look up an objective and find assignments, projects, and assessments they may use in classroom instruction. This is in line with Krupa (2011) when she says there should be an online community created with resources and activities that relate to the CCSS. If teachers know curriculum based, teacher tested projects have been created then they are more likely to have greater success than they would with projects developed through a textbook company. "Carefully aligned assessments and expectations, with input from teachers and others, add to the value teachers give to these documents and their willingness to make sense of these documents" (Webb, 1997 p. 2).

Hence in this chapter we have reviewed research literature that provides insight into the use of projects for implementing the CCSS. The CCSS have introduced a change in Utah to not only what is being taught, but students are now expected to think and act with a higher level of math reasoning than before. Teachers can use projects to help facilitate this change as most projects cover multiple pieces of mathematical content and learning levels. Projects can be used in multiple ways, such as a background reference tool, a launch into a new mathematical idea, a formative evaluation of the current unit or a final assessment. Fennema's five different ways that mathematical understanding is developed provide a framework to describe how projects enhance student understanding in the classroom. A project has the ability to relate to all five: constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating what one knows, and making mathematical knowledge one's own. With tasks that can be solved in multiple ways with varied amounts of investigation added to the prompts there is an enhanced level of understanding of mathematical constructs. There is currently a great need for these projects to be easily accessible to teachers. The Jordan-Granite Consortium has solicited resources and has organized them by standards. This report adds an additional five projects which will be found on the Jordan-Granite website.

The following chapter is a summary of the solicitation, analysis and classification process used to collect and develop the projects in this report. It describes the steps taken to narrow down fifteen projects to five, and various methods employed to complete the analysis. Chapter 4 follows with a description of each project along with a summary of which Common Core State Standards for Mathematics are addressed in each.

CHAPTER 3

METHODS AND PROCEDURES

As a current secondary math teacher, I have an immediate connection to the CCSS as I am required to lead my students to achieve these standards in my practice. Many teachers have spent time building, developing and improving units and projects for previous Utah core curriculum topics; I have collected, enhanced and aligned these classroom projects to the CCSS as my master's degree project.

A four-part plan was put into place for completing my project: solicitation, analysis, classification, and follow-up. Each section below details individual steps I accomplished to compile a collection of teacher generated projects and align and adjust them so as to be relevant to the new core.

SOLICITATION

The first part of my plan centered on spreading the word of my master's project to as many secondary teachers as possible. Using the USU Journal Club as a springboard to help spread the word, I then contacted through email every math teacher in Box Elder, Cache and Logan School district that had ties to USU or myself. The goal was to receive 8-10 projects from teachers, narrow the projects down to four that would be published for this report. When not enough projects were submitted, more emails were sent out to other secondary teachers along with follow-up emails and phone calls to those who were most likely to submit projects. Eventually fifteen projects were received and four were chosen to be part of this report. I also chose one project of my own to analyze and improve and incorporate into this report. Narrowing the projects received down to the five selected was done using three different methods. First, the CCSS, I wanted the project to be able to be used in the realm of the current curriculum. Second, Fennema's five ways mathematical understanding emerges. Finally, my own judgment as a teacher in choosing projects which covered different mathematical content, could be differentiated based on the experience and proficiency of the students, and would be engaging such that it would gain the students' attention.

ANALYSIS

After receiving the different projects, I read and evaluated them. A few looked like they were copied from enrichment workbooks so I eliminated them from the five. I found that a couple submissions overlapped in math objectives, and some addressed topics I knew were not explicitly covered in the CCSS. After narrowing down my search I chose five different math projects that looked like they were original. Because of the change to the new common core, an emphasis was placed on projects that specifically covered higher-order thinking skills as well as mathematical modeling.

For each project I created an implementation plan that outlined the materials needed, the common core objectives covered, and any student worksheets needed for the activity. The task of creating each plan was unique in the amount of materials provided to work from, and will be described in more depth in Chapter 4. A rubric was created for each student tasksheet based on the objectives.

After reading each project I wrote learning objectives for each project based on Cangelosi's cognitive domains (see Table 2). These domains are referenced parenthetically after the mathematical objectives to clarify the main type of learning the student is expected to accomplish. In my view, using these cognitive domains to classify learning objectives helps to realize the eight mathematical practices of the CCSS that every student should be working on. For example: construct a concept correlates with modeling, comprehension and communication relates with construct viable arguments and critique the reasoning of others, while discover a relation goes hand in hand with look for and make use of structure.

Construct a Concept	Students achieve an objective at the construct-a-concept learning level by using inductive reasoning to distinguish examples of a particular concept from non-examples of that concept.
Discover a Relation	Students achieve an objective at the discover-a-relationship learning level by using inductive reasoning to discover that a particular relationship exists or why the relationship exists.
Simple Knowledge	Students achieve an objective at the simple-knowledge learning level by remembering a specified response (but not multiple-step process) to a specified stimulus.
Comprehension and Communication	Students achieve an objective at the comprehension-and-communication level by (i) extracting and interpreting meaning from expression, (ii) using the language of mathematics, (iii) communication with and about mathematics.
Algorithmic Skill	Students achieve an objective at the algorithmic-skill level by remembering and executing a sequence of steps in a specific procedure.
Application	Students achieve an objective at the application level by using deductive reasoning to decide how to utilize, if at all, a particular mathematical content to solve problems
Creative Thinking	Students achieve an objective at the creative-thinking learning level by using divergent reasoning to view mathematical content from unusual and novel ways.

Table 2: Cangelosi's cognitive domains relevant to learning objectives and assessment items. (Cangelosi, 2003)

Additionally, I also wrote clarifying questions for each teacher who submitted a project to help assist with the implementation of the project in the classroom and publication of the project and materials. From my correspondence with the contributors, I was able to understand the intentions the teachers had for the original projects as I began to find ways to improve upon them. The teachers' responses also helped with creating rubrics suited to their classroom needs.

CLASSIFICATION

I used the objectives I wrote after reading the projects, and then found where these objectives were located in the CCSS. To find each, I had to familiarize myself with the content of the CCSS and understand the indexing of each standard. The standards for grades 1 through 8 are first indexed by the grade level, then a domain is specified indicating the category of mathematics. Hence, standard 7.SP.1 references the first standard under Statistics and Probability for 7th grade.

The High School CCSS is broken into six conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. Each of the six topics are broken into large subtopics called Domains, with each Domain broken down into more specific mathematical statements of achievement called Standards. For instance, one objective I wrote for the Celebrity Project is as follows, "Students will be able to use the slope-intercept form of a line to write the equation of their regression line (Algorithmic Skill)." The closest matching standard from the CCSS is indexed by F-IF.6, and states that students "Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph." It seems to be the best match for this objective in the sense that given a line, this skill will allow students to formulate the appropriate function. The initial F in the index indicates this is a function standard, then IF refers to the domain "Interpreting Functions."

Course outlines are provided in the CCSS Appendix. Looking up in the math appendix of the common core under Functions, Domain-Interpreting Functions, Standard 6, shows this standard belongs in Secondary Math I. Most projects had many objectives which were covered in more than one math grade level. Looking first in the general CCSS document for the objective and then in the appendix where courses are described, allowed project objectives to be aligned with corresponding standards, and to be classified in the correct course level.

FOLLOW-UP

I continued corresponding with teachers who submitted projects. As answers to my follow-up questions returned, each project was refined, an electronic copy was sent to the original author for verification, and then each project was uploaded to the Jordan-Granite Consortium as each one finalized. The final products (see the Appendix) contain a list of objectives tied to the common core, project task sheets ready to be printed and used in the classroom, a set of instructions including tips on how to make it run smoothly and a scoring rubric.

CHAPTER 4

PROJECTS

After projects were chosen because of the mathematical content and creative learning activities they brought to the classroom, five original teacher-created projects were analyzed and enhanced. Each project presented here has its own lesson style, but to standardize their presentation, learning objectives have been created and listed along with corresponding standards from the CCSS. I have made recommendations for implementing projects, and found that time constraints for most of the projects are flexible. Based on my analysis of their alignment with the CCSS, I found that the projects range from being most appropriate for 6^{th} grade through Secondary Math II and they are consequently described below by ascending grade level.

INVENTION PROJECT (Recommended for 6^{th} grade)

The invention project is a statistics project, submitted by Ashley Ramsten, where students create their own invention. The students then produce, distribute and run analysis on the survey questions they generate based on their invention. There were a few different statistics projects submitted for review, but this one stood out as being something different in its fostering of student creative thinking. It allows the students to concoct a unique invention of their own and then to create and analyze survey questions based on their own inquiries. As Fennema points out it this allows the students to both articulate what one knows and make mathematical knowledge one's own. Also from a teacher point of view this project could be easily contracted or expanded depending on the time frame of a class.

As a precursor to the lesson the teacher should provide some kind of model by creating an invention of their own, for example one invention could be a student desk that flips over and has an iPad on the other side. The teacher can then ask questions and show the students what is expected on the survey questions. The teacher would demonstrate to the students which questions are pertinent to the project and which are not. A question like “How much would you be willing to pay for my invention?” would be appropriate whereas a question such as “Do you think if I made a million dollars on this I could meet Justin Bieber?” would not. There are nine teacher directed questions to answer after the survey. Depending on what the teacher has taught, these questions could easily be expanded by requiring more depth and details per question or shortened by removing some questions to fit the schedule of each class.

