January 1970

Formulation and Solution of Transient Flow of Water From an Infiltrometer Using the Kirchhoff Transformation

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FORMULATION AND SOLUTION OF TRANSIENT FLOW OF WATER FROM AN INFILTROMETER USING THE KIRCHHOFF TRANSFORMATION

by

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INTRODUCTION

In the progress report[1] dealing with methods for solving the three dimensional axisymmetric transient flow from infiltrometers, it was suggested that the numerical solution may be improved for situations containing regions of nearly saturated flow by transforming the hydraulic head (the dependent variable in the partial differential equation) by means of the Kirchhoff Transformation. While this transformation does not completely linearize the basic nonlinear partial differential equation, there are indications, as pointed out in the progress report, that solutions to the initial-boundary value problem might be more readily obtained for those situations in which the equation begins to change from parabolic to elliptic type.

Since writing the progress report, a computer program has been developed which implements the Kirchhoff transformation as suggested in the progress report. Use of the Kirchhoff Transformation in the formulation of the mathematical problem of partially saturated unsteady flow from an infiltrometer results in improved solution capabilities, particularly for problems in which a portion of the region approaches unit saturation. This paper describes the solution methods used in the computer program and presents example solution results. Consequently, this paper is a supplement to the prior report. The parameters and notations used herein are more fully defined on the previous report.

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[1] Jeppson, R. W., "Transient Flow of Water from Infiltrometers--Formulation of Mathematical Model and Preliminary Numerical Solutions and Analyses of Results." Report PRWG-59c-2, Utah Water Research Laboratory, Utah State University, Logan, Utah, June 1970.

FORMULATION

The Darcian based differential equation describing axisymmetric flow in partially saturated porous media is

$$\nabla \cdot (K_r \nabla h) + \frac{K_r}{r} \frac{\partial h}{\partial r} = \frac{\eta}{K_o} \frac{\partial S}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which h is the hydraulic head, K_r is the relative hydraulic conductivity, η is the soil porosity, S is the saturation, and K_o is the saturated hydraulic conductivity.

Upon applying the Kirchhoff Transformation

$$\xi = \frac{1}{\rho g} \int_{p_o}^{p} K_r(p)\, dp$$

Eq. 1 becomes

$$\frac{\partial^2 \xi}{\partial r^2} + \frac{\partial^2 \xi}{\partial z^2} + \frac{1}{K_r} \frac{\partial K_r}{\partial p_h} \frac{\partial \xi}{\partial z} + \frac{1}{r} \frac{\partial \xi}{\partial r} = \frac{\eta}{K_r} \frac{\partial S}{\partial p_h} \frac{\partial \xi}{\partial \tau} \quad . \quad . \quad . \quad (2)$$

in which p is the pressure, p_h is the pressure head, and $\tau = K_o t$ (t is real time).

The formulation of the initial-boundary value problem for the transient movement of infiltrating water is depicted in Fig. 1. The boundary conditions are given by an equation by each boundary on this figure.

NUMERICAL SOLUTION

The alternating direction implicit method (ADI) used in the previous report has been applied to obtain the solution to the problem depicted in Fig. 1. The finite difference operator for the first portion of the time step in the ADI method is

$$- \xi^*_{i-1,j} + (\zeta + 2)\xi^*_{i,j} - \xi^*_{i+1,j} = \xi^n_{i+1,j} + \xi^n_{i-1,j} + 2(\xi^n_{i,j+1} + \xi^n_{i,j-1})$$

$$+ (\zeta - 6)\xi^n_{i,j} - \alpha \Delta s (\xi^n_{i,j+1} - \xi^n_{i,j-1}) + \frac{\Delta s}{r} (\xi^n_{i+1,j} - \xi^n_{i-1,j}) \quad . \quad . \quad (3)$$

