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AN OPTIMAL CONTROL MODEL FOR ANALYSIS OF

TIMBER RESOURCE UTILIZATION IN SOUTHEAST ASIA

By

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An Optimal Control Model for Analysis of Timber Resource

Utilization in Southeast Asia

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ABSTRACT

A discrete time optimal control forestry model is built and a shooting method solution algorithm identified. The applicability of the model and algorithm to public policies that affect forestry resources is demonstrated in an application of the model to examine the development of wood processing capacity in Southeast Asia. The necessary conditions of the optimal control model are manipulated to identify a difference equation problem with initial and terminal conditions. The solution to this boundry value problem is identified using a search routine that repetitively, numerically evaluates (shoots) the difference equations. The solution is the trajectory that satisfies the initial and terminal conditions.

Introduction

Many nations that have extensive forest resources use public policy to achieve specific objectives. Analysis of the achievement of these objectives is complicated by the stock-flow characteristics of the resource. In the near term the forest resources are relatively fixed but over the longer run the forests are renewable. The policies, therefore, have impacts on the utilization of current stocks, regeneration of the forest, and the utilization of these regenerated forests. These topics can be analyzed both theoretically and numerically using discrete time optimal control (DTOC) theory. In this paper we introduce a new method to numerically solve this problem and apply it to the public policy of developing a wood processing capacity in Southeast Asia.

Traditionally, Southeast Asia has been a source of tropical hardwood logs which have been processed into lumber and plywood elsewhere. In this paper we use a control theory model to examine the effects of the implementation of a log export ban upon the accumulation of domestic processing capital stock (plywood production capacity). The intertemporal implications of such a policy on mill wood prices and harvests are also examined. Such a policy is of interest because Indonesia, the region's largest single source of commercial tropical hardwoods, has recently imposed such an export ban. Other Southeast Asian countries are also considering similar actions to limit log exports.

The hectares of timber by age group and the stock of plywood production capital are state variables in our DTOC model while the annual harvests, annual expenditure on variable factors of plywood production and annual investment in plywood production capital are the control variables.

The first order conditions from the maximum principle are manipulated to give a set of difference equations which are solved using an iterative shooting technique. While the DTOC theory has been applied to timber harvest scheduling problems, these studies use different solution techniques.

DTOC theory has been applied to the forest utilization problem by McDonough and Park (1975), Cohan (1982), and Lyon and Sedjo (1983). McDonough and Park developed an optimal control forestry model and wrote a gradient method algorithm to implement their model. They implemented their model using illustrative data originally used by Walker (1971). Their primary purposes were to show the usefulness of a new computer language and the usefulness optimal control theory.

David Cohen (1982) developed a detailed theoretical model of forest management and timber supply using optimal control theory. His model is solved using a two-step procedure which first selects total harvest quantities and management activities and second selects actual harvests to minimize costs. The algorithm uses an approach known as generalized equilibrium modeling which is essentially a form of successive approximations with relaxiation to solve systems of nonlinear simultaneous equations.

Lyon and Sedjo (1983) developed and applied a Supply Potential Optimal Control (SPOC) model which examines the potential long-term supply of timber harvest using a discrete time optimal control techique. Their model incorporates features that allows it to deal with the problems of finding the economically optimal rate of drawdown of existing old growth stands, as well as to project the optimum harvest levels after the transition has been completed and a steady state

achieved. The problem was solved by using an optimal control theory algorithm which uses a gradient technique.

Below the DTOC forestry model that we use will be developed, the difference equation problem will be identified from the necessary conditions, our application of the shooting technique described and finally our application to Southeast Asian forests presented and discussed.

Model Description

The objective function of this model is the discounted present value of the stream of net surplus (i.e., consumers' and producers' surplus) for the plywood industry. The function is maximized subject to a set of constraints. The constraints include the initial conditions, the laws of motion for the system, and the production function of plywood. The initial conditions are hectares of forest by age group and capital stock in the plywood industry.

The laws of motion for the system include a difference equation that controls the aging of age groups of trees. In addition, because a selective harvesting scheme is used, the commercial stand of trees is naturally regenerated from the trees that are left; thus, hectares of trees harvested in one year become hectares of newly regenerated trees in the next year. The other law of motion is a difference equation that determines the evolution of plywood processing capital stock as a function of depreciation and gross investment.

The state variables are hectares of trees by age group and the capital stock in the plywood industry, and the control variables are the harvest levels, the production of plywood, and the level of investment in the plywood industry in each year.

We structure the problem so that it evolves to the stationary state; hence, the computer program of optimization first solves for the optimal length of the rotation period, the mill-wood price of timber, the volumes of harvested timber, the quantity and price of produced plywood, the level of capital stock and investment for the plywood industry, and the shadow value of processing capacity in the stationary state. Then the optimal time profiles of these same variables are calculated for the transition period. This is done by solving the difference equation problem identified by the laws of motion, the first order conditions, the initial conditions, and the terminal conditions. The initial and terminal conditions are hectares of forest by age group, and capital stock in the plywood industry for the initial and terminal years, respectively; where the initial conditions were determined by past events, and the terminal conditions are to be determined by the stationary state solution which was mentioned above.

The role of discrete optimal control theory lies in the identification of the laws of motion and the equations and equalities for the necessary conditions. These are used to identify the difference equation problem that is iteratively solved to numerically solve the problem.

Model Formulation

The model used is a modification of the Supply Potential Optimal Control model developed by Lyon and Sedjo (1983), which examines the potential long-term supply of timber harvest. The new model includes an activity that accounts for developing the plywood industry in the region over time, and a different solution technique.

The net surplus in year j can be written as

$$s_j = \int_0^{L_j} D(n)\,dn - C_j \qquad\qquad (j = 0,1,\ldots,J-1)$$

where L_j is the quantity of plywood produced in year j, and is a function of capital stock, timber production, and a composite input; $D(L_j)$ is the inverse form of the demand function for plywood in year j; and C_j is the total cost (expenditures) in year j.

The total costs are the sum of harvesting and transportation cost (CH), and plywood production expenditures (CL). Harvesting and transportation costs in year j depend on the total volume harvested (T_j),

$$CH_j = f(T_j).$$

The plywood production expenditures in year j can be written as

$$CL_j = v_j + y_j$$

where v_j is both the expenditure on and the level of the composite input in plywood production. This equality of expenditure and level exists because we scale the composite input so that its price is one dollar. In addition, y_j is the level of investment in the plywood industry. It is a scalar control variable.

x_j is a state vector of hectares of trees in different age groups. Its elements x_{ij} indicate the hectares of trees in year j that were regenerated i years ago. For notational simplification, call x_{ij} the hectares of trees in age group i in year j. The length of the x vector is M with M equal to or greater than one plus the longest rotation period in any year. In addition, let X_j be a diagonal matrix of the elements of x_j.

The control variable of harvesting timber is u_{ij} which denotes the portion of age group i harvested in year j. The control vector of these elements is denoted by u_j.

In this model, the yield of merchantable volume of timber in cubic meters per hectare is a function of age of tree. The vector of yield by age is denoted as t.

