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ABSTRACT

The feasibility of optimizing large regional water resource planning problems by means of integer programming algorithms is analyzed. Two types of integer programming models are developed: (1) A water supply model including 23 separate but geographically related community systems; and (2) A river basin water quality model including 15 point sources of wastewater, 4 types of pollutants, 6 surveillance points, and 7 alternative treatment processes. The water supply model was structured as a mixed integer problem (some continuous variables included) while the water quality model was an all integer problem.

Four integer programming algorithms were tested on the sample problems as follows: (1) MXINT - The Burroughs B6700 TEMPO package algorithm; (2) FMPS-MIP - The UNIVAC 1108 MPS package algorithm; (3) GMINT - A proprietary algorithm authored by A. M. Geoffrion and R. D. McBride; and (4) AIP - A 0,1 algorithm which uses the Balas additive concept.

Several versions (sizes) of both problems were successfully solved by one or more of the algorithms with computational efforts ranging from less than 1 to more than 40 minutes of CPU time.

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CHAPTER I

INTRODUCTION

Scope and Objectives of Phase I

This report describes an investigation of the capability of existing integer programming algorithms in solving water resource problems. The work consists of a combination of separate lines of inquiry on two types of example problems in water resource planning and management:

Regional Planning of Water Supply—An Integer Planning Approach
Principal Investigators—Trevor C. Hughes and Calvin G. Clyde

An Interactive Simulation—Optimization Model for River Basin Management
Principal Investigators—William J. Grenney and A. Bruce Bishop.

These two studies are related in that they both propose the use of integer programming (IP) as their optimizing tool. In order to verify the computational feasibility of IP solutions to problems of the size envisioned in these proposals, OWRT supported this limited, combined initial study.

The objectives of this phase of the work are:

1. Review and evaluate existing IP algorithms and identify those which appear adaptable to the two types of water resource problems involved.

2. Structure example problems of the types outlined in the original proposals.

3. Test the selected algorithms on the example problems to determine their run times, costs, and capabilities in terms of number of variables and constraints.

4. Select the best algorithms for application to the actual case study problems proposed for follow-on research and evaluate the limiting size for proposed types of models to which integer programming can currently be applied.

Integer Programming Concepts

Integer programming problems can be categorized as either mixed integer (MIP) or all integer (AIP) types. MIP problems include both integer and continuous type variables. These would be linear programming (LP) problems except for the requirement that some of the variables can assume only discrete (integer) magnitudes. IP was in fact developed as an extension of LP, and virtually all the modern algorithms still use the simplex algorithms as the optimizing tool within the IP framework.

AIP problems are those in which *all* the variables are constrained to integer values. In many problems a further restriction is possible which limits the variables to either 0 or 1 values. This characteristic allows greater computational efficiency and many AIP algorithms are coded to accept only this structure of problem. The more general algorithms accept upper bounds of greater than unity. Any integer variable, however, can be defined in terms of a combination of 0, 1 variables, by using a binary expansion (McMillan, 1975), so that the 0, 1 codes can also be used for problems with higher upper bounds.

Clearly AIP problems can be solved with an MIP algorithm (a special case in which the number of noninteger variables is zero) but the reverse is not true.

The two types of IP models developed herein represent a good combination for evaluating IP algorithms. They have very different characteristics which collectively will test the capabilities of different types of algorithms. The water supply model is a mixed integer problem. It has some integer upper bounds greater than one but not enough greater that binary expansions are difficult to use; therefore, both types of MIP codes are easily usable for this model. The waste treatment model is structured as a strictly 0, 1 problem; therefore, both MIP and AIP codes with any upper bounds are applicable to this problem solution.

Integer Programming–State of the Art Summary

The following discussion identifies the various basic approaches to IP which have evolved and some of the recent additions and improvements to the algorithms which have some importance in regard to improving computational efficiency. None of the concepts are described in detail here. The literature content, however, is identified to the extent necessary to assist the reader who is interested in such details in locating relevant publications.

The most comprehensive discussion of IP algorithms in the literature was written by Geoffrion and Marsten (1972). This state of the art survey develops a general IP framework by which the various steps in an algorithm can be identified and compared with the related approach of other algorithms. The framework is then used as a format for a detailed discussion of nine branch-and-bound type algorithms, three Benders decomposition type, two cutting plane type approaches and a group theoretic approach.

The branch-bound approach was characterized by Geoffrion and Marsten as the concept for general purpose IP problems with by far the largest and most successful practical computational experience on large problems. In 1972 the state of the art included such improvements as: (1) Using surrogate constraints (redundant linear combinations of existing constraints) as an improved fathoming device in order to "capture more of the joint logical implications of the entire set of original constraints" (developed by Glover (1965)). (2) Other means of improving "simple penalties" such as psuedocosts and adding Gomory cuts to determine variable bounds in branch-bound algorithms.

Geoffrion later updated the earlier state of the art paper by discussing "recent practical advances in integer linear programming" (Geoffrion, 1975). Since virtually all modern algorithms use LP as their optimizing tool, several of the improvements discussed in the 1975 paper are related to recent improvements in the simplex algorithm, such as: Generalized upper bounding; improved representations of the inverse; and interactive implementations of full scale mathematical programming systems.

Geoffrion's discussion of modeling principles emphasizes that IP model structuring is still very much an art rather than a science. This aspect is addressed in the following quotations (Geoffrion, 1975):

> The computational tractability of any given IP application is strongly dependent on both the *content* (assumptions) of the model and the way in which the model is *represented* mathematically (the distinction here is important). It is essential to recognize that some of the guiding principles from linear programming can be downright dangerous if applied absent-mindedly to integer programs.
>
> * * *
>
> The main lesson of Williams (1974) . . . is an important one: one should examine the various possible mathematical representations of a model which are equivalent in a logical sense and select the one which seems likely to give the tightest bound when relaxed in the usual way to an ordinary LP. The reason is that better bounds imply less need for branching, thereby shifting the balance of work to the relatively more efficient machinery of linear programming (as opposed to enumeration). Williams gives five specific examples to illustrate various ways of achieving "equivalent" formulations yielding better LP bounds. . . . William's other examples illustrate instances in which new constraints can be added that are redundant in an IP sense but are not so for the associated LP relaxation. In the second example these constraints can be discovered by graphically examining two or three-dimensional components of the problem. For the remaining examples they can be discovered by "disaggregating" existing constraints, as by writing $x_1 \leq x_4$; $x_2 \leq x_4$; and $x_3 \leq x_4$ instead of $x_1 + x_2 + x_3 \leq 3x_4$ when the variables are 0-1.
>
> * * *
>
> Another technique for generating useful "redundant" constraints is to explicitly derive the convex hull of a select (and relatively simple) subset of the set of all constraints. An illustration of this technique is to be found in Geoffrion and McBride (1972).
>
> * * *
>
> Thus the integer programming modeler must learn that economizing on the number of constraints in the respresentation of a model can be a sin rather than a virtue. Economizing on the number of integer variables, however, is usually very desirable.

The continuing trend toward almost exclusive use of brand-and-bound type algorithms is characterized by Geoffrion (1975) as follows:

> . . . Discussion will largely be limited to the context of LP-based branch-and-bound, as virtually all commercially available IP software is of this type.

Geoffrion's own algorithm which was used in this study, however, is a hybrid in that the basic branch-and-bound algorithm has had a cutting-plane option added to it.

Many of the IP concepts mentioned previously are described in considerable detail in two recent textbooks (Garfinkel and Nemhauser, 1972, and McMillan, 1975), and in a collection of IP papers (Balinski, 1974).

CHAPTER II

SPECIFIC IP ALGORITHMS USED IN THIS STUDY

MXINT

The Burroughs B6700 computer at Utah State University includes as part of its TEMPO mathematical programming package, a mixed integer programming algorithm referred to as MXINT. This is a branch-and-bound algorithm which was developed by Driebeek (1966) and modified by Beale and Small (1965). The algorithm accepts integer variables with upper bounds greater than unity. This is the only algorithm encountered in this study which has this desirable capability (which eliminates the need for manual binary expansions of such variables).

The TEMPO package also includes the generalized upper bounding (GUB) capability in its LP algorithm, however, this capability is apparently not available for use in conjunction with MXINT.

A brief description of the MXINT algorithm is included in Appendix A. One of the important aspects of an IP algorithm in regarding applications to large problems is flexibility in setting and adjusting the tolerance levels by which the algorithm operates. Computational effort can become totally unreasonable unless some minimum discrete intervals for parameter improvement are selected. MXINT provides for user selection of the following parameters:

1. If the objective function for the optimal solution is known to exceed some value, that value is used as a lower bound. This reduces the size of the branch-bound structure for the problem.

2. If an integer variable assumes a magnitude within a certain tolerance of an integer value it is assumed to be integer. The standard tolerance is ±1 percent.

3. After an integer solution is obtained, the only other solutions which are considered are those which improve the objective function by at least the selected amount (1 percent for example).

MXINT also provides a choice of 4 back tracking criteria for the branch-bound search.

GMINT

GMINT algorithm was develop by Arthur M. Geoffrion and Richard D. McBride at the Western Management Science Institute, UCLA. It is a mixed integer (0, 1 integer only) code which evolved as described in the User Instructions (Geoffrion and McBride, 1975) as follows:

> GMINT uses a highly developed branch-and-bound procedure with linear programming as the primary relaxation. It is an evolutionary descendant of the widely distributed RIP30C code developed almost a decade ago at RAND and described in Ref. 1 (see also Ref. 2). The general conceptual framework within which the code should be viewed is given in Ref. 3 [See Geoffrion and Marsten, 1972.] (see especially Sec. 3.1.5). Numerous refinements have been incorporated since these references were written, including: an all-new linear programming subroutine with the GUB feature (Ref. 4) and a linked list data structure which makes extremely efficient use of core (cf. Sec. 2.2.6 of Ref. 5), a streamlined re-implementation of logical fathoming devices, and much-improved branching and feasibility-seeking design.
>
> The user will find GMINT to be far more efficient than any commercial mixed integer linear programming package for most problems.

GMINT is a proprietary algorithm which is being marketed by the authors. Details of the algorithm are therefore not available.

FMPS-MIP

The mathematical programming package on the UNIVAC 1108 computer in Salt Lake City includes a mixed integer branch-and-bound type algorithm referred to as FMPS-MIP. This code accepts only 0, 1 variables in the integer sector. It provides the following alternate strategies:

1. Try to obtain the true optimum integer solution.

2. Try to obtain an integer solution as fast as possible (even if the objective function value of that solution is not very good).

3. Try to obtain a "good" integer solution (not proved optimum—but certainly not worse than a supplied CUTOFF value) fairly quickly (within a time that is expected to be between 1 and 2, nearer to 3).

A detailed discussion of these strategies and various node and integer variable selection options is given in the FMPS manual (Sperry, 1975). The manual does not list the algorithm's authors. It apparently was developed by combining concepts from several different algorithms which have been described in the literature.

AIP

The only algorithm used in this study which is not of the branch-and-bound type is an all integer algorithm which was included in Edition I of *Mathematical Programming* by McMillan (1970). It consists basically of the Balas additive algorithm (Balas, 1965) but with the addition of the use of LP for generating "strongest surrogate constraints" in order to improve the efficiency (Geoffrion, 1969). The Balas algorithm is an enumeration scheme by which many possible solutions are enumerated only implicitly and dismissed, so that only a relative few are examined explicitly.

CHAPTER III

REGIONAL WATER SUPPLY MODEL

Nature of the Planning Problem

In spite of the tremendous size on a national scale of the annual investment in both construction and operation of municipal and rural water supply facilities, the large majority of this investment is still based on planning which is limited to individual municipal boundaries. Typical results of this limited planning scope are: (1) Several parallel supply lines and other facilities from a single water source which serve different communities. (2) A single community develops all of the local high quality water sources such as spring flow and wastes what it doesn't use, while neighboring communities search for other less attractive sources.

The obvious disadvantages of these planning problems are: (1) The tremendous diseconomies of scale due to several small pipelines, reservoirs, treatment plants, etc., rather than common larger facilities. (2) The loss of scarce high quality water due to the lack of interconnections between systems which would allow use and/or storage by one community when another community's supply exceeds its storage capacity. (3) Rural residents are forced to construct individual wells or to haul water to cisterns at great costs because service to areas outside municipal boundaries are not considered by planners.

Regions which include Indian reservations experience these same planning problems plus the additional diseconomies resulting from the institutional and traditional myopia inherent in separate planning for Caucasian and Indian water users.

The potential in the water supply field for savings on both capital investments and operational costs due to economies of scale is tremendous. For example, Higgins (1972) indicates that the construction cost of ground level reservoirs varies approximately as the square root of their capacity; so that doubling the cost buys four times the capacity. The scale effect for elevated tanks is even more while that of treatment plants and pipelines is only sightly less. If by proper regional planning, advantage could be taken of such scale effects, the cost savings on a nationwide basis would be in the multi-billion dollar category.

The Corporate Boundary Perspective

With such potential savings as a real possibility, it is significant that at the present time, municipal water supply systems are largely being planned on the basis of individual corporate boundaries. Planning engineers for individual cities are expected, indeed are usually directed, to limit the scope of their studies to the existing city boundary or to possible modifications to those boundaries due to annexation of the immediate peripheral areas. City fathers typically are not interested in interconnections between their water supply and that of other communities or with Indian reservations, nor are they likely to favor service to surrounding rural areas by their system. There are several apparent reasons for this lack of interest in regional planning:

1. Regional planning costs money and individual cities are not interested in paying for planning which includes areas beyond their probable future boundaries.

2. The major regional planning effort which has recently been supported by state planning agencies and financed by the federal government (through HUD) has been the 701 type comprehensive Master Plans. The value of these plans to municipal water supply engineers is essentially zero. The scope is such that the plans are limited geographically by county boundaries rather than natural hydrologic basins; but even more importantly, the water supply section of these plans is typically a brief discussion of generalities such as:

> As the community grows, the water system will need to be upgraded. It is suggested that this also be studied with the regional implication,. . . (Planning and Research Associates, 1972)

An appropriate question seems to be, "Why wasn't the water supply question studied with the regional implication as part of this major planning effort?"

3. Even if the necessary fiscal and institutional resources were available for regional water supply planning, much work needs to be done in developing the planning capability. What is needed is a systems approach which is easily adaptable to any basin and

simple enough to be used by planning engineers who are not mathematical programming specialists.

Project Oriented Regional Planning

As a result of disinterest in regional water supply planning by municipalities, the bulk of such planning in the western U.S. has been oriented toward supporting particular large scale multi-purpose projects. For examples, large irrigation projects planned by the Bureau of Reclamation which use M&I revenue to help repay their costs. The problem with this sort of regional planning is that it is not analyzed from the standpoint of determining the optimal way to provide public water supply in a region. Rather, it is considered in the framework of how much revenue can be obtained from the water supply portion of the project to help amortize total project costs. Often, for example, local groundwater would better serve M&I demand in an area, but its use will not contribute to paying for a large importation project and it is therefore not considered.

Research Objectives

The overall objective of this program is to develop methodology for optimal planning of water resource systems on a regional basis. The sub-objectives of the initial phase of the study in relation to the water supply component are as follows:

1. Develop an integer programming water supply model which incorporates a least-cost objective function and all necessary constraints in order to allow evaluation of the regional system alternatives including:
    a. Scale of each facility
    b. Interconnections between communities
    c. Service for individual rural connections between or near communities.

2. Test the capability of selected IP algorithms by using them to produce optimal solutions to the problems represented by various forms of the model.

Nature of Model Input Data

The water supply model developed herein is not a hypothetical problem. Rather, it represents a reasonably accurate definition of existing and potential water supply and projected demands for each municipal and rural domestic system in Cache Valley, Utah. Better resolution of these parameters will be obtained in future phases of this research (such as more accurate data on seasonal variations in supply and demand, better analysis of optimal well and pump sizing in various aquifers, and potential for additional spring development). However, the best possible real world estimates within the existing time constraints were made for this initial study. This attempt to approximate the actual parameter levels and number and types of sources was made in order to insure algorithm tests in a realistic setting.

Much of Cache Valley has an abundance of good quality groundwater and therefore most of the future source facilities included in the model are wells. Treatment plants were not considered except in one zone (where additional groundwater is not available) because of the much higher unit costs. The traditional sources of municipal water for most communities have been springs in nearby canyons. Most demands, however, are now beginning to exceed natural spring flows and many systems are being supplemented by pumped groundwater.

Model Structure

The water supply model developed for this study is basically a transportation problem which requires demands from each of 23 service zones (cities or rural areas) to be satisfied by flow from existing or potential springs, wells, and/or treatment plants. Interzonal transfers of water are considered by including conduits of two alternate sizes between adjacent cities.

The objective function is structured to provide the desired quality of service at least annual cost. Fixed (capital investment) and variable (O&M) costs are defined separately. Fixed cost coefficients are associated with integer (usually 0 or 1) variables which represent construction of new production or transfer facilities. Variable cost coefficients are associated with continuous variables which represent seasonal flow through each existing or new production or transfer facility.

The activity levels of the continuous variables insure that average seasonal operating costs are included in the objective function. The two season model considers average summer flow (season 1 includes June through September) separately from the lower level colder month flows. This allows use factors (and therefore unit costs) to vary independently from investment costs.

The level of capital investment required to satisfy the summer season demand is not adequate to meet the peak day demand during an average year and therefore clearly is inadequate for the peak day during an unusually high demand and/or low supply day. Provision for chance constrained programming is therefore included in the model. The stochastic portion of the model basically repeats the demand and supply constraints but with constants representing the peak day levels at the desired recurrence interval. In this example, demands are all simply increased 30 percent and supplies are decreased 10 percent. These levels, however, will be varied independently for each zone and source in the final version of the model. The purpose of the peak day constraints is to require the appropriate capital investment.

The simplified form of the model is as follows:

Minimize total annual cost = $C_1 I + C_2 X + C_3 X^p$

in which

I = vector of integer variables

X = vector of continuous seasonal variables

X^p = vector of peak day continuous variables

C_1, C_2, C_3 = cost coefficients

Subject to the following seasonal constraints:

$X \geqslant d$ (supply to each zone $\geqslant$ demand)

$X \leqslant b$ (flow from each existing production facility $\leqslant$ its capacity)

$X \leqslant AI$ (flow from each new facility $\leqslant$ its capacity) (I = number of units built; A = capacity of each single unit)

$I \leqslant 1$ (forces no more than one of the alternate sizes of each facility to be built)

Peak day constraints

$X^p + A^p I \geqslant d^p$ (demand constraints on peak day)

$X^p \leqslant b^p$ (existing facility supply constraints on peak day)

$X^{pe} \leqslant A^p I$ (zonal transfers $\leqslant$ pipe capacities on peak day)

Detailed Model Description Cache Valley Application

A detailed description of the sample problem is developed by the scalar equations in Table 1. The model notation requires triple subscripting of variables according to the following indexes:

i = service zone index (1,2 ... 23)

Each community is represented by a single number except for Logan City which has a high elevation Zone (1) and a low elevation Zone (2). The key map for this index is given on Figure 1.

j = facility type and size index as follows:

j	
1	Existing well
2	Existing spring
3	Future well
4	Future spring
5	Future treatment plant size A
6	Future treatment plant size B
7	Future treatment plant size C
8	Future pipeline size A
9	Future pipeline size B

k = Season index (1 or 2)
1 = summer 4 months;
2 = other 8 months

i' = usually i+1 or i-1 but may be any service zone with potential direct connection to zone i (see Figure 1)

ii' = implies a flow from zone i to adjacent zone i' (and conversely i'i represents flow into zone i)

Integer variables:

I_{ij} = integer variable denoting development of a new well, spring, or treatment plant (j = 3, ... 7) in zone i. Activity level indicates the number of facilities built. Usual values are 0 or 1 but higher integers are possible where more than one potential well exists in a zone

$I_{ii'j}$ = 0 or 1 variable denoting construction of a particular size (j = 8 or 9) of pipeline between zone i and adjacent zone i'

Table 3. Applications of optimization models.

Programming Method	Purpose of Optimization	References
Linear	Least cost combination of unit processes to remove a given amount of BOD	Lynn, Logan & Charnes (1962)
Linear	Stage development over time of wastewater treatment systems	Lynn (1964)
Linear	Least cost of wastewater collection and treatment and staging of construction for a region	Deininger & Su (1973)
Nonlinear	Least cost combination of inputs to production function to remove BOD	Marsden, Pingry, & Whinston (1972)
Nonlinear	Least cost regional wastewater planning	Young and Pisano (1970)
Dynamic	Sequential capacity expansion of plants	Kirby (1971)
Dynamic	Multistage capacity expansion of water treatment systems	Hinomoto (1972)
Dynamic	Least cost combinations of unit processes to remove a given amount of BOD	Evenson, Orlob & Monser (1969)
Dynamic	Serial multistage system of industrial waste treatment for BOD	Shih & Krishnan (1969)
Dynamic	Minimum total annual cost to meet given treatment requirements	Shih & DeFilippi (1970)
Dynamic	Sequencing of water supply projects to meet capacity requirements over time	Butcher, Haimes & Hall (1969)
Approximate & Incomplete Dynamic	Capacity expansion of large multilocation wastewater treatment systems	Erlenkotter (1973)
Integer	Location and size of wastewater treatment plants and trunk sewers	Wanielista & Bauer (1972)
Integer	Least cost selection of treatment levels to meet river quality standards using zones of uniform treatment level	Liebman & Marks (1968)
Nonlinear-Decomposition & Multilevel Approach	Minimization of overall regional treatment costs to meet desired river quality standards. Determination of effluent charge pricing level.	Haimes (1971) Haimes (1972a) Haimes (1972b) Haimes, Kaplan, & Husar (1972)

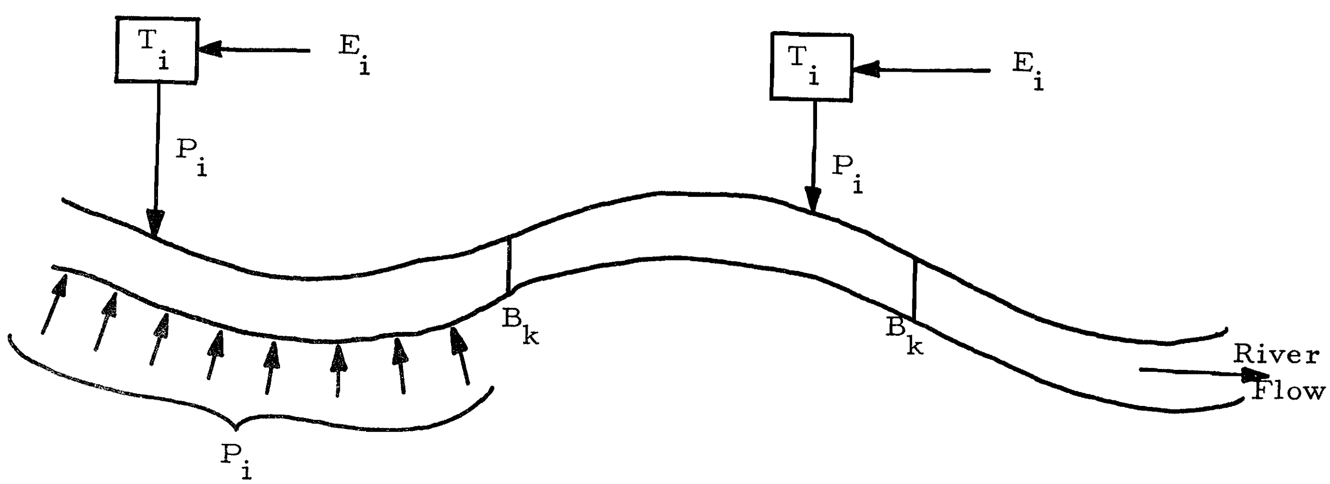

Figure 3. Schematic representation of treatment cost optimization in basin wide water quality management.

from the stream water quality simulation model based on biophysical processes and river channel characteristics. Let the concentrations of the constituents in the river be signified by:

$Y_k = (y_j)_k$ a vector of concentrations in the river at surveillance point k.

The river concentrations (y_j) are related to the load concentrations (p_j) and river distance by the simulation and include biophysical processes, river channel characteristics, lateral inflow, diffuse source loading as well as point loads. Next define:

$P_i^o = (p_j^o)_i$ a vector of initial conditions for constituent concentrations in the effluent of load i.

$Y_k^o = (y_j^o)_k$ a vector of constituent concentrations in the river resulting from P_i^o.

It can be shown that for equations linear in P that the change in river concentration, $Y_k^o - Y_k$, can be related to the change in effluent concentration, $P_i^o - P_i$, by the D matrix as follows:

$$Y_k - Y_k^o = \sum_i D_{ik} (P_i - P_i^o) \quad . \ . \ . \ . \ (2)$$

when

$$d_{jm} = \left. \frac{\partial y_i}{\partial p_m} \right|_{p_m^o} \qquad \begin{matrix} j = 1, 2, ..., J \\ m = 1, 2, ..., J \end{matrix}$$

For nonlinear expressions Equation 2 is not strictly valid; however, it does represent an approximate relationship.

Finally define:

$C_i = (c_\ell)_i$ a row vector of total costs for treatment levels, ℓ, at load i.

Note that the cost function for a given treatment process can incorporate economies of scale.

With the variables and coefficients thus defined, the management alternative resulting in minimum basinwide cost can be structured as follows:

$$\text{Minimize Total Cost} = \sum_i C_i T_i \quad . \ . \ . \ (3)$$

Subject to the sets of constraints:

(1) Water quality standards.

$$\sum_i D_{ik} E_i T_i \leqslant B_k - Y_k^o + \sum_i D_{ik} P_i^o \quad k = 1,2, ..., K \quad (4)$$

(2) Integer solution for treatment levels, i.e., only one treatment level per load point i.

$$\sum_\ell (t_\ell)_i = 1 \qquad i = 1,2, ..., I \quad . \ . \ . \ (5)$$

and

$t_\ell = 0$ or 1 for all values of ℓ.

Therefore, the problem is one of choosing a t_ℓ at each load i such that the cost is a minimum subject to the water quality standards at each of the surveillance points, k. A number of efficient solution methods are available for the integer programming problem thus formulated.

Nonlinearities in cost functions are accounted for since costs are described for treatment levels for which unit costs are constants for a specified flow rate at a discharge point or zone. Use of the simulation model to generate constraint coefficients accounts for nonlinearities in biophysical assimilative processes and stream characteristics, and gives the model a more dynamic, as opposed to steady-state characteristic.

Simulation Model Structure

The mathematical model selected for this study was the stream simulation and assessment model (SSAM) which has been applied in six river basin studies in the Intermountain West. The model can be applied to a river system with diffuse surface inflow, diffuse groundwater inflow (or outflow) and any reasonable number of tributaries, point loads, and point diversions. The river channel must be divided into "reaches" representing lengths of river which can be assumed to have uniform physical characteristics. The equations shown here are simplified to represent only the mechanisms of interest in this study. A complete description of the model can be found in Grenney and Porcella (1975).

The water quality equations shown here represent two phenomenon occurring in a slug of water as it travels downstream (dispersion is neglected):

1. Mass being added or removed from the water due to sources or sinks distributed along the stream channel.

2. Biochemical reactions and interactions among constituents.

Descriptions of symbols used in the equations are shown in Table 4.

