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## A Theoretical Study of Infiltration into Range and Forest Soils

Joel E. Fletcher

Yehia Z. El-Shafei

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**A THEORETICAL STUDY OF INFILTRATION  
INTO RANGE AND FOREST SOILS**

**A Final Technical Report**

**by**

**Joel E. Fletcher and Yehia Z. El-Shafei**

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**Utah Water Research Laboratory  
College of Engineering  
Utah State University  
Logan, Utah 84321**

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## ABSTRACT

More than 400 rainfall simulator experiments were examined to detect which soil properties could be used to compute infiltration time relationships. Three theoretical equations were tested to determine their efficacy for calculating infiltration time relationships from soil and site characteristics. It was shown that both the modified Green and Ampt and Fletcher equations could be successfully used.

Darcian type equations were developed on laboratory type samples which would show the relation between soil, solution and rainfall properties and infiltration. These latter equations have not been tested on undisturbed soils but give excellent agreement between measured and computed values for time before flooding and infiltration time relationships.

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## LIST OF SYMBOLS

a	=	depth of water on the soil surface, L
A	=	area of the soil surface, L <sup>2</sup>
A'(θ <sub>i</sub> )	=	depends on the initial moisture content dimensionally inconsistent
α	=	angle of contact between water and the soil particle, dimensionless
b	=	constant representing the slope of the cumulative infiltration curve on log-log paper, dimensionless
c	=	$\frac{\pi g \rho}{8 \eta}$
C	=	constant representing d at unit time of flooded infiltration, dimensionally inconsistent
C'	=	constant = $\left( \frac{R}{\theta_T - \theta_i} \right)^{n'}$
ψ	=	matric or capillary potential, L
d	=	cumulative depth of infiltration, L
η	=	coefficient of viscosity, mL <sup>-1</sup> t <sup>-2</sup>
$\frac{g}{g}$	=	acceleration due to gravity, Lt <sup>-2</sup>
$\frac{g}{g}$	=	the gravitational term being equal to ΔZ for vertical infiltration
ν	=	surface tension, mt <sup>-2</sup>
h	=	depth from water surface to the wetting front, L
h <sub>c</sub>	=	capillary potential head, L
h <sub>T</sub>	=	head loss in the transmission zone extrapolated to the wetting front, L
h <sub>w</sub>	=	pressure potential loss in the transmission zone, L
H	=	specific moisture capacity
i	=	refers to distance as a subscript or superscript
j	=	refers to time as a subscript or superscript
K	=	hydraulic conductivity, Lt <sup>-1</sup>
K <sub>C</sub>	=	capillary conductivity, Lt <sup>-1</sup>
K <sub>G</sub>	=	constant depending on capillary forces at the wetting front = π r <sub>p</sub> ν cos α
ln	=	natural logarithm
m	=	slope of the parallel lines formed by plotting C' as a function of R, dimensionless
n	=	porosity, dimensionless
n'	=	constant having values between 0.80 and 1.00 depending on the soil properties
N	=	(bC), constant representing q at unit time, dimensionally inconsistent
p	=	$\frac{\sum r^4}{A p}$
P	=	constant = cp
Φ	=	the potential = ψ + Z
π	=	constant
q	=	infiltration rate, L
r	=	particle radius or modal particle radius, L
r <sub>p</sub>	=	pore or capillary radius, L
ρ <sub>p</sub>	=	density of water solution, mL <sup>-3</sup>
R	=	rain intensity, Lt <sup>-1</sup>
s	=	net increment of the degree of saturation or the degree of saturation after wetting minus the degree of saturation before wetting as a fraction of the total porosity S = ns, dimensionless
S	=	pore space not filled with water which could be filled under flooded infiltration $\frac{\theta_m - \theta_i}{\theta_i}$



- t = time, t
- $\theta$  = moisture content volume basis, dimensionless
- $\theta_i$  = initial moisture content volume basis, dimensionless
- $\theta_m$  = maximum moisture content obtained under flooded infiltration, dimensionless
- $\theta_T$  = moisture content of the transmission zone volume basis, dimensionless
- u = (b-1) represents the slope of the infiltration rate curve on log-log paper, dimensionless
- V = vertical flow velocity at the soil surface,  $Lt^{-1}$
- Z = depth from the soil surface to the wetting front, L
- $\left(\frac{dZ}{dt}\right)_R$  = advance rate of the wetting front for rain infiltration,  $Lt^{-1}$

## INTRODUCTION

Despite the recognition of the importance of the infiltration process to range and watershed management for many years, research has not completely solved the problems of infiltration under all environmental conditions nor has it solved all of the problems in its application to runoff forecasting even when infiltration is known. Hickok and Osborn (1969) outlined some of the problems and limitations in the use of infiltration in watershed estimates from the classic paper of Horton (1933) to the present.

In the laboratory, Green and Ampt (1911) considered the soil to be a bundle of cylindrical capillaries and developed equations for estimating infiltration from various parameters. Since the time of Green and Ampt (1911) literally thousands of papers on the subject of infiltration have been written. No attempt will be made at this time to review the extensive literature. The reader is referred to such reviews as those given by Keller (1967), Parr and Bertrand (1959), Richards (1952), Davidson (1940), Chow (1964), Muchler and Hermsmeier (1965), Chebotarev (1962), and Neyestani (1969) to note that the capillary theory, the diffusivity theory and the porous media flow theory are still with us. The problem still remains. "How do you actually estimate infiltration for a particular soil in the field?"

The theoretical studies made during the present investigation were directed toward shedding light on the answer to this problem with the development in two principal directions. The first of these two developments was through an extension of the capillary flow theory and the second development was through Darcian porous media flow theory.

Briefly, the soil and solution properties which were considered are grouped under the following headings for the theoretical study:

### Gradient factors:

1. depth of water on the surface
2. depth of wetting
3. tortuosity
4. capillary sorptivity
  - a. surface tension
  - b. wettability

- c. pore size distribution
- d. moisture content
5. stratification

### Water supply factors:

1. particulates (clear water, silty water, etc.)
2. depth per unit area
3. intensity of supply (rain, snowmelt or sprinkler)
4. soil aggregate stability

### Conduction factors:

1. viscosity
2. capillary conductivity
3. degree of saturation
4. dispersability of the soil

Most of the foregoing factors vary with time in any soil due to biological and climatic factors.