Using all these definitions, we can write the equation for the volume of timber produced in year j as

$$T_j = u_j^{'} x_j t$$

The quantity of plywood produced in year j can be written as

$$L_j = \min[g(k_j, v_j), \frac{T_j}{b}]$$

where $g(k_j, v_j) = \tilde{A} k_j^a v_j^{1-a}$, v_j is the level of the composite input, and k_j is the total capital stock for the plywood industry in year j. The selection of this production function and the calculation of the parameters in this composite fixed-variable proportion production function will be described in the application section. The maximization problem can be written as

Maximize

$$S_0[x_0, k_0, u, T, v, y] = s_0 + \rho s_1 + \ldots + \rho^{J-1} s_{J-1} + \rho^J S_J^*[x_J, k_J] \qquad (1)$$

subject to a set of constraints

(a) the laws of motion

$$x_{j+1} = (A + BU_j)x_j \qquad (j = 0,1,\ldots,J-1) \qquad (2)$$

$$k_{j+1} = (1 - \delta)k_j + y_j \qquad (j = 0,1,\ldots,J-1) \qquad (3)$$

(b) the composite fixed-variable proportion production function of plywood

$$L_j = \min[g(k_j, v_j), \frac{T_j}{b}] \qquad (4)$$

where $g(k_j, v_j) = \tilde{A} k_j^a v_j^{1-a}$

The production function can be written as a pair of constraints

$$\frac{T_j}{b} - L_j \geq 0, \text{ or}$$

$$T_j - bL_j \geq 0, \text{ and} \tag{4b}$$

$$\tilde{A} \, k_j^a \, v_j^{1-a} - L_j \geq 0. \tag{4c}$$

(c)   the additional constraints are

$$e \geq u_j \geq 0 \tag{5a}$$

$$k_j \geq 0 \tag{5b}$$

$$ymx \geq y_j \geq 0 \tag{5c}$$

(5a) states that the portions of hectares harvested are constrained to be nonnegative and less than or equal to one, (5b) shows that the capital stock is constrained to be nonnegative, and (5c) states that investments in the plywood industry are constrained to be nonnegative and less than maximum gross investment (ymx). ymx is included so that the growth rate of capital stock in the plywood industry may be constrained.

In the above statement of the problem, ρ is the discount factor, which is equal to exp(-r), where r is the market rate of interest; δ is the depreciation rate of capital in the plywood industry. $S_0(\cdot)$ states that the present value of net surplus stream at time zero depends upon the initial conditions (x_0, k_0) and the time paths for the control variables (u, T, v, y). The super asterisk is used to indicate optimal quantities; thus the constrained solution at time zero would be denoted by $S_0^*(x_0, k_0)$, and the term $S_J^*(x_J, k_J)$ is an optimal terminal value function and can be viewed as S_0^* was. In addition,

$$A = \begin{bmatrix} 0 & 0 & 0 & . & . & . & . & . & 0 \\ 1 & 0 & 0 & . & . & . & . & . & 0 \\ 0 & 1 & 0 & . & . & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 & . & . & . & . & . & 1 \\ -1 & 0 & 0 & . & . & . & . & . & 0 \\ 0 & -1 & 0 & . & . & . & . & . & 0 \\ 0 & 0 & -1 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \quad e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix}$$

where A, B, and U are M-square matrices, U_j is a diagonal matrix using the elements of u_j.

The product Ax_j moves x_{ij} to $x_{i+j,j+1}$. Each year each age group becomes older by one year. The product BU_jx_j subtracts the area harvested from the redefined quantities, and places them in the one-year old category (newly regenerated category).

In each time period, the following Hamiltonian is maximized[1] with respect to u_j, y_j, and v_j subject to the constraints (equations 4b, 4c, 5a, 5b, and 5c).

The Hamiltonian for year j is

$$H_j = \int_0^{L_j} D(n)dn - f(T_j) - (v_j + y_j) + \lambda'_{j+1}[(A + BU_j)x_j]$$
$$+ \psi_{j+1}[(1 - \delta)k_j + y_j] \tag{8a}$$

where

$$\lambda_j = \rho \frac{dS_j^*(x_j,k_j)}{dx_j}, \qquad (j = 1,. . . ., J)$$

using the envelope theorem we get

$$\lambda_j = \rho[-f'(T_j^*)U_j^* t + (A + BU_j^*)'\lambda_{j+1} + \phi_j U_j^* t]$$

$$= \rho [\phi_j - f'(T_j^*)]U_j^* t + (A + BU_j^*)'\lambda_{j+1} \tag{9a}$$

In addition,

$$\psi_j = \rho \, \frac{dS_j^*(x_j, k_j)}{dk_j} \; ,$$

which by the envelope theorem is

$$\psi_j = \rho[\eta_j a \cdot \tilde{A} k_j^{a-1} v_j^{1-a} + (1 - \delta)\psi_{j+1}] \tag{9b}$$

where ϕ is the Langrangean multiplier for equation (4b) and is the shadow value of timber delivered at the processing mill (mill-wood price), and η is the Langrangean multiplier for equation (4c) and is the shadow value of the processing capacity. The derivatives with respect to a vector are gradient vectors. The λ_j and the ψ_j are costate variables. They identify the shadow values of the hectares of the forest in each age group, and the capital stock of the plywood industry, respectively, in year j.

Since the Hamiltonian is maximized in year j over u_j, y_j, L_j, and v_j subject to equations (4b-5c), the Langrangean function and the Kuhn-Tucker conditions of this problem are relevant. These are

$$\angle_j = H_j + \phi_j(T_j - bL_j) + \eta_j(Ak_j^a v_j^{1-a} - L_j) + \xi_j(e - u_j)$$
$$+ \Upsilon_j(ymx - y_j)$$

$$\begin{aligned} L_j \\ = \int_0^{} D(n)dn - f(u_j X_j t) - (v_j + y_j) + \lambda_{j+1}[(A + BU_j)x_j] \end{aligned}$$

$$+ \psi_{j+1}[(1 - \delta)k_j + y_j] + \phi_j(T_j - bL_j) + \eta_j(\tilde{A}k_j^a v_j^{1-a} - L_j)$$

$$+ \xi_j(e - u_j) + \Upsilon_j(ymx - y_j) \tag{10}$$

$$\frac{d\mathcal{L}_j}{du_j} = -f'(T_j)\dot{X}_j t + X_j B' \lambda_{j+1} + \phi_j X_j t - \xi_j \leq 0$$

$$= [\phi_j - f'(T_j)]X_j t + X_j B' \dot{\lambda}_{j+1} - \xi_j \leq 0 \tag{11a}$$

$$\left(\frac{\partial \mathcal{L}_j}{\partial u_{ij}}\right) u_{ij} = 0 \tag{11b}$$

$$\frac{\partial \mathcal{L}_j}{\partial L_j} = D(L_j) - \phi_j b - n_j \leq 0 \tag{11c}$$

$$\left(\frac{\partial \mathcal{L}_j}{\partial L_j}\right) L_j = 0 \tag{11d}$$

$$\frac{\partial \mathcal{L}_j}{\partial y_j} = -1 + \psi_{j+1} - \gamma \leq 0 \tag{11e}$$

$$\left(\frac{\partial \mathcal{L}_j}{\partial y_j}\right) y_j = 0 \tag{11f}$$

$$\frac{\partial \mathcal{L}_j}{\partial v_j} = -1 + n_j(1 - a)\tilde{A}k_j^a v_j^{-a} \leq 0 \tag{11g}$$

$$\left(\frac{\partial \mathcal{L}_j}{\partial v_j}\right) v_j = 0 \tag{11h}$$

$$\frac{\partial \mathcal{L}_j}{\partial \phi_j} = T_j - bL_j \geq 0 \tag{11i}$$

$$\left(\frac{\partial \mathcal{L}_j}{\partial \phi_j}\right) \phi_j = 0 \tag{11j}$$

$$\frac{\partial \mathcal{L}_j}{\partial n_j} = \tilde{A}k_j^a v_j^{1-a} - L_j \geq 0 \tag{11k}$$

$$\left(\frac{\partial L_j}{\partial \eta_j}\right)\eta_j = 0 \tag{11l}$$

$$\frac{\partial L_j}{\partial \xi_j} = e - u_j \geq 0 \tag{11m}$$

$$\left(\frac{\partial L_j}{\partial \xi_j}\right)\xi_j = 0 \tag{11n}$$

$$\frac{\partial L_j}{\partial \gamma_j} = ymx - y_j \geq 0 \tag{11o}$$

$$\left(\frac{\partial L_j}{\partial \gamma_j}\right)\gamma_j = 0 \tag{11p}$$

These Kuhn-Tucker conditions, the laws of motion for the state variables (equations 2 and 3), and the laws of motion for the costate variables (equations 9a and 9b) identify the two-point boundary value problem to be solved.

