A PASSIVITY-BASED APPROACH TO SPACECRAFT FORMATION FLYING

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Abstract

In this paper we present a generalized coupled dynamics approach to formation flying. The approach is completely passivity-based. It requires no velocity information. It is decentralized in that very little communication among spacecraft is required.

1 Introduction

Travel to neighboring galaxies would require space voyages lasting thousands of years. As a result further space exploration can only be practically achieved by indirect observation of astronomical objects. Much can be determined about a space object from the emitted light. To make such delicate observations, space-based interferometers with baselines on the order of one to ten kilometers would be needed.

Large monolithic space-based interferometers are not physically feasible. In [Decou, 1991a] and [Decou, 1991b] a free-flying multiple spacecraft interferometer is proposed. A proposed free-flying multiple spacecraft interferometer would consist of three spacecraft. Two spacecraft would move within an observation plane to sample light from an astronomical body. The light is then reflected to the third spacecraft which observes the interference pattern from the two different light paths (see Figure 1).

In [Joshi, 1998] the van Cittert-Zernike result is described which is the basis of interferometric imaging. Define the $\nu - \eta$ plane as the observation plane. The location of the two collector spacecraft are $(\nu_1, \eta_1)$ and $(\nu_2, \eta_2)$ respectively. The star is located at $(x, y)$ a distance $z$ away from the observation plane. We can define new coordinates

$\begin{align*}
u &= \frac{\eta_1 - \eta_2}{z\lambda} \\
v &= \frac{\nu_1 - \nu_2}{z\lambda},
\end{align*}$

where $\lambda$ is the wave length of the observed light. These new coordinates define the $U - V$ plane.

Observation of the interference pattern for different values of $(u, v)$ allows for measurement of the complex mutual coherence function $\mu(u,v)$. The van Cittert-Zernike result is that the inverse Fourier transform of $\mu(x,y)$ gives the desired irradiance pattern $I(x,y)$ of the celestial body.

To adequately sample the U-V plane the spacecraft formation must make many measurements from different formation orientations and relative spacings. Formation reorientations and expansions fit into the class of motion that we define as elementary formation maneuvers. Elementary formation maneuvers (EFMs) are formation translations, reorientations and resizing. The focus of this work is the development of controls to implement EFMs.

In order to perform this series of measurements sensor lock must be initialized among pairs of spacecraft to monitor the relative distance and orientation to neighboring spacecraft. Once this sensor lock is established a series of measurements can be made to sample the U-V plane. While each measurement is made the spacecraft must maintain relative formation to within centimeters. On board machinery would then reduce these relative formation errors down to the order of nanometers as required for interferometry applications. While moving the formation to the next observation the relative position and alignment errors must be kept small enough to maintain sensor lock among spacecraft. If sensor lock is lost, the costly process (in terms of fuel and time) of formation initialization must be repeated.

In the literature there exists three approaches
Soon we will publish results that integrate the position and attitude control problems.

The paper is organized as follows. In Section 2 we develop a modified definition of an elementary formation maneuver. This definition is amenable to nonlinear formation control. We present the passivity based coupled dynamics control in Section 3. Section 4 gives some simulations. We summarize our results in Section 5.

2 Elementary Formation Maneuvers

Knowledge of the formation task can be used to simplify the spacecraft coordination problem. For interferometry problems the group of spacecraft must move in unison performing a series of reorienting, resizing and repointing maneuvers. This allows the group of spacecraft to sample the UV plane. It is natural to restrict our attention to the following Elementary Formation Maneuvers (EFMs): formation resizing, reorientations and translations (see Figure 2).