If used to the full extent, this project addresses a variety of mathematical objectives; however, it can be easily altered to expand or contract to fit different lengths of time depending on the class’s needs. After analyzing the materials, I created the following list of objectives for the Invention Project:

- Given certain parameters the student will create, distribute and tally a survey based on their individual inventions. (Comprehension and Communication)
- Given a set of raw data students will apply mean, median, mode, range, and standard deviation procedures to solve real world problems by hand. (Algorithmic Skill)
- Given a set of categorical data, students will create bar graphs and pie graphs. (Algorithmic Skill)

- When given a pie chart, students will be able to determine the approximate percentage of individuals for each category. (Simple Knowledge)
- Students will be able to create and explain key components of a box and whisker plot when given a set of raw data. (Comprehension and Communication)
- Given a set of raw data, students will create a histogram by hand. (Algorithmic Skill)
- Given a histogram students will describe and interpret the distribution. (Comprehension and Communication)
- Students will be able to calculate by hand the standard deviation and compute it for a set of data. (Algorithmic Skill)
- Students will be able to create a frequency table. (Simple Knowledge)

These objectives were then used as the basis for my comparison with the CCSS (Table 3). I found that several of the objectives are covered in the 6th grade core and then reappear again in the Secondary Math I while other objectives no longer explicitly appear in the Secondary CCSS (USOE). For example, dot plots, histograms, and box plots all appear in both sets of standards while pie charts no longer exist in Secondary math (CCSS, 2011). Although this project covers 6th, 7th and Secondary Math I objectives, the project most closely aligns with the 6th grade because of age level and core requirements.

6th Grade	* 6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. * 6.SP.5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations.

	b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
7th Grade	* 7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences
Secondary Math I	* S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). * S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Table 3. Standards for the CCSS addressed in the Invention Project.

This project can be used for formative evaluation, summative evaluation or to motivate a unit of instruction on statistics and data representation. At the end of the Invention Project, I have added a rubric created for teachers who would like to use this project as a type of evaluation. This is found in the appendix and is broken down by individual questions and corresponds to the learning objectives listed above. For example if a teacher wanted to evaluate the Secondary Math I objective, they could shorten the task and require their students to only complete prompts 4,7, and 9.

Overall Mrs. Ramsten's Invention Project brought the world of statistics out of the textbook and into the hands of the students. It covered several different areas of the CCSS and it allows for the teachers to employ statistical methods of data gathering and display in a meaningful way for students. The project can be used as an extended piece of an entire instructional unit or over a few days to summarize already learned techniques.

DILATION (Recommended for 7th grade)

This project was submitted by the researcher, Lynette Checketts. The original lesson plan consists of a teacher purchasing two identical greeting cards and cutting one of them into one-inch square pieces. The students then take a piece of the greeting card, a ruler, and a four-by-four inch square and dilate the greeting card picture onto their blank square. The teacher explains to them to how to measure along the edges of the greeting card and using the scale factor of 4 they multiply and make mark along the white piece of paper. Any shapes on the inside of the square will need to be measured, proportions set up and new dimensions determined.

This project was originally chosen because it covers a specific area of emphasis of the CCSS – namely dilations which falls under geometric transformations. It can be expanded or contracted to fit multiple time slots for teachers, and can be extended to include 3-dimensional objects easily. This project allows for a variety of different learning methods based on the needs of each teacher. As Fennema promoted it allows the students to extend and apply mathematical knowledge, and helps the student to make mathematical knowledge one's own. From the standards for mathematical practice this project teaches students to use appropriate tools strategically and to attend to precision and accuracy in drawing and measurement.

Only one mathematical learning objective was created for this project:

- Using proportions and similar figures, students will expand a greeting card by a scale factor of 4. (Application)

As seen from Table 4, this project fits well in both the Math 7 and the Secondary Math II Core Curriculum.

Math 7	* 7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. * 7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
Secondary Math II	* G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Table 4: Standards for the CCSS addressed on the Dilation Project.

There are several ways this project can be used in the classroom efficiently and effectively. It can be used as a launching point to teach a unit on dilations. After a lesson has been taught on ratios and scale factors this could serve as an enrichment project. Finally, if little to no directions were given aside from the prompt, it could be used as an evaluation tool. This project can also be expanded over the course of a few days or contracted down to 30 minutes depending on the timetables of the teacher.

INTERIOR AND EXTERIOR ANGLES PROJECT (Recommended for 8th grade)

Using the Triangle Sum Theorem as background knowledge students discover the formula for the sum of interior angles of polygons by breaking different polygons into triangles and adding the sum of the vertices together and finding pattern. Students then build on that knowledge to find the sum of exterior angles formula.

This project was submitted by Kathy Norman and included a PowerPoint, lesson plan, and student handouts. Originally this project was chosen due to the content area, the organization of the lesson, and the belief of the author that it could be well-executed in the classroom. It also allowed students to construct relationships and extend and apply mathematical knowledge according to Fennema's five ways that mathematical knowledge emerges.

Looking more in depth at the student handouts brought to light that they were cut and pasted from various textbooks. Since this project was to be posted online the time was taken to redo the student handouts with new unique problem sets. A rubric was created and all materials are attached at the end of the appendix.

After a careful read through of all the materials, the lesson plan was aligned to include exterior angles. Both mathematical and learning objectives were constructed based off of the PowerPoint and student handouts.

- Students will discover the formula for the sum of the interior angles of a polygon. (Discover a relationship)
- Students will use the interior angles formula to solve for missing angles of polygons. (Algorithmic Skill)

- Students will discover the sum of the measures of the exterior angles of any polygon is 360 degrees. (Discover a relationship)
- Students will solve for exterior angles. (Algorithmic Skill)

Looking over the mathematical objectives in the CCSS, there came about a realization of a significant piece of information. Although these objectives were taught in the 2007 Utah Geometry Core, they are no longer an explicit part of any Secondary Math Core. For this project I found ties to Math 8, Secondary Math I and II. The way this project is currently written fits best with the Math 8 objectives.

This lesson now became an extension lesson of the Triangle Sum Theorem. Teachers can use it to show students how the Triangle Sum Theorem is applicable to different shapes and can help find different measurements. Table 5 shows the different standards from the CCSS that this project may be used to instruct (CCSS, 2011).

Math 8	* 8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
Secondary Math I	* A-CED.2. Create equations in two or more variables to represent relationships between quantities;
Secondary Math II	* G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Table 5. Standards from the CCSS addressed on the Interior and Exterior Angle Project.

Originally, I selected the Interior and Exterior Angles Project because of the clear organization and the large supply of teacher-driven materials: PowerPoint, lesson plan, student worksheet. These materials are available in the appendix, but they have been dramatically altered to make them distinct from published work. I have completely recrafted items that appear in the handouts, rewritten the lesson plan to cover the topics completely, and supplied a scoring rubric with complete answers to the student task sheets. This project is recommended as an extension project to the Triangle Sum Theorem covered in 8th grade as it helps to expound on that topic.

CELEBRITY PROJECT (Recommended for Secondary Math I)

The celebrity project, submitted also by Ashley Ramsten (see the Invention Project), allows students to study regression lines with data that they generate themselves. Using the applet created at http://www.ruf.rice.edu/~lane/stat_sim/reg_by_eye/index.html the teacher can discuss regression lines with students, and would be able to quickly show many examples of what would make both good and poor choices for a line of best fit. After an understanding of what is to be accomplished in class is clear, students will begin by guessing the ages of several different celebrities or people of interest which become the y-coordinate in an ordered pair. The teacher then supplies the class with the x-coordinate, the accurate ages of the celebrities. After the data is gathered the students are able to draw their individual scatterplots and find the line of best fit. Hence, students compute a regression line for the relationship between their guessed ages of the celebrities and the celebrities' actual ages.

This project was chosen to be a part of this report due to its use of technology and my belief that it would be motivational to students based on their interests in pop-culture and celebrities. Mrs. Ramsten submitted an outline of a lesson plan, an Applet website, a student worksheet, and provided a PowerPoint used to guide students through the activity. As with the other projects, I wrote specific lesson objectives, evaluated the match-up between these objectives and the CCSS, and created rubrics for scoring the task sheets she provided.

Initially, little was altered from what Mrs. Ramsten originally submitted, I made formatting edits of her plans and task sheets, and reworded the lesson plan to make it easier for teachers to use. I also wrote the following lesson objectives:

- Students will be able to estimate a regression line by picking two points that they feel would be on that line. (Simple Knowledge)
- Students will be able to use the slope-intercept form of a line to write the equation of their regression line.(Algorithmic Skill)
- Students will analyze and explain regression lines and their relationship to the data set. (Comprehension and Communication)

But after further study of the intended use of the project and how the project might best be used with the new standards, there were several adjustments to the student handout to include more depth and discussion. The original student handout only focused on arithmetic, where the revised worksheet adds a layer of student understanding. The current version requires students to compare answers with each other and explain why there are different lines of best fit. The lesson objectives which were originally created still stayed the same. The standards the objectives were aligned to can be found in Table 6 below.

Secondary Math I	* F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. * S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for scatter plots that suggest a linear association. * S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. * S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

Table 6. Standards from the CCSS addressed in the Celebrities Project.

A list of the celebrities birthdays were added to help save time for teachers as they worked through this project. All materials needed for this project can be found at the end of this report in the appendix.

The regression line project was chosen for a few key reasons. It has the potential to grab the students' attention. Teachers who know what their students might find particularly engaging could change the pictures as wanted. An example of this would be including a picture of the school principal or the current skateboard guru.

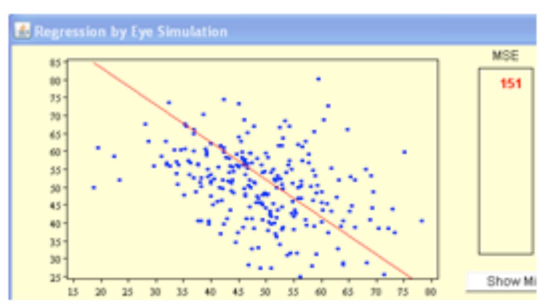

Figure 1: Shows a guess where the regression line is on the applet.