The Difference Equation Problem

The Difference Equations

The difference equations to be solved are Equations (2), (3), (9a), (9b), and an equation derived from (11a) and (11b). Equations (2) and (3) are the laws of motion for hectares of forest by age group and capital stock for the plywood industry, respectively. These have initial conditions dictated by the starting point of the problem. Equations (9a) and (9b) are the laws of motion of the costate variables (shadow values of the state variables). Note that these are backward moving difference

equations, with the calculations beginning in the terminal time period, J, and move backward through time to the first time period. These have terminal conditions identified by the stationary state because we build this in as the end point of the evolution of the system.

Manipulation of Equations (11a) and (11b) yield a difference equation for the net price or stumpage price (shadow value) of timber. This difference equation will have a terminal condition identified by the stationary state. We use a shooting technique to select the initial value of the stumpage price of timber such that equations (2), (3), (9a), (9b), (11c) through (11p) and the initial and terminal conditions are simultaneously satisfied.

To identify the difference equation for the stumpage price of timber, write the elements of equation (11a) as

$$P_j x_{ij} t_i + x_{ij}(\lambda_{1,j+1} - \lambda_{i+1,j+1}) - \xi_j \leq 0 \tag{12}$$

where

$$P_j = \phi_j - f'(T_j)$$

with P_j the stumpage price of timber. It is equal to the shadow value of timber delivered at the processing mill (ϕ_j) minus the marginal harvesting and transportation cost of timber [$f'(T_j)$]. With a concave yield function the oldest trees will be harvested first; thus there will be a youngest age group of trees harvested in year j. Call it m.

In equation (12) $\lambda_{1,j+1}$ is the shadow value of trees that are 1 year old in year j + 1, i.e., trees regenerated in year j. Examination of equation (9a) indicates that it is the discounted value of the actual harvest of these trees in the future. The costate variable $\lambda_{i+1,j+1}$ is the discounted value of age group i from next year. For age group m it can be written (see Appendix A for details).

$$\lambda_{m+1,j+1} = [\lambda_{1,j+2} + P_{j+1} t_{m+1}]$$

which states that the opportunity cost of harvesting m year old trees in year j is the discounted value of the trees that could be regenerated a year in the future and the stumpage price of timber next year times the volume of timber on that hectare one year in the future. From this equation and equation (12) we can derive

$$p_{j+1} = (p_j t_m + \lambda_{1,j+1} - \rho\lambda_{1,j+2} - \xi_{mj}/x_{mj})/\rho t_{m+1} \tag{12a}$$

which is the other difference equation.

Terminal Conditions

We assume that the system evolves to the stationary state (SS) because this state is as reasonable as any other terminal state and it can be identified. We define the SS to have the characteristic that all years are alike. The solution for the SS is found by first solving the differential equations for the costate variables where all years are alike then we simultaneously solve the laws of motion for the state variables and the first order conditions for the control variables. For details see Appendix B.

The solution to the SS problem identifies the terminal conditions for all of the difference equations.

Solution Algorithm

We find the "solution" time paths for the control, state, and costate variables using a three-step procedure. The first finds an initial feasible time path of control and state variables but not the costate variables, which are calculated using a backward moving difference equation that requires a feasible time path of the state variables. This is achieved by solving a difference equation problem that excludes the costate

variables. This problem includes in the place of equation (12a) the difference equation

$$p_{j+1} = \frac{p_j t_m}{p t_{m+1}}$$
(13)

which is derived from equation (12) by ignoring the shadow value of hectares of trees harvested in the future. In addition, the laws of motion for the costate variables are dropped, and capital is held at its stationary state value. The adjusted system of equations is used to determine the initial time path of the state and control variables from the initial conditions to a long-run equilibrium. This difference equation problem is solved using a shooting method which is described below.

Using the time profiles of the state variables, the next step is to calculate the costate variables moving from long-run equilibrium backward to year one by using the backward moving laws of motion, equations (9a) and (9b).

Based on the results of the second step, we solve, using a shooting method, the difference equations (12a), (2), and (3) subject to the necessary conditions, equations (11c) through (11p), to determine the new time profiles of state and control variables from the initial conditions to a long-run equilibrium. These last two steps, which calculate the costate variables in the second step and determine the new time profiles of state and control variables in the third step, can be repeated until a satisfactory "solution" is determined.

The shooting method is a search for a particular element of a set of solutions to the difference equations and initial conditions. This element is the one that satisfies the terminal conditions.

The first step is to arbitrarily select the mill-wood or stumpage price of timber at the first time period. The mathematical relationships given by the first order conditions, equations (11c) through (11p), are then used to calculate the static market clearing values of the plywood produced, the composite input for the plywood industry, timber harvested, the price of plywood, and the shadow value of the processing capacity. This static problem in a particular year can be solved by recognizing the plywood demand function and the plywood production function. Based on the information of the forest initial conditions and the timber yield function, we harvest the oldest age group of trees first, and determine the hectares of trees harvested and the youngest age group harvested. Having these results, we can iteratively solve the difference equation problem. Evaluation of the difference equations yields calculations of the hectares of trees by age group, including those in the newly regenerated class and the capital stock in the plywood industry from one time period to the next. This process of statically solving the first order conditions and dynamically evaluating the difference equations yields a time profile of price. The process is iterated until a particular time period when we realize that the price in the initial time period is too low or too high, or until the time horizon is reached. If the time profile of price over (under) utilizes the forest resources, the price in the initial time period was too low (high). In this case, the process needs to be repeated. A new price in the initial time period is calculated by selecting the midpoint between the lowest price that is known to be high, and the highest price that is known to be low.

The iterative process stops when the difference between the two levels of price which bracket the optimum level becomes sufficiently small that continued iterations yield no significant information.

Figure 1 shows the flow diagram of the shooting method implemented in this model.

Application to Southeast Asian Forests

We applied the model and algorithm to the forests of Southeast Asia assuming a log export ban that is completely effective. All processing is assumed to take the form of plywood production. We report below the resulting time profiles of some of the variables for two scenarios. We report time profiles for plywood production capital stock, the mill-wood price of timber and volume of timber harvested. These variables allows one to monitor an important characteristic of the plywood industry, the value of units of the base resource, timber, and the harvests of this resource.

The scenarios differ only with respect to the allowed rate of change of the capital stock. In the constrained capital, CC, scenario we constrain the growth rate of capital, and in the perfectly mobile capital, PMC, scenario we allow instantaneous capital flows into or out of the region. The model identifies potential values of the variables that would be forthcoming from price-taker firms; thus, the scenarios identify potential time paths for different systems of constraints. Even though these scenarios probably are not precisely correct, they serve to illustrate the impacts upon the potential harvests of policies that affect the mobility of plywood production resources. Policies that adversely affect this mobility such as tarrifs, taxes, and the security of capital will yield predictions like those of the CC scenario.