EFMs allow for a nice division between the centralized and decentralized formation tasks. Selection of the EFM to be implemented would be done by a formation supervisor. Control gains would be selected for each spacecraft based on the saturation limits of each vehicle and the desired distance to travel. This would be initiated upon completion of a previous EFM and any necessary measurements. The execution of the EFM itself can be implemented with a decentralized control.
2.1 Formation Translations

For formation translations there exists a trade-off between maintaining formation and arriving at a final goal. Consider the example shown in Figure 3. The left triangle represents the desired formation for the group of spacecraft at the initial time \( t_1 \). Each vertex being a desired spacecraft position. The right triangle represents the desired formation of the spacecraft at time \( t_2 \). The initial location of each spacecraft is shown in the figure. In this example a group of three spacecraft are to move in formation to the right. However, one of the spacecraft begins the maneuver out of formation. In order to correct this problem it has two conflicting objectives:

- move right to arrive at the final goal,
- move left to regain formation.

If it moves left it will likely overshoot the formation, which is moving right, and if it moves right it will take longer to regain formation as the others “catch up”.

![Formation Translation](image)

Figure 3: Bottom right member of the formation is initially too far to the right

Consider a group of \( N \) spacecraft with indices comprising the set \( \mathcal{I} = \{1, 2, \ldots, N\} \). Each spacecraft has double integrator dynamics

\[
m_i \ddot{r}_i = u_i,
\]

where \( m_i \) is the spacecraft mass \( r_i, u_i \in \mathbb{R}^3 \). Note that no requirement is made that these vehicles be real vehicles. In fact by choosing a formation leader to be a virtual vehicle, all of the results of this paper extend to virtual structure control.

Let \( e_G, e_F : \mathbb{R} \to \mathbb{R} \) be positive definite, symmetric continuously differentiable functions. Additionally we will require that \( e_F \) be monotonically increasing and invertible on the domain \([0, \infty)\). Furthermore let

\[
k_G(x) = \frac{de_G(x)}{dx}, \quad k_F(x) = \frac{de_F(x)}{dx}.
\]

We wish to create an error function that incorporates both absolute convergence and formation keeping. First we develop an error function \( E_G \) that expresses absolute convergence. Let \( \mathcal{L} \neq \emptyset \subseteq \mathcal{I} \) and let

\[
E_G = \sum_{i \in \mathcal{L}} \sum_{j=1}^{3} e_G(z_{ij}),
\]

where \( r_i = (r_{i1}, r_{i2}, r_{i3})^T, r_{id} = (r_{id1}, r_{id2}, r_{id3})^T \) and \( z_i = r_i - r_{id} \) (see Figure 3).

Next, we define the the function \( E_F \) as the spacecraft formation keeping error. Let \( \mathcal{P} \subseteq \mathcal{I} \times \mathcal{I} \) were \( \mathcal{P} \) is the edge set for a connected graph \( G \) with vertices \( V(G) = \mathcal{I} \) (see [Gould, 1988]). The formation error is defined by

\[
E_F = \sum_{(i,j) \in \mathcal{P}} \sum_{k=1}^{N} e_F(z_{ik} - z_{jk}).
\]

By maintaining the quantity \( E_F \) small during the entire maneuver, the spacecraft will equalize the distance that they need to go to reach the final goal. Note that \( E_F = 0 \) if and only if \( z_i = z_j \) for all \((i,j) \in \mathcal{P}\). This is equivalent to saying that \( r_i - r_j = r_{id} - r_{jd} \). Since \( G \) is a connected graph this will only be true if all spacecraft are in the same relative formation that they will have at the end of the maneuver. Therefore when \( E_F = 0 \), the spacecraft will be keeping formation, but they will not necessarily be at their final desired positions.

If \( E_G = E_F = 0 \) then the spacecraft in the set \( \mathcal{L} \) will converge to their final desired positions and since all spacecraft will be keeping formation, the remaining spacecraft will converge as well.

We can now define a total error for the formation problem as a weighted sum of \( E_G \) and \( E_F \)

\[
E = k_F E_F + k_G E_G,
\]

where \( k_F \) and \( k_G \) weight the relative importance of formation keeping versus absolute convergence.
The control objective is to cause $E \to 0$ asymptotically given the spacecraft equations of motion (1).