The mass of a constituent being added or removed due to diffuse sources or sinks located along the channel can be expressed as follows:

Table 4. **Model coefficients used in problem I.**

Water Quality Constituent	Coef-ficient	Description	Units	Reach 1	2	3	4	5	6
Biochemical Oxygen	$K_{1,a}$	Oxidation rate	day^{-1}	0.25	0.25	0.32	0.32	0.32	0.32
Demand	$K_{1,b}$	Benthic contribution	$g/m^2/day$	0.0	0.0	0.02	0.04	0.10	0.10
Ammonia	$K_{2,a}$	Nitrification rate	day^{-1}	0.30	0.30	0.33	0.35	0.35	0.35
	$K_{2,b}$	Benthic contribution	$g/m^2/day$	0.0	0.0	0.01	0.03	0.08	0.08
Phosphorus	$K_{3,b}$	Benthic contribution	$g/m^2/day$	0.0	0.0	0.0	0.0	0.0	0.0
Dissolved	$K_{4,a}$	Reaeration coefficient	day^{-1}	1.5	2.0	1.5	1.5	1.5	1.5
Oxygen	$K_{4,b}$	Benthic contribution	$g/m^2/day$	0.0	0.0	0.02	0.06	0.06	0.06
Deficit	$K_{4,2}$	Algae respiration	$mg\text{-}O_2/mgP/day$	2.0	2.0	2.0	2.0	2.0	2.0

$$S_j = \frac{Q_s(Y_{sj} - Y_j)}{\bar{A}} + \frac{K_{j,b}}{\bar{D}} \quad . . . . . (6)$$

where the first term on the right-hand side represents diffuse surface inflow and the second represents contributions from the stream bottom. Y_j is the concentration of constituent j(mg/l), Q_s is lateral inflow ($m^3/m/min$), Y_{sj} is the concentration of constituent j in the lateral inflow (mg/l, $\bar{A}$ is the average cross sectional area (m^2), $K_{j,b}$ is a coefficient for constituent j($g/m^2/min$), and $\bar{D}$ is average depth (m).

The model equations used in this study are as follows

(j = 1) Biochemical oxygen demand. The rate change in concentration is a function of first-order decay (oxidation), leaching from bottom deposits, mass input from lateral inflow, and point loads.

$$\frac{dY_1}{dt} = - K_{1,a} Y_1 + S_1 \quad . . . . (7)$$

$$K_{1,a} = K_{1,1}\ 1.047^{(T-20)} \quad . . . . (7a)$$

$K_{1,1}$ is the first order decay rate at 20°C and T is temperature in °C.

(j = 2) Ammonia. The rate change in concentrations is a function of first-order decay (nitrification), leaching from bottom deposits, mass input from lateral inflow, and point loads.

$$\frac{dY_2}{dt} = - K_{2,a} Y_2 + S_2 \quad . . . . (8)$$

$$K_{2,a} = K_{2,1}\ 1.047^{(T-20)} \quad . . . . . (8a)$$

$K_{2,1}$ is the first order decay rate at 20° C.

(j = 3) Total phosphorus. This constituent is represented as a conservative substance. The rate change in concentration is a function of leaching from the bottom deposits, mass input from lateral inflow, and point loads.

$$\frac{dY_3}{dt} = S_3 \quad . . . . (9)$$

(j = 4) Dissolved oxygen deficit. The rate change in DO deficit is a function of reaeration, BOD oxidation, nitrification, benthic uptake, mass input from lateral inflows, and point loads. For purposes of this example it was desirable to link the dissolved oxygen deficit with phosphorus. Therefore it was assumed that the phytoplankton concentration was directly proportional to the phosphorus concentration (Y_3), and further that algal respiration occurring at night (when photosynthesis is zero) would add to the oxygen deficit. The reasonableness of this model is limited to a stretch of river which has a travel time less than the night time hours.

$$\frac{dY_4}{dt} = - K_{4,a} Y_4 + K_{1,a} Y_1 + 4.22\, K_{2,a} Y_2 + K_{4,2} Y_3 + S_4 \quad . . . . (10)$$

$$K_{4,a} = K_{4,1}\ 1.0159^{(T-20)} \quad \ldots\ldots \quad (10a)$$

$$\text{Do concentration: } Z_4 = Y_{sat} - Y_4 \quad \ldots \quad (10b)$$

where:

$$Y'_{sat} = 24.8 - 0.4259T_f + 0.003734T_f^2 - 0.00001328T_f^3 \quad \ldots\ldots\ldots \quad (10c)$$

$$T_f = \frac{T}{0.556} + 32.0 \quad \ldots\ldots\ldots \quad (10d)$$

$$Y_{sat} = Y'_{sat}\left\{\exp\left[-\frac{0.03419\ EL}{288.0 - 0.006496\ EL}\right]\right\} \quad (10e)$$

$K_{4,1}$ is the reaeration rate (per minute) at 20°C,

$K_{4,2}$ is the oxygen uptake due to algae respiration at night (mg/l/min),

Y_{sat} is the saturation concentration of dissolved oxygen at temperature T (°C) and elevation EL (M).

In order to incorporate point loads in the solution, a new stream reach is always defined at the location of a point source (also point diversions and tributary junctions).

Example Problem

Figure 4 is a diagram of the river system used in this simplified example. It consists of a main river with a major tributary, four point loads, five surveillance points, and six river reaches having different hydraulic characteristics. Each point load is discharging four water quality constitutents (pollutants): (1) Biochemical chemical oxygen demand, (2) ammonia, (3) total phosphorus, and (4) dissolved oxygen deficit. Each point load may be subjected to one of several levels of treatment, each level having different removal efficiencies for the various constituents (pollutants). Table 5 summarizes the system. Table 6 shows the physical characteristics of the system. Table 7 shows the water quality initial conditions and boundary conditions. The headwaters and diffuse lateral inflow into the system are contributing pollutants as well as the point loads. Although the optimization modeling technique developed in the previous section is capable of including the control of diffuse sources, no diffuse source control will be considered in this simplified example.

Stream water quality standards: Vectors (B_k)

The stream standards are sown in Table 8 along with the resulting $(b_j)_k$ vectors.

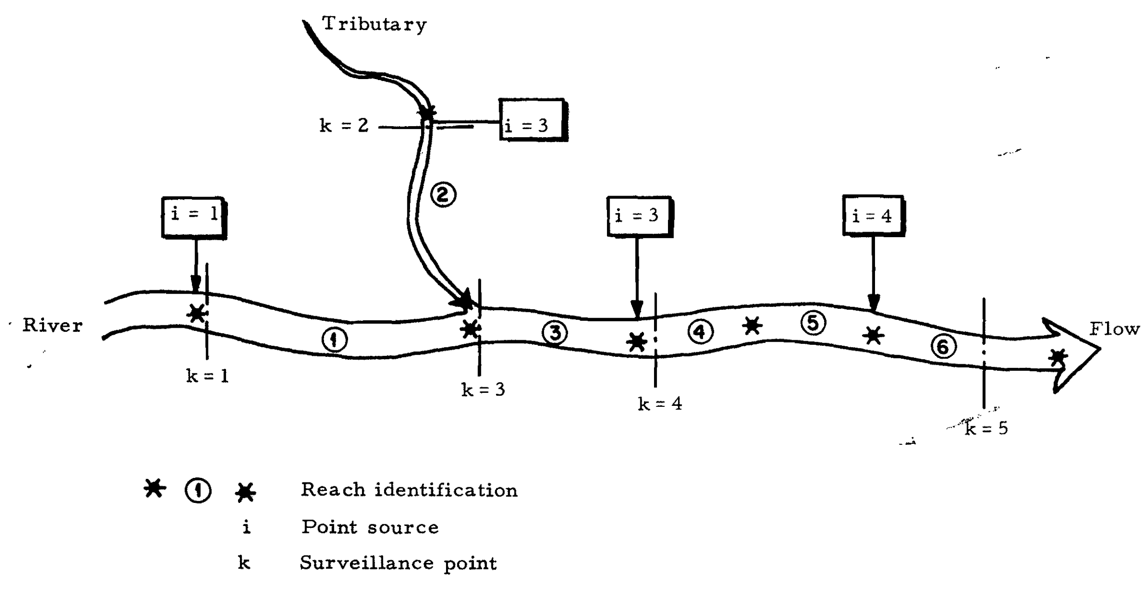

Figure 4. River system layout.

Table 5. Index identification.

Index	Description
Index identification:	
i	Index on point loads i = 1,2,3 ... I
j	Index on water quality constituent j=1,2,3...J
k	Index on surveillance points k = 1,2,3 ... K
ℓ	Index on treatment level ℓ = 1,2,3 ... L
	Total number of combinations = L^I
Water quality constituents:	
Index j	Description
1	Biochemical oxygen demand mg/l
2	Ammonia mg/l
3	Total phosphorus mg/l
4	Dissolved oxygen deficit (mg/l)
Treatment levels:	
Secondary treatment is currently in operation at all point discharges.	
Index ℓ	Description
1	No additional treatment (i.e., remain at secondary)
2	Ammonia removal; nitrification
3	Phosphorus removal; chemical precipitation in secondary
4	Phosphorus removal; tertiary precipitation
5	BOD and SS removal; tertiary sand filter
6	Ammonia and phosphorus removal; nitrification plus tertiary phosphorus
7	Reverse osmosis + aeration

Initial effluent conditions:
Vectors $(P_j^o)_i$

The effluent concentrations for initial conditions at each point load, i, are given in the vectors P_i^o (Table 9). These values correspond to the effluent concentrations shown in Table 6.

Effluent quality at various treatment levels: Matrices $(e_{j\ell})_i$

The effluent concentration of a constituent (row j) for a particular treatment level (column ℓ) is given in the matrices E_i for each point load i in Table 10. For example the ammonia concentration (j = 2) in the effluent at point load i = 4 would be 3 mg/l if nitrification (treatment level ℓ = 2) was installed.

Costs for various treatment levels at each load: $(c_\ell)_i$

Total present worth in thousands of dollars (based on a capital recovery factor of 0.08) is shown in Table 11 for each treatment level at each point load. For this example it was assumed that all plants had secondary treatment (treatment level 1) operating.

Costs were based on the following formulas where the design flow (Q) is expressed in millions of gallons per day (MGD).

Treatment level 2: (nitrification)

Capital cost = $(26.4 \times 10^3)Q^{0.87}$
Operation and maintenance (O and M) = $(6.2 \times 10^3)Q^{0.94}$
(Klemetson and Grenney, 1975).

Treatment level 3: (Chemical precipitation of phosphorus in the secondary system).

Capital and O and M = $5380 + 41{,}200\,Q + 4620\,Q^{0.594}$
(EPA, 1974).

Treatment level 4: (Tertiary precipitation of phosphorus)

Capital and O and M = $5380 + 41{,}400\,Q + 4620\,Q^{0.594} + 15{,}200\,Q^{0.865}$
(EPA, 1974).

Treatment level 5: (Tertiary sand filter)

Capital cost = $14{,}320\,Q^{0.660}$
O and M cost = $47{,}000\,Q^{0.636}$
(Klemetson and Grenney, 1975).

Treatment level 6: (Nitrification plus tertiary phosphorus precipitation).

The sum of 2 and 4.

Treatment level 7: (Reverse osmosis and aeration)

Capital and O and M = $99{,}700\,(2.87 - \log_{10}Q)\,Q$
(EPA, 1974).

Initial conditions in the river system at surveillance points: $(y_j)_k$

Concentrations in the river can be calculated at surveillance points for the initial boundary conditions

Table 6. River system layout and hydraulics.

Description	Location km	Hydraulic Coefficients						
		Input Flow (m^3/min)	Lateral Inflow for Reach (m^3/min/km)	River Flow (m^3/min)	Dilution Factor W	Velocity for Reach (m/min)	Ave. Depth $\bar{D}$	Ave. Area $\bar{A}$
Head of reach 1 (headwater)	200.	300.	1.0	300		16	3.1	18.8
Point discharge (i = 1)	200.	50.		350	0.14			
Surveillance point (k = 1)	200.			350				
Head of reach 2 (headwater)	220.	100.	0.20	100		22	1.2	4.5
Point discharge (i = 2)	220.	20.		120	0.17			
Surveillance point (k = 2)	220.			120				
Head of reach 3 (confluence)	170.		1.0	510		14	4.7	36.5
Surveillance point (k = 3)	170.			510				
Head of reach 4	130.		0.5	550		12	5.8	53.
Point discharge (i = 3)	130.	70.		620	0.11			
Surveillance point (k = 4)	130.			620				
Head of reach 5	110.		0.2	630		12	6.1	55.
Head of reach 6	90.		0.2	634		12	6.3	60.
Point discharge (i = 4)	90.	70.		704	0.10			
Surveillance point (k = 5)	70.			710				

Table 7. **River system water quality characterization.**

Description	Biochemical Oxygen Demand (mg/l)	Ammonia (mg/l)	Total Phosphorus (mg/l)	Dissolved Oxygen Deficit (mg/l)	Temperature (°C)	Elevation (m)	Y_{sat} (mg/l)
Headwater (Reach 1)	2.0	1.0	0.0	1.0	9	1000	10.1
Reach 1, lateral inflow	0.0	0.5	0.005	2.0			
Point discharge (i = 1)	30.0	25.0	20.0	Y_{sat}			
Headwater (Reach 2)	1.0	1.0	0.0	0.5	7	1000	10.7
Reach 2, lateral inflow	0.0	0.0	0.0	1.0			
Point discharge	20.0	25.0	15.0	Y_{sat}			
Reach 3, lateral inflow	1.5	0.3	0.80	2.0	10	900	10.2
Reach 4, lateral inflow	0.9	1.2	0.01	2.0	11	885	10.2
Point discharge (i = 3)	25.0	20.0	20.0	Y_{sat}			
Reach 5, lateral inflow	0.5	0.4	0.01	1.5	12	875	10.1
Reach 6, lateral inflow	0.8	1.0	0.05	1.0	12	865	10.1
Point discharge (i = 4)	30.0	15.0	15.0	Y_{sat}			

specified in Tables 5 and 6 by means of the water quality model. Table 12 contains the initial river water quality conditions at the surveillance points. Note that stream standards (Table 8) are exceeded in several instances.

Usually numerical computer techniques are required to solve water quality models. However, in order to better demonstrate the theory in this example, Equations 7 through 10 were selected so that exact solutions could be obtained. The solutions are contained in Appendix K.

Linking matrix: $(d_{mj})_{ik}$

The elements in the D_{ik} matrices link an incremental change in water quality at the load to a resulting incremental change in stream water quality at a surveillance point. The elements can be calculated mathematically by:

$$d_{jm} = \left.\frac{\partial Y_j}{\partial P_m}\right|_{P_m^o} \qquad \begin{array}{l} j = 1,2,\ldots J \\ m = 1,2,\ldots J \end{array} \quad . . (11)$$

Applying the operation of Equation 11 to the solutions of the model equations (Appendix K) results in the functions presented in Appendix L. Evaluation of these functions at the conditions of P^o results in the values given in Table 13. For example, the removal of 1 mg/l ammonia (j = 2) at load i = 1 will result in a 0.05 mg/l reduction in dissolved oxygen deficit at surveillance point k = 3.

These relationships are exact because the water quality model (Equations 7 through 10) are linear. This is not generally the case, and in Phase II applications, an iterative technique will be required between the optimization model and the simulation model.

Although the number and size of the matrices seems awkward in this example, it should be emphasized that each matrix shown here in detail is actually generated conveniently by a computer program and stored on disk for quick and efficient data handling. Because of large input data requirements a computer program (ASSEM) was written to generate the data in the proper format and store it on disk for use by the TEMPO program. A listing of ASSEM is shown in Appendix E.

Table 8. Water quality stream standards: B_k.

BOD standard: $Y_1 \leqslant 5.0$ mg/l
Ammonia Standard: $Y_2 \leqslant \infty$ (No ammonia standard)
Total Phosphorus: $Y_3 \leqslant 1.0$ (mg/l) at k = 1 and 3
$\leqslant 0.8$ (mg/l) at k = 2
$\leqslant 1.2$ (mg/l) at k = 4 and 5
Dissolved Oxygen: $Z_4 \geqslant 6.0$ (mg/l) at k = 1 and 2
$\geqslant 4.0$ (mg/l) at k = 3, 4, and 5

$$B_1 = \begin{bmatrix} 5.0 \\ \infty \\ 1.0 \\ Y_{sat} - 6.0 \end{bmatrix}_1 = \begin{bmatrix} 5.0 \\ \infty \\ 1.0 \\ 4.1 \end{bmatrix}_1$$

$$B_2 = \begin{bmatrix} 5.0 \\ \infty \\ 0.8 \\ Y_{sat} - 6.0 \end{bmatrix}_2 = \begin{bmatrix} 5.0 \\ \infty \\ 0.8 \\ 4.7 \end{bmatrix}_2$$

$$B_3 = \begin{bmatrix} 5.0 \\ \infty \\ 1.0 \\ Y_{sat} - 4.0 \end{bmatrix}_3 = \begin{bmatrix} 5.0 \\ \infty \\ 1.0 \\ 6.2 \end{bmatrix}_3$$

$$B_4 = \begin{bmatrix} 5.0 \\ \infty \\ 1.2 \\ Y_{sat} - 4.0 \end{bmatrix}_4 = \begin{bmatrix} 5.0 \\ \infty \\ 1.2 \\ 6.2 \end{bmatrix}_4$$

$$B_5 = \begin{bmatrix} 5.0 \\ \infty \\ 1.2 \\ Y_{sat} - 4.0 \end{bmatrix}_5 = \begin{bmatrix} 5.0 \\ \infty \\ 1.2 \\ 6.1 \end{bmatrix}_5$$

Table 9. Initial effluent conditions: P_i.

$$P_1^o = \begin{bmatrix} BOD & 30 \\ NH_4 & 25 \\ P & 20 \\ DOD & Y_{sat} \end{bmatrix}_1 = \begin{bmatrix} 30 \\ 25 \\ 20 \\ 10.1 \end{bmatrix}_1$$

$$P_2^o = \begin{bmatrix} BOD & 20 \\ NH_4 & 25 \\ P & 15 \\ DOD & Y_{sat} \end{bmatrix}_2 = \begin{bmatrix} 20 \\ 25 \\ 15 \\ 10.7 \end{bmatrix}_2$$

$$P_3^o = \begin{bmatrix} BOD & 25 \\ NH_4 & 20 \\ P & 20 \\ DOD & Y_{sat} \end{bmatrix}_3 = \begin{bmatrix} 25 \\ 20 \\ 20 \\ 10.2 \end{bmatrix}_3$$

$$P_4^o = \begin{bmatrix} BOD & 30 \\ NH_4 & 15 \\ P & 15 \\ DOD & Y_{sat} \end{bmatrix}_4 = \begin{bmatrix} 30 \\ 15 \\ 15 \\ 10.1 \end{bmatrix}_4$$

Table 10. Effluent matrix: E_i.

		1	2	3	4	5	6	7
E_1 =	BOD	30	25	20	5	5	5	0
	NH_4	25	5	20	20	20	4	0
	P	20	15	2	0.5	10	0.5	0
	DOD	10	10	10	10	10	10	0
E_2 =	BOD	20	15	10	5	5	5	0
	NH_4	25	5	20	20	20	4	0
	P	15	10	2	0.3	8	0.3	0
	DOD	11	11	11	11	11	11	0
E_3 =	BOD	25	20	15	5	5	5	0
	NH_4	20	3	10	10	10	2	0
	P	20	15	2	0.5	10	0.5	0
	DOD	10	10	10	10	10	10	0
E_4 =	BOD	30	25	10	5	5	5	0
	NH_4	15	3	10	10	10	2	0
	P	15	10	2	0.3	10	0.3	0
	DOD	10	10	10	10	10	10	0

Table 11. Cost per year in thousands of 1974 dollars (capital recovery factor = 0.08) for each treatment level at each point load.

Load Point (i)	Flow Q(i) MGD	Treatment Level (ℓ)						
		1	2	3	4	5	6	7
1	19	0	441	815	1007	406	1448	3014
2	7.6	0	196	334	422	225	618	1507
3	26.6	0	594	1134	1391	504	1985	3832
4	26.6	0	594	1134	1391	504	1985	3832

Table 12. Initial river conditions at surveillance points: Y_k.

$$Y_1^o = \begin{matrix} BOD \\ NH_4 \\ P \\ DOD \end{matrix} \begin{bmatrix} 5.9 \\ 4.4 \\ 2.8 \\ 2.3 \end{bmatrix}$$

$$Y_2^o = \begin{matrix} BOD \\ NH_4 \\ P \\ DOD \end{matrix} \begin{bmatrix} 4.2 \\ 5.1 \\ 2.6 \\ 2.2 \end{bmatrix}$$

$$Y_3^o = \begin{matrix} BOD \\ NH_4 \\ P \\ DOD \end{matrix} \begin{bmatrix} 3.8 \\ 2.9 \\ 2.5 \\ 7.3 \end{bmatrix}$$

$$Y_4^o = \begin{matrix} BOD \\ NH_4 \\ P \\ DOD \end{matrix} \begin{bmatrix} 4.5 \\ 3.5 \\ 4.3 \\ 10.0 \end{bmatrix}$$

$$Y_5^o = \begin{matrix} BOD \\ NH_4 \\ P \\ DOD \end{matrix} \begin{bmatrix} 3.1 \\ 1.7 \\ 5.2 \\ 9.8 \end{bmatrix}$$

Table 13. Values in the linking matrix.

Matrix		BOD	NH_4	P	DOD
$D_{1,1}$ =	BOD	0.14	0	0	0
	NH_4	0	0.14	0	0
	P	0	0	0.14	0
	DOD	0	0	0	0.14
$D_{1,2}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0	0
	DOD	0	0	0	0
$D_{1,3}$ =	BOD	0.06	0	0	0
	NH_4	0	0.06	0	0
	P	0	0	0.09	0
	DOD	0.01	0.05	0.14	0.02
$D_{1,4}$ =	BOD	0.03	0	0	0
	NH_4	0	0.03	0	0
	P	0	0	0.09	0
	DOD	0	0	0.19	0
$D_{1,5}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0.09	0
	DOD	0	0	0.25	0
$D_{2,1}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0	0
	DOD	0	0	0	0
$D_{2,2}$ =	BOD	0.17	0	0	0
	NH_4	0	0.17	0	0
	P	0	0	0.17	0
	DOD	0	0	0	0.17
$D_{2,3}$ =	BOD	0.03	0	0	0
	NH_4	0	0.03	0	0
	P	0	0	0.04	0
	DOD	0	0.03	0.04	0

Table 13. Continued.

Matrix		BOD	NH_4	P	DOD
$D_{2,4}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0.04	0
	DOD	0	0	0.05	0
$D_{2,5}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0.03	0
	DOD	0	0	0.06	0
$D_{3,1}$, $D_{3,2}$, $D_{3,3}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0	0
	DOD	0	0	0	0
$D_{3,4}$ =	BOD	0.11	0	0	0
	NH_4	0	0.11	0	0
	P	0	0	0.11	0
	DOD	0	0	0	0.11
$D_{3,5}$ =	BOD	0.04	0	0	0
	NH_4	0	0.04	0	0
	P	0	0	0.11	0
	DOD	0.01	0.04	0.14	0
$D_{4,1}$, $D_{4,2}$, $D_{4,3}$, $D_{4,4}$ =	BOD	0	0	0	0
	NH_4	0	0	0	0
	P	0	0	0	0
	DOD	0	0	0	0
$D_{4,5}$ =	BOD	0.07	0	0	0
	NH_4	0	0.07	0	0
	P	0	0	0.10	0
	DOD	0.02	0.06	0.11	0.02

CHAPTER V

COMPUTATIONAL EXPERIENCE WITH ALGORITHMS

MXINT—Application to Water Supply Model

As defined previously, the basic version of the Cache Valley Water Supply Model consisted of 258 variables of which 54 were integer and 204 were continuous. One objective function and 278 constraint equations were used to define the model.

To determine the minimum pipe size (size A) for Interzonal transfers the assumption that a zone's demand was to be totally supplied by zonal transfer was made. Standard pipe flow equations were then used to determine a normal pipe diameter for this zonal transfer. The next larger standard pipe diameter (about double capacity) was used for those zones with two pipe size options (e.g., if size A = 6" dia. then size B = 8" dia.). The rationale for this lower size criteria was that it won't be efficient to build a pipeline unless a substantial proportion of the zone's demand is supplied through it (at least during peak days). The larger size selection assumes that more than one zone may demand flow through the pipe.

Upper bounds were placed on all variables as follows:

A. Integer Variables
   1. Future wells/Zone ⩽ 1. Except Zone 18 which required 2 wells to avoid an infeasible solution.
   2. Future springs/Zone ⩽ 1. Only Zone 2 had the potential for a future spring.
   3. Future treatment facilities/Zone ⩽ 1. Only Zone 11 had the potential for future treatment facilities.
   4. Zonal transfer facilities (pipe)/Zone ⩽ 1.

B. Continuous Variables
   1. Flow from existing wells and springs was limited to the maximum capacity of that facility or the water rights filed for at that facility or which ever was least if both applied.
   2. Flow from future wells, springs, and treatment plants was limited to design capacity determined from past studies with the assumption that water rights would be granted.
   3. Zonal transfers were limited to the maximum capacity (both seasons and peak day) of the largest alternate pipeline.

Original model (year 2000 real world supply and demand levels)

Two computer runs were made of the model with the previously described upper bounds and with year 2000 projections of supply and demand. Run No. 1 was set up as Batchmode and let run to the first and second integer solutions. The resulting branch node system was saved for future restart. Restart was made via interactive (timeshare) mode and the TEMPO-MXINT algorithm allowed to make a complete search of the branch nodes. The system determined CUTOFF[1] from the last best integer solution. The criteria for improvement of the objective function was anything greater than zero for this run. The first integer solution was $183,665.97 and the optimum (16th) solution found was $174,148.24 (an improvement of $9,517.73 or about 5 percent). The total CPU time for the run was 43.2 minutes.

Run No. 2 was run indentical to Run No. 1 except that after each integer solution the cutoff was manually set to allow for about a 1 percent improvement in the last best integer solution. The same optimum integer solution was reached, however, only 5 integer solutions were found and a reduction of CPU time of about 24 percent was realized. Total CPU time for this run was 37.8 minutes. The complete solution is given in Appendix C.

Increased upper bounds on integer variables (Revision No. 1)

The original model was revised to test how the number of potential active integer variables effects the CPU run time.

[1]Projected integer solutions with an objective function value greater than cutoff are discarded.

Revision No. 1 changes the original model in the following ways:

A. Integer variable upper bounds for future wells in zones were increased from one to three except for Zone 18 which was increased from two to three. A change of one in the upper bound of an integer variable is essentially equivalent to adding one more integer variable to the problem. The number of integer variables (in the 0, 1 variable sense) in Revision No. 1 was raised to 82 (e.g., there were 55 defined integer variables, and 14 zones have future wells, therefore Revision No. 1 total 0, 1 variables = 55 + 2 x 13 + 1 = 82).

Revision No. 1 was run identical to Run No. 2 of the original model with cutoff being manually set. The optimum solution was reached at the third integer solution with the same value as the original model ($174,148.24). Total CPU time for this revision was 45.7 minutes.

