An Efficient Mechanism to Control Correlated Externalities: Redistributive Transfers and the Coexistence of Regional and Global Pollution Permit Markets

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Abstract: We examine joint tradable permit markets as a self-enforcing mechanism to control correlated externality problems. By "correlated" we mean multiple pollutants that are jointly produced by a single source but which simultaneously cause differentiated regional and global externalities (e.g. smog and global warming). By "self-enforcing" we mean a mechanism that accounts for the endogeneity that exists between competing jurisdictions in the setting of environmental policy within a federation of regions. We find that joint domestic and international permit markets are Pareto efficient for a wide class of preferences.

Keywords: correlated externality, joint emissions permit markets, Pareto efficiency.

JEL Classification: C72, D62, D78, H41, H77, Q28.

1. Introduction

Mechanisms to control transboundary externalities have received a great deal of attention in the recent literature, driven by the onset of such global problems as climate change, atmospheric ozone depletion, and biodiversity loss, as well as problems associated with acid rain. The mechanisms share two attributes. First, they explicitly account for jurisdictional control over the policy instruments used to mitigate the externality. For example, regional governments are endowed with the authority to independently levy emissions taxes or select abatement levels while a central government, e.g., the Global Environmental Facility (GEF) envisioned in the Kyoto Protocol, enacts transfers between the various regions of the federation or determines initial allocations of pollution permits (cf., [5, 6, 7, 8, 12, 15]).[1] Second, the mechanisms are designed to control the emissions of a single pollutant, e.g., carbon-equivalent gases in the case of global warming and sulfur dioxide in the case of acid rain.

In reality, a single source of emissions is typically comprised of multiple pollutants that cause simultaneous localized and global externality problems, and pollution abatement in turn jointly reduces the flows of these pollutants. For example, the burning of fossil fuels generates carbon dioxide (CO_2), ozone, nitrogen oxide (NO_x), and sulfur dioxide (SO_2) which create global or multiregional externalities, as well as carbon monoxide (CO), volatile organic compounds, and particulate matter, which have more localized, regional, external effects. Abatement technologies, on the other hand, are typically coarse in that they have joint effects on a multiplicity of

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[1] The GEF was established to operate the protocol's financial mechanism, including promoting monetary transfers to participant nations to help them defray the costs of desirable pollution control projects. As we show below, such an agency is of paramount importance for our ideal mechanism. It should be endowed with the authority to effect ex post interregional income transfers.

pollutants.[2] Consider the case of air pollution control. Jet scrubbers used to remove dust particles from a gas stream with a dispersed liquid – e.g. in the steel, chemical, and foundry industries – also remove gaseous pollutants [1, 13]. Absorption technologies, which create residual molecular forces at the surface of solids to attract molecules of gases and vapors, also provide good examples, since they lead to simultaneous removal of dust and gaseous pollutants from a gas mixture. Examples of these technologies include the absorption of SO_2 , NO_x, hydrogen flouride, and hydrogen chloride, as well as particulate matter from stack gases [1,13].

The facts that large numbers of single sources emit multiple pollutants and current abatement technologies are coarse imply that a number of important pollution problems are correlated. This motivates us to use the terminology “correlated externalities” to characterize such problems. In the presence of correlated externalities, previous mechanisms designed to control single externalities are generally incomplete. Their adoption would typically create inefficiencies, since correlated effects are neglected. For example, Chichilnisky, et al. [8] consider the creation of a single global permit market to control carbon dioxide emissions, where the GEF determines ex ante the initial allocation of permits across nations. Caplan, et al. [6] and Caplan and Silva [2] similarly examine a noncooperative "global warming game" concerning the allocation of carbon dioxide emissions, where the GEF determines ex post redistributive transfers. These studies, however, do not consider the correlation of regional and global externalities.[3]

In this paper, we demonstrate how a global permit market to control carbon emissions (along the lines of Chichilnisky [8]) in concert with ex post redistributive transfers made by a GEF

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[2] Fine abatement technologies would single out pollutants, enabling the controlling sources to deal with each pollutant separately.

[3] Hoel [9] and Michaelis [11] consider multiple, yet uncorrelated pollutants. See Jensen [10], Vis [14], and Yamin [16] for recent attempts at linking domestic and global tradable permit markets on the basis of the correlation that exists between domestic and global pollutants.

(along the lines of Caplan, et al. [6]) can be linked with regional permit markets to control localized pollutants such as smog. The contributions of the paper are twofold. First, we demonstrate that a decentralized mechanism that induces regional governments to simultaneously *and* endogenously control the regional and global externalities at efficient levels requires the governments to condition their regional pollution permit markets on the global pollution permit quotas they all, selfishly, choose under the international agreement.[4] Second, we show that an independent and benevolent GEF should enact interregional income transfers *after* the regions have chosen their respective endowments of the global pollutant, as this will induce the regions to fully internalize their contributions to the global externality.[5] Such a combination of joint permit markets and interregional transfers is self-enforcing in the sense that regions that voluntarily participate in the mechanism independently choose an efficient allocation of regional and global externalities – they are induced by the mechanism itself to do so.

The next section presents the basic correlated externality model with joint abatement technology and characterizes the Pareto efficient solution. Section 3 presents the case of joint emissions permit markets – separate markets to control the regional externalities and one international market to control the global externality – and shows that this mechanism is efficient for a wide class of preferences under decentralized leadership within a global federation. This section also examines the conditions under which the efficient mechanism is implementable. Section 4 provides some simple three-country examples of the joint permit market model. Section 5 concludes.

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[4] Concurrent research on the effectiveness of joint emissions taxes in controlling correlated externalities is on-going. See Caplan [4] for further details.

[5] However, as we show below, the central government can "move" before the regions choose their respective endowments of the regional pollutant.

2. A Correlated Externality Model

Consider a global economy consisting of $J \geq 2$ regions indexed by j, and $I_j > 1$ energy firms indexed by ij. [6] Assume n_j consumers are located in region j. The utility of a representative consumer in region j is $u^j(x_j,y_j,g_j,e)$, where x_j, y_j, g_j, and $e = \sum_{j=1}^{J} e_j$ are respectively the quantities consumed of a numeraire good, energy, a regional pollutant, and a global pollutant.[7] For the sake of illustration we call the global pollutant "carbon" and the local pollutant "smog". We assume that u^j is strictly quasi-concave, increasing in the first two arguments, and decreasing in the last two arguments. Net emissions of smog and carbon are, for example, harmful to each individual's health.

