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NEW CONCEPTS FOR PRELIMINARY HYDROPOWER DESIGN:

THE POWERMAX SLOPE, BINARY TURBINE SIZING,

AND STATIC REGAIN

by

Frank W. Haws
Eugene K. Israelsen

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ABSTRACT

        In Utah during the 1960s, the cost of producing electrical
energy was as much, or in some cases more, by hydroelectric
generation than by plants using steam from coal fired boilers.
The relatively high hydropower cost was generally attributed to
maintenance and replacement costs associated with plants that had
been built in the 1920s. Utah Power & Light Company during the
1960 period decided not to renew power licenses and to abandon
many small hydroplants. Since 1973, rising coal and related
fossil fuel costs have caused steam generation costs to acceler-
ate and have made hydroelectric generation relatively more
attractive. However, the capital cost of replacing deteriorated
pipelines and restoring plants to production capability is high,
and the prospect of large capital investment during periods of
high interest rates creates a hesitancy to renovate existing or
to construct new small hydro units.

        The cost analysis to replace abandoned plants or to con-
struct new plants has been generally based on restoring an
existing configuration or building to design standards in use at
the time of the original structure. The traditional design
method was to design a pipeline on a flat slope with a relatively
large pipe diameter. This method maximized head, but minimized
the flow. The resultant energy was therefore less than the
potential, but constant. This method also confined the varia-
tions in flow to a range that could be handled by a single, or at
most two, variable geometry turbines. The flow point on the
typical flow duration curve for western mountain streams where
the ratio of maximum to minimum flow variation is 4 to 1 or less
is at or near the 25 percent exceedance level.

        It is shown in this report that the same diameter pipeline
as used in traditional design can be sloped to maximize the power
output of the plant (powermax slope) and thus increase the annual
energy production by 149 to 186 percent, the difference being
dependent upon the amount of energy recovered by the static
regain in pressure pipelines when flows are reduced below the
maximum. This optimizes flow and head without changing the cost
of the pipeline. The effect is to reduce the unit cost of energy
produced.

        The higher flow at the powermax slope has a greater vari-
ability and will therefore require turbines with greater vari-
ability. It is demonstrated that multiple fixed geometry tur-
bines sized in binary steps can effectively span flow variability
ratios from 10 to 1 or greater and be installed at less cost than
custom designed variable geometry units. Thus, designing at the

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point on the flow duration curve corresponding to the 10 percent or lower exceedance level is economically feasible.

Combining the powermax concept for pipelines with the concept of using binary sized turbines and a pressure system to use the static regain concept can result in hydro plant designs that utilize a greater portion of the potential energy at a given site and reduce the unit cost of energy.

ACKNOWLEDGMENTS

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SYMBOLS AND DEFINITIONS

MW = megawatts

kwh = kilowatt hours

L = pipe length in feet

H_G = total head available in feet

H_f = head lost in friction

H = total hydraulic head

H_m = minor head losses

H_T = sum of H_f + H_m

k = sum of coefficients for minor losses

P = power in kw

Q = flow rate in cfs

D = pipe diameter

e = roughness element

f = friction factor

Re = Reynolds number

T = time interval

Q^* = flow which achieves maximum power

D^* = diameter needed to produce maximum power

P_{max} = maximum power

N = number of turbines

C = number of interger combinations

λ = kinematic viscosity

S = hoop strength

p = internal water pressure

D = average diameter

t = wall thickness

W = weight

Di = inside diameter

Do = outside diameter

GWH/yr = gigawatt hours per year

V = velocity

g = acceleration due to gravity

E = efficiency

w = specific weight

INTRODUCTION

The boom period for hydroelectric power development in Utah peaked during the 1920s. In the Utah service area of Utah Power & Light Co., there were 95 separate sites with about 207 MW installed capacity. Fifty years later, as federal energy licenses expired and existing pipelines, penstocks, and electromechanical equipment needed repair or replacement, the decision was made to abandon many of these small hydroplants in favor of larger steam-electric plants. This was particularly true of the large utilities such as Utah Power & Light Company whose demand for power soon exceeded the combined output of all its hydroplants. Utah Power & Light Company had at one time upwards of 40 separate hydrosites, many of them with multiple turbines. The total installed power was about 175 megawatts (MW). With the retirement or abandonment of 17 small plants the installed UP&L hydro capacity has declined to the present 124 MW. This compares with UP&L total generating capacity in June 1983, of 3343 MW.

During the 1960s, it was determined by UP&L that the energy generated by hydroplants could be replaced by energy from steam generating plants for about 3 mills per kwh. The operation and maintenance cost of many small worn out hydrosystems was approaching or exceeding 3 mills per kwh, and when cost of modernizing the units was added to the O&M costs, the hydroplants were not considered economical. Thus decisions were made to abandon.

The increasing value of electric power today and the escalating costs of fuel to fire the steam plants, coupled with the national goal to reduce dependence upon imported oil is causing utilities to take a second look at the hydropower potential and to perhaps construct new hydropower plants or to reinstate some of the older plants. Furthermore, hydropower is clean, has little environmental impact, can be operated at high efficiency, and does not use up a limited fuel resource. The problem is, however, that first costs of building new or refurbishing old plants are high. This is particularly true if long pipelines are needed to conduct the water to the turbine. In some cases, the cost of replacing the existing pipeline to a small generating unit can exceed $1500 per installed kilowatt which is the cost generally attributed to a completely new installation including electromechanical equipment. The last 400 MW coal fired plant (Hunter 3) to come on line for UP&L cost in 1983 dollars about $1100 per installed kilowatt.

The purpose of this study was to examine the production of electrical energy from falling water in nontraditional ways and to determine if changes in design philosophy, the use of new materials, or construction methods could reduce the cost of energy to a point where utilities would again choose to invest in hydropower. The cost of energy can be reduced by decreasing the cost of the facility, increasing the power produced by a given sized facility, or by a combination of the two.

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Run of the river hydropower systems in mountainous terrain typically consist of: 1) a diversion structure with pipe inlet and trash rack, 2) a long conveyance pipe and penstock, 3) a turbine-generator system, 4) tailrace, and 5) appurtenant controls, regulators, safety devices and transmission lines. Each of these components might profitably be examined for ways of reducing costs, but this study considers in depth only two, the pipelines and the turbine-generators. The first four are briefly described.

1. Diversion. Some method must be used to get the water out of the natural stream into the pipeline while excluding unwanted debris. Screens are normally installed to obstruct the passage of debris into the pipeline and either manual or mechanical rakes are used to clean the screens. This obviously requires energy and represents a high cost in operation and maintenance. Automated mechanical rakes usually require shelters and some method to dispose of the raked debris. High capital costs are also associated with this type of trash rack. Additionally, a diversion structure must provide flow control devices, capacity to pass high spring runoff, and ability to handle winter icing conditions. Traditional methods have not satisfactorily solved these problems without unproductive use of energy.

2. Conveyance. The water must be transported from the diversion to the water turbine in such a manner so as to conserve energy and make it available to the associated generating equipment. Traditional methods have used open ditches, flumes, or low head pipes, all installed on the "grade" line or that location that will convey the water to the site with the least decrease in elevation below the diversion. This "grade" line concept is graphically defined in Figure 1(a). At the site, the water is then dropped through a steep penstock to the turbine. This "grade" line method uses a low velocity flow and requires a large cross-sectional area conveyance structure, which in turn economically limits the system water flows. The head at the turbine under this method is constant. The unit cost of these systems is higher per unit of water flow than are systems which are placed to use greater pressures in the pipe. Placing the pipe on a steeper slope than the "grade" line permits smaller diameter pipes or larger flows and also a greater recovery of static pressure at low flows. It can be shown that there exists in every pipeline system a slope that will produce maximum power. This slope is defined as the "powermax" slope and is shown graphically on Figure 1(b). Since the cost of the pipe represents such a large proportion of the total project cost, it should be practical to use the size that will give the greatest energy output. This is not always in keeping with traditional practices.