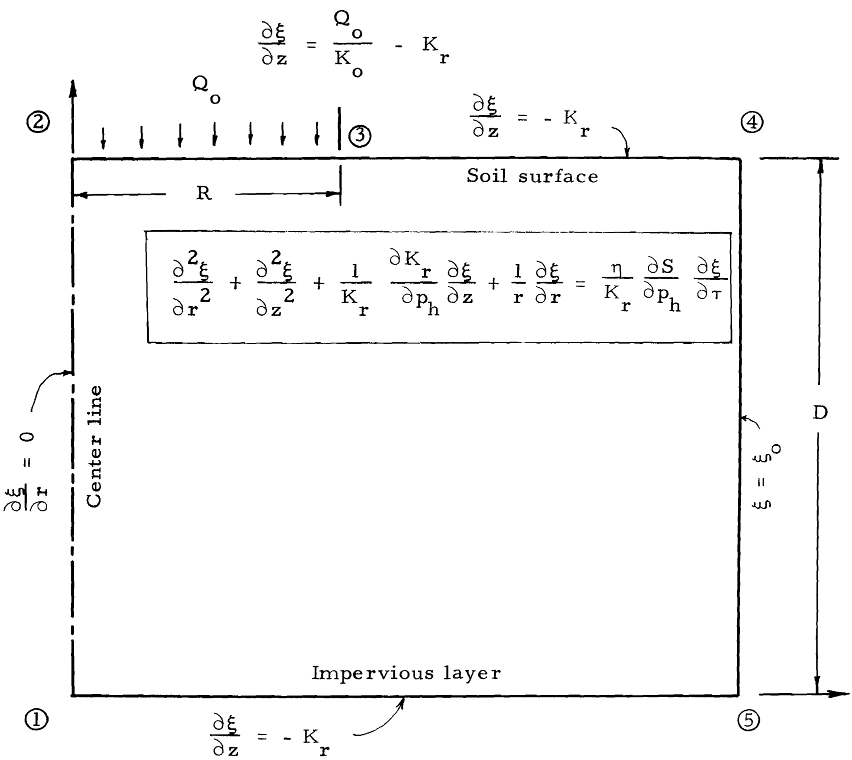

Fig. 1. Formulation of the initial-boundary value problem for the transient flow of water from a circular infiltrometer after the introduction of a new dependent variable through the Kirchhoff Transformation to replace the hydraulic head.

and for the second portion of the time step is

$$-\xi_{i,j-1}^{n+1} + (\zeta+2)\xi_{i,j}^{n+1} - \xi_{i,j+1}^{n+1} = \zeta\,\xi_{i,j}^{*} + 2\,\xi_{i,j}^{n} - \xi_{i,j+1}^{n} - \xi_{i,j-1}^{n} \quad \ldots\ldots \quad (4)$$

In Eqs. 3 and 4 $\Delta s = \Delta r = \Delta z$, $\zeta = \frac{2\eta\Delta s^2}{\Delta\tau K_r}\frac{\partial S}{\partial p}$, $\alpha = \frac{1}{K_r}\frac{\partial K_r}{\partial p}$, the superscripts denote the time step and the subscripts the space increments such that $i = r/\Delta r + 1$ and $j = N_y - z/\Delta z$ (N_y = depth of soil/Δz + 1).

The procedure for obtaining the numerical solution of Eqs. 3 and 4 with the accompanying boundary condition operators is the same as described in the earlier progress report, with the exception of the operator for the boundary ① to ⑤ . The method for handling this boundary had to be modified to overcome difficulties resulting from a poor approximation of the finite differences in describing the actual behavior of the function ξ. The reasons for these difficulties and the methods used to overcome these difficulties are described below in detail.

Operator for Boundary ① to ⑤

The boundary condition for ① to ⑤ in Fig. 1 is derived from the condition that the normal derivative of the hydraulic head along this boundary vanishes, namely $\partial h/\partial z = 0$, as shown in the equation below.

$$\frac{\partial h}{\partial z} = \frac{\partial p}{\partial z} + 1 = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial z} + 1 = \frac{1}{K_r}\frac{\partial \xi}{\partial z} + 1 = 0 \quad \ldots\ldots \quad (5a)$$

$$\frac{\partial \xi}{\partial z} = -K_r \quad \ldots\ldots \quad (5b)$$

A second order central difference approximation of Eq. 5b leads to

$$\xi_{i,N_y+1} - \xi_{i,N_y-1} = -2\Delta z K_r \quad \ldots\ldots \quad (6)$$

The usual manner for developing the finite difference operator for the boundary grid points along ① to ⑤ on Fig. 1 would combine Eq. 6 with the operators for the interior grid points in such a way as to eliminate the value ξ_{i,N_y+1} at the nonexistent grid points outside the boundary. This combination leads to the following operator for the first portion of the time step.