The Applet allows for students to develop intuition about regression lines as a minimization of the sum of squared errors. The mean squared error is listed for students in the applet to help them as they find the regression line. The diagrams to the right allow the teacher to use the applet in a way to discuss which line was a better regression line. Some of the best teaching moments are made with incorrect answers. Finally the project reviews slope and solving equations, two basic algebra concepts that the students need to review time and time again. The project materials presented here could be expanded to include further analysis of residuals and a study of correlation coefficients.

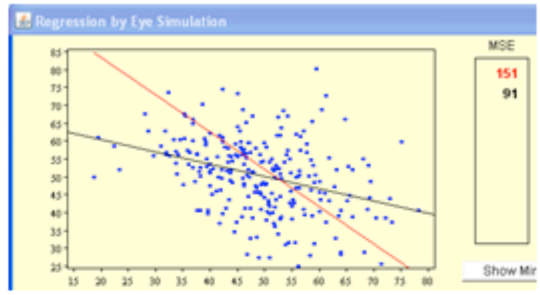

Figure 2: Shows the guess and the actual regression line.

KITE FLYING PROJECT (Recommended for Secondary Math II)

This project, which helps to solidify students application of the Pythagorean theorem, was submitted by Megan Bushnell. The kite flying project allows students a chance to relate math with real-world application. Students fly kites and take horizontal and diagonal measurements as the kite holds steady in the air. Using the Pythagorean theorem they can estimate the height of the kite based on measurements taken along the ground and of the kite string. After they finish, reports are written about what they learned, what may have altered their findings, and whether or not they believe the accuracy of their results.

This project was originally chosen due to the differentiated instruction it offers, by allowing the students to practice and apply their skills to an authentic learning situation. Mrs. Bushnell sent the original Kite Flying Project Worksheet and a student sample, both attached in the appendix. A lesson plan was created along with three mathematical objectives for this project:

- Students will be able to apply the Pythagorean theorem to find the height of an object. (Algorithmic Skill)
- Students will be able to assess their work and the accuracy of their answers. (Application)
- Students will be able to explain the Pythagorean theorem. (Comprehension and Communication)

The mathematical objectives were evaluated on the CCSS website and then cross-referenced under the appendix to see which grade level they are to be taught. As seen in Table 7 below (CCSS, 2011) this project objects overlap several different years, however the Pythagorean theorem only appears explicitly in the 8th and Secondary Math II core. However, for this project it is the belief of the researcher that this project best fits in the Secondary Math II grade level as it uses Pythagorean theorem, similar triangles, and congruence throughout the project.

8th Grade	* 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Secondary Math I	* A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. * A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
Secondary Math II	* G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. * G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Secondary Math III	* A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. * A-REI.2. Solve simple rational and radical equations in one

	variable, and give examples showing how extraneous solutions may arise.

Table 7. Standards for the CCSS addressed on the Kite Flying Project.

Significant time was spent evaluating the original form and I decided that there needed to be two different forms for this given project to allow teachers to choose which one is best suited for their students. The first form titled 'Kite Flying Project' outlines how to complete the project with step-by-step instructions for the students. It informs the students that they will be using the Pythagorean theorem to solve the problems, how to draw the picture, take the measurements, and to calculate the height of the kite. The second form titled 'Kite Flying Enhanced' asks the same questions as the first, but it allows the students to decide for themselves how to best solve the question at hand. In other words, it does not tell them to use the Pythagorean theorem, it allows them to think about different ways that they would try to solve the problem. They are still required to draw a picture and take measurements but how they go about this is up depends on the students to differentiate between what they have learned in the classroom and what they now have to accomplish with tangible objects. With the enhanced worksheet the teacher needs to be ready to accept a range of correct mathematical work. For instance, the student may calculate the height of the kite by finding the angle between the string and the ground then use the tangent function to solve for the height of the kite rather than utilizing the Pythagorean theorem.

An alternate assignment was also created for students that were not going to participate in the kite flying activity. It follows along the lines of the original project by instructing students to find different items around the school, such as library books and ladders leaning against a wall, and to use Pythagorean theorem to solve for missing lengths. A rubric was designed from the objectives defined in the lesson plan. It emphasizes the process the students took in solving the problems along with the answers they calculated. All forms are included in the appendix of this report.

This project covers three of Fennema's ideas: extending and applying mathematical knowledge, reflecting about experiences, and articulating what one knows. Mrs. Bushnell has done this project in the past and said because of the report at the end of the lesson, and the students' reflection on their work, she believes they come to know and understand mathematics at a deeper level than before.

Mrs. Bushnell's Kite Flying Project allows students to solve problems in a genuine application through the use of this one activity. It gives the students a chance to relate math to an activity outside of the classroom while reteaching basic mathematical practices.

CHAPTER 5

CONCLUSION

Based on my literature review I set out to accomplish three things: 1) Obtain different mathematical projects from teachers that they have used in their classrooms and I would have the potential to use in my own classroom. 2) Become familiar with the CCSS and align the projects to the new core. 3) Enhance the projects in some way before sharing them with teachers across the state.

Fennema gave us five ways mathematical understanding emerges: constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating what one knows, and making mathematical knowledge one's own. Combining this with the eight standards for mathematical practice of the CCSS that students are to be working on in every mathematical classroom, experts are clearly making a strong case for a project-based classroom approach. Each of the five projects that have been submitted in this report have been described by these learning categories.

Each investigation allows the students to think through a problem themselves and to reach a solution. Some projects, such as the Kite Flying Project. have been revised in a way to allow the students to devise creative solutions. Several of the projects also include a probing analytical piece where the students have the opportunity to write about the math they are experiencing. Allowing the students time to reflect on their work and articulate about the mathematics they are engaged with helps to solidify the concepts and relationships between mathematical topics.

The five projects contained in this report span grades 6 through 10 (with Secondary Math II). They cover five different mathematical topics: lower level statistics, dilations, interior and exterior angles, regression lines, and the Pythagorean theorem. The projects can be altered by teachers for individual classroom use based on time and particular student needs.

Projects were chosen based on three sets of criteria. Fennema's five ways mathematical understanding emerges, the CCSS, and my knowledge as a teacher. After a project was chosen it was broken down into subgroups. I wanted to make it easy for a teacher to pick up the project and have the materials needed already available for them. Each project was organized and formatted into lesson outlines, objectives, handouts, rubrics, and any supplementary materials such as Powerpoints or figures used for demonstrations such as the 2D shapes in the Interior and Exterior Angles Project.

Aligning the projects to the common core standards became an intensive activity. As these projects were not originally created for this specific core, they can potentially be used in many different classrooms for many different mathematical learning outcomes. However, for each project, I have tried to list the closest fit to the author's original goal when they created the project. For example, the Invention project is listed as a 6th grade project but has components that fit both the 7th and Secondary Math I core as well.

My third goal was to enhance the projects in some way before putting them online. At multiple points, the judgment call to keep the worksheets like they were or change them came into play. Eventually, the decision was made based on the goals I had in the beginning, I wanted to enhance the projects in someway if possible. This was never to say that I made perfect projects aligned to the core every time, but only improved the materials that were given to me. As detailed in Table 8, in the end every project has been altered.

Invention Project	* Minor adjustments to the student worksheet as I deleted out unnecessary pages and wording. * Added and aligned mathematical and learning level objectives. * Included a rubric that related back to the objectives.
Dilation Project	* Created a lesson outline. * Created and aligned mathematical and learning level objectives.
Interior and Exterior Angles Project	* Created a new student worksheet. * Created a rubric that related back to the objectives, * Added and aligned mathematical and learning level objectives. * Added to the teacher's lesson plan to include exterior angles.
Celebrity Project	* Added and aligned mathematical and learning level objectives. * Changed the student worksheet to include a Comprehension and Communication learning level instead of purely algorithmic skill.
Kite Flying Project	* Created an Enhanced student worksheet and an Alternate student worksheet. * Added and aligned mathematical and learning level objectives. * Created a rubric that related back to the objectives that emphasizes the process as much as the answers.

Table 8. Enhancements made to each project.

Spending time, enhancing projects for teachers to use in their classrooms, is something I plan to continue by asking other secondary mathematics teachers for their classroom projects. Time is one of the most valuable commodities for teachers in the public school arena. Creating worksheets, classroom outlines and rubrics based from activities teachers have found effective in their classrooms has reinforced in me ways I can better enhance my own teaching. These projects will be used in my classroom and will be placed on the Jordan-Granite Consortium website to share with people across the state as I believe they will enhance student learning.

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APPENDIX

Invention Project
Statistics end-of-unit

Materials needed: Copies of the handouts for each student
Colored Pencils or markers (not required)
Core Standards: Math 6: 6.SP.4; 6.SP.5
CCMI; S-ID.1; S-ID.2

Objectives:

- Given certain parameters the student will create, distribute and tally a survey based on their individual inventions. (Comprehension and Communication)
- Given a set of raw data students will apply mean, median, mode, range, and standard deviation procedures to solve real world problems by hand. (Algorithmic Skill)
- Given a set of categorical data, students will create bar graphs and pie graphs. (Algorithmic Skill)
- When given a pie chart, students will be able to determine the percentage of individuals for each category. (Simple Knowledge)
- Students will be able to create and explain key components of a box and whisker plot when given a set of raw data. (Comprehension and Communication)
- Given a set of raw data, students will create a histogram by hand. (Algorithmic Skill)
- Given a histogram students will describe the distributions shape. (Comprehension and Communication)
- Students will be able to calculate by hand the standard deviation and apply it to a set of data. (Algorithmic Skill)
- Students will be able to create a frequency table. (Simple Knowledge)

Before the lesson:
Create your own invention and survey some people (or make up the answers) and complete the assignment.

