An Innovative Method for Measuring Drag on Small Satellites

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The need to accurately predict satellite positions is a leading aspect of space situational awareness and presents increased challenges in the specification of the spacecraft environment in low earth orbit. Atmospheric drag is the most important environmental perturbation for low orbiting spacecraft and the most difficult one to model and predict precisely. The author will present a method for the characterization of satellite drag through the use of a dual-instrument in-situ approach. The elements of this method include a novel acceleration measurement suite and a small Wind and Temperature Spectrometer in order to measure both atmospheric density and wind. A sophisticated error model has been developed to evaluate this method and the results show that it is possible to improve our ability to characterize satellite drag by at least 10-14%. Furthermore, ground testing indicates that the instrument hardware will meet the requirements necessary to produce an improved data product. In order to evaluate this approach in orbit, students at the University of Colorado at Boulder have developed a small spacecraft called the Drag and Atmospheric Neutral Density Explorer. This small (under 50 kg), spherical satellite addresses important needs of the defense and civilian community by measuring quantities which are crucial to the determination of atmospheric drag on spacecraft. This paper describes the measurement process, as well as a method of computing the satellite drag coefficient. We then present the design and testing of the instruments and summarize the results of the error model.

Introduction

SPECIFYING and predicting the location of low-earth orbit (LEO) space assets is of great importance to both the private and military space-flight sectors. The requirements for predicting precise orbital parameters are driven by the need to catalogue and track an ever increasing number of objects, precise-positioning for rendezvous and formation missions, as well as re-entry prediction and collision avoidance efforts. Drag due to atmospheric density is the dominant perturbation in the motion of most man-made objects orbiting below 500 km altitude. Since atmospheric density has been observed to vary up to 800% during geomagnetic events, the ability to model this density has a significant impact on our awareness of present and future spacecraft locations. This paper presents a new method for characterizing atmospheric density and drag with the intent of improving predictive models which serve the spacecraft community. The measurement technique was designed for a small satellite mission at low cost and low systems level impact (mass, power, size). The mission scalability inherent in this approach means that multiple density-observing “Nanosats” could be placed in orbit on a single launch or on multiple launches of opportunity. The result would not only be an increase in the amount of data available but also the separation of spatial variabilities from temporal ones.

Measurements of drag can be performed by observing the satellite’s semi-major axis and using this knowledge to deduce the drag acceleration. The drag acceleration may also be determined with spaceborne accelerometers by modeling the contribution of non-atmospheric perturbations and subtracting it from the measured acceleration. This acceleration can be written as

$$\vec{a} = \frac{A_{sc} \cdot C_D \cdot \rho \cdot \|\vec{V}_w - \vec{V}_{sc}\|^2}{M_{sc}} (-\vec{V}_T)$$  \hspace{1cm} (1)

where

$$-\vec{V}_T = \left(\vec{V}_w - \vec{V}_{sc}\right)$$  \hspace{1cm} (2)

and $\vec{a}$ is the drag acceleration vector ($m/s^2$), $A_{sc}$ is the projected area of the object ($m^2$), $C_D$ is the
unitless coefficient of drag, $\rho$ is the mass density ($kg/m^3$), $V_w$ and $V_{sc}$ are the atmospheric wind and spacecraft velocity vectors respectively ($m/s$), and $M_{sc}$ is the spacecraft mass ($kg$). The relative orientation of drag acceleration and spacecraft motion is shown in Figure 1 for a spherically symmetric object. By measuring the drag acceleration and estimating or measuring the other parameters, one can solve for density, $\rho$, in Equation 1.