Technical Data

The forests included are those of Malaysia (Peninsular Malaya, Sarawak, and Sabah), the Philippines (Luzon, Visayas, Mindanao, and
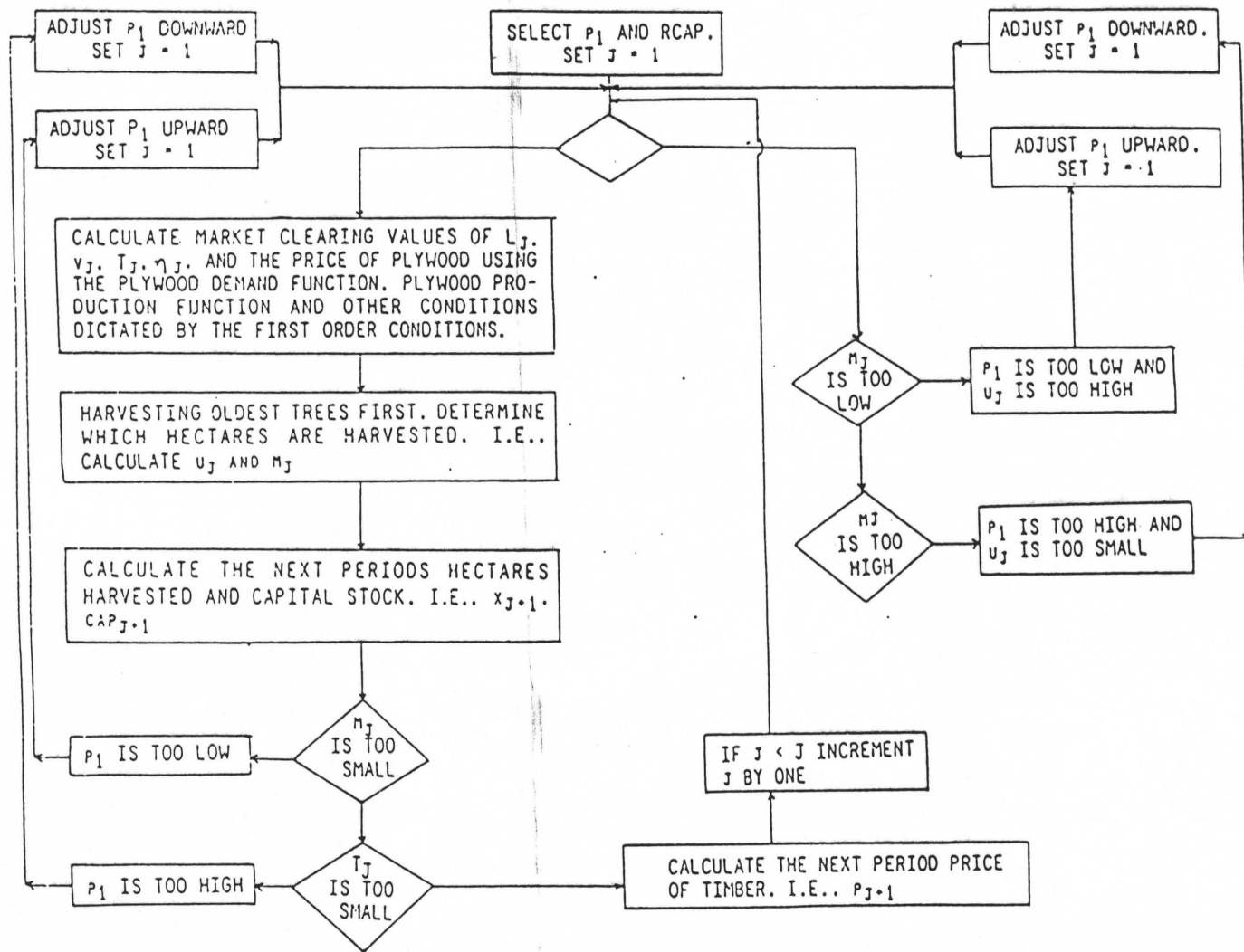
Figure 1. Flow diagram of the shooting method.

Palawan), and Indonesia (Sumatra and Kalimantan). These forests supply a large part, 80 percent in 1978, of the world's harvest of tropical hardwood. The harvesting is carried out using a selective cutting system where only trees over 50 centemeters in diameter at breast height are harvested. The remaining trees of the commercial species are left to grow and generate the subsequent harvest.

The yield function incorporated in the model is

$$t = a_0 \exp(b_0/age^2)$$

where age is length of time since the last cutting, and exp( ) is the natural exponential function. The exponential function was selected because it fits yield data well [Kao and Brodie (1980) and Lyon and Sedjo (1983)]. The parameters, a_0 and b_0 were selected to give an optimum rotation of 40 years and to yield a mean annual increment one cubic meter per hectare at age 40. According to Keil (1978) and Ross (1983), the rotation period used in these forests is 35 years with a yield of one to two cubic meters per hectare per year. The numbers selected are on the conservative side. The values used are a_0, 108.72, and b_0, -1600.

The inventory data for hectares of trees by age group were constructed from inventory data reported by FAO (1981) for total hectares of forests and logged-over hectares of forests and from harvest data reported by FAO (1945-1981). We assumed the logged-over hectares were distributed through time in the same way as were the harvesters.

The parameters of the plywood production function, equation (4), were calculated from data reported by Takeuchi (1982). The input-output ratio, b, was calculated from the average wood recovery rate for the region, and the exponents in the Cobb-Douglas production function were calculated using the fact that these exponents are factor shares in total costs of processing. The values used are b, 2.01, A, 0.0073, and a, 0.2.

For the harvesting and transportation cost functions we assumed constant marginal costs at $20 and $5 per cubic meter, respectively. Keil (1978) reported logging costs of between $25 and $28 per cubic meter of logs harvested.

We assumed a linear demand function for plywood with parameters that yielded prices in the range $245 to $318 per cubic meter of plywood and had an elasticity of -2.34 at the stationary state solution. The price range is consistent with the prices reported by Takenchi (1982) which range from $256 to $290. The resulting mill-wood price per cubic meter of timber ranged from $42 to $73 while Takenchi (1982) reported a range of $45 to $80.

We report the results for a 4 percent real interest rate; however, we also used a 6 percent interest rate. As expected the higher interest rate with its associated higher implicit user cost of capital yielded a lower level of capital stock in each time period. The relative shapes of the time profiles of the variables, however, were not altered by the choice of the interest rate; thus, either interest rate could be used to make the points we make below.

The Time Profiles of the Variables

Application of the model yielded time profiles for the control, state and costate variables in the model. In executing the algorithm a time horizon of 200 years was used; however we report only the first 100 years. Figures 2, 3, and 4 show the time profiles of plywood processing capital stock, the mill-wood price of timber, and the volume of timber harvested, respectively.

