

Magnetic Attitude Control for Small Satellites with Orbit-Independent Missions and Modest Pointing Constraints

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ABSTRACT

As the need for small satellite missions increases, the practice of launching multiple satellites from a single launch vehicle is also likely to increase. Small satellite missions are often flexible enough that a variety of orbits will be satisfactory, which increases the possibility of coordinated, multi-satellite launches; unfortunately, various satellite subsystems may impose restrictions on the possible orbit parameters. Since magnetic attitude control algorithms are typically tuned to a particular orbit, the performance of the control system is often adversely affected if the selected orbit is not particularly close to the orbit parameters for which the system was originally designed.

This paper describes the expansion of a linear quadratic regulator routine. The routine has been adapted using a magnetic-inclination-dependent gain scheduling technique that allows a single algorithm to be used in a wide range of orbits. The system presented has been shown to be effective over a range of inclinations and altitudes. The inclusion of internal momentum bias is not necessary; but it does significantly improve the performance.

INTRODUCTION

Magnetic attitude control—attitude control through the use of magnetic torque coils or torque rods—has been demonstrated to be an economical, dependable and effective form of attitude control for satellites with modest pointing constraints. The most significant restriction of magnetic attitude control is the fact that the resulting torques are, by definition, orthogonal to the magnetic field vector of the earth local to the satellite. A variety of algorithms for calculating the desired control torques have been proposed. Most have the potential to be effective over a range of orbits; however, the control system typically must be tuned to a particular set of orbit parameters and is especially sensitive to changes in the inclination.

The work was motivated by the USU TOROID mission. The TOROID satellite is a micro-satellite designed to take measurements of the low-latitude ionosphere in the hopes of obtaining a better understanding of ionospheric scintillations and eventually being able to predict possible outages of GPS and other satellite-to-ground or ground-to-satellite communications.

Since the area of interest for the TOROID satellite measurements will be near the equator, possible orbit inclinations range from 47° (assuming a ground station in Logan, Utah) to 90° and possible altitudes range from 300 km to 1000 km. In order to increase the possibility of a coordinated launch, the attitude control system

must also be as flexible as possible. It is assumed that all subsystems of the satellite must be completed before the orbit parameters are known.

Besides flexibility in orbit parameters, the attitude control system must also be shown to be effective with or without internal momentum bias. This is necessary to accommodate communication both before and after the initialization of a rotating science instrument.

SATELLITE ATTITUDE DYNAMICS

The attitude dynamics of a spacecraft can be described using Euler's equation for rotational dynamics¹

$$\mathbf{T} = \dot{\mathbf{h}}_i = \dot{\mathbf{h}} + \boldsymbol{\omega}_{sc} \times \mathbf{h} \quad (1)$$

and the kinematic equation for rotational motion, which can be written in quaternion form as

$$\dot{\mathbf{q}} = \frac{1}{\gamma} \begin{bmatrix} \cdot & \omega_{sc,z} & -\omega_{sc,y} & \omega_{sc,x} \\ -\omega_{sc,z} & \cdot & \omega_{sc,x} & \omega_{sc,y} \\ \omega_{sc,y} & -\omega_{sc,x} & \cdot & \omega_{sc,z} \\ -\omega_{sc,x} & -\omega_{sc,y} & -\omega_{sc,z} & \cdot \end{bmatrix} \mathbf{q} \quad (2)$$

Although these equations are useful for the simulation of the attitude of the spacecraft, linearization of the system dynamics and kinematics is useful for control

system design. The resulting control system will then be verified using the nonlinear equations.

