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A NOTE ON THE OPTIMAL FOREST ROTATION
WHEN NON-OPERATORS DEPLETE RESOURCE STOCK

The disparity between theory and practice has definitely contributed to a continuing interest in the determination of the optimal harvest age for a growing forest. The Faustmann model has played a key role in this respect. The optimal rotation problem, as viewed by Faustmann (1849), is a timber management problem when knowledge of the forest operator about the resource biomass is certain and depends only on the natural biological growth characteristics of the tree population.

However, in many parts of the world, especially in the third world countries, local inhabitants of forested areas illegally (in terms of property rights) and indiscriminately fell trees. Thus, contributing to an unnatural depletion of tree population. In most countries in the Old World, poor communities in the forested rural sector are directly dependent on nearby forests for the requirements of day-to-day life: wood for agricultural implements and building huts, thorns for fencing, bark for rope making, land for grazing cattle, and so on. Firewood is the only source of energy in the domestic sector of many of these poor communities. But it is often too scarce or expensive for the poor to buy. As the population grows and, with it, the number of livestock, the demand grows for more (grazing) land, as well as for more wood for fuel, housebuilding, etc. The temptation to cut down the tree population at any age of its growth may become irresistible.

From the viewpoint of a forest operator, whether private or social, this is a depletion of the forest resource stock caused by extraneous, non-operator human agents. To prevent this depletion and to exclude the
non-operators from harvesting the forest, the forest-operator has to incur a flow of prevention (or exclusion or policing) costs.

In this note, a simple model is proposed to capture this phenomenon in a Faustmann framework. This model is then utilized to examine the question of when a forest should be harvested.

I

The optimal forest rotation is an economic decision based on maximizing the present value of all future net returns to be obtained from a piece of forested land. The decision must include the biological rate of growth of the forest stand, the value of harvested timber, and the harvesting costs. And since Faustmann our calculations must also include the appropriate cost of capital (rate of discount) and the opportunity cost of land (foregone return to grow new forest on the cleared land).

Having taken all of the above into his calculations, and abstracting from planting costs and forest maintenance cost, Faustmann found the optimal life of the forest. The harvesting date will come before the forest achieves its maximum stumpage value, assuming that the forest will be clear-cut.

To keep the analysis simple, the present note advances one new aspect over the basic Faustmann model. It is the depletion of forest resource stock (tree population) perpetrated by non-operators and the concomitant costs incurred by the forest operator to prevent it. We are abstracting from any multiple uses or nontimber values of a forest stand.
Let the stumpage value (net of harvesting costs) of a homogeneous tree stock at time \( t = T \), \( G_T \), be a function of the age of the forest, such that \( G_T = G(T) \). Given that the timber price is constant overtime and the underlying biological characteristics of tree growth, \( G'(T) = 0 \) (Figure 1). It is assumed that the depletion of the forest-stock by the non-operator human agents is age-independent and occurs at each age of the tree population, until the forest-stock is voluntarily harvested (clear-cutting) by the forest operator. To prevent occurrence of such incidents the forest-operator resorts to some (continuous) preventive measures including policing the area under operation. Let a fixed amount of prevention cost, \( C_P \), be incurred at each point of time to stop the non-operator induced depletion of the resource stock. Suppose that even if a prevention cost is incurred at each age \( t \), some fixed proportion, \( k_t \), of the stumpage value is lost from a growing forest stand. Thus, at time \( t = T \), the accumulated loss of stumpage value is, \( \int_0^T kG(t) \). This could be realized by the forest operator, had the trees at each age been allowed to survive and attain age \( T \).