The ability to obtain atmospheric density from accelerometer and semi-major axis measurements is limited by uncertainties in the estimation of thermospheric winds, spacecraft drag coefficients, and spacecraft cross-sectional area. Thermospheric winds at high latitudes have been found to exceed 1,000 m/s, a number which is non-negligible relative to the orbital velocity of 7,800 m/s of a LEO satellite. Since velocity contributes a squared term in Equation 1, the estimation of wind is of particular importance in deducing atmospheric density. Furthermore, the drag coefficient is a function of atmospheric gas interaction with the satellite surfaces and varies as a function of altitude and solar conditions. The previous use of constant drag coefficients has introduced an 8-10% altitude-dependent bias into earlier atmospheric models. Therefore, correctly estimating this parameter is important and presents a non-trivial task. Similarly, the cross-sectional area of spacecraft can vary significantly, inducing uncertainties into the density determination. This parameter is particularly sensitive to the attitude of long-spacecraft such as CHAMP and GRACE which have been used to derive densities from on-board accelerometers. The most direct challenge to obtaining drag and density is the measurement of acceleration. The magnitude of the measured drag is on the order of several $\mu g$ with variations of around 100 $\mu g$ and is in some cases swamped by spacecraft noise. In accelerometer-carrying systems such as CHAMP, the estimation of accelerometer scale factor and bias has presented significant difficulties.

Although past missions (Atmospheric Explorer, Dynamics Explorer and CHAMP for example) have measured either wind vectors or mass density, there have been none which have measured the two parameters together while having a well determined coefficient of drag. By measuring both these properties, one can deduce the contribution of the $V_w$ and $C_D$ components and thus significantly reduce the uncertainty in the density measurement.

The measurement method presented in this paper was originally developed for the Drag and Atmospheric Neutral Density Explorer (DANDE) but can be applied, either in whole or in part, to other spacecraft missions. DANDE is an 18 inch spherical spacecraft developed at the University of Colorado, Boulder by graduate and undergraduate engineering students. The results of these efforts will be to aid in the development of better first principles and assimilative models as well as to answer some important scientific questions about our atmosphere at around 350 km altitude in the 2011-2012 time frame.

The author’s role on the project was to define the measurement technique, analyze the measurement approach including the accelerometer error budget and drag coefficient analysis, and lead the development of the wind sensor including the ion optics design and testing.

**Overview of Science Requirements**

Top level science requirements, defined for the atmospheric measurements, are presented in Table 1. These system independent science requirements drove the development of the instruments and spacecraft bus. The primary instruments which are responsible for collecting in-situ data in this scheme per was originally developed for the Drag and Atmospheric Neutral Density Explorer (DANDE) but can be applied, either in whole or in part, to other spacecraft missions. DANDE is an 18 inch spherical spacecraft developed at the University of Colorado, Boulder by graduate and undergraduate engineering students. The results of these efforts will be to aid in the development of better first principles and assimilative models as well as to answer some important scientific questions about our atmosphere at around 350 km altitude in the 2011-2012 time frame.

![Figure 1. Drag induced acceleration on a spherically symmetric object.](image)

**Table 1. Minimum success science requirements.**

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<tr>
<th>Requirement</th>
<th>Precision</th>
<th>Accuracy</th>
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<td>Density</td>
<td>$2E-13kg/m^3$</td>
<td>$1E-12kg/m^3$</td>
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<tr>
<td>Wind$^*$</td>
<td>100 m/s</td>
<td>100 m/s</td>
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<tr>
<td>Drag Coeff.</td>
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<td>0.2</td>
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<tr>
<td>Cadence</td>
<td>Horizontal resolution of 500 km or approx 64 seconds flight time in 350 km circular orbit</td>
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$^*$Wind refers to both the along-orbit and cross-orbit components
Figure 2. Engineering diagram of the DANDE spacecraft assembly.

are the accelerometer system, and the Wind and Temperature Spectrometer (WTS). The spacecraft structure is also a part of this measurement system as it houses the instruments and interacts directly with the atmospheric gas. Accordingly, requirements on the level of cross-sectional area variation were derived from the science analysis. The allowable center of gravity offset was determined by the maximum allowable drag torques for performing acceleration measurements.

The DANDE Spacecraft and Mission

The DANDE spacecraft addresses the aforementioned measurement challenges in its design (by having a determined coefficient of drag, cross-sectional area, and by allowing for the removal of the effects of in-track winds). In January of 2009, the satellite team won the nation University Nanosat Program (UNP)\textsuperscript{9} competition and was the first University Nanosat winner with a mission focused entirely on collecting scientific measurements. The spacecraft is under 50 kg in mass and 18 inches in diameter conforming to EELV Secondary Payload Adapter\textsuperscript{10} requirements. The primary instruments, accelerometers and WTS, are aligned together to enable velocity vector scanning in the nominal attitude state. The instrument locations and relative sizes are shown in Figure 2 (accelerometers are callout 1 and the WTS is callout 11).