If the total momentum is separated into internal and body momentum; then solving Euler's equation for the angular acceleration yields

$$\dot{\boldsymbol{\omega}}_{sc} = \mathbf{I}^{-1} \left[\mathbf{T} - \dot{\mathbf{h}}_{\theta} - \boldsymbol{\omega}_{sc} \times (\mathbf{I}\boldsymbol{\omega}_{sc} + \mathbf{h}_{\theta}) \right] \quad (3)$$

where \mathbf{I} is the inertia tensor of the satellite and \mathbf{h}_{θ} is the momentum of internal devices relative to the body of the spacecraft. For a spacecraft with a diagonal inertia tensor, this can be simplified to

$$\begin{aligned} \dot{\omega}_{sc,x} &= \frac{1}{I_{xx}} \left[T_x - \dot{h}_{\theta,x} - \left(\omega_y I_{zz} - \omega_z I_{yy} \right) - \left(\omega_y h_{\theta,z} - \omega_z h_{\theta,y} \right) \right] \\ \dot{\omega}_{sc,y} &= \frac{1}{I_{yy}} \left[T_y - \dot{h}_{\theta,y} - \left(\omega_z I_{xx} - \omega_x I_{zz} \right) - \left(\omega_z h_{\theta,x} - \omega_x h_{\theta,z} \right) \right] \\ \dot{\omega}_{sc,z} &= \frac{1}{I_{zz}} \left[T_z - \dot{h}_{\theta,z} - \left(\omega_x I_{yy} - \omega_y I_{xx} \right) - \left(\omega_x h_{\theta,y} - \omega_y h_{\theta,x} \right) \right] \end{aligned} \quad (4)$$

For the general case, these equations are still nonlinear; however, for the case of a nadir-pointing satellite in a near-circular orbit, it can be assumed that $\omega_x \omega_z \approx 0$, $\omega_x \omega_y \approx \omega_x \omega_0$, and $\omega_z \omega_y \approx \omega_z \omega_0$, where ω_0 is the orbital rate for a circular orbit. With these assumptions, the dynamic equations of motion simplify to

$$\begin{aligned} \dot{\omega}_{sc,x} &= \frac{1}{I_{xx}} \left[T_x - \dot{h}_{\theta,x} + \omega_z \left[\left(I_{yy} - I_{zz} \right) + h_{\theta,y} \right] - \omega_y h_{\theta,z} \right] \\ \dot{\omega}_{sc,y} &= \frac{1}{I_{yy}} \left[T_y - \dot{h}_{\theta,y} - \omega_z h_{\theta,x} + \omega_x h_{\theta,z} \right] \\ \dot{\omega}_{sc,z} &= \frac{1}{I_{zz}} \left[T_z - \dot{h}_{\theta,z} + \omega_x \left[\left(I_{xx} - I_{yy} \right) - h_{\theta,y} \right] + \omega_y h_{\theta,x} \right] \end{aligned} \quad (5)$$

For the TOROID satellite, the only source of internal momentum is a science instrument that rotates about the y-axis. The equations of angular motion with and without momentum bias about the y-axis are

$$\begin{aligned} \dot{\omega}_{sc,x} &= \frac{1}{I_{xx}} \left\{ T_x + \omega_z \left[\omega \cdot \left(I_{yy} - I_{zz} \right) + h_{\omega,y} \right] \right\} \\ \dot{\omega}_{sc,y} &= \frac{1}{I_{yy}} \left[T_y - \dot{h}_{\omega,y} \right] \\ \dot{\omega}_{sc,z} &= \frac{1}{I_{zz}} \left\{ T_z + \omega_x \left[\omega \cdot \left(I_{xx} - I_{yy} \right) - h_{\omega,y} \right] \right\} \end{aligned} \quad (6)$$

and

$$\begin{aligned} \dot{\omega}_{sc,x} &= \frac{1}{I_{xx}} \left\{ T_x + \omega_z \omega \cdot \left(I_{yy} - I_{zz} \right) \right\} \\ \dot{\omega}_{sc,y} &= \frac{T_y}{I_{yy}} \\ \dot{\omega}_{sc,z} &= \frac{1}{I_{zz}} \left\{ T_z + \omega_x \omega \cdot \left(I_{xx} - I_{yy} \right) \right\} \end{aligned} \quad (7)$$