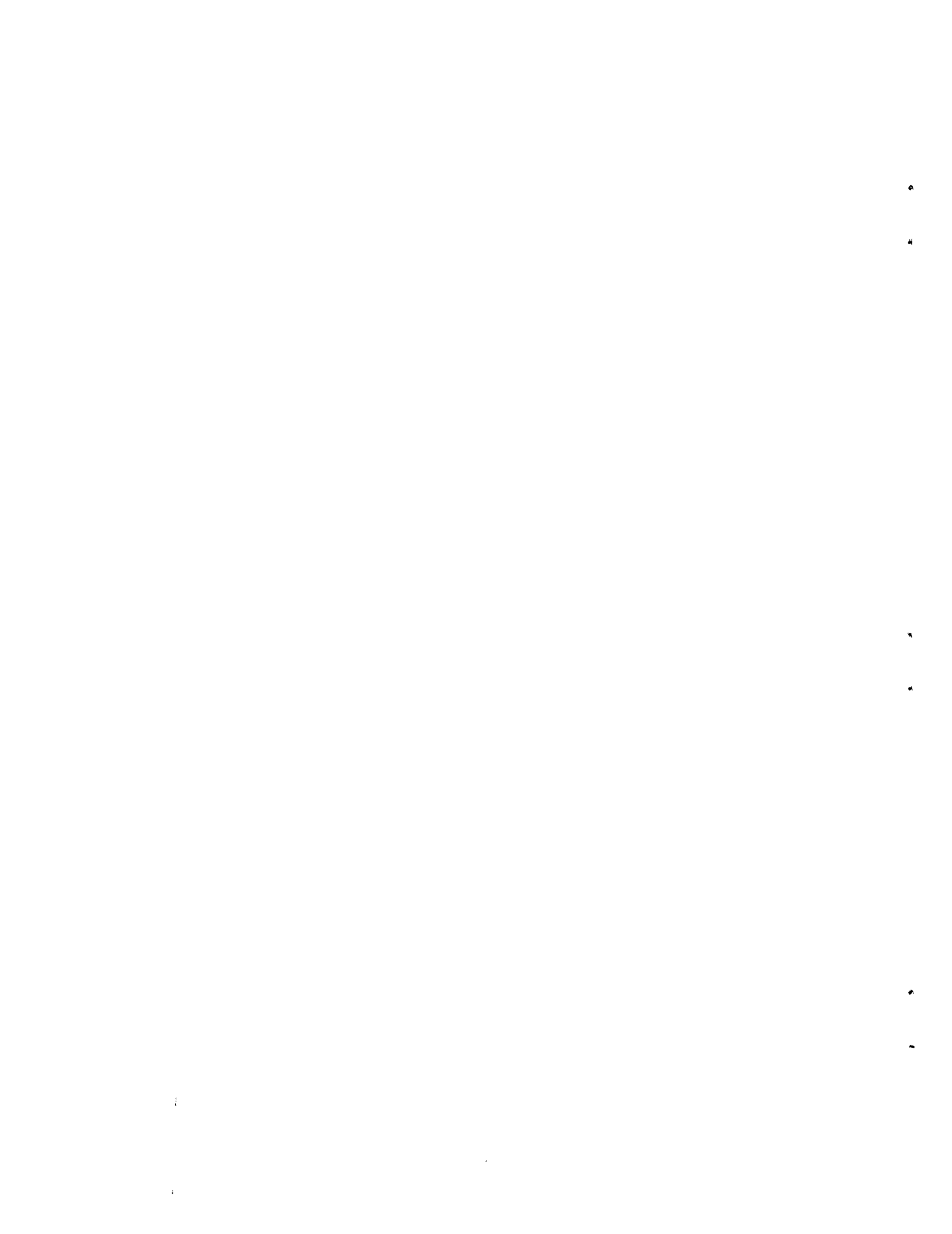
### Objectives

The objectives of this investigation were as follows:

1. To develop theoretically sound relationships between infiltration and physical factors such as hydraulic conductivity, soil porosity, soil waterholding capacity, antecedent soil moisture content, capillary potential and others.
2. To find the relation between infiltration and the derived infiltration relationships to actual physical and biological factors found in the field.
3. To find a relation between infiltration and watershed retention which can be used to forecast runoff relations of ungaged watersheds.

Briefly the plan of the investigation was as follows:

1. A survey of available infiltrometer data to ascertain how much of the data, if any, includes the necessary physical and biological data needed to compute infiltration by different equations in the literature or derived during the course of this investigation. Data needed consist of such items as temperature, hydraulic conductivity, wettability, soil porosity, soil moisture, soil moisture at saturation and capillary permeability.
2. Test existing or derived relationships utilizing the above data to test the validity of the relationships.



## WORK ACCOMPLISHED AND FINDINGS

A survey was made of the available infiltrometer data. Generally data on soil and vegetation were lacking but some data taken by the Soil Conservation Service in Arizona and New Mexico had most of the needed parameters so they were processed for study.

The infiltrometer data contained the following information:

1. rainfall intensity
2. plot size and slope
3. soil moisture content at the beginning and end of a run
4. depth of moisture penetration following the run
5. temperature of the water, air, and soil at the beginning and end of each run
6. soil porosity
7. mechanical analyses of the soil
8. organic matter content of the soil
9. times for unit volumes of runoff
10. time when first flooding occurred
11. time when runoff started
12. times when depression storage was 25%, 50%, 75%, and 100% filled
13. vegetation kind and density
14. soil classification and description
15. apparent specific gravity of soil by horizons
16. any other pertinent data observed

The infiltrometer data were punched, programs were written, and the log-log plots of mass infiltration against time in seconds were made. Data points for the runoff curves, depression storage curves, and the surface detention curves were tabulated. A typical example of the original field measurement data sheet can be seen in Figure 1, and a typical infiltration-time relation curve as plotted on the computer is shown in Figure 2. Table 1 shows the properties of the soils used.

From a study of the infiltration time curves, it appeared that they could be classified into three distinct categories. First, those completely linearized by the log-log plot; second, those which produced more than one linear portion and third, those which were linear only after the first 5 or 10 minutes of a run being convex upward during the first portion of a run. Examples of each of the three types of curves may be seen in Figure 3.

Utilizing the relation  $Q/S$ , wherein  $Q$  is the mass infiltration in inches and  $S$  is the fraction of the pore space not filled with water, to compute the depth to the wetting front, each break in the curve corresponds to either a new stratum in the soil in the case of abrupt breaks, or a sufficiently gradual change in the organic matter content to reflect a change in the wettability.

### Capillary Flow Type Equations

#### Green and Ampt (1911)

The equation these authors suggested on the basis of their capillary tube theory was as follows:

$$\frac{P}{S} t = Z - (2 + K_G) \ln (1 + Z/a + K_G) \quad \dots \dots \dots (1)$$

in which

- $a$  = depth of water on the soil surface, L
- $Z$  = depth from the soil surface to the wetting front, L
- $K_G$  = a constant depending on the capillary forces on the moving water-soil boundary
- $P$  = constant =  $cp$
- $c$  =  $\frac{\pi g \rho}{8 \eta}$
- $p$  =  $\frac{\sum r_p^4}{A}$
- $r_p$  = pore or capillary radius, L
- $A$  = area of soil surface,  $L^2$
- $S$  = pore space available for infiltration or pore space not filled with water which could be filled under flooded infiltration,  $\theta_m - \theta_i$ , dimensionless
- $t$  = time, t
- $\eta$  = coefficient of viscosity,  $mL^{-1} t^{-1}$
- $g$  = acceleration due to gravity,  $Lt^{-2}$
- $\rho$  = density of the water,  $mL^{-3}$

U. S. DEPARTMENT OF AGRICULTURE - SOIL CONSERVATION SERVICE  
 DIVISION OF RESEARCH PROJECT ARIZ-R-1  
 RAINFALL SIMULATOR EXPERIMENTS

Notes: RRG  
 Runoff: DA  
 11:00 a.m.  
 Date Feb. 23, 1939  
 City Farm  
 Site 10 Plot 2 Run 50 Slope 2.00%  
 Dry

Avg. Intensity Duration of Mass Mass Run-  
 Inches Per Hour 3.30 Application 30 min. Rain in. 1.650 off, in. 1.124

Soil Gila fine sandy loam. Moist to 6 inches from rains.

Cover None

TIME		RUNOFF		INFILTRATION	REMARKS
After Start min.	Between Read. sec.	Mass cu. ft.	Rate in./hr.	in./hr.	
Before app. started		Soil	70°F.	Air 75°F. Water 60°F.	
0:00					Application started
1:05					Water movement
2:35		0.000	0.00		Runoff started
2:54			0.39		Soil moisture
3:13	38	0.050			0-6" = 2.8%
3:22			0.83		Moisture Equiv.
3:31	18	0.100			0-6" = 29.5%
3:43			1.20		
3:56	25	0.200			
4:05			1.58		Sample #1 @ 4:00
4:15	19	0.300			
4:23			1.82		
4:31 $\frac{1}{2}$	16 $\frac{1}{2}$	0.400			
4:38			2.00		
4:46 $\frac{1}{2}$	15	0.500			
4:53			2.14		
5:00 $\frac{1}{2}$	14	0.600			
5:07			2.14		
5:14 $\frac{1}{2}$	14	0.700			
5:21			2.07		

Figure 1. Typical field measurement data sheet summarizing the soil, vegetation, temperature, and rainfall simulator measurements.

SITE 20 RUN 105

SECONDS	DA	QR	FR
2700	0.06271	1.72	1.43000
2710	0.05452	1.50	1.24709
2732	0.03942	1.20	0.99767
2762	0.02370	0.86	0.71500
2792	0.01256	0.60	0.49884
2842	0.00213	0.22	0.18291
2880	0.0	0.0	0.0

FFR	F	FEE
0.05214	1.28150	1.29207

FC AT 2700 SECONDS # 1.29207

SECONDS	DELTA P	DELTA Q	DELTA F	FC
2605	0.08312	0.04542	0.03771	1.25436
2432	0.15137	0.08333	0.06804	1.18632
2256	0.15400	0.08333	0.07067	1.11565
2080	0.15400	0.08333	0.07067	1.04499
1902	0.15575	0.08333	0.07242	0.97257
1719	0.16012	0.08333	0.07679	0.89578
1541	0.15575	0.08333	0.07242	0.82336
1369	0.15050	0.08333	0.06717	0.75620
1194	0.15312	0.08333	0.06979	0.68640
1005	0.16537	0.08333	0.08204	0.60436
910	0.08312	0.04167	0.04146	0.56290
815	0.08312	0.04167	0.04146	0.52145
723	0.08050	0.04167	0.03883	0.48261
628	0.08312	0.04167	0.04146	0.44115
608	0.01750	0.00833	0.00917	0.43199
589	0.01662	0.00833	0.00829	0.42370
570	0.01662	0.00833	0.00829	0.41540
549	0.01837	0.00833	0.01004	0.40536
528	0.01837	0.00833	0.01004	0.39532
506	0.01925	0.00833	0.01092	0.38440
483	0.02012	0.00833	0.01179	0.37261
457	0.02275	0.00833	0.01442	0.35820
441	0.01400	0.00417	0.00983	0.34836
420	0.01837	0.00417	0.01421	0.33415
380	0.03500	0.00417	0.03083	0.30332
328	0.04550	0.00275	0.04275	0.26057
300	0.02450	0.00142	0.02308	0.23749
195				0.17062

AT 195 SECONDS T#F# 0.17062

Figure 2b. Tabulated rainfall simulator data for the curve in Figure 2a.