DANDE is spin stabilized around the orbit normal vector, meaning the instruments aligned with the “equator” of the spinning sphere will scan the velocity vector at a predictable rate (see Figure 3). The nominal spin magnitude is 10 RPM. At the worst case solar maximum density conditions and a nominal 350 km circular, near-polar orbit, the spacecraft is expected to spend approximately 3 months in orbit before reentry.\textsuperscript{11}

One prominent feature visible in Figure 3 is that DANDE’s spherical shape is modified by flat facets which provide the mounting areas for small photovoltaic cells. The effect of these facets on the drag coefficient will be described in a later section.

Instrument Detail

In order to fully characterize the drag equation (Equation 1), the spacecraft velocity will be estimated using radar tracking and orbit determination while the acceleration forces will be measured using the unique accelerometer system developed at the University of Colorado, Boulder. The in-situ horizontal wind vector will be measured using the DANDE Wind and Temperature Spectrometer. These two instruments are shown in Figure 4. In addition to measuring drag with accelerometers, the spacecraft will be tracked by radar and an orbit-averaged drag acceleration will be generated for validation of the accelerometer measurements.
Drag Coefficient Computation

The drag coefficient is determined by the interaction of atmospheric gases with the spacecraft surface and is primarily a function of the gas composition, atmospheric and surface temperature, the relative velocity $V_T$, and the amount of energy which incoming molecules loose as they collide with the surface. The extent to which a particle’s energy is transferred to the surface can be described by the accommodation coefficient, $\alpha$, defined as

\[ \alpha = \frac{T_i - T_r}{T_i - T_w} \]  

(3)

where $T_i$ is the kinetic temperature carried to the surface by a particle, $T_r$ is the kinetic temperature of the reflected particle, and $T_w$ is the kinetic temperature the particle would have if it was re-emitted at the temperature of the surface.\(^{12}\) Kinetic temperature is defined as\(^{12}\)

\[ T_i = \frac{mv_i^2}{3k_b} \]  

(4)

where $v_i$ is the magnitude of velocity and $k_b$ is the Boltzmann constant. We may write the reflected kinetic temperature at the surface as the following equation.

\[ T_r = \frac{m}{3k_b} v_i^2 (1 - \alpha) + \alpha T_w \]  

(5)

Note that for diffuse reflection, the velocity distribution of reflected particles is centered around the surface normal vector. For quasi-specular reflection, the particles are reflected in a narrow lobe which is centered around the specular reflection direction. When computing the drag coefficient, a successful technique\(^{13}\) is the adaptation of the drag coefficient models of Sentman (diffuse)\(^{14}\) and Schamberg (quasi-specular)\(^{15}\) with the use of empirical accommodation coefficients profiles for the former and Goodman’s model\(^{16}\) for the latter. Sentmann’s model adapts the diffuse (or Maxwellian) energy distribution with a variable accommodation coefficient. The angular probability distribution follows the cosine of the angle at the surface. Meanwhile Schamberg’s quasi-specular model reflects the majority of particles near a direction such that the angle of reflection (as defined from the surface) is slightly larger than the incident angle. These reflection models are described in Figure 5. The accommodation coefficient for the quasi-specular case is described by Goodman’s models which takes into account the molecular properties of the surface.
menta are subtracted to find the total momentum change. To convert this momentum change to a force, a simulation time is defined by comparing the simulated number of particles with the flux at a given surface given a Maxwellian velocity distribution. This flux is computed using the following equation,

$$\dot{N}_i = \frac{n}{2\sqrt{\pi}} \left[ \exp(-s^2 \cos \theta^2) + \sqrt{\pi} s \cos \theta (1 + \text{erf}(s \cos \theta)) \right]$$ \hspace{1cm} (6)

where the speed ratio, $s$, is

$$s = \left| \frac{\vec{V}_T}{\beta} \right|$$ \hspace{1cm} (7)

and \text{erf}() denotes the error function defined as

$$\text{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^x \exp(-t^2), dt$$ \hspace{1cm} (8)