To facilitate the dicussion of the time profiles, we divide the 100 year reported time horizon into three consecutive subperiods with dividing
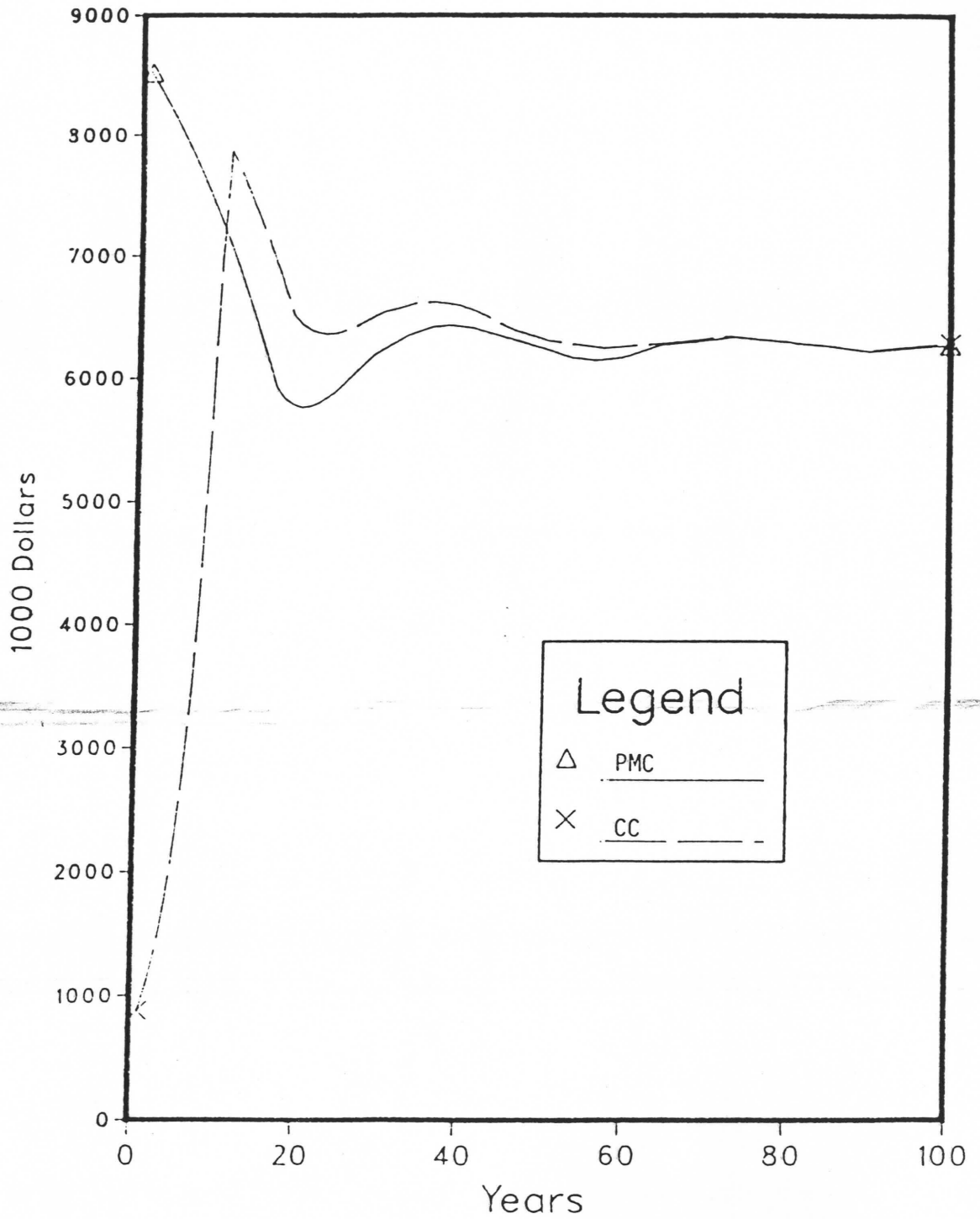
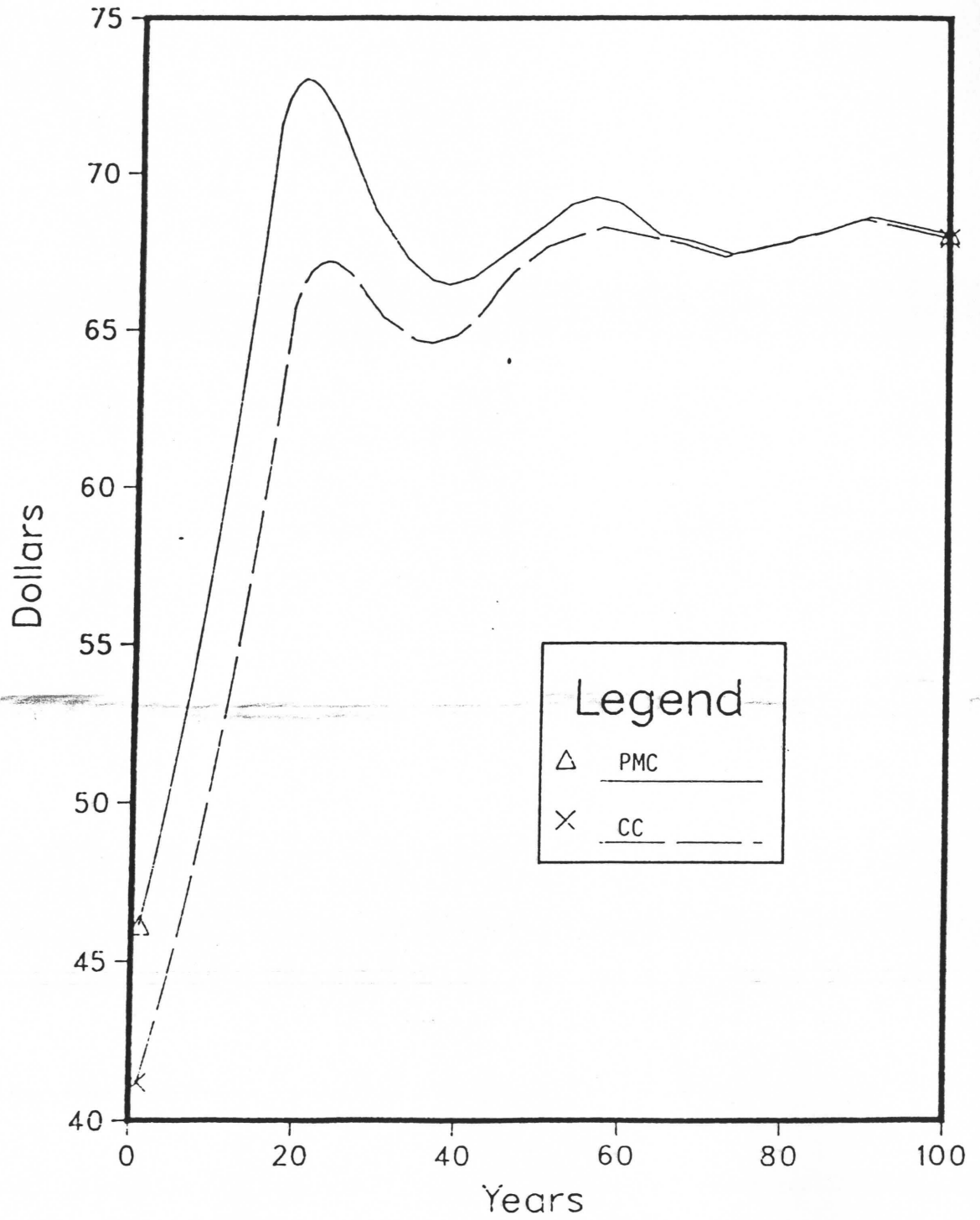Figure 2.   Time profiles of capital stock.
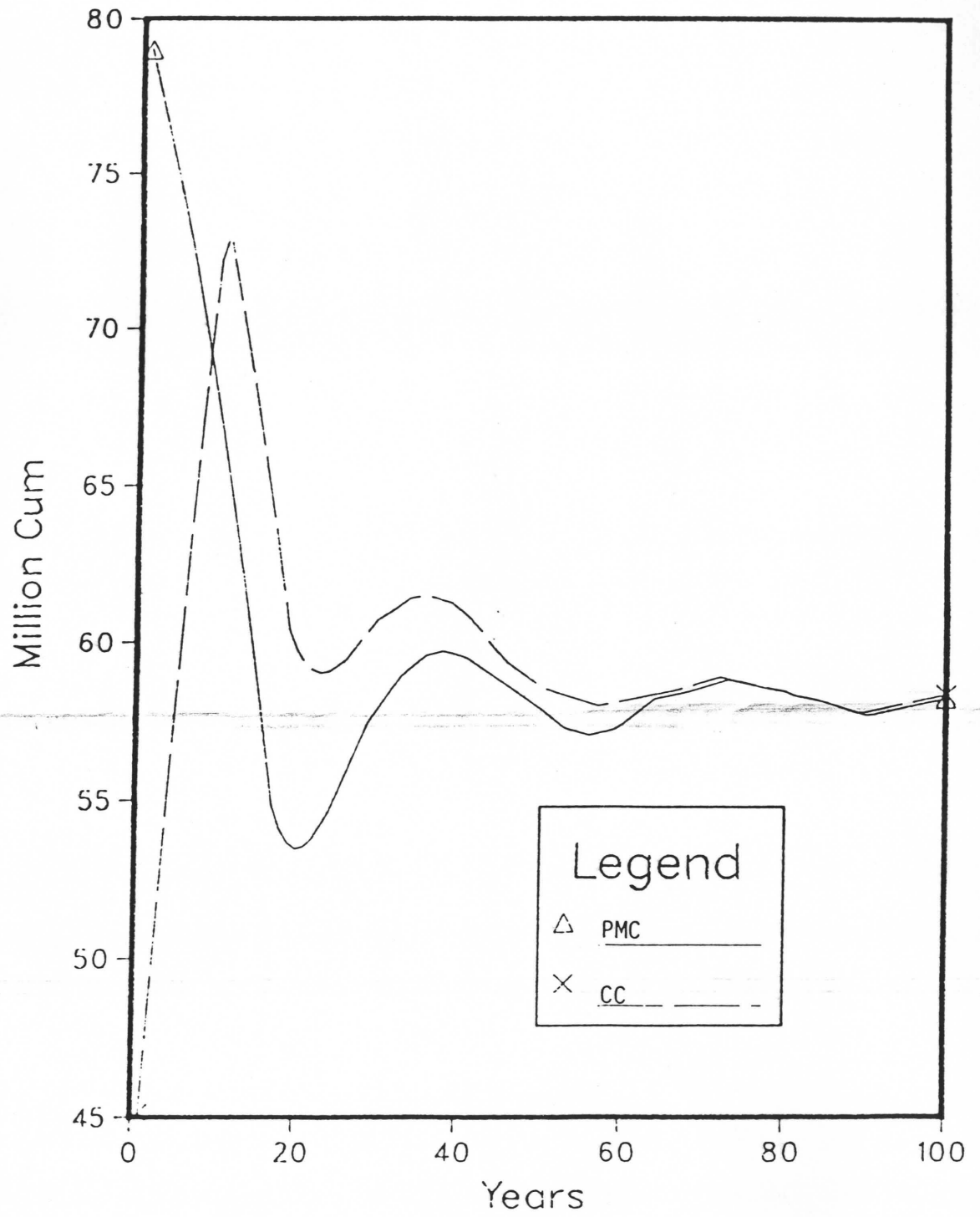
Figure 3. Time profiles of mill-wood price.