**Table 1. Properties of field soils used for confirmation.**

Soil Type	Whitehouse Gr SL No. 1		Sonoita Gr SL		Comoro Gr SL		Whitehouse Gr SL No. 2	
Horizons (inches depth)	a. 0"-10" b. 10"-23" c. 23"-33"		0"-13" 13"-26"		0"-10" 10"-18" 18"-29"		0"-6" 6"-16" 16"-36"	
Volume Weight (V) (ratio)	1.593		1.647		1.680		1.662	
Water at saturation (%)	a. 48.93 b. 46.20 c. 50.15		27.57 36.58		24.97 33.45 36.24		21.49 39.53 49.53	
Pores not filled with water (S) (ml./ml.)	0.109		0.111		0.100		0.118	
Modal particle radius (r) (cm.)	0.0360		0.0437		0.0533		0.0237	
Porosity (n) (ml./ml.)	.3766		.3470		.3412		.3710	
Cosine of the contact angle $\alpha$ (ratio)	0"-0.7" 0.7"-1.0" 1.0"-3.0" 3.0"->5.0"	0.267 0.387 0.757 1.000	0"-1.0" 1.0"-1.5" 1.5"-2.1" 2.1"-9.0"	0.098 0.152 0.169 0.193	0"- .8" 0.8"-1.2" 1.2"-2.8" 2.8"-4.5" 4.5"-8.8"	0.029 0.063 0.111 0.165 0.307	0"- .6" 0.6"-1.3" 1.3"-3.8" 3.8"-11.5"	0.372 0.120 0.227 0.600
Hydraulic conductivity (K) (in./min.)	2.50x10 <sup>-4</sup>		6.18x10 <sup>-4</sup>		12.77x10 <sup>-5</sup>		6.69x10 <sup>-4</sup>	

Equation (1) was not suitable for use directly since the parameters, as needed to solve the equation, were not available. However, if a few assumptions are made the equation can be modified to accommodate the use of field measurement data on hand. These assumptions are as follows:

1. The cross section of a pore is a 3 cusped hypocycloid and the modular particle diameter represents all of the particles.
2. The depth of ponding on the surface to be negligibly small compared to the depth of wetting.
3. The soil is essentially saturated between the surface and the wetting front so that  $d/S = Z$ .
4. The parameters  $\eta$ ,  $g$ , and  $\nu$  the surface tension, remain constant at .01 poises, 980 centimeters per second squared, and 72 dynes per centimeter throughout the estimation and that other temperature effects are negligible and the soil is vertically and horizontally homogeneous within each stratum.

Then

$$h = Z = \frac{d}{S} \quad \text{as } a \rightarrow 0$$

$$K_G = \pi r_p \nu \cos \alpha$$

$$P = \frac{\sum r_p^4}{A} \times \frac{\pi g \rho}{8 \eta}$$

$$r_p^2 = .051 r^2$$

in which

$$r = \text{particle radius}$$

Substituting back in Equation (1) yields

$$t = \frac{8A\eta}{\pi g \rho (0.051 r^2)^2} \left[ d - \pi r S \nu \cos \alpha \ln \left( 1 + \frac{d}{\pi r S \nu \cos \alpha} \right) \right] \dots \dots \dots (2)$$

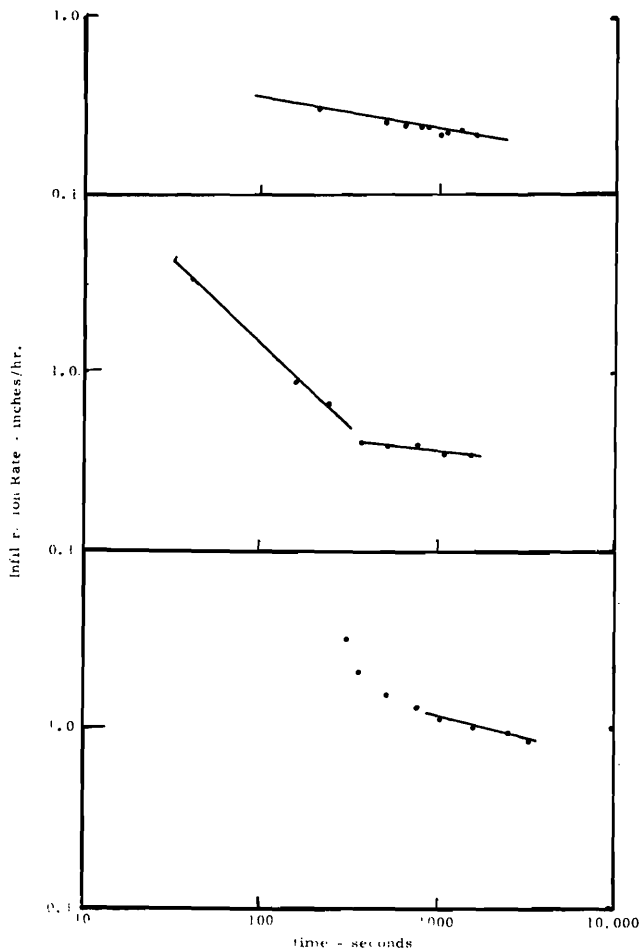


Figure 3. Types of relations between infiltration rate and time from 400 infiltrometer runs.

Equation (4) was to compute the log mass infiltration-log time curves for the four soils in Table 1. The results may be seen in Figures 4, 5, 6, and 7. The agreement may be considered to be reasonable.

Fletcher (1949)

Fletcher derived an equation for a single cylindrical capillary which would reflect the physical interactions between a soil and a solution. His equation was as follows:

$$q = \frac{r_p^3 (\rho g h r_p + v \cos \alpha)}{8Z\eta} \dots (5)$$

in which

- q = volume of flow per unit of time per pore or infiltration rate
- r<sub>p</sub> = radius of the pore
- g<sup>p</sup> = acceleration of gravity
- ρ = density of the water solution
- h = depth from water surface to the wetting front
- v = surface tension between water and solid phase
- α = contact angle between water and solid
- Z = depth from soil surface to wetting front
- η = coefficient of viscosity

Equation (5) was modified to a unit area of soil surface and to consider the pore cross section as a 3 cusped hypocycloid and to have the pore dimensions in terms of the particle radii, r. It was further assumed that the depth of the liquid on the surface was small so the difference between h and Z becomes negligible. Then if d is the total depth of infiltration in a unit of time and 0.577/r<sup>2</sup> is the number of pores in a square centimeter

$$r_p^2 = \frac{0.16 r^2}{\pi}$$

and

$$Z = h = \frac{d}{S}$$

then

$$\frac{d^2}{t} = \frac{0.577 \pi (0.51r^2)^2}{8r^2\eta} (0.16r^2 d g + \pi r S v \cos \alpha) \dots (6)$$

and

$$t = \frac{10^{-2}}{Sr^4} \left[ d - 226.2rS \cos \alpha \ln \left( 1 + \frac{d}{226.2rS \cos \alpha} \right) \right] \dots (3)$$

with d and r in centimeters. Converting d to inches yields

$$t = \frac{3.93 \times 10^{-3}}{Sr^4} \left[ d - 574rS \cos \alpha \ln \left( 1 + \frac{d}{574rS \cos \alpha} \right) \right] \dots (4)$$

converting d to inches and substituting values for constants as follows: c = 1, g = 980, v = 72, π = 3.1416 and neglecting temperature, Equation (6) becomes