The $\theta$ in Equation 6 is the acute angle between the reference surface normal and the bulk velocity while $\beta = \sqrt{m/(2k_b T_i)}$ where $T_i$ is the kinetic temperature of the atmosphere. The number of simulated particles impacting the reference surface divided by the area of the surface and the theoretical flux results in the scaled time step for the simulation. The total momentum change over this time step is equal to the force and is computed separately for each surface element in the mesh. Accommodation coefficients at a 350 km altitude were chosen to be 0.89 during solar minimum and 0.97 during solar maximum.\(^3\) The total force on the object is obtained by summing the force from all impacts. Finally, the drag coefficient is the force in the direction of the incoming gas divided by the area of the cross section which is normal to this flow direction. The numerical results are compared to analytical expressions for the drag coefficient of a smooth sphere.\(^12\)

$$C_{D, \text{sphere}} = \frac{2s^2 + 1}{\sqrt{\pi} s} \exp(-s^2) + \frac{4s^4 + 4s^4 - 1}{2s^4} \text{erf}(s) + \frac{2(1-\epsilon)}{3s} \sqrt{T_r/T_i};$$ \hspace{1cm} (9)

The reflected temperature, $T_r$, is computed using Equation 5 and $\epsilon$ is the fraction particles which are specularly reflected. The drag coefficient of a sphere computed using Equation 9 without specular reflection is 2.36 using an accommodation coefficient of 0.89 (solar minimum) and 2.21 for an accommodation coefficient of 0.97 (solar maximum). If the fraction of specularly reflected particles is 0.35, the corresponding spherical drag coefficients are 2.24 and 2.15.

Table 2 shows that the drag coefficient of the DANDE sphere is not much different from that of a perfect sphere. The previously reported variability in the drag coefficient of Starshine\(^3\) can be explained by the fact that the cross sectional area of that spacecraft varied by 5.8% above the average and −6.7% below the average. This is primarily a result of the launch vehicle adapter remaining with the Starshine spheroids. In contrast, the DANDE spacecraft will jettison its launch adapter before beginning the science mission. The expected cross sectional area variation for DANDE is ±1 − 2% which will more than meet the precision requirement in Table 1. Furthermore, while the Starshine satellites had no knowledge of their attitude state and hence no knowledge of cross-sectional area, DANDE will spin around a known axis and will telemeter attitude data to the ground. In order to estimate $C_D$ accuracy, we take the partially quasi-specular results as the lower bounds of the drag coefficient estimate and subtract this value from the 100% diffusely reflected case.\(^17\) The justification for doing this is that the

<table>
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<th>$C_{DP}$ 100% Diffuse</th>
<th>$C_{DP}$ 35% Quasi-Specular</th>
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<tr>
<td>Solar Max</td>
<td>2.22</td>
<td>2.12</td>
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Figure 6. Simplified geometry of the DANDE spacecraft.

Table 2. DANDE drag coefficients calculated at 350 km for solar minimum ($\alpha = 0.89$) and maximum ($\alpha = 0.97$).
gas surface interaction is expected to be nearly 100% diffuse below 400 km near solar maximum (personal communication with Drs. Kenneth Moe and Mildred Moe). Using this approach, the accuracy of the DANDE drag coefficient is estimated to be between 0.10 and 0.15 and within the required value of 0.20.