Model decomposition (Revisions No. 2 and No. 3)

The original model was then split into a northern half (Revision No. 2) and a southern half (Revision No. 3) to again test the effect of number of integer variables on run time.

Revision No. 2 included Zones 1, 2, and 13 through 23, or 13 zones. The data were identical to that of the original model for the noted zones. This model consisted of 136 variables of which 27 were integer and 109 were continuous. One objective function and 150 constraint equations were used to define the model. Revision No. 2 was run identical to Run No. 2 of the original model with cutoff being manually set. The optimum solution of $140,494.78 was reached at the third integer solution. The Revision No. 2 solution was identical to that comparitive portion of the original model solution. Total CPU time for Revision No. 2 was 1.3 minutes.

Revision No. 3 included Zones 1, 2, and 3 through 12, or 12 zones. Zones 1 and 2 were used again to allow for Revision No. 2 and No. 3 to be manually interfaced as a comparison with the original model solution. Also Zone 2 is a possible major supply zone for both the north and south areas and should be included in both. The data were identical to that of the original model for the noted zones. This model consisted of 145 variables of which 30 were integer and 115 were continuous. One objective function and 148 constraint equations were used to define the model.

Revision No. 3 was run identical to Run No. 2 of the original model with cutoff being manually set. The optimum solution of $84,876.10 was reached at the third integer solution. The Revision No. 3 solution was identical to that comparative portion of the original model solution. Total CPU time for Revision No. 3 was 1.2 minutes.

When Revisions No. 2 and No. 3 are put together and costs manually adjusted to eliminate overlap at Zone 2, the total objective function costs and the activity levels of all decision variables were identical to the original model optimal solution. Complete solutions are given in Appendixes C and D.

Increased demands and supply capacities (Revision No. 4)

Revision No. 4 was made in order to attempt to define the upper limits of the feasible MXINT computational capability. Increasing the model size by defining new variables would require extensive restructuring of the model. However, much the same effect can be accomplished by simply increasing the number of possible integer solutions. This approach was used in Revision No. 1 (increasing the upper bounds on integer variables) and a significant increase computation load resulted. However, the demands were not increased concurrently with the potential supply so that most of the increase in number of potential solutions was apparently dismissed by means of implicit enumeration.

In order to devise a more difficult problem for the algorithm, Revision No. 4 includes an extensive increase in demand as well as potential supply and commensurate pipe sizes so that many more probable active integer variable values are brought into the problem. These revisions to the capacities and demands are as follows:

A. Zonal season demands and zonal peak day demands were doubled.
B. Integer variable upper bounds for future wells were raised to 3 except Zone 18 where they were raised to 6.
C. The minimum and maximum zonal transfer pipe sizes were increased in selected zones to allow for additional transfer of water.
D. Continuous variable upper bounds for the potential flow from future wells and zonal transfers were increased to reflect the above changes.

The above changes increased the potential effective number of active integer variables (equivalent number of 0, 1 variables) from 55 to 85.

Revision No. 4 Run No. 1 was run identical to Run No. 2 of the original model with cutoff being manually set. The branch system expanded much faster than any other run with the algorithm reaching 500 branch nodes after approximately 1.5 hours of run time. The algorithm cannot maintain more than 500 branch nodes and discards the worst 10 percent of the projected solutions. However, the optimum solution may be in this set of discarded branch nodes and therefore, a procedure for avoiding this problem was undertaken. The best integer solution found up to the point of branch node discarding was $484,457.27.

Revision No. 4 Run No. 2 was run to test the procedure for avoiding the discarding of branch nodes and to improve run time for this large model.

The following procedure was used and run as a batch job:

A. Solve problem to first integer solution.
B. Save branch node system immediately after integer solution.
C. Set cutoff at 96 percent of last integer solution or a 4 percent improvement for next integer solution (compared to 1 percent in previous runs).
D. Restart search of branch nodes for the next best integer solution. When integer solution is found return to Step B.
E. Repeat Steps B-D until branch nodes are exhausted.
F. Restart at last integer solution but set cutoff at the last percentage + 1 percent (next step would be 97 percent or 3 percent improvement) go to Step D and loop as in Step E except with the new percent improvement.
G. Keep making the percent improvement smaller until the algorithm completes a search at 99 percent or 1 percent improvement. When 1 percent is completed assume this integer solution is optimal and output.

The above technique was used for Run No. 2 and found to be successful.

The optimum integer solution found was $483,101.97 and the total CPU time was 160 minutes. The 500 node limit was never reached during this run due to the larger required objective improvement and smaller number of active nodes at any point in time.

Elimination of upper bounds of continuous variables

Revision No. 5 was created to test the relationship between upper bounds on continuous variables and CPU time. Revision No. 5 is identical to Revision No. 4 except all upper bounds on the continuous variables were removed (allowing the bounds to go to infinity). This increases the size of the solution space, thereby requiring additional computational effort.

Revision No. 5 was run exactly as Revision No. 4 Run No. 2 had been run using the techniques developed in Revision No. 4.

The optimum integer solution found was identical to Revision No. 4. However, the CPU time was greatly increased. The CPU time was 305 minutes for Revision No. 5 compared to 160 minutes for Revision No. 4 or an increase of about 91 percent in CPU time due to removal of continuous variable upper bounds.

GMINT—Application to Water Supply Model

The input data for the model form referred to previously as the original model was converted to the format required by GMINT. Since this is a proprietary algorithm the data deck was mailed to the owners of the code and runs were made by them (Arthur M. Geoffrion and Richard D. McBride) on an IBM 360 model 158 computer at the University of Southern California.

A solution defined as optimal by the code was produced after 1.7 minutes with an objective function value of $178,074. The algorithm tolerance indicated that this was within $1,000 of the true optimal solution; however, the previous algorithm, MXINT, produced a solution of $174,148 which is $3,926 or 2.3 percent better than the GMINT solution. The essential difference between the two solutions was that GMINT had selected the larger pipe size between two pairs of service zones (16 to 23 and 18 to 19) in which the smaller sizes would have met all of the constraints and saved $3,900.

The problem was rerun on GMINT to determine why the better solution was missed. Analysis of the computation procedure revealed that the apparent true optimal solution was implicitly deleted by the Gomory mixed integer cuts prior to obtaining the initial LP solution. An attempted solution without the Gomory cuts became unstable and appeared headed for a large computation effort and therefore was aborted.

The GMINT code apparently works well on problems in which the initial LP solution is only a few percent smaller than the optimal IP solution. This is apparently not a serious limitation because most models can be modified to decrease this difference by adding constraints to force the LP solution closer to the IP solution. Such constraints are redundant to the total model including IP constraints but are not redundant to the LP problem while integer constraints are being ignored. Examples of this type of model revision for computational efficiency were mentioned in the literature review section. Such model revisions were not accomplished for the water supply model Phase I tests because of time and budget limitations. This concept should be pursued, however, in Phase II of the research. See Appendix I for the GMINT solution.

FMPS—Application to Water Supply Model

Because of the large computation times required for the water quality model solutions on the UNIVAC algorithm (to be discussed in a later section), no attempt was made to run the full water supply model with this algorithm. However, since the nature of the two models is very different, it appeared worthwhile to run one half of the decomposed model (referred to in the previous MXINT discussion as model Revision No. 3).

Initial runs on the decomposed model using the first node selection option (global optimum) required about 10 minutes to achieve the optimal solution. This time was decreased, however, by almost an order of magnitude (1.5 minutes) by using the second node selection alternate and by reordering the integer variables according to decreasing unit cost. Apparently the reordering of variables was the prime factor in improving computation efficiency. The improved run times are slightly higher than those achieved by MXINT. This was surprising in view of the fact that the UNIVAC computer is usually much faster than the Burroughs on identical programs. A principal reason for the longer than expected run times on the UNIVAC may be that the FMPS code uses double precision (72 bits) while MXINT uses single precision (48 bits). The only way to compare the algorithms themselves would be to run them both on the same computer (which is obviously not feasible).

MXINT—Application to the Basin Planning IP Model

Summary of model description

The smaller of the two example problems (described in Chapter IV) consisted of five surveillance points along the stream; four wastewater contaminants of interest; and four point sources of impaired water quality, with seven possible treatment process alternatives (one of which must be chosen at each source). The objective function is structured to seek the minimum total cost of treatment. There are a total of 28 decision variables corresponding to the four sources and seven treatment levels. The model has 24 total constraints. Restrictions on each of the four contaminants at each of the five surveillance points added up to a total of 20 constraints. The technique used to select only one of the seven possible treatment alternatives at a given source, required one additional constraint for each source, thus producing four constraints for the four discharge points. This problem will be referred to as Problem I hereafter.

An expanded model of the problem with 105 variables and 39 constraints was developed to further test the algorithm. This example had fifteen point sources of wastewater, four pollutants and six surveillance points. With seven alternative treatment processes available at each source, the problem became one of choosing a process at each source such that the total cost is a minimum. The quality standards imposed on each pollutant at each of the surveillance points produced 24 constraints. The selection of only one of the seven treatment processes at each source is accomplished by putting in one constraint corresponding to each source. These 15 additional rows resulted in 39 total constraints. This model will be designated as Problem II henceforth.

Problem I

The small model required 0.14 minute (8.4 seconds) of CPU time for the MXINT optimal solution (objective function = $2,291.7). The solution included two integer variables with slightly non-integer values (.99231 and .00769). These result from the default tolerance on integer approximations of ± 1 percent. The values given are within this tolerance, however, they result in a slight nonconformance to water quality standards and therefore other runs were made with this tolerance supposedly lower but the solution was unchanged. This is apparently a minor system problem which has been referred to Burroughs representatives. The solution is given in Appendix F.

Problem II

The large model in original form required 2.9 minutes to produce an optimal solution (objective function = $9,950). In order to further test the algorithm's capability without extensive model restructuring, Problem II was rerun after arbitrarily increasing (strengthening) the water quality standards constraints. This had the effect of increasing substantially the number of possible "good" solutions which required explicit enumeration (even though the total number of variables remained at 105). The run time for this more difficult problem was 3.8 minutes (32 percent increase in CPU time). The solution is given in Appendix G.

AIP–Application to the Basin Planning IP Model

Problem I

The small model required 0.21 minutes (12.8 seconds) to produce an optimal solution (objective function = $2,371). The solution is slightly different than the MXINT solution because all of the variable activity levels are precisely integers. The slight constraint problem resulting from the tolerance limit in MXINT caused the two solutions to differ by one treatment level at one location (see Appendix J).

Problem II

Attempts to solve the large model with the AIP algorithm failed. Some array dimension problems were encountered. The computation effort required for the small model appeared to be considerably greater than for the other two algorithms. The work necessary to modify the algorithm therefore, did not appear to be justified.

FMPS-MIP–Application to the Basin Planning IP Model

Problem I

The small model required 0.099 minutes (using the UNIVAC 1108) to produce the same optimal solution (objective function = $2,371) as the AIP algorithm (see Appendix H).

Problem II

The original version of the large model required 10.1 minutes to produce the same optimal solution as MXINT (objective function = $9,950). This time could likely be improved substantially by reordering the integer variables as described previously for the water supply problem. This was not done, however, because of contract time and budget constraints.

Use of Interactive Mode

Much of the work with the MXINT algorithm was accomplished while in interactive mode using a Texas Instruments Model 725 portable data terminal which communicated with the Burroughs B6700 computer over a dedicated telephone line. Several advantages in regard to integer programming which derive from the interactive capability became apparent during this research. The interactive mode was particularly valuable during preliminary runs while the operator was becoming familar with the algorithm control language and with the order of magnitude of run times to expect.

The advantages of an interactive mode in controlling the operation of a mixed integer algorithm include: (1) The potential to decrease CPU run time. (2) Familiarization with a new model. (3) Ability to revise data with respect to infeasibilities and equation constants in order to restart the problem quickly. (4) Assurance of a global optimum solution. (5) Develop control language techniques for a model to enable later runs to be made unattended (batch). (6) Ability to interact with other algorithms where the solution to one problem may be the data for another.

1. Run time may be decreased in several ways. By setting or revising algorithm tolerances for integer variables, (if an activity of an integer variable is within a set tolerance of an integer value it is assumed to be an integer) solutions to the degree of accuracy desired can be obtained, the smaller the tolerance the longer the run time. Cutoff values can be adjusted to substantially reduce run times by skipping interim solutions that do not give acceptable solution value improvements and also reduce the number of branch nodes that must be carried, thereby reducing memory requirements. If an optimal feasible solution rather than the global optimal feasible solution is acceptable one can terminate the run when a satisfactory solution is obtained thereby reducing run time.

2. One may not be familiar with the solution space of a new model and data refinements may have to be made. The interactive mode allows one to analyze the output as the run is going and make these refinements prior to final model formulation.

3. If infeasibilities occur or data errors are noted at the beginning of a run they can be modified and the run immediately restarted without waiting for long turnaround times from batch operations.

4. For many problems a global optimum is a necessity and must be assured. Since the interactive mode allows one to monitor and guide the solution direction a global optimum can be guaranteed.

5. Since the results of a control language change on the interactive mode can be seen almost immediately many changes can be tried to discover

the best sequence of the control language to suit this particular model or family of models. Future runs can then be made by batch mode if desired to take advantage of special late night or long turnaround time rates.

6. The interactive mode offers the possibility, not yet explored by this study, of the results of one model being used as the data of another model. Solutions of the data model can be input to the second model with the results of the second model used to modify the data model.

Analysis and Conclusions

MXINT algorithm/water supply model

The full size problem was run only on MXINT and GMINT. The only algorithm which produced what is apparently the global optimal solution was the Burroughs algorithm, MXINT. Run times for various versions of the model have been described in a previous section and are summarized in Figure 5. The real world version of the model required 38 minutes of CPU time on the Burroughs computer. The current cost of both the Burroughs and UNIVAC computers at standard priority is \$.08 per second. At this rate three 38 minute runs at different planning horizons would cost \$547 plus IO and other miscellaneous charges. However, a very low cost rate (10 percent of normal priority rates) is available on the USU Burroughs computer for evening unattended runs on large problems of this sort. Therefore, the computer rates were not at all excessive for runs made during this first phase, and the same rates should be available for future applications of this methodology. The MXINT algorithm has a format that is convenient to work with, and variable and row names provide for rapid error searches and proof reading. This format (the TEMPO MPS package format) is identical except for upper bound definitions to the UNIVAC FMPS format and therefore provides for eash conversion of a model from one computer to the other.

The MXINT capability of handling integer values greater than unity is a very desirable feature in relation to future water supply model versions which may have several variables with upper bounds of 3 or 4.

The MXINT algorithm appeared to solve the real world version of the model (55 equivalent 0, 1 variables) without difficulty. However, when the number of 0, 1 variables was increased to 85 (about 300 total variables) the number of nodes exceeded the capability of the code (500) at the standard objective improvement tolerance. This problem was overcome by varying the objective tolerance and thereby decreasing the number of nodes to be examined. This revised model version (which is larger than any version anticipated in follow-on real world applications) would appear to be approaching the upper limit of the algorithm/computer combination on which it was run.

If some unforeseen future addition to the model requires a substantial number of integer variables, the problem could still be solved successfully by decomposing the model as was done in Revisions 2 and 3. The total computation time for the decomposed model halves was an order of magnitude less than for the full model (see Figure 5). It is therefore significant that after easy manual adjustment for duplication in Zone 2 the combined solution for the model halves precisely equalled the full model optimal solution. This provides a viable alternate approach for situtations where computational effort for the entire model may exceed the computer cost budget.

GMINT algorithm/water supply model

As discussed previously the GMINT algorithm on an IBM 370 Model 158 computer produced a rapid problem solution, but because of round-off type errors introduced by 15 preliminary Gomory cuts, the true optimal solution was excluded from the branch-and-bound solution space. The authors of this algorithm have designed it for particularly efficient use on models which have had special redundant constraints added which force the LP solution to approach as closely as possible, the IP solution. Because of severe time limitations in Phase I of this research such model structure revisions were not incorporated into the GMINT model. The LP solution objective function (without the Gomory cuts) was about \$129,000 which was 74 percent of the optimal IP solution. This large difference apparently represented a difficult computational problem for the GMINT code. GMINT's performance on the water supply model in its present form was disappointing; however it appears to be a fast code for problems which have been structured to take advantage of its strong points. The GMINT authors believe that this model could be restructured so as to run efficiently on their code. It would appear to be worthwhile to do this for future applications of the model. A rational comparison would appear to involve comparison of reduced computer costs versus personnel time required to restructure the model.

Basin planning model

The all integer water model appears to represent an easier computational problem than the

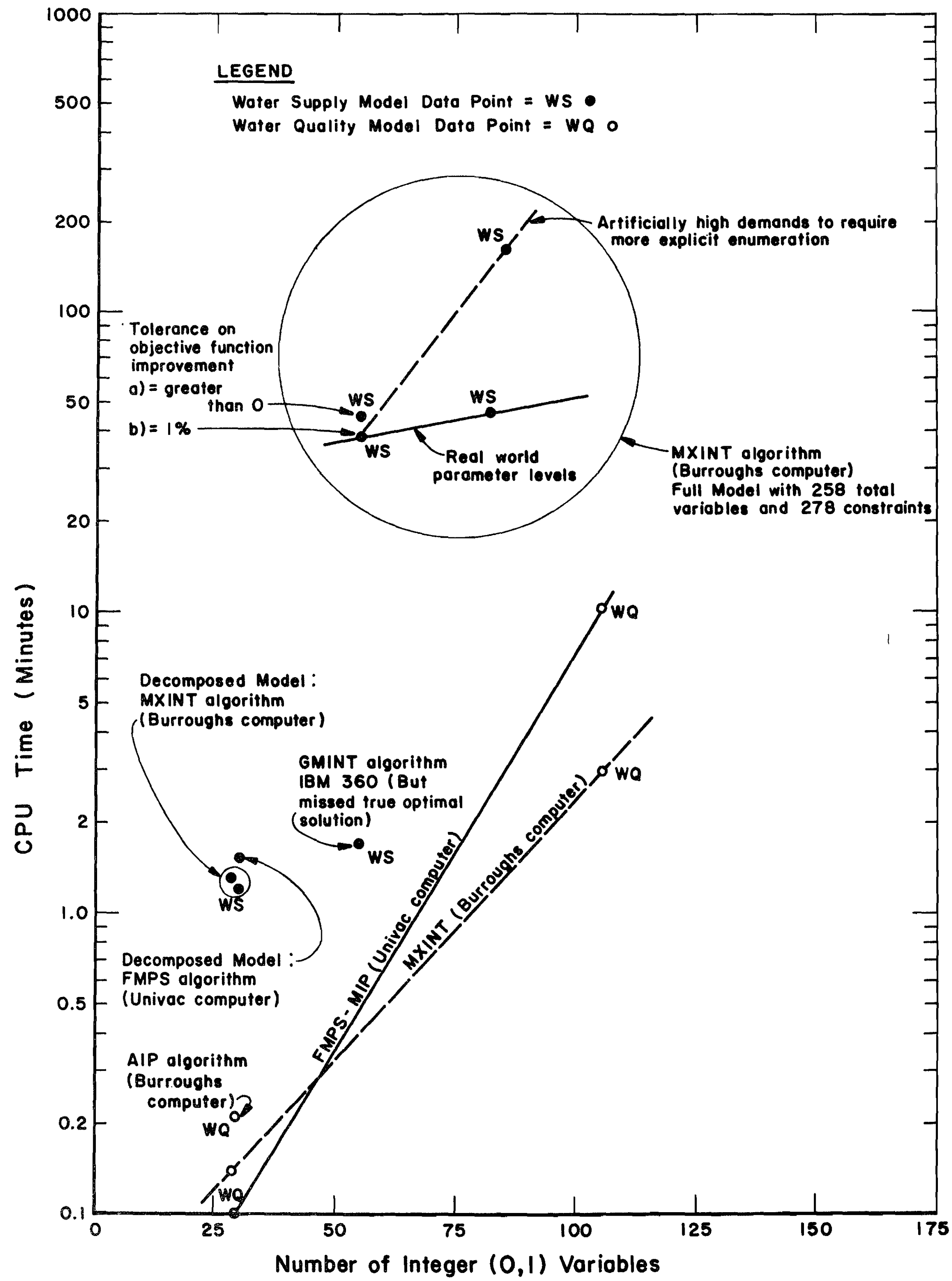

Figure 5. Algorithm computational experience summary.

mixed integer water supply model. The larger quality problem has more integer variables than the supply problem (105 compared to 55) but fewer total variables (105 compared to 258) and a much smaller number of constraints (39 compared to 278). As shown in Figure 5 the MXINT algorithm produced the most efficient solution; however, the FMPS solution could likely be improved by incorporating techniques described in the discussion of the water supply model. This model was not run on GMINT nor on AIP.

Conclusions

1. The algorithm tests described herein have clearly demonstrated that modern integer programming codes exist which are capable of optimizing both regional water supply and water quality planning models at reasonable costs.

2. The MXINT algorithm included in the TEMPO mathematical programming package on the Burroughs 6700 computer at Utah State University appears to be the best of the four algorithms tested for both models in their present form.

3. The GMINT algorithm being marketed by Geoffrion and McBride should be evaluated further after restructuring the water supply model to take better advantage of this code's special capabilities.

4. Proper use of the UNIVAC 1108 FMPS Package can apparently produce optimal solutions with computation efforts only slightly greater than those achieved by the Burroughs TEMPO package. However, to date, such comparable run times have been verified only for the decomposed version of the water supply model, not for either full sized model.

5. The all integer algorithm tested was less efficient than the mixed integer algorithms on small versions of the all integer (water quality problem) and was not capable of solving the larger version.

6. Easy decomposition of the water supply model is a viable alternative for IP solutions of problems which approach the size of the problems solved herein.

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APPENDIXES

Appendix A

Description of MXINT Algorithm from Burroughs TEMPO Manual

Algorithm Description from Sperry Manual

Starting off from an optimal solution to the linear programming problem (integrality constraints are ignored) the algorithm proceeds as follows:

(1) Choose one of the integer variables violating the integrality requirement. Suppose it has a value of 2.4, an upper bound of 6 and a lower bound of 1. Construct two subproblems (both are linear programming problems). In the first subproblem this variable has bounds (1, 2) and in the second (3, 6). Estimate which subproblem (called a branch) is more likely to lead to a "good" integer solution. Store the less "good" problem on a work file. It may be necessary to return to it later.

(2) Optimize the chosen branch (solve the linear problem with the new bounds). There are four possibilities:

  (a) The subproblem is infeasible.
  (b) the objective is above a cutoff (either user inputted or determined by a previously found integer solution).
  (c) The subproblem is optimal and is the best integer solution found to date. A new cutoff is now available (the solution value).
  (d) The subproblem is optimal and some integer activities violate the integrality constraints.

  In cases (a), (b) and (c) proceed to Step 3. In case (d) return to Step (1).

(3) Pick the "best" available branch on the work file and go to Step (2). Only branches with projections below the current cutoff are viable (assuming minimization). If there are no viable branches on the file TERMINATE. The best integer solution found to date (if found) is the optimal solution to the integer programming problem.

The efficiency of the algorithm is strongly influenced by:

(a) The branching strategy used in Step (1). In general, it is best to branch on the important variables (those leading to the greatest degradation in the objective) before branching on the less important variables.

(b) The backtrack strategy used in Step (3). There are conflicting ends in the strategy used to return to unexplored nodes. It is desirable to return to nodes with the best projected objective. However, strong adherence to this end will often lead to a large number of unexplored nodes. As a compromise, return to *deep* nodes with "good" projected objectives. (See the parameter ZBACK.)

The optimization procedure used by TEMPO (Step 2) is a parametric on the bound, and algorithm akin to PARRHS and DUAL. A projected objective is hence available at each parametric iteration so that a branch can be dropped as soon as its projection is above the cutoff.

MXINT uses the following parameters and has no available modifiers.

(a) ZBIOBJ–if the solution to the integer model is known to exceed some value, ZBIOBJ should be set to that value. This reduces the size of the tree structure for the problem. ZBIOBJ is updated as MXINT finds better integer solutions.

(b) ZTOLIN–if an integer variable and integer value differ by less than ZTOLIN, the variable is assumed to be integer. Standard value is .01.

(c) ZTOLOB–if an integer solution differs from the solution to the continuous problem by less than ZTOLOB times the continuous solution, MXINT terminates with the current solution as the best solution. Standard value is .01.

(d) ZTOLIM–after an integer solution is obtained, better integer solutions which differ by less than ZTOLIM times the difference between the continuous solution and ZBIOBJ are skipped. Standard value is .01.

