A 1D Plasmasphere Refilling Model through the Application of the Flux-Corrected Transport Method

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Abstract

The refilling of the plasmasphere following a geomagnetic storm remains one of the longstanding problems in ionosphere-magnetosphere coupling research. In this report, we present the development of a 1D refilling model using the flux-corrected transport method, a numerical method that is extremely well-suited to handling problems with shocks and discontinuities. The developed methodology has been validated against exact, analytical solutions and preliminary results are also seen to be consistent with refilling results reported in the literature. The ultimate objective of this research is the development of a 3D multi-ion model for the plasmasphere refilling problem and with additional development the methodology could potentially be applied to the study of other complex space plasma coupling problems in closed flux tube geometries.

Introduction

There are two kinds of ionospheric outflow in nature – the refilling of the plasmasphere following a geomagnetic storm [*Carpenter & Park*;1973] which occurs along closed field lines and polar wind outflow [*Khazanov,* 2009] which occurs along open lines. At its essence, these problems involve plasma escaping into vacuum in the presence of gravitational, electrostatic and magnetic force fields and collisional effects. The development of a hydrodynamic model for the refilling of the plasmasphere following a geomagnetic storm is the subject of this report. However, a brief literature survey of the numerical models that have been developed for the modeling of ionosphere-magnetosphere coupling problems is provided in the following section.

Diffusion and Hydrodynamic Outflow Models: A Brief Literature Survey

Over the last several years, numerical studies have been undertaken to model and quantify ionospheric outflow and these studies have led to the development of ionosphere-plasmasphere coupling models. These models based on the solution of the plasma transport equations, fall within two broad categories. In one of these two categories, the nonlinear inertial terms in the plasma transport equations are neglected and thus low-speed, diffusion dominated flow can be modeled. Included in this category are the Sheffield University Plasmasphere Ionosphere Model (SUPIM) [*Bailey, et al.,* 1997], the Ionosphere-Plasmasphere Model (IPM) [*Schunk et al.,* 2004], and the Field-Line Interhemispheric Plasma (FLIP) model [*Young et al.*, 1980]. The FLIP model has recently been integrated into the Ionosphere Plasmasphere Electrodynamics (IPE) model developed at the National Oceanic and Atmospheric Administration/Space Weather Prediction Center (NOAA/SWPC) to facilitate a better understanding of the connection between terrestrial and space weather.

The second category of models that exists in the literature is the so-called "hydrodynamic model," where the nonlinear inertial terms are retained in the plasma transport equations. It was introduced by *Banks et al.* [1971] and has subsequently been worked on by many researchers [*Khazanov et al.,* 1984; *Singh et al.,* 1986; *Rasmussen & Schunk,* 1988]. The most well-developed hydrodynamic model of the low-latitude ionosphere is SAMI2/SAMI3 developed by *Huba & Joyce* [2000] and *Krall & Huba* [2013]. In the next section, a brief description of the 'flux-corrected transport' (FTC) method [*Boris & Book,* 1976; *Kuzmin et al.,* 2012] used in this work, is provided.

Flux-Corrected Transport Method: A Brief Description

Ionospheric outflow problems are characterized by sharp discontinuities in the ion concentration profiles, resulting in gradients in the pressure as well as electric field profiles. Had it not been for these discontinuities, a second-order scheme such as the Lax-Wendroff [*Hoffmann and Chiang,* 2000] would have been adequate, where the numerical method itself does not introduce any diffusion in the problem. The fundamental philosophy behind the FTC method is that "diffusion" is artificially introduced "only" at spatial points where shocks and discontinuities are present. In partial fulfillment of the requirements of K. Chatterjee's doctoral dissertation (to be completed by the end of 2018), an FTC-based solution methodology has been developed for the plasmasphere refilling problem (including multiple ions and neutrals) following a geomagnetic storm. In this report, we discuss briefly the analytical benchmark problems that have been solved to validate the methodology. An extended discussion of the results, along with associated figures, will be presented in a peer-reviewed American Geophysical Union (AGU) journal.

Discussion of Results

The following results were obtained for the analytical benchmarks that were established to validate the refilling model:

A. The developed methodology was used to solve the problem of the propagation of a square wave with a constant velocity in a 1D problem domain. The problem is an "approximation" of plasma transport under the assumption of constant drift velocity. It was seen that "flux-correction" remedied the problem of oscillations present at the two edges of the square wave, when a second-order scheme such as the Lax-Wendroff [*Hoffmann and Chiang,* 2000] was used

B. The second benchmark problem solved was the problem of a single-ion plasma escaping into vacuum, and the numerical solution agreed with the "self-similar" solution given in *Schunk & Nagy* [2009].

C. In the third application example, plasmasphere refilling after a geomagnetic storm is modeled as a single-stream isothermal flow of H^+ ions, governed by mass and momentum conservation equations, under the assumption of constant gravitational force and neglecting the curvature of the field line, while choosing the length of the line to be the same as that of the '*L=4*' line. Even under these simplifying assumptions, the results obtained from the model was consistent with literature [*Singh et al.,* 1986; *Rasmussen & Schunk,* 1988]. High supersonic drifts velocities were observed in the early hours of refilling, which agreed with the result in Benchmark Problem B, obtained under the assumption of zero gravity. Also observed was a shock formed at the equator within half an hour of the onset of refilling, consistent with *Singh et al.* [1986]. The formation of the shock was a consequence of the fact that the plasma was modeled as a single stream, as opposed to separate two streams originating from the northern and southern hemispheres. Finally, steady-state was reached in about 20 hours consistent with *Singh et al.* [1986], and the steady-state numerical solution matched with the analytical result.

Summary and Future Work

Summarizing, we have developed a 1D refilling model for the plasmasphere following a geomagnetic storm using the FTC method. The model has been validated by exact, analytical solutions and the results obtained are also consistent with those reported in the literature. In this work, a single stream flow model has been assumed for modeling the flow of H^+ ions. Our future work in this area would involve the improvement and enhancement of this model through incorporating plasma flow along dipolar field lines as well as modeling the plasma flow as two separate streams originating from conjugate ionospheres. It is known that two-stream models do not produce the shocks that are produced in single-stream models, and it needs to be further explored if these shocks are physical or not. In addition, our model will also be adapted to include multiple ion species as well as boundary conditions different from this present work. The ultimate objective of this research is the extension of this methodology to two and three dimensions and the modeling of plasma transport across field lines.

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