Figure 4. Time profiles of timber harvested.

years of 23 and 70. The first subperiod is the most interesting of the three with the shape of the time profiles being dominated by the harvesting of the old-growth forest and the capital stock constraint. This constraint affects capital accumulation, the harvests and the value of the forests. During the second subperiod all three of the time profiles or both scenarios have a damping oscillatory characteristic, and after about year 70 all of the time profiles continue in an oscillatory version of the stationary state. The oscillations are caused by the discrete nature of both the difference equations and the variables.

The perfectly mobile capital (PMC) scenario has an immediate jump in the capital stock followed by a declining time path through the first subperiod. The constrained capital (CC) scenario however, has capital increasing rapidly during the first eleven years then decreasing for the remainder of the first subperiod. With the completely effective log export ban and a large mature forest of tropical hardwood the shadow value of plywood production capital is high, and in both scenarios the capital stock increases rapidly.

The time profiles of the mill-wood price of timber are dominated by the old growth forests. During the first subperiod the mill-wood price rises rapidly. This rise is due to the role of the mill-wood price as rationer of the inventory of mature timber. For the harvests of mature timber to be spread over time the increase in mill-wood price must be sufficiently large to reward the owner to hold the inventory. The two scenarios have the same stationary state mill-wood price; however, during subperiods 1 and 2 the PMC scenario has the higher mill-wood price indicating that the value of the base resource is higher for this scenario. The value of the timber is adversely affected by policies that impede the mobility of capital.

The time profiles of timber harvested have the same general shape as those for capital for the respective scenarios. The harvests are initially higher for the PMC scenario; however, by year eleven the harvests from the CC scenario have surpassed those of the PMC scenario. The harvest levels evolve along oscillatory time paths to a common stationary state level. The cumulative harvests over, say, the first 60 years are the same for the two scenarios. Only the timing is different with those for the PMC scenario yielding the higher value of timber and higher present value of net surplus.

Summary

A shooting method for solving discrete time optimal control forestry problems has been identified and demonstrated. The application analyzed effects of a log export ban in Southeast Asia to demonstrate the applicability of the model and algorithm to the analysis of public policy analysis.

The maximum principle of optimal control theory was used to identify a difference equation problem to be solved to identify the optimal time profiles of the state variables, hectares of trees by age group and plywood production capital stock, and the control variables, timber harvested, gross investment in the plywood industry, and the level of the composite input. We find the solution time profiles using a three-step procedure. The first finds an initial feasible time path for the control and state variables Second, the costate variables are calculated using backward moving difference equations that require a feasible path of the state variables. Third, a solution to the boundary value problem is found by repetitively evaluating (shooting) the difference equations until a trajectory that satisfies the initial and terminal conditions is found.

The model results indicate the optimum rates of capital accumulation under constrained and nonconstrained conditions. The results indicate that different time paths of capital accumulation can have profound effects on the levels of harvest and mill-wood prices in the near term. However, in the longer term, the two systems converge.

Appendix A

The Laws of Motion for λ, ψ and P

If we define

$$\alpha_j = [\phi_j - f'(T_j)]U_j t \tag{A1}$$

then from equation (9a) we can write

$$\lambda_J = \rho \frac{dS_J}{dx_J} \tag{A2a}$$

$$\lambda_j = \rho[\alpha_j + (A + BU_j)'\lambda_{j+1}] \tag{A2b}$$

$$\lambda_1 = \rho[\alpha_1 + \rho(A + BU_1)'\alpha_2 + \rho^2(A + BU_1)'(A + BU_2)'\alpha_3 +$$

$$\ldots + \rho^{J-2}(A + BU_1)'(A + BU_2)' \ldots (A + BU_{J-2})'\alpha_{J-1} +$$

$$\rho^{J-1}(A + BU_1)'(A + BU_2)' \ldots (A + BU_{J-1})(\frac{dS_J}{dx_J})] \tag{A2c}$$

Since the oldest trees will be harvested first, there will be at most for each year one u_{ij} that is not either zero or one. Let m be age of the youngest age group harvested in year j and let $u_{ij} = 1$ for $i > m$. This explains that we are harvesting in year j all existing trees for which the age is greater than m. We can express u_j as

$$u_j = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ u_{mj} \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{A3a}$$

and $(A + BU_j)'$ as

$$(A + BU_j)' = \begin{bmatrix} 0 & 1 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{mj} & 0 & \cdot & \cdot & \cdot & \cdot & (1-u_{mj}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} \tag{A3b}$$

Thus,

$$\alpha_j = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \alpha_{m,j} \\ \alpha_{m+1,j} \\ \cdot \\ \cdot \\ \alpha_{M,j} \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ [\phi_j - f'(T_j)]u_{m+1,j}t_m \\ [\phi_j - f'(T_j)]t_{m+1} \\ \cdot \\ \cdot \\ [\phi_j - f'(T_j)]t_M \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ p_j u_{mj}t_m \\ p_j t_{m+1} \\ \cdot \\ \cdot \\ p_j t_M \end{bmatrix} \qquad (A3c)$$

where p_j is called net price or stumpage price (shadow value) of timber. Using equations (A2b), (A3b), and (A3c) we get

$$\lambda_{ij} = \rho\lambda_{i+1,j+1} \quad \text{for } i < m \qquad (A4a)$$

since for $i < m$ the upper partition of matrices α_j and $(A + BU_j^*)$ are 0 and 1, respectively.