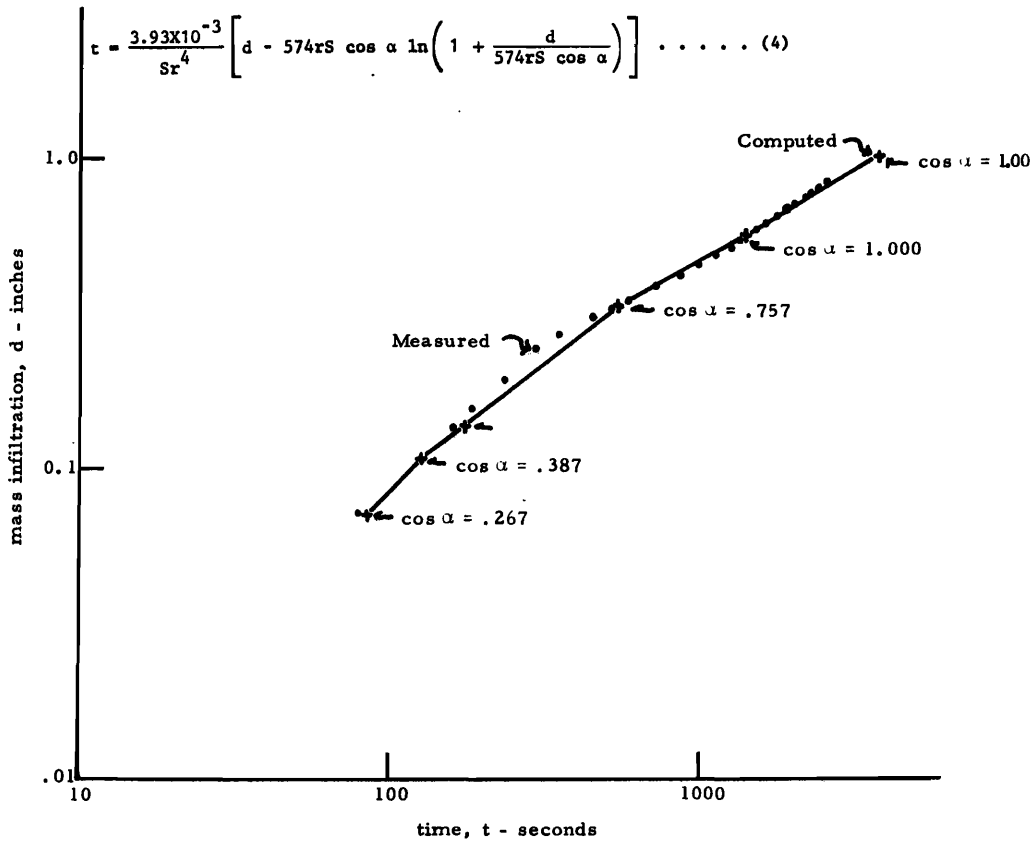


Figure 4. A comparison between the log mass infiltration-log time relations as observed in the field and computed using soil properties on Whitehouse Gr SL No. 1 soil and a modified Green and Ampt Equation (4).

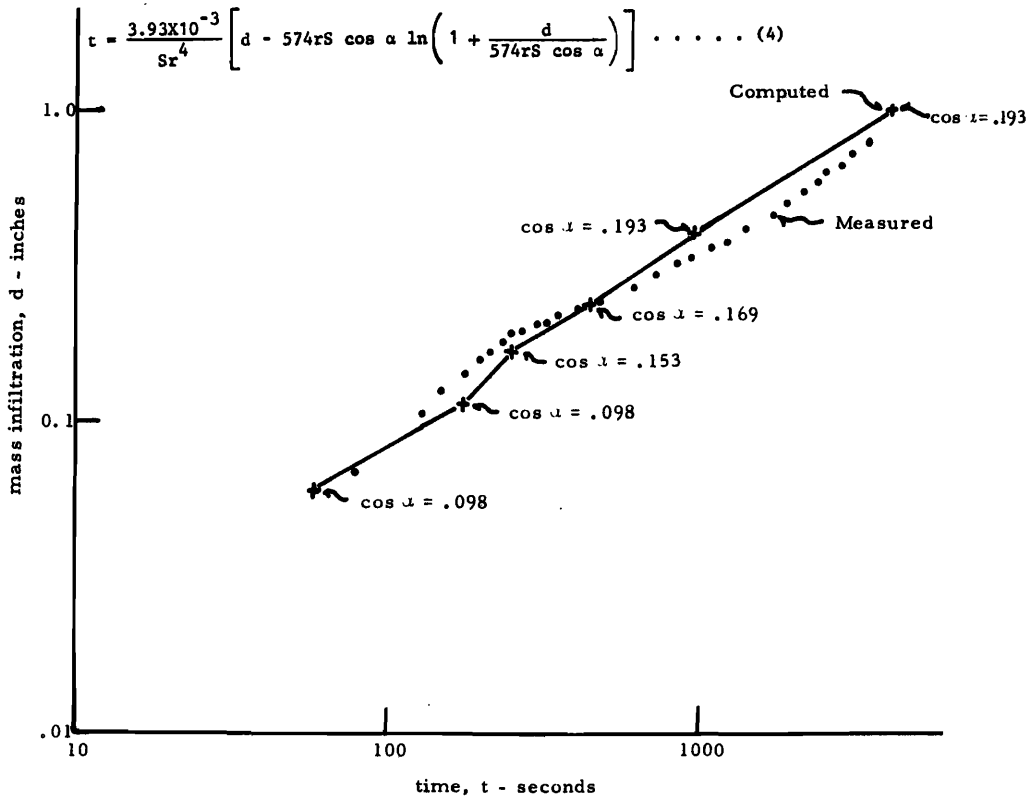


Figure 5. A comparison between the log mass infiltration-log time relations for Sonoita Gr SL soil as observed in the field and as computed from the soil properties using Green and Ampt Equation (4) as modified herein.

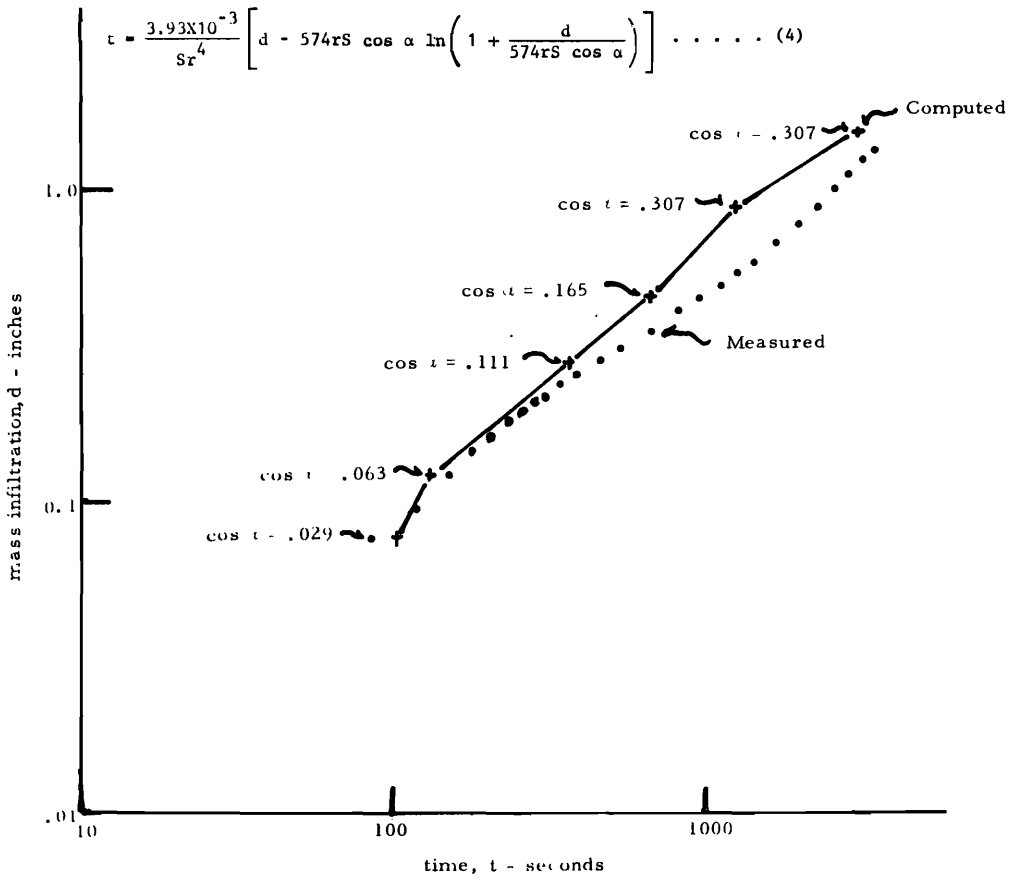


Figure 6. A comparison between the log mass infiltration-log time relations for Comoro Gr SL soil as observed in the field and as computed from soil properties using a modified Green and Ampt Equation (4).