The lesson:
Show the students your example of what an invention should look like and spend the needed amount of time discussing how to complete each task for part 3.

CCS 1 Name:____________________

Invention Project-Unit 2

Part 1: The Invention

You are going to invent a product. It can do whatever you want it to do (school appropriate). In the box below, describe what your invention does in some detail. Then in the space below the box, you will draw a picture of your invention. Please color your picture with crayons or markers or colored pencils (must be colored for full points).

CCS 1 Name:____________________

Part 2: The Survey

Next you will create 10 questions about your product. You will be surveying 10 people about their opinions on your product. You must survey **5 of your friends, 3 of your teachers, and 2 other adults.**

Questions 1 thru 5 must all be questions with **numerical answers** (i.e. How much would you be willing to pay for my invention?).

Questions 6 thru 8 must all be questions with **categorical answers** (i.e. What color would you like it to be?).

Questions 9 and 10 must all be questions with a **yes or a no answer** (i.e. Would you buy my product?).

Fill out the chart on the back of this page with your questions, names of the people you surveyed, and their answers.

Part 3: The Statistics and Graphs

1) Using the results of your survey of question #1, find the mean, median, mode, and range. (Show your work for credit.)
    a. Mean

    b. Median

    c. Mode

    d. Range

    e. Explain which method of central tendency best fits your survey data.

2) Make a frequency table with your data from question #10.

Title:

Category	Yes	No
Frequency		

3) Make a frequency table for your data from question #5.

Title:

Number/ Interval										
Frequency										

4) Make a line plot of the data from your question #7. (Be sure to label your line plot and use a ruler to make clean lines.)

5) Make a circle graph for the data from your question #8. Be sure to label the graph and create a legend using colors. Show all the work you did to figure out the percentages of the circle. (Use a compass to get a nice circle.)

6) Make a bar graph for your question #6. Remember to label your axes and your graph. (Use a ruler to get straight lines.)

7) Make a box and whisker plot for your question #2. Show all your work to find the medians and quartiles. Don't forget to label and title. (Use a ruler to get straight lines.)

   a. What is the IQR?

   b. Do the quartiles infer anything meaningful about the invention? Explain.

8) Make a histogram for your question #4 data. Choose appropriate intervals and label and title your graph. (Use a ruler to get straight lines.)
    a. Is your histogram uniform, normal, or skewed in shape?

9) Find the standard deviation of your data for question #3. Show all your work for credit.
    a. Within how many standard deviations does all of your data fall? What does this mean for your invention?

Invention Project Rubric

1.		
	+1 0	Choose Mean, Median or Mode
	+1 0	Correct mathematical computation
	+2 +1 0	Explanation is clear for why they chose the measure of central tendency.
2.		
	+1 0	Frequency table is accurate
3.		
	+1 0	Frequency table is accurate
4.		
	+1 0	Line plot is labeled
	+1 0	Line plot is constructed accurately
5.		
	+1 0	Created complete legend
	+1 0	Percentages are calculated accurately
	+1 0	Percentages around the circle are relatively accurate
6.		
	+1 0	Axes are labeled
	+1 0	Bar graph is accurate
7.		
	+2 +1 0	Box and Whisker plot is accurate
	+1 0	50% of the data lies in the IQR.
	+1 0	Explanation of quartiles is clear
	+1 0	Explanation relates back to the invention.
8.		
	+1 0	Histogram is labeled
	+1 0	Intervals are equal and named properly
	+1 0	Histogram's shape is named properly
	+2 +1 0	Explanation of how it impacts the invention demonstrates understanding of distributions shape.
9.		
	+1 0	Standard Deviation is calculated accurately
	+2 +1 0	Describes the impact of finding standard deviation relative to the given invention.

Total ______/26

Dilation Project

Materials needed:	Rulers, Markers, Colored pencils, 2 identical greeting cards (One cut into 1'' by 1'' pieces) 4'' by 4'' cut squares (One for each student)
Core Standards:	Math 7; 7.G.1; 7.G.2 CCMII; G-SRT.1

Objectives:

- Using proportions and similar figures students will expand a greeting card by a scale of 4. (Application)

Before the lesson:
Buy two identical greeting cards that have a lot going on in the picture (A plain blue sky would not give the students anything to dilate.) Cut the first card into one inch square pieces. Number the pieces so it's easier to put back together. Cut out four inch long squares of white paper that the students will use to dilate their piece onto.

The lesson:
Show your students the second greeting card and tell them they are going to dilate it by a scale factor of 4. Give each student a one inch square and a 4 inch square along with a ruler. Explain to them how to measure along the edges of the greeting card square and they will need to multiply it by 4 and make the same mark along the white piece of paper. Any shapes on the inside of the square will need to be measured with proportions set up and new dimensions determined.

Examples:

Figure 1; 1 in x 1 in greeting card square

Figure 2; Students Dilated image by a scale factor of 4

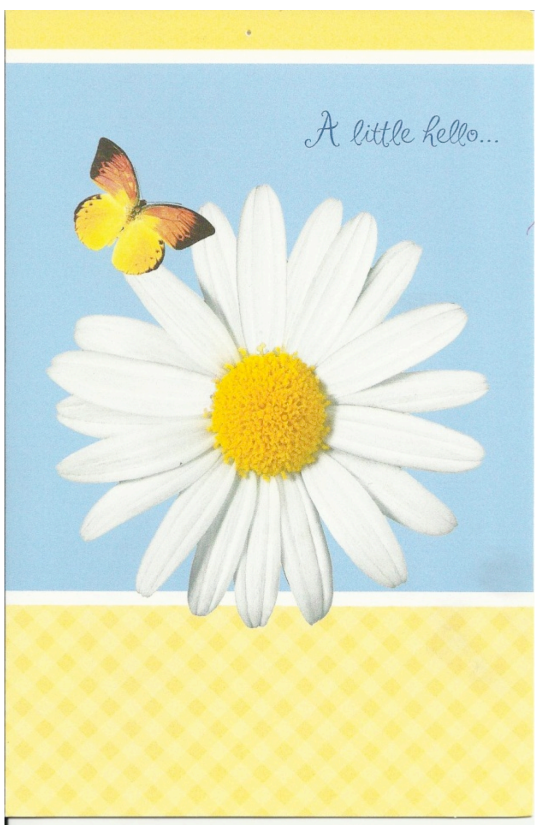


Figure 3; Original Card

Figure 4; Original and Dilated Card

Interior and Exterior Angles of a Polygon

Materials needed:	Copies of the handouts for each student White Board Markers for each group Paper Towels Copies of Shapes inside plastic sleeves
Core Standards:	Math 8: 8.G.5; CCMI: A-CED.2 CCMII: G-CO.10

Objectives:

- Students will discover the formula for the sum of the interior angles of a polygon. (Discover a relationship)
- Students will use the interior angles formula to solve for missing angles of polygons. (Algorithmic Skill)
- Students will discover the sum of the measures of the exterior angles of any polygon is 360 degrees. (Discover a relationship)
- Students will solve for exterior angles. (Algorithmic Skill)

Before the lesson:
Group the students into pairs.
Make a set of copies for each group.

The lesson:
Allow the groups of students to work together on this lesson as they discover for themselves the correct formulas for solving interior and exterior angles of polygons.

Interior and Exterior Angles of a Polygon

Name: ____ Date:___________

Discover for yourself!
It is your responsibility to find out as much as you can about each polygon. You may work together and share your ideas. Good luck!

Step 1: Choose one vertex on the pentagon, hexagon or heptagon you have been given. Draw all possible diagonals from that vertex.

Step 2: How many triangles are formed? Write your response in the space provided in the chart.

Step 3: What is the sum of the interior angles? Write your response in the space provided in the chart. (Hint: Remember the sum of three angles of a triangle.)

Step 4: When you are finished, wipe off the marks you made on the polygon with a paper towel and switch polygons with someone.

Step 5: Repeat this process and complete the chart.

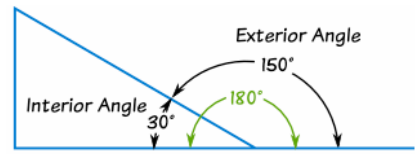
Convex Polygon	Number of Sides	Number of Diagonals from one Vertex	Number of Triangles	Sum of Interior Angles
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
n-gon				

Step 6: Now that you have completed the chart, make a conjecture as to what you think each formula is:

The sum of the interior angles of a polygon with n sides is: ________________________

The measure of an interior angle of a regular polygon with n sides is: ________________

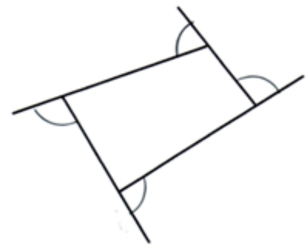
Exterior Angles:



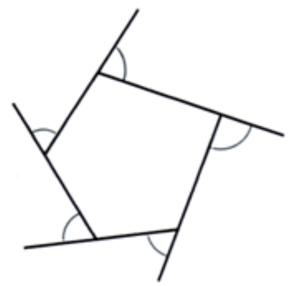
The exterior angle is the angle between any side of a shape, and a line extended from the next side.

Using this information and the knowledge you know about interior angles of a polygon to answer the following questions.

What is the sum of the exterior angles of the quadrilateral to the right?