Appendix B

MXINT Solution to Water Supply Model—Original Version

ORIGINAL MODEL
SOLUTION

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
1	OBJECT	BS	174148.23577	-174148.23577	NONE	NONE	1.00000
2	D1-1	LL	1148.00000	.	1148.00000	NONE	-35.26000
3	D1-2	LL	1312.00000	.	1312.00000	NONE	-35.60000
4	D2-1	LL	364.00000	.	364.00000	NONE	-34.10000
5	D2-2	LL	486.00000	.	486.00000	NONE	-34.10000
6	D3-1	LL	241.00000	.	241.00000	NONE	-45.27000
7	D3-2	LL	161.00000	.	161.00000	NONE	-18.90000
8	D4-1	LL	248.00000	.	248.00000	NONE	-59.42000
9	D4-2	LL	248.00000	.	248.00000	NONE	-6.60000
10	D5-1	LL	51.00000	.	51.00000	NONE	-68.78000
11	D5-2	LL	60.00000	.	60.00000	NONE	-18.60000
12	D6-1	LL	86.00000	.	86.00000	NONE	-9.18000
13	D6-2	LL	98.00000	.	98.00000	NONE	-10.80000
14	D7-1	LL	10.00000	.	10.00000	NONE	-18.91000
15	D7-2	LL	19.00000	.	19.00000	NONE	-18.91000
16	D8-1	LL	124.00000	.	124.00000	NONE	-3.00000
17	D8-2	LL	150.00000	.	150.00000	NONE	-3.00000
18	D9-1	LL	22.00000	.	22.00000	NONE	-18.91000
19	D9-2	LL	26.00000	.	26.00000	NONE	-18.91000
20	D10-1	LL	7.00000	.	7.00000	NONE	-18.91000
21	D10-2	LL	8.00000	.	8.00000	NONE	-18.91000
22	D11-1	LL	79.00000	.	79.00000	NONE	-3.00000
23	D11-2	LL	94.00000	.	94.00000	NONE	-3.00000
24	D12-1	LL	53.00000	.	53.00000	NONE	-54.43000
25	D12-2	LL	66.00000	.	66.00000	NONE	-6.60000
26	D13-1	LL	304.00000	.	304.00000	NONE	-45.92000
27	D13-2	LL	364.00000	.	364.00000	NONE	-45.92000
28	D14-1	LL	87.00000	.	87.00000	NONE	-25.66000
29	D14-2	LL	115.00000	.	115.00000	NONE	-25.66000
30	D15-1	LL	402.00000	.	402.00000	NONE	-25.66000
31	D15-2	LL	290.00000	.	290.00000	NONE	-25.66000
32	D16-1	LL	61.00000	.	61.00000	NONE	-63.33000
33	D16-2	LL	108.00000	.	108.00000	NONE	-65.51000
34	D17-1	LL	7.00000	.	7.00000	NONE	-203.33000
35	D17-2	LL	9.00000	.	9.00000	NONE	-240.54000
36	D18-1	LL	138.00000	.	138.00000	NONE	-29.04000
37	D18-2	LL	91.00000	.	91.00000	NONE	-6.60000
38	D19-1	LL	206.00000	.	206.00000	NONE	-48.49000
39	D19-2	LL	290.00000	.	290.00000	NONE	-6.60000
40	D20-1	LL	19.00000	.	19.00000	NONE	-3.00000
41	D20-2	LL	25.00000	.	25.00000	NONE	-3.00000
42	D21-1	LL	21.00000	.	21.00000	NONE	-28.19000
43	D21-2	LL	28.00000	.	28.00000	NONE	-28.19000
44	D22-1	LL	39.00000	.	39.00000	NONE	-97.42000

ORIGINAL MODEL

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
45	D22-2	LL	46.00000	.	46.00000	NONE	-6.60000
46	D23-1	LL	54.00000	.	54.00000	NONE	-127.78000
47	D23-2	LL	46.00000	.	46.00000	NONE	-6.60000
48	FW1-1	BS	.	2364.00000	NONE	2364.00000	.
49	FW1-2	BS	.	4727.00000	NONE	4727.00000	.
50	FW2-1	BS	778.00000	96.00000	NONE	874.00000	.
51	FW2-2	BS	223.00000	1526.00000	NONE	1749.00000	.
52	FW3-1	BS	188.00000	.	NONE	188.00000	.
53	FW3-2	BS	161.00000	215.00000	NONE	376.00000	.
54	FW4-1	BS	35.00000	205.00000	NONE	240.00000	.
55	FW4-2	BS	.	480.00000	NONE	480.00000	.
56	FW8-1	BS	.	437.00000	NONE	437.00000	.
57	FW8-2	BS	.	874.00000	NONE	874.00000	.
58	FW9-1	BS	.	10.00000	NONE	10.00000	.
59	FW9-2	BS	.	19.00000	NONE	19.00000	.
60	FW13-1	BS	192.00000	53.00000	NONE	245.00000	.
61	FW13-2	BS	140.00000	350.00000	NONE	490.00000	.
62	FW14-1	BS	48.00000	57.00000	NONE	105.00000	.
63	FW14-2	BS	37.00000	173.00000	NONE	210.00000	.
64	FW15-1	BS	12.00000	215.00000	NONE	227.00000	.
65	FW15-2	BS	79.00000	375.00000	NONE	454.00000	.
66	FW16-1	BS	46.00000	.	NONE	46.00000	.
67	FW16-2	BS	90.00000	.	NONE	90.00000	.
68	FW21-1	BS	21.00000	2.00000	NONE	23.00000	.
69	FW21-2	BS	28.00000	17.00000	NONE	45.00000	.
70	FS2-1	BS	787.00000	.	NONE	787.00000	.
71	FS2-2	BS	1575.00000	.	NONE	1575.00000	.
72	FS4-1	BS	234.00000	.	NONE	234.00000	.
73	FS4-2	BS	249.00000	220.00000	NONE	469.00000	.
74	FS5-1	BS	30.00000	.	NONE	30.00000	.
75	FS5-2	BS	59.00000	.	NONE	59.00000	.
76	FS6-1	BS	46.00000	.	NONE	46.00000	.
77	FS6-2	BS	63.00000	.	NONE	63.00000	.
78	FS8-1	BS	164.00000	228.00000	NONE	392.00000	.
79	FS8-2	BS	185.00000	165.00000	NONE	350.00000	.
80	FS11-1	BS	97.00000	148.00000	NONE	245.00000	.
81	FS11-2	BS	94.00000	396.00000	NONE	490.00000	.
82	FS12-1	BS	35.00000	.	NONE	35.00000	.
83	FS12-2	BS	66.00000	4.00000	NONE	70.00000	.
84	FS13-1	BS	112.00000	.	NONE	112.00000	.
85	FS13-2	BS	224.00000	.	NONE	224.00000	.
86	FS14-1	BS	39.00000	.	NONE	39.00000	.
87	FS14-2	BS	78.00000	.	NONE	78.00000	.
88	FS15-1	BS	119.00000	.	NONE	119.00000	.
89	FS15-2	BS	238.00000	.	NONE	238.00000	.

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ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
90	FS18-1	BS	105.00000	.	NONE	105.00000	.
91	FS18-2	BS	91.00000	119.00000	NONE	210.00000	.
92	FS19-1	BS	200.00000	.	NONE	200.00000	.
93	FS19-2	BS	290.00000	810.00000	NONE	1100.00000	.
94	FS20-1	BS	23.00000	37.00000	NONE	60.00000	.
95	FS20-2	BS	25.00000	95.00000	NONE	120.00000	.
96	FS22-1	BS	35.00000	.	NONE	35.00000	.
97	FS22-2	BS	46.00000	24.00000	NONE	70.00000	.
98	FS23-1	BS	32.00000	.	NONE	32.00000	.
99	FS23-2	BS	46.00000	18.00000	NONE	64.00000	.
100	FS2F-1	UL	.	.	NONE	.	27.50000
101	FS2F-2	UL	.	.	NONE	.	27.50000
102	FW1F-1	BS	.	.	NONE	.	.
103	FW1F-2	UL	.	.	NONE	.	4.28374
104	FW2F-1	BS	.	.	NONE	.	.
105	FW2F-2	UL	.	.	NONE	.	4.28378
106	FW3F-1	UL	.	.	NONE	.	26.36000
107	FW3F-2	BS	.	.	NONE	.	.
108	FW4F-1	UL	.	.	NONE	.	.
109	FW4F-2	UL	.	.	NONE	.	7.45788
110	FW5F-1	UL	.	.	NONE	.	43.12000
111	FW5F-2	BS	.	.	NONE	.	.
112	FW6F-1	BS	.	.	NONE	.	.
113	FW6F-2	UL	.	.	NONE	.	7.45900
114	FW7F-1	BS	-78.00000	78.00000	NONE	.	.
115	FW7F-2	BS	-156.00000	156.00000	NONE	.	.
116	FW8F-1	BS	.	.	NONE	.	.
117	FW8F-2	UL	.	.	NONE	.	10.28464
118	FW9F-1	BS	-4.00000	4.00000	NONE	.	.
119	FW9F-2	BS	-26.00000	26.00000	NONE	.	.
120	FW10F-1	BS	-19.00000	19.00000	NONE	.	.
121	FW10F-2	BS	-44.00000	44.00000	NONE	.	.
122	FW13F-1	BS	.	.	NONE	.	.
123	FW13F-2	UL	.	.	NONE	.	.
124	FW14F-1	BS	.	.	NONE	.	.
125	FW14F-2	UL	.	.	NONE	.	7.45924
126	FW15F-1	UL	.	.	NONE	.	.
127	FW15F-2	BS	-630.00000	630.00000	NONE	.	.
128	FW18F-1	BS	-137.00000	137.00000	NONE	.	.
129	FW18F-2	BS	-350.00000	350.00000	NONE	.	.
130	FT11ABC1	BS	.	.	NONE	.	.
131	FT11ABC2	UL	.	.	NONE	.	47.85145
132	Z1+3*1	BS	-34.00000	34.00000	NONE	.	.
133	Z1+3*2	BS	-175.00000	175.00000	NONE	.	.
134	Z3+1*1	BS	-87.00000	87.00000	NONE	.	.

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ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
135	Z3+1*2	BS	-175.00000	175.00000	NONE	.	.
136	Z3+4*1	BS	.	.	NONE	.	.
137	Z3+4*2	BS	.	.	NONE	.	.
138	Z4+3*1	BS	.	.	NONE	.	.
139	Z4+3*2	UL	.	.	NONE	.	2.42000
140	Z4+5*1	BS	-66.00000	66.00000	NONE	.	.
141	Z4+5*2	BS	-174.00000	174.00000	NONE	.	.
142	Z5+4*1	BS	-87.00000	87.00000	NONE	.	.
143	Z5+4*2	BS	-175.00000	175.00000	NONE	.	.
144	Z5+6*1	BS	.	.	NONE	.	.
145	Z5+6*2	BS	.	.	NONE	.	.
146	Z6+5*1	UL	.	.	NONE	.	45.00000
147	Z6+5*2	BS	.	.	NONE	.	.
148	Z6+7A1	BS	.	.	NONE	.	.
149	Z6+7A2	UL	.	.	NONE	.	12.00000
150	Z6+8*1	BS	-87.00000	87.00000	NONE	.	.
151	Z6+8*2	BS	-175.00000	175.00000	NONE	.	.
152	Z8+6*1	BS	-47.00000	47.00000	NONE	.	.
153	Z8+6*2	BS	-140.00000	140.00000	NONE	.	.
154	Z8+9A1	BS	.	.	NONE	.	.
155	Z8+9A2	UL	.	.	NONE	.	24.57143
156	Z9+10A1	BS	.	.	NONE	.	.
157	Z9+10A2	UL	.	.	NONE	.	13.71429
158	Z8+11*1	BS	.	.	NONE	.	.
159	Z8+11*2	BS	.	.	NONE	.	.
160	Z11+12A1	BS	-69.00000	69.00000	NONE	.	.
161	Z11+12A2	BS	-175.00000	175.00000	NONE	.	.
162	Z2+13*1	BS	-262.00000	262.00000	NONE	.	.
163	Z2+13*2	BS	-525.00000	525.00000	NONE	.	.
164	Z13+2*1	BS	-262.00000	262.00000	NONE	.	.
165	Z13+2*2	BS	-525.00000	525.00000	NONE	.	.
166	Z13+14*1	BS	.	.	NONE	.	.
167	Z13+14*2	UL	.	.	NONE	.	6.38298
168	Z14+13*1	BS	.	.	NONE	.	.
169	Z14+13*2	BS	.	.	NONE	.	.
170	Z14+15*1	BS	.	.	NONE	.	.
171	Z14+15*2	UL	.	.	NONE	.	8.76190
172	Z15+14*1	BS	.	.	NONE	.	.
173	Z15+14*2	BS	.	.	NONE	.	.
174	Z15+18*1	BS	.	.	NONE	.	.
175	Z15+18*2	UL	.	.	NONE	.	1.02370
176	Z18+19*1	BS	-158.00000	158.00000	NONE	.	.
177	Z18+19*2	BS	-329.00000	329.00000	NONE	.	.
178	Z20+21A1	BS	-87.00000	87.00000	NONE	.	.
179	Z20+21A2	BS	-175.00000	175.00000	NONE	.	.

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ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
180	Z18+20*1	BS	.	.	NONE	.	.
181	Z18+20*2	BS	.	.	NONE	.	.
182	Z20+22A1	BS	-83.00000	83.00000	NONE	.	.
183	Z20+22A2	BS	-175.00000	175.00000	NONE	.	.
184	Z23+22A1	BS	.	.	NONE	.	.
185	Z23+22A2	UL	.	.	NONE	.	28.00000
186	Z16+23*1	BS	-65.00000	65.00000	NONE	.	.
187	Z16+23*2	BS	-175.00000	175.00000	NONE	.	.
188	Z16+17A1	BS	-80.00000	80.00000	NONE	.	.
189	Z16+17A2	BS	-166.00000	166.00000	NONE	.	.
190	Z15+16*1	BS	-120.00000	120.00000	NONE	.	.
191	Z15+16*2	BS	-302.00000	302.00000	NONE	.	.
192	TRPLANT	BS	.	1.00000	NONE	1.00000	.
193	PZ11+3	BS	1.00000	.	NONE	1.00000	.
194	PZ13+4	BS	.	1.00000	NONE	1.00000	.
195	PZ14+5	BS	1.00000	.	NONE	1.00000	.
196	PZI5+6	BS	.	1.00000	NONE	1.00000	.
197	PZI6+8	BS	1.00000	.	NONE	1.00000	.
198	PZI8+11	BS	.	1.00000	NONE	1.00000	.
199	PZI2+13	BS	1.00000	.	NONE	1.00000	.
200	PZI13+14	BS	.	1.00000	NONE	1.00000	.
201	PZI14+15	BS	.	1.00000	NONE	1.00000	.
202	PZI15+18	BS	.	1.00000	NONE	1.00000	.
203	PZI18+19	BS	1.00000	.	NONE	1.00000	.
204	PZI18+20	BS	.	1.00000	NONE	1.00000	.
205	PZI16+23	BS	1.00000	.	NONE	1.00000	.
206	PZI15+16	BS	1.00000	.	NONE	1.00000	.
207	PD1	LL	12.28000	.	12.28000	NONE	-0.48000
208	PD2	LL	3.90000	.	3.90000	NONE	-0.47000
209	PD3	LL	1.29000	.	1.29000	NONE	-0.19000
210	PD4	LL	2.66000	.	2.66000	NONE	-0.59000
211	PD5	LL	0.55000	.	0.55000	NONE	-0.68000
212	PD6	LL	0.92000	.	0.92000	NONE	-0.32000
213	PD7	BS	0.72000	-0.64500	0.07500	NONE	.
214	PD8	LL	1.33000	.	1.33000	NONE	-0.26000
215	PD9	LL	0.23400	.	0.23400	NONE	-0.19000
216	PD10	BS	0.21600	-0.14100	0.07500	NONE	.
217	PD11	LL	0.84000	.	0.84000	NONE	-0.03000
218	PD12	LL	0.56200	.	0.56200	NONE	-0.54000
219	PD13	LL	3.26000	.	3.26000	NONE	-0.59000
220	PD14	LL	0.94000	.	0.94000	NONE	-0.26000
221	PD15	LL	4.31000	.	4.31000	NONE	-0.26000
222	PD16	LL	0.65500	.	0.65500	NONE	-0.64000
223	PD17	LL	0.07500	.	0.07500	NONE	-2.05000
224	PD18	LL	1.48000	.	1.48000	NONE	-3611.11111

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ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
225	PD19	LL	2.21000	.	2.21000	NONE	-3611.30111
226	PD20	LL	0.20600	.	0.20600	NONE	-0.03000
227	PD21	LL	0.22500	.	0.22500	NONE	-0.66000
228	PD22	LL	0.41200	.	0.41200	NONE	-0.97000
229	PD23	LL	0.58000	.	0.58000	NONE	-1.26000
230	PS1	BS	10.96000	6.55000	NONE	17.51000	.
231	PS2	BS	5.83000	.	NONE	5.83000	.
232	PS3	BS	1.29000	0.11000	NONE	1.40000	.
233	PS4	BS	2.99000	0.51000	NONE	3.50000	.
234	PS5	BS	0.22000	.	NONE	0.22000	.
235	PS6	BS	0.33000	.	NONE	0.33000	.
236	PS8	BS	1.92000	4.23000	NONE	6.15000	.
237	PS9	BS	0.01800	0.05200	NONE	0.07000	.
238	PS11	BS	1.14200	0.67800	NONE	1.82000	.
239	PS12	BS	0.26000	.	NONE	0.26000	.
240	PS13	BS	2.65000	.	NONE	2.65000	.
241	PS14	BS	0.94000	0.13000	NONE	1.07000	.
242	PS15	BS	2.45000	0.12000	NONE	2.57000	.
243	PS16	BS	0.34000	.	NONE	0.34000	.
244	PS18	BS	0.77000	.	NONE	0.77000	.
245	PS19	BS	1.48000	.	NONE	1.48000	.
246	PS20	BS	0.41300	0.02700	NONE	0.44000	.
247	PS21	BS	0.17000	.	NONE	0.17000	.
248	PS22	BS	0.26000	.	NONE	0.26000	.
249	PS23	BS	0.23000	.	NONE	0.23000	.
250	P1+3	BS	-0.72000	0.72000	NONE	.	.
251	P3+1	BS	-0.72000	0.72000	NONE	.	.
252	P3+4	BS	.	.	NONE	.	.
253	P4+3	BS	.	.	NONE	.	.
254	P4+5	BS	-0.39000	0.39000	NONE	.	.
255	P5+4	BS	-0.72000	0.72000	NONE	.	.
256	P5+6	UL	.	.	NONE	.	0.21000
257	P6+5	BS	.	.	NONE	.	.
258	P6+7A	BS	.	.	NONE	.	.
259	P6+8	BS	-0.13000	0.13000	NONE	.	.
260	P8+6	BS	-0.72000	0.72000	NONE	.	.
261	P8+9A	BS	.	.	NONE	.	.
262	P9+10A	BS	.	.	NONE	.	.
263	P8+11	BS	.	.	NONE	.	.
264	P11+12A	BS	-0.41800	0.41800	NONE	.	.
265	P2+13	BS	-1.54000	1.54000	NONE	.	.
266	P13+2	BS	-2.15000	2.15000	NONE	.	.
267	P13+14	BS	.	.	NONE	.	.
268	P14+13	BS	.	.	NONE	.	.
269	P14+15	BS	.	.	NONE	.	.

ORIGINAL MODEL

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
270	P15+14	BS	.	.	NONE	.	.
271	P15+18	UL	.	.	NONE	.	3610.49111
272	P18+19	BS	-0.62000	0.62000	NONE	.	.
273	P20+21A	BS	-0.66500	0.66500	NONE	.	.
274	P18+20	BS	.	.	NONE	.	.
275	P20+22A	BS	-0.56800	0.56800	NONE	.	.
276	P23+22A	BS	.	.	NONE	.	.
277	P16+23	BS	-0.37000	0.37000	NONE	.	.
278	P16+17A	BS	-0.64500	0.64500	NONE	.	.
279	P15+16	BS	-0.61000	0.61000	NONE	.	.

ORIGINAL MODEL

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
289	I1WF	IV	.	7500.00000	.	1.00000	.
290	I2WF	IV	.	7500.00000	.	1.00000	.
291	I2SF	IV	.	83000.00000	.	1.00000	-89443.23600
292	I3WF	IV	.	4700.00000	.	1.00000	-3603.89400
293	I4WF	IV	.	4700.00000	.	1.00000	.
294	I5WF	IV	.	4700.00000	.	1.00000	-8884.56800
295	I6WF	IV	.	4700.00000	.	1.00000	.
296	I7WF	IV	1.00000	2600.00000	.	1.00000	2600.00000
297	I8WF	IV	.	3600.00000	.	1.00000	.
298	I9WF	IV	1.00000	1500.00000	.	1.00000	1499.95896
299	I10WF	IV	1.00000	1500.00000	.	1.00000	1500.00000
300	I11ATF	IV	.	11100.00000	.	1.00000	3587.30303
301	I11BTF	IV	.	24500.00000	.	1.00000	.
302	I11CTF	IV	.	32400.00000	.	1.00000	5220.28017
303	I13WF	IV	.	4700.00000	.	1.00000	4698.46600
304	I14WF	IV	.	4700.00000	.	1.00000	.
305	I15WF	IV	1.00000	4700.00000	.	1.00000	4699.32400
306	I18WF	IV	2.00000	2600.00000	.	2.00000	.
307	I1+3A	IV	1.00000	1200.00000	.	1.00000	1200.00000
308	I1+3B	IV	.	1700.00000	.	1.00000	1700.00000
309	I3+4A	IV	.	1700.00000	.	1.00000	903.82000
310	I3+4B	IV	.	2200.00000	.	1.00000	929.50000
311	I4+5A	IV	1.00000	700.00000	.	1.00000	700.00000
312	I4+5B	IV	.	1100.00000	.	1.00000	1100.00000
313	I5+6A	IV	.	1500.00000	.	1.00000	-2415.15120
314	I5+6B	IV	.	2100.00000	.	1.00000	-5280.28350
315	I6+7A	IV	.	2100.00000	.	1.00000	.
316	I6+8A	IV	1.00000	700.00000	.	1.00000	700.00000
317	I6+8B	IV	.	1100.00000	.	1.00000	1100.00000
318	I8+9A	IV	.	4300.00000	.	1.00000	.
319	I9+10A	IV	.	2400.00000	.	1.00000	.
320	I8+11A	IV	.	4500.00000	.	1.00000	4500.00000
321	I8+11B	IV	.	6300.00000	.	1.00000	6300.00000
322	I11+12A	IV	1.00000	4200.00000	.	1.00000	4200.00000
323	I2+13A	IV	1.00000	4000.00000	.	1.00000	4000.00000
324	I2+13B	IV	.	4800.00000	.	1.00000	4800.00000
325	I13+14A	IV	.	1500.00000	.	1.00000	382.97872
326	I13+14B	IV	.	2100.00000	.	1.00000	.
327	I14+15A	IV	.	3600.00000	.	1.00000	717.33333
328	I14+15B	IV	.	4600.00000	.	1.00000	.
329	I15+18A	IV	.	6500.00000	.	1.00000	1289.03869
330	I15+18B	IV	.	8300.00000	.	1.00000	.
331	I18+19A	IV	1.00000	4200.00000	.	1.00000	4200.00000
332	I18+19B	IV	.	5400.00000	.	1.00000	5400.00000
333	I20+21A	IV	1.00000	3100.00000	.	1.00000	3100.00000
334	I18+20A	IV	.	6000.00000	.	1.00000	6000.00000
335	I18+20B	IV	.	8400.00000	.	1.00000	8400.00000

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
336	I20+22A	IV	1.00000	4000.00000	.	1.00000	4000.00000
337	I23+22A	IV	.	4900.00000	.	1.00000	.
338	I16+23A	IV	1.00000	4000.00000	.	1.00000	4000.00000
339	I16+23B	IV	.	5700.00000	.	1.00000	5700.00000
340	I16+17A	IV	1.00000	4500.00000	.	1.00000	4500.00000
341	I15+16A	IV	.	3300.00000	.	1.00000	3300.00000
342	I15+16B	IV	1.00000	4600.00000	.	1.00000	4600.00000
343	X1W1	LL	.	47.60000	.	2364.00000	12.34000
344	X1W2	LL	.	47.60000	.	4727.00000	12.00000
345	X2W1	BS	778.00000	34.10000	.	874.00000	.
346	X2W2	BS	223.00000	34.10000	.	1749.00000	.
347	X3W1	UL	188.00000	18.90000	.	188.00000	-26.37000
348	X3W2	BS	161.00000	18.90000	.	376.00000	.
349	X4W1	BS	35.00000	59.42000	.	240.00000	.
350	X4W2	LL	.	59.42000	.	480.00000	52.82000
351	X8W1	LL	.	25.66000	.	437.00000	22.66000
352	X8W2	LL	.	25.66000	.	874.00000	22.66000
353	X9W1	LL	.	18.91000	.	10.00000	.
354	X9W2	LL	.	18.91000	.	19.00000	.
355	X13W1	BS	192.00000	45.92000	.	245.00000	.
356	X13W2	BS	140.00000	45.92000	.	490.00000	.
357	X14W1	BS	48.00000	25.66000	.	105.00000	.
358	X14W2	BS	37.00000	25.66000	.	210.00000	.
359	X15W1	BS	12.00000	25.66000	.	227.00000	.
360	X15W2	BS	79.00000	25.66000	.	454.00000	.
361	X16W1	UL	46.00000	17.22000	.	46.00000	-46.11000
362	X16W2	UL	90.00000	17.22000	.	90.00000	-48.29000
363	X21W1	BS	21.00000	28.19000	.	23.00000	.
364	X21W2	BS	28.00000	28.19000	.	45.00000	.
365	X1WF1	LL	.	47.60000	.	875.00000	12.34000
366	X1WF2	LL	.	47.60000	.	1750.00000	16.28374
367	X2WF1	LL	.	34.10000	.	875.00000	.
368	X2WF2	LL	.	34.10000	.	1750.00000	4.28378
369	X2S1	UL	787.00000	6.60000	.	787.00000	-27.50000
370	X2S2	UL	1575.00000	6.60000	.	1575.00000	-27.50000
371	X2SF1	BS	.	6.60000	.	4700.00000	.
372	X2SF2	BS	.	6.60000	.	1570.00000	.
373	X3WF1	BS	.	18.91000	.	315.00000	.
374	X3WF2	LL	.	18.91000	.	630.00000	0.01000
375	X4WF-1	BS	.	59.42000	.	315.00000	.
376	X4WF-2	LL	.	59.42000	.	630.00000	60.27788
377	X4S-1	UL	234.00000	6.60000	.	234.00000	-52.82000
378	X4S-2	BS	249.00000	6.60000	.	469.00000	.
379	X5WF-1	BS	.	25.66000	.	315.00000	.
380	X5WF-2	LL	.	25.66000	.	630.00000	7.06000

ORIGINAL MODEL

COLUMNS SECTION

	NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
	381	X5S-1	UL	30.00000	6.60000	.	30.00000	-62.18000
	382	X5S-2	UL	59.00000	6.60000	.	59.00000	-12.00000
	383	X6WF-1	LL	.	23.97000	.	315.00000	14.79000
	384	X6WF-2	LL	.	23.97000	.	630.00000	20.62900
	385	X6S-1	UL	46.00000	6.60000	.	46.00000	-2.58000
	386	X6S-2	UL	63.00000	6.60000	.	63.00000	-4.20000
	387	X7WF-1	BS	10.00000	18.91000	.	88.00000	.
	388	X7WF-2	BS	19.00000	18.91000	.	175.00000	.
	389	X8WF-1	LL	.	25.66000	.	175.00000	22.66000
	390	X8WF-2	LL	.	25.66000	.	350.00000	32.94464
	391	X8S-1	BS	164.00000	3.00000	.	392.00000	.
	392	X8S-2	BS	185.00000	3.00000	.	350.00000	.
	393	X9WF-1	BS	22.00000	18.91000	.	26.00000	.
	394	X9WF-2	BS	26.00000	18.91000	.	52.00000	.
	395	X10WF-1	BS	7.00000	18.91000	.	26.00000	.
	396	X10WF-2	BS	8.00000	18.91000	.	52.00000	.
	397	X11TA-1	LL	.	61.40000	.	79.00000	58.40000
	398	X11TA-2	LL	.	83.80000	.	157.00000	128.65145
	399	X11TB-1	LL	.	56.90000	.	216.00000	53.90000
	400	X11TB-2	LL	.	68.50000	.	512.00000	113.35145
	401	X11TC-1	LL	.	52.80000	.	284.00000	49.80000
	402	X11TC-2	LL	.	63.60000	.	568.00000	108.45145
	403	X11S-1	BS	97.00000	3.00000	.	245.00000	.
	404	X11S-2	BS	94.00000	3.00000	.	490.00000	.
	405	X12S-1	UL	35.00000	6.60000	.	35.00000	-47.83000
	406	X12S-2	BS	66.00000	6.60000	.	70.00000	.
A	407	X13WF-1	LL	.	45.92000	.	315.00000	.
	408	X13WF-2	BS	.	45.92000	.	630.00000	.
	409	X13S-1	UL	112.00000	6.60000	.	112.00000	-39.32000
	410	X13S-2	UL	224.00000	6.60000	.	224.00000	-39.32000
A	411	X14WF-1	LL	.	25.66000	.	315.00000	.
	412	X14WF-2	LL	.	25.66000	.	630.00000	7.45924
	413	X14S-1	UL	39.00000	6.60000	.	39.00000	-19.06000
	414	X14S-2	UL	78.00000	6.60000	.	78.00000	-19.06000
	415	X15WF-1	BS	315.00000	25.66000	.	315.00000	.
A	416	X15WF-2	LL	.	25.66000	.	630.00000	.
	417	X15S-1	UL	119.00000	6.60000	.	119.00000	-19.06000
	418	X15S-2	UL	238.00000	6.60000	.	238.00000	-19.06000
	419	X18WF-1	BS	39.00000	29.04000	.	176.00000	.
	420	X18WF-2	LL	.	29.04000	.	350.00000	22.44000
	421	X18S-1	UL	105.00000	6.60000	.	105.00000	-22.44000
	422	X18S-2	BS	91.00000	6.60000	.	210.00000	.
	423	X19S-1	UL	200.00000	6.60000	.	200.00000	-41.89000
	424	X19S-2	BS	290.00000	6.60000	.	1100.00000	.
	425	X20S-1	BS	23.00000	3.00000	.	60.00000	.