### 2.2 Formation Resizing and Reorientations

A formation resize or reorientation can be formulated as a formation translation in cylindrical coordinates with translations along the radial axis corresponding to resizing, and translations along the polar axis corresponding to reorientations. A polar plane translation in an arbitrary direction would be a spiral.

First we define the $r_{i3}$-axis of the coordinate system as the axis of reorientation or resize, and express the spacecraft position in terms of cylindrical coordinates $(\rho_i, \phi_i r_{i3})$. The cylindrical coordinates relate to the Cartesian coordinates via

$$
\begin{align*}
    r_{i1} &= \rho_i \cos(\phi_i) \\
    r_{i2} &= \rho_i \sin(\phi_i) \\
    r_{i3} &= r_{i3}.
\end{align*}
$$

(3)

Differentiating $r_i$ we get

$$
\dot{r}_i = \begin{bmatrix}
    \dot{r}_{i1} \\
    \dot{r}_{i2} \\
    \dot{r}_{i3}
\end{bmatrix} = R(\phi_i) \begin{bmatrix}
    1 & 0 & 0 \\
    0 & \rho_i & 0 \\
    0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    \dot{\rho}_i \\
    \dot{\phi}_i \\
    \dot{r}_{i3}
\end{bmatrix},
$$

where $R(\phi_i)$ is the rotation matrix

$$
R(\phi_i) = \begin{bmatrix}
    \cos(\phi_i) & -\sin(\phi_i) & 0 \\
    \sin(\phi_i) & \cos(\phi_i) & 0 \\
    0 & 0 & 1
\end{bmatrix}.
$$

Define the formation goal error by

$$
E_G = \sum_{i \in \mathcal{L}} \sum_{k=1}^{3} e_G(z_{ik})
$$

and the formation keeping error by

$$
E_F = \sum_{(i,j) \in \mathcal{P}} \sum_{k=1}^{3} e_F(z_{ij} - z_{i,k}),
$$

where

$$
\begin{align*}
    \dot{\rho}_i &= \rho_i - \rho_{id} \\
    \dot{\phi}_i &= \phi_i - \phi_{id} \\
    \dot{r}_{i3} &= \dot{r}_{i3} - \dot{r}_{i3d} \\
    z_i &= [\dot{\rho}_i, \dot{\phi}_i, \dot{r}_{i3}].
\end{align*}
$$

Again let $E = k_F E_F + k_G E_G$. For example, if we pick $\rho_{id} = \rho_i$ and choose $\phi_{id} = \phi_i + \pi/4$, forcing $E \to 0$ will result in a formation rotation of $\pi$ radians. Alternatively letting $\rho_{id} = \rho_i + 5$ and $\phi_{id} = \phi_i$ will result in a formation expansion by five units. Similarly if we vary both $\phi_i$ and $\rho_i$ simultaneously, then the rotation will spiral outward.

The formation resize/reorient control problem can be stated as driving the function $E \to 0$ given the double integrator dynamics of equation (1).

### 3 Coupled Dynamics Control

Given the general error function presented in the previous section we can develop a whole class of formation controls. Specifically we wish to develop a Passivity-Based control that requires no velocity information. Lemma 3.1 will show the evolution of the the trajectory of $E$ over time. This will be used in the proof of Theorem 3.1.

**Lemma 3.1** [Open Loop Error Kinematics] If we define the sets $\mathcal{L} = \mathcal{I}$ and $\mathcal{P} = \{(1,2),(2,3),\ldots,(N-1,N),(N,1)\}$. The derivative of $E$ is given by

$$
\dot{E} = \sum_{i=1}^{N} \dot{r}_i^T F^T(\xi_i)
$$

$$
\begin{bmatrix}
    k_G(z_{i1}) + k_F(z_{i1} - z_{i+1,1}) + k_F(z_{i1} - z_{i-1,1}) \\
    k_G(z_{i2}) + k_F(z_{i2} - z_{i+1,2}) + k_F(z_{i2} - z_{i-1,2}) \\
    k_G(z_{i3}) + k_F(z_{i3} - z_{i+1,3}) + k_F(z_{i3} - z_{i-1,3})
\end{bmatrix}
$$