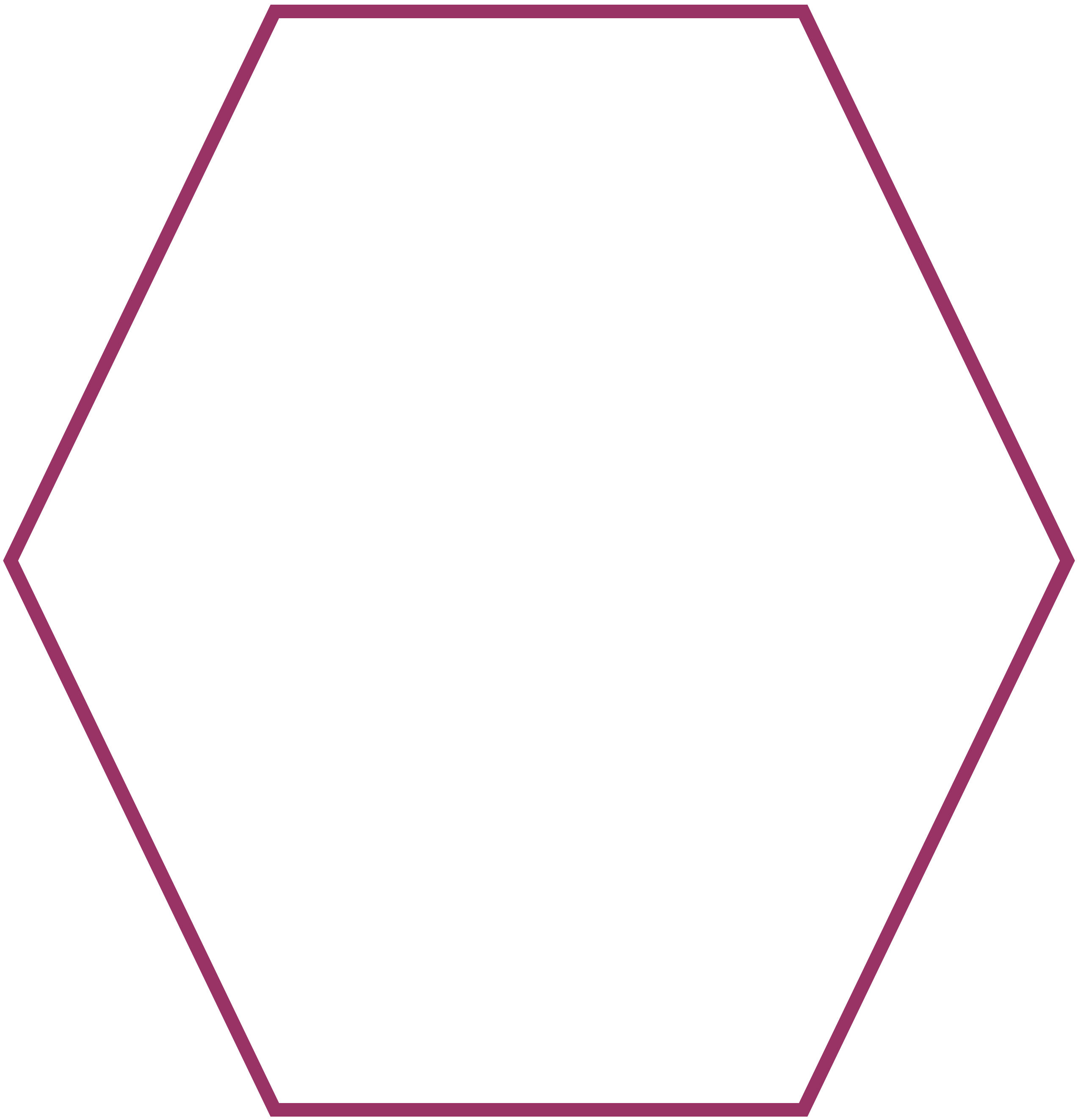
What is the sum of the exterior angles of the pentagon to the right?



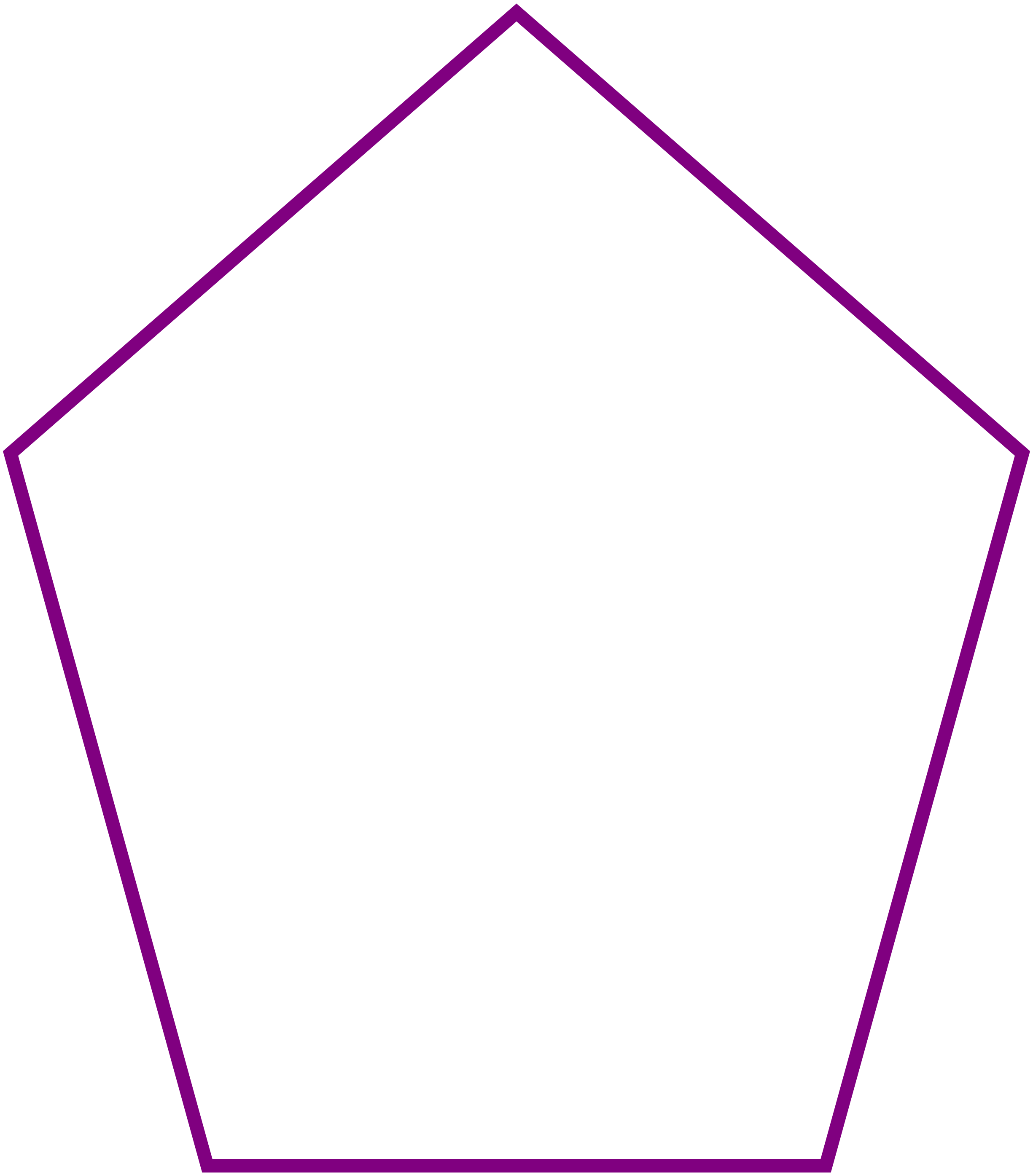
The sum of the measure of the exterior angles of a polygon with n sides is: ___________

The measure of an exterior angle of a regular polygon with n sides is: _______________