ORIGINAL MODEL

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
426	X20S-2	BS	25.00000	3.00000	.	120.00000	.
427	X22S-1	UL	35.00000	6.60000	.	35.00000	-90.82000
428	X22S-2	BS	46.00000	6.60000	.	70.00000	.
429	X23S-1	UL	32.00000	6.60000	.	32.00000	-121.18000
430	X23S-2	BS	46.00000	6.60000	.	64.00000	.
431	Z1+2-1	LL	.	18.85000	.	7000.00000	20.01000
432	Z1+2-2	LL	.	19.26000	.	7000.00000	20.76000
433	Z2+1-1	BS	1201.00000	1.16000	.	7000.00000	.
434	Z2+1-2	BS	1312.00000	1.50000	.	7000.00000	.
435	Z1+3-1	BS	53.00000	10.01000	.	164.00000	.
436	Z1+3-2	LL	.	12.09000	.	329.00000	28.79000
437	Z3+1-1	LL	.	25.58000	.	164.00000	35.59000
438	Z3+1-2	LL	.	30.66000	.	329.00000	13.96000
439	Z3+4-1	LL	.	51.05000	.	262.00000	36.90000
440	Z3+4-2	LL	.	53.77000	.	525.00000	66.07000
441	Z4+3-1	LL	.	7.16000	.	262.00000	21.31000
442	Z4+3-2	BS	.	9.88000	.	525.00000	.
443	Z4+5-1	BS	21.00000	9.36000	.	164.00000	.
444	Z4+5-2	BS	1.00000	12.00000	.	329.00000	.
445	Z5+4-1	LL	.	43.12000	.	164.00000	52.48000
446	Z5+4-2	LL	.	45.76000	.	329.00000	57.76000
447	Z5+6-1	LL	.	14.60000	.	164.00000	74.20000
448	Z5+6-2	LL	.	18.94000	.	329.00000	26.74000
449	Z6+5-1	BS	.	14.60000	.	164.00000	.
450	Z6+5-2	LL	.	18.94000	.	329.00000	11.14000
451	Z6+7-1	LL	.	56.59000	.	87.00000	46.86000
452	Z6+7-2	LL	.	56.59000	.	175.00000	60.48000
453	Z6+8-1	LL	.	18.84000	.	164.00000	25.02000
454	Z6+8-2	LL	.	20.46000	.	329.00000	28.26000
455	Z8+6-1	BS	40.00000	6.18000	.	164.00000	.
456	Z8+6-2	BS	35.00000	7.80000	.	329.00000	.
457	Z8+9-1	LL	.	96.92000	.	87.00000	81.01000
458	Z8+9-2	LL	.	118.32000	.	175.00000	126.98143
459	Z9+10-1	LL	.	82.57000	.	87.00000	82.57000
460	Z9+10-2	LL	.	105.32000	.	175.00000	119.03429
461	Z8+11-1	LL	.	45.65000	.	164.00000	45.65000
462	Z8+11-2	LL	.	57.88000	.	329.00000	57.88000
463	Z11+12-1	BS	18.00000	51.43000	.	87.00000	.
464	Z11+12-2	LL	.	63.60000	.	175.00000	60.00000
465	Z2+13-1	LL	.	12.17000	.	360.00000	0.35000
466	Z2+13-2	LL	.	15.46000	.	721.00000	3.64000
467	Z13+2-1	LL	.	17.23000	.	360.00000	29.05000
468	Z13+2-2	LL	.	20.52000	.	721.00000	32.34000
469	Z13+14-1	LL	.	14.46000	.	164.00000	34.72000
470	Z13+14-2	LL	.	17.54000	.	329.00000	44.18298

ORIGINAL MODEL

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
471	Z14+13-1	LL	.	34.72000	.	164.00000	14.46000
472	Z14+13-2	LL	.	37.80000	.	329.00000	17.54000
473	Z14+15-1	LL	.	12.12000	.	262.00000	12.12000
474	Z14+15-2	LL	.	19.47000	.	525.00000	28.23190
475	Z15+14-1	LL	.	12.12000	.	262.00000	12.12000
476	Z15+14-2	LL	.	19.47000	.	525.00000	19.47000
477	Z15+18-1	LL	.	36.31000	.	262.00000	32.93000
478	Z15+18-2	LL	.	60.88000	.	525.00000	80.96370
479	Z18+19-1	BS	6.00000	19.45000	.	262.00000	.
480	Z18+19-2	LL	.	22.91000	.	525.00000	22.91000
481	Z20+21-1	LL	.	63.36000	.	87.00000	38.17000
482	Z20+21-2	LL	.	75.63000	.	175.00000	50.44000
483	Z18+20-1	LL	.	117.28000	.	164.00000	143.32000
484	Z18+20-2	LL	.	144.71000	.	329.00000	148.31000
485	Z20+22-1	BS	4.00000	94.42000	.	87.00000	.
486	Z20+22-2	LL	.	110.29000	.	175.00000	106.69000
487	Z23+22-1	LL	.	101.71000	.	87.00000	132.07000
488	Z23+22-2	LL	.	121.12000	.	175.00000	149.12000
489	Z16+23-1	BS	22.00000	64.45000	.	164.00000	.
490	Z16+23-2	LL	.	88.76000	.	329.00000	147.67000
491	Z16+17-1	BS	7.00000	140.00000	.	87.00000	.
492	Z16+17-2	BS	9.00000	175.03000	.	175.00000	.
493	Z15+16-1	BS	44.00000	37.67000	.	164.00000	.
494	Z15+16-2	BS	27.00000	39.85000	.	329.00000	.
495	PE1	BS	10.96000	0.48000	.	17.50000	.
496	PE2	UL	5.83000	0.34000	.	5.83000	-0.13000
497	PE3	BS	1.29000	0.19000	.	1.40000	.
498	PE4	BS	2.99000	0.59000	.	3.50000	.
499	PE5	UL	0.22000	0.26000	.	0.22000	-0.42000
500	PE6	UL	0.33000	0.24000	.	0.33000	-0.08000
501	PE8	BS	1.92000	0.26000	.	6.15000	.
502	PE9	BS	0.01800	0.19000	.	0.07000	.
503	PE11	BS	1.14200	0.03000	.	1.82000	.
504	PE12	UL	0.26000	0.07000	.	0.26000	-0.47000
505	PE13	UL	2.65000	0.46000	.	2.65000	-0.13000
506	PE14	BS	0.94000	0.26000	.	1.07000	.
507	PE15	BS	2.45000	0.26000	.	2.60000	.
508	PE16	UL	0.34000	0.17000	.	0.34000	-0.47000
509	PE18	UL	0.77000	0.29000	.	0.77000	-3610.82111
510	PE19	UL	1.48000	0.07000	.	1.48000	-3611.23111
511	PE20	BS	0.41300	0.03000	.	0.44000	.
512	PE21	UL	0.17000	0.28000	.	0.17000	-0.38000
513	PE22	UL	0.26000	0.07000	.	0.26000	-0.90000
514	PE23	UL	0.23000	0.07000	.	0.23000	-1.21000
515	PZ1+2	LL	.	0.19000	.	30.00000	0.20000

ORIGINAL MODEL

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
516	PZ2+1	BS	1.32000	0.01000	.	30.00000	.
517	PZ1+3	LL	.	0.10000	.	1.35000	0.39000
518	PZ3+1	LL	.	0.29000	.	1.35000	.
519	PZ3+4	LL	.	0.51000	.	2.15000	0.11000
520	PZ4+3	LL	.	0.07000	.	2.15000	0.47000
521	PZ4+5	BS	0.33000	0.09000	.	1.35000	.
522	PZ5+4	LL	.	0.43000	.	1.35000	0.52000
523	PZ5+6	LL	.	0.15000	.	1.35000	0.72000
524	PZ6+5	BS	.	0.15000	.	1.35000	.
525	PZ6+7	LL	.	0.57000	.	0.72000	0.89000
526	PZ6+8	LL	.	0.19000	.	1.35000	0.25000
527	PZ8+6	BS	0.59000	0.06000	.	1.35000	.
528	PZ8+9	LL	.	0.97000	.	0.72000	1.04000
529	PZ9+10	LL	.	0.83000	.	0.72000	1.02000
530	PZ8+11	LL	.	0.46000	.	1.35000	0.69000
531	PZ11+12	BS	0.30200	0.51000	.	0.72000	.
532	PZ2+13	BS	0.61000	0.12000	.	2.96000	.
533	PZ13+2	LL	.	0.17000	.	2.96000	0.29000
534	PZ13+14	LL	.	0.15000	.	1.35000	0.48000
535	PZ14+13	LL	.	0.35000	.	1.35000	0.02000
536	PZ14+15	LL	.	0.12000	.	2.15000	0.12000
537	PZ15+14	LL	.	0.12000	.	2.15000	0.12000
538	PZ15+18	BS	.	0.36000	.	2.15000	.
539	PZ18+19	BS	0.73000	0.19000	.	2.15000	.
540	PZ20+21	BS	0.05500	0.63000	.	0.72000	.
541	PZ18+20	LL	.	1.17000	.	1.35000	3612.25111
542	PZ20+22	BS	0.15200	0.94000	.	0.72000	.
543	PZ23+22	LL	.	1.02000	.	0.72000	1.33000
544	PZ16+23	BS	0.35000	0.64000	.	1.35000	.
545	PZ16+17	BS	0.07500	1.41000	.	0.72000	.
546	PZ15+16	BS	0.74000	0.38000	.	1.35000	.

Appendix C

MXINT Solution to Water Supply Model—Revision 2 (Decomposed Model—North Half)

REV 2
SOLUTION

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
1	OBJECT	BS	140494.78597	-140494.78597	NONE	NONE	1.00000
2	D1-1	LL	1148.00000	.	1148.00000	NONE	-35.26000
3	D1-2	LL	1312.00000	.	1312.00000	NONE	-35.60000
4	D2-1	LL	364.00000	.	364.00000	NONE	-34.10000
5	D2-2	LL	486.00000	.	486.00000	NONE	-34.10000
6	D13-1	LL	304.00000	.	304.00000	NONE	-45.92000
7	D13-2	LL	364.00000	.	364.00000	NONE	-45.92000
8	D14-1	LL	87.00000	.	87.00000	NONE	-25.66000
9	D14-2	LL	115.00000	.	115.00000	NONE	-25.66000
10	D15-1	LL	402.00000	.	402.00000	NONE	-25.66000
11	D15-2	LL	290.00000	.	290.00000	NONE	-25.66000
12	D16-1	LL	61.00000	.	61.00000	NONE	-63.33000
13	D16-2	LL	108.00000	.	108.00000	NONE	-65.51000
14	D17-1	LL	7.00000	.	7.00000	NONE	-203.33000
15	D17-2	LL	9.00000	.	9.00000	NONE	-240.54000
16	D18-1	LL	138.00000	.	138.00000	NONE	-29.04000
17	D18-2	LL	91.00000	.	91.00000	NONE	-6.60000
18	D19-1	LL	206.00000	.	206.00000	NONE	-48.49000
19	D19-2	LL	290.00000	.	290.00000	NONE	-6.60000
20	D20-1	LL	19.00000	.	19.00000	NONE	-3.00000
21	D20-2	LL	25.00000	.	25.00000	NONE	-3.00000
22	D21-1	LL	21.00000	.	21.00000	NONE	-28.19000
23	D21-2	LL	28.00000	.	28.00000	NONE	-28.19000
24	D22-1	LL	39.00000	.	39.00000	NONE	-97.42000
25	D22-2	LL	46.00000	.	46.00000	NONE	-6.60000
26	D23-1	LL	54.00000	.	54.00000	NONE	-127.78000
27	D23-2	LL	46.00000	.	46.00000	NONE	-6.60000
28	FW1-1	BS	.	2364.00000	NONE	2364.00000	.
29	FW1-2	BS	.	4727.00000	NONE	4727.00000	.
30	FW2-1	BS	725.00000	149.00000	NONE	874.00000	.
31	FW2-2	BS	223.00000	1526.00000	NONE	1749.00000	.
32	FW13-1	BS	192.00000	53.00000	NONE	245.00000	.
33	FW13-2	BS	140.00000	350.00000	NONE	490.00000	.
34	FW14-1	BS	48.00000	57.00000	NONE	105.00000	.
35	FW14-2	BS	37.00000	173.00000	NONE	210.00000	.
36	FW15-1	BS	227.00000	.	NONE	227.00000	.
37	FW15-2	BS	79.00000	375.00000	NONE	454.00000	.
38	FW16-1	BS	46.00000	.	NONE	46.00000	.
39	FW16-2	BS	90.00000	.	NONE	90.00000	.
40	FW21-1	BS	21.00000	2.00000	NONE	23.00000	.
41	FW21-2	BS	28.00000	17.00000	NONE	45.00000	.
42	FS2-1	BS	787.00000	.	NONE	787.00000	.
43	FS2-2	BS	1575.00000	.	NONE	1575.00000	.
44	FS13-1	BS	112.00000	.	NONE	112.00000	.

REV 2

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
45	FS13-2	BS	224.00000	.	NONE	224.00000	.
46	FS14-1	BS	39.00000	.	NONE	39.00000	.
47	FS14-2	BS	78.00000	.	NONE	78.00000	.
48	FS15-1	BS	119.00000	.	NONE	119.00000	.
49	FS15-2	BS	238.00000	.	NONE	238.00000	.
50	FS18-1	BS	105.00000	.	NONE	105.00000	.
51	FS18-2	BS	91.00000	119.00000	NONE	210.00000	.
52	FS19-1	BS	200.00000	.	NONE	200.00000	.
53	FS19-2	BS	290.00000	810.00000	NONE	1100.00000	.
54	FS20-1	BS	23.00000	37.00000	NONE	60.00000	.
55	FS20-2	BS	25.00000	95.00000	NONE	120.00000	.
56	FS22-1	BS	35.00000	.	NONE	35.00000	.
57	FS22-2	BS	46.00000	24.00000	NONE	70.00000	.
58	FS23-1	BS	32.00000	.	NONE	32.00000	.
59	FS23-2	BS	46.00000	18.00000	NONE	64.00000	.
60	FS2F-1	UL	.	.	NONE	.	27.50000
61	FS2F-2	UL	.	.	NONE	.	27.50000
62	FW1F-1	BS	.	.	NONE	.	.
63	FW1F-2	UL	.	.	NONE	.	4.28374
64	FW2F-1	BS	.	.	NONE	.	.
65	FW2F-2	UL	.	.	NONE	.	4.28378
66	FW13F-1	BS	.	.	NONE	.	.
67	FW13F-2	BS	.	.	NONE	.	.
68	FW14F-1	BS	.	.	NONE	.	.
69	FW14F-2	UL	.	.	NONE	.	7.45924
70	FW15F-1	BS	-215.00000	215.00000	NONE	.	.
71	FW15F-2	BS	-630.00000	630.00000	NONE	.	.
72	FW18F-1	BS	-137.00000	137.00000	NONE	.	.
73	FW18F-2	BS	-350.00000	350.00000	NONE	.	.
74	Z2&13*1	BS	-262.00000	262.00000	NONE	.	.
75	Z2&13*2	BS	-525.00000	525.00000	NONE	.	.
76	Z13&2*1	BS	-262.00000	262.00000	NONE	.	.
77	Z13&2*2	BS	-525.00000	525.00000	NONE	.	.
78	Z13&14*1	BS	.	.	NONE	.	.
79	Z13&14*2	UL	.	.	NONE	.	6.38298
80	Z14&13*1	BS	.	.	NONE	.	.
81	Z14&13*2	BS	.	.	NONE	.	.
82	Z14&15*1	BS	.	.	NONE	.	.
83	Z14&15*2	BS	.	.	NONE	.	.
84	Z15&14*1	BS	.	.	NONE	.	.
85	Z15&14*2	BS	.	.	NONE	.	.
86	Z15&18*1	BS	.	.	NONE	.	.
87	Z15&18*2	BS	.	.	NONE	.	.
88	Z18&19*1	BS	-158.00000	158.00000	NONE	.	.
89	Z18&19*2	BS	-329.00000	329.00000	NONE	.	.

REV 2

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
90	Z20&21A1	BS	-87.00000	87.00000	NONE	.	.
91	Z20&21A2	BS	-175.00000	175.00000	NONE	.	.
92	Z18&20*1	BS	.	.	NONE	.	.
93	Z18&20*2	BS	.	.	NONE	.	.
94	Z20&22A1	BS	-83.00000	83.00000	NONE	.	.
95	Z20&22A2	BS	-175.00000	175.00000	NONE	.	.
96	Z23&22A1	BS	.	.	NONE	.	.
97	Z23&22A2	UL	.	.	NONE	.	28.00000
98	Z16&23*1	BS	-65.00000	65.00000	NONE	.	.
99	Z16&23*2	BS	-175.00000	175.00000	NONE	.	.
100	Z16&17A1	BS	-80.00000	80.00000	NONE	.	.
101	Z16&17A2	BS	-166.00000	166.00000	NONE	.	.
102	Z15&16*1	BS	-120.00000	120.00000	NONE	.	.
103	Z15&16*2	BS	-302.00000	302.00000	NONE	.	.
104	PZI2&13	BS	1.00000	.	NONE	1.00000	.
105	PZI13&14	BS	.	1.00000	NONE	1.00000	.
106	PZI14&15	BS	.	1.00000	NONE	1.00000	.
107	PZI15&18	BS	.	1.00000	NONE	1.00000	.
108	PZI18&19	BS	1.00000	.	NONE	1.00000	.
109	PZI18&20	BS	.	1.00000	NONE	1.00000	.
110	PZI16&23	BS	1.00000	.	NONE	1.00000	.
111	PZI15&16	BS	1.00000	.	NONE	1.00000	.
112	PD1	LL	12.28000	.	12.28000	NONE	-0.48000
113	PD2	LL	3.90000	.	3.90000	NONE	-0.47000
114	PD13	LL	3.26000	.	3.26000	NONE	-0.59000
115	PD14	LL	0.94000	.	0.94000	NONE	-0.26000
116	PD15	LL	4.31000	.	4.31000	NONE	-0.26000
117	PD16	LL	0.65500	.	0.65500	NONE	-0.64000
118	PD17	LL	0.07500	.	0.07500	NONE	-2.05000
119	PD18	LL	1.48000	.	1.48000	NONE	-3611.11111
120	PD19	LL	2.21000	.	2.21000	NONE	-3611.30111
121	PD20	LL	0.20600	.	0.20600	NONE	-0.03000
122	PD21	LL	0.22500	.	0.22500	NONE	-0.66000
123	PD22	LL	0.41200	.	0.41200	NONE	-0.97000
124	PD23	LL	0.58000	.	0.58000	NONE	-1.28000
125	PS1	BS	10.96000	6.55000	NONE	17.51000	.
126	PS2	BS	5.83000	.	NONE	5.83000	.
127	PS13	BS	2.65000	.	NONE	2.65000	.
128	PS14	BS	0.94000	0.13000	NONE	1.07000	.
129	PS15	BS	2.45000	0.12000	NONE	2.57000	.
130	PS16	BS	0.34000	.	NONE	0.34000	.
131	PS18	BS	0.77000	.	NONE	0.77000	.
132	PS19	BS	1.48000	.	NONE	1.48000	.
133	PS20	BS	0.41300	0.02700	NONE	0.44000	.
134	PS21	BS	0.17000	.	NONE	0.17000	.

HCWS SECTION

REV 2

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
135	PS22	BS	0.26000	.	NONE	0.26000	.
136	PS23	BS	0.23000	.	NONE	0.23000	.
137	P2&13	BS	-1.54000	1.54000	NONE	.	.
138	P13&2	BS	-2.15000	2.15000	NONE	.	.
139	P13&14	BS	.	.	NONE	.	.
140	P14&13	BS	.	.	NONE	.	.
141	P14&15	BS	.	.	NONE	.	.
142	P15&14	BS	.	.	NONE	.	.
143	P15&18	UL	.	.	NONE	.	3610.49111
144	P18&19	BS	-0.62000	0.62000	NONE	.	.
145	P20&21A	BS	-0.66500	0.66500	NONE	.	.
146	P18&20	BS	.	.	NONE	.	.
147	P20&22A	BS	-0.56800	0.56800	NONE	.	.
148	P23&22A	BS	.	.	NONE	.	.
149	P16&23	BS	-0.37000	0.37000	NONE	.	.
150	P16&17A	BS	-0.64500	0.64500	NONE	.	.
151	P15&16	BS	-0.61000	0.61000	NONE	.	.

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
193	I1WF	IV	.	7500.00000	.	1.00000	.
194	I2WF	IV	.	7500.00000	.	1.00000	.
195	I2SF	IV	.	83000.00000	.	1.00000	-89443.23600
196	I13WF	IV	.	4700.00000	.	1.00000	4698.46600
197	I14WF	IV	.	4700.00000	.	1.00000	.
198	I15WF	IV	1.00000	4700.00000	.	1.00000	4699.32400
199	I18WF	IV	2.00000	2600.00000	.	2.00000	.
200	I2&13A	IV	1.00000	4000.00000	.	1.00000	4000.00000
201	I2&13B	IV	.	4800.00000	.	1.00000	4800.00000
202	I13&14A	IV	.	1500.00000	.	1.00000	382.97872
203	I13&14B	IV	.	2100.00000	.	1.00000	.
204	I14&15A	IV	.	3600.00000	.	1.00000	3600.00000
205	I14&15B	IV	.	4600.00000	.	1.00000	4600.00000
206	I15&18A	IV	.	6500.00000	.	1.00000	1625.83700
207	I15&18B	IV	.	8300.00000	.	1.00000	537.44411
208	I18&19A	IV	1.00000	4200.00000	.	1.00000	4200.00000
209	I18&19B	IV	.	5400.00000	.	1.00000	5400.00000
210	I20&21A	IV	1.00000	3100.00000	.	1.00000	3100.00000
211	I18&20A	IV	.	6000.00000	.	1.00000	6000.00000
212	I18&20B	IV	.	8400.00000	.	1.00000	8400.00000
213	I20&22A	IV	1.00000	4000.00000	.	1.00000	4000.00000
214	I23&22A	IV	.	4900.00000	.	1.00000	.
215	I16&23A	IV	1.00000	4000.00000	.	1.00000	4000.00000
216	I16&23B	IV	.	5700.00000	.	1.00000	5700.00000
217	I16&17A	IV	1.00000	4500.00000	.	1.00000	4500.00000
218	I15&16A	IV	.	3300.00000	.	1.00000	3300.00000
219	I15&16B	IV	1.00000	4600.00000	.	1.00000	4600.00000
220	X1W1	LL	.	47.60000	.	2364.00000	12.34000
221	X1W2	LL	.	47.60000	.	4727.00000	12.00000
222	X2W1	BS	725.00000	34.10000	.	874.00000	.
223	X2W2	BS	223.00000	34.10000	.	1749.00000	.
224	X13W1	BS	192.00000	45.92000	.	245.00000	.
225	X13W2	BS	140.00000	45.92000	.	490.00000	.
226	X14W1	BS	48.00000	25.66000	.	105.00000	.
227	X14W2	BS	37.00000	25.66000	.	210.00000	.
228	X15W1	UL	227.00000	25.66000	.	227.00000	.
229	X15W2	BS	79.00000	25.66000	.	454.00000	.
230	X16W1	UL	46.00000	17.22000	.	46.00000	-46.11000
231	X16W2	UL	90.00000	17.22000	.	90.00000	-48.29000
232	X21W1	BS	21.00000	28.19000	.	23.00000	.
233	X21W2	BS	28.00000	28.19000	.	45.00000	.
234	X1WF1	LL	.	47.60000	.	875.00000	12.34000
235	X1WF2	LL	.	47.60000	.	1750.00000	16.28374
236	X2WF1	LL	.	34.10000	.	875.00000	.
237	X2WF2	LL	.	34.10000	.	1750.00000	4.28378
238	X2S1	UL	787.00000	6.60000	.	787.00000	-27.50000
239	X2S2	UL	1575.00000	6.60000	.	1575.00000	-27.50000

COLUMNS SECTION

REV 2

	NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
	240	X2SF1	BS	.	6.60000	.	4700.00000	.
	241	X2SF2	BS	.	6.60000	.	1570.00000	.
A	242	X13WF-1	LL	.	45.92000	.	315.00000	.
A	243	X13WF-2	LL	.	45.92000	.	630.00000	.
	244	X13S-1	UL	112.00000	6.60000	.	112.00000	-39.32000
	245	X13S-2	UL	224.00000	6.60000	.	224.00000	-39.32000
A	246	X14WF-1	LL	.	25.66000	.	315.00000	.
	247	X14WF-2	LL	.	25.66000	.	630.00000	7.45924
	248	X14S-1	UL	39.00000	6.60000	.	39.00000	-19.06000
	249	X14S-2	UL	78.00000	6.60000	.	78.00000	-19.06000
	250	X15WF-1	BS	100.00000	25.66000	.	315.00000	.
A	251	X15WF-2	LL	.	25.66000	.	630.00000	.
	252	X15S-1	UL	119.00000	6.60000	.	119.00000	-19.06000
	253	X15S-2	UL	238.00000	6.60000	.	238.00000	-19.06000
	254	X18WF-1	BS	39.00000	29.04000	.	176.00000	.
	255	X18WF-2	LL	.	29.04000	.	350.00000	22.44000
	256	X18S-1	UL	105.00000	6.60000	.	105.00000	-22.44000
	257	X18S-2	BS	91.00000	6.60000	.	210.00000	.
	258	X19S-1	UL	200.00000	6.60000	.	200.00000	-41.89000
	259	X19S-2	BS	290.00000	6.60000	.	1100.00000	.
	260	X20S-1	BS	23.00000	3.00000	.	60.00000	.
	261	X20S-2	BS	25.00000	3.00000	.	120.00000	.
	262	X22S-1	UL	35.00000	6.60000	.	35.00000	-90.82000
	263	X22S-2	BS	46.00000	6.60000	.	70.00000	.
	264	X23S-1	UL	32.00000	6.60000	.	32.00000	-121.18000
	265	X23S-2	BS	46.00000	6.60000	.	64.00000	.
	266	Z1&2-1	LL	.	18.85000	.	7000.00000	20.01000
	267	Z1&2-2	LL	.	19.26000	.	7000.00000	20.76000
	268	Z2&1-1	BS	1148.00000	1.16000	.	7000.00000	.
	269	Z2&1-2	BS	1312.00000	1.50000	.	7000.00000	.
	270	Z2&13-1	LL	.	12.17000	.	360.00000	0.35000
	271	Z2&13-2	LL	.	15.46000	.	721.00000	3.64000
	272	Z13&2-1	LL	.	17.23000	.	360.00000	29.05000
	273	Z13&2-2	LL	.	20.52000	.	721.00000	32.34000
	274	Z13&14-1	LL	.	14.46000	.	164.00000	34.72000
	275	Z13&14-2	LL	.	17.54000	.	329.00000	44.18298
	276	Z14&13-1	LL	.	34.72000	.	164.00000	14.46000
	277	Z14&13-2	LL	.	37.80000	.	329.00000	17.54000
	278	Z14&15-1	LL	.	12.12000	.	262.00000	12.12000
	279	Z14&15-2	LL	.	19.47000	.	525.00000	19.47000
	280	Z15&14-1	LL	.	12.12000	.	262.00000	12.12000
	281	Z15&14-2	LL	.	19.47000	.	525.00000	19.47000
	282	Z15&16-1	LL	.	36.31000	.	262.00000	32.93000
	283	Z15&18-2	LL	.	60.88000	.	525.00000	79.94000
	284	Z18&19-1	BS	6.00000	19.45000	.	262.00000	.