$$-\xi^{*}_{i-1,N_y} + (\zeta + 2)\xi^{*}_{i,N_y} - \xi^{*}_{i+1,N_y} = \xi^{n}_{i+1,N_y} + \xi^{n}_{i-1,N_y} + (\zeta - 6)\xi^{n}_{i,N_y} + 4\xi^{n}_{i,1}$$

$$+ (2\alpha\Delta s^2 - 4\Delta s)K_r + \frac{\Delta s}{r}(\xi^{n}_{i+1,N_y} - \xi^{n}_{i-1,N_y}) \quad . \; . \; . \; . \; .$$

The above approach is generally valid. However, for the problem of infiltration, the parabolic approximation of the function given by Eq. 6 which leads to Eq. 7 does not approximate the derivative $\partial\xi/\partial z$ at $z = 0$ close enough to use the operator Eq. 7 for the boundary ① to ⑤ . Its use causes the value at ξ to be less (accompanied by a decrease in the hydraulic head) than the initializing values even though the wetting front has not reached the boundary. After a number of time steps this influence spreads to the interior grid points with a resulting further decrease in values of ξ at the boundary. This occurrence was observed in first attempts at obtaining a solution in which changes in the third digit beyond the decimal point were observed after the first time step.

To examine the accuracy of a second order polynomial approximation of the function ξ at $z = 0$, note that ξ varies according to the following equation provided that the hydraulic head is constant.

$$\xi = \frac{p_b}{\alpha - 1} - \frac{p_b^{\alpha}}{(\alpha - 1)(p_o + z)^{\alpha - 1}} = C_1 - \frac{C_2}{(p_o + z)^{\alpha - 1}} \quad . \; . \; . \; . \; .$$

in which p_o is the pressure at the boundary $z = 0$. The first and second derivatives of Eq. 8 are respectively

$$\frac{\partial\xi}{\partial z} = \frac{p_b^{\alpha}}{(p_o + z)^{\alpha}} = \frac{(\alpha - 1)C_2}{(p_o + z)^{\alpha}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

and

$$\frac{\partial^2 \xi}{\partial z^2} = - \frac{\alpha p_b^{\alpha}}{(p_o + z)^{\alpha+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

An indication of the accuracy of Eq. 6 in approximating the derivative $\partial\xi/\partial z$ at $z = 0$ can be had by comparing the values obtained from a second degree polynomial with values obtained from the latter part of Eq. 8 in which C_1 and C_2 may take on slightly different values than those defined by the first part of Eq. 8. To do this note that under the assumption C_1 and C_2 are constant adjacent to and on the boundary ① to ⑤ , that

$$\xi_{i,N_y} = C_1 - \frac{C_2}{p_o^{\alpha-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

and

$$\xi_{i,N_y} = C_1 - \frac{C_2}{(p_o + \Delta s)^{\alpha-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

Subtracting Eq. 12 from Eq. 11 and solving for C_2 gives

$$C_2 = - \frac{\xi_{i,N_y} - \xi_{i,N_y-1}}{\frac{1}{p_o^{\alpha-1}} - \frac{1}{(p_o + \Delta s)^{\alpha-1}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

Upon substituting Eq. 13 into Eq. 9 gives

$$\left.\frac{\partial \xi}{\partial z}\right|_{z=0} \approx - \frac{(\alpha - 1)(\xi_{i,N_y} - \xi_{i,N_y-1})}{p_o - \frac{p_o^{\alpha}}{(p_o + \Delta s)^{\alpha-1}}} \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

Since $\partial\xi/\partial z = - K_r$ along ① to ⑤

$$\xi_{i,N_y} - \xi_{i,N_y-1} = \frac{K_r}{\alpha - 1}\left(p_o - \frac{p_o^{\alpha}}{(p_o + \Delta s)^{\alpha-1}}\right) \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

The difference in the right hand sides of Eqs. 6 and 15 illustrates the lack of good approximation by a second degree polynomial of the derivative $\partial\xi/\partial z$ at $z = 0$.