Let net emissions of carbon in region j be $e_j = \sum_{i=1}^{I_j} e_{ij} = \sum_{i=1}^{I_j} \left(Y_{ij} - \left(a_{ij}^e + \gamma^e a_{ij}^g\right)\right)$, where Y_{ij} is the total quantity of energy produced by energy firm i in region j, a_{ij}^e is total amount of abatement of carbon dioxide produced by energy firm i in region j, and a_{ij}^g is the total amount of abatement of smog produced by energy firm i in region j. The term $\gamma^e \in (0,1]$ represents the fraction of firm ij's abatement effort of smog that also reduces carbon dioxide emissions. For example, γ^e may be the fraction of carbon dioxide emissions removed by firm ij's application of a jet scrubber technology to remove dust particles at level a_{ij}^g.

Similarly, net emissions of smog in region j is defined as

$g_j = \sum_{i=1}^{I_j} g_{ij} = \sum_{i=1}^{I_j} \left(Y_{ij} - \left(a_{ij}^g + \gamma^g a_{ij}^e\right)\right)$, where in this case $\gamma^g \in (0,1]$ represents the fraction of firm

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[6] We can think of an energy firm as representing any type of firm that produces embodied energy.

[7] In keeping with the extant literature, we identify regions with "j" superscripts on functions and subscripts on variables.

ij's abatement effort of carbon that also reduces smog emissions. For example, (^{g} may be the fraction of dust particles removed by firm ij's application of an absorption technology to remove carbon emissions at level a^e_{ij}.

We assume throughout that all markets are competitive. We also normalize the price of numeraire good to one. We will focus our analysis on the operation of three types of markets, namely, the regional markets for energy and smog permits and the global market for carbon permits.

Region j's total income is represented by

$$w_j = x_j^0 + \sum_{i=1}^{I_j} \pi^{ij} + \tau_j, \quad \square\, j,k \,\square\, J,\ j \,\square\, k \tag{1}$$

where x_j^0 is an initial endowment of the numeraire good (e.g. money), B^{ij} is energy firm ij's profit (defined below), and ϑ_j is the central authority's transfer remitted to region j (if positive) or sent from region j (if negative). As mentioned earlier, an example of a central authority in the context of this model is the GEF established by the Kyoto Protocol. Ideally, the GEF should have the authority to enact redistributive international transfers between and within Annex I and Annex II countries. Hence, the GEF's transfers should satisfy the constraint, $\sum_{j=1}^{J} \tau_j = 0$.

Since residents are identical within each region, each consumer in region j faces a budget constraint,

$$x_j + r_j y_j = \frac{w_j}{n_j}, \quad \square\, j \,\square\, J \tag{2}$$

where r_j is the price of energy in region j. The representative consumer's problem is to maximize u^j by choosing $\{x_j,y_j\}$ subject to (2), taking r_j, g_j, w_j and e as given. The solution is given by (2) and

$$\frac{u_y^j}{u_x^j} = r_j, \ \square\, j \,\square\, J \tag{3}$$

i.e., the standard consumer-maximization result where the marginal rate of substitution is set equal to the price ratio (subscripts on functions denote the associated partial derivatives). Equations (2) and (3) can be used to implicitly define the consumer's demand functions $x_j \equiv x_j(r_j, w_j, g_j, e)$ and $y_j \equiv y_j(r_j, w_j, g_j, e)$.

In region j, firm i's profit from energy production is defined as $\pi^{ij} = r_j Y_{ij} - c^{ij}\left(Y_{ij}, a_{ij}^g, a_{ij}^e\right) + p\left(\bar{e}_{ij} - e_{ij}\right) + v_j\left(\bar{g}_{ij} - g_{ij}\right)$, where the cost function c^{ij} is strictly increasing and convex in each term; p equals the competitively determined price of a carbon emissions permit; v_j equals the competitively determined price of a smog permit in region j; and the quantities $\bar{e}_{ij}$ and $\bar{g}_{ij}$ are the amounts of carbon emissions permits and smog permits initially allocated to the firm. Note that $\sum_{i=1}^{I_j} \bar{e}_{ij} = \bar{e}_j$, $\sum_{j=1}^{J} \bar{e}_j = \bar{e}$ and $\sum_{i=1}^{I_j} \bar{g}_{ij} = \bar{g}_j$.[8]

In region j, each energy firm i maximizes B^{ij} by choosing $\left\{Y_{ij}, a_{ij}^g, a_{ij}^e\right\}$, taking $\bar{g}_{ij}, \bar{e}_{ij}$ and all prices as given. The first order conditions can be written as follows:

$$p = r_j - v_j - c_Y^{ij} = c_{a^e}^{ij} - \gamma^g v_j = \frac{c_{a^g}^{ij} - v_j}{\gamma^e}, \ \square\, i \,\square\, I_j; \square\, j \,\square\, J \tag{4}$$

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[8] We further assume that $c_{YY}^{ij}(c_{a^g a^g}^{ij} + c_{a^e a^e}^{ij}) - \left(c_{Ya^g}^{ij} + c_{Ya^e}^{ij}\right)^2 > 0$, which along with the quasi-concavity condition for the representative agents' utility functions ensures concave programming problems for each of the ensuing games analyzed below. Note that although the regional governments are responsible for determining an initial allocation of smog and carbon permits across their respective firms, the total endowments of these permits are determined in the equilibrium in the policy game described in Section 3.

namely, the standard profit-maximization conditions associated with the choices of energy output and abatement of smog and carbon. Equations (4) enable us to implicitly define the demand functions of each energy firm i in every region j, $Y_{ij} \equiv Y_{ij}\left(r_j, p, v_j\right)$, $a_{ij}^{g} \equiv a_{ij}^{g}\left(r_j, p, v_j\right)$, $a_{ij}^{e} \equiv a_{ij}^{e}\left(r_j, p, v_j\right)$, $e_{ij} \equiv e_{ij}\left(r_j, p, v_j\right)$, and $g_{ij} \equiv g_{ij}\left(r_j, p, v_j\right)$. Given these functions, one can immediately obtain each firm's indirect profit function $\pi^{ij} \equiv \pi^{ij}\left(r_j, p, v_j, \overline{e}_{ij}, \overline{g}_{ij}\right)$.