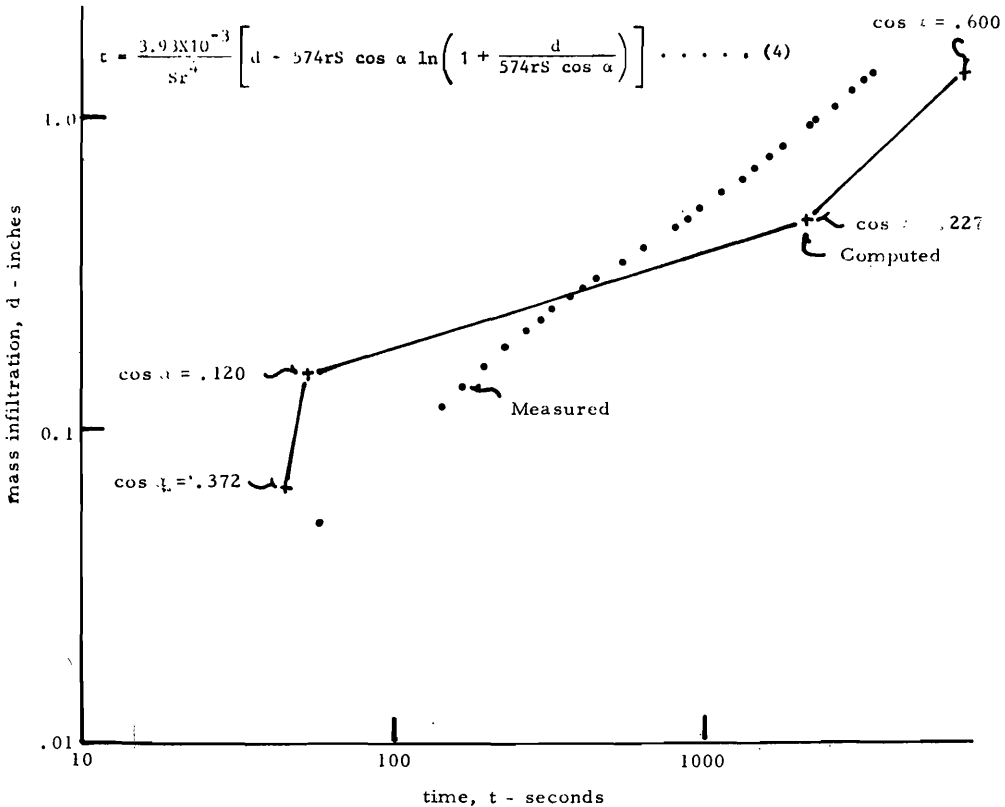


Figure 7. A comparison between the log infiltration-log mass time relations for Whitehouse Gr SL No. 2 as observed in the field and as computed from the soil properties using a modified Green and Ampt Equation (4).

$$\frac{d^2}{t} = 13.31 r^3 S \cos \alpha + 9.24 d r^4 \dots \dots \dots (7)$$

and

$$t = \frac{d^2}{85.90 r^3 S \cos \alpha + 23.47 d r^4} \dots \dots \dots (8)$$

If the observed log mass infiltration-log time relationships for the soils in Table 1 are compared to the curves computed using Equation (8), the curves shown in Figures 8, 9, 10, and 11 are obtained. These computed points appear to adequately represent infiltration on the soils tested.

**Fok and Hansen (1965)**

Fok and Hansen applied the Darcy equation and continuity equation to obtain an equation for the accumulative-infiltration time relationship. Their equation may be termed a combination capillary and Darcian type equation, and may be expressed as

$$\frac{d}{nsh_T} = \ln \left( 1 + \frac{d}{nsh_T} \right) = \frac{Kt}{nsh_T} \dots \dots (9)$$

in which

- d = accumulative depth of infiltration = Q
- n = porosity
- s = net increment of the degree of saturation or the degree of saturation after wetting minus the degree of saturation before wetting as a fraction of the total porosity so S = ns
- h<sub>T</sub> = the head loss in the transmission zone extrapolated to the wetting front h<sub>T</sub> = h<sub>o</sub> + h<sub>c</sub> - h<sub>w</sub> with
  - h<sub>o</sub> = depth of water on surface
  - h<sub>c</sub> = capillary potential head
  - h<sub>w</sub> = pressure potential loss in the wetting zone
- K = hydraulic conductivity of the transmission zone
- ln = base of natural logarithms

If the terms given in Equation (9) are converted to the terms used earlier in this paper, the Fok and Hansen equation becomes

$$t = \frac{nS}{K} \left( \frac{90.5d}{S} + \frac{130.5 \cos \alpha}{r} \right)$$

$$\left[ \frac{d}{nS \left( \frac{90.5d}{S} + \frac{130.5 \cos \alpha}{r} \right)} - \ln \left( 1 + \frac{d}{nS \left( \frac{90.5d}{S} + \frac{130.5 \cos \alpha}{r} \right)} \right) \right] \dots (10)$$

with t in seconds and d in inches. Using the soils and soil properties tabulated in Table 1, the relations shown in Figures 12, 13, 14, and 15 are derived. The fit is apparently reasonable but less satisfactory than the other equations on these soils. It must be remembered, however, that this equation was intended to apply to vertically homogeneous soils and the four soils here are not only mechanically stratified, but the carbon content of the surface decreases almost exponentially with depth thus affecting the wettability and a single value represents K.

**Darcian Type Equations**

The processes taking place during infiltration may logically be divided into two groups, namely those going on before flooding of the soil surface takes place and those going on after flooding of the surface. These may be simply designated sprinkling and flooded infiltration respectively.

**Assumptions**

The following assumptions were made in developing the Darcian type equations dealing with infiltration into a vertical soil column:

1. The soil is a semi-infinite, homogeneous, isotropic body whose bulk density is uniform and remains so during watering.
2. One dimensional flow occurs in the system.
3. The initial moisture content is uniform throughout the profile.
4. The soil air is a continuous phase essentially at atmospheric pressure.
5. The water application rate is constant throughout watering and at a rate high enough to eventually cause flooding.
6. The kinetic energy of the falling rain drops is sufficiently small that surface disturbance of the soil is negligible.

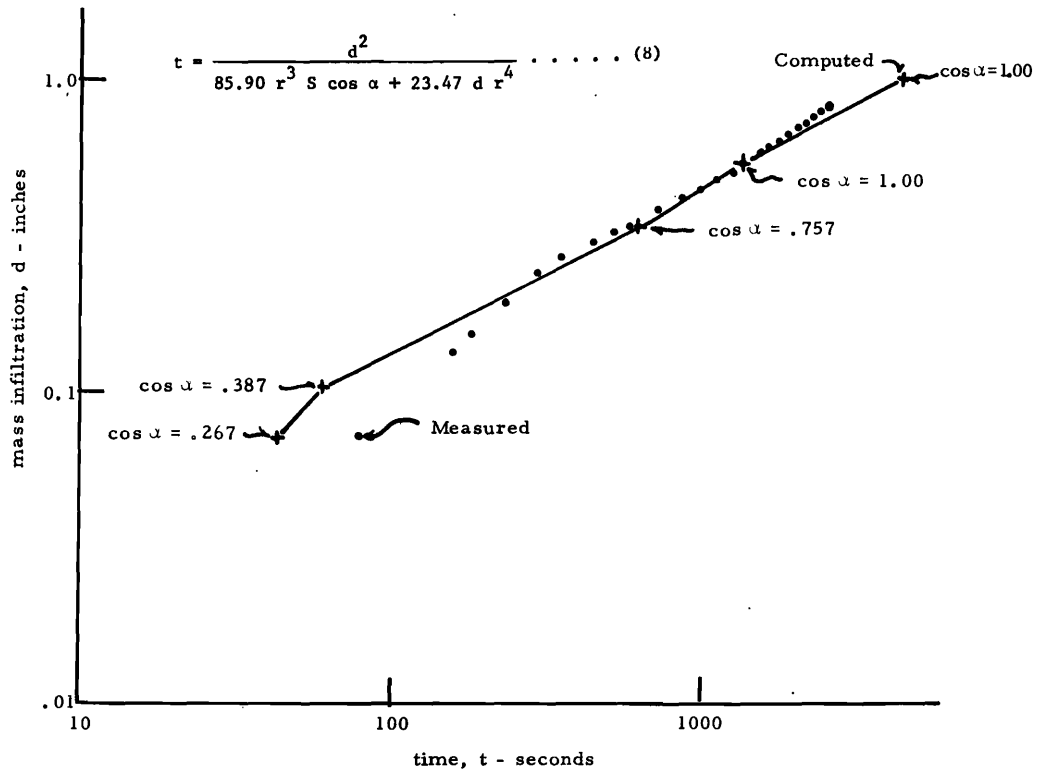


Figure 8. A comparison between the log mass infiltration-log time relationships for Whitehouse Gr SL No. 1 soil as observed in the field and as computed from soil properties by Equation (8).