REV 2

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
285	Z18&19-2	LL	.	22.91000	.	525.00000	22.91000
286	Z20&21-1	LL	.	63.36000	.	87.00000	38.17000
287	Z20&21-2	LL	.	75.63000	.	175.00000	50.44000
288	Z18&20-1	LL	.	117.28000	.	164.00000	143.32000
289	Z18&20-2	LL	.	144.71000	.	329.00000	148.31000
290	Z20&22-1	BS	4.00000	94.42000	.	87.00000	.
291	Z20&22-2	LL	.	110.29000	.	175.00000	106.69000
292	Z23&22-1	LL	.	101.71000	.	87.00000	132.07000
293	Z23&22-2	LL	.	121.12000	.	175.00000	149.12000
294	Z16&23-1	BS	22.00000	64.45000	.	164.00000	.
295	Z16&23-2	LL	.	88.76000	.	329.00000	147.67000
296	Z16&17-1	BS	7.00000	140.00000	.	87.00000	.
297	Z16&17-2	BS	9.00000	175.03000	.	175.00000	.
298	Z15&16-1	BS	44.00000	37.67000	.	164.00000	.
299	Z15&16-2	BS	27.00000	39.85000	.	329.00000	.
300	PE1	BS	10.96000	0.48000	.	17.50000	.
301	PE2	UL	5.83000	0.34000	.	5.83000	-0.13000
302	PE13	UL	2.65000	0.46000	.	2.65000	-0.13000
303	PE14	BS	0.94000	0.26000	.	1.07000	.
304	PE15	BS	2.45000	0.26000	.	2.60000	.
305	PE16	UL	0.34000	0.17000	.	0.34000	-0.47000
306	PE18	UL	0.77000	0.29000	.	0.77000	-3610.82111
307	PE19	UL	1.48000	0.07000	.	1.48000	-3611.23111
308	PE20	BS	0.41300	0.03000	.	0.44000	.
309	PE21	UL	0.17000	0.28000	.	0.17000	-0.38000
310	PE22	UL	0.26000	0.07000	.	0.26000	-0.90000
311	PE23	UL	0.23000	0.07000	.	0.23000	-1.21000
312	PZ1&2	LL	.	0.19000	.	30.00000	0.20000
313	PZ2&1	BS	1.32000	0.01000	.	30.00000	.
314	PZ2&13	BS	0.61000	0.12000	.	2.96000	.
315	PZ13&2	LL	.	0.17000	.	2.96000	0.29000
316	PZ13&14	LL	.	0.15000	.	1.35000	0.48000
317	PZ14&13	LL	.	0.35000	.	1.35000	0.02000
318	PZ14&15	LL	.	0.12000	.	2.15000	0.12000
319	PZ15&14	LL	.	0.12000	.	2.15000	0.12000
320	PZ15&18	BS	.	0.36000	.	2.15000	.
321	PZ18&19	BS	0.73000	0.19000	.	2.15000	.
322	PZ20&21	BS	0.05500	0.63000	.	0.72000	.
323	PZ18&20	LL	.	1.17000	.	1.35000	3612.25111
324	PZ20&22	BS	0.15200	0.94000	.	0.72000	.
325	PZ23&22	LL	.	1.02000	.	0.72000	1.33000
326	PZ16&23	BS	0.35000	0.64000	.	1.35000	.
327	PZ16&17	BS	0.07500	1.41000	.	0.72000	.
328	PZ15&16	BS	0.74000	0.38000	.	1.35000	.

Appendix D

MXINT Solution to Water Supply Model–Revision 3 (Decomposed Model–South Half)

REV 3
SOLUTION

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
1	OBJECT	BS	84876.09930	-84876.09930	NONE	NONE	1.00000
2	D1-1	LL	1148.00000	.	1148.00000	NONE	-35.26000
3	D1-2	LL	1312.00000	.	1312.00000	NONE	-35.60000
4	D2-1	LL	364.00000	.	364.00000	NONE	-34.10000
5	D2-2	LL	486.00000	.	486.00000	NONE	-34.10000
6	D3-1	LL	241.00000	.	241.00000	NONE	-45.27000
7	D3-2	LL	161.00000	.	161.00000	NONE	-18.90000
8	D4-1	LL	248.00000	.	248.00000	NONE	-59.42000
9	D4-2	LL	248.00000	.	248.00000	NONE	-6.60000
10	D5-1	LL	51.00000	.	51.00000	NONE	-68.78000
11	D5-2	LL	60.00000	.	60.00000	NONE	-18.60000
12	D6-1	LL	86.00000	.	86.00000	NONE	-9.18000
13	D6-2	LL	98.00000	.	98.00000	NONE	-10.60000
14	D7-1	LL	10.00000	.	10.00000	NONE	-18.91000
15	D7-2	LL	19.00000	.	19.00000	NONE	-18.91000
16	D8-1	LL	124.00000	.	124.00000	NONE	-3.00000
17	D8-2	LL	150.00000	.	150.00000	NONE	-3.00000
18	D9-1	LL	22.00000	.	22.00000	NONE	-18.91000
19	D9-2	LL	26.00000	.	26.00000	NONE	-18.91000
20	D10-1	LL	7.00000	.	7.00000	NONE	-18.91000
21	D10-2	LL	8.00000	.	8.00000	NONE	-18.91000
22	D11-1	LL	79.00000	.	79.00000	NONE	-3.00000
23	D11-2	LL	94.00000	.	94.00000	NONE	-3.00000
24	D12-1	LL	53.00000	.	53.00000	NONE	-54.43000
25	D12-2	LL	66.00000	.	66.00000	NONE	-6.60000
26	FW1-1	BS	.	2364.00000	NONE	2364.00000	.
27	FW1-2	BS	.	4727.00000	NONE	4727.00000	.
28	FW2-1	BS	778.00000	96.00000	NONE	874.00000	.
29	FW2-2	BS	223.00000	1526.00000	NONE	1749.00000	.
30	FW3-1	BS	188.00000	.	NONE	188.00000	.
31	FW3-2	BS	161.00000	215.00000	NONE	376.00000	.
32	FW4-1	BS	35.00000	205.00000	NONE	240.00000	.
33	FW4-2	BS	.	480.00000	NONE	480.00000	.
34	FW8-1	BS	.	437.00000	NONE	437.00000	.
35	FW8-2	BS	.	874.00000	NONE	874.00000	.
36	FW9-1	BS	10.00000	.	NONE	10.00000	.
37	FW9-2	BS	.	19.00000	NONE	19.00000	.
38	FS2-1	BS	787.00000	.	NONE	787.00000	.
39	FS2-2	BS	1575.00000	.	NONE	1575.00000	.
40	FS4-1	BS	234.00000	.	NONE	234.00000	.
41	FS4-2	BS	249.00000	220.00000	NONE	469.00000	.
42	FS5-1	BS	30.00000	.	NONE	30.00000	.
43	FS5-2	BS	59.00000	.	NONE	59.00000	.
44	FS6-1	BS	46.00000	.	NONE	46.00000	.

REV 3

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
45	FS6-2	BS	63.00000	.	NONE	63.00000	.
46	FS8-1	BS	164.00000	228.00000	NONE	392.00000	.
47	FS8-2	BS	185.00000	165.00000	NONE	350.00000	.
48	FS11-1	BS	97.00000	148.00000	NONE	245.00000	.
49	FS11-2	BS	94.00000	396.00000	NONE	490.00000	.
50	FS12-1	BS	35.00000	.	NONE	35.00000	.
51	FS12-2	BS	66.00000	4.00000	NONE	70.00000	.
52	FS2F-1	UL	.	.	NONE	.	27.50000
53	FS2F-2	UL	.	.	NONE	.	27.50000
54	FW1F-1	BS	.	.	NONE	.	.
55	FW1F-2	UL	.	.	NONE	.	4.28374
56	FW2F-1	BS	.	.	NONE	.	.
57	FW2F-2	UL	.	.	NONE	.	4.28378
58	FW3F-1	UL	.	.	NONE	.	26.36000
59	FW3F-2	BS	.	.	NONE	.	.
60	FW4F-1	UL	.	.	NONE	.	.
61	FW4F-2	UL	.	.	NONE	.	7.45788
62	FW5F-1	UL	.	.	NONE	.	43.12000
63	FW5F-2	BS	.	.	NONE	.	.
64	FW6F-1	BS	.	.	NONE	.	.
65	FW6F-2	UL	.	.	NONE	.	7.45900
66	FW7F-1	BS	-78.00000	78.00000	NONE	.	.
67	FW7F-2	BS	-156.00000	156.00000	NONE	.	.
68	FW8F-1	BS	.	.	NONE	.	.
69	FW8F-2	UL	.	.	NONE	.	10.28464
70	FW9F-1	BS	-14.00000	14.00000	NONE	.	.
71	FW9F-2	BS	-26.00000	26.00000	NONE	.	.
72	FW10F-1	BS	-19.00000	19.00000	NONE	.	.
73	FW10F-2	BS	-44.00000	44.00000	NONE	.	.
74	FT11ABC1	BS	.	.	NONE	.	.
75	FT11ABC2	UL	.	.	NONE	.	47.85145
76	Z1&3*1	BS	-34.00000	34.00000	NONE	.	.
77	Z1&3*2	BS	-175.00000	175.00000	NONE	.	.
78	Z3&1*1	BS	-87.00000	87.00000	NONE	.	.
79	Z3&1*2	BS	-175.00000	175.00000	NONE	.	.
80	Z3&4*1	BS	.	.	NONE	.	.
81	Z3&4*2	BS	.	.	NONE	.	.
82	Z4&3*1	BS	.	.	NONE	.	.
83	Z4&3*2	UL	.	.	NONE	.	2.42000
84	Z4&5*1	BS	-66.00000	66.00000	NONE	.	.
85	Z4&5*2	BS	-174.00000	174.00000	NONE	.	.
86	Z5&4*1	BS	-87.00000	87.00000	NONE	.	.
87	Z5&4*2	BS	-175.00000	175.00000	NONE	.	.
88	Z5&6*1	BS	.	.	NONE	.	.
89	Z5&6*2	BS	.	.	NONE	.	.

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
90	Z6&5*1	UL	.	.	NONE	.	45.00000
91	Z6&5*2	BS	.	.	NONE	.	.
92	Z6&7A1	BS	.	.	NONE	.	.
93	Z6&7A2	UL	.	.	NONE	.	12.00000
94	Z6&8*1	BS	-87.00000	87.00000	NONE	.	.
95	Z6&8*2	BS	-175.00000	175.00000	NONE	.	.
96	Z8&6*1	BS	-47.00000	47.00000	NONE	.	.
97	Z8&6*2	BS	-140.00000	140.00000	NONE	.	.
98	Z8&9A1	BS	.	.	NONE	.	.
99	Z8&9A2	UL	.	.	NONE	.	24.57143
100	Z9&10A1	BS	.	.	NONE	.	.
101	Z9&10A2	UL	.	.	NONE	.	13.71429
102	Z8&11*1	BS	.	.	NONE	.	.
103	Z8&11*2	BS	.	.	NONE	.	.
104	Z11&12A1	BS	-69.00000	69.00000	NONE	.	.
105	Z11&12A2	BS	-175.00000	175.00000	NONE	.	.
106	TRPLANT	BS	.	1.00000	NONE	1.00000	.
107	PZ11&3	BS	1.00000	.	NONE	1.00000	.
108	PZI3&4	BS	.	1.00000	NONE	1.00000	.
109	PZI4&5	BS	1.00000	.	NONE	1.00000	.
110	PZI5&6	BS	.	1.00000	NONE	1.00000	.
111	PZI6&8	BS	1.00000	.	NONE	1.00000	.
112	PZI8&11	BS	.	1.00000	NONE	1.00000	.
113	PD1	LL	12.28000	.	12.28000	NONE	-0.48000
114	PD2	LL	3.90000	.	3.90000	NONE	-0.47000
115	PD3	LL	1.29000	.	1.29000	NONE	-0.19000
116	PD4	LL	2.66000	.	2.66000	NONE	-0.59000
117	PD5	LL	0.55000	.	0.55000	NONE	-0.68000
118	PD6	LL	0.92000	.	0.92000	NONE	-0.32000
119	PD7	BS	0.72000	-0.64500	0.07500	NONE	.
120	PD8	LL	1.33000	.	1.33000	NONE	-0.26000
121	PD9	LL	0.23400	.	0.23400	NONE	-0.19000
122	PD10	BS	0.21600	-0.14100	0.07500	NONE	.
123	PD11	LL	0.84000	.	0.84000	NONE	-0.03000
124	PD12	LL	0.56200	.	0.56200	NONE	-0.54000
125	PS1	BS	10.35000	7.16000	NONE	17.51000	.
126	PS2	BS	5.83000	.	NONE	5.83000	.
127	PS3	BS	1.29000	0.11000	NONE	1.40000	.
128	PS4	BS	2.99000	0.51000	NONE	3.50000	.
129	PS5	BS	0.22000	.	NONE	0.22000	.
130	PS6	BS	0.33000	.	NONE	0.33000	.
131	PS8	BS	1.92000	4.23000	NONE	6.15000	.
132	PS9	BS	0.01800	0.05200	NONE	0.07000	.
133	PS11	BS	1.14200	0.67800	NONE	1.82000	.
134	PS12	BS	0.26000	.	NONE	0.26000	.

REV 3

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
135	P1&3	BS	-0.72000	0.72000	NONE	.	.
136	P3&1	BS	-0.72000	0.72000	NONE	.	.
137	P3&4	BS	.	.	NONE	.	.
138	P4&3	BS	.	.	NONE	.	.
139	P4&5	BS	-0.39000	0.39000	NONE	.	.
140	P5&4	BS	-0.72000	0.72000	NONE	.	.
141	P5&6	UL	.	.	NONE	.	0.21000
142	P6&5	BS	.	.	NONE	.	.
143	P6&7A	BS	.	.	NONE	.	.
144	P6&8	BS	-0.13000	0.13000	NONE	.	.
145	P8&6	BS	-0.72000	0.72000	NONE	.	.
146	P8&9A	BS	.	.	NONE	.	.
147	P9&10A	BS	.	.	NONE	.	.
148	P8&11	BS	.	.	NONE	.	.
149	P11&12A	BS	-0.41800	0.41800	NONE	.	.

REV 3

COLUMNS SECTION

	NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
	193	I1WF	IV	.	7500.00000	.	1.00000	.
	194	I2WF	IV	.	7500.00000	.	1.00000	.
	195	I2SF	IV	.	83000.00000	.	1.00000	-89443.23600
	196	I3WF	IV	.	4700.00000	.	1.00000	-3603.89400
	197	I4WF	IV	.	4700.00000	.	1.00000	.
	198	I5WF	IV	.	4700.00000	.	1.00000	-8884.56800
	199	I6WF	IV	.	4700.00000	.	1.00000	.
	200	I7WF	IV	1.00000	2600.00000	.	1.00000	2600.00000
	201	I8WF	IV	.	3600.00000	.	1.00000	.
	202	I9WF	IV	1.00000	1500.00000	.	1.00000	1499.95896
	203	I10WF	IV	1.00000	1500.00000	.	1.00000	1500.00000
	204	I11ATF	IV	.	11100.00000	.	1.00000	3587.30303
	205	I11BTF	IV	.	24500.00000	.	1.00000	.
	206	I11CTF	IV	.	32400.00000	.	1.00000	5220.28017
	207	I1&3A	IV	1.00000	1200.00000	.	1.00000	1200.00000
	208	I1&3B	IV	.	1700.00000	.	1.00000	1700.00000
	209	I3&4A	IV	.	1700.00000	.	1.00000	903.82000
	210	I3&4B	IV	.	2200.00000	.	1.00000	929.50000
	211	I4&5A	IV	1.00000	700.00000	.	1.00000	700.00000
	212	I4&5B	IV	.	1100.00000	.	1.00000	1100.00000
	213	I5&6A	IV	.	1500.00000	.	1.00000	-2415.15120
	214	I5&6B	IV	.	2100.00000	.	1.00000	-5280.28350
	215	I6&7A	IV	.	2100.00000	.	1.00000	.
	216	I6&8A	IV	1.00000	700.00000	.	1.00000	700.00000
	217	I6&8B	IV	.	1100.00000	.	1.00000	1100.00000
	218	I8&9A	IV	.	4300.00000	.	1.00000	.
	219	I9&10A	IV	.	2400.00000	.	1.00000	.
	220	I8&11A	IV	.	4500.00000	.	1.00000	4500.00000
	221	I8&11B	IV	.	6300.00000	.	1.00000	6300.00000
	222	I11&12A	IV	1.00000	4200.00000	.	1.00000	4200.00000
	223	X1W1	LL	.	47.60000	.	2364.00000	12.34000
	224	X1W2	LL	.	47.60000	.	4727.00000	12.00000
	225	X2W1	BS	778.00000	34.10000	.	874.00000	.
	226	X2W2	BS	223.00000	34.10000	.	1749.00000	.
	227	X3W1	UL	188.00000	18.90000	.	188.00000	-26.37000
	228	X3W2	BS	161.00000	18.90000	.	376.00000	.
	229	X4W1	BS	35.00000	59.42000	.	240.00000	.
	230	X4W2	LL	.	59.42000	.	480.00000	52.82000
	231	X8W1	LL	.	25.66000	.	437.00000	22.66000
	232	X8W2	LL	.	25.66000	.	874.00000	22.66000
A	233	X9W1	UL	10.00000	18.91000	.	10.00000	.
A	234	X9W2	LL	.	18.91000	.	19.00000	.
	235	X1WF1	LL	.	47.60000	.	875.00000	12.34000
	236	X1WF2	LL	.	47.60000	.	1750.00000	16.28374
A	237	X2WF1	LL	.	34.10000	.	875.00000	.
	238	X2WF2	LL	.	34.10000	.	1750.00000	4.28378
	239	X2S1	UL	787.00000	6.60000	.	787.00000	-27.50000

REV 3

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
240	X2S2	UL	1575.00000	6.60000	.	1575.00000	-27.50000
241	X2SF1	BS	.	6.60000	.	4700.00000	.
242	X2SF2	BS	.	6.60000	.	1570.00000	.
243	X3WF1	BS	.	18.91000	.	315.00000	.
244	X3WF2	LL	.	18.91000	.	630.00000	0.01000
245	X4WF-1	BS	.	59.42000	.	315.00000	.
246	X4WF-2	LL	.	59.42000	.	630.00000	60.27788
247	X4S-1	UL	234.00000	6.60000	.	234.00000	-52.82000
248	X4S-2	BS	249.00000	6.60000	.	469.00000	.
249	X5WF-1	BS	.	25.66000	.	315.00000	.
250	X5WF-2	LL	.	25.66000	.	630.00000	7.06000
251	X5S-1	UL	30.00000	6.60000	.	30.00000	-62.18000
252	X5S-2	UL	59.00000	6.60000	.	59.00000	-12.00000
253	X6WF-1	LL	.	23.97000	.	315.00000	14.79000
254	X6WF-2	LL	.	23.97000	.	630.00000	20.62900
255	X6S-1	UL	46.00000	6.60000	.	46.00000	-2.58000
256	X6S-2	UL	63.00000	6.60000	.	63.00000	-4.20000
257	X7WF-1	BS	10.00000	18.91000	.	88.00000	.
258	X7WF-2	BS	19.00000	18.91000	.	175.00000	.
259	X8WF-1	LL	.	25.66000	.	175.00000	22.66000
260	X8WF-2	LL	.	25.66000	.	350.00000	32.94464
261	X8S-1	BS	164.00000	3.00000	.	392.00000	.
262	X8S-2	BS	185.00000	3.00000	.	350.00000	.
263	X9WF-1	BS	12.00000	18.91000	.	26.00000	.
264	X9WF-2	BS	26.00000	18.91000	.	52.00000	.
265	X10WF-1	BS	7.00000	18.91000	.	26.00000	.
266	X10WF-2	BS	8.00000	18.91000	.	52.00000	.
267	X11TA-1	LL	.	61.40000	.	79.00000	58.40000
268	X11TA-2	LL	.	83.80000	.	157.00000	128.65145
269	X11TB-1	LL	.	56.90000	.	216.00000	53.90000
270	X11TB-2	LL	.	68.50000	.	512.00000	113.35145
271	X11TC-1	LL	.	52.80000	.	284.00000	49.80000
272	X11TC-2	LL	.	63.60000	.	568.00000	108.45145
273	X11S-1	BS	97.00000	3.00000	.	245.00000	.
274	X11S-2	BS	94.00000	3.00000	.	490.00000	.
275	X12S-1	UL	35.00000	6.60000	.	35.00000	-47.83000
276	X12S-2	BS	66.00000	6.60000	.	70.00000	.
277	Z1&2-1	LL	.	18.85000	.	7000.00000	20.01000
278	Z1&2-2	LL	.	19.26000	.	7000.00000	20.76000
279	Z2&1-1	BS	1201.00000	1.16000	.	7000.00000	.
280	Z2&1-2	BS	1312.00000	1.50000	.	7000.00000	.
281	Z1&3-1	BS	53.00000	10.01000	.	164.00000	.
282	Z1&3-2	LL	.	12.09000	.	329.00000	28.79000
283	Z3&1-1	LL	.	25.58000	.	164.00000	35.59000
284	Z3&1-2	LL	.	30.66000	.	329.00000	13.96000

REV 3

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
285	Z3&4-1	LL	.	51.05000	.	262.00000	36.90000
286	Z3&4-2	LL	.	53.77000	.	525.00000	66.07000
287	Z4&3-1	LL	.	7.16000	.	262.00000	21.31000
288	Z4&3-2	BS	.	9.88000	.	525.00000	.
289	Z4&5-1	BS	21.00000	9.36000	.	164.00000	.
290	Z4&5-2	BS	1.00000	12.00000	.	329.00000	.
291	Z5&4-1	LL	.	43.12000	.	164.00000	52.48000
292	Z5&4-2	LL	.	45.76000	.	329.00000	57.76000
293	Z5&6-1	LL	.	14.60000	.	164.00000	74.20000
294	Z5&6-2	LL	.	18.94000	.	329.00000	26.74000
295	Z6&5-1	BS	.	14.60000	.	164.00000	.
296	Z6&5-2	LL	.	18.94000	.	329.00000	11.14000
297	Z6&7-1	LL	.	56.59000	.	87.00000	46.86000
298	Z6&7-2	LL	.	56.59000	.	175.00000	60.48000
299	Z6&8-1	LL	.	18.84000	.	164.00000	25.02000
300	Z6&8-2	LL	.	20.46000	.	329.00000	28.26000
301	Z8&6-1	BS	40.00000	6.18000	.	164.00000	.
302	Z8&6-2	BS	35.00000	7.80000	.	329.00000	.
303	Z8&9-1	LL	.	96.92000	.	87.00000	81.01000
304	Z8&9-2	LL	.	118.32000	.	175.00000	126.98143
305	Z9&10-1	LL	.	82.57000	.	87.00000	82.57000
306	Z9&10-2	LL	.	105.32000	.	175.00000	119.03429
307	Z8&11-1	LL	.	45.65000	.	164.00000	45.65000
308	Z8&11-2	LL	.	57.88000	.	329.00000	57.88000
309	Z11&12-1	BS	18.00000	51.43000	.	87.00000	.
310	Z11&12-2	LL	.	63.60000	.	175.00000	60.00000
311	PE1	BS	10.35000	0.48000	.	17.50000	.
312	PE2	UL	5.83000	0.34000	.	5.83000	-0.13000
313	PE3	BS	1.29000	0.19000	.	1.40000	.
314	PE4	BS	2.99000	0.59000	.	3.50000	.
315	PE5	UL	0.22000	0.26000	.	0.22000	-0.42000
316	PE6	UL	0.33000	0.24000	.	0.33000	-0.08000
317	PE8	BS	1.92000	0.26000	.	6.15000	.
318	PE9	BS	0.01800	0.19000	.	0.07000	.
319	PE11	BS	1.14200	0.03000	.	1.82000	.
320	PE12	UL	0.26000	0.07000	.	0.26000	-0.47000
321	PZ1&2	LL	.	0.19000	.	30.00000	0.20000
322	PZ2&1	BS	1.93000	0.01000	.	30.00000	.
323	PZ1&3	LL	.	0.10000	.	1.35000	0.39000
324	PZ3&1	LL	.	0.29000	.	1.35000	.
325	PZ3&4	LL	.	0.51000	.	2.15000	0.11000
326	PZ4&3	LL	.	0.07000	.	2.15000	0.47000
327	PZ4&5	BS	0.33000	0.09000	.	1.35000	.
328	PZ5&4	LL	.	0.43000	.	1.35000	0.52000
329	PZ5&6	LL	.	0.15000	.	1.35000	0.72000

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
330	PZ6&5	BS	.	0.15000	.	1.35000	.
331	PZ6&7	LL	.	0.57000	.	0.72000	0.89000
332	PZ6&8	LL	.	0.19000	.	1.35000	0.25000
333	PZ8&6	BS	0.59000	0.06000	.	1.35000	.
334	PZ8&9	LL	.	0.97000	.	0.72000	1.04000
335	PZ9&10	LL	.	0.83000	.	0.72000	1.02000
336	PZ8&11	LL	.	0.46000	.	1.35000	0.69000
337	PZ11&12	BS	0.30200	0.51000	.	0.72000	.

