An experimental study of internal wave generation through evanescent regions

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Abstract

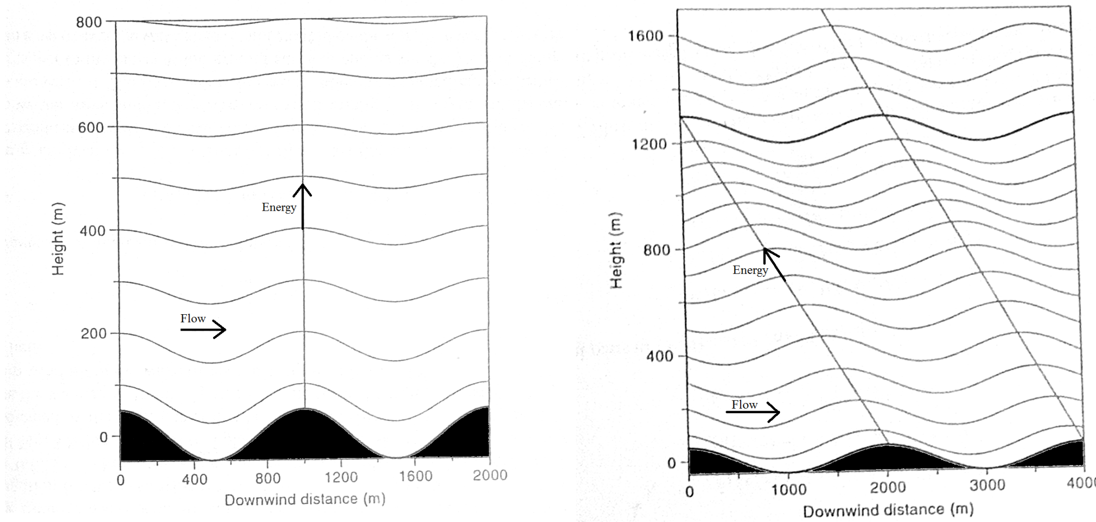
Internal waves are a complex phenomena that affect both the atmosphere and the ocean. Internal wave generation occurs uniquely in fluids which are stratified, or have varying density with respect to height. The natural frequency (N) is a fluid property that is defined by the rate of density change within the fluid. Internal waves are generated in stratified fluids when an excitation frequency (ω) less than the local natural frequency occurs. If the reverse is true, then only evanescent waves, or waves with an exponentially decaying amplitude, are generated. Within the ocean the natural frequency is not constant and in some locations can decrease with depth [1]. In these locations it has been assumed that no propagating waves will be generated and instead only evanescent waves will be formed. However, recent research has indicated that evanescent waves can become propagating internal waves when the natural frequency increases past the value of the original excitation frequency. Laboratory experiments were performed to validate this theory as well as investigate the amount of kinetic energy found in the propagating internal waves.

1 Introduction

Internal waves play a critical role in understanding atmospheric and oceanic dynamics. In the ocean, these waves propagate away from their generation sites and can transmit energy upward from the ocean floor when generated over topography, or downward from the ocean surface when generated by strong winds at the surface. When internal waves eventually steepen and break, mixing is induced which helps to maintain healthy oceans [2]. The same principles apply to the atmosphere where internal waves can affect both weather and climate patterns [3].

A fluid is termed stratified when the density of the fluid varies with respect to height. When a continuous, uniformly stratified fluid is disturbed through a mean flow, shear stress, or flow over a topography etc., internal waves are generated. This is similar to how free surface waves are generated when a rock is thrown into a pond. The ocean and atmosphere are both stably stratified, allowing for the constant generation of internal waves. Peterson first recognized internal waves in 1908 and they have been a topic of much investigation since then [4].