Equilibrium clearing conditions for the regional energy and smog permit markets, and the global carbon permit market, respectively, are as follows:

$$n_j y_j\left(r_j, w_j, \overline{g}_j, \overline{e}\right) = \sum_{i=1}^{I_j} Y_{ij}\left(r_j, p, v_j\right), \;\square\, j \,\square\, J \tag{5a}$$

$$\sum_{i=1}^{I_j} g_{ij}\left(r_j, p, v_j\right) = \overline{g}_j, \;\square\, j \,\square\, J \tag{5b}$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I_j} e_{ij}\left(r_j, p, v_j\right) = \overline{e}. \tag{5c}$$

The 2J+1 equations represented by (5a) - (5c) define $r_j \equiv r_j\left(w_1, \ldots, w_J, \overline{g}_1, \ldots, \overline{g}_J, \overline{e}\right) \;\square\, j \,\square\, J$, $v_j \equiv v_j\left(w_1, \ldots, w_J, \overline{g}_1, \ldots, \overline{g}_J, \overline{e}\right) \;\square\, j \,\square\, J$, and $p \equiv p\left(w_1, \ldots, w_J, \overline{g}_1, \ldots, \overline{g}_J, \overline{e}\right)$.

Before we analyze the making of environmental policy, it is useful to consider the conditions that characterize the set of Pareto efficient allocations. For a fixed set of social welfare weights θ = {2_j | 0 < 2_j < 1, j = 1,....J, $\sum_{j=1}^{J} \theta_j = 1$}, an interior Pareto efficient allocation can be obtained as a solution to the following problem:

$$\underset{\left\{x_j, y_j, \left\{Y_{ij}, a_{ij}^{g}, a_{ij}^{e}\right\}_{ij}\right\}_j}{\text{Max}} \sum_{j=1}^{J} \theta_j u^j \left(x_j, y_j, \sum_{i=1}^{I_j}\left(Y_{ij} - \left(a_{ij}^{g} + \gamma^{g} a_{ij}^{e}\right)\right), \sum_{j=1}^{J} \sum_{i=1}^{I_j}\left(Y_{ij} - \left(a_{ij}^{e} + \gamma^{e} a_{ij}^{g}\right)\right)\right)$$

$$\text{s.t.: } \sum_{j=1}^{J}\left(n_j x_j + \sum_{i=1}^{I_j} c^{ij}\left(Y_{ij}, a_{ij}^{g}, a_{ij}^{e}\right) - x_j^0\right) \le 0 \ , \ \sum_{j=1}^{J}\left(n_j y_j - \sum_{i=1}^{I_j} Y_{ij}\right) \le 0.$$

Besides the binding constraints, the first-order conditions are:

$$\frac{\theta_j u_x^j}{n_j} = \frac{\theta_k u_x^k}{n_k}, \ \text{any } j,k \ \square \ J, \ j \square k \tag{6}$$

and $\square$ j $\square$ J and $\square$ i $\square$ I_j

$$\frac{n_j u_g^j}{u_x^j} + \sum_{j=1}^{J} \frac{n_j u_e^j}{u_x^j} + \frac{u_y^j}{u_x^j} - c_Y^{ij} = 0 \tag{7}$$

$$\frac{n_j u_g^j}{u_x^j} + \gamma^e \sum_{j=1}^{J} \frac{n_j u_e^j}{u_x^j} + c_{a^g}^{ij} = 0 \tag{8}$$

$$\frac{\gamma^g n_j u_g^j}{u_x^j} + \sum_{j=1}^{J} \frac{n_j u_e^j}{u_x^j} + c_{a^e}^{ij} = 0. \tag{9}$$

Equations (6) show that the marginal utilities of the numeraire good (normalized by the welfare weights and populations) are equated across all regions. Equations (7) – (9) are modified Samuelson conditions for impure public bad (energy) and goods (abatement of smog and carbon), each equating the marginal social benefits of an additional unit of the economic activity with its associated social marginal cost. Equations (7) tell us how the energy outputs should be determined in each region j. Equations (8) and (9), on the other hand, show us how we should determine regional and global abatement levels for smog and carbon emissions, respectively. [9] The two binding constraints and equations (6) - (9), therefore, characterize the set of Pareto efficient solutions.

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[9] Note that equations (8) and (9) imply the familiar cost-minimization result of equalized marginal costs of abatement across firms within a given region is not satisfied by a Pareto-efficient allocation. This result occurs because of the joint-abatement cost configuration of our problem.

We will now compare the efficient allocation above with an allocation that emerges from a non-cooperative, command-and-control environmental policy system, which serves as the benchmark status quo. Suppose the regional governments choose their two vectors of abatement levels, $\left\{\mathbf{a}_{ij}^{g}, \mathbf{a}_{ij}^{e}\right\}$, independently of one another, where the vectors are defined over all firms ij in region j. To begin with, firm ij's necessary condition for profit maximization, equation (4), results in $r_j - c_Y^{ij} = 0$, which, appealing to (3), violates efficiency condition (7). The violation occurs because there is no mechanism (e.g. market) to induce the firm to internalize the negative externalities associated with its energy production.