Appendix E

Source Deck Listing for Program "Assem"

```
$ SET BCD
FILE  20=TEMPOCARDS
FILE  7=INTORCARDS
FILE  5=DUM
C
C                    MAIN PROGRAM ASSEMBLE
C
      COMMON C(7,15),D(5,7,15,20),E(5,7,15),P(5,15),B(5,20),R(5,20)
     *      ,NKA(15),KA(15,20),X(20)
     *      ,IOPE,IOPM,IOPWP,IOPP,NI,NJ,NK,NL,NRB,NRR,NRP,NRC,NRE,NRD
     *      ,NWRT
      NWRT=20
      NRB=5
      NRR=5
      NRP=5
      NRC=5
      NRE=5
      NRD=5
      READ(5,500) IOPE,IOPM,IOPWP,IOPP,IOPUG
  500 FORMAT(5I2)
      WRITE(6,502)IOPE,IOPM,IOPWP,IOPP,IOPUG
  502 FORMAT(1H ,'ECHO OF INPUT DATA FOR PROGRAM ASSEMBLE'/
     *       1HO,'IOPE=',I2,',    IOPM=',I2,',    IOPWP=',I2
     *      ,',    IOPP =',I2,',    IOPUG =',I2)
C
C                    READ THE B(J,K), R(J,K), P(J,I), C(L,I), E(J,L,I),
C                    AND THE D(J,M,I,K) MATRICIES
C
      CALL READ1
C
C                    CALCULATE THE RIGHT HAND SIDE OF THE CONSTRAINT
C                    EQUATIONS AND STORE IN R(J,K)
C
      DO 2 K=1,NK
      DO 1 IX=1,20
    1 X(IX)=0.0
      DO 4 I=1,NI
      DO 6 IA=1,NJ
      DO 8 IB=1,NJ
      X(IA)=D(IA,IB,I,K)*P(IB,I)+X(IA)
    8 CONTINUE
    6 CONTINUE
    4 CONTINUE
      DO 10 J=1,NJ
      R(J,K)=X(J)+B(J,K)-R(J,K)
   10 CONTINUE
    2 CONTINUE
C
C                    CALCULATE COEFFICIENTS FOR THE CONSTRAINT EQUATIONS
C                    AND STORE IN THE D(J,L,I,K) MATRIX
      DO 12 K=1,NK
      DO 14 I=1,NI
      DO 16 IA=1,NJ
      DO 18 IX=1,NL
   18 X(IX)=0.0
      DO 20 ID=1,NL
      DO 22 IB=1,NJ
      X(ID)=X(ID)+D(IA,IB,I,K)*E(IB,ID,I)
   22 CONTINUE
   20 CONTINUE
      DO 24 ID=1,NL
```

```
      D(IA,ID,I,K)=X(ID)
   24 CONTINUE
   16 CONTINUE
   14 CONTINUE
   12 CONTINUE
C
C                       PRINT OUTPUT - MATRICIES  R(J,K) AND D(J,L,I,K)
C                       ON THE LINE PRINTER
      IF(IOPM.GT.0) CALL WRITEM
C
C                       WRITE OUTPUT ON FILE NWRT IN THE PROPER
C                       FORMAT FOR I.P. PROGRAM
C
      IF(IOPWP.GT.0) CALL WRITEF
      LOCK 20
C
C                       WRITE OUTPUT IN PROPER FORMAT ON LISTING
C                       SHEET AND PUNCH CARDS FOR --ZERONE PROGRAM
C
      IF(IOPUG .GT. 0) CALL ZERONE
      LOCK 7                                                            00000560
      END
      SUBROUTINE READ1
      COMMON C(7,15),D(5,7,15,20),E(5,7,15),P(5,15),B(5,20),R(5,20)
     *      ,NKA(15),KA(15,20),X(20)
     *      ,IOPE,IOPM,IOPWP,IOPP,NI,NJ,NK,NL,NRB,NRR,NRP,NRC,NRE,NRD
     *      ,NWRT
      READ(5,500) NI,NJ,NK,NL
  500 FORMAT(4I5)
      WRITE(6,502)NI,NJ,NK,NL
  502 FORMAT(1H0,'NI =',I3,',     NL =',I3,',     NK =',I3,',     NL =',I3)
      IF(IOPE.GT.0) WRITE(6,503)
  503 FORMAT(1H0 / 1H0,'WATER QUALITY STANDARDS (BY J)')
      DO 2 K=1,NK
      READ(NRB,504) (B(J,K),J=1,NJ)
  504 FORMAT(10F8.0)
      IF(IOPE.GT.0) WRITE(6,506) K,(B(J,K),J=1,NJ)
  506 FORMAT(1H ,'K=',I2,',',5X,10F10.3)
    2 CONTINUE
      IF(IOPE.GT.0) WRITE(6,512)
  512 FORMAT(1H0 /1H,'BASE WATER QUALITY (BY J)')
      DO 6 K=1,NK
      READ(NRR,504) (R(J,K),J=1,NJ)
      IF(IOPE.GT.0) WRITE(6,506) K,(R(J,K),J=1,NJ)
    6 CONTINUE
      IF(IOPE.GT.0) WRITE(6,514)
  514 FORMAT(1H0,1H ,'BASE EFFLUENT CONCENTRATIONS (BY J)')
      DO 8 I=1,NI
      READ(NRP,504) (P(J,I),J=1,NJ)
      IF(IOPE.GT.0) WRITE(6,510) I,(P(J,I),J=1,NJ)
    8 CONTINUE
      IF(IOPE.GT.0) WRITE(6,508)
  508 FORMAT(1H0 / 1H ,'TREATMENT COSTS (BY L)')
      DO 4 I=1,NI
      READ(NRC,504) (C(L,I),L=1,NL)
      IF(IOPE.GT.0) WRITE(6,510) I,(C(L,I),L=1,NL)
  510 FORMAT(1H ,'I=',I2,',',5X,10F10.3)
    4 CONTINUE
      IF(IOPE.GT.0) WRITE(6,516)
  516 FORMAT(1H0,1H ,'EFFLUENT CONCENTRATIONS FOR VARIOUS TREATMENT'
     *              ,' LEVELS (BY L)')
      DO 10 I=1,NI
      IF(IOPE.GT.0) WRITE(6,518) I
```

```
 518 FORMAT(1H0,'I =',I3)
     DO 12 J=1,NJ
     READ(NRE,504) (E(J,L,I),L=1,NL)
     IF(IOPE.GT.0) WRITE(6,520) J,(E(J,L,I),L=1,NL)
 520 FORMAT(1H ,3X,'J=',I2,',',5X,10F10.3)
  12 CONTINUE
  10 CONTINUE
     IF(IOPE.GT.0) WRITE(6,522)
 522 FORMAT(1H0 / 1H ,'THE D MATRICIES (BY J)')
     DO 1 I=1,NI
     DO 1 J=1,NJ
     DO 1 M=1,NJ
     DO 1 K=1,NK
     D(J,M,I,K)=0.0
   1 CONTINUE
     DO 14 I=1,NI
     IF(IOPE.GT.0) WRITE(6,518) I
     READ(NRD,532) NKADUM,(KA(I,IK),IK=1,20)
532  FORMAT(21I3)
     NKA(I)=NKADUM
     IF(IOPE.GT.0) WRITE(6,526) I,NKA(I)
 524 FORMAT(21I3)
 526 FORMAT(1H ,'NKA(',I2,') =',I3)
     IF(NKADUM.LT.1) GO TO 14
 530 FORMAT(1H0,5X,'AFFECT OF I=',I3,',  ON  K =',I3)
     DO 16 IK=1,NKADUM
     K=KA(I,IK)
     IF(IOPE.GT.0) WRITE(6,530)  I,K
     DO 18 J=1,NJ
     READ(NRD,504) (D(J,M,I,K),M=1,NJ)
     IF(IOPE.GT.0) WRITE(6,528) J,(D(J,M,I,K),M=1,NJ)
 528 FORMAT(1H ,8X,'J=',I2,',',5X,10F10.3)
  18 CONTINUE
  16 CONTINUE
  14 CONTINUE
     RETURN
     END
     SUBROUTINE WRITEM
     COMMON C(7,15),D(5,7,15,20),E(5,7,15),P(5,15),B(5,20),R(5,20)
    *       ,NKA(15),KA(15,20),X(20)
    *       ,IOPE,IOPM,IOPWP,IOPP,NI,NJ,NK,NL,NRB,NRR,NRP,NRC,NRE,NRD
    *       ,NWRT
     WRITE(6,500)
 500 FORMAT(1H1,'COST COEFFICIENTS' / 1H0)
     DO 2 I=1,NI
     WRITE(6,502) I,(C(L,I),L=1,NL)
 502 FORMAT(1H ,'I=',I2,',',5X,10F10.3)
   2 CONTINUE
     WRITE(6,504)
 504 FORMAT(1H1,'CONSTRAINT COEFFICIENTS AND RIGHT-HAND-SIDE VALUES' /
    *       ,1H0)
     DO 4 K=1,NK
     WRITE(6,506) K
 506 FORMAT(1H0 / 1H0,'K ,',I3)
     DO 6 I=1,NI
     WRITE(6,508) I
 508 FORMAT(1H0,3X,'I =',I3)
     DO 8 J=1,NJ
     WRITE(6,510) J,(D(J,L,I,K),L=1,NL)
 510 FORMAT(1H ,8X,'J=',I2,',',5X,7F12.3)
   8 CONTINUE
   6 CONTINUE
     WRITE(6,514)
```

```
514 FORMAT(1H0)
    DO 10 J=1,NJ
    WRITE(6,512) J,K,R(J,K)
512 FORMAT(1H ,3X,'RHS(J=',I2,',K=',I2,')',F12.3)
 10 CONTINUE
  4 CONTINUE
    RETURN
    END
    SUBROUTINE WRITEF
    COMMON C(7,15),D(5,7,15,20),E(5,7,15),P(5,15),B(5,20),R(5,20)
   *       ,NKA(15),KA(15,20),X(20)
   *       ,IOPE,IOPM,IOPWP,IOPP,NI,NJ,NK,NL,NRB,NRR,NRP,NRC,NRE,NRD
   *       ,NWRT
    WRITE(NWRT,500)
500 FORMAT('NAME',10X,'WLA'/'ROWS' / ' N  COST')
    DO 2 K=1,NK
    DO 4 J=1,NJ
    IF(R(J,K).LT.0.0) GOTO 3
    WRITE(NWRT,502) K,J
502 FORMAT(' L  ROWK',I2,'J',I1)
    GOTO 4
  3 WRITE(NWRT,503) K,J
503 FORMAT(' G  ROWK',I2,'J',I1)
  4 CONTINUE
  2 CONTINUE
    DO 6 I=1,NI
    WRITE(NWRT,504) I
504 FORMAT(' E  ROWI',I2)
  6 CONTINUE
    WRITE(NWRT,506)
506 FORMAT('COLUMNS' / '    ABC',7X,8H'MARKER',17X,8H'BIVORG')
    DO 8 I=1,NI
    DO 10 L=1,NL
    WRITE(NWRT,508) L,I,C(L,I)
508 FORMAT('    T',I1,'I',I2,5X,'COST',6X,F12.4)
    DO 12 K=1,NK
    DO 14 J=1,NJ
    IF(ABS(D(J,L,I,K)).LT.1.0E-6) GOTO 14
    IF(R(J,K).GE.0.0) GOTO 16
    D(J,L,I,K)=D(J,L,I,K)*(-1.0)
 16 WRITE(NWRT,510) L,I,K,J,D(J,L,I,K)
510 FORMAT('    T',I1,'I',I2,5X,'ROWK',I2,'J',I1,2X,F12.3)
 14 CONTINUE
 12 CONTINUE
    WRITE(NWRT,511) L,I,I
511 FORMAT('    T',I1,'I',I2,5X,'ROWI',I2,10X,'1.0')
 10 CONTINUE
  8 CONTINUE
    WRITE(NWRT,512)
512 FORMAT('    DEF',7X,8H'MARKER',17X,8H'BIVEND' / 'RHS')
    DO 18 K=1,NK
    DO 20 J=1,NJ
    R(J,K)=ABS(R(J,K))
    WRITE(NWRT,514) K,J,R(J,K)
514 FORMAT('    QVECT',5X,'ROWK',I2,'J',I1,2X,F12.4)
 20 CONTINUE
 18 CONTINUE
    DO 24 I=1,NI
    WRITE(NWRT,515) I
515 FORMAT('    QVECT',5X,'ROWI',I2,10X,'1.0')
 24 CONTINUE
    WRITE(NWRT,516)
516 FORMAT('ENDATA')
```

```
      RETURN
      END
      SUBROUTINE ZERONE
      DIMENSION ABC(200), JJ(3), II(3), XX(3)
      COMMON C(7,15),D(5,7,15,20),E(5,7,15),P(5,15),B(5,20),R(5,20)
     *       ,NKA(15),KB(15,20),X(20)
     *       ,IOPE,IOPM,IOPWP,IOPP,NI,NJ,NK,NL,NRB,NRR,NRP,NRC,NRE,NRD
     *       ,NRWT
      WRITE (6,400)
 400  FORMAT ('1', 10X, 'OBJECTIVE FUNCTION COSTS' / )
      WRITE (6,500) ((C(L,I),L=1,NL),I=1,NI)
      WRITE (7,500) ((C(L,I),L=1,NL),I=1,NI)
 500  FORMAT (7F10.3)
      DO 50 I = 1,NI
 50   ABC(I) = 1.0
      WRITE (6,401)
 401  FORMAT (//// 10X, 'RIGHT HAND SIDE VALUES' / )
      WRITE(6,500) ((R(J,K),J = 1,NJ),K = 1,NK), (ABC(I), I = 1,NI)
      WRITE(7,500) ((R(J,K),J = 1,NJ),K = 1,NK), (ABC(I), I = 1,NI)
      WRITE(6,402)
 402  FORMAT(//// 10X, 'MATRIX A(I,J) COMPONENTS' // 3( 1X, 'ROW',2X,
     1 'COL', 5X, 'VALUE' 3X) / )
      KK = 0
      LL = 0
      NN = NK * NJ
      DO 100 I = 1,NI
      NN = NN + 1
      DO 101 L = 1,NL
      LL = LL + 1
      MM = 0
      DO 102 K = 1,NK
      DO 103 J = 1,NJ
      MM = MM + 1
      IF(ABS(D(J,L,I,K )) .LT. 1.0E-6) GO TO 103
      KK = KK + 1
      JJ(KK) = LL
      II(KK) = MM
      XX(KK) = D(J,L,I,K) * (-1.0)
      IF (KK .NE. 3) GO TO 103
      WRITE(6,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
      WRITE(7,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
 501  FORMAT (3(2X,I3,2X,I3,F10.3))
      KK = 0
 103  CONTINUE
 102  CONTINUE
      KK = KK + 1
      JJ(KK) = LL
      II(KK) = NN
      XX(KK) = -1.0
      IF(KK .NE. 3) GO TO 101
      WRITE(6,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
      WRITE(7,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
      KK = 0
 101  CONTINUE
 100  CONTINUE
      IF(KK .EQ. 0) GO TO 110
      WRITE(6,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
      WRITE(7,501) (II(KA), JJ(KA), XX(KA),KA = 1,KK)
 110  RETURN
      END
  DATA
```

Appendix F

MXINT Solution to Small Water Quality Model
(Problem I)

XXXX

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
1	COST	BS	2291.72308	-2291.72308	NONE	NONE	1.00000
2	ROW1	BS	2.80000	0.50000	NONE	3.30000	.
3	ROW2	BS	2.80000	96.30000	NONE	99.10000	.
4	ROW3	BS	0.28000	0.72000	NONE	1.00000	.
5	ROW4	BS	1.40000	1.84000	NONE	3.24000	.
6	ROW5	BS	1.70000	2.50000	NONE	4.20000	.
7	ROW6	BS	3.40000	95.75000	NONE	99.15000	.
8	ROW7	BS	0.34000	0.41000	NONE	0.75000	.
9	ROW8	BS	1.87000	2.44900	NONE	4.31900	.
10	ROW9	BS	1.50000	2.10000	NONE	3.60000	.
11	ROW10	BS	1.80000	97.55000	NONE	99.35000	.
12	ROW11	BS	0.26000	0.64000	NONE	0.90000	.
13	ROW12	BS	2.36000	2.44200	NONE	4.80200	.
14	ROW13	BS	2.25000	1.90000	NONE	4.15000	.
15	ROW14	BS	1.70000	97.75000	NONE	99.45000	.
16	ROW15	BS	0.48000	1.02000	NONE	1.50000	.
17	ROW16	BS	1.58000	0.29200	NONE	1.87200	.
18	ROW17	BS	2.68923	2.31077	NONE	5.00000	.
19	ROW18	BS	1.44731	98.70269	NONE	100.15000	.
20	ROW19	UL	1.95000	.	NONE	1.95000	872.30769
21	ROW20	BS	4.78362	4.61838	NONE	9.40200	.
22	ROW21	EQ	1.00000	.	1.00000	1.00000	-972.01538
23	ROW22	EQ	1.00000	.	1.00000	1.00000	-386.33846
24	ROW23	EQ	1.00000	.	1.00000	1.00000	-1325.90769
25	ROW24	EQ	1.00000	.	1.00000	1.00000	-1308.46154

XXXX

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
49	T111	IV	.	.	.	1.00000	598.13846
50	T211	IV	.	441.00000	.	1.00000	646.60000
51	T311	IV	1.00000	815.00000	.	1.00000	.
52	T411	IV	.	1007.00000	.	1.00000	74.23846
53	T511	IV	.	406.00000	.	1.00000	219.06154
54	T611	IV	.	1446.00000	.	1.00000	515.23846
55	T711	IV	.	3014.00000	.	1.00000	2041.96462
56	T112	IV	.	.	.	1.00000	6.20000
57	T212	IV	.	196.00000	.	1.00000	71.35385
58	T312	IV	1.00000	334.00000	.	1.00000	.
59	T412	IV	.	422.00000	.	1.00000	43.51231
60	T512	IV	.	225.00000	.	1.00000	48.01538
61	T612	IV	.	618.00000	.	1.00000	239.51231
62	T712	IV	.	1507.00000	.	1.00000	1120.66154
63	T113	IV	.	.	.	1.00000	593.16923
64	T213	IV	.	594.00000	.	1.00000	1646.96923
65	T313	IV	1.00000	1134.00000	.	1.00000	.
66	T413	IV	.	1391.00000	.	1.00000	113.06923
67	T513	IV	.	504.00000	.	1.00000	137.63077
68	T613	IV	.	1985.00000	.	1.00000	707.06923
69	T713	IV	.	3832.00000	.	1.00000	2506.09231
70	T114	IV	0.99231	.	.	1.00000	.
71	T214	IV	.	594.00000	.	1.00000	157.64615
72	T314	IV	0.00769	1134.00000	.	1.00000	.
73	T414	IV	.	1391.00000	.	1.00000	108.70769
74	T514	IV	.	504.00000	.	1.00000	67.64615
75	T614	IV	.	1985.00000	.	1.00000	702.70769
76	T714	IV	.	3832.00000	.	1.00000	2523.53846

ESTIMATED CHARGE FOR B6700 USAGE

CPU TIME	3.44 SECS	$ 0.68	MEMORY	415.63 KW-SEC	$	0.69
I/O TIME	16.27 SECS	$ 0.81	LINES PRINTED	326	$	0.20
CARDS READ	224	$ 0.11	CARDS PUNCHED	0	$	0.00

SURCHARGE OF (+10%) 0.25
*** TOTAL COST = $2.74

Appendix G

MXINT Solution to Large Water Quality Model (Problem II)

ROWS SECTION

NUMBER	NAME	STATUS	ACTIVITY	SLACK ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL ACTIVITY
1	COST	BS	17100.00000	-17100.00000	NONE	NONE	1.00000
2	ROWK 1J1	BS	13.40000	8.40000	NONE	21.00000	.
3	ROWK 1J2	BS	14.07000	2.73000	NONE	16.00000	.
4	ROWK 1J3	BS	12.06000	9.74000	NONE	21.00000	.
5	ROWK 1J4	BS	14.73000	0.27000	NONE	15.00000	.
6	ROWK 2J1	BS	16.35000	3.85000	NONE	20.20000	.
7	ROWK 2J2	BS	17.43000	7.77000	NONE	25.20000	.
8	ROWK 2J3	BS	13.44000	6.76000	NONE	20.20000	.
9	ROWK 2J4	BS	24.47000	0.53000	NONE	25.00000	.
10	ROWK 3J1	BS	19.00000	8.50000	NONE	27.50000	.
11	ROWK 3J2	BS	21.10000	1.40000	NONE	22.50000	.
12	ROWK 3J3	BS	14.30000	13.20000	NONE	27.50000	.
13	ROWK 3J4	BS	24.80000	0.20000	NONE	25.00000	.
14	ROWK 4J1	BS	15.60000	9.90000	NONE	25.50000	.
15	ROWK 4J2	BS	23.55000	6.95000	NONE	30.50000	.
16	ROWK 4J3	BS	8.50000	8.00000	NONE	16.50000	.
17	ROWK 4J4	BS	29.50000	0.50000	NONE	30.00000	.
18	ROWK 5J1	BS	16.55000	13.95000	NONE	30.50000	.
19	ROWK 5J2	BS	22.95000	2.55000	NONE	25.50000	.
20	ROWK 5J3	BS	12.50000	18.00000	NONE	30.50000	.
21	ROWK 5J4	BS	29.95000	0.05000	NONE	30.00000	.
22	ROWK 6J1	BS	12.10000	16.40000	NONE	28.50000	.
23	ROWK 6J2	BS	11.75000	15.75000	NONE	27.50000	.
24	ROWK 6J3	BS	6.50000	21.00000	NONE	27.50000	.
25	ROWK 6J4	BS	12.90000	12.10000	NONE	25.00000	.
26	ROWI 1	EQ	1.00000	.	1.00000	1.00000	-1500.00000
27	ROWI 2	EQ	1.00000	.	1.00000	1.00000	.
28	ROWI 3	EQ	1.00000	.	1.00000	1.00000	-1200.00000
29	ROWI 4	EQ	1.00000	.	1.00000	1.00000	-1500.00000
30	ROWI 5	EQ	1.00000	.	1.00000	1.00000	-567.00000
31	ROWI 6	EQ	1.00000	.	1.00000	1.00000	-600.00000
32	ROWI 7	EQ	1.00000	.	1.00000	1.00000	-1500.00000
33	ROWI 8	EQ	1.00000	.	1.00000	1.00000	-2267.00000
34	ROWI 9	EQ	1.00000	.	1.00000	1.00000	.
35	ROWI10	EQ	1.00000	.	1.00000	1.00000	-1500.00000
36	ROWI11	EQ	1.00000	.	1.00000	1.00000	-1133.00000
37	ROWI12	EQ	1.00000	.	1.00000	1.00000	-600.00000
38	ROWI13	EQ	1.00000	.	1.00000	1.00000	-3000.00000
39	ROWI14	EQ	1.00000	.	1.00000	1.00000	-1133.00000
40	ROWI15	EQ	1.00000	.	1.00000	1.00000	-300.00000

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
49	T1I 1	IV	.	.	.	1.00000	-1500.00000
50	T2I 1	IV	.	750.00000	.	1.00000	-750.00000
51	T3I 1	IV	1.00000	1500.00000	.	1.00000	.
52	T4I 1	IV	.	2250.00000	.	1.00000	750.00000
53	T5I 1	IV	.	3000.00000	.	1.00000	1500.00000
54	T6I 1	IV	.	3750.00000	.	1.00000	2250.00000
55	T7I 1	IV	.	5500.00000	.	1.00000	4000.00000
56	T1I 2	IV	1.00000	.	.	1.00000	.
57	T2I 2	IV	.	567.00000	.	1.00000	567.00000
58	T3I 2	IV	.	1133.00000	.	1.00000	1133.00000
59	T4I 2	IV	.	1700.00000	.	1.00000	1700.00000
60	T5I 2	IV	.	2267.00000	.	1.00000	2267.00000
61	T6I 2	IV	.	2833.00000	.	1.00000	2833.00000
62	T7I 2	IV	.	4000.00000	.	1.00000	4000.00000
63	T1I 3	IV	.	.	.	1.00000	-1200.00000
64	T2I 3	IV	.	300.00000	.	1.00000	-900.00000
65	T3I 3	IV	.	600.00000	.	1.00000	-600.00000
66	T4I 3	IV	.	900.00000	.	1.00000	-300.00000
67	T5I 3	IV	1.00000	1200.00000	.	1.00000	.
68	T6I 3	IV	.	1500.00000	.	1.00000	300.00000
69	T7I 3	IV	.	2000.00000	.	1.00000	800.00000
70	T1I 4	IV	.	.	.	1.00000	-1500.00000
71	T2I 4	IV	.	750.00000	.	1.00000	-750.00000
72	T3I 4	IV	1.00000	1500.00000	.	1.00000	.
73	T4I 4	IV	.	2250.00000	.	1.00000	750.00000
74	T5I 4	IV	.	3000.00000	.	1.00000	1500.00000
75	T6I 4	IV	.	3750.00000	.	1.00000	2250.00000
76	T7I 4	IV	.	5500.00000	.	1.00000	4000.00000
77	T1I 5	IV	.	.	.	1.00000	-567.00000
78	T2I 5	IV	1.00000	567.00000	.	1.00000	.
79	T3I 5	IV	.	1133.00000	.	1.00000	566.00000
80	T4I 5	IV	.	1700.00000	.	1.00000	1133.00000
81	T5I 5	IV	.	2267.00000	.	1.00000	1700.00000
82	T6I 5	IV	.	2833.00000	.	1.00000	2266.00000
83	T7I 5	IV	.	4000.00000	.	1.00000	3433.00000
84	T1I 6	IV	.	.	.	1.00000	-600.00000
85	T2I 6	IV	.	300.00000	.	1.00000	-300.00000
86	T3I 6	IV	1.00000	600.00000	.	1.00000	.
87	T4I 6	IV	.	900.00000	.	1.00000	300.00000
88	T5I 6	IV	.	1200.00000	.	1.00000	600.00000
89	T6I 6	IV	.	1500.00000	.	1.00000	900.00000
90	T7I 6	IV	.	2000.00000	.	1.00000	1400.00000
91	T1I 7	IV	.	.	.	1.00000	-1500.00000
92	T2I 7	IV	.	750.00000	.	1.00000	-750.00000
93	T3I 7	IV	1.00000	1500.00000	.	1.00000	.
94	T4I 7	IV	.	2250.00000	.	1.00000	750.00000
95	T5I 7	IV	.	3000.00000	.	1.00000	1500.00000