3. Conversion. The energy in falling water is converted to electrical energy through rotating turbines and generators. The turbines can utilize either the velocity (impulse type), or the pressure (reaction type) of the water, and can be fixed or variable geometry configurations. In cases where the natural streamflow varies over a wide range of flow, as is the case in most mountain streams, multiple turbines can be used. There are some nontraditional ways of selecting the number and
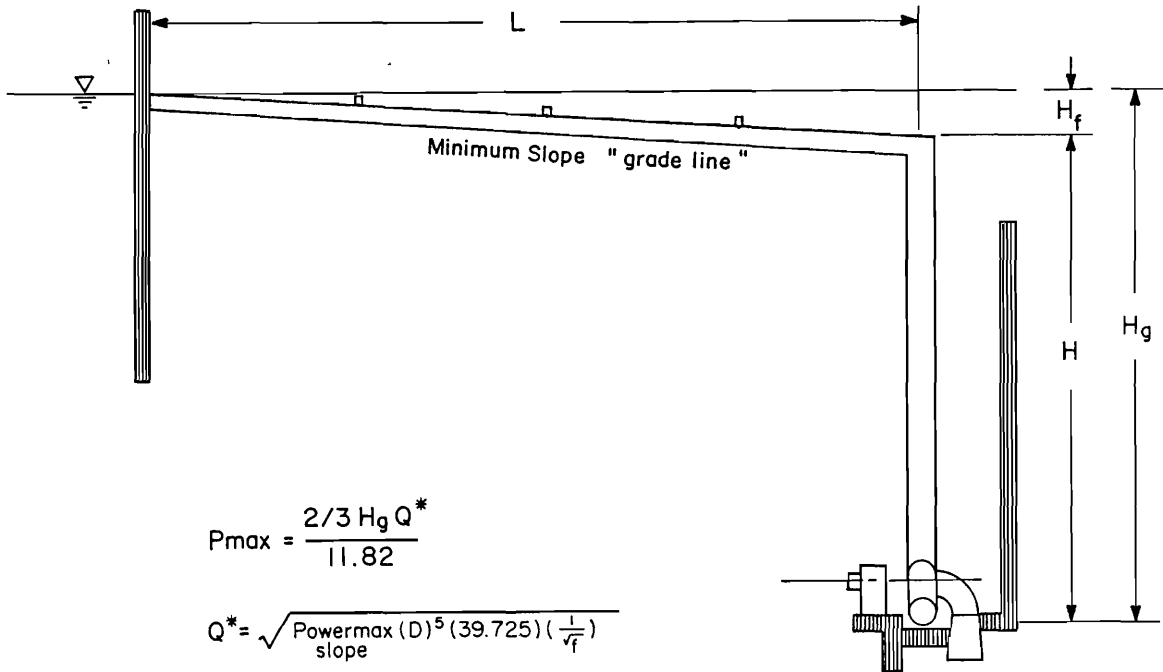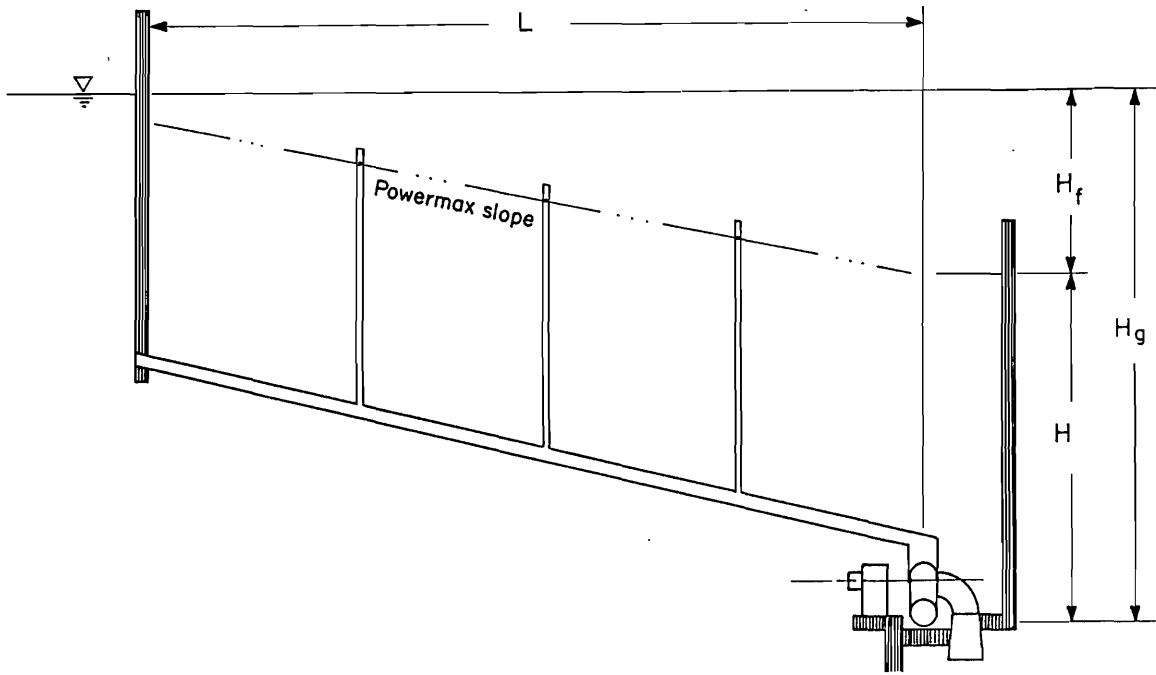
$$P_{max} = \frac{2/3\ H_g\ Q^*}{11.82}$$

$$Q^* = \sqrt{\frac{Powermax\ (D)^5\ (39.725)\ (\frac{1}{\sqrt{f}})}{slope}}$$

$$Powermax\ slope = 1/3\ \frac{H_g}{L}$$

$$\frac{1}{\sqrt{f}} = 1.14\ \ 2\ log_{10}(\frac{9.35\ \frac{1}{\sqrt{f}}}{R_e} + \frac{e}{d})$$

Figure 1. Diagram showing head loss at powermax slope and "grade line" slope.

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size of turbines that can be used to increase output and decrease costs, as will be discussed later.

4. <u>Outlet</u>. The outlet works are usually associated with the conversion portion of the system and must be rationally designed but usually are not a significant portion of overall cost.

There has been little change in the design philosophy of any of these segments since the early days of hydro-development. The existence today of large interconnecting grids of power sources and distribution lines should allow a different philosophy. This study therefore looks at power development as if all generated sources can be utilized and that the goal should be to maximize energy production at the lowest cost. The long pipelines and the use of multiple generating units will receive prime consideration.

The benefits from hydropower generation are derived from the amount of energy produced which is determined by the quantity of flowing water and the total available head. The price of electrical energy also plays a part but is not determined by the generation site.

Energy from falling water can be expressed in simple terms as a function of the rate of flow of available water, Q, the total hydraulic head, H, and the time interval during which the event is measured, T, or

$$Energy = f(Q, H, T)$$

The flow rate is dependent upon precipitation, temperature, size of watershed area, infiltration rate, vegetative cover, and all other factors which affect the runoff from a watershed. Since energy recovery systems are fixed in size and location it is necessary to know the historical occurrence of water at the point of recovery and to assess the probable risks associated with dependence upon a particular flow. One of the first tasks then necessary for energy evaluation at a site is a flow-duration analysis, reproduced in a format that is easily understood and used. There is usually very little that can be done to alter the hydrologic setting to change the flow either in magnitude or temporal distribution, although weather modification, water imports, and watershed management may be useful. In most cases an energy site is fixed by nature and must be evaluated historically and statistically.

The hydraulic head or the vertical distance that the water can be made to fall depends largely on the topography of the site. A high dam with storage, a high dam without significant storage, or a diversion dam with a long penstock may be necessary to create the difference in hydraulic head between the two sides of the turbine.

When a long penstock is used, part of the elevation difference between source and outlet works must be used to overcome the friction losses in the pipe. The remaining difference or head, H, is used to drive the turbine. Increasing the head lost in friction, H_f, increases the flow rate, Q, but reduces the head available for power generation, H. Since power is the product of Q and H, the pipe diameter should be chosen so the Q-H combination will result in the maximum power. This will automatically result in the lowest power cost per unit of flow. Any additional power generated by oversize pipe diameters must be justified on the basis of the lowest alternative cost available to the producer.

The third parameter, the time interval over which power generation takes place, is important because of the variability of the flow. A constant Q can sometimes be maintained when storage is available, but without storage, as in a run of the river operation, Q is a variable. Since the flow system completes a cycle with each revolution of the earth around the sun, the analysis of river flow on an annual basis is most commonly used. The flow between extremes of high and low as well as the variation between extremes are represented on a flow-duration curve. If available head can be superimposed upon the flow, an energy-duration

curve can be constructed and used to evaluate the total energy available for the year or for time intervals within the year.

The energy parameters listed, Q, H, and T, are in reality fixed by nature. Very little can be done by man to change the hydrology and meteorology which affects Q, or the topography to change the slope of the river bed, or the cycle of the earth around the sun. The extraction of energy from falling water is dependent, therefore, upon the devices used to capture a given Q, to manipulate the given H, and to fit the equipment to function over a long period of time, and over a wide variation in Q.

DESIGN CRITERIA

At any given hydropower site, there are many alternative physical configurations to choose among. Decisions must be made as to the placement of the turbines with respect to the gross head available and the length of penstock needed; the diameter of the penstock and the material of which it is constructed; and the number, type, and size of turbines required. Each decision affects the number of dollars invested in the physical plant and the capacity of the plant to produce energy which in turn is valued in dollars. The most economical system will be that one which maximizes energy production and minimizes plant cost.

After carefully analyzing traditional hydrostructures in Utah and the factors upon which logical design can be made, the writers recommend the following design criteria. If these are followed, the effective cost of hydroplants can be decreased. That is, the capital cost can be reduced, or the generated output can be increased over traditional methods.

1. Do not limit design flow rates to some pre-selected point on the flow duration curve. Flows greater than the traditional 25 percent exceedance values may be economical and can be utilized if chosen properly.

The duration curve is constructed by plotting a flow value versus the percentage of the time which the flow value is equaled or exceeded according to the total historical record. Figure 2 shows the duration curve for Beaver Creek, Utah, in southern Utah, and is used to illustrate the logic and fallacy of using the 25 percent exceedance point for design. There are two parameters to

consider: 1) the diameter of the pipe and 2) the flow range to be spanned by the turbines. If, as was the practice in the 1920s, the design intent was to maximize head, the wood stave pipe diameter necessary to carry 42 cfs (25 percent exceedance) at 2 percent head loss would be 48 inches. This probably represents the maximum diameter wood stave pipe that could economically be built on such a steep hillside. The flow range of 4 to 1 could easily be met with not more than two turbines. But, the energy still available in the higher stream flows occurring less frequently is substantial. At the 10 percent exceedance level (Q = 119 cfs) the recoverable energy would be 139 percent of the energy at 25 percent exceedance, and at the 7 percent exceedance level (150 cfs) the recoverable energy is 151 percent of the 25 percent level. Although there are substantial amounts of energy in the high flows of mountain streams, it was not really available to the 1920 designers. Besides the technical difficulties of building large diameter pipelines and spanning the wide flow range with turbines, the designers had no market for power produced for such a short time during the year. The intertie to an extensive power grid was not yet available.

2. Do not design the diameter of long pipelines on the basis of pre-selected velocity limits. High velocities in pipelines (at least to 20 fps) can be tolerated and designed for.