Taking the energy price and firms' decisions as given, regional government j's problem is therefore,[10]

$$\underset{\left\{\mathbf{a}_{ij}^{g}, \mathbf{a}_{ij}^{e}\right\}}{\text{Max}} \quad u^j\left(x_j, y_j, g_j, e\right),$$

subject to (1) and (2), where $g_j = \sum_{i=1}^{I_j}\left(Y_{ij} - a_{ij}^{g} - \gamma^g a_{ij}^{e}\right)$ and $e = \sum_{j=1}^{J}\sum_{i=1}^{I_j}\left(Y_{ij} - a_{ij}^{e} - \gamma^e a_{ij}^{g}\right)$. The first-order conditions for this problem are,

$$\frac{n_j u_g^j}{u_x^j} + \gamma^e \frac{n_j u_e^j}{u_x^j} + c_{a^g}^{ij} = 0, \ \forall\, i \in I_j \text{ and } \forall\, j \in J \qquad \text{(10a)}$$

$$\gamma^g \frac{n_j u_g^j}{u_x^j} + \frac{n_j u_e^j}{u_x^j} + c_{a^e}^{ij} = 0, \ \forall\, i \in I_j \text{ and } \forall\, j \in J \qquad \text{(10b)}$$

Conditions (10a) and (10b) violate efficiency conditions (8) and (9) because there is no mechanism to induce the regions to account for the transboundary benefits associated with their choices of $\left\{\mathbf{a}_{ij}^{g}, \mathbf{a}_{ij}^{e}\right\}$. Since the regional governments do not make interregional transfers, the allocation also fails to satisfy efficiency condition (6). Thus, a non-cooperative, command-and-

[10] The regional government is aware, however, of the marginal effects of a_{ij}^{g} and a_{ij}^{e} on c^{ij} and thus their associated marginal effects on π^{ij}.

control environmental policy fails on each margin except with respect to the representative consumer's utility maximizing choice. For future reference, let $\{U^{jD}\}_j$ be the set of regional welfares that result from the non-cooperative, command-and-control environmental policy game.

3. Joint Emissions Permit Markets

We now analyze the allocation of resources under a game with permit markets for both smog and carbon. This game consists of three stages. In the first stage, each regional government agency charged with formulating its region's policy to control carbon – henceforth, the "carbon agency" – decides on its region's total endowment of carbon emissions permits, taking as given the price of energy and each other regional government's decision (i.e. as the result of a Nash game). Having observed the regional governments' decisions concerning their respective carbon endowments, the GEF decides in the second stage of the game the levels of the budget-balanced interregional income transfers. In the third and final stage, each regional governmental agency charged with formulating its region's policy to control the local pollutant – henceforth, the "smog agency" – decides its region's total endowment of smog permits, taking as given the price of energy and each other regional government's decision (i.e., also as a result of a Nash game).[11] Since we assume that this game is "played" in three distinct stages, the equilibrium concept used for its ultimate solution is sub-game perfection.

It is important to note that since the GEF and the regional governmental authorities take as given the decisions of the consumers and energy firms in each region, equations (1) – (5c)

[11] Note that the smog and carbon agencies need not be separate agencies within any given region. They are assumed so here strictly for expository purposes. Further, without loss of generality we assume that the smog and carbon agencies are also responsible for choosing the initial distribution of the smog and carbon permits, respectively, across firms, as this has no effect on the firms' marginal decisions at an interior solution.

naturally obtain in this game's equilibrium. Further, as Caplan and Silva [3] show, the solution for this game is interior for a wide class of preferences.

3.1 The Third Stage of the Game

Through backward induction, we start at the last stage of the game. In this stage, region j's smog agency chooses $\bar{g}_j$ to maximize its regional welfare. Formally stated, its problem is,

$$\underset{\{\bar{g}_j\}}{\text{Max}} \; u^j\left(x_j, y_j, \bar{g}_j, \bar{e}\right),$$

subject to the budget constraint (2) for the representative consumer in region j,

$$x_j = \frac{w_j}{n_j} - r_j y_j,$$

where w_j is defined according to (1). This results in the set of first-order optimality conditions

$$v_j = -\frac{n_j u_g^j}{u_x^j}, \; \forall j \in J. \tag{11}$$

Equations (11) reveal that each smog agency chooses the level of smog up to the point where the price of a permit just equals the value of the region's aggregate welfare loss associated with an additional unit of smog. The regional markets for smog therefore work as they should – the equilibrium price of a permit reflects the social marginal damage associated with an additional unit of smog in each respective region. Furthermore, equations (11) define the smog agencies' respective smog-endowment response functions $\bar{g}_j = \bar{g}_j\left(\tau_j, \bar{e}_j\right)$, $\forall j \in J$. These response functions are derived by totally differentiating (11), which, given $\sum_{j=1}^{J} \tau_j = 0$, results in,[12]

[12] Section 4 provides a simple three-country example of this result that helps to clarify the notation for (12a) and (12b).

$$\frac{\partial \bar{g}_j}{\partial \bar{e}_j} = p\frac{\partial \bar{g}_j}{\partial \tau_j} - \frac{\Gamma_e^j}{\Omega_g^j} \quad \square\ j \ \square\ J_{-k},\ J_{-k} \ \square\ J,$$

(12a)

$$\frac{\partial \bar{g}_k}{\partial \bar{e}_k} = -p\frac{\partial \bar{g}_k}{\partial \tau_j} - \frac{\Gamma_e^k}{\Omega_g^k} \quad \text{some } k \ \square\ J,\ k \ \square\ J_{-k},\ k \neq j, \tag{12b}$$

where $\Gamma_e^j = v_j u_{xe}^j + n_j u_{ge}^j$, $\Omega_g^j = \frac{v_j^2 u_{xx}^j}{n_j} + 2v_j u_{xg}^j + n_j u_{gg}^j$, □ j □ J, and J_{-k} is the subset of J not including region k.

As anticipated, the smog agencies' responses to their respective region's $\bar{e}_j$ and τ_j are roughly proportional to each other. In a sub-game perfect equilibrium, these response functions are endogenized (or, "correctly guessed") by the other players (i.e. the GEF and the respective carbon agencies) in the earlier stages of the game. Even though $\Omega_g^j < 0$, □ j □ J, is necessary for the second-order conditions to satisfy a global maximum, the sign of Γ_e^j, □ j □ J, is ambiguous. For this game, we assume $\Gamma_e^j = 0$, □ j □ J, and discuss the implications of this assumption in Section 4.