As an example to illustrate the magnitudes involved consider a probl with the following specifications: $h_o = 1.0$ ft, $\lambda = 1.28$, $\alpha = 5.84$, $p_b = 0.$ and $\Delta s = 1.0/14.0$. These specifications lead to $\xi_{i,N_y} = 0.063120$, $\xi_{i,N_y-1} = 0.099164$, and $(2\alpha\Delta s^2 - 4\Delta s)K_r = 0.257704$. On the other hanc an examination of Eq. 7 indicates that under the initial condition $h_o = -1.$ the last quantity should equal $4(\xi^n_{i,N_y-1} - \xi^n_{i,N_y}) = 0.252480$. The differe is 0.005224. Since the magnitude ζ in this example equals 8.0694, this difference leads to a value of ξ^*_{i,N_y} upon solving Eq. 7, equal to 0.063767 which deviates from the correct value ξ_{i,N_y} in the fourth digit beyond the decimal point.

From the above discussion it is obvious that an alternative to Eq. 3 must be developed as the finite difference operator along the boundary ① to ⑤ . The operator has been developed by noting from Eqs. 9 and 10 that along the boundary ① to ⑤ (at least when $h_{i,N_y} = h_o$)

$$\frac{\partial^2\xi}{\partial z^2} + \alpha\frac{\partial\xi}{\partial z} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

and therefore the differential equation can be reduced to

$$\frac{\partial^2\xi}{\partial r^2} + \frac{1}{r}\frac{\partial\xi}{\partial r} = \Gamma\frac{\partial\xi}{\partial\tau} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

Approximating the derivations in Eq. 17 by appropriate central differences as needed for the first portion of the time step for the alternating direction implicit method leads to

$$-\xi^*_{i-1,N_y} + (\zeta+2)\xi^*_{i,N_y} - \xi^*_{i+1,N_y} = \xi^n_{i+1,N_y} + \xi^n_{i-1,N_y} + (\zeta-2)\xi^n_{i,N_y}$$

$$+\frac{\Delta s}{r}(\xi_{i+1,N_y} - \xi_{i-1,N_y}) \quad . \; . \; . \; . \; . \; . \; . \; . \; .$$

and at point ① this equation reduces to

$$(\zeta + 1)\xi^{*}_{2,N_y} - \xi^{*}_{3,N_y} = \xi^{n}_{3,N_y} + (\zeta - 1)\xi^{n}_{2,N_y} + \frac{\Delta s}{r}(\xi^{n}_{3,N_y} - \xi^{n}_{2,N_y}) \quad . \quad . \qquad (19)$$

For the second portion of the time step the usual approach of combining Eq. 6 with Eq. 4 has been used and gives

$$(\frac{\zeta}{2} + 1)\xi^{n+1}_{i,N_y} - \xi^{n+1}_{i,N_y} = \frac{\zeta}{2}\xi^{*}_{i,N_y} + \xi^{n}_{i,N_y} - \xi^{n}_{i,N_y-1} \quad . \quad . \quad . \quad . \qquad (20)$$

The usual second degree polynomial approximation has been used along boundary ③ to ④ without apparent difficulties.

RESULTS FROM A SAMPLE SOLUTION

The problem, selected to illustrate the solution capability using the Kirchhoff Transformation to the infiltration problem, specified a soil with the following parameters in the Brooks-Corey equations: $S_r = 0.120$, $\lambda = 1.28$, $\eta = .240$, $p_b = 0.92$. The initial condition was specified as a constant value of the hydraulic head $h_o = -1.0$ ft. The solution results which include data giving values for ξ, saturation and the hydraulic head throughout the flow field within the wetting front for a number of time steps are given in Table 1.

Table 1. Solution output for problem of infiltration from circular infiltrometer. The output consists of value for ξ, saturation and hydraulic head within the wetting front.

```
N2X=  7 NX= 20 NY= 15 NT=   40 PI=  -1.000 DY=   1.00 DELT=    .0025 Q=    .500000
   2    5    2    0    0    2
SR=    .120 LAMBDA=   1.28 POROSITY=    .240 PB=    .9200
RADIUS OVER WHICH INFILTRATION OCCURS    .42857
       1.10341          .41714          .42857         -.12378          .07143        4.84000        .20661
```