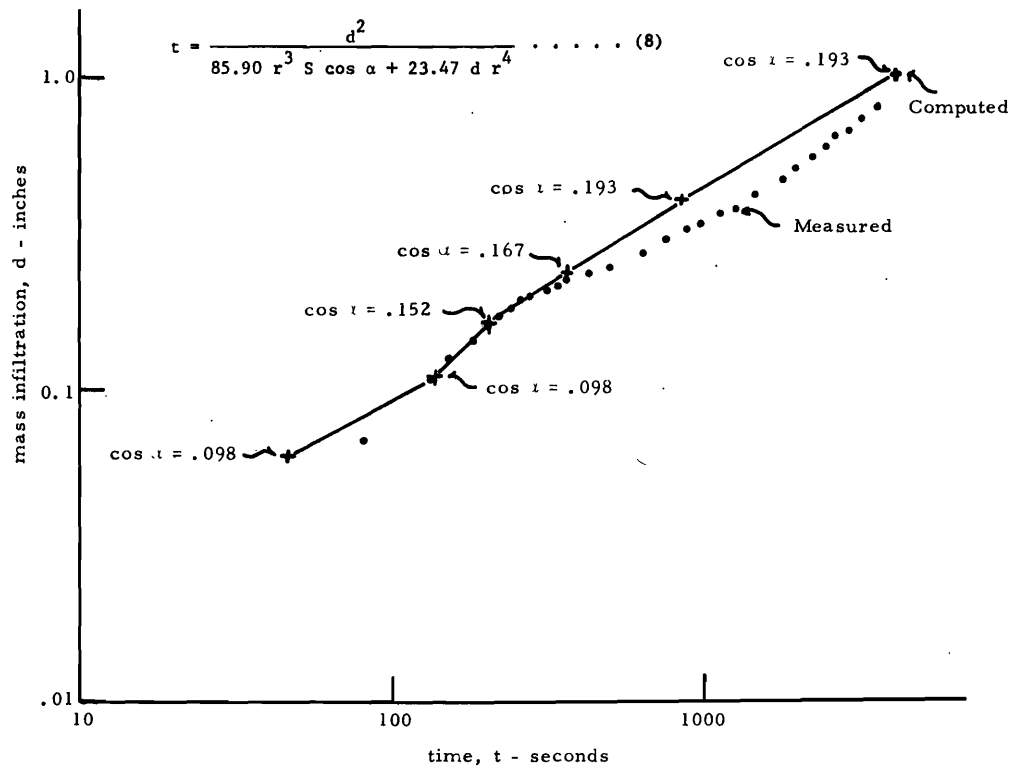


Figure 9. A comparison between the log mass infiltration-log time relations for Sonoita Gr SL soil as observed in the field and as computed from soil properties by Equation (8).

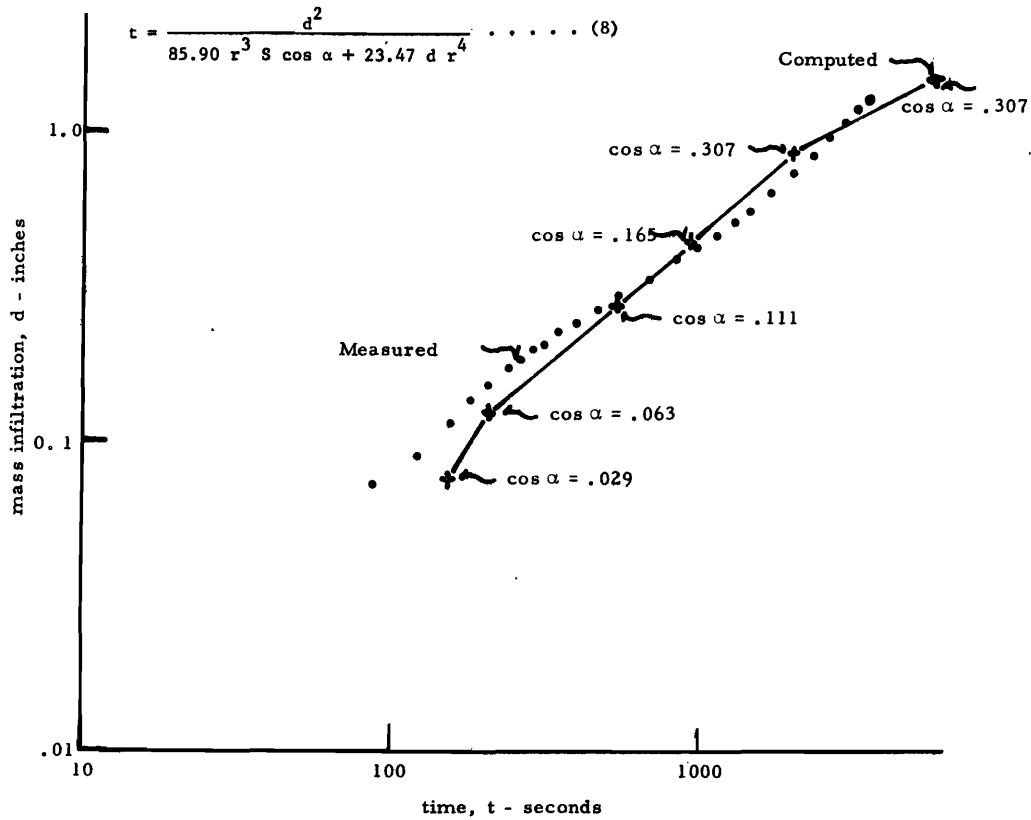


Figure 10. A comparison between the log mass infiltration-log time relations for Comoro Gr SL soil as observed in the field and as computed from soil properties by Equation (8).

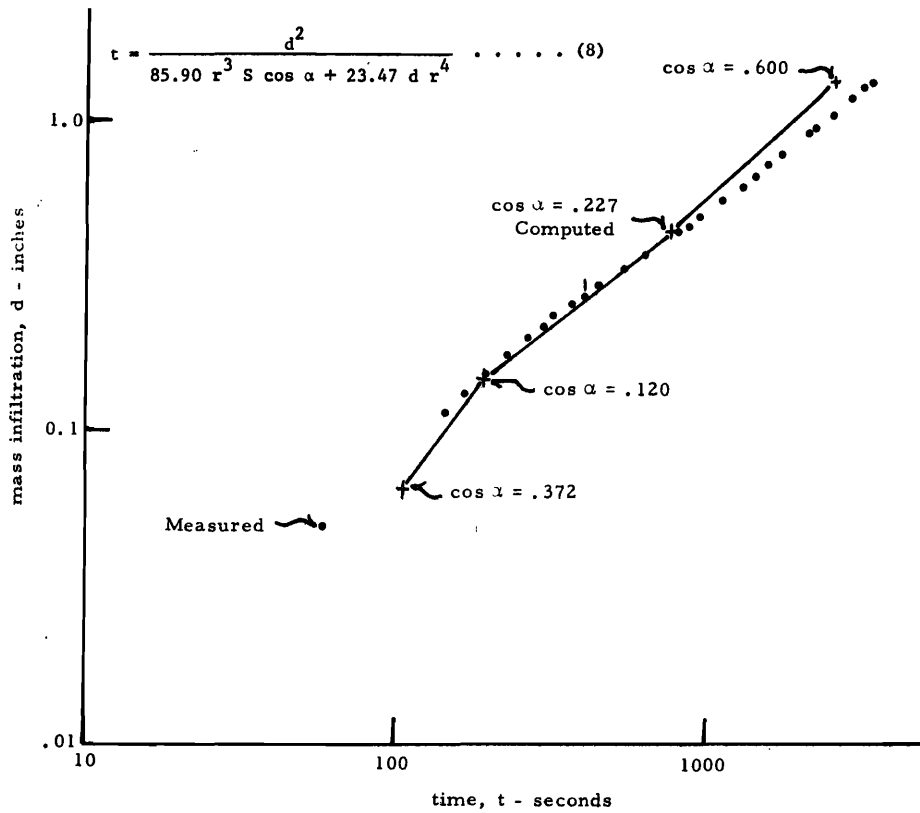


Figure 11. A comparison between the log mass infiltration-log time relations for Whitehouse Gr SL No. 2 as observed in the field and as computed from Equation (8).