Given $\Gamma_e^j = 0$, □ j □ J, (12a) and (12b) may be rewritten as

$$\frac{\partial \bar{g}_j}{\partial \bar{e}_j} = p\frac{\partial \bar{g}_j}{\partial \tau_j}, \quad \square\ j \ \square\ J_{-k},\ J_{-k} \ \square\ J, \tag{13a}$$

$$\frac{\partial \bar{g}_k}{\partial \bar{e}_j} = -p\frac{\partial \bar{g}_k}{\partial \tau_j}, \quad \text{some } k \ \square\ J,\ k \ \square\ J_{-k},\ k \neq j. \tag{13b}$$

3.2 The Second Stage of the Game

In this stage, we assume that the GEF's objective function is a weighted global welfare function as follows:

$$W\left(\left\{u^j\right\}\right)=\sum_{j=1}^{J}\theta_j u^j\left(x_j, y_j, \bar{g}_j\left(\tau_j, \bar{e}_j\right), \bar{e}\right)$$

that is, the same objective function as in the Pareto efficiency problem examined above. The GEF takes $\left\{r_j, v_j, p, x_j, y_j, \left\{Y_{ij}, a_{ij}^{g}, a_{ij}^{e}\right\}_{ij}, \bar{e}\right\}_j$ as given and chooses the set $\left\{\tau_j\right\}_j$ to maximize W({u^j})

subject to (1), (2), $\sum_{j=1}^{J}\tau_j = 0$, and

$$u^j\left(x_j, y_j, \bar{g}_j\left(\tau_j, \bar{e}_j\right), \bar{e}\right) \geq U^{jD}, \ \forall j \in J, \tag{14}$$

where the set of variables $\left\{x_j, y_j, \bar{g}_j, \bar{e}\right\}_j$ in (14) is evaluated at the game's equilibrium.

Equations (14) represent the participation constraints for this game. Voluntary participation is necessary for the effectiveness of the game's agreement. Since there is potential for the agreement to Pareto improve upon the status quo (i.e. the non-cooperative, command-and-control benchmark solution, U^{jD}), all participation constraints may be satisfied nonbinding in the sub-game perfect equilibrium for this game. If the equilibrium allocation is Pareto efficient, there will be a range of θ values under which all regions will be strictly better off by participating in the agreement.

We shall make it our working hypothesis that the participation constraints are satisfied slack in the equilibrium for the second stage and later show that this is indeed a possibility.

The first-order conditions for this problem result in

$$\frac{\theta_j u_x^j}{n_j}+\left(\frac{v_j u_x^j}{n_j}-u_g^j\right)\frac{\partial \bar{g}_j}{\partial \tau_j}=\frac{\theta_k u_x^k}{n_k}+\left(\frac{v_k u_x^k}{n_k}-u_g^k\right)\frac{\partial \bar{g}_k}{\partial \tau_j}, \ \forall j \in J_{-k}, J_{-k} \subset J, \text{ some } k \in J, k \notin J_{-k}, k \neq j,$$

which, after applying (11), results in (6). Thus, the GEF is able to choose its transfers such that the marginal utilities of the numeraire good (normalized by the welfare weights and populations) are equated across all regions. As shown in the Appendix, total differentiation of equations (6) result in the GEF's transfer response functions $\tau_j(\bar{e}_1,\ldots,\bar{e}_J)$, □ j □ J. Given (13a) and (13b) these response functions imply,

$$\frac{\partial \tau_j}{\partial \bar{e}_k} - \frac{\partial \tau_j}{\partial \bar{e}_j} = p\,,$$ □ j □ J_{-k}, J_{-k} □ J, some k □ J, k □ $J_{-k,}$ $k \neq j$.

(15)

3.3. The First Stage of the Game

In this stage, region j's carbon agency chooses $\bar{e}_j$ to maximize its regional welfare. In doing so, the carbon agency correctly guesses the GEF's transfer-response function for its region, as well as the smog agency's permit-allocation response function. Formally stated, its problem is,

$$\underset{\{\bar{e}_j\}}{\text{Max}}\ u^j\left(x_j, y_j, \bar{g}_j\left(\tau_j\left(\bar{e}_1,\ldots,\bar{e}_J\right), \bar{e}_j\right), \bar{e}\right),$$

subject to (1) and (2). This results in the set of first-order optimality conditions

$$\frac{u_x^j\left(v_j\left(\frac{\partial \bar{g}_j}{\partial \bar{e}_j} + \frac{\partial \bar{g}_j}{\partial \tau_j}\frac{\partial \tau_j}{\partial \bar{e}_j}\right) + p + \frac{\partial \tau_j}{\partial \bar{e}_j}\right)}{n_j} + u_g^j\left(\frac{\partial \bar{g}_j}{\partial \bar{e}_j} + \frac{\partial \bar{g}_j}{\partial \tau_j}\frac{\partial \tau_j}{\partial \bar{e}_j}\right) + u_e^j = 0,$$ □ j □ J,

which, after applying (11), results in,

$$p + \frac{\partial \tau_j}{\partial \bar{e}_j} = -\frac{n_j u_e^j}{u_x^j},$$ □ j □ J_{-k}, J_{-k} □ J

(16a)

$$p - \sum_{j=1}^{J} \frac{\partial \tau_j}{\partial \bar{e}_k} = -\frac{n_k u_e^k}{u_x^k},$$ some k □ J, k □ $J_{-k,}$ $k \neq j$. (16b)

Summing (16a) and (16b) results in,

$$p = -\sum_{j=1}^{J} \frac{n_j u_e^j}{u_x^j}. \qquad (17)$$

Equation (17) reveals that the carbon agencies jointly choose the level of the carbon emissions up to the point where the equilibrium price of a permit just equals the value of the *global* welfare loss associated with an additional unit of carbon. The market for carbon therefore works as it should – the equilibrium price of a permit reflects the global marginal damage associated with an additional unit of carbon.

The following proposition summarizes the results of the equilibrium analysis for the joint permit market game.

Proposition: Provided (a) the participation constraints are satisfied slack and (b) $\Gamma_e^j = 0, \forall j \in$ J, the sub-game perfect equilibrium for joint emissions permit markets is Pareto efficient.

Proof: First, note that equations (1) - (6) hold for the game. Substituting equations (11) and (17) recursively into the three equalities of equations (4) for each respective region results in equations (7) – (9). Therefore, each of the necessary conditions for a Pareto efficient solution is satisfied for this game for any set of weights consistent with the participation constraints being satisfied slack.