**Unique Accelerometer Suite**

In order to achieve mass density measurements with a precision of \( \pm 2\% \) \((1 - \sigma)\) or better while keeping cost down, a unique acceleration measuring technique is employed on the DANDE spacecraft. Six off-the-shelf accelerometers are integrated to custom filtering hardware and software to reduce the measurement noise. The accelerometer sensors are Honeywell QA-2000 accelerometers which employ an actively centered proof mass on a deflection beam to measure the acceleration. These devices include an internal temperature sensor which allows for the computation of a temperature dependant bias and scale factor. The spinning spacecraft will modulate the drag input from each of the six accelerometers by rotating the instruments (see Figure 7) through the velocity vector at a known rate. The filters on the spacecraft reduce the noise in all frequency bands except for a narrow band pass around the spacecraft spin frequency. Assuming a sinusoidal signal model, a linear least squares fit is performed on the data from each accelerometer signal. The six resulting sinusoidal amplitudes indicate the drag acceleration. Finally, the results from each of the six accelerometers are averaged to produce a data product with the required precision. The benefit of using deflection type accelerometers is that calibration constants which define the bias and scale factors can be determined on the ground. Furthermore, by modulating the drag signal and looking only for the amplitude of that modulation, the system is insensitive to changes in accelerometer bias. An additional benefit of this approach is that the loss of one or two out of six accelerometers results in only a partial loss of useful data allowing the system to degrade gracefully. The accelerometer suite fits inside a volume of 4.0\( \times \)4.5\( \times \)2.5 inches and has a mass of 1.3 kg.

As mentioned above, an analog band-pass filter is applied to the output in order to reject signals outside of a frequency band surrounding the spacecraft spin frequency. This is possible because the noise spectral density curve (see Figure 8) for the accelerometer sensors is flat around the spin-frequency and across the filter band pass. In fact, the spacecraft spin-frequency was chosen to locate the band-pass inside the flat region of the accelerometer noise power spectral density, where the noise is at minimum, while at the same time meeting gyroscopic stability requirements.

The accelerometer subsystem is simulated by specifying an analog filter output precision which is then used to add random errors on top of the acceleration measurements (including the error in internal temperature sensors). The resulting accelerations over 10 spacecraft rotations are processed by the least squares algorithm. Next, the post-processed acceleration is time-stamped and stored for interpretation by the ground segment. The system’s ability to filter a signal at the spacecraft spin frequency was demonstrated by adding an emulated drag sinusoid to the noise output of a single accelerometer. Spectral results showing that the filtering method achieves its objective can be seen in Figure 9.
Figure 9. Accelerometer system measured noise and inserted signal.

Figure 10. Section views of the WTS instrument showing the path of a molecule in blue.

Wind and Temperature Spectrometer

The wind and temperature spectrometer is capable of determining the horizontal wind vector, temperature, and atomic oxygen (O) and molecular nitrogen (N₂) number densities in the atmosphere. This instrument contains a Small-Deflection Energy Analyzer (SDEA) for energy selection (semi-circular component in Figures 4 and 10). It is a coarse analyzer (ΔE/E ≈ 0.1) with the ability to resolve the kinetic energy of neutral species entering the aperture at approximately 7,800 m/s. This corresponds to kinetic energies of about 5 eV (8.5 × 10⁻¹⁹ J) for atomic oxygen and 9 eV (14.9 × 10⁻¹⁹ J) for molecular nitrogen. The detector is a Micro-Channel Plate which amplifies charged particle impacts by setting off a cascade of electrons across the plate. The achieved gain in the charge value is approximately 3 × 10⁷. Anodes behind the MCP detect the electron shower and electronics connected to each anode record these pulses at different values of energy selection. The WTS instrument fits inside a volume of 4x4x3 inches and has a mass of 1.6 kg.

Figure 10 describes the motion of a molecule through the WTS instrument and the corresponding sequence of the measurement process is described below.

1. Neutral particle enters the collimator
2. Neutral particle is ionized inside of a field free electron bombardment region
3. Neutral particle enters the energy selector (SDEA) and undergoes acceleration toward the exit slit
4. Once outside of the selector, the particle is accelerated abruptly by a -3kV potential toward the Micro-Channel Plate detector
5. The impact on the MCP causes a cascade of electrons to travel toward one of the anodes which measures the impact (which anode is triggered depends on the angle at which the neutral particle entered the collimator)

To simulate the data product from the WTS instrument, a Maxwellian distribution was generated across the area corresponding to the collimator entrance. Equation 6 was used to obtain the flux through the rectangular aperture. From this generated distribution, only particles which are within the collimator 32° × 3.7° field-of-view are selected. A triangular bandpass with the characteristics of the instrument energy resolution is then swept across the resulting spectrum to model the SDEA energy scan. The resulting spectra for various angular distributions (anodes) is plotted in Figure 11. By performing a nonlinear least squares fit of two superimposed Gaussian curves to the resulting spectra in both the energy and angular axes, the incoming wind angle, energy, and temperature may be recovered. As the mass of the primary constituents is known, the resulting characteristic energy peaks seen in Figure 11...
are an indication of the magnitude of the wind. Likewise, the center of the distributions in the angular direction is used to deduce the wind direction with respect to the spacecraft coordinates. Due to the predominance of the 5eV atomic oxygen peak, this feature will be used primarily for wind and temperature determination.