$$
= \sum_{i=1}^{N} \dot{r}_i^T F^T(\xi_i) K(z_i, z_{i+1}, z_{i-1}),
$$

(4)

where

$$
F(\xi) = \begin{cases}
    I_3 & \text{for Cartesian coordinates} \\
    \text{diag}(1,1,1/\rho_i) R(-\phi) & \text{for cylindrical coordinates}.
\end{cases}
$$

(5)

**Proof:** Since $E = E_G + E_G$, $\dot{E} = \dot{E}_G + \dot{E}_F$. To calculate $\dot{E}$ we can consider $E_G$ and $E_F$ separately. First

$$
\dot{E}_F = \sum_{i=1}^{N} \sum_{j=1}^{3} k_F(z_{ij} - z_{i+1,j})(z_{ij} - z_{i+1,j})
$$

$$
= \sum_{i=1}^{N} \sum_{j=1}^{3} \dot{z}_{ij}(k_F(z_{ij} - z_{i+1,j}) + k_F(z_{ij} - z_{i-1,j})),
$$

(6)
Similarly for $\dot{E}_G$ we get
\begin{equation}
\dot{E}_G = \sum_{i=1}^{N} \sum_{j=1}^{3} \dot{z}_{ij} k_G(z_{ij}).
\end{equation}

Summing Equation (6) and Equation (7) we get
\begin{align*}
\dot{E} &= \dot{E}_F + \dot{E}_G \\
&= \sum_{i=1}^{N} \sum_{j=1}^{3} \dot{z}_{ij} (k_F(z_{ij} - z_{i+1,j}) + k_F(z_{ij} - z_{i-1,j}) \\
&\quad + k_G(z_{ij})) \\
&= \sum_{i=1}^{N} z_i^T K(z_i, z_{i+1}, z_{i-1}) \\
&= \sum_{i=1}^{N} r_i^T F^T(\xi) K(z_i, z_{i+1}, z_{i-1})
\end{align*}

Thus no velocity measurement is required for the passivity based control.

**Proof:**

1. Let $D = d_G I_N + d_F C^T C$, where the i, jth component of $C$ is defined by
\[
e_{ij} = \begin{cases} 
1 & \text{for } j = i \\
-1 & \text{for } j = i+1 \\
0 & \text{otherwise}
\end{cases}
\]

Observe that $D$ is positive definite and symmetric. Likewise $D^{-1}$ will also be positive definite and symmetric. Define $x, y, z$ respectively as the vectors $x_i, y_i, z_i$ stacked end on end. The series of state space equations (9) can be written as a single state space equation
\begin{align}
\dot{x} &= (I_N \otimes A)x + (D \otimes B)\dot{z} \\
y &= (I_N \otimes B^T P)x,
\end{align}

where $\otimes$ is the Kronecker product operator.

Consider the Lyapunov function candidate
\[V = \frac{1}{2} E + \frac{1}{2} \sum_{i=1}^{N} m_i r_i^T \dot{r}_i + x^T (D^{-1} \otimes P)x.\]

Differentiating gives that
\begin{align*}
\dot{V} &= \sum_{i=1}^{N} r_i^T \{ u_i + F^T(\xi_i) K(z_i, z_{i+1}, z_{i-1}) - \frac{1}{2} x^T(D^{-1} \otimes Q)x \\
&\quad + \frac{1}{2} x^T (D^{-1} \otimes Q)x + \sum_{i=1}^{N} \dot{z}_i T y_i \\
&\quad + \frac{1}{2} x^T(D^{-1} \otimes Q)x \}
\end{align*}