Propagating internal waves are only found in regions where the excitation frequency (ω) is less than the natural frequency (N). The natural frequency is described by $N^2 = (-g/\rho_0)(d\rho/dz)$, where g is the gravity constant, ρ_0 is a reference density for the fluid, and $d\rho/dz$ describes density as a function of depth. If the value of the excitation frequency is greater than the natural frequency ($\omega > N$), the bulk fluid cannot react to the excitation frequency and only evanescent waves are formed. Evanescent waves move vertically away from their generation sites and the amplitude of the waves die away at an exponential rate. However, when the excitation frequency is less than the natural frequency ($\omega < N$), propagating internal waves are generated. These waves propagate away from their generation sites at a angle away from the vertical and there is little to no attenuation of the amplitude. Figure 1 shows the generation of both evanescent and propagating waves as fluid flows



(a) Evanscent Waves

(b) Internal Waves

Figure 1: Isopycnals, or lines of constant density, are shown for evanescent waves created by flow over sinusoidal hills in (a). The amplitude of the waves die away quickly as the waves propagate vertically away from the generation location. In (b), propagating internal waves are formed by flow over a topography. Note the energy of the wave propagates up and away to the left, while the crest and troughs of the wave seem to move to the right. The crests and troughs of internal waves are always orthogonal to the direction of energy propagation. Adapted from Nappo [3].

from left to right over a topography. Evanescent wave energy moving vertically upwards with a decreasing amplitude is shown in Figure 1a, while the propagating waves in Figure 1b have no attenuation as they propagate at an angle from the vertical.

There are locations in the ocean where the natural frequency of the water decreases, and it can decrease below the M2 tidal frequency, which is a well known generator of internal waves. These regions are known as evanescent regions, as only evanescent waves can be formed. Usually these regions are not included as researchers estimate the amount of internal wave energy generated throughout the ocean. However, recent research has shown the possibility of internal waves being formed out of evanescent regions [1]. Numerical simulations have shown that if evanescent wave energy reaches a turning depth, or the location where an evanescent region meets a propagating region, then that energy can become a propagating internal wave. This idea is depicted in Figure 2 as flow over a topography in an evanescent region generates an evanescent wave which moves upward until it reaches the turning depth (the dashed line) where it then becomes a propagating internal wave. To further explore this possibility, experimental investigations were needed. This paper will detail the results of the experiments performed to validate this theory.

2 Experimental setup

Internal waves are formed uniquely in stratified fluids, or fluids whose density vary with height. Within the laboratory, adding salt to water is the primary method of varying the density. To generate a specific stratification, a variation of the "double-bucket method" as detailed by Hill [5] is used. This method takes fresh water and systemically adds it to a salt water bucket using a peristaltic pump. The salt water is then slowly added to the experimental tank to create the layered density system needed to perform experiments. Figure 3 depicts the overall setup as water transfers from the fresh water bucket to the salt water bucket and then into the actual experimental tank. The experimental tank is

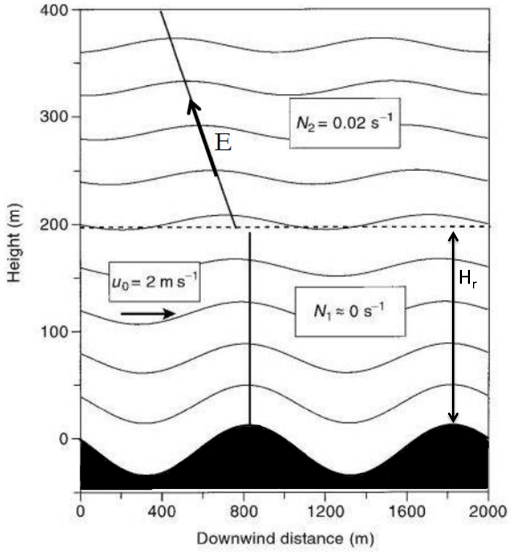


Figure 2: Internal waves are formed only when the evanescent wave energy reaches the turning depth when the natural frequency is greater than the excitation frequency. The lower portion is labeled as section 1, while the section above the turning depth is section 2. Adapted from Nappo [3].

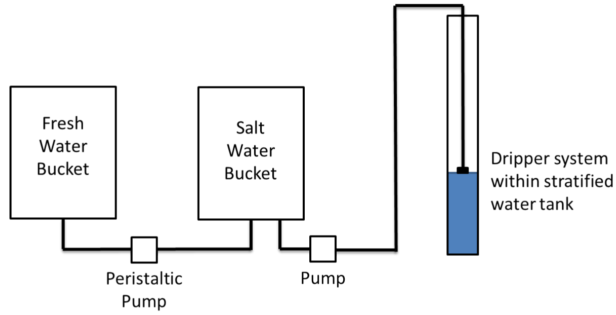


Figure 3: This setup uses a programmable peristaltic pump to control the volume flow rate of fresh water entering the salt water bucket in order to create specific stratification profiles.