In order to validate the energy resolution of the DANDE-WTS, the author led an effort to measure spectra in a vacuum chamber at NASA Goddard. The test setup is illustrated in Figure 12. An external ion source was placed in front of the collimator entrance at the left side of the schematic. On the left, filament F generates an electron beam which is collected at plate C. Neutrals inside the chamber are ionized inside of the beam and are then drawn out by the potential drop between IS and $E_X$ as well as between $E_X$ and CM which is at ground. The collimator, CM, is at a constant potential and directs particles in to the Small Energy Deflection Analyzer (SDEA). Once inside, the ionized particles are deflected by a potential applied to PD, the plate deflector (this is a part of the SDEA). The particles which exit the SDEA correspond to a certain kinetic energy proportional to the voltage on PD. Part of this experiment is to identify the relationship between voltage PD and the incoming ion energy.

The data were collected by connecting one of the center anodes to a pico-ammeter via a coax cable. The output of the pico-ammeter was routed to a digital oscilloscope. A function generator was then used to drive a sawtooth voltage wave on the SDEA plate deflector. Figure 13 shows the resulting spectra at various values of $V_I$ which is equal to the ionizer potential IS. The plots conclusively show that the SDEA acts as a kinetic-energy band-pass filter for the incoming ions. The high frequency component of the raw data (blue) is a result of ions moving around the SDEA and impacting the MCP. The noise to signal level was also exacerbated by the fact that the impact rate (signal) during testing was around 10 to 20 times smaller than that expected in orbit (2000 counts per energy spectrum). This issue was mitigated by adding shielding to the ion-source and by averaging multiple spectra together. Data analysis was performed by averaging five spectra for every input energy and extractor plate setting. The resulting average spectrum was then passed through a low-pass filter to remove the high frequency noise. Next, a nonlinear least-squares fit to the low-passed data is used to identify the center of the peak and the half-peak width of each averaged spectrum. In future tests, additional shielding will decrease the contributions from external ion impacts in order to eliminate the need to average multiple spectra.

The peak width, $dV$, is a measure of the energy resolution or the spectrometer’s $\Delta E/E$ or $\Delta m/m$. The ratio of the peak width to the peak potential is plotted as a function of incoming energy for three values of $E_X$ in Figure 14. Consider increasing the energy of ions entering into the instrument. At lower energies (and thus potential drops), the relative thermal dispersion is high resulting in wide peaks. As energy is increased the potential drop which the ions undergo is increased and the ion beam becomes more coherent relative to the total energy. Thus, the $\Delta E/E$ can be expected to become arbitrarily small at higher energies. A finite energy resolution of the SDEA prevents this from occurring and eventually the peak width is limited by the ion optics of the instrument. The flat portions of each curve in Figure 14 indicate an instrument resolution of 0.08 to 0.10 $\Delta E/E$. This meets the required resolution in Table 1 by a factor of three.

The analysis also resulted in several calibration curves for the instrument which can be used to calculate the relationship between ion-kinetic energy
Figure 14. Measured resolution at various extractor plate potentials.

Figure 15. Measured calibration curves at various extractor plate potential settings.