Proposition 1 tells us that the redistributive transfers implemented by the GEF, in concert with an international permit market for carbon and separate smog permit markets, may be powerful enough to nullify each region's incentive to ignore the negative externalities caused by its own emissions. Since the modified Samuelson conditions are satisfied in the equilibrium of the game, equations (11) and (17) clearly demonstrate that each firm in each respective region faces its set of Lindahl prices when it chooses how much carbon and smog to emit (these prices are the carbon and smog permit prices, respectively). This implies that each firm (and thus each region) has no unilateral incentive to deviate from fully internalizing both the regional and global externalities. The mechanism therefore induces not only an efficient allocation of carbon and smog, but it is also self-enforcing.

It is important to reiterate that Proposition 1 holds if the participation constraints are satisfied slack in equilibrium. Since the non-cooperative (i.e. status quo) allocation is inefficient and the equilibrium allocation for this game is efficient when the participation constraints are ignored, there exists a range of θ values under which all regions can be made better off by participating in the game – i.e. the equilibrium allocation is such that each region's welfare is not less than its non-cooperative welfare U^{jD}. We shall assume henceforth that the designers of the game – that is, the regional governments themselves – agree before ratification on a distribution of θ parameters that will make all regions better off upon completion of the game. Such an agreement may emerge, for example, from a Nash bargaining game played by the regions prior to the commencement of the game.

We now turn our attention to the types of preferences that satisfy the third-stage assumption $\Gamma_e^j = 0$, □ j □ J, and therefore satisfy our efficiency proposition. As will be shown, satisfying this assumption is a *sufficient* condition for Pareto efficiency. To accomplish this goal, and to illuminate the previous analysis of the joint permit market game, we provide a simple three-region example of the correlated externality model.

4. Examples

In this section we begin by showing that a simple Cobb-Douglas specification of preferences satisfies the efficiency proposition of Section 3. We are then in a position to show why a completely separable preference specification also satisfies the efficiency proposition, but partially separable preferences do not.

Assume a simple three-region, I_j-firm world. The consumer in region j, j=1,2,3, is characterized by the Cobb-Douglas utility function,

$$u_j = x_j^{\sigma_j} y_j^{\delta_j} \bar{g}_j^{\alpha_j} \bar{e}^{\beta_j},$$

and income and budget constraints (1) and (2), respectively, where $0 < \Phi_j > 0$, $0 < *_j < 1$, $\forall_j < 0$, and $\exists_j < 0$. The consumer's utility maximization problem therefore results in,

$$\frac{\delta_j x_j}{\sigma_j y_j} = r_j, \ \ j=1,2,3. \qquad (3^{\mathbb{Z}})$$

The profit function for firm i, inclusive of its specified joint cost function, is

$$\pi^{ij} = \begin{cases} r_j Y_{ij} - \left(Y_{ij}^2 + \dfrac{\left(a_{ij}^g\right)^2}{2} + \dfrac{\left(a_{ij}^e\right)^2}{2} \right) & \text{if Pareto efficient solution} \\ r_j Y_{ij} - \left(Y_{ij}^2 + \dfrac{\left(a_{ij}^g\right)^2}{2} + \dfrac{\left(a_{ij}^e\right)^2}{2} \right) + v_j \left(\bar{g}_{ij} - g_{ij} \right) + p \left(\bar{e}_{ij} - e_{ij} \right) & \text{if mechanism with joint permits} \end{cases}$$

The firm's profit maximization problem therefore results in,

$$p = r_j - v_j - 2Y_{ij} = a_{ij}^e - \gamma^g v_j = \frac{a_{ij}^g - v_j}{\gamma^e}, \ \ \square \text{ i } \square \text{ I}_j; \ j = 1,2,3. \qquad (4^{\mathbb{Z}})$$

Together, the representative consumers' utility functions and the firms' profit functions define concave programming problems under both the Pareto-efficient and pollution-permit mechanisms. It is now straightforward to show that the Pareto efficient solution satisfies the following conditions,

$$\frac{\sigma_1 \theta_1 u_1}{n_1 x_1} = \frac{\sigma_2 \theta_2 u_2}{n_2 x_2} = \frac{\sigma_3 \theta_3 u_3}{n_3 x_3} \qquad (6^{\mathbb{Z}})$$

$$\frac{\alpha_j n_j x_j}{\sigma_j \bar{g}_j} + \sum_{j=1}^{3} \frac{\beta_j n_j x_j}{\sigma_j \bar{e}} = -\left(r_j - 2Y_{ij} \right), \ \ \square \text{ i } \square \text{ I}_j \text{ and j=1,2,3} \qquad (7^{\mathbb{Z}})$$

$$\frac{\alpha_j n_j x_j}{\sigma_j \bar{g}_j} + \gamma_j^e \sum_{j=1}^{3} \frac{\beta_j n_j x_j}{\sigma_j \bar{e}} = -a_{ij}^g, \ \ \square \text{ i } \square \text{ I}_j \text{ and j=1,2,3} \qquad (8^{\mathbb{Z}})$$

$$\frac{\gamma_j^g \alpha_j n_j x_j}{\sigma_j \bar{g}_j} + \sum_{j=1}^{3} \frac{\beta_j n_j x_j}{\sigma_j \bar{e}} = -a_{ij}^e, \ \ \square \text{ i } \square \text{ I}_j \text{ and j=1,2,3.} \qquad (9^{\mathbb{Z}})$$

For the three-stage joint-permit market game, note that conditions (1), (2), $(3^{\mathbb{Z}})$, $(4^{\mathbb{Z}})$, and (5a) – (5c) all hold. Following the procedure for Stage 3 described in Section 3, the smog agencies' problems result in,

$$v_j = -\frac{\alpha_j n_j x_j}{\sigma_j \bar{g}_j}, \text{ j=1,2,3.} \tag{$11^{\mathbb{Z}}$}$$