What then is the optimum cutting age of trees? It is assumed that the objective of the forest operator is to maximize the present value of the stream of receipts minus expenditures flowing from the continued use of the land in timber growing. For its intuitive appeal and use as a basis of further comparisons, consider a model in which the planning horizon runs through one cutting of the forest.\(^1\) (A more realistic many-cycle infinite planning horizon approach is developed

\(^1\) This generalizes the "Fisherian" model. See Hirshleifer (1970, pp. 82-87) or Samuelson (1976).
Figure 1. Stumpage Value [G(t)] Growth Curve
In this case, mathematically, the problem is to choose $T$ to maximize

$$V_1(T) = G(T)e^{-rt} - \int_0^T kG(t)e^{-rt}dt - \int_0^T C_T e^{-rt}dt$$

where $r > 0$ is the discount rate and $T$ is the harvest age to be chosen by the operator. The first-order condition for an interior maximum (assuming it exists) is

$$V_1'(T) = [G'(T) - rG(T) - kG(T) - C_T e^{-rT}] = 0$$

which reduces to

$$G'(T) = (r + k)G(T) + C_T$$

or

$$G'(T)/G(T) = (r + k) + C_T/G(T).$$

The second order condition (after simplification is)

$$G''(T) < (r + k)G'(T)$$

Hence, for an interior maximum $G'(T)$ must intersect $(r + k)G(T) + C_T$ from above.

The optimality condition (3) can be interpreted in the following manner. On the right is the interest (revised upward by the factor $k$, acting as a sort of risk premium to take account of the known risk of involuntary depletion) foregone plus the prevention cost incurred by postponing the harvest for one period. On the left is the gain (in stumpage value) from postponing the harvest one period.
In the absence of non-operator induced (involuntary) harvesting, \( k = C_T^p = 0 \), and (4) reduces to the well known Fisherman rule that a forest should be harvested when its rate of growth equals the discount rate. With involuntary depletion of tree stock, however, \( k > 0 \) and \( C_T^p / G(T) > 0 \), and the forest should be harvested when the rate of growth is more than the discount rate. This is achieved only by hastening the harvest. This is shown graphically in Figure 2.

II

Now, consider a model where the planning horizon runs through an infinite sequence of harvests. The objective now is to maximize

\[
V(T) = \sum_{k=1}^{\infty} V_k(T) = G(T) \left[ e^{-rT} + e^{-2rT} + e^{-3rT} + \ldots \right]
\]

\[= \int_0^T e^{-rt} G(t) \, dt \left[ 1 + e^{-rt} + e^{-2rt} + \ldots \right] - \int_0^T e^{-rt} C_T^p \, dt \left[ 1 + e^{-rt} + e^{-2rt} + \ldots \right]
\]

\[= G(T) e^{-rt} \int_0^T kG(t)e^{-rt} \, dt - \int_0^T C_T e^{-rt} \, dt - \int_0^T C_T e^{-rt} \, dt \]

\[= \frac{G(T)e^{-rt} \int_0^T kG(t)e^{-rt} \, dt - \int_0^T C_T e^{-rt} \, dt}{1-e^{-rT}}
\]

The first-order condition of maximization of \( V(T) \) is

\[2 \text{ This generalizes the Faustmann solution. See Samuelson (1976) or Hirshleifer (1970, pp 88-90).} \]
Figure 2. Optimal Rotation Age: Simple Fisherian ($T_F$) and Generalized Fisherian ($T_*$).
\( (7) \quad V'(T) = \frac{e^{-rT}}{1 - e^{-rT}} \{ G'(T) - rG(T) - kG(T) - C^\rho_T \} \)

\[
\begin{align*}
    r[G(T) e^{-rT} - k \int_0^T G(t)e^{-rt} dt - C^\rho_T \int_0^T e^{-rt} dt] \\
    \frac{-}{1 - e^{-rT}} \{ G(T) - G(T)e^{-rT} \} = 0
\end{align*}
\]

This simplifies to

\[
(8) \quad \frac{G'(T)}{G(T)} = r \left[ \frac{1}{1 - e^{-rT}} - \left( \frac{k \int_0^T G(t)e^{-rt} dt}{(1 - e^{-rT}) G(T)} + \frac{C^\rho_T \int_0^T e^{-rt} dt}{(1 - e^{-rT}) G(T)} \right) \right] + k + \frac{C^\rho_T}{G(T)}
\]

In the absence of involuntary harvesting \( k = C^\rho_T = 0 \), and (8) reduces to the well known simple Faustmann rule,

\[
(9) \quad \frac{G'(T)}{G(T)} = \frac{r}{1 - e^{-rT}}
\]