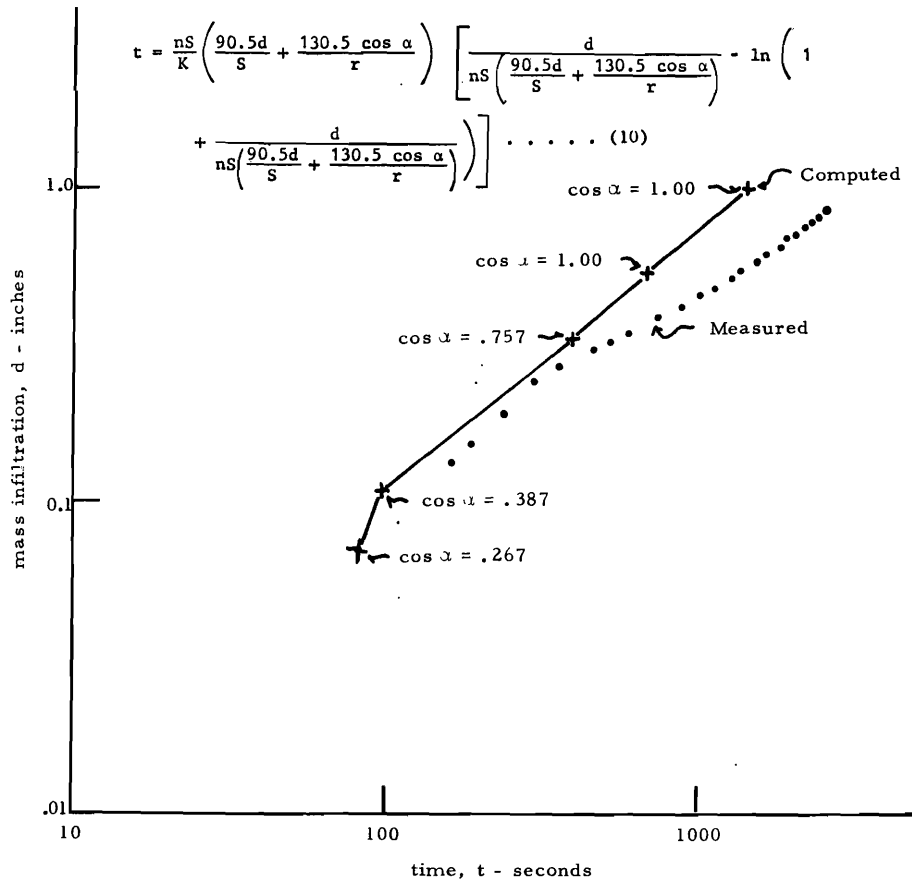


Figure 12. A comparison between the log mass infiltration-log time relationships for Whitehouse Gr SL soil No. 1 as observed in the field and as computed from soil properties by the Fok and Hansen Equation (10).

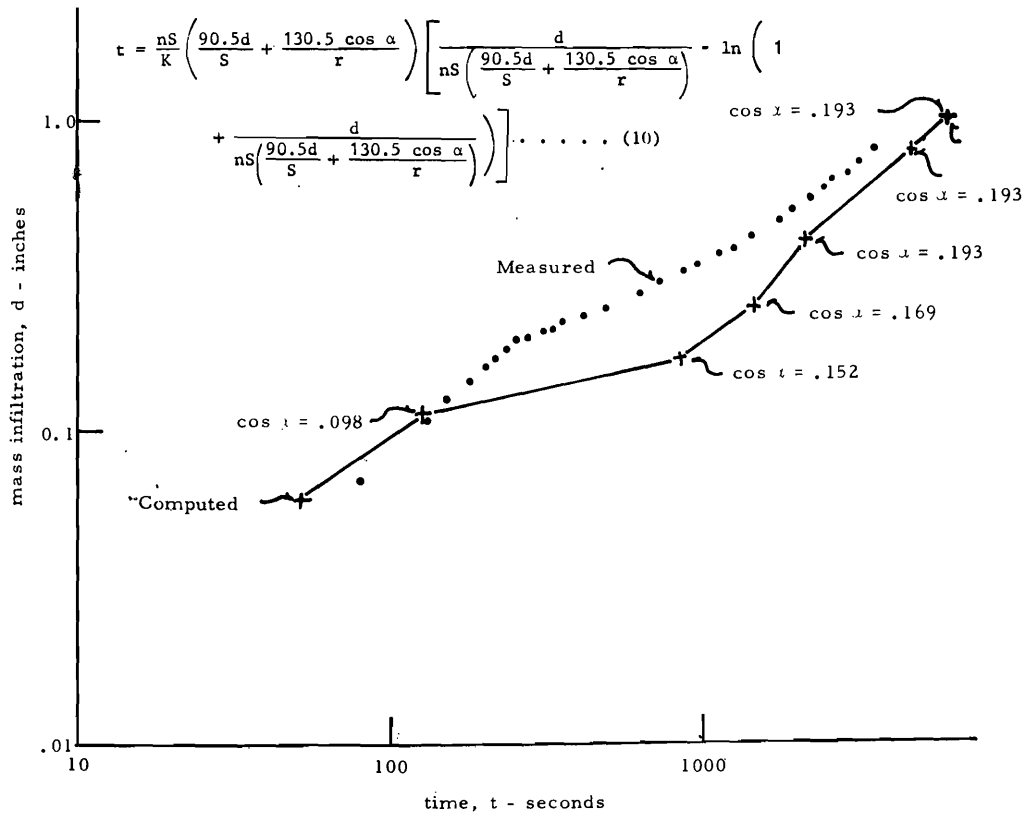


Figure 13. A comparison between the log mass infiltration-log time relationships on Sonoita Gr SL soil as observed in the field and as computed by Equation (10).

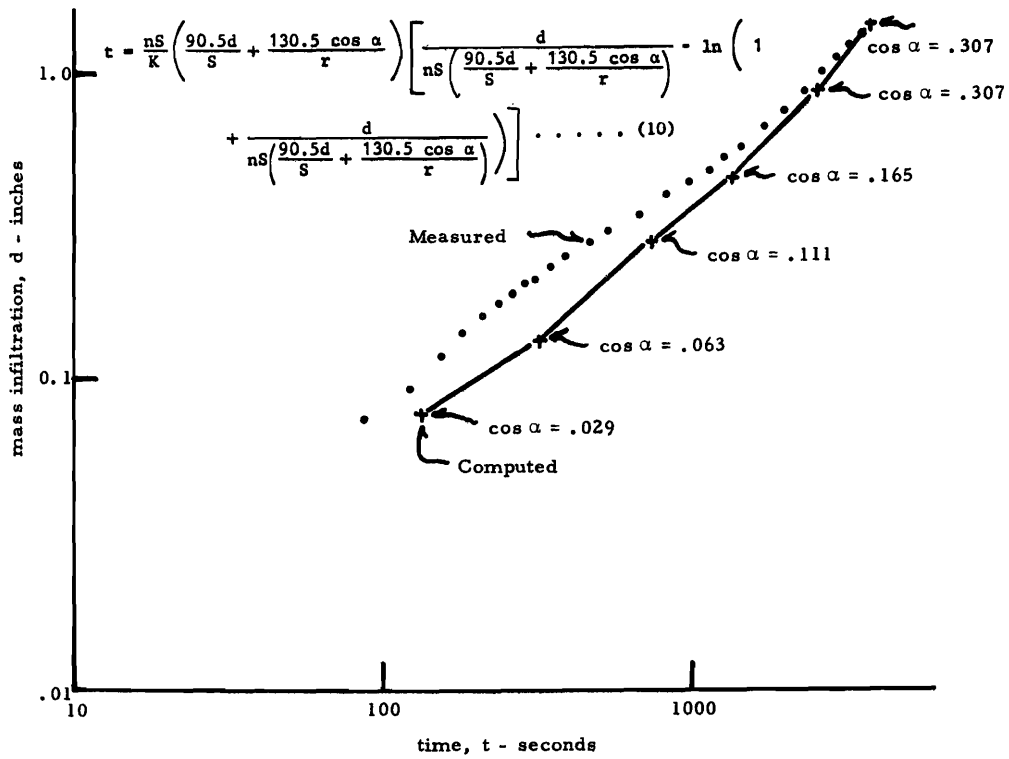


Figure 14. A comparison between the log mass infiltration-log time relationships of Comoro Gr SL soil as observed in the field and as computed from soil properties by Equation (10).