(wind) and the voltage applied to the SDEA. These curves are shown in Figure 15. The extractor-plate field is seen to modify the ion optics of the instrument and cannot be ignored during experimentation. This is most likely the result of a penetration electric field emanating from the extractor plate. The calibration curves allow us to deduce a measured plate factor, or P factor. The plate factor is the relationship between applied voltage and selected energy. We use the curve with the least extractor penetration ($E_X = 0.5\,V$) field to compute this number by inverting the slope of the line shown in Figure 15. The resulting plate factor was 3.4. Dr. Fred Herrero at NASA Goddard performed a numerical study to determine the plate factor as a function of geometric parameters and found it to be between 3.3 and 3.4. This agreement between calculation and observed calibration factor is a great vindication of the instrument design and of our understanding of the ion optics. A plate factor of 3.4 means that 1.47V applied to the SDEA results in selection of 5 eV of kinetic energy (5.0 eV = 1.47 eV × 3.4) through the analyzer. The electronics have been designed and confirmed to sweep SDEA voltage from 0.1V to 14V. From this we can compute the effective energy range of the DANDE-WTS to be 0.4eV to 48eV. The wind resolution will be verified in a future round of testing.

System Error Model

In order to make full use of the atmospheric information presented by drag and in-situ data gathering spacecraft, one must be aware of the accuracy and precision of the measurements. Therefore, a numerical model was applied to evaluate the errors inherent to the proposed technique. The end-to-end system model was written in MATLAB and uses random number generators to add noise to simulated wind and density signals. The Horizontal Wind Model, HWM-93, and a The Mass Spectrometer Incoherent Scatter density model, NRLMSISE-00, were compiled and interfaced to the MATLAB interpreter. The user specifies the initial start date and end dates, the initial attitude state (yaw, pitch, roll, yaw rate, pitch rate, roll rate) along with the inertia matrix, and the initial orbital parameters (inclination, perigee altitude, apogee altitude, right ascension of the ascending node, argument of perigee, and true anomaly). The position of each accelerometer in the body frame is specified in a series of direction vectors as is the analog to digital precision used later to simulate truncation errors. The accelerometer specific temperature correction terms for both scale factor and bias, are also provided by the user.

The accelerometer error is evaluated by integrating the PSD shown in Figure 8 between the band pass filter cutoff frequencies and the resulting noise value is 0.60 $\mu$g. The wind detection error estimates are a result of work done at NASA Goddard with a model of the instrument selector mentioned in the previous section. The study found the wind error to be 45 m/s for a peak count of 100 molecules and 15 m/s for a peak count of 1000. Since the expected peak count may drop below 1000 during night time, a wind precision of 30 m/s was entered into the model. The wind errors are assumed to adhere to a zero-mean normal distribution. The error inputs are further defined in Table 3.

The simulation time, $t$, is incremented in seconds from the start date until the end date and controls the progression of integrating the equations of motion (attitude dynamics and orbital dynamics modules). The position solution along with the time is used to obtain local atmospheric conditions.
from NRLMSISE-00 and HWM-93. The attitude state determines the drag coefficient and feeds this variable back to the orbital dynamics package. The spacecraft bus module is a simple package which drives the temperature state of the instruments in a periodic way corresponding with the sunlit and eclipsed portions of the orbit. The spacecraft time generator module is used to add random error and time-dependent bias to the on-board time measurements.

The modules described above send their data to the accelerometer system simulator and mass spectrometer system simulator. Each of these modules acts on incoming data with a cadence determined by the input sample rate. The accelerometer data processing is performed in the accelerometer module at pre-specified cadence, 60 seconds nominally. The outputs of the program are simulated in-situ, data-products of the neutral wind, number density, and acceleration. Based on simulations done by the DANDE engineering team, predictions of the tracking uncertainties\textsuperscript{11} and attitude determination errors\textsuperscript{21} are used to estimate ground sector determination and propagation errors in the ground-processing simulation module. This module contains orbit and attitude simulations used to propagate the spacecraft dynamics for the duration of the simulated measurements. The ground processing module uses the predicted satellite state along with instrument data to calculate mass density, number densities of atmospheric constituents, as well as neutral wind velocity. The final data output is tabulated for a particular spacecraft time.

Figure 16, shows the post-processed drag acceleration and the input winds along four orbits at solar maximum input conditions. The corresponding ground module output is shown in Figure 17. Note that information from the WTS instrument has not yet been applied in Figure 17. There is a visible bias which is mostly due to a modeled error in drag coefficient estimation. The measured and input densities also cross at high latitudes which is indicative of wind-induced errors in the near-polar thermospheric circulation cells.\textsuperscript{8} In other words, an increase in wind magnitude along the direction of the satellites orbit is causing a significant increase in the observed drag. Without knowledge of the in-situ winds, the ground module incorrectly interprets this drag increase as an increase in density.