There should be flexibility in design and the opportunity to examine many alternatives. Unfortunately, past "experience" has crept into textbook
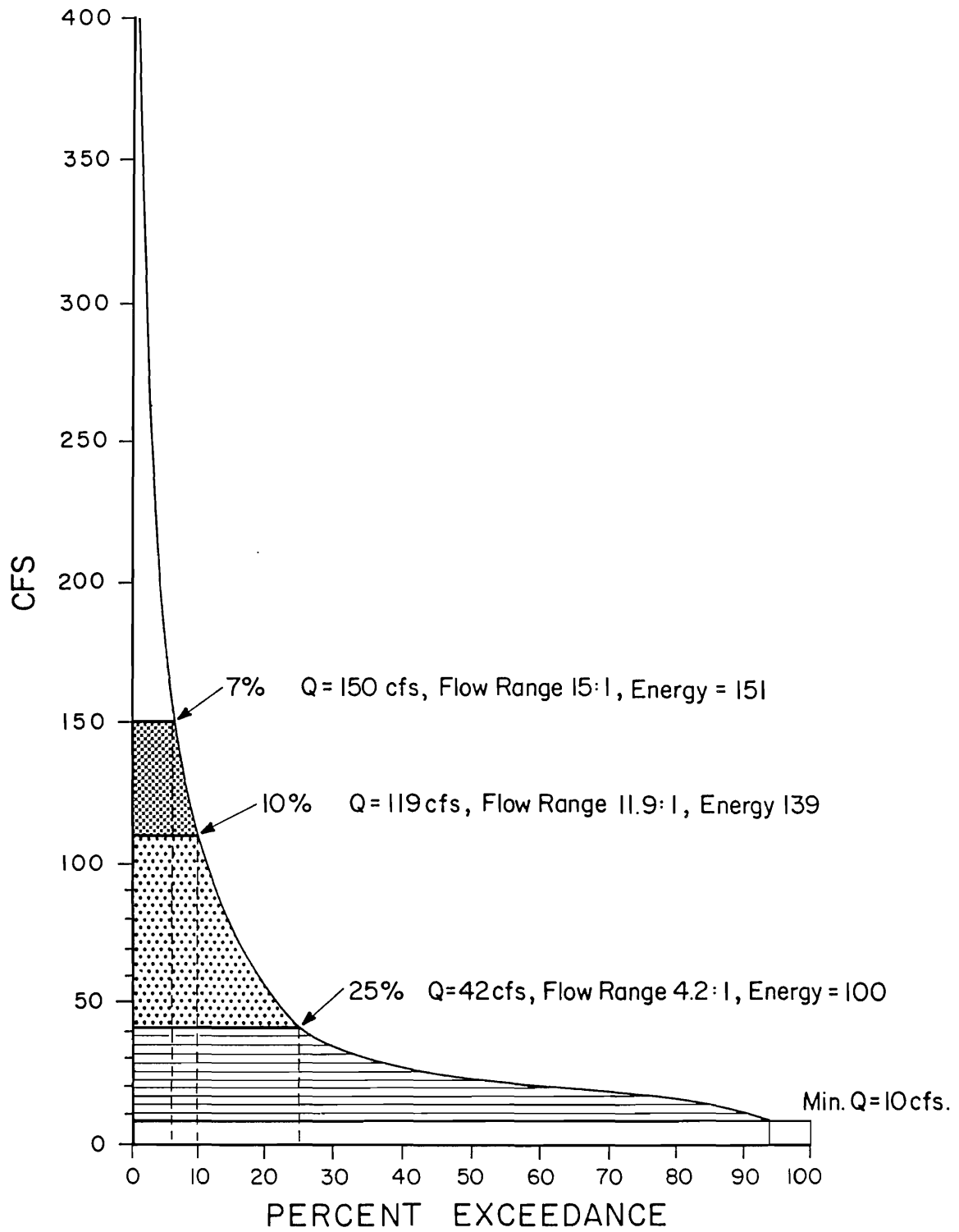
Figure 2. Flow duration curve for Beaver Creek, Utah.

instruction and placed limitations on the velocity of flow in a pipeline. The expression, "it has been shown that ..." is used to limit the velocity to about 10 feet per second. With the flow rate, Q, defined by the 25 percent exceedance level of the duration curve, and the velocity limited to 10 fps, the pipe size is automatically defined and the power potential limited.

3. Reduce the diameter of long pipelines by using a pressure pipeline and design for optimum power production utilizing a hydraulic gradeline equal to the "powermax slope" using the equations shown herein.

The hydropower equation is quite simple and straightforward, and can be written as

$$P = QH_GE/11.82 \quad . \quad . \quad . \quad . \quad (1)$$

in which

P = power in kw
Q = flow in cfs
H_G = total head available in feet
E = efficiency

When long pipelines are used to convey water to the turbine, the total head for power generation is reduced by the friction loss in the pipe. The power equation then becomes:

$$P = \frac{QE \left(H_G - \dfrac{LQ^2 f}{39.725 \, D^5} \right)}{11.82} \quad . \quad . \quad (2)$$

in which

f = friction factor
L = pipe length in feet
D = pipe diameter in feet

The power equation in a system with no pipeline head loss is a straight line function between flow and total head. When pipelines are used, the

power equation is not linear and includes D to the fifth power and Q to the third power. By incrementing Q and solving for P in the pipeline power equation for a given diameter and length, a flow is found that gives a maximum power value. Flows higher or lower than this value produce less power. This behavior can be expected from the form of the actual power equation. The writers noted this behavior in the power equation while computing the power for a chosen river on a daily flow basis. It is thus implied that there is a maximum power point in any real system. Since maxima and minima are determined by differentiating an equation and setting the differential equal to zero and solving for the desired variable, this was done with respect to flow for the actual power equation. The resulting equation is:

$$Q* = \sqrt{\frac{39.725 \, D^5 \, H_G}{3L}} \cdot \frac{1}{\sqrt{f}} \quad . \quad . \quad (3)$$

where

Q^* = the flow which achieves maximum power for the defined conditions of H_G, L, D, and f.

The relationship between power and Q in a pipeline with friction losses is shown graphically in Figure 3.

It should be noted that H_G/L is the average slope of the streambed between diversion and turbine, and that Q^* reaches maximum when the hydraulic gradline slope is $1/3 \, H_G/L$. The power equation for pipeline systems can now be written

$$P = \frac{2/3 \, Q^*H_GE}{11.82} \quad . \quad . \quad . \quad . \quad (4)$$

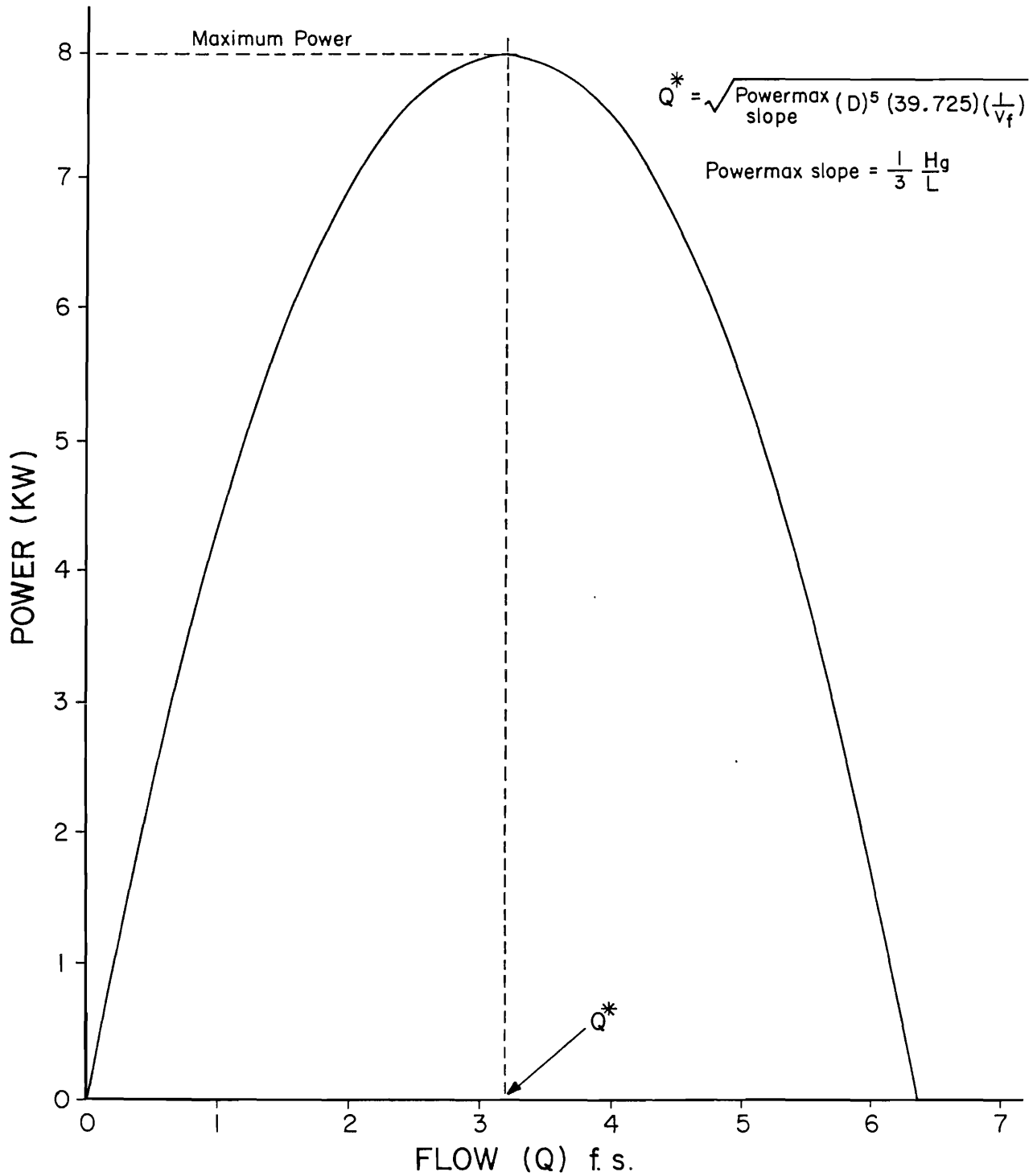The derivation of these equations is shown in the appendix.

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Figure 3. Relationship between power potential and rate of flow with given pipe diameter, length, and slope.