$$\lambda_{mj} = \rho[(\lambda_{1,j+1} + p_j t_m)u_{mj} + (1 - u_{mj})\lambda_{m+1,j+1}] \quad \text{for } i = m \qquad (A4b)$$

$$\lambda_{ij} = \rho[\lambda_{1,j+1} + p_j t_i] \quad \text{for } i > m \qquad (A4c)$$

since for $i > m$, $u_{ij} = 1$

From equation (9b), if we define

$$\beta_j = n_j aAk_j^{a-1}v_j^{1-a} \qquad (A5)$$

where β_j is the value of marginal product of capital, η_j is the shadow value of the plywood processing capacity, and $a\tilde{A}k_j^{a-1} v_j^{1-a}$ is the marginal product of capital, then equation (9b) can be written as

$$\psi_J = \rho \, \frac{dS_J}{dk_J} \tag{A6a}$$

$$\psi_j = \rho[\beta_j + (1 - \delta)\psi_{j+1}] \tag{A6b}$$

$$\psi_1 = \rho[\beta_1 + \rho(1 - \delta)\beta_2 + \rho^2(1 - \delta)^2\beta_3 + \ldots +$$

$$\rho^{J-2}(1 - \delta)^{J-2}\beta_{J-1} + \rho^{J-1}(1 - \delta)^{J-1}(\frac{dS_J}{dk_J})] \tag{A6c}$$

Since the element

$$\beta_j = {}_j a\tilde{A}k_j^{a-1} v_j^{1-a}$$

then

$$\psi_j = \rho[\eta_j a\tilde{A}k_j^{a-1} v_j^{1-a} + (1 - \delta)\psi_{j+1}] \tag{A7}$$

Using the same techniques, the elements of $\dfrac{dL_j}{du_j}$ in equation (11a) can be derived as

$$[\phi_j - f'(T_j)]X_j t + X_j'B'\lambda_{j+1} - \xi_j \leq 0$$

Since

$$X_j'B = \begin{bmatrix} x_{1j} & \cdot & \cdot & \cdot & 0 \\ 0 & x_{2j} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & x_{Mj} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & \cdot & \cdot & 0 \\ 1 & 0 & -1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & -1 \\ 1 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} =$$

$$\begin{bmatrix} x_{1j} & -x_{1j} & 0 & \cdot & \cdot & 0 \\ x_{2j} & 0 & -x_{2j} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -x_{M-1,j} \\ x_{Mj} & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

then

$$X_j'B\lambda_{j+1} = \begin{bmatrix} x_{1j} & -x_{1j} & \cdot & \cdot & & 0 \\ x_{2j} & 0 & -x_{2j} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ x_{M-1,j} & \cdot & \cdot & \cdot & 0 & -x_{M-1,j} \\ x_{M,j} & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} \lambda_{1,j+1} \\ \lambda_{2,j+1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{M,j+1} \end{bmatrix}$$

$$
= \begin{bmatrix} x_{1j}(\lambda_{1,j+1}) - x_{ij}(\lambda_{2,j+1}) \\ x_{2j}(\lambda_{1,j+1}) - x_{2j}(\lambda_{3,j+1}) \\ \cdot \\ \cdot \\ \cdot \\ x_{M-1,j}(\lambda_{1,j+1}) - x_{M-1,j}(\lambda_{M,j+1}) \\ x_{M,j}(\lambda_{1,j+1}) \end{bmatrix} = \begin{bmatrix} x_{1j}(\lambda_{1,j+1} - \lambda_{2,j+1}) \\ x_{2j}(\lambda_{1,j+1} - \lambda_{3,j+1}) \\ \cdot \\ \cdot \\ \cdot \\ x_{M-1,j}(\lambda_{1,j+1} - \lambda_{M,j+1}) \\ x_{M,j}(\lambda_{1,j+1}) \end{bmatrix}
$$

Therefore, the elements in equation (11a) can be written as

$$
p_j x_{ij} t + x_{ij}(\lambda_{1,j+1} - \lambda_{i+1,j+1}) - \xi_j \leq 0 \tag{A8}
$$

Substituting equation (A4c) into (A8), letting i equal m, and solveing for p_{j+1} yields

$$
p_{j+1} = (p_j t_m + \lambda_{1,j+1} - \rho\lambda_{1,j+2} - \xi_{mj}/x_{mj})/\rho t_{m+1} \tag{A8a}
$$

Appendix B

Terminal Conditions

To identify the terminal conditions we solve for the solution values of the control variables in the stationary state. To achieve this we let all years of the current analysis be in the stationary state; and to prevent further complications of the notation, we let the time periods range from 1 to J as we did above.

In the stationary state we let m be the age of the youngest age group harvested and let $u_{mj}^* = 1$ where the asterisk indicates solution value. The relevant equations for this state can be simplified since $u_j^* = u_{j+1}^*$, $\alpha_j^* = \alpha_{j+1}^*$, $y_j^* = y_{j+1}^*$, and $\beta_j^* = \beta_{j+1}^*$ for all j and j+1 in the stationary state.

Equation (14c) can be modified as

$$\lambda_1 = \rho[I + \rho(A + BU_j^*)' + \rho^2 (A + BU_j^*)'^2 +$$

$$\rho^2\{(A + BU_j^*)'\}^3 + \ldots\ldots + \rho^{J-2} (A + BU_j^*)'^{J-2}]\alpha_j$$

$$+ \rho[\rho^{J-1}\{(A + BU_j^*)'\}^{J-1} (\frac{dS_J^*}{dx_J})] \tag{B1}$$

where years 1 through J are assumed to be in the stationary state. Since $0 \leq \rho \leq 1$ and J is very large, we can ignore the last term of equation (B1).

Since $u_{mj}^* = 1$, and $\rho[\rho^{J-1}\{(A + BU_j^*)'\}^{J-1}\frac{dS_J^*}{dx_J}] = 0$, then equation (B1) can be written as

$$\lambda_1 = \begin{bmatrix} \rho^m \Delta \alpha_{mj} \\ \rho^{m-1} \Delta \alpha_{mj} \\ \cdot \\ \cdot \\ \rho \Delta \alpha_{mj} \\ \rho(\Delta - 1)\alpha_{mj} + \rho \alpha_{m+1,j} \\ \cdot \\ \cdot \\ \rho(\Delta - 1)\alpha_{mj} + \rho \alpha_{M,j} \end{bmatrix} \qquad (m^{th} \text{ row}) \qquad (B2)$$

where α_{mj} are defined in equation (A3c), and

$$\Delta = 1 + \rho^m + \rho^{2m} + \ldots$$

Thus if m and y* were known, λ_1 could be calculated.

In the stationary state, equation (A6c) can be modified as

$$\psi_1 = \rho[1 + \rho(1 - \delta) + \rho^2(1 - \delta)^2 + \rho^3(1 - \delta)^3 + \ldots$$

$$\rho^{J-2}(1 - \delta)^{J-2}]\beta_j + \rho[\rho^{J-1}(1 - \delta)^{J-1}] \frac{dS_J^*}{dk_J} \qquad (B3)$$

As in the case above, the last term will be dropped; therefore, then we can write equation (B3) as

$$\psi_1 = \rho[1 + \rho(1 - \delta) + \rho^2(1 - \delta)^2 + \ldots + \rho^{J-2}(1 - \delta)^{J-2}]\beta_j \qquad (B4)$$

If we define

$$\Omega = 1 + \rho(1 - \delta) + \rho^2(1 - \delta)^2 + \rho^3(1 - \delta)^3 + \ldots$$

we get

$$\Omega = \frac{1}{1 - \rho(1 - \delta)}$$

Equation (B4) becomes

$$\psi_1 = \frac{\rho}{1 - \rho(1 - \delta)} \beta_j \qquad (B5)$$

ψ_1 can be calculated if m and y^* were known.