VALUES OF THE XI FOR TIME=   .00250

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	.1670	.1670	.1670	.1670	.1670	.1670	.1683	.1849	.1856	.1856	.1856	.1856	.1856
2	.1832	.1832	.1832	.1832	.1832	.1832	.1833	.1847	.1848	.1848	.1848	.1848	.1848
3	.1836	.1836	.1836	.1836	.1836	.1836	.1836	.1837	.1837	.1837	.1837	.1837	.1837
4	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824	.1824
5	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807	.1807
6	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786	.1786

VALUES OF SATURATION

1	.6241	.6241	.6241	.6241	.6241	.6237	.6162	.4591	.4457
2	.4862	.4862	.4862	.4862	.4862	.4861	.4838	.4622	.4612
3	.4801	.4801	.4801	.4801	.4801	.4801	.4798	.4782	
4	.4967	.4967	.4967	.4967	.4967	.4967	.4967	.4965	
5	.5167	.5167	.5167						

VOLUME OF WATER ABSORBED =   .0014855

VALUES FOR HYDRAULIC HEAD

	1	2	3	4	5	6	7	8	9	10	11	12
1	-.422	-.422	-.422	-.422	-.422	-.423	-.439	-.938	-1.000	-1.000	-1.000	-1.000
2	-.896	-.896	-.896	-.896	-.896	-.897	-.906	-.996	-1.000	-1.000	-1.000	-1.000
3	-.992	-.992	-.992	-.992	-.992	-.992	-.993	-1.000	-1.000	-1.000	-1.000	-1.000
4	-.999	-.999	-.999	-.999	-.999	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
5	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
6	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000

LUES OF THE XI FOR TIME= .02375

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
.0776	.0776	.0787	.0812	.0861	.0953	.1146	.1628	.1793	.1844	.1854	.1856	.1856	.1856	.1
.1040	.1040	.1054	.1086	.1145	.1248	.1418	.1662	.1793	.1836	.1846	.1848	.1848	.1848	.1
.1322	.1322	.1335	.1364	.1414	.1495	.1608	.1734	.1807	.1831	.1836	.1837	.1837	.1837	.1
.1566	.1566	.1574	.1592	.1622	.1668	.1725	.1781	.1811	.1821	.1824	.1824	.1824	.1824	.1
.1703	.1703	.1707	.1715	.1728	.1747	.1771	.1791	.1802	.1806	.1807	.1807	.1807	.1807	.1
.1748	.1749	.1749	.1752	.1757	.1764	.1773	.1780	.1784	.1785	.1786	.1786	.1786	.1785	.1
.1745	.1745	.1745	.1746	.1748	.1751	.1754	.1756	.1758	.1758	.1758	.1758	.1758	.1758	.1
.1717	.1717	.1718	.1718	.1719	.1720	.1721	.1722	.1722	.1722	.1722	.1722	.1722	.1722	.1
.1673	.1673	.1673	.1673	.1674	.1674	.1674	.1675	.1675	.1675	.1675	.1675	.1675	.1675	.16
.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.1611	.16
.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.1525	.15

ILUES OF SATURATION

.8860	.8860	.8840	.8793	.8703	.8520	.8092	.6466	.5316	.4682	.4497	.4463	.4457
.8337	.8337	.8305	.8233	.8094	.7833	.7324	.6283	.5325	.4797	.4645	.4617	.4613
.7625	.7625	.7586	.7499	.7337	.7051	.6566	.5822	.5177	.4879	.4799	.4784	.4781
.6761	.6761	.6724	.6644	.6497	.6253	.5887	.5439	.5127	.5005	.4973	.4967	.4965
.6035	.6035	.6012	.5960	.5867	.5723	.5531	.5337	.5223	.5182	.5170	.5168	
.5719	.5719	.5708	.5685	.5644	.5584	.5511	.5444	.5408	.5394	.5390		
.5744	.5744	.5740	.5732	.5717	.5697	.5673	.5652	.5640	.5636	.5635		
.5941	.5941	.5940	.5937	.5932	.5926	.5918	.5912	.5908	.5907			
.6220	.6220	.6220	.6219	.6218	.6216	.6213	.6211					
.6553	.6553	.6553	.6552	.6552	.6551							