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4. Locate pipeline in most accessible route. There is no need to build the pipeline on a steep sidehill.

Since Q^* occurs with considerable head loss, the velocity in the pipe is usually greater than 10 fps. This can only be accomplished by using a pressure pipeline, which also permits the route location to be chosen to fit the more accessible areas and to not be confined to steep, inaccessible sidehills. Another advantage of using pressure pipe is that during low flows the head loss is reduced and the effective head for producing power is increased. This recovered energy is termed the "static regain" principle and is illustrated in the example later.

5. Select number and type of turbines to match maximum flow range, sizing the turbines using a binary scheme as explained herein.

To extract the energy from a stream using the high flows occurring at the lower exceedance levels means providing turbines that can operate efficiently over a wide range of flows. A single Francis turbine with adjustable wicket gates can efficiently operate over a 2:1 flow range. This means that the minimum flow which the turbine can efficiently accept is about 50 percent of the design or maximum flow. Kaplan turbines which have adjustable blades can operate over a 3:1 range and impulse type turbines can operate over a 5:1 range. If the streamflow varies over a 15:1 flow range and if equal size turbines are used in multiples, it would require eight Francis turbines, five Kaplan turbines or three impulse turbines. Obviously the wide span in river flows would be costly to handle in this manner. By using unequal size turbines and by sizing the turbines in a binary sequence, the number can be greatly reduced.

In a binary set of numbers, every number in the sequence is an integer power of the number 2. The sequence is

2^0, 2^1, 2^2.....2^n or $1,2,4...2^n$. The maximum number of turbine combinations is the sum of the binary numbers or:

$$C = 2^N - 1 \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

where C is the number of integer combinations and N is the number of turbines. The negative one is included since zero is not considered to be a useful combination. For example if four turbines are used, the binary sequence is 1,2,4,8 and the sum of these integers ($2^n - 1$) is 15. Applying the binary sequence to the example above of a stream which varies in flow 15:1, it would take three, instead of eight, Francis turbines in the sequence 1,2,4. There are seven combinations in this sequence and if Francis turbines span 2:1, the total span of the system would be 7 x 2 which is 14:1. This is not quite up to the 15:1 specified, but close enough to question the value of adding a fourth unit. Using Kaplan turbines the required number of combinations would be 5, (15/3 = 5). This can be done using turbines in the sequence 1,2,2. Impulse turbines would require a combination of flows equal to 3, (15/5 = 3). This can be done with two units as 1,2. Thus it can be seen that a wide range of flows in a stream can be handled by few turbines. Very seldom would it be necessary to exceed four units.

When multiple units of unequal sizes are used, the desirable overlap between the maximum flow of one turbine and the minimum flow of the next turbine is important and dependent upon the flow range of the turbines. Turbines with flow ranges equal to 2:1 or larger will have sufficient overlap to effectively use all the flows with no spills between the sizes. When the flow range is less than 2:1 there will be some water spilled or used at efficiencies less than desirable. However, these spills occur at the lower flows. The narrower the turbine flow range the greater this spill becomes. The magnitude of the energy lost, however, may

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not be sufficient to pay for additional turbines.

6. Compare several pipe materials as they relate to cost, strength, flow characteristics, durability, and ease of installation.

In selecting a pipe for long hydro penstocks, the pipe must be chosen on the basis of three properties: 1) its hydraulic smoothness, 2) its structural integrity and 3) its chemical resistance. Ease of installation is also a factor when costs are tabulated, but is not discussed in detail here.

Hydraulic

Flow through a pipe, is a function of the slope of the pipe, the diameter, and the friction factor. The Q for maximum power is:

$$Q* = \sqrt{39.725 \cdot \frac{slope}{3} \cdot D^5} \cdot \frac{1}{\sqrt{f}} \quad (6)$$

The parameter $1/\sqrt{f}$ is a function of the smoothness of the pipe, the velocity of the fluid, and the fluid temperature and viscosity. The smoothness of the pipe is measured by the size of its wall roughness elements, e. The velocity, temperature and viscosity are represented by the Reynolds numbers, Re, which can be expressed by the term:

$$Re = \frac{4Q}{\pi D \nu} \quad \text{where } \nu \text{ is the kinematic viscosity}$$

The Moody diagram is usually used to display the relationship between f, Re, and the relative roughness e/D. Equations can also be solved, usually through an iterative process, to give more precise values. The equation to be used in most hydro power cases where flow is always turbulent is:

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10} \left(e/D + \frac{9.35 \left(\frac{1}{\sqrt{f}} \right)}{Re} \right)$$

$$\cdots \cdots \cdots \quad (7)$$

The precision is usually not justified for more than three digits and hence few iterations are necessary to solve for $1/\sqrt{f}$. The third iteration using calculated values for next estimate is usually sufficient.

For example, suppose you wish to solve the equations for Beaver Creek in Utah using 36 inch diameter steel pipe. Q = 104 cfs, e = 0.0018 inches. Since $1/f$ will vary between 7.5 and 10.0, for the first iteration, use $1/\sqrt{f}$ = 8.30 as initial estimate. First solution gives $1/\sqrt{f}$ = 9.39. Using this value the second solution gives $1/\sqrt{f}$ = 9.35. The third solution also gives $1/\sqrt{f}$ = 9.35. The second and third iterations give values differing only in the fourth digit and give an f value of 0.0114.

The size of the surface roughness element, e, and the ratio e/D determines the smoothness of the pipe and the energy loss in friction. Hence a low value of e means a greater flow capacity for a given diameter. Reducing the diameter required generally means a reduction in cost.

The roughness element, e, is shown for various pipe material in Table 1.

Structural

A long pipe conveying water from a stream to a power plant is subject to structural forces which the pipe material must be able to resist. The most obvious force is the bursting pressure of the water inside the pipe. Other internal and external forces are present or may become present at some time during the life of the pipe. Forces created by the velocity of the water, the hydro static pressure of the water, and its external loads created by back fill material, dynamic loading, beam supports, temperature differences including contraction, expansion, and freezing must all be considered. In addition, the possibility of vandalism

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Table 1. Roughness element, e, for various pipe materials and the Reynolds number for different water temperatures.

Material	e	
	Inches	Feet
Riveted steel	0.036 to 0.36	0.003 to 0.03
Concrete	0.012 to 0.12	0.001 to 0.01
Wood stave	0.0072 to 0.036	0.006 to 0.003
Cast iron	0.0102	0.00085
Galvanized iron	0.006	0.0005
Asphalt-dipped C.I.	0.0048	0.0004
Welded steel, commercial steel or wrought iron	0.0018	0.00015
PVC	0.000084	0.000007
PE	0.000084	0.000007
FRP	0.00090	0.000075
RPM	0.00090	0.000075
Drawn tubing, brass, lead, glass	0.000060	0.000005

Water Temp.	Re
40°	76500 Q/D
50°	90300 Q/D
60°	104600 Q/D
70°	120230 Q/D

to exposed pipes or to destruction from falling debris on steep mountain slopes are important items. This treatise does not intend to be a manual of pipe design but only to indicate there are many things that influence the selection of pipe material and pipe location and that because many of these are site specific the selection and comparisons of pipes cannot be generalized. It would probably be helpful, however, to be able to make some comparison of pipe materials so that cost advantages would be more readily available. The criteria for comparison will be the internal bursting strength of pipe material.

The ultimate strength of the material in hoop tension resisting the internal water pressure is a unit force called hoop stress. It is calculated by the simple relationship:

$$S = pD/2t \quad \ldots \quad \ldots \quad (8)$$

where

S = hoop strength in pounds/sq. inch. (psi)

p = internal water pressure (psi)

D = diameter in inches (average diameter between inside and outside dimensions)

t = wall thickness of the pipe (inches)

Solving for wall thickness, the relationship is:

$t = pD/2S$ or $t = \dfrac{p(Di + t)}{}$, inside

diameter, and $t = \dfrac{p + (Do - t)}{}$,

outside diameter

Pipe manufacturers generally specify the allowable hoop stress for their material which is generally the tensile strength divided by a safety factor which ranges from 2 to 6 depending on the material.

When the wall thickness is known and the material is specified, the weight per foot of pipe can be calculated. The weight is:

$$W = tw (Di + t) (12) \qquad . \quad . \qquad (9)$$

where

W = weight in #/ft
t = wall thickness, in
w = specific weight of material #/in^3
Di = inside diameter of pipe

When outside diameter is given the relationship is:

$$W = tw (Do - t) (12)$$

The values of S for hoop stress and W for specific weight of some of the material used in the manufacture of pipes for hydropower penstocks are given in Table 2.

Chemical Resistance

When comparing cost of a power pipeline, the life of the material within the site environment must be considered. Some materials are very active chemically and react to corrosive elements in the water, air or soil of the site environment. Steel, for example, is nearly always coated internally with asphalt, enamels, or epoxys, and wrapped externally with asphalt impregnated paper or fabric. Other materials, such as high molecular weight polyethelene and polyvinyl chloride, are naturally immune to corrosion without further treatment.

Table 2. Allowable hoop stress and specific weight for pipe materials.

Material	Allowable Hoop stress (psi)	Specific Weight (pounds/cu.in)
Commercial Steel	18,000	0.2833
PVC	2,000	0.0498
PE	800	0.034
FRP (RTR)	6,000	0.064 to .070

To show how the design criteria can be used in evaluating a specific hydrosite the following example is described. Utah Power & Light Co. has a hydrosite located in Beaver Canyon in southern Utah. The original power plant, built by Telluride Power, consisted of a 36-inch diameter wood stave pipeline running on "grade" 12,309 feet from the diversion point on the river to a surge pipe above the powerhouse. The wood stave pipe continued for another 410 feet and then joined a 775 foot steel pipe penstock which delivered water to the turbine nozzles. The vertical drop at this point was 484 feet. A new pressure pipeline installed in the bottom of the canyon would be about 12,720 feet long. The capacity of the wood stave pipeline with f = 0.02 and h_f = 8.2 feet is calculated to be 17.7 cfs. The power house is equipped with two pelton wheels with a total rated capacity of 600 kw.