Let $\sum_{j=1}^{3} \tau_j = 0 \Rightarrow \tau_2 = \tau_1 - \tau_3$. Totally differentiating $(11^{\mathbb{Z}})$ for j = 1,2,3, respectively, results in

$$\frac{\partial \bar{g}_j}{\partial \bar{e}_j} = p \frac{\partial \bar{g}_j}{\partial \tau_j}, \text{ j = 1 and 3} \tag{$13a^{\mathbb{Z}}$}$$

$$\frac{\partial \bar{g}_2}{\partial \bar{e}_2} = -p \frac{\partial \bar{g}_2}{\partial \tau_1} = -p \frac{\partial \bar{g}_2}{\partial \tau_3}. \tag{$13b^{\mathbb{Z}}$}$$

In the second stage, the central government chooses $\{\tau_j\}_{j=1,2,3}$ to maximize

$$W\left(\left\{u^j\right\}\right) = \sum_{j=1}^{3} \theta_j u^j \left(x_j, y_j, \bar{g}_j\left(\tau_j, \bar{e}_j\right), \bar{e}\right), \text{ subject to (1), (2), and } \sum_{j=1}^{3} \tau_j = 0 \Rightarrow \tau_2 = -\tau_1 - \tau_3.$$

This problem results in

$$\frac{\theta_1 \sigma_1 u^1}{n_1 x_1} + \theta_1 \left(\frac{\sigma_1 v_1 u^1}{n_1 x_1} + \frac{\alpha_1 u^1}{\bar{g}_1} \right) \frac{\partial \bar{g}_1}{\partial \tau_1} = \frac{\theta_2 \sigma_2 u^2}{n_2 x_2} + \theta_2 \left(\frac{\sigma_2 v_2 u^2}{n_2 x_2} + \frac{\alpha_2 u^2}{\bar{g}_2} \right) \frac{\partial \bar{g}_2}{\partial \tau_1}, \text{ and}$$

$$\frac{\theta_3 \sigma_3 u^3}{n_3 x_3} + \theta_3 \left(\frac{\sigma_3 v_3 u^3}{n_3 x_3} + \frac{\alpha_3 u^3}{\bar{g}_3} \right) \frac{\partial \bar{g}_3}{\partial \tau_3} = \frac{\theta_2 \sigma_2 u^2}{n_2 x_2} + \theta_2 \left(\frac{\sigma_2 v_2 u^2}{n_2 x_2} + \frac{\alpha_2 u^2}{\bar{g}_2} \right) \frac{\partial \bar{g}_2}{\partial \tau_3}.$$

Applying $(11^{\mathbb{Z}})$ to these equations results in $(6^{\mathbb{Z}})$. It is now straightforward, albeit algebraically messy, to show that totally differentiating the first equality in $(6^{\mathbb{Z}})$ with respect to $\vartheta_1, \bar{e}_1$, and $\bar{e}_2$, and the second equality by $\vartheta_3, \bar{e}_3$, and $\bar{e}_2$ results in

$$\frac{\partial \tau_1}{\partial \bar{e}_2} - \frac{\partial \tau_1}{\partial \bar{e}_1} = \frac{\partial \tau_3}{\partial \bar{e}_2} - \frac{\partial \tau_3}{\partial \bar{e}_3} = p. \tag{$15^{\mathbb{Z}}$}$$

Following the procedure for Stage 1 described in Section 3, the carbon agencies' problems result in,

$$\frac{\sigma_j u_j \left(v_j \left(\frac{\partial \bar{g}_j}{\partial \bar{e}_j} + \frac{\partial \bar{g}_j}{\partial \tau_j} \frac{\partial \tau_j}{\partial \bar{e}_j} \right) + p + \frac{\partial \tau_j}{\partial \bar{e}_j} \right)}{n_j x_j} + \frac{\alpha_j u_j \left(\frac{\partial \bar{g}_j}{\partial \bar{e}_j} + \frac{\partial \bar{g}_j}{\partial \tau_j} \frac{\partial \tau_j}{\partial \bar{e}_j} \right)}{\bar{g}_j} + \frac{\beta_j u_j}{\bar{e}} = 0, \ j = 1,2,3.$$

Applying ($11^{\mathbb{Z}}$) to these equations results in,

$$p + \frac{\partial \tau_j}{\partial \bar{e}_j} = -\frac{\beta_j n_j x_j}{\sigma_j \bar{e}}, \ j = 1 \text{ and } 3 \tag{16a$^{\mathbb{Z}}$}$$

$$p - \left(\frac{\partial \tau_1}{\partial \bar{e}_2} + \frac{\partial \tau_3}{\partial \bar{e}_2} \right) = -\frac{\beta_2 n_2 x_2}{\sigma_2 \bar{e}}. \tag{16b$^{\mathbb{Z}}$}$$

Summing ($16a^{\mathbb{Z}}$) and ($16b^{\mathbb{Z}}$) and applying ($15^{\mathbb{Z}}$) results in

$$p = -\sum_{j=1}^{3} \frac{\beta_j n_j x_j}{\sigma_j \bar{e}}. \tag{17$^{\mathbb{Z}}$}$$

As in the proof of the proposition in Section 3, substituting equations ($11^{\mathbb{Z}}$) and ($17^{\mathbb{Z}}$) recursively into the three equalities of equations ($4^{\mathbb{Z}}$) for each of the three regions results in equations ($7^{\mathbb{Z}}$) – ($9^{\mathbb{Z}}$). Therefore, each of the necessary and sufficient conditions for a Pareto efficient solution is satisfied for this game.

It is instructive to note that the additional sufficient condition $\Gamma_e^j = v_j u_{xe}^j + n_j u_{ge}^j = 0$, j = 1,2,3, holds for this game. To see this, note that $v_j u_{xe}^j = v_j \beta_j \sigma_j x_j^{\sigma_j - 1} y^{\delta_j} \bar{g}^{\alpha_j} \bar{e}^{\beta_j - 1}$ and $n_j u_{ge}^j = n_j \alpha_j \beta_j x_j^{\sigma_j} y^{\delta_j} \bar{g}^{\alpha_j - 1} \bar{e}^{\beta_j - 1}$, j = 1,2,3. Now applying ($11^{\mathbb{Z}}$) to $v_j u_{xe}^j + n_j u_{ge}^j$ reveals that $\Gamma_e^j = 0$, j = 1,2,3.