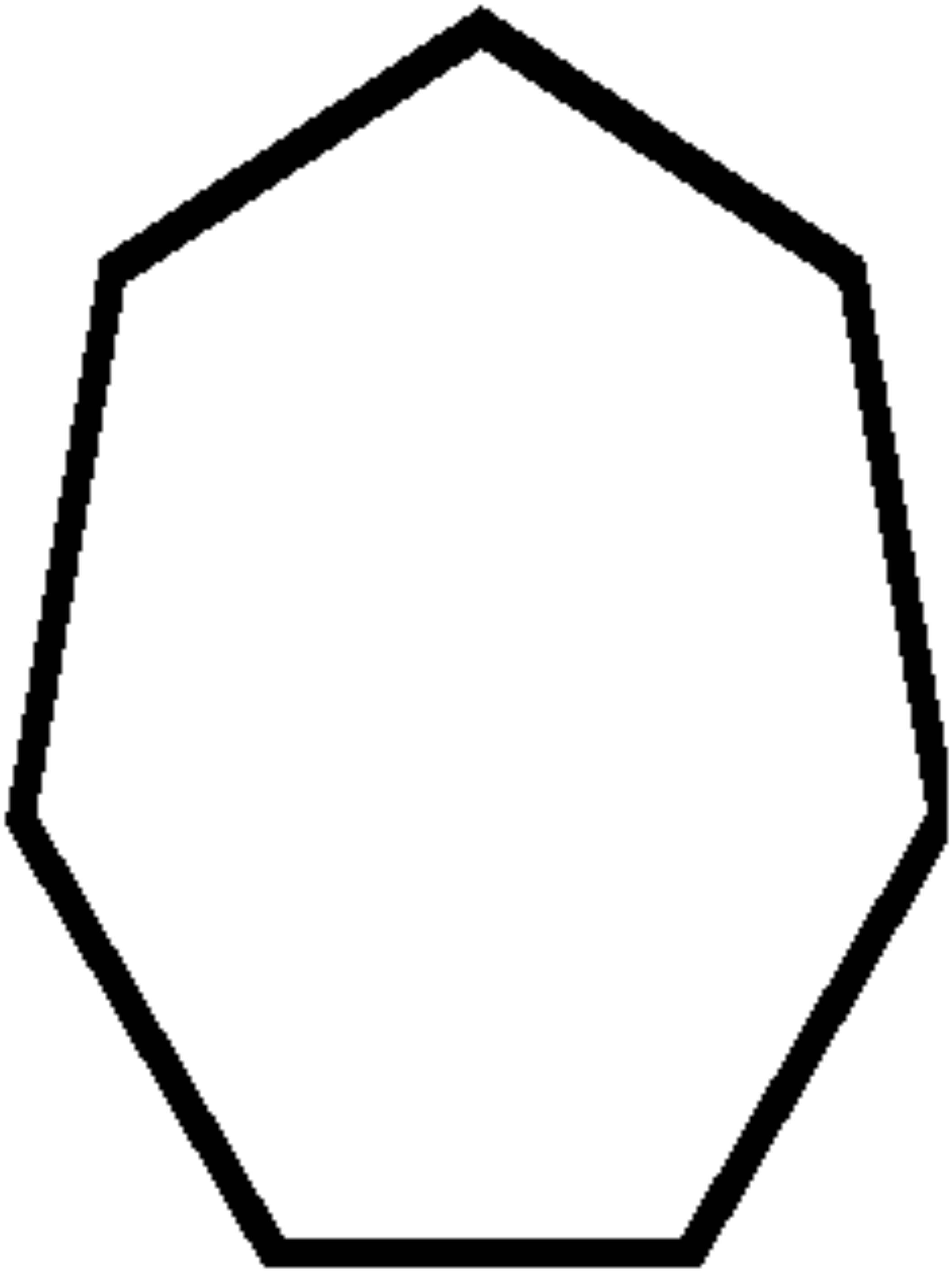
Hexagon



Pentagon



Heptagon

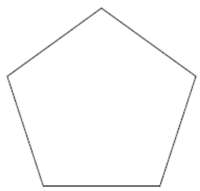


Name: ______________________________________ Date: __________

Find the Interior and Exterior Angles:

Find the measure of one interior angle and the sum of the interior angles for each polygon. Round your answer to the nearest tenth if necessary.

1)



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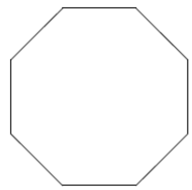

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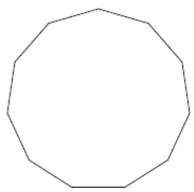
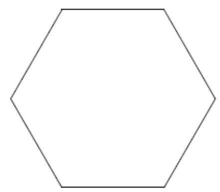
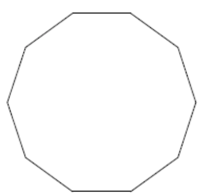


4)

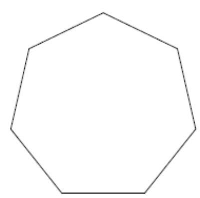


Find the measure of one exterior angle and the sum of the exterior angles for each polygon. Round your answer to the nearest tenth if necessary.

5)



6)



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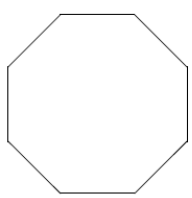

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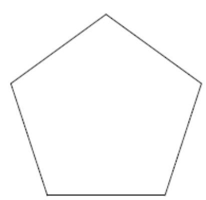
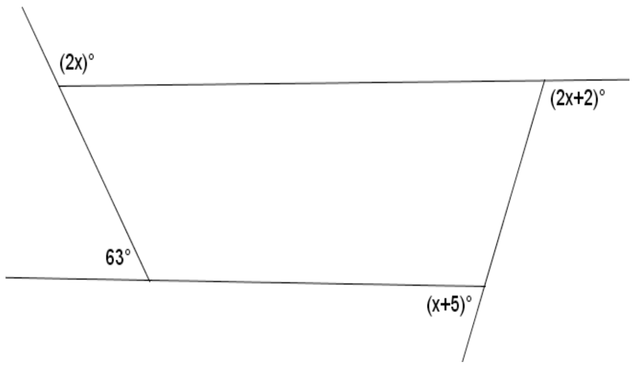


9. Explain how the formula 180(n-2) accurately finds the sum of the interior angles of a polygon. Will it work for any type of polygon?

10. Explain how to find an exterior angle of a regular polygon. What does *regular polygon* mean and how would that influence your solution?

Solve for x and find the missing interior or exterior angles.



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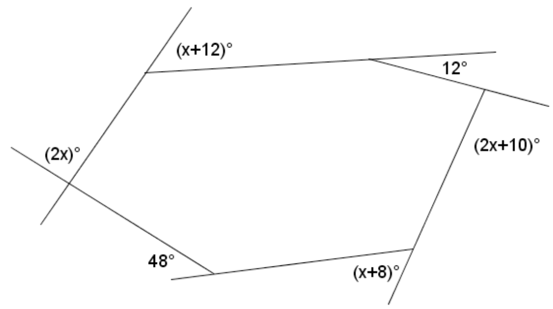

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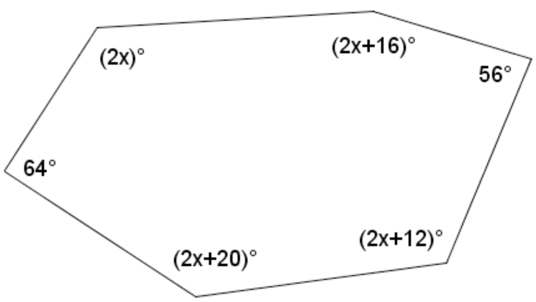

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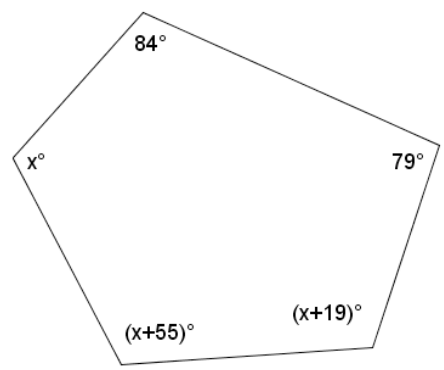

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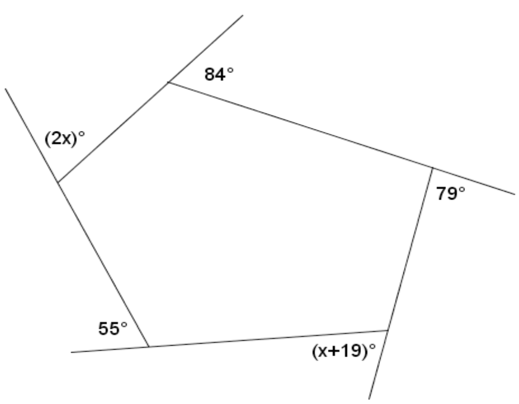

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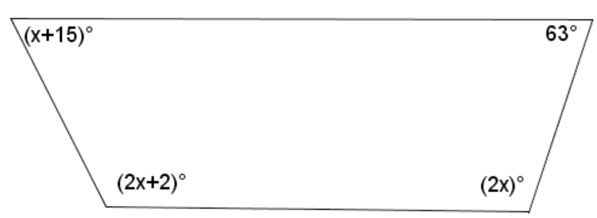

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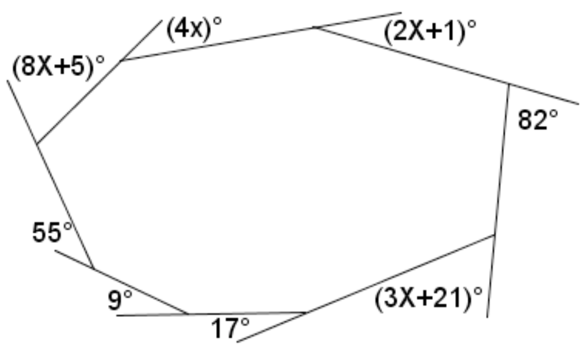
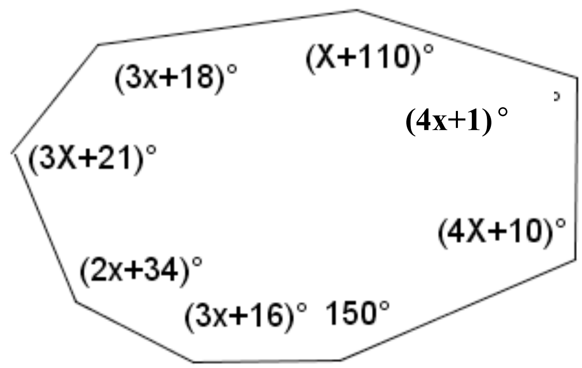




17.

18.