We now examine the determination of m by combining equations (A8), (B2) and (A3c), where $u_{mj}^* = 1$ implying $\xi = 0$. Thus, we get from equation (A8)

$$p_j x_{ij} t + x_{ij}(\lambda_{1,j+1} - \lambda_{i+1,j+1}) \leq 0$$

We assume that $x_{ij} > 0$ for i equal m and m+1, and, substituting the values of λ_j from equation (B2), we get

$$p_j t_m + [\rho^m \Delta \alpha_{mj} - \rho(\Delta - 1)\alpha_{mj} - \rho\alpha_{m+1,j}] \leq 0$$

substituting the values of α_j from equation (A3c)

$$p_j t_m + \rho^m \Delta p_j t_m - \rho(\Delta - 1)p_j t_m - \rho p_j t_{m+1} \leq 0$$

$$p_j t_m[1 + \rho^m \Delta - \rho(\Delta - 1) - \rho\frac{t_{m+1}}{t_m}] \leq 0 \qquad (B6)$$

and

$$p_j t_{m+1}[1 + \rho^{m+1}\Delta - \rho(\Delta - 1) - \rho\frac{t_{m+2}}{t_{m+1}}] > 0$$

This last equation dictates that we will harvest all hectares in age group m+1.

We can calculate k^* and v^* using equations (11e), (B5), (A5), (11g), and the results of equation (B6).

We now turn our attention to k^* and v^* which are calculated using the stationary state rotation period (m). Using the stationary state

rotation period (m) calculated from equation (B6), we can calculate the volume of timber produced in the stationary state (T_s) from

$$T_s = \frac{\text{Total Hectares}}{m} \cdot t_m \tag{B7}$$

The volume of plywood produced in the stationary state (L_s) can be calculated using modified equation (4b)

$$L_s = \frac{T_s}{b} \tag{B7a}$$

From equation (11e)

$$\psi_{j+1} - 1 \leq 0$$

substitute the values of ψ_1 from equation (B5)

$$\frac{\rho}{1 - \rho(1 - \delta)} \beta_j - 1 \leq 0$$

and then substitute the values of β_j from equation (A5) to get

$$\frac{\rho}{1 - \rho(1 - \delta)} \, \eta_j a \tilde{A} k_j^{a-1} v_j^{1-a} - 1 = 0, \tag{B8}$$

From equation (11g)

$$\eta_j(1 - a)\tilde{A} k_j^{a} v_j^{-a} - 1 = 0, \tag{B9}$$

and equation (11k), where we substitute the value of plywood produced in the stationary state, we get

$$\tilde{A} k_j^{a} v_j^{1-a} - L_s = 0 \tag{B10}$$

The solution values of v, k, and η in the stationary state can be solved using equations (B8), (B9), and (B10).

Finally, we can calculate the level of investment in the stationary state (y^*) by using equation (3)

$$k_{j+1} = (1 - \delta)k_j + y_j$$

then we get

$$y^* = \delta k^*$$

It is obvious that the gross investment is equal to the depreciation in the stationary state.

From equation (B10)

$$\tilde{A}k_j^a v_j^{1-a} - L_s = 0$$

we get

$$k^a = \frac{L_s}{\tilde{A}v_j^{1-a}}$$

$$k = [\frac{L_s}{\tilde{A}v_j^{1-a}}]^{1/a} = [\frac{L_s}{\tilde{A}} v^{a-1}]^{1/a} \qquad \text{(B11a)}$$

Substitute the value of k from equation (B11a) into equation (B9)

$$n_j(1 - a)\tilde{A} \frac{L_s}{\tilde{A}v_j^{1-a}} \cdot v_j^{-a} - 1 = 0$$

$$n_j(1 - a)L_s v^{-a} = v_j v_j^{-a}$$

$$v_j = n_j(1 - a)L_s \qquad \text{(B11b)}$$

Substitute the value of k from equation (B11a) into equation (B8)

$$\frac{\rho}{1 - \rho(1 - \delta)} j a\tilde{A}[\frac{L_s}{A v_j^{1-a}}]^{\frac{a-1}{a}} v_j^{1-a} - 1 = 0$$

$$\frac{\rho}{1 - \rho + \rho\delta} \, n_j a \tilde{A}^{1-\frac{a-1}{a}} L_s^{\frac{a-1}{a}} v_j^{-(1-a)\frac{(a-1)}{a} + (1-a)} = 1$$

$$\frac{\rho}{1 - \rho + \rho\delta} \, n_j a \tilde{A}^{\frac{1}{a}} L_s^{\frac{a-1}{a}} v_j^{\frac{-(a-1)}{a}} = 1$$

$$v_j^{\frac{a-1}{a}} = \frac{\rho}{1 - \rho + \rho\delta} \, n_j a \tilde{A}^{\frac{1}{a}} L^{\frac{a-1}{a}}$$

$$v_j = [\frac{\rho}{1 - \rho + \rho\delta} \, n_j a]^{\frac{a}{a-1}} \tilde{A}^{\frac{1}{a-1}} L_s \qquad (B11c)$$

Equating equations (B11b) and (B11c)

$$j(1 - a)L_s = [\frac{\rho}{1 - \rho + \rho\delta} \, n_j a]^{\frac{a}{a-1}} \tilde{A}^{\frac{1}{a-1}} L_s$$

$$n_j^{\frac{-1}{a-1}} = (1-a)^{-1} (\frac{\rho}{1 - \rho + \rho\delta})^{\frac{a}{a-1}} a^{\frac{a}{a-1}} \tilde{A}^{\frac{1}{a-1}}$$

$$\eta_j = \frac{(1 - a)^{a-1}}{\tilde{A}} [\frac{1 - \rho + \rho\delta}{a\rho}]^a \qquad (B11d)$$

Substitute equation (B11d) to equation (B11b)

$$v_j = [\frac{(1 - a)^{a-1}}{\tilde{A}} (\frac{1 - \rho + \rho\delta}{a\rho})^a](1 - a)L_s$$

$$v_j = (1 - a)^a (\frac{1 - \rho + \rho\delta}{a\rho})^a \frac{L_s}{\tilde{A}} \qquad (B11e)$$

Substitute equation (B11e) to equation (B11a)

$$k_j = [\frac{L_s}{\tilde{A}}(\frac{[1-a]^a}{\tilde{A}} [\frac{1 - \rho + \rho\delta}{a\rho}]^a L_s)^{a-1}]^{\frac{1}{a}}$$

$$k_j = (1 - a)^{a-1} [\frac{1 - \rho + \rho\delta}{a\rho}]^{a-1} \frac{L_s}{\tilde{A}} \tag{B11f}$$

Therefore, we can calculate T_s, L_s, k_s, v_s, and n_s in the stationary state.

Using the demand function for plywood, we can calculate the price of plywood in the stationary state. Then using equation (11c), we can calculate the shadow value of timber delievered at the processing mill (mill-wood price) in the stationary state (ϕ_s)

$$\phi_s = \frac{D(L_s)}{b} - \frac{n_s}{b} \tag{B12}$$

Footnotes

[1]In general, the necessary conditions require only that the Hamiltonian be stationary (Jackson and Horn 1964, p. 390); however, a stationary value will be a maximum value subject to the constraints because the constraints are linear and equation (8a) is quasi-concave at a stationary point, i.e., at a point where the quasi-saddle-point conditions of the associated Lagrangean function are satisfied.

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