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
96	T6I 7	IV	.	3750.00000	.	1.00000	2250.00000
97	T7I 7	IV	.	5500.00000	.	1.00000	4000.00000
98	T1I 8	IV	.	.	.	1.00000	-2267.00000
99	T2I 8	IV	.	567.00000	.	1.00000	-1700.00000
100	T3I 8	IV	.	1133.00000	.	1.00000	-1134.00000
101	T4I 8	IV	.	1700.00000	.	1.00000	-567.00000
102	T5I 8	IV	1.00000	2267.00000	.	1.00000	.
103	T6I 8	IV	.	2833.00000	.	1.00000	566.00000
104	T7I 8	IV	.	4000.00000	.	1.00000	1733.00000
105	T1I 9	IV	.	.	.	1.00000	.
106	T2I 9	IV	1.00000	300.00000	.	1.00000	300.00000
107	T3I 9	IV	.	600.00000	.	1.00000	600.00000
108	T4I 9	IV	.	900.00000	.	1.00000	900.00000
109	T5I 9	IV	.	1200.00000	.	1.00000	1200.00000
110	T6I 9	IV	.	1500.00000	.	1.00000	1500.00000
111	T7I 9	IV	.	2000.00000	.	1.00000	2000.00000
112	T1I10	IV	.	.	.	1.00000	-1500.00000
113	T2I10	IV	.	750.00000	.	1.00000	-750.00000
114	T3I10	IV	1.00000	1500.00000	.	1.00000	.
115	T4I10	IV	.	2250.00000	.	1.00000	750.00000
116	T5I10	IV	.	3000.00000	.	1.00000	1500.00000
117	T6I10	IV	.	3750.00000	.	1.00000	2250.00000
118	T7I10	IV	.	5500.00000	.	1.00000	4000.00000
119	T1I11	IV	.	.	.	1.00000	-1133.00000
120	T2I11	IV	.	567.00000	.	1.00000	-566.00000
121	T3I11	IV	1.00000	1133.00000	.	1.00000	.
122	T4I11	IV	.	1700.00000	.	1.00000	567.00000
123	T5I11	IV	.	2267.00000	.	1.00000	1134.00000
124	T6I11	IV	.	2833.00000	.	1.00000	1700.00000
125	T7I11	IV	.	4000.00000	.	1.00000	2867.00000
126	T1I12	IV	.	.	.	1.00000	-600.00000
127	T2I12	IV	.	300.00000	.	1.00000	-300.00000
128	T3I12	IV	1.00000	600.00000	.	1.00000	.
129	T4I12	IV	.	900.00000	.	1.00000	300.00000
130	T5I12	IV	.	1200.00000	.	1.00000	600.00000
131	T6I12	IV	.	1500.00000	.	1.00000	900.00000
132	T7I12	IV	.	2000.00000	.	1.00000	1400.00000
133	T1I13	IV	.	.	.	1.00000	-3000.00000
134	T2I13	IV	.	750.00000	.	1.00000	-2250.00000
135	T3I13	IV	.	1500.00000	.	1.00000	-1500.00000
136	T4I13	IV	.	2250.00000	.	1.00000	-750.00000
137	T5I13	IV	1.00000	3000.00000	.	1.00000	.
138	T6I13	IV	.	3750.00000	.	1.00000	750.00000
139	T7I13	IV	.	5500.00000	.	1.00000	2500.00000
140	T1I14	IV	.	.	.	1.00000	-1133.00000

COLUMNS SECTION

NUMBER	NAME	STATUS	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
141	T2I14	IV	.	567.00000	.	1.00000	-566.00000
142	T3I14	IV	1.00000	1133.00000	.	1.00000	.
143	T4I14	IV	.	1700.00000	.	1.00000	567.00000
144	T5I14	IV	.	2267.00000	.	1.00000	1134.00000
145	T6I14	IV	.	2833.00000	.	1.00000	1700.00000
146	T7I14	IV	.	4000.00000	.	1.00000	2867.00000
147	T1I15	IV	.	.	.	1.00000	-300.00000
148	T2I15	IV	1.00000	300.00000	.	1.00000	.
149	T3I15	IV	.	600.00000	.	1.00000	300.00000
150	T4I15	IV	.	900.00000	.	1.00000	600.00000
151	T5I15	IV	.	1200.00000	.	1.00000	900.00000
152	T6I15	IV	.	1500.00000	.	1.00000	1200.00000
153	T7I15	IV	.	2000.00000	.	1.00000	1700.00000

[illegible] TIME--PROCESSOR = 4.01 ELAPSED = 10.70

OLD ENTRY WLA1 DELETED ON ZPROF (OR ZSOLF)
NEW ENTRY WLA1 ENTERED ON ZPROF (OR ZSOLF)
BASIS WLA1 SAVED

[illegible] SYSTEM FILE DIRECTORIES

PROBLEMS ON ZPROF

ZNAME	DATE	NO ROWS	NO COLS	NO RECS
WLA1	09/23/75	40	105	2
XXXX	09/26/75	25	28	1
WLA2	10/07/75	40	105	2

BASES ON ZPROF

ZBASNM	DATE	ZNAME	NO RECS
WLA2	10/07/75	WLA2	1
WLA1	10/08/75	WLA2	1

TOTAL RECORDS = 10
WASTED RECORDS = 3

ENDRUN TIME--PROCESSOR = 4.02 ELAPSED = 10.81

Appendix H

FMPS-MIP Solution to Small Water Quality Model (Problem I)

SECTION 1 - ROWS

PRIMAL-DUAL OUTPUT

NUMBER	..NAME..	AT	...ACTIVITY...	SLACK ACTIVITY	.LOWER LIMIT	.UPPER LIMIT	.DUAL ACTIVITY	..INPUT COST..	.REDUCED COST.
1	COST	FR	2371.000000	-2371.000000	NONE	NONE	1.000000	.000000	1.000000
2	ROW1	BS	2.800000	.500000	NONE	3.30000	.000000	.000000	.000000
3	ROW2	BS	2.800000	96.299999	NONE	99.10000	.000000	.000000	.000000
4	ROW3	BS	.280000	.720000	NONE	1.00000	.000000	.000000	.000000
5	ROW4	BS	1.400000	1.840000	NONE	3.24000	.000000	.000000	.000000
6	ROW5	BS	.850000	3.350000	NONE	4.20000	.000000	.000000	.000000
7	ROW6	BS	3.400000	95.749999	NONE	99.15000	.000000	.000000	.000000
8	ROW7	BS	.051000	.699000	NONE	.75000	.000000	.000000	.000000
9	ROW8	BS	1.870000	2.449000	NONE	4.31900	.000000	.000000	.000000
10	ROW9	BS	1.350000	2.250000	NONE	3.60000	.000000	.000000	.000000
11	ROW10	BS	1.800000	97.549999	NONE	99.35000	.000000	.000000	.000000
12	ROW11	BS	.192000	.708000	NONE	.90000	.000000	.000000	.000000
13	ROW12	BS	2.292000	2.510000	NONE	4.80200	.000000	.000000	.000000
14	ROW13	BS	2.250000	1.900000	NONE	4.15000	.000000	.000000	.000000
15	ROW14	BS	1.700000	97.750000	NONE	99.45000	.000000	.000000	.000000
16	ROW15	BS	.412000	1.088000	NONE	1.50000	.000000	.000000	.000000
17	ROW16	BS	1.495000	.377000	NONE	1.87200	.000000	.000000	.000000
18	ROW17	BS	2.700000	2.300000	NONE	5.00000	.000000	.000000	.000000
19	ROW18	BS	1.450000	98.700000	NONE	100.15000	.000000	.000000	.000000
20	ROW19	BS	1.909000	.041000	NONE	1.95000	.000000	.000000	.000000
21	ROW20	BS	4.698000	4.704000	NONE	9.40200	.000000	.000000	.000000
22	ROW21	EQ	1.000000	.000000	1.00000	1.00000	.000000	.000000	.000000
23	ROW22	EQ	1.000000	.000000	1.00000	1.00000	-196.000000	.000000	-196.000000
24	ROW23	EQ	1.000000	.000000	1.00000	1.00000	.000000	.000000	.000000
25	ROW24	EQ	1.000000	.000000	1.00000	1.00000	-504.000000	.000000	-504.000000

SECTION 2 - COLUMNS

PRIMAL-DUAL OUTPUT

NUMBER	..NAME..	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
26	T1I1	IT	.000000	.000000	.000000	1.000000	.000000
27	T2I1	IT	.000000	441.000000	.000000	1.000000	441.000000
28	T3I1	IT	1.000000	815.000000	.000000	1.000000	815.000000
29	T4I1	IT	.000000	1007.000000	.000000	1.000000	1007.000000
30	T5I1	IT	.000000	406.000000	.000000	1.000000	406.000000
31	T6I1	IT	.000000	1448.000000	.000000	1.000000	1448.000000
32	T7I1	IT	.000000	3014.000000	.000000	1.000000	3014.000000
33	T1I2	IT	.000000	.000000	.000000	1.000000	-196.000000
34	T2I2	IT	.000000	196.000000	.000000	1.000000	.000000
35	T3I2	IT	.000000	334.000000	.000000	1.000000	138.000000
36	T4I2	IT	1.000000	422.000000	.000000	1.000000	226.000000
37	T5I2	IT	.000000	225.000000	.000000	1.000000	29.000000
38	T6I2	IT	.000000	618.000000	.000000	1.000000	422.000000
39	T7I2	IT	.000000	1507.000000	.000000	1.000000	1311.000000
40	T1I3	IT	.000000	.000000	.000000	1.000000	.000000
41	T2I3	IT	.000000	594.000000	.000000	1.000000	594.000000
42	T3I3	IT	1.000000	1134.000000	.000000	1.000000	1134.000000
43	T4I3	IT	.000000	1391.000000	.000000	1.000000	1391.000000
44	T5I3	IT	.000000	504.000000	.000000	1.000000	504.000000
45	T6I3	IT	.000000	1985.000000	.000000	1.000000	1985.000000
46	T7I3	IT	.000000	3832.000000	.000000	1.000000	3832.000000
47	T1I4	IT	1.000000	.000000	.000000	1.000000	-504.000000
48	T2I4	IT	.000000	594.000000	.000000	1.000000	90.000000
49	T3I4	IT	.000000	1134.000000	.000000	1.000000	630.000000
50	T4I4	IT	.000000	1391.000000	.000000	1.000000	887.000000
51	T5I4	IT	.000000	504.000000	.000000	1.000000	.000000
52	T6I4	IT	.000000	1985.000000	.000000	1.000000	1481.000000
53	T7I4	IT	.000000	3832.000000	.000000	1.000000	3328.000000

Appendix I

GMINT Solution to Water Supply Model—Original Version

CACHE VALLEY MOD

LEAST Z BEFORE VARIABLE CHANGE = 1.78074711D 05

0	0	0	0	0	0	0	8	0	10	11	0	0	0	0
0	17	18	19	20	0	0	0	0	0	26	0	0	0	30
0	0	0	0	35	0	37	0	0	0	0	0	0	0	45
46	0	0	49	0	0	52	53	0	55	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0											

FRACTIONAL VARIABLES

58	0.778D 03	59	0.223D 03	60	0.188D 03	61	0.161D 03	62	0.140D 02	68	0.192D 03
69	0.140D 03	70	0.480D 02	71	0.370D 02	72	0.120D 02	73	0.790D 02	74	0.460D 02
75	0.900D 02	76	0.210D 02	77	0.280D 02	82	0.787D 03	83	0.158D 04	90	0.234D 03
91	0.248D 03	94	0.300D 02	95	0.590D 02	98	0.460D 02	99	0.630D 02	100	0.100D 02
101	0.190D 02	104	0.185D 03	105	0.186D 03	106	0.220D 02	107	0.260D 02	108	0.700D 01
109	0.800D 01	116	0.970D 02	117	0.940D 02	118	0.350D 02	119	0.660D 02	122	0.112D 03
123	0.224D 03	126	0.390D 02	127	0.780D 02	128	0.315D 03	130	0.119D 03	131	0.238D 03
132	0.390D 02	134	0.105D 03	135	0.910D 02	136	0.200D 03	137	0.290D 03	138	0.230D 02
139	0.250D 02	140	0.350D 02	141	0.460D 02	142	0.320D 02	143	0.460D 02	146	0.120D 04
147	0.131D 04	148	0.530D 02	162	0.210D 02	163	0.100D 01	168	0.610D 02	169	0.360D 02
176	0.180D 02	192	0.600D 01	198	0.400D 01	202	0.220D 02	204	0.700D 01	205	0.900D 01
206	0.440D 02	207	0.270D 02	208	0.110D 02	209	0.583D 01	210	0.129D 01	211	0.264D 01
212	0.220D 00	213	0.330D 00	214	0.225D 01	215	0.180D-01	216	0.114D 01	217	0.260D 00
218	0.265D 01	219	0.940D 00	220	0.246D 01	221	0.340D 00	222	0.770D 00	223	0.148D 01
224	0.420D 00	225	0.170D 00	226	0.260D 00	227	0.230D 00	229	0.132D 01	237	0.330D 00
240	0.920D 00	244	0.302D 00	245	0.610D 00	252	0.730D 00	253	0.600D-01	255	0.150D 00
257	0.350D 00	258	0.750D-01	259	0.745D 00						

TOTAL TIME 104.520 6.916 WORK FACTOR TO TERM
TIME IN LOGICAL 5.965
TIME TO START OF FIRST LP 1.891
TIME TO END OF FIRST LP 13.203

```
TIME TO FIRST FEASIBLE SOLUTION        28.906
TIME TO LAST  FEASIBLE SOLUTION        58.270        3.413 WORK FACTOR TO OPT

LP CALLS (TOTAL, PCT FATHOM)              124         38.7
TIME IN LP MODE (SEC, PCT TOTAL)       71.066         68.0
                                                                 PENALTY TIME   =  10.551 SEC
                                                                 CUM ROUND TIME =   2.340 SEC

FEASIBLE SOLUTIONS FOUND, TOTAL          5
    NATURAL LP                                        2
    ROUNDED LP SOLUTIONS                              3
VALUE FIRST LP          0.14785493D 06             17.0 PCT SUBOPTIMAL
VALUE FIRST FEAS SOL    0.18095347D 06              1.6 PCT SUBOPTIMAL

PARTIAL SOLUTIONS CONSIDERED           136                   5.8 PCT LOGICAL FATHOM
AUGMENTATIONS                           54                 TIME IN AUG  =   0.238 SEC

FATHOMINGS                              62
    AUGMENTATION IMPOSSIBLE              0                   0.0 PCT
    DUE TO RELAX 0                       0                   0.0 PCT
    S-CONSTRAINT INFEASIBLE              8                  12.9 PCT
    REGULAR CONSTRAINT INFEASIBLE        0                   0.0 PCT
    DUE TO LP, TOTAL                    48                  77.4 PCT
         BY INFEASIBILITY                             10
         BY VALUE                                     20
         BY NATURAL INTEGRALITY                        2
         BY VALUE+PMAX                                14
         BY VALUE+OPEN                                 2
    DUE TO LBD                           6                   9.7 PCT
PEGGED FREE VARIABLES, TOTAL           163                                  RATIO =  1.181
    BY RELAX 0                                         0     0.0 PCT
    BY S-CONSTRAINTS                                  46    28.2 PCT
    BY REGULAR CONSTRAINTS                            38    23.3 PCT
    BY LP PENALTY                                     79    48.5 PCT

NKODE ARRAY

   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24

   0   1   0   5   0   0   0   3   0   0   0   0  53  14   0   0   0   0   0   0   2   0  51  13


OBAA7600      .......END JOB  70.......9.52.26 AM  10 OCT 75... RM2.PR1   ....OBAA7600....RICHARD MCBRIDE    .......OBAA7600
OBAA7600      .......END JOB  70.......9.52.26 AM  10 OCT 75... RM2.PR1   ....OBAA7600....RICHARD MCBRIDE    .......OBAA7600
OBAA7600      .......END JOB  70.......9.52.26 AM  10 OCT 75... RM2.PR1   ....OBAA7600....RICHARD MCBRIDE    .......OBAA7600
OBAA7600      .......END JOB  70.......9.52.26 AM  10 OCT 75... RM2.PR1   ....OBAA7600....RICHARD MCBRIDE    .......OBAA7600
OBAA7600      .......END JOB  70.......9.52.26 AM  10 OCT 75... RM2.PR1   ....OBAA7600....RICHARD MCBRIDE    .......OBAA7600

                  $.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.
                  .                                                           $
                  $          U C C   J O B   C H A R G E S                     .
                  .                                                           $
                  $     INPUT  CHARGES                                         .
                  .         JOB SURCHARGE              $        .07           $
                  $                                                            .
                  .     TOTAL STEP CHARGES             $      13.55           $
                  $                                                            .
                  .     OUTPUT  CHARGES                                       $
                  $                                                            .
                  .                                 ***************           $
                  $     JOB  TOTAL                 *  $     13.62   *          .
                  .                                 ***************           $
                  $                                                            .
                  .                                                           $
                  $.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.$.
```

Appendix J

AIP Solution to Small Water Quality Model
(Problem I)

OBJECTIVE FUNCTION

X(1)	X(2)	X(3)	X(4)	X(5)
X(6)	X(7)	X(8)	X(9)	X(10)
X(11)	X(12)	X(13)	X(14)	X(15)
X(16)	X(17)	X(18)	X(19)	X(20)
X(21)	X(22)	X(23)	X(24)	X(25)
X(26)	X(27)	X(28)	X(	
0.000	441.000	815.000	1007.000	406.000
1448.000	3014.000	0.000	196.000	334.000
422.000	225.000	618.000	1507.000	0.000
594.000	1134.000	1391.000	504.000	1985.000
3832.000	0.000	594.000	1134.000	1391.000
504.000	1985.000	3832.000		

CONSTRAINT NUMBER — CONSTRAINTS

CONSTRAINT NUMBER					
1	-4.200	-3.500	-2.800	-0.700	-0.700
	-0.700	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	3.300	
2	-3.500	-0.700	-2.800	-2.800	-2.800
	-0.560	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	99.100	
3	-2.800	-2.100	-0.280	-0.070	-1.400
	-0.070	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	1.000	
4	-1.400	-1.400	-1.400	-1.400	-1.400
	-1.400	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	3.214	
5	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-3.400	-2.550	-1.700
	-0.850	-0.850	-0.850	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	4.200	
6	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-4.250	-0.850	-3.400
	-3.400	-3.400	-0.680	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	99.150	
7	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-2.550	-1.700	-0.340
	-0.051	-1.360	-0.051	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.750	
8	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-1.870	-1.870	-1.870
	-1.870	-1.870	-1.870	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	4.319	
9	-1.800	-1.500	-1.200	-0.300	-0.300
	-0.300	0.000	-0.600	-0.450	-0.300
	-0.150	-0.150	-0.150	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	3.600	
10	-1.500	-0.300	-1.200	-1.200	-1.200
	-0.240	0.000	-0.750	-0.150	-0.600
	-0.600	-0.600	-0.120	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	99.350	

11	-1.800	-1.350	-0.180	-0.045	-0.900
	-0.045	0.000	-0.600	-0.400	-0.080
	-0.012	-0.320	-0.012	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.900	

12	-4.550	-2.800	-1.680	-1.320	-2.650
	-0.520	0.000	-1.350	-0.550	-0.680
	-0.612	-0.920	-0.132	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	4.802	

13	-0.900	-0.750	-0.600	-0.150	-0.150
	-0.150	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	-2.750
	-2.200	-1.650	-0.550	-0.550	-0.550
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	4.150	

14	-0.750	-0.150	-0.600	-0.600	-0.600
	-0.120	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	-2.200
	-0.330	-1.100	-1.100	-1.100	-0.220
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	99.450	

15	-1.800	-1.350	-0.180	-0.045	-0.900
	-0.045	0.000	-0.600	-0.400	-0.080
	-0.012	-0.320	-0.012	0.000	-2.200
	-1.650	-0.270	-0.055	-1.100	-0.055
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	1.500	

16	-3.800	-2.850	-0.380	-0.095	-1.900
	-0.095	0.000	-0.750	-0.500	-0.100
	-0.015	-0.400	-0.015	0.000	-1.100
	-1.100	-1.100	-1.100	-1.100	-1.100
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	1.872	

17	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	-1.000
	-0.800	-0.600	-0.200	-0.200	-0.200
	0.000	-2.100	-1.750	-0.700	-0.350
	-0.350	-0.350	0.000	5.000	

18	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	-0.800
	-0.120	-0.400	-0.400	-0.400	-0.080
	0.000	-1.050	-0.210	-0.700	-0.700
	-0.700	-0.140	0.000	100.150	

19	-1.800	-1.350	-0.180	-0.045	-0.900
	-0.045	0.000	-0.450	-0.300	-0.060
	-0.009	-0.240	-0.009	0.000	-2.200
	-1.650	-0.220	-0.055	-1.100	-0.055
	0.000	-1.500	-1.000	-0.200	-0.030
	-1.000	-0.030	0.000	1.950	

20	-5.000	-3.750	-0.500	-0.125	-2.500
	-0.125	0.000	-0.900	-0.600	-0.120
	-0.018	-0.480	-0.018	0.000	-3.650
	-2.420	-0.830	-0.520	-1.650	-0.200
	0.000	-3.350	-1.980	-1.220	-0.933
	-2.000	-0.455	0.000	9.402	

21	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-1.000	

22	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-1.000	

23	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	1.000
	1.000	1.000	1.000	1.000	1.000
	1.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-1.000	

24	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000
	0.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	-1.000	

SOLUTION FOUND Z= 2371.00000000

X(3) = 1

X(11) = 1

X(17) = 1

X(22) = 1

ALL OTHER VARIABLES EQUAL ZERO

ESTIMATED CHARGE FOR B6700 USAGE

CPU TIME	12.81 SECS	$	1.02	MEMORY	866.20 KW-SEC	$	1.44
I/O TIME	19.65 SECS	$	0.98	LINES PRINTED	917	$	0.55
CARDS READ	660	$	0.33	CARDS PUNCHED	0	$	0.00

Appendix K

Exact Solutions for the Water Quality Simulation Model

Exact solutions can be obtained for Equations 7 through 10 for a particular reach of stream as follows:

$$Y_1 = \frac{\beta_{1,2}}{\beta_{1,1}}\left[1 - e^{-\beta_{1,1}t}\right] + \left[(1-w)Y_1^o + P_1 w\right] e^{-\beta_{1,1}t} \quad \text{(K-1)}$$

$$w = Q_p/(Q + Q_p) \quad \text{(K-1a)}$$

$$\beta_{1,1} = K_{1,a} + \frac{Q_s}{\bar{A}} \quad \text{(K-1b)}$$

$$\beta_{1,2} = \frac{Q_s Y_{s1}}{\bar{A}} + \frac{K_{1,b}}{\bar{D}} \quad \text{(K-1c)}$$

in which Y_1 is the concentration at any point in the reach, t is the travel time, Y_1^o is the concentration in the river at the head of the reach (mg/l), P_1 is the concentration in the point source (if present) (mg/l), Q_p is the flow of the point source (m^3/min), Q is the flow in the river at the head of the reach (m^3/min), and w is the dilution factor at the point source.

$$Y_2 = \frac{\beta_{2,2}}{\beta_{2,1}}\left[1 - e^{-\beta_{2,1}t}\right] + \left[(1-w)Y_2^o + P_2 w\right] e^{-\beta_{2,1}t} \quad \text{(K-2)}$$

$$\beta_{2,1} = K_{2,a} + \frac{Q_s}{\bar{A}} \quad \text{(K-2a)}$$

$$\beta_{2,2} = \frac{Q_s Y_{s2}}{\bar{A}} + \frac{K_{2,b}}{\bar{D}} \quad \text{(K-2b)}$$

$$Y_3 = \frac{\beta_{3,2}}{\beta_{3,1}}\left[1 - e^{-\beta_{3,1}t}\right] + \left[(1-w)Y_3^o P_3 w\right] e^{-\beta_{3,1}t} \quad \text{(K-3)}$$

$$\beta_{3,1} = \frac{Q_s}{\bar{A}} \quad \text{(K-3a)}$$

$$\beta_{3,2} = \frac{Q_s Y_{s3}}{\bar{A}} + \frac{K_{3,b}}{\bar{D}} \quad \text{(K-3b)}$$

$$Y_4 = \frac{\beta_{4,2}}{\beta_{4,1}} + \frac{\beta_{4,3}}{\beta_{4,1} - \beta_{1,1}} e^{-\beta_{1,1}t} + \frac{\beta_{4,4}}{\beta_{4,1} - \beta_{2,1}} e^{-\beta_{2,1}t} + \frac{\beta_{4,5}}{\beta_{4,1} - \beta_{3,1}} e^{-\beta_{2,1}t} + \left[(1-w)Y_4^o + P_4 w - \frac{\beta_{4,5}}{\beta_{4,1}} - \frac{\beta_{4,3}}{\beta_{4,1} - \beta_{1,1}} - \frac{\beta_{4,4}}{\beta_{4,1} - \beta_{2,1}} - \frac{\beta_{4,5}}{\beta_{4,1} - \beta_{3,1}}\right] e^{-\beta_{4,1}t} \quad \text{(K-4)}$$

$$\beta_{4,1} = K_{4,a} + \frac{Q_s}{\bar{A}} \quad \text{(K-4a)}$$

$$\beta_{4,2} = \frac{Q_s Y_{s4}}{\bar{A}} + \frac{K_{4,b}}{\bar{D}} + \frac{K_{1,a}\beta_{1,2}}{\beta_{1,1}} + \frac{2.44\, K_{2,a}\beta_{2,2}}{\beta_{2,1}} + \frac{K_{4,2}\beta_{3,2}}{\beta_{3,1}} \quad \text{(K-4b)}$$

$$\beta_{4,3} = \left[Y_1^o + P_1 w - \frac{\beta_{1,2}}{\beta_{1,1}}\right] K_{1,a}\ (1.0) \quad \text{(K-4c)}$$

$$\beta_{4,4} = \left[Y_2^o + P_2 w - \frac{\beta_{2,2}}{\beta_{2,1}}\right] (4.22)\, K_{2,a} \quad \text{(K-4d)}$$

$$\beta_{4,5} = \left[Y_3^o + P_3 w - \frac{\beta_{3,2}}{\beta_{3,1}}\right] K_{4,2}\ (1.0) \quad \text{(K-4e)}$$

Appendix L

Exact Solutions for Elements in the Linking Matrix: D_{ik}

D Matrix

Elements for the D matrices are obtained from the partial derivatibes of Equations K-1 through K-4 with respect to changes in effluent concentrations.

$$\frac{\partial Y_1}{\partial P_1} = we^{-\beta_{1,1}t} + \text{(effects of upstream reaches)} \quad \text{(L-1)}$$

$$\frac{\partial Y_1}{\partial P_j} = 0; j = 2, 3, 4 \quad \text{(L-1a)}$$

$$\frac{\partial Y_2}{\partial P_2} = we^{-\beta_{2,1}t} + \text{(effects of upstream reaches)} \quad \text{(L-2)}$$

$$\frac{\partial Y_2}{\partial P_j} = 0; j = 1, 3, 4 \quad \text{(L-2a)}$$

$$\frac{\partial Y_3}{\partial P_3} = we^{-\beta_{3,1}t} + \text{(effects of upstream reaches)} \quad \text{(L-3)}$$

$$\frac{\partial Y_3}{\partial P_j} = 0; j = 1, 2, 4 \quad \text{(L-3a)}$$

$$\frac{\partial Y_4}{\partial P_4} = we^{-\beta_{4,1}t} + \text{(effects of upstream reaches)} \quad \text{(L-4)}$$

$$\frac{\partial Y_4}{\partial P_1} = \frac{wK_{1,a}}{\beta_{4,1} - \beta_{1,1}} \left[e^{-\beta_{1,1}t} - e^{-\beta_{4,1}t} \right] + \text{(effects of upstream reaches)}$$

$$= \frac{wK_{1,a}}{K_{4,a} - K_{1,a}} \left[e^{-\beta_{1,1}t} - e^{-\beta_{4,1}t} \right] \quad \text{(L-4a)}$$

$$\frac{\partial Y_4}{\partial P_2} = \frac{4.22\ wK_{2,a}}{K_{4,a} - K_{2,a}} \left[e^{-\beta_{2,1}t} - e^{-\beta_{4,1}t} \right] + \text{(effects of upstream reaches)} \quad \text{(L-4b)}$$

$$\frac{\partial Y_4}{\partial P_3} = \frac{wK_{4,2}}{K_{4,a}} \left[e^{-\beta_{3,1}t} - e^{-\beta_{4,1}t} \right] + \text{(effects of upstream reaches)} \quad \text{(L-4c)}$$