Observe that Filter (9) can be implemented as
\begin{align*}
\dot{z}_i &= Ax_i + B \{ d_G \dot{z}_i + d_F(z_i - z_{i+1}) + d_F(z_i - z_{i-1}) \} \\
y_i &= B^T P A x_i \\
&\quad + B^T P B \{ d_G \dot{z}_i + d_F(z_i - z_{i+1}) + d_F(z_i - z_{i-1}) \}.
\end{align*}

Thus we see that $V$ is a Lyapunov function. To show convergence let $\Omega = \{z_i, \dot{r}_i | V = 0\}$.
and let $\bar{\Omega} \subset \Omega$ be the largest invariant set of $\Omega$. On the set $\bar{\Omega}$, $x(t) = 0$. In light of Equation (11) $y(t) = 0, z(t) = 0$ and $\dot{z}(t) = 0$. Since $F(\xi)$ is full rank we further have $\dot{\xi}(t) = 0$ which implies that $u_i(t) = 0$. Therefore we arrive at the condition that

$$F^T(\xi)K(z_i, z_{i+1}, z_{i-1}) = 0.$$ 

Due to the fact that $F(\xi)$ is full rank and the assumptions of the theorem we have that $z_i = 0$ on the set $\bar{\Omega}$. By LaSalle's invariance principle $z_i \to 0$ asymptotically.

2. Since $V(t)$ is a Lyapunov function we have that $E_G(0) = V(0) \geq V(t) \geq E_F(t) \geq e_F(z_i - z_{i+1,1}) + e_F(z_i, z_{i+1,1}) + e_F(z_i, z_{i-1})$.

4 Simulations

For the linear control problem we take $e_G(x) = \frac{1}{2}k_G x^2$ and $e_F(x) = \frac{1}{2}k_F x^2$ then

$$K(z_i, z_{i+1}, z_{i-1}) = k_G z_i + k_F (z_i - z_{i+1}) + k_F (z_i - z_{i-1}).$$

This can be written in matrix form as

$$\{k_G I_3 + k_F (C^T C \otimes I_3)\} z = 0.$$ 

The coefficient of $z$ is positive definite and symmetric since it is the sum of a positive definite symmetric matrix and a positive semi-definite symmetric matrix. Therefore since the coefficient matrix is full rank, $z = 0$. Therefore for a linear formation control law Theorem 3.1 is valid.

Now we will run a few simulations to observe different features of the linear control. We assume that we have a constellation of three spacecraft. The mass of each spacecraft are given by $500Kg, 300Kg$ and $300Kg$ respectively. The spacecraft begin the maneuver in a triangular formation. The formation is then rotated by $\pi/6$ radians a shown in Figure 4.

In the first simulation we start in perfect formation. The first spacecraft starts moving at time $t = 0$ and the second spacecraft start moving at $t = 1$ sec and the third starts at $t = \infty$. This illustrates the ability of the control law to maintain formation even when one or more of the spacecraft do not start moving at the same time. The failure could be due to poor synchronization (i.e the two spacecraft starting one second apart) or due to spacecraft failure (the third spacecraft starting at $t = \infty$). Figure 5 plots the formation error between the second spacecraft and each of its two neighbors.

![Figure 4: Spacecraft Rotation Example.](image-url)

![Figure 5: Formation Rejection of Poor Synchronization.](image-url)

In the second simulation we start the maneuver with some formation error in the direction of the polar angle. This could be due to finishing the previous maneuver with some formation error, or due to poor synchronization. This simulation illustrates the ability of the formation to recover from some initial error and complete the maneuver. Figure 6 shows the convergence of each spacecraft to its final desired position. Figure 7 shows the convergences of the relative position of the second spacecraft with respect to the first and third...
spacecraft.

Figure 6: Rejection of Absolute Position Error.

Figure 7: Rejection of Formation Error.

5 Summary

We have developed a velocity-free formation control. This control only requires position information of neighboring spacecraft. Simulations demonstrate the ability of the formation to withstand poor synchronization, spacecraft failure and initial formation error. Future work will integrate the position and attitude formation problems.

References


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