The 36-inch diameter wood stave pipe has been abandoned because of badly deteriorated material and collapsed sections. The plant has not operated for several years. The question is: Does the site have the potential to economically support a new power generating system?

We next examine the site on the basis of the criteria mentioned.

Flow-Duration Curve

The historical record of Beaver Creek was used to construct the flow-duration curve of Figure 2. The high flows occur for a small percentage of the time, typical of snowmelt fed mountain streams, and the ratio between high and low flows are in excess of 40:1. The curve was plotted in 1 percent time increments, therefore, the high value shown as occurring or being exceeded 1 percent of the time is approximately 400 cfs. Short term peaks could exceed this but not appear on the graph. The minimum flow of record is about 10 cfs and is represented on the curve as occurring or being exceeded 100 percent of the time.

Traditionally the design flow for hydropower would be that flow which occurs or is exceeded 25 to 30 percent of the time. For Beaver Creek this would be a flow between 42 and 34 cfs. Actually the designers of Beaver Creek Power Plant were much more conservative than this and designed the plant for 17.7 cfs which occurs or is exceeded about 77 percent of the time. Apparently a steady, consistent power supply was needed at that time.

Three energy duration curves can be derived from the flow duration curve depending on how the head loss is conceived. Curve A shown in Figure 4 assumes that no head loss occurs as if a pipe of infinite diameter were used. This represents the maximum potential energy that could be produced. Curves B and C assume that the pipe is laid on a slope so that the product of Q and H is maximum. With increasing slope, Q increases and H decreases. The product, QH, reaches a maximum when the slope is 1/3 the gross slope between reservoir and tailrace, or expressed in terms of head the head loss, H_L, is 1/3 the total head, H_G, or

$$H_L = \frac{H_G}{3} \qquad . \quad . \quad . \quad . \quad . \quad (10)$$
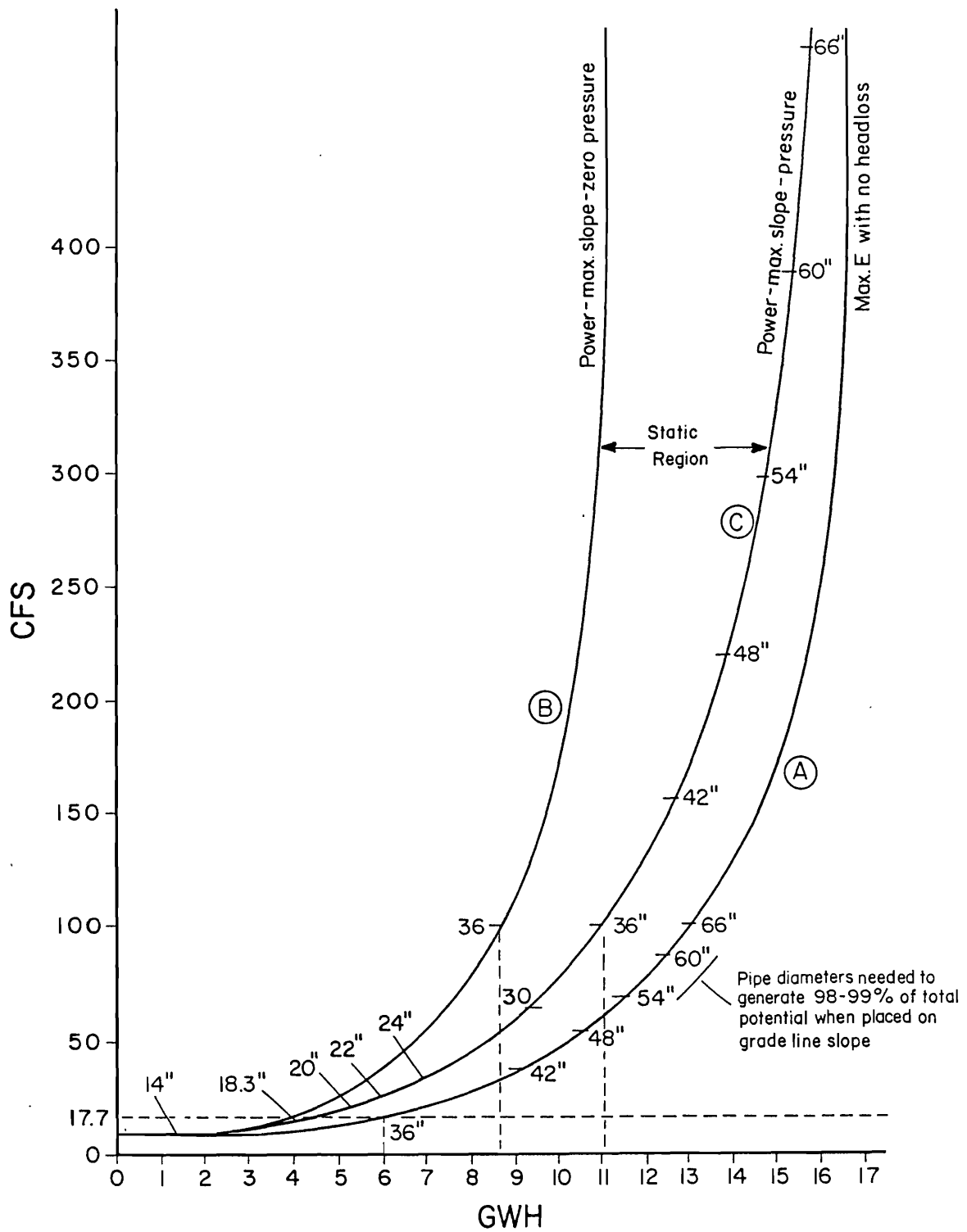
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Figure 4. Energy duration curve for Beaver Creek, Utah.

The derivation of this equation is shown in the appendix. The power produced with this headloss or at the "powermax slope" is

$$P = 2/3 \frac{Q*H}{11.82} \text{ (kw)} \quad . \quad . \quad . \quad . \quad (11)$$

Curve B shows the energy available when the delivery pipe is placed on the powermax slope and discharged into a penstock open to the atmosphere. In other words the conveyance is either an open flume or a pipe under zero pressure.

When the flow in a closed conduit which is capable of withstanding internal pressure is reduced, the friction loss is reduced in accordance with the Darcy-Weisbach equation and the difference between the initial headloss and the new smaller value is added back into the system as head available for power generation. The energy recoverable with this "static-regain" head is shown for Beaver Creek in curve C of Figure 4. Again the powermax slope is used.

To illustrate what this means in terms of numbers and magnitudes the following is extracted from the curves. The original pipeline was placed on a very flat slope so that the head available for power was about 98.3 percent of the total head. The maximum energy available at the pipeline capacity of 17.7 cfs was 5.91 GWH/yr. Replacing the wood stave pipe with steel pipe could increase the recovery rate slightly because of the differences in friction. The new rate would be 5.94 GWH/yr, which is 99 percent of the potential.

There are other alternatives. The same diameter pipeline, but made of steel, could be placed on a different slope such as the powermax slope and without increasing the cost for pipe[1], the recoverable energy would be greatly multiplied. This 36-inch pipe placed ๐n the powermax slope but operated Ꭵithout pressure, that is discharging ᴉnto an open penstock, would be capable of utilizing 103.6 cfs of flow and recovering 8.6 GWH/yr of energy. Using a closed penstock and utilizing the static regain head the recoverable energy would increase to 11.08 GWH/yr. These two options represent energy increases of 149 and 186 percent without an increase in pipeline cost over the flat grade line system. The 103.6 cfs of flow made possible by increasing the slope occurs or is exceeded about 11 percent of the time. The recoverable energy can be further increased by increasing the diameter of the pipeline. A 42-inch pipe on the powermax slope would be capable of recovering 12.66 GWH/yr at a flow of 155 cfs. This represents an exceedance level of 6 percent and an energy increase of 213 percent. A 48-inch pipe would be capable of recovering 13.87 GWH/yr at a flow rate of 219 cfs at an exceedance level of 4 percent and an increase in energy of 234 percent. However, costs of pipe and turbines are increasing for these options and therefore must be compared with the attendant benefits.

Table 3 shows the comparison of several options available for Beaver Creek, and the ratio of energy available to energy produced by the existing 36-inch pipe diameter, but with steel material instead of wood stave. It is to be noted that a 22-inch diameter pipe placed on the more accessible powermax slope will duplicate in power and energy what a 36-inch pipe would do on the gradeline slope.

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[1]It is assumed wood stave pipe is obsolete and would not be used as a replacement. The wall thickness of steel pipe in this example is the minimum for handling rigidity and therefore costs of steel pipe on grade line slope and powermax slope would be equal.

Table 3. Power and energy at various pipe diameters, Beaver Creek lower power plant.

Exceedance %	Q	Diameter (inches)	H_f	Ratio H_f/H_G	Maximum E potential	kw	kw/kw_o	GWH/yr	$\frac{GWH/yr}{GWH_o/yr}$
77	17.7	36[a]	8.3	0.983	6.00	712	0.858	5.91	0.995
77	17.7	36[b]	5.7	0.988	6.00	716	1.000	5.94	1.000
77	17.7	24	41.9	0.913	6.00	662	0.924	5.49	0.924
77	17.7	22	64.6	0.867	6.00	628	0.877	5.21	0.877
77	17.7	20	104	0.785	6.00	569	0.795	4.72	0.795
77	17.7	18.326	161.3	0.666	6.00	483	0.675	4.01	0.675
60	22.2	20	161.3	0.666	6.958	606	0.851	4.98	0.838
39	28.6	22	161.3	0.666	8.013	780	1.096	6.00	1.010
31	35.9	24	161.3	0.666	8.968	980	1.376	6.91	1.163
19	64.3	30	161.3	0.666	11.214	1755	2.456	9.17	1.544
11	103.5	36	161.3	0.666	13.295	2828	3.972	11.08	1.865
6	155	42	161.3	0.666	14.767	4226	5.935	12.66	2.131
4	219	48	161.3	0.666	15.707	5985	8.406	13.87	2.335
2	298	54	161.3	0.666	16.238	8133	11.423	14.75	2.483
1	390	60	161.3	0.666	16.432	10516	14.769	15.33	2.581
<1	497	66	161.3	0.666	16.432	12008	16.865	15.67	2.638

existing H_G = 484 ft., Q water right = 17.7 cfs, D = 36" wood stave
 plant   Rated Output − 600 kw. Average actual GWH − 4.00. L = 12,720 ft.