The dotted line in Figure 18 shows the percent error in the accelerometer-derived density measurement. The high-latitude wind-induced error over the span of two orbits is seen above 60° latitude and is as high as 14%. The solid blue line is the improved data-product error which results from including the wind measurements in the determination of density. The inclusion of wind information has removed the

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<tr>
<td>Apogee Altitude</td>
<td>350 km</td>
<td>0.08 km</td>
</tr>
<tr>
<td>Inclination</td>
<td>96.9°</td>
<td>0.00°</td>
</tr>
<tr>
<td>RAAN, Ω</td>
<td>99.9°</td>
<td>0.00°</td>
</tr>
</tbody>
</table>

**Table 3. Inputs to the error-budget simulation.**

![Figure 16. Modeled acceleration and wind measurements.](image1)

![Figure 17. Input and measured density.](image2)
peak errors. Furthermore, the resulting $1 - \sigma$ deviation in density is around $\pm 1\%$ as indicated by the horizontal dotted lines. The dual instrument approach results in an error of $2.05 \times 10^{-12} \text{kg/m}^3$ and a precision of $\pm 1.97 \times 10^{-13} \text{kg/m}^3$ which meet the requirements in table 1 within uncertainty.

Conclusion and Lessons Learned

The determination of satellite drag and atmospheric density is made difficult by the high precision with which acceleration must be characterized (tens of nano-g’s). The often unquantified contribution of atmospheric wind, and the difficulty in estimating an appropriate drag coefficient while maintaining a known cross-sectional area also increases the error in observations of drag and atmospheric density. To address these challenges, we have presented a novel approach for drag determination and have shown a numerical model for error propagation used to evaluate this approach. The model has been applied to the University of Colorado satellite, DANDE, and the resulting data product fidelities match the science objectives of the mission. DANDE will have the unique opportunity to test this new approach in orbit during the 2011-2012 time period.

The results of our analysis indicate that winds at high latitude (above 60°) can have a 10%-14% impact on the error in density determinations from satellite drag. This error could be further exacerbated during geomagnetic storms and the range of effected latitudes could also be increased. The dual instrument package presented here has the ability to remove wind induced error and thus enables the study of densities at all latitudes and solar conditions. The testing of the accelerometer suite and Wind and Temperature Spectrometer have confirmed this analysis. The noise introduced in testing was a result of inadequate shielding which allowed stray ions to move around the ion-optics and impact the detector. This has motivated the design of an additional shield in front of the detector.

By rotating the accelerometers into and out of the drag acceleration, the system effectively removes accelerometer bias. The accelerometer scale factor is temperature compensated in flight via temperature sensors inside the accelerometers. The team will take special care to characterize the scale factors prior to launch. We have shown that the primary source of density bias is the drag coefficient uncertainty estimate which is based on our knowledge of the gas-surface interactions in low-earth orbit. The Monte Carlo method has the ability to compute the drag coefficients of complex geometries and the resulting uncertainty estimate is within acceptable limits. To further reduce the uncertainty in atmospheric density however, gas-surface interactions and the energy accommodation coefficients must be better characterized.

To meet the increasing needs of orbit prediction and space situational awareness, our ability to determine the environment in which our space-assets fly must continually improve. The measurement technique presented in this paper and the instruments which make it possible are an important step in the improvement of operational drag models. We plan to use the on-orbit data resulting from this experiment to validate and improve existing and future models for use by the spacecraft community. This implementation will not only improve the predictive drag models, but also enable the construction of small-satellite “weather stations” capable of observing the drag environment at a reduced cost.

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Brandon Gilles performed the detailed and challenging electronics design for this instrument and helped support the vacuum test. Spacecraft tracking will be provided through collaboration with the Air Force Space Command, Space Analysis Office, and the author thanks Bruce Bowman for presenting this opportunity.

References