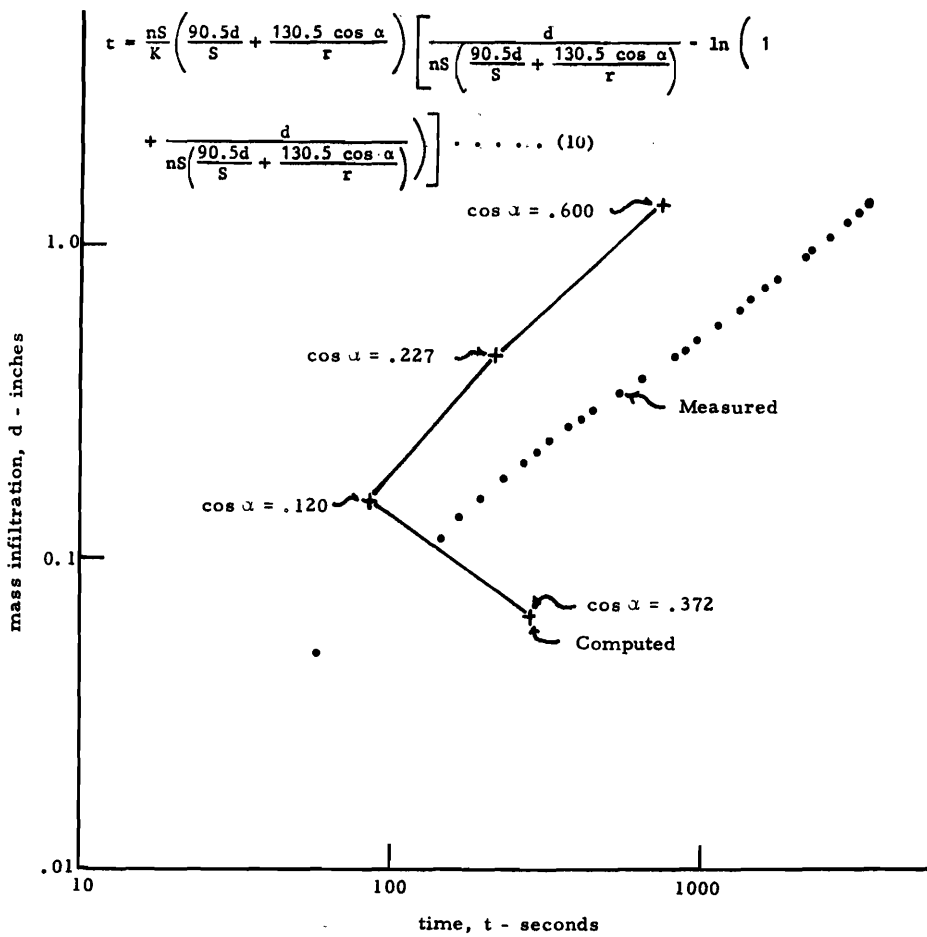


Figure 15. A comparison between the log mass infiltration-log time relationships for Whitehouse Gr SL No. 2 soil as observed in the field and as computed from soil properties by Equation (10).







```

226*      SUMA=0.
227*      DO 131 I=2,K
228*      SUM1=W(I)+SUM1
229*      SUM2=Y(I)+SUM2
230*      IF (ABS(SUM1-SUM2)-ABS(SUM3)) 131,131,130
231* 130    SUM3=SUM1-SUM2
232* 131    CONTINUE
233*      IF (ABS(SUM3)-ABS(CONO))134,134,132
234* 132    IF (DELT-DETT) 134,134,133
235* 133    DELT=DELT*0.5
236*      GO TO 38
237* 134    WFRD=(B(1)+((H(1)-H(2)+G(1)-G(2))/2.0+GRAVY))/DELT
238*      CWF=(SUM1-PIT)*DELT
239*      WFRDD=(SUM1-SUM2)*DELT/DELT
240*      WFRU=(B(K)+((H(K)-H(KK)+G(K)-G(KK))/2.0+GRAVY))/DELT
241*      CUMS=WFRD*DELT+CUMS
242*      CUMB=WFRU*DELT+CUMB
243*      CUMM=WFRDD*DELT+CUMM
244*      CWF LX=(SUM1-SUM2)*DELT
245* 700    CONTINUE
246* 706    IF (EOR-D.0)136,136,135
247* 135    RUNOF=(EOR-WFRD)*DELT+RUNOF
248* 136    TIME=TIME+DELT
249*      IF (LL-MM) 138,137,137
250* 137    CALL PLOT (KK,WATH,W,DD)
251*      WRITE (6,166) (H(I),I=1,KK)
252*      LL=0
253*      WRITE (6,184)
254* 138    WRITE (6,166) TIME,CWF,EOR,DELT,RUNOF,WFRU
255*      IF (SUM3-D.0) 139,141,139
256* 139    TW=ABS(CONO*DELT/SUM3)
257*      IF (TW-CTM) 152,140,140
258* 140    IF (TW-D.1*DETT) 141,142,142
259* 141    TW=DETT*0.1
260*      GO TO 144
261* 142    IF (TW-100.0*DETT) 144,144,143
262* 143    TW=100.0*DETT
263* 144    DELT=TW
264* C---TEST TO SEE IF EVAP OR RAIN INTENSITY (EOR) HAS CHANGED
265*      I=1
266* 145    IF (TIME-V(I+1)) 148,147,146
267* 146    I=I+2
268*      GO TO 145
269* 147    CALL PLOT (KK,WATH,W,DD)
270*      WRITE (6,166) (H(I),I=1,KK)
271*      WRITE (6,166) TIME,CWF,EOR,DELT,RUNOF,WFRU
272*      DELT=DETT
273*      EOR=V(I+2)
274*      W(1)=W(2)
275*      H(1)=H(2)
276*      GO TO 151
277* 148    IF (TIME+DELT-V(I+1)) 150,150,149
278* 149    DELT=V(I+1)-TIME
279*      IF (DELT-CTM) 147,147,150
280* 150    EOR=V(I)
281* 151    LL=LL+1
282*      IF (TIME-CUMT) 153,152,152
283* 152    IF (ML-LMM) 162,162,1

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```

284* 153 Y (1)=(W(1)+Y(1))*0.5
285*      J=(Y(1)-T(1))/DELW+1.0
286*      BR=(Y(1)-T(J))/DELW
287* 155 G (1)=(P(J+1)-P(J))*RB*(J)
288* 156 DO 161 I=2,KK
289*      TW=(W(I)-Y(I))+W(I)
290*      IF (TW-WATH) 157,157,159
291* 157 IF (TW-WATL) 158,160,160
292* 158 TW=WATL
293*      GO TO 160
294* 159 TW=WATH
295* 160 Y (I)=W (I)
296*      W (I)=TW
297*      G (I)=H (I)
298* 161 CONTINUE
299*      KCH=1
300*      NIT=0
301*      GO TO 16
302* 162 STOP
303* C---
304* 105  FORMAT(7F10.4)
305* 500  FORMAT(E8.2)
306* 163  FORMAT(20I3)
307* 164  FORMAT(20I3)
308* 165  FORMAT(10E8.2)
309* 166  FORMAT(10E12.4)
310* 167  FORMAT(80I1)
311* 168  FORMAT(2X,80I1)
312* 169  FORMAT(11H K MM IFR)
313* 171  FORMAT(43H WFTING PRESURE STARTING WITH LOWEST VALUE)
314* 172  FORMAT(53H HDRY HWET WATL WATH CR)
315* 173  FORMAT(41H CONDUCTIVITY STARTING WITH LOWEST VALUE)
316* 179  FORMAT(93H FLUX1 TIME 1 FLUX 2 TIME 2 FLUX
317* 1 3 TIME 3 FLUX 4 TIME 4)
318* 180  FORMAT(66H DELX NETT GRAVY CONG DELW)
319* 1 TIME)
320* 181  FORMAT(54H TT CUMT TAA CCNSH CTH)
321* 183  FORMAT(45H DIFFUSIVITY DATA--SUMMATION OF D TIMES DELW)
322* 184  FORMAT(68H TIME CWF EOR DELT SUN)
323* 10F WFRU)
324* 185  FORMAT(55H WLO WHI DPUT DTGP SINK)
325*      END

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**APPENDIX B**  
**Subroutine Program for Plotting**

```

1*      SUBROUTINE PLOT(N,WMAX,WVALUE,XVALUE)
2*      DIMENSION ALINE(101),WVALUE(99),XVALUE(99)
3*      DATA FILL,AXIS,CHAR/1H .1H.,1HM/
4*      WRITE(6,7) WMAX
5*      DO 1 J=1,101
6* 1     ALINE(J)=AXIS
7*      WRITE(6,8) (ALINE(K),K=1,101)
8*      DO 2 J=1,101
9* 2     ALINE(J)=FILL
10*     ALINE(1)=AXIS
11*     DO 4 L=1,N
12*       J=100.0*(WVALUE(L)/WMAX)+1.5
13*       IF (J.LT.1.OR.J.GT.101) GO TO 12
14* 10    ALINE(J)=CHAR
15* 12    WRITE(6,9) XVALUE(L),WVALUE(L),(ALINE(K),K=1,101)
16*     ALINE(J)=FILL
17*     55 ALINE(1)=AXIS
18* 4     CONTINUE
19*     DO 5 J=1,101
20* 5     ALINE(J)=AXIS
21*     WRITE(6,8) (ALINE(K),K=1,101)
22*     RETURN
23* 7     FORMAT(15H XVALUE WVALUE,5X,17H MAX WAT CONT IS,F7.4)
24* 8     FORMAT(31X,101A1)

25* 9     FORMAT(1H ,F6.1,F9.4,7H           ,101A1)
26*     END

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