[a]wood stave, all other pipes in steel.
[b]base, KW_o, GWH_o, for comparisons.

Multiple Turbines

One of the most critical problems to consider in run of the river hydropower generation is the number and size of turbines required to extract the most power from highly variable river flows. The objective of selecting turbine numbers and size is to cover the largest range of flows that occur in the stream. It is not uncommon to have mountain streamflows vary over a usable ten-fold range. The flow range for different types of turbines is shown in Figure 5.

The method to determine the size and spacing of turbines can be illustrated with the Beaver example. With a 36-inch diameter pipe the flow capacity is 103.6 cfs. The minimum flow is 10 cfs. This is a flow range ratio of 10.36 to 1 (10.36:1). It would thus be possible to capture essentially all of the flow with a single, multiple jet impulse turbine. The rated head may not be large enough, however, to make this choice practical. (This is according to the literature, not practical application, as the existing turbines at this site are Pelton wheels.) Consider next a cross flow type with a range of flow of 5:1. With a streamflow range of 10:1 and a turbine flow range of 5:1 two combinations of turbines are needed [(10:1/5:1) = 2]. Two equal size cross flow turbines, each rated at 51.5 cfs, would cover the needed range[2]. A Kaplan turbine with a flow span of 3:1 gives a turbine combination number of 3.33 [(10:1/3:1) = 3.33]. Since we are dealing with whole digits, this could be approximated with a 1,2 combination giving (3 x 3 = 9) a 9:1 range of flow with 11.5 cfs as the minimum. The two turbines would be rated at 34.5 and 69.0 cfs respectively. Francis turbines have a flow range of 2:1 so that for a combined flow span of 10:1 the turbine combination number

would be 5[(10:1/2:1) = 3]. This could be accomplished with a 1,2,2 combination giving a flow of 20.7 cfs for the first unit and 41.4 cfs for each of the other two units. Using fixed geometry turbines, the flow range is narrower. If the turbine flow range is 1.43:1 the number of combinations is 10/1.43 = 7. This can be accomplished with a 1,2,4 combination and the first unit would be 14.8 cfs. The other units would be 29.6 and 59.2 cfs. If the fixed units chosen have a narrower range, say 1.11:1, the combination would be 10/1.11 = 9. This could be arranged as 1,2,4,2, with the small unit being 11.5 and the others, 23.0, 23.0, and 46.0 cfs. The options for Beaver Creek are summarized in Table 4.

When multiple units are used, the amount of overlap between the maximum flow of one turbine and the minimum flow of the next turbine is important and dependent upon the flow range of the turbines. This is illustrated for Beaver Creek on Figure 6 showing the part of the duration curve affected by these non-overlapping turbines. The case is for a 36-inch pipe with a capacity of 103 cfs and using fixed geometry turbines with a flow span of 1.43:1. There are two areas where overlap does not occur at the specified efficiency of 85 percent. Also at the very bottom end of the minimum flow level the turbines must extend below the 85 percent efficiency level to pick up all the flow. In all, it is estimated the total energy loss would not exceed 0.6 percent of the total recoverable. A fourth turbine would solve the overlap but may cost more than the additional energy is worth. Francis turbines at the same site with a flow range of 2:1 would have no overlap problem, but because the Francis is much more expensive than fixed geometry machines, it is doubtful if the three units required to cover the full range of flow would be

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[2]Binary numbers separated by commas will be used here to represent the number and relative size of the turbines. The foregoing combination of turbines would be designated as 1,1; meaning two equal sized turbines.
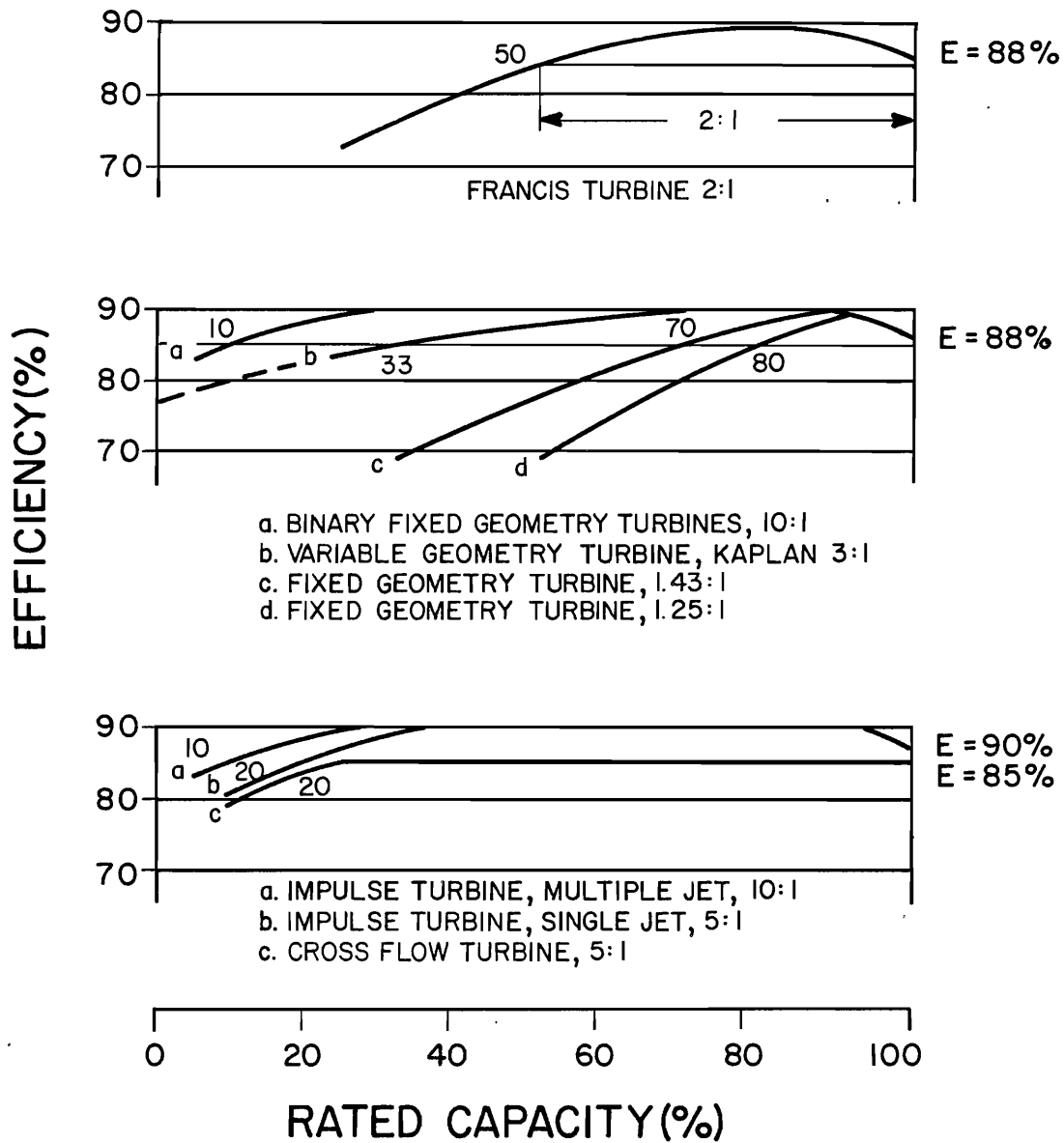
Figure 5. Flow range for different types of turbines. Range is defined by the portion of the curve above 85 percent efficiency.

Table 4. Binary combination of turbines for selected ranges in streamflow.

| Pipe Diameter, Streamflow, and Flow Range	Turbine Type	Efficient Flow Range	N	Combination	Range (N x flow range)	Sizes (cfs)	Min. Flow (cfs)
36" - 103 cfs	Multiple jet impulse	10:1	1	1,	10:1	103	10.3
	Impulse/cross flow	5:1	2	1,1	10:1	51.5, 51.5	10.3
$\frac{max\ Q}{min\ Q}$ = 10.3	Kaplan	3:1	3	1,2	9:1	34.3, 68.7	11.4
	Francis	2:1	5	1,2,2	10:1	20.6, 41.2, 41.2	10.3
	Fixed Geometry	1.43:1	7	1,2,4	10:1	14.7, 29.4, 58.8	10.3
		1.11:1	9	1,2,4,2	10:1	11.5, 22.9, 22.9, 45.7	10.3
		1.11:1	15	1,2,4,8	16.6:1	6.9, 13.7, 27.5, 54.9	6.2
		1.25:1	8	1,2,4,1	10:1	12.9, 25.8, 51.5, 12.9	10.3
48" - 250 cfs	Fixed Geometry	1.43:1	18	1,2,4,8,3	25.7:1	13.9, 27.8, 55.6, 111.1, 41.7	9.7
$\frac{max\ Q}{min\ Q}$ = 25	Francis	2:1	12	1,2,4,5	24:1	21.2, 42.5, 85, 106.3	10.6
				1,3,4	16:1	31.9, 95.7, 127.5	16.0
	Kaplan	3:1	8	1,2,4,1	24:1	31.9, 63.8, 127.5, 31.9	10.6
	Impulse/cross flow	5:1	5	1,4	25:1	51, 204	10.2
				1,2,2	25:1	51, 102, 102	10.2
	Multijet Impulse	10:1	2.5	1,1, 1/2	25:1	102, 102, 51	5.1
			2.	1,1	20:1	127.5, 127.5	12.8