2m x 1m x .15m (length, height, width). The small width allows for the generation of two-dimensional waves while the longer length and height prevent wave reflection from disturbing the experiment.

To generate internal waves, a single-peak, Gaussian-shaped topography was created. This topography moves along a track from left to right at the top of the tank at a constant speed for approximately 0.6m and then stops. The speed is varied for different experiments and can range from 0.5-2.5 cm/s. A technique called synthetic schlieren was used to visualize and analyze the internal waves generated in each experiment [6]. Similar to the classical schlieren technique, this method uses the relationship between changes of index of refraction and changes in density to follow the movement of internal waves. A front and side view of the experimental setup, including camera placement is shown in Figure 4. The experiments in this paper used a two-layer linear stratification. The first layer is an evanescent region near the topography and is labeled with N_1, while the second layer, N_2, is the propagating region. The location where the two layers meet is the turning depth. The topography was placed at the top of the tank because the density variation is small enough within the tank that downward propagating waves will exhibit the same dynamic properties as upward propagating waves [7]. The next section will describe the results of the tests that were performed.

3 Results

Using the setup explained in the previous section, five different tests were performed. Each test varied only by the speed at which the topography was, which ranged from $0.9 - 2.3 cm/s$. The velocities were chosen to ensure that the only evanescent waves would be formed in the first layer of the tank. While only one set of results will be discussed in detail, all of the tests showed similar results. Figure 5 shows the topography moving from right to left at a speed u_o while generating waves. The turning depth is at approximately 0.15m from the surface.

In order to estimate the kinetic energy of the propagating wave, it is necessary to know the frequency with units of $1/s$ of the wave as well as the horizontal wave number with units of $1/m$. The software package Digiflow was used to implement the synthetic schlieren technique. This program outputs values of $\Delta N^2(1/s)$ for the entire viewing window. These values are directly related to the change in density and can be used to calculate frequencies and wave numbers, as

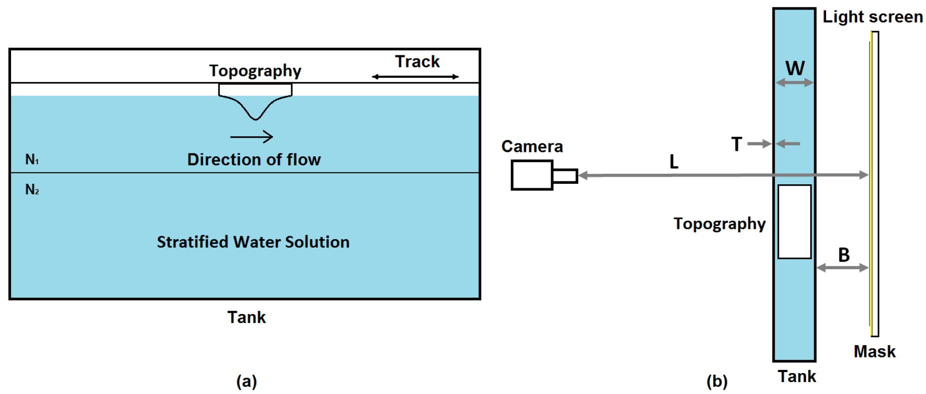


Figure 4: In (a) the topography moves left to right and generates evanescent waves in the first layer. These waves then become propagating internal waves in the second layer. The top own view showed in (b) shows the camera placement with the light screen and mask needed for the synthetic schlieren process behind the experiment tank.

well as estimate kinetic energy using a Fourier analysis. The topography used for these experiments is a Gaussian shape, which is a combination of multiple wave numbers. However, there is a dominant wave number that is matched to the dominant frequency. Figure 6 is a contour plot comparing frequency and wavelength for the different values of ΔN^2. The greatest peak corresponds to a frequency of $0.53(1/s)$ with a wave number of $30(1/m)$; these values were expected based on the topography shape and towing speed. All five tests showed internal waves being formed in the propagating region while there were no internal waves in the evanescent region. These tests experimentally confirmed the simulations performed by Paoletti et. al [1].