UME OF WATER ABSORBED = .0155637

LUES FOR HYDRAULIC HEAD

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-.025	-.025	-.027	-.032	-.042	-.062	-.114	-.374	-.666	-.898	-.981	-.997	-1.000	-1.000
-.155	-.155	-.159	-.167	-.185	-.219	-.293	-.484	-.734	-.922	-.985	-.998	-1.000	-1.000
-.319	-.319	-.325	-.337	-.362	-.408	-.497	-.664	-.854	-.961	-.993	-.999	-1.000	-1.000
-.531	-.531	-.538	-.553	-.582	-.633	-.719	-.842	-.942	-.985	-.997	-1.000	-1.000	-1.000
-.754	-.754	-.760	-.773	-.796	-.833	-.886	-.945	-.981	-.995	-.999	-1.000	-1.000	-1.000
-.906	-.906	-.909	-.915	-.926	-.943	-.964	-.984	-.994	-.999	-1.000	-1.000	-1.000	-1.000
-.970	-.970	-.972	-.974	-.978	-.983	-.990	-.995	-.998	-1.000	-1.000	-1.000	-1.000	-1.000
-.992	-.992	-.992	-.993	-.994	-.995	-.997	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-.998	-.998	-.998	-.998	-.998	-.999	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-.999	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000

VALUES OF THE XI FOR TIME= .05500

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.0532	.0532	.0552	.0590	.0656	.0768	.0982	.1485	.1684	.1781	.1828	.1848	.1854	.1856	.1856	.1856
2	.0683	.0683	.0710	.0762	.0948	.0983	.1189	.1473	.1556	.1762	.1815	.1838	.1845	.1847	.1848	.1848
3	.0849	.0849	.0880	.0939	.1030	.1159	.1328	.1518	.1664	.1757	.1806	.1827	.1834	.1837	.1837	.1837
4	.1023	.1023	.1054	.1111	.1196	.1308	.1441	.1577	.1687	.1759	.1799	.1815	.1821	.1823	.1824	.1824
5	.1194	.1194	.1222	.1273	.1345	.1435	.1535	.1632	.1710	.1761	.1789	.1801	.1805	.1807	.1807	.1807
6	.1348	.1348	.1372	.1413	.1469	.1537	.1607	.1672	.1723	.1756	.1773	.1781	.1784	.1786	.1786	.1786
7	.1471	.1471	.1489	.1519	.1558	.1604	.1650	.1690	.1720	.1740	.1750	.1755	.1757	.1758	.1758	.1758
8	.1549	.1549	.1560	.1579	.1604	.1632	.1659	.1683	.1700	.1711	.1717	.1720	.1722	.1722	.1722	.1722
9	.1575	.1575	.1582	.1593	.1608	.1624	.1639	.1652	.1662	.1668	.1672	.1674	.1675	.1675	.1675	.1675
10	.1555	.1555	.1559	.1565	.1573	.1582	.1591	.1598	.1604	.1608	.1610	.1611	.1611	.1612	.1612	.1611
11	.1493	.1493	.1495	.1499	.1503	.1508	.1513	.1517	.1521	.1523	.1524	.1525	.1525	.1525	.1525	.1525
12	.1387	.1387	.1388	.1390	.1393	.1396	.1399	.1401	.1403	.1404	.1405	.1406	.1406	.1406	.1406	.1405
13	.1226	.1226	.1227	.1228	.1229	.1231	.1233	.1234	.1235	.1236	.1237	.1237	.1237	.1237	.1237	.1236
14	.0987	.0987	.0988	.0988	.0989	.0990	.0991	.0992	.0992	.0993	.0993	.0993	.0993	.0993	.0993	.0993
15	.0630	.0630	.0630	.0630	.0630	.0631	.0631	.0631	.0631	.0631	.0632	.0632	.0632	.0632	.0631	.0631