Interior and Exterior Angles Rubric

Name:__________________________

0 +1	1. An interior angle 108°
0 +1	Sum of the interior angles 540°
0 +1	2. An interior angle 135°
0 +1	Sum of the interior angles 1080°
0 +1	3. An interior angle 147.3°
0 +1	Sum of the interior angles 1620°
0 +1	4. An interior angle 720°
0 +1	Sum of the interior angles 720°
0 +1	5. An exterior angle 36°
0 +1	Sum of the exterior angles 360°
0 +1	6. An exterior angle 51.4°
0 +1	Sum of the exterior angles 360°
0 +1	7. An exterior angle 45°
0 +1	Sum of the exterior angles 360°
0 +1	8. An exterior angle 72°
0 +1	Sum of the exterior angles 360°
0 +1 +2	9. Explanation of formula includes triangle sum theorem and is accurate and precise.
0 +1	No. For example concave polygons.
0 +1	10. 360° divided by number of sides (angles)
0 +1 +2	Regular: Congruent sides and angles
0 +1	11. x=44
0 +1	88°, 90°, 49°,63°
0 +1	12. x=45
0 +1	100°, 57°, 12°, 90°, 48°, 53°
0 +1	13. x=69
0 +1	138°, 154°, 56°, 150°, 158°, 64°
0 +1	14. x=101
0 +1	101°, 84°, 79°, 120°, 156°
0 +1	15. x=41
0 +1	82°, 84°, 79°, 60°, 55°
0 +1	16. x=56
0 +1	71°, 63°, 112°, 114°
0 +1	17. x=10
0 +1	85°, 40°, 21°, 82°, 51°, 17°, 9°, 55°
0 +1	18. x=36
0 +1	126°, 146°, 145°, 154°, 150°, 124°, 106°, 129°

Total ______/38

CELEBRITY PROJECT

Overview: Students will explore regression lines by hand.

Items needed: Handouts for each student
Spaghetti noodles one for each student (optional)

Objectives:

- Students will be able to estimate a regression line by picking two points that they feel would be on that line. (Simple Knowledge)
- Students will be able to use the slope-intercept form of a line to write the equation of their regression line.(Algorithmic Skill)
- Students will analyze and explain regression lines and their relationship to the data set. (Comprehension and Communication)

Connection to Core Curriculum: CCMI: F-IF.6; S-ID.6; S-ID.7; S-ID.8

Technology: Internet Applet
http://www.ruf.rice.edu/~lane/stat_sim/reg_by_eye/index.html
PowerPoint presentation

Before the Lesson:
Find the actual ages of the celebrities. Their birthdays are listed at the end of the PowerPoint.

Plan:

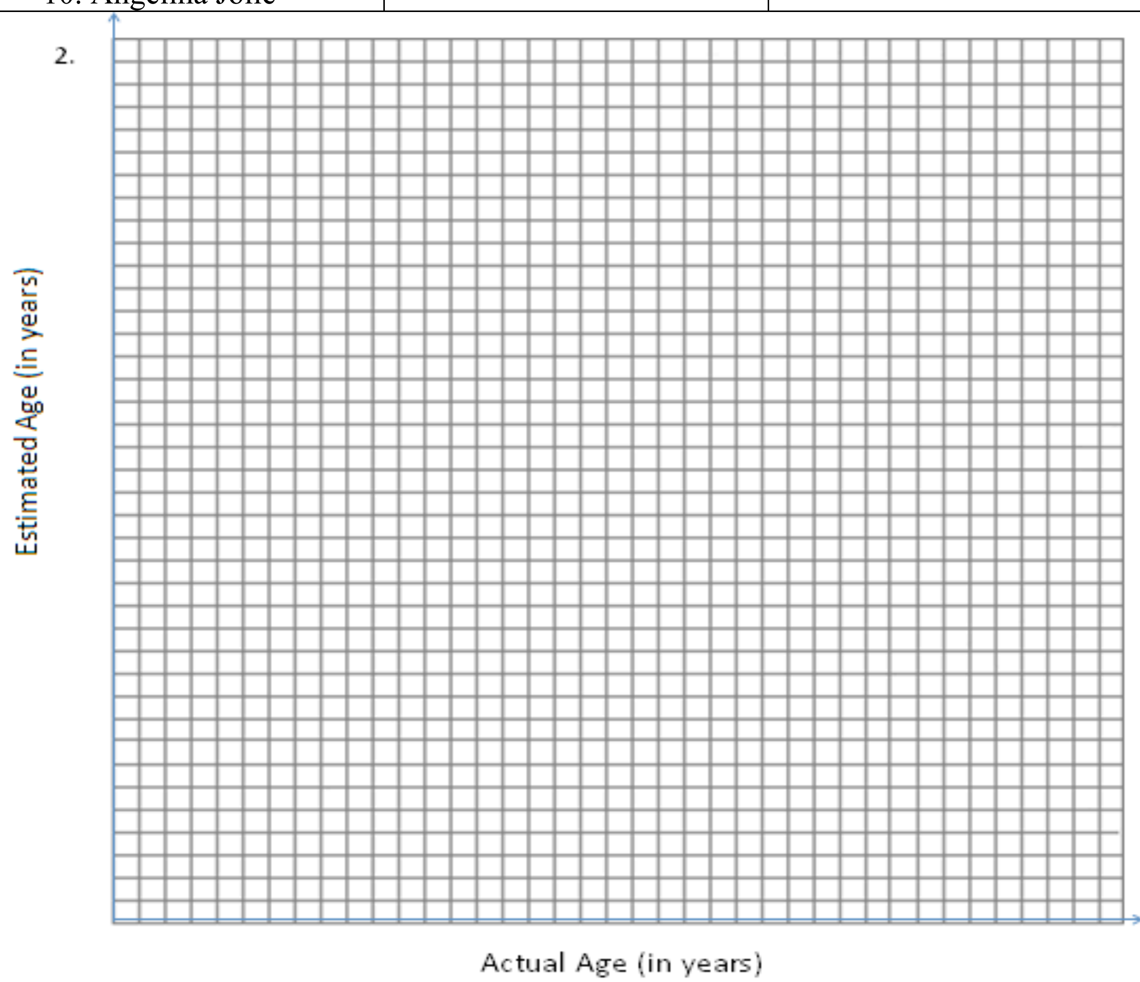
- Start off by looking at a couple of scatterplots on the applet and discuss what a regression line is. ie: A regression line is going to be a line that estimates the points. It won't go through all the points because they are not on a line but they follow the general shape of a line.
- Explain to the student how to choose a line that comes as close to as many points as possible.
- Ask some students to volunteer and draw a line that they think is the best representation of the data on the applet.
- Display the PowerPoint. Give each student a printout packet and have them on the 1st page guess the age of each celebrity.
- Inform the students of the real ages.
- Have them plot it on the given graph with actual age on x-axis and estimated age on the y-axis.
- Show the students how to take a spaghetti noodle and place it on their graph where they think the line of best fit would be and then have them pick two points that lie on the spaghetti noodle (if no points lie on the noodle have them pick two points that would lie on the spaghetti noodle.
- Have the students compute the slope of their best fit line and then solve for the y-intercept.
- Proceed to the rest of the questions on the handouts.

CELEBRITY PROJECT **Name:**

1. As we go through the PowerPoint, guess the age of each celebrity. Record it in the appropriate column. The actual ages will be given at the end.

Celebrity	Actual Age (x)	Estimated Age (y)
1. Tiger Woods		
2. Clint Eastwood		
3. Ashley Tisdale		
4. Justin Timberlake		
5. Madonna		
6. Leonardo DiCaprio		
7. Hilary Duff		
8. Jay Leno		
9. Toby Keith		
10. Angelina Jolie		

2.



2. Take your spaghetti noodle and place it on your graph where you think it best estimates the line created by your points. Draw your line on the graph using your spaghetti noodle as a guide.

   a. Pick two points that are on your line created by your spaghetti noodle.

Point 1 (x_1, y_1)	Point 2 (x_2, y_2)

   b. Find the equation of the line in slope intercept form. This equation is your line of best fit or the regression line.

3. Write down the equations of the students sitting in front and behind you. Are their equations the same as yours? Why or why not?

Use all three regression equations to answer questions 4-6. Keep your answers organized.

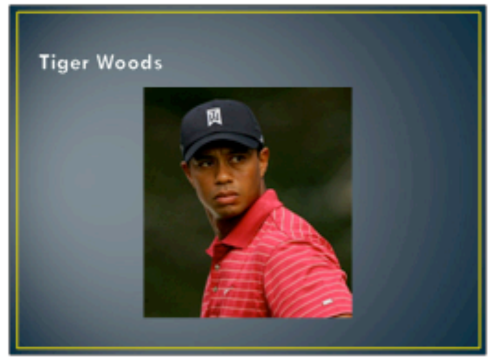
4. If someone was actually 37 years old, what would each equation estimate for their age? Show work below.

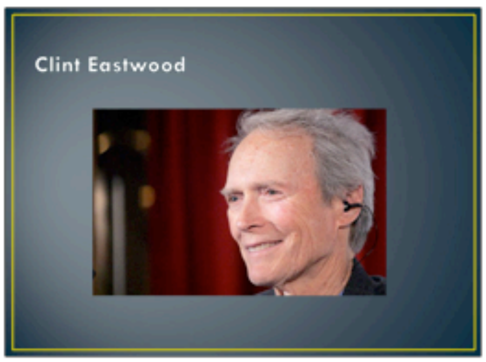
5. If you had guessed that someone was 52 years old, what would each of the equations tell you their real age was? Show work below.

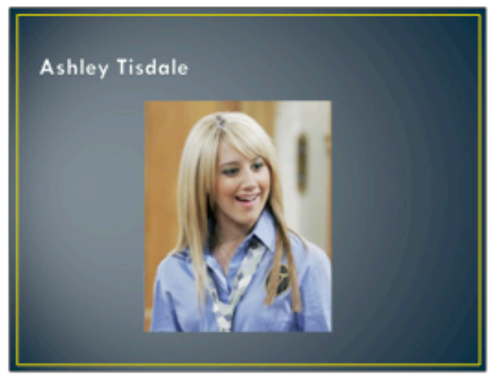
6. What is your age?
   Based on your actual age, what do the different equations predict your age as? Show work below.