We see immediately that $\Gamma_e^j = 0$ for completely separable preferences, since u_{xe}^j and u_{ge}^j are zero by definition. Thus, we know that complete separability also results in Pareto efficiency for this game.[13] These results for Cobb-Douglas and completely separable preferences imply that our efficiency proposition also holds for constant-elasticity-of-substitution (CES) preferences. To the

[13] See Caplan and Silva [3] for a full example of this result.

contrary, partially separable preferences of the form $u_j = x_j^{\sigma_j} y_j^{\delta_j} + \bar{g}_j^{\alpha_j} \bar{e}^{\beta_j}$ do not satisfy $\Gamma_e^j = 0$. To see this, note that the smog agencies' first-order conditions become

$$v_j = -\frac{\alpha_j n_j \bar{g}_j^{\alpha_j - 1} \bar{e}^{\beta_j}}{\sigma_j x_j^{\sigma_j - 1} y_j^{\delta_j}}, \text{ j=1,2,3.} \qquad (11^{\mathbb{Z}\mathbb{Z}})$$

In this case, $v_j u_{xe}^j = 0$ and $n_j u_{ge}^j = n_j \alpha_j \beta_j \bar{g}^{\alpha_j - 1} \bar{e}^{\beta_j - 1}$, j = 1,2,3. Now applying $(11^{\mathbb{Z}\mathbb{Z}})$ to $v_j u_{xe}^j + n_j u_{ge}^j$ reveals that $\Gamma_e^j \neq 0$, j = 1,2,3.

5. Conclusions

This paper represents an initial examination of the effectiveness of joint permit markets in controlling correlated externalities with joint abatement technology when control over the relevant policy instruments is shared by a hierarchy of independent governments and governmental agencies. Perhaps most restrictive is the full-information sub-game equilibrium concept used to determine the model's outcomes. Future research might therefore incorporate both uncertainty and alternative game-theoretic equilibrium concepts in order to further test the robustness of the paper's main finding that joint permit markets are Pareto efficient and self-enforcing under jurisdictional competition. Future research might also focus on a more thorough delineation of what types of preferences lead to the Pareto efficient result in this framework.

Though the framework for this paper is both restrictive and idealistic, the model accounts for three important constraints inherent in the control of transboundary pollution problems. These constraints are (1) the correlated nature of regional and global pollutants, (2) technologies that provide various degrees of joint abatement, and (3) the inescapable fact that a hierarchy of governmental institutions – often with competing objectives – are jointly responsible for enacting the policies and enforcement mechanisms that ultimately determine the levels at which these pollution problems are controlled. Having forgotten these constraints, proponents of a single permit markets to control global externalities may be overlooking a crucial objective, that market-based mechanisms not only induce an efficient outcome, but also one that is self-enforcing.

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Appendix

This appendix shows how the GEF's transfer response functions are derived, and also why $\Gamma_e = 0$, □ j □ J is sufficient for condition (15). For ease of exposition, we consider a simple two-region example, which naturally generalizes to J regions. Given (6) and $\tau_1 + \tau_2 = 0$, total differentiation of (6) results in,

$$\Psi_1 d\tau_1 + \Psi_2 d\bar{e}_1 + \Psi_3 d\bar{e}_2 = 0$$

where $\Psi_1 = \left(\frac{\theta_1 n_2 u^1_{xx}}{n_1} + \left(\frac{v_1 \theta_1 n_2 u^1_{xx}}{n_1} + \theta_1 n_2 u^1_{xg} \right) \frac{\partial \bar{g}_1}{\partial \tau_1} + \frac{\theta_2 n_1 u^2_{xx}}{n_2} - \left(\frac{v_2 \theta_2 n_1 u^2_{xx}}{n_2} + \theta_2 n_1 u^2_{xg} \right) \frac{\partial \bar{g}_2}{\partial \tau_1} \right)$,

$$\Psi_2 = \left(\frac{\theta_1 n_2 u^1_{xx} p}{n_1} + \left(\frac{v_1 \theta_1 n_2 u^1_{xx}}{n_1} + \theta_1 n_2 u^1_{xg} \right) \frac{\partial \bar{g}_1}{\partial \bar{e}_1} + \theta_1 n_2 u^1_{xe} - \theta_2 n_1 u^2_{xe} \right), \text{ and}$$

$$\Psi_3 = \left(\theta_1 n_2 u^1_{xe} - \left(\frac{v_2 \theta_2 n_1 u^2_{xx}}{n_2} + \theta_2 n_1 u^2_{xg} \right) \frac{\partial \bar{g}_2}{\partial \bar{e}_2} - \frac{\theta_2 n_1 u^2_{xx} p}{n_2} - \theta_2 n_1 u^2_{xe} \right).$$

Applying (12a) and (12b) to this expression results in,

$$\frac{\partial \tau_1}{\partial \bar{e}_2} - \frac{\partial \tau_1}{\partial \bar{e}_1} = p - \left\{ \frac{\Gamma^1_e}{\Omega^1_g} \left(\frac{\theta_1 n_2 u^1_{xx} v_1}{n_1} + \theta_1 n_2 u^1_{xg} \right) - \frac{\Gamma^2_e}{\Omega^2_g} \left(\frac{\theta_2 n_1 u^2_{xx} v_2}{n_2} + \theta_2 n_1 u^2_{xg} \right) \right\} / \Psi_1 . \quad \text{(A1)}$$

It is now readily apparent that $\Gamma^1_e = \Gamma^2_e = 0 \Rightarrow \Gamma^j_e = 0$, □ j □ J is sufficient for (15). Appealing to (A1), the necessary condition for (15) is therefore,

$$\frac{\Gamma^j_e}{\Omega^j_g} \left(\frac{\theta_j n_k u^j_{xx} v_j}{n_j} + \theta_j n_k u^j_{xg} \right) + \frac{\Gamma^k_e}{\Omega^k_g} \left(\frac{\theta_k n_j u^k_{xx} v_k}{n_k} + \theta_k n_j u^k_{xg} \right) = 0$$ □ j □ J-k, J-k □ J, some k □ J,

k □ J-k, $k \neq j$.