Except for the term in brackets, (8) is the same as (4). The term in brackets may be treated as a "correction factor" for the interest rate. This term can be re-expressed as

\[
(10) \quad \frac{G(T) - [k \int_0^T G(t)e^{-rt} dt + C^\rho_T \int_0^T e^{-rt} dt]}{G(T) - G(T)e^{-rT}}.
\]

Since, from (1), \( G(T) e^{-rT} > [k \int_0^T G(t)e^{-rt} dt + C^\rho_T \int_0^T e^{-rt} dt] \), the expression in brackets is greater than one. This results in an "effective interest rate" (the interest rate multiplied by the "correction factor")\(^3\) in (8) which is greater than the interest rate appearing in (4). Following the

\[\] 3 These expressions are borrowed from Hartman (1976), for their aptness.
earlier logic this implies a shorter optimal harvest age relative to the model with a one-harvest horizon.

Now, equation (8) can be rewritten as

\[
\frac{G'(T)}{G(T)} = r\left[\frac{1}{1 - e^{-rT}}\right] + k\left\{1 - \frac{\int_0^T G(t)e^{-rt}dt}{(1 - e^{-rT}) G(T)}\right\},
\]

since \( r \frac{\int_0^T e^{-rt}dt}{(1 - e^{-rT}) G(T)} = CP / G(T). \) The term \( r \frac{\int_0^T e^{-rt}dt}{(1 - e^{-rT}) G(T)} \) in (11) may be written as \( \lambda \int_0^T G(t)e^{-rt}dt \), where \( \lambda = 1 - e^{-rT}/r = \int_0^T e^{-rt}dt \) is the present value of a dollar stream of return for \( T \) years. Division of \( \int_0^T G(t)e^{-rt}dt \), the discounted total return for \( T \) years, by \( \lambda \) converts it to an annual basis. If this annual value happens to be equal to the stock value of harvested timber \( G(T) \), the term in braces in (11) becomes zero and (11) and (9) imply the same optimum rotation period. If \( \frac{1}{\lambda} \left[\int_0^T G(t)e^{-rt}dt\right] > G(T) \), the rotation period implied by (11) will be longer than that implied by (9). However, it is very plausible to assume that \( \frac{1}{\lambda} \left[\int_0^T G(t)e^{-rt}dt\right] < G(T) \), and hence the optimal rotation period implied by (11) will be shorter than that implied by the simple Faustmann rule (9).

III

The main conclusion of this analysis is that the presence of involuntary and non-operator induced depletion of tree stock and costs incurred to prevent it may have an important impact on when a forest should be harvested. Incorporation of this widely observed phenomenon (particularly in the third world countries) in a formal model, lends the decision of when to harvest a forest further generality. However, the
models considered in this note are particularly simple. Any realistic model should incorporate regeneration costs and multiple use characteristics of a standing forest.\textsuperscript{4}

The cumulated value of timber expropriated by the non-operators may be treated as a transfer of income or wealth. But this harvesting process is essentially suboptimal because it involves some real resources and time which could, otherwise, be utilized more efficiently elsewhere. Non-optimal behavior of the non-operators imposes a premium \((k)\) on the operators, compelling them to choose a shorter optimal rotation age, devaluing society of the benefits of economically more mature volume of tree population. A standing forest contributes a large flow of economic value besides timber. The flow of these services is an increasing function of the age of the forest (Hartman, 1976). Choice of shorter rotation age by the operators and deforestation by non-operators entail a loss of such multiple use values of a standing forest. To realize these values, a solution may be to enhance the living conditions of the poor and/or providing them access to alternative resources to meet their day to day needs. An alternative policy may be to provide the operators with some sort of subsidy to protect the growing forest from non-operator intervention or provide sufficient compensation to induce delaying harvest till such time as is optimal for society. In any event, this problem requires further theoretical and empirical investigation to help quantify gains and losses such that appropriate corrective action may be taken.

\textsuperscript{4} See, e.g., Hartman (1976), Strang (1983), Berck (1981), or Bhattacharyya (1985).
REFERENCES