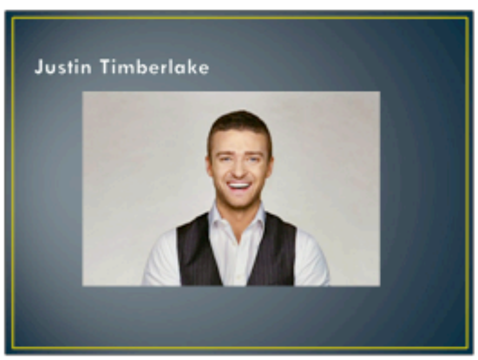
7. What were the similarities or differences between the equations? Which equation answered number 6 the most accurately? Now try all three equations with one of your parents or guardians ages. Is the same equation still the most accurate? Why or why not?

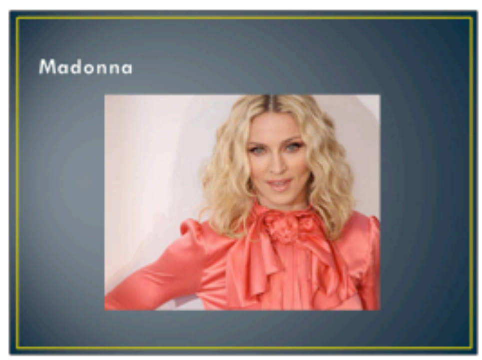
PowerPoint Slides:

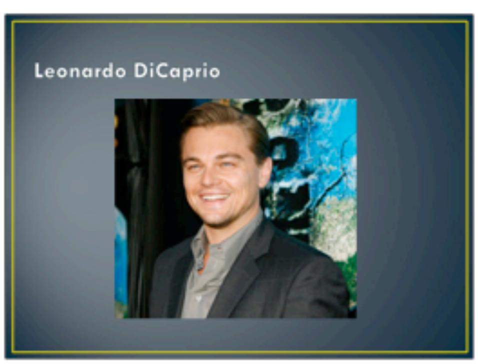


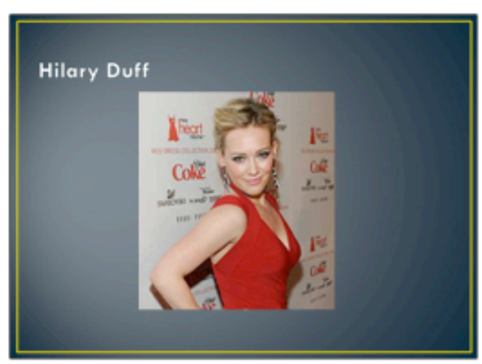












Celebrities Birthdays:

Tiger Woods	12/30/1975
Clint Eastwood	05/31/1930
Ashley Tisdale	07/02/1985
Justin Timberlake	01/31/1981
Madonna	08/16/1958
Leonardo DiCaprio	11/11/1974
Hilary Duff	09/28/1987
Jay Leno	04/28/1950
Toby Keith	07/08/1961
Angelina Jolie	06/04/1975

Kite Flying Project
Pythagorean Theorem

Materials needed: Copies of Student handouts
Rulers, Tape, Yard Stick (measuring tape)
Kites (Students supply)

Core Standards: 8th Grade: 8.G.7
Secondary Math I: A-CED.1; A-CED.4
Secondary Math II: A-CED.4; G-SRT.5; G-SRT.8
Secondary Math III: A-CED.4; A-REI.2

Objectives:

- Students will be able to apply the Pythagorean Theorem to find the height of an object. (Algorithmic Skill)
- Students will be able to assess their work and the accuracy of their answers. (Application)
- Students will be able to explain Pythagorean Theorem. (Comprehension and Communication)

Before the lesson:
Give yourself enough time to allow the students turn in the kites and to have the appropriate weather needed to fly kites. Find some time in class where the students can figure out how long their individual stride length is.

The lesson:
This lesson is an application of Pythagorean Theorem. There are two different forms which will allow you to choose the one which is best suited for your classroom. The second form asks the same questions as the first, but it allows the students to decide for themselves how to best solve the question at hand. In other words, it doesn't tell them to use Pythagorean Theorem, it allows them to think about different ways that they would try to solve the problem at hand.

We will be applying the Pythagorean Theorem this spring by flying kites. You will need a partner to do this project. You must bring your kite and string to school and leave it in the classroom until the conditions are right and we can go outside to fly them. If you are unable to purchase a kite, you need to have a parent signature below and will do a worksheet instead of this assignment.

Everybody needs to bring their kites to school by: ____________________
We hope to fly our kites by the week of: ____________________

Parent Signature: __________________________ My student can/cannot bring a kite to school.

Pre-Flying Questions:

1. Who will be your partner?
2. Who will be the one to bring the kite with string to school?
3. Both of you need to practice pacing off your steps so that you can calculate the distances for this project. (We will practice this in class.)
   One step = ________feet

Once you have flown your kite, you need to write a one page report and answer the following questions.

1. Draw a picture of the right triangle that relates to the kite. Tell me the distances of the string and how far your partner walked until he/she was directly under the kite (a perpendicular line).

The distance from Partner A to underneath the kite is ________.

Mark off your kite string with a piece of tape once Partner B is under the kite. Then measure the string.
The string is ___________ feet.
THE DRAWING SHOULD BE NO LARGER THAN 1/3 OF THE PAGE.

2. Now using your data, calculate how high your kite went using the Pythagorean Theorem. (Please show your work.)

3. How exact do you think your answer is? What are some possible errors that could make this number not very accurate? What were the weather conditions like?

***Also in your report please include the following:

- The names of partners and your class hour.
- How much one step equaled in feet for both of you.

Remember that this is a group effort. You will be grading each other on how well you participated in this project as well as writing the report.

The report is due on: ________________________

We will be reviewing math concepts this spring by flying kites. You will need a partner to do this project. You must bring your kite and string to school and leave it in the classroom until the conditions are right and we can go outside to fly them. If you are unable to purchase a kite, you need to have a parent signature below and will do a worksheet instead of this assignment.

Everybody needs to bring their kites to school by: ____________________
We hope to fly our kites by the week of: ____________________

Parent Signature: __________________________ My student can/cannot bring a kite to school.

Pre-Flying Questions:

1. Who will be your partner?
2. Who will be the one to bring the kite with string to school?
3. Both of you need to practice pacing off your steps so that you can calculate the distances for this project. (We will practice this in class.)
   One step = ________feet

Once you have flown your kite, you need to write a one page report and answer the following questions.

4. How are you going to find the height of the kite? Tell me the length of the string you let out. Mark off the kite string with a piece of tape to help you measure it.

5. If your partner walked until he/she was directly under the kite what kind of line would that make with the ground? What was the distance from Partner A to underneath the kite ________?

   Make a drawing to help explain how you solved this problem. It should be no larger than 1/3 of the page.

6. Now using your data, calculate how high your kite flew. (Please show your work.)

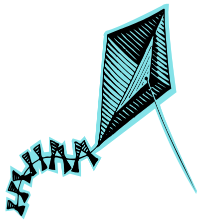
7. How exact do you think your answer is? What are some possible errors that could make this number not very accurate? What were the weather conditions like?

***Also in your report please include the following:

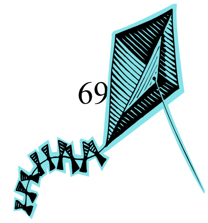
- The names of partners and your class hour.
- How much one step equaled in feet for both of you.

Remember that this is a group effort. You will be grading each other on how well you participated in this project as well as writing the report.

The report is due on: ________________________



Alternate Kite Flying Assignment



Name:____ Date:_____________

1. Go the library and find a book that is tilted at an angle. Take a ruler and measure the distance along the bookshelf from the book to the where it should be propped up against. Measure the length of the book. Use Pythagorean Theorem to find how high up the bookshelf the book is resting.

2. Take the same book and continue to slide the base as far as you can away so that it is still leaning, but won't fall. Complete the steps above and find the new height of the book on the bookshelf.

3. Go outside and find the length of a car's shadow using Pythagorean Theorem. (Be careful measuring not to scratch the car.)

Once you have measured the shadow, you need to write a one page report and answer the following questions.

Draw a picture of the right triangle that relates to the shadow. Tell me the lengths of all three sides and how you found them.

THE DRAWING SHOULD BE NO LARGER THAN 1/3 OF THE PAGE.

4. Now using your data, calculate the length of the shadow using the Pythagorean Theorem. (Please show your work.)

5. How exact do you think your answer is? What are some possible errors that could make this number not very accurate? What were the weather conditions like?

6. List four other objects you could have found the length of using Pythagorean Theorem.

The report is due on: ________________________

Kite Flying Scoring Rubric

Name(s):

Pythagorean Theorem:

+1 0	Height is accurate
+1 0	Students step length was measured accurately and relates to the kites height
+ 3 +2 +1 0	Explanation of how Pythagorean Theorem relates to the kite is clear and concise.

Drawing:

+1 0	3 sides labeled correctly and the student explains why they are accurate
+1 0	Right angle marked (some indication that Pythagorean Theorem can be used)

Errors:

+2 +1 0	Possible errors are listed and not erroneous. (Students have thought through if answers are plausible.)

Weather:

+1 0	Weather was included in the report and how that influenced the final height of the kite.

Total ____/10

Comments

Kite Flying Scoring Rubric

Name(s):

Pythagorean Theorem:

+1 0	Height is accurate
+1 0	Students step length was measured accurately and relates to the kites height
+ 3 +2 +1 0	Explanation of how Pythagorean Theorem relates to the kite is clear and concise.

Drawing:

+1 0	3 sides labeled correctly and the student explains why they are accurate
+1 0	Right angle marked (some indication that Pythagorean Theorem can be used)

Errors:

+2 +1 0	Possible errors are listed and not erroneous. (Students have thought through if answers are plausible.)

Weather:

+1 0	Weather was included in the report and how that influenced the final height of the kite.

Total ____/10

Comments