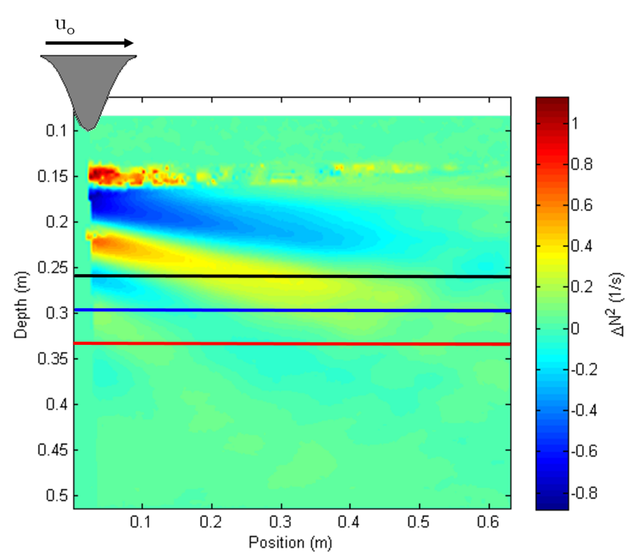


Figure 5: As the topography moves to the right, internal waves are generated below the turning depth, located approximately 0.15m from the surface. Note that the wave energy moves down and to the right. The three lines in black, blue and red are locations where kinetic energy was calculated and will discussed in the Analysis section.

4 Analysis

While confirming the presence of internal wave generation out of evanescent regions is an interesting new discovery further research into the amount of energy transfered is needed. An initial analysis of kinetic energy transfer based on these results was performed by using a method described by Wunsch and Brandt [8] for linear, Boussinesq waves. This method combines the Fourier amplitudes of ΔN^2 with the Navier-Stokes equations to estimate kinetic energy using the following equation:

$$KE = \frac{\omega^2 N^2}{k^2(N^2-\omega^2)+(\omega\partial_z N^2/N^2)}\left|\frac{\Delta N_o^2}{N^2}\right|^2 \tag{1}$$

where ω is the excitation frequency, ΔN_o^2 is the local change in the natural frequency, N is the

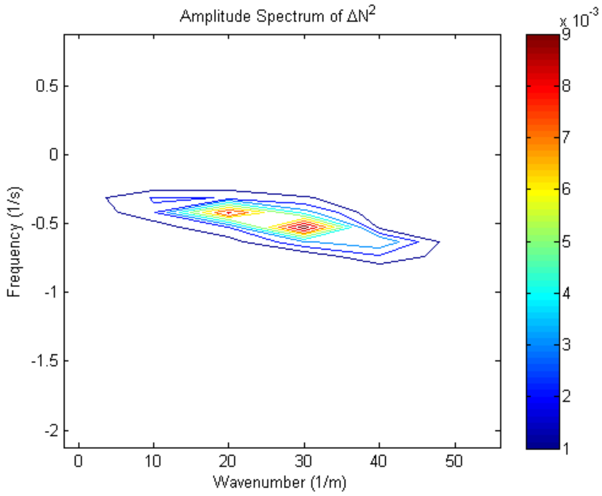


Figure 6: This contour plot shows the wide range of frequencies and wave numbers of the generated propagating wave.

natural frequency of the fluid and k is the horizontal wavenumber.

Kinetic energy was calculated at three different locations within the viewing frame indicated by the three horizontal lines in Figure 5. These locations were chosen by taking into consideration how the wave moved through the frame, as well as some distortions near the turning depth that occurred part of the way through each of the five tests. Using Equation 1, the average kinetic energy through the test was calculated at each line. These values were then normalized based on the peak kinetic energy value. Figure 7 shows that the values of peak kinetic energy corresponded to the expected dominant wave frequencies. The sharp drop between the three locations is greater than was originally expected because internal waves experience little viscous dissipation as they propagate. Also, while there is some dispersion of the waves as they propagate because of the difference in wavelengths, the short distance the waves have propagated should not lead to such a large discrepancy in kinetic energy values. However, the calculated energy values were averaged through the full experiment time, and the bottom two lines had less wave energy traveling through them for that full time compared to the top line. Thus, the decrease in kinetic energy is more likely due to decreased amount of time that the wave was passing through those lines rather than a dissipation effect. Unfortunately, different lines could not be chosen as the higher locations within the viewing window experienced some degradation of clarity due to the presence of the turning depth. To improve on this scenario, it would be necessary to look at the wave movements for either shorter times, or over a shorter horizontal distance to more accurately estimate the kinetic energy of the propagating waves.