VALUES OF SATURATION

1	.9268	.9268	.9238	.9176	.9068	.8874	.8461	.7087	.6158	.5437	.4907	.4615	.4503	.4469	.4460	.4458
	.4457															
2	.9023	.9023	.9977	.8985	.8727	.8459	.7987	.7131	.6316	.5606	.5072	.4776	.4560	.4624	.4615	.4613
	.4612															
3	.8724	.8724	.8665	.8550	.8360	.8062	.7609	.6960	.6274	.5647	.5179	.4924	.4824	.4792	.4784	.4782
4	.8374	.8374	.8306	.8175	.7969	.7667	.7247	.6711	.6139	.5626	.5263	.5074	.4999	.4975	.4968	.4966
5	.7975	.7975	.7902	.7765	.7557	.7266	.6890	.6444	.5991	.5611	.5363	.5240	.5191	.5174	.5169	.5168
6	.7547	.7547	.7475	.7342	.7145	.6885	.6569	.6225	.5902	.5656	.5507	.5434	.5405	.5394	.5390	
7	.7138	.7138	.7073	.6957	.6793	.6586	.6353	.6120	.5922	.5782	.5700	.5661	.5644	.5637	.5635	
8	.6835	.6835	.6786	.6700	.6584	.6445	.6299	.6164	.6055	.5984	.5942	.5921	.5911	.5908		
9	.6721	.6721	.6690	.6637	.6567	.6488	.6409	.6339	.6285	.6249	.6228	.6217	.6212	.6210		
10	.6809	.6809	.6792	.6764	.6727	.6685	.6647	.6612	.6585	.6568	.6558	.6552	.6549			
11	.7058	.7058	.7050	.7036	.7018	.6998	.6979	.6962	.6949	.6941	.6935	.5933				
12	.7427	.7427	.7423	.7416	.7407	.7398	.7389	.7381	.7374	.7370						
13	.7892	.7892	.7890	.7887	.7883	.7878	.7874	.7870	.7867							
14	.8450	.8450	.8449	.8448	.8446	.8444	.8442									
15	.9111	.9111	.9111													

VOLUME OF WATER ABSORBED = .0347149

VALUES FOR HYDRAULIC HEAD

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	.015	.015	.012	.007	-.004	-.024	-.069	-.259	-.440	-.628	-.807	-.927	-.978
2	-.080	-.080	-.085	-.094	-.111	-.141	-.198	-.324	-.477	-.651	-.819	-.931	-.979
3	-.183	-.183	-.189	-.202	-.224	-.260	-.321	-.424	-.557	-.711	-.853	-.944	-.983
4	-.294	-.294	-.302	-.317	-.344	-.385	-.448	-.540	-.659	-.788	-.897	-.961	-.988
5	-.414	-.414	-.424	-.442	-.472	-.516	-.579	-.664	-.765	-.864	-.937	-.976	-.992
6	-.545	-.545	-.555	-.576	-.507	-.651	-.711	-.783	-.858	-.923	-.965	-.986	-.995
7	-.680	-.680	-.693	-.710	-.740	-.779	-.826	-.878	-.925	-.961	-.982	-.993	-.998
8	-.803	-.803	-.812	-.828	-.851	-.878	-.909	-.939	-.964	-.981	-.991	-.997	-.999
9	-.896	-.896	-.901	-.912	-.925	-.941	-.957	-.972	-.984	-.991	-.996	-.999	-1.000
10	-.951	-.951	-.954	-.959	-.965	-.974	-.981	-.988	-.993	-.996	-.999	-1.000	-1.000
11	-.979	-.979	-.980	-.982	-.985	-.989	-.992	-.995	-.997	-.999	-1.000	-1.000	-1.000
12	-.991	-.991	-.992	-.993	-.994	-.996	-.997	-.998	-.999	-1.000	-1.000	-1.000	-1.000
13	-.997	-.997	-.997	-.997	-.998	-.998	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
14	-.999	-.999	-.999	-.999	-.999	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
15	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000

	14	15	16	17
1	-.994	-.998	-.999	-1.000
2	-.995	-.999	-1.000	-1.000
3	-.995	-.999	-1.000	-1.000
4	-.997	-.999	-1.000	-1.000
5	-.998	-.999	-1.000	-1.000
6	-.999	-1.000	-1.000	-1.000
7	-.999	-1.000	-1.000	-1.000
8	-1.000	-1.000	-1.000	-1.000
9	-1.000	-1.000	-1.000	-1.000
10	-1.000	-1.000	-1.000	-1.000
11	-1.000	-1.000	-1.000	-1.000
12	-1.000	-1.000	-1.000	-1.000
13	-1.000	-1.000	-1.000	-1.000
14	-1.000	-1.000	-1.000	-1.000
15	-1.000	-1.000	-1.000	-1.000