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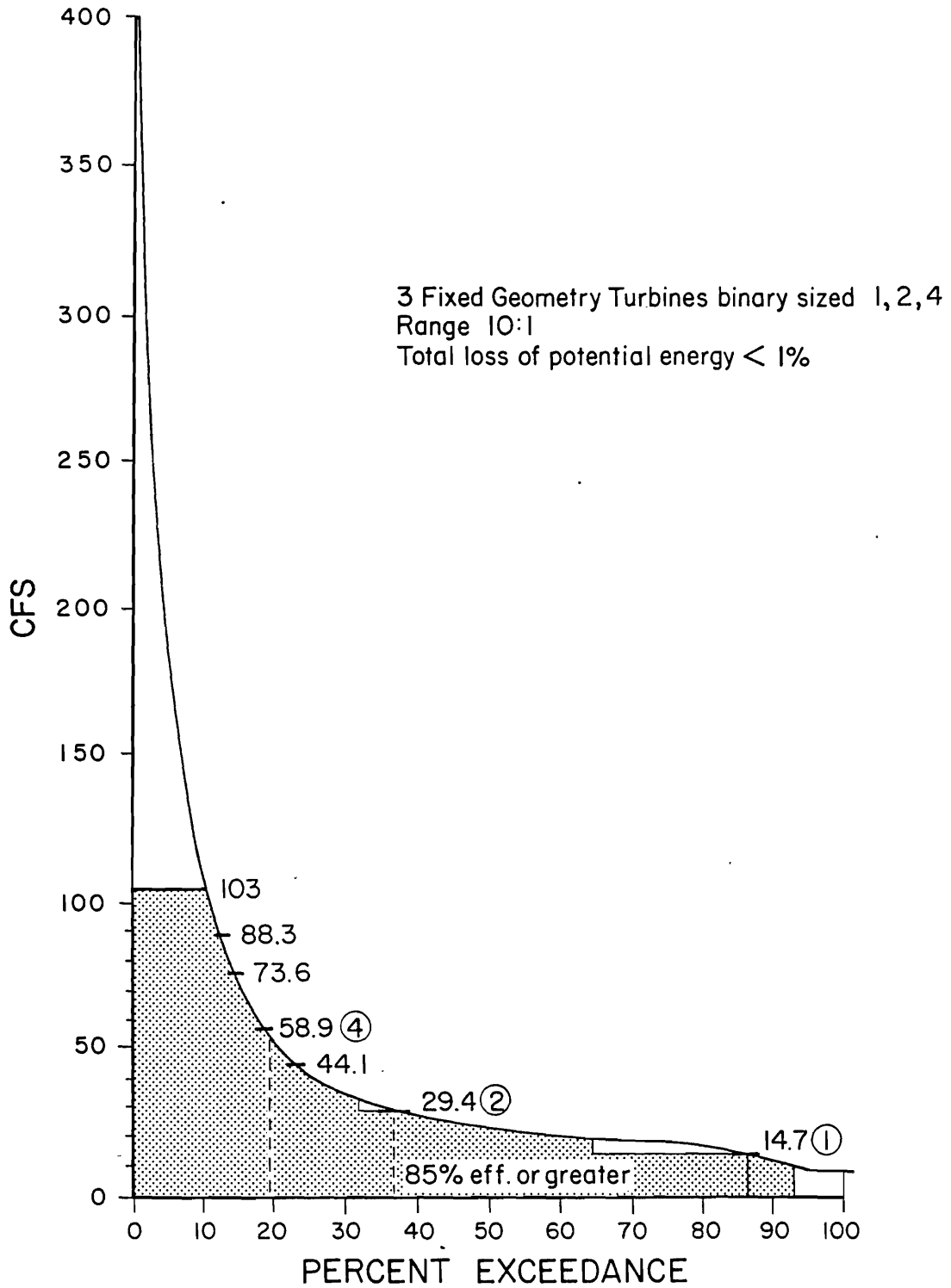
Figure 6. Flow duration curve for Beaver Creek, Utah, showing energy missed by multiple fixed geometry turbine sized binarily.

economical. Two Francis turbines would cover a range of 6:1 and if sized to fit the maximum 103 cfs point would have a minimum flow of 17.1 cfs at 85 percent efficiency and would have to drop to 75 percent efficiency to get the minimium flow of 10 cfs. This represents a loss in energy of about 1 percent and is illustrated in Figure 7.

In the final analysis the choice of turbines will be based on cost, but binary spacing will allow broad flow ranges to be covered by less expensive fixed geometry units or fewer of the expensive movable geometry units. The binary sizing for combinations from 1 to 15 are illustrated in Table 5.

Pipe Materials

Since the cost of pipe is usually a linear multiple of the unit weight of the material, the relative cost of pipe can be compared by relative unit weights. When comparing diameters of pipe of the same material this relationship is direct. For instance, 36-inch diameter steel pipe with a wall thickness of 0.134 inches (10 gage) weighs 51.7 pounds per foot. A 22-inch diameter steel pipe with the same wall thickness weighs 31.7 pounds. The ratio of cost between 36 and 22 inch pipe is therefore 51.7/ 31.7 = 1.63. This is interesting because at the Beaver site each of these pipes will produce the same kw and GWH if the larger pipe is laid on a flat grade line slope and the smaller pipe is laid on the powermax slope. Comparing these pipes on the basis of pounds per foot per energy unit would give (51.7/5.94) ÷ (31.7/6.0) = 1.65, which means that it costs 165 percent more to use 36-inch pipe at the grade line than 22-inch at the powermax slope. An additional cost advantage to the powermax slope is the more favorable route selection, being able to choose a course located in the canyon bottom rather than on a steep hillside.

Table 5. Binary[1]/ sizing for combinations of turbines from 1-16.

C	Combinations	N
1	1	1
2	1,1	2
3	1,2	2
4	1,2,1	3
5	1,2,2	3
6	1,2,3	3
7	1,2,4	3
8	1,2,4,1	4
9	1,2,4,2	4
10	1,2,4,3	4
11	1,2,4,4	4
12	1,2,4,5	4
13	1,2,4,6	4
14	1,2,4,7	4
15	1,2,4,8	4
16	1,2,4,8,1	5

[1]/Some of the numbers used in the combinations are not binary, but are digits which can increase the flow span in small units and would cost less than the next binary number turbine.
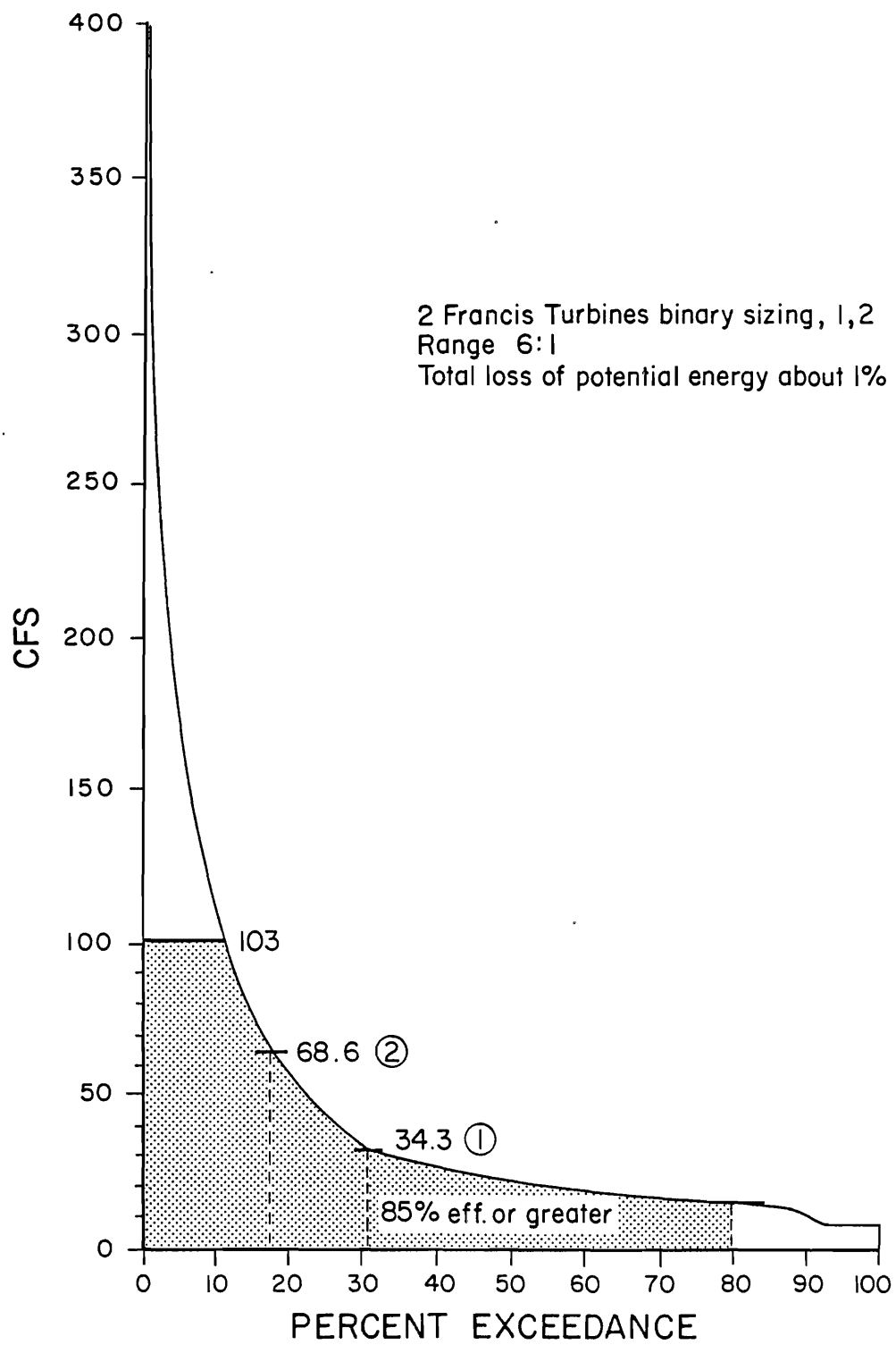
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Figure 7. Flow duration curve for Beaver Creek, Utah, showing energy missed by Francis turbine sized binarily.