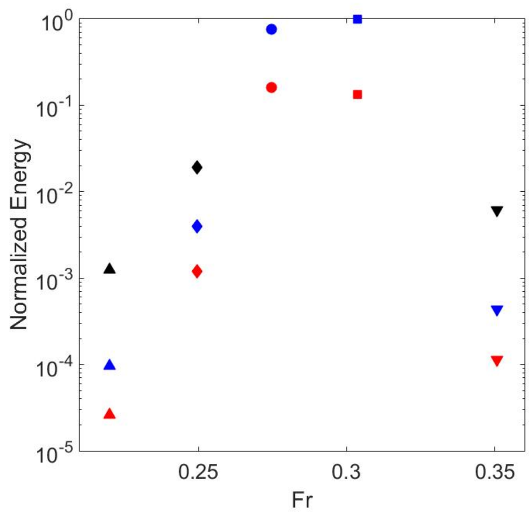


Figure 7: The normalized kinetic energy of the propagating internal waves at three different locations below the turning depth. The energy values were normalized based on the maximum kinetic energy in this test. Notice how the peak energy for all three locations correspond to the same frequency of 0.53 (1/s).

All five tests were analyzed using this same method and then compared based upon their Froude numbers (Fr) where $Fr = u_o k/N_2$. Again, these energy values were normalized by the maximum energy value seen through the five tests. The results of these tests are shown in Figure 8. For two of the tests, only two depths were analyzed, as opposed to the three depths in the other tests. As the Froude number increases, the kinetic energy of the wave increases. This trend is logical as velocity is directly proportional to Froude number. The final test does indicate a decrease in overall kinetic energy, which can also sometimes be seen with internal waves generated in a propagating region. As the Froude number increases for a propagating wave, the wave steepens and is more likely to overturn, break, and mix, thus dissipating kinetic energy [2]. However, without further testing at higher Froude numbers, a general trend for beyond this point in this scenario cannot be stated conclusively.

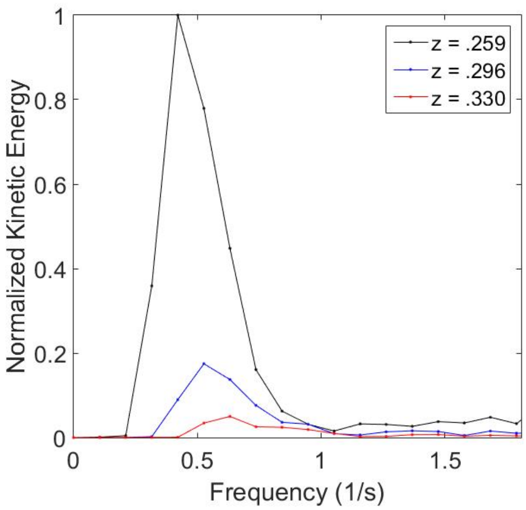


Figure 8: The five different tests are compared using the Froude number against a log scale of the normalized kinetic energy. Each test is depicted by a different symbol, with each color corresponding to the same relative location within the viewing window.

5 Conclusion

The results presented here confirm the numerical simulations presented by Paoletti et. al [1] and show that evanescent wave energy can become propagating internal waves. Further investigation showed a correlation between increasing Froude number and increasing kinetic energy. All tests were performed in a two layer linear stratification with the same topography shape. Continued research in this area should include a study of how different topography shapes will change the amount of energy seen in the propagating region. Other variables, such as the distance from the topography to the turning depth should also be explored to fully understand the amount of energy that can be transferred from an evanescent region into a propagating region.

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