Pipes of materials other than steel can be similarly compared when multiplied by the cost per pound of material. Several pipe materials are compared in Table 6 for diameters of 36-inch and 22-inch based on the Beaver example.

It would appear that steel pipe is the cheaper pipe in these instances, but the corrosive nature of steel and the handling ease of lighter material may shift these unit costs.

Effect of Increased Flow Range on Turbine Selection

The larger diameter pipes and pipes on the powermax slope will increase the power rating of the turbine-generators rather dramatically. Moving the 36-inch pipe from the flat slope to the power-max slope increases the kw rating by 397 percent. This means an increase in the cost of turbines and generators and will tend to offset some of the savings in pipe costs. Turbine costs vary greatly depending upon type, head, and flow range and it is therefore difficult to generalize on unit turbine costs. A recent example made known to the writers through correspondence with Allis Chalmers, hydro turbine division, gave a cost range of 325 to 800 dollars per installed kilowatt at the same plant using different types of turbines. The high value in their case was a single Francis turbine, the low value was for multiple-fixed geometry, reverse pumps. Putting as much cost information together as was available gives the following ratios and should cause designers to look at the more favorable options:

Type of Turbine	Relative Cost
Fixed blade propeller	1.00
Francis	1.13
Kaplan	1.22
Reverse pumps	0.46

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Table 6. Weight and cost of pipe material-Beaver Creek hydro powermax slope.

Material	Density (#/cu. in.)	Inside Diameter (inches)	Wall Thickness (inches)	Weight (#/foot)	Cost $/#	Cost per Foot
Steel	0.2833	36	0.134	51.7	0.30	15.51
PVC	0.050	36	0.920	64.0	0.90	57.6
PE	0.034	36	1.650	79.6	0.60	47.76
FRP*	0.0676	36	0.140-0.200	18.5	2.43	44.96
Steel	0.2833	22	0.134	31.7	0.30	8.64
PVC	0.050	22	0.500	21.2	0.90	19.08
PE	0.034	22	0.875	25.7	0.60	15.42
FRP*	0.065	22	0.252	13.7	2.00	27.40

*The cost of FRP is not linear with weight because glass/resin ratio differs. High strength pipe may have smaller wall thickness than low pressure pipe.

SUMMARY

Hydropower generation on steep mountainous streams where storage is impractical can usually be made economical if:

1. The flow duration curve shows sufficient energy available in the high flows to merit extending use to the low end of the exceedance curve--10 percent or less.

2. The powermax slope and static regain concept are utilized to size and position the long conveyance pipe line.

3. Multiple turbines are installed in binary sequence to extend flow range over entire flow duration curve to the 10 percent or less exceedance level.

4. Pipe materials meet hydraulic, structural, and corrosion needs without excessive over design.

ADDITIONAL RESEARCH

One area where further economies could be made in capital expenditure and in annual maintenance expense is the inlet trash racks. Automatic cleaning racks are expensive, requiring a high first cost; and they require continual maintenance and energy. The recent experience of Murray City, Utah, in rebuilding its hydro plant is a case in point. About 10 percent of the capital budget was required for construction and equipment installed to divert water and remove trash. This represents in excess of $160 per installed kw--or about one-third the cost of the turbine and generators. The automatic trash rake is a mechanical device and subject to wear and breakdown and requires energy to operate. The raked debris must be disposed of and the structure is a visible part of the mountain landscape. The advantages of a self-cleaning, gravity device are apparent.

Nature has been effective in filtering debris from groundwater and discharging clear clean water into surface springs and artesian flows. Slow-rate sand filters have been effective as cleaning mechanisms for domestic water and the "plugging" effect is a surface phenomenon, easily removed by

disturbance and/or replacement of a thin surface layer.

Research should provide ways of duplicating the works of nature. Additional research on slow-rate filters for larger flows and filter media that would be automatically regenerated by induced turbulent surface flows is needed. The potential benefits are great enough to warrant the expenditure of research money in this direction. Some of the benefits are:

1. Extended life of turbine equipment without corrosive particles in flow.

2. Elimination of moss and vegetal debris clogging turbines.

3. Elimination of environmental impact of unsightly debris removing structures and disposal sites.

4. Reduction in annual maintenance costs.

5. Savings in energy lost as debris cloggs screens and reduces flow.

APPENDIX

The Power Equation

Power in kilowatts from falling water can be expressed as:

$$P = QH \left(\frac{62.4}{550} \cdot 0.7457\right) \quad . \quad . \quad (1)$$

or

$$P = \frac{QH}{11.82}$$

where

    Q = flow in cubic feet per second
    H = vertical distance or usable head in feet

    $\left(\frac{62.4}{550} \cdot 0.7457\right)$ converts cfs to pounds per sec, foot-pounds per second to horsepower and horsepower to kilowatts.

The head, H, can be further defined as

$$H = H_G - (H_f + H_m) \quad . \quad . \quad . \quad (2)$$

where

    H_G = total elevation different or gross head
    H_f = head loss by friction in the pipe
    H_m = head loss by elbows, contractions, valves, etc., commonly called minor losses

H_f can be futher defined by the Darcy equation as:

$$H_f = f \frac{L}{D} \frac{V}{2 g} \quad (3)$$

where

    f = a friction factor
    L = length of pipe
    D = diameter of pipe
    V = velocity of pipe
    g = acceleration due to gravity

Since $Q = AV$ (area times velocity) and

$$A = \frac{\pi D^2}{4}$$

the equation can be rewritten as:

$$V = \frac{Q}{\frac{\pi D^2}{4}} \; , \quad V^2 = \frac{Q^2}{\frac{\pi^2 D^4}{16}}$$

substituting in Equation 3

$$H_f = f \frac{L}{D^5} \frac{Q^2}{\frac{(\pi^2 2 g)}{16}} \quad . \quad . \quad . \quad (4)$$

Similarly

$$H_m = k \frac{Q^2}{D^4 \left(\frac{\pi^2 2 g}{16}\right)} \quad . \quad . \quad . \quad (5)$$

where

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k = sum of coefficients for the minor losses

The power equation can now be written

$$P = \frac{Q \ (H_G - H_f - H_m)}{11.82}$$

or

$$P = (Q \ H_G - f \ \frac{L}{D^5} \ \frac{Q^3}{(\frac{\pi^2 \ 2 \ g}{16})} - k \ \frac{Q^3}{D^4(\frac{\pi^2 \ 2 \ g}{16})}) \ (\frac{1}{11.82}) \ . \quad (6)$$

Differentiating the power equation and equating to zero will give maximum power

$$\frac{dP}{dQ} = \frac{H_G}{11.82} - \frac{3fL \ Q^2}{D^5(\frac{\pi^2 \ 2 \ g}{16})} \ (\frac{1}{11.82})$$

$$- 3 \ k \ \frac{Q^2}{D^4(\frac{\pi^2 \ 2 \ g}{16})} \ (\frac{1}{11.82}) = 0$$

or

$$H_G = \frac{3 \ Q^2}{(\frac{\pi^2 \ 2 \ g}{16}) \ D^4} \ (\frac{fL}{D} + k)$$

Multiply both sides by D and solve for Q

$$Q = \sqrt{\frac{H_G \ (\frac{\pi^2 \ 2 \ g}{16}) \ (D^5)}{3(fL + Dk)}} \quad . \ . \ (7)$$

This is the Q that will give maximum power with a pipe of given material and diameter on a given slope. From here on then, Q will be designated

Q^* to indicate the flow for maximized power in the given system.

Going to the head loss equation:

Total head loss,

$$H_T = Hf + H_m$$

$$= f \ \frac{L}{D} \ \left(\frac{Q^2}{(\frac{\pi^2 \ 2 \ g}{16}) \ (D^4)} \right) + k \left(\frac{Q^2}{(\frac{\pi^2 \ 2 \ g}{16}) \ (D^4)} \right)$$

Substituting for Q^2 and eliminating terms

$$H_T = \frac{fL \ H_G}{3(fL+kD)} + \frac{K \ H_G \ D}{3(fL+kD)}$$

which reduces to:

$$H_T = H_G/3 \qquad (8)$$

The interesting relationship shown in Equation 8, which the writers have not found in the literature, gives an easy way to evaluate potential hydrosites and to determine if a detailed cost analysis of a proposed project is warranted. The equation says that the maximum power that can be generated at a site with a given diameter and pipe material will occur when the head consumed is equal to 1/3 of the total head available. This means that each pipe size at a given site has a maximum flow, Q, to produce a maximum power P. Any flow less than this or more than this will produce less power. The power equation can now be written:

$$P_{max} = \frac{2/3 \ H_G \ Q^*}{11.82} \qquad . \ . \ . \ . \quad (9)$$

and expressions for D^*, the diameter needed to produce maximum power with given slope, Q, and L; and Q^* can be shown to be:

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$$D^* = \left(\frac{3}{H_G} \frac{Q^2}{39.725} fL\right)^{1/5} \quad . \quad . \quad . \quad (10)$$

and

$$Q^* = \sqrt{\frac{H_G}{3} \frac{39.725 \, D^5}{L}} \left(\frac{1}{\sqrt{f}}\right) \quad . \quad (11)$$

The friction can be evaluated by the following formula:

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10}\left(\frac{9.35 \left(\frac{1}{\sqrt{f}}\right)}{Re} + \frac{e}{D}\right)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where

e = size of the pipe roughness elements

Re = Reynolds number = $\dfrac{4 \, Q}{D \, V}$

V = kinematic viscosity

Re for 50° water = 90300 Q/D so that,

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10}\left(\frac{9.35 \left(\frac{1}{\sqrt{f}}\right) D}{90,300 \, Q} + \frac{e}{D}\right)$$

$$. \quad . \quad . \quad . \quad . \quad (13)$$

for most western mountain hydrosites.

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