For the kinematic equation, the quaternion is written as

$$\mathbf{q} = \begin{bmatrix} \bar{\mathbf{q}} \\ q_{\epsilon} \end{bmatrix} \quad (8)$$

where

$$\bar{\mathbf{q}} = \hat{e} \sin\left(\frac{\theta}{\sqrt{v}}\right), \quad q_{\epsilon} = \cos\left(\frac{\theta}{\sqrt{v}}\right) \quad (9)$$

and \hat{e} is a unit vector in the direction of the axis about which the spacecraft (or vector, or object) is rotated and θ is the angle of rotation. In this form, an error quaternion can be defined as

$$\delta \mathbf{q} = \hat{e} \sin\left(\frac{\theta}{\sqrt{v}}\right) q_{\epsilon} \quad (10)$$

and the kinematic equation simplifies to

$$\delta \dot{\mathbf{q}} = \frac{1}{\sqrt{v}} \mathbf{I} \boldsymbol{\omega} \quad (11)$$

For small angles, the error quaternion is an approximation of the Euler angles.

ENVIRONMENTAL DISTURBANCE TORQUES

There are several environmental effects that may cause small disturbance torques on the satellite. Some of the most commonly considered torques include: gravity gradient torques, aerodynamic torques, solar radiation torques and magnetic residuals torques.

Gravity Gradient Torques

The effects of the gravity gradient on the spacecraft can be written in the following form:

$$\mathbf{T}_g = \frac{\sqrt{\mu}}{\sqrt{r^3}} \begin{bmatrix} \left(I_{zz} - I_{yy} \right) \sin(\nu\phi_e) \cos^{\nu}(\theta_e) \\ \left(I_{zz} - I_{xx} \right) \sin(\nu\theta_e) \cos^{\nu}(\phi_e) \\ \left(I_{xx} - I_{yy} \right) \sin(\nu\theta_e) \cos^{\nu}(\phi_e) \end{bmatrix} \quad (12)$$

μ is the gravitational constant, r is the distance from the center of the earth to the satellite, and ϕ_e and θ_e are the x and y Euler angles between the local-vertical-local-horizon axis and the body axis of the spacecraft.¹

The TOROID satellite structure is designed such that the gravity gradient torque will be favorable for the desired nadir-pointing orientation, but it will be significantly smaller than the other external torques.

Aerodynamic Torques

For low altitudes the aerodynamic torque is likely to be the largest disturbance torque acting on the spacecraft. A simple equation for estimating the effects of the aerodynamic disturbance torque is

$$\bar{\mathbf{T}}_{d,a} = \frac{1}{2} \rho C_d A_a |\mathbf{V}|^2 (\mathbf{C}_{p,a} - \mathbf{C}_m) \quad (13)$$

where A_a is the reference surface area, V is the velocity of the spacecraft, $C_{p,a}$ is the center of aerodynamic pressure, and C_m is the center of mass. An estimate of the density of the atmosphere, ρ , can be found using the data in Figure 1.

Although the density of the air taken from the US Standard atmosphere shown in Figure 1 is useful for mission planning and simulation, the actual density of the atmosphere at high altitudes is highly dependent on the activity of the sun, and can change by orders of magnitude over a period of several years.

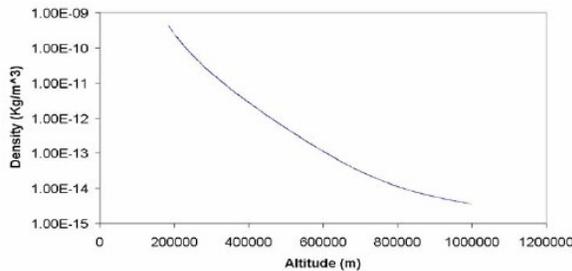


Figure 1: Change in atmospheric density with altitude. Data taken from the US Standard Atmosphere, 1976.²

The drag coefficient, C_d , varies slightly with the dimensions of the satellite. For a near-rectangular satellite, Table 1 can be used.

Because the atmospheric density will change with time as the satellite moves into and out of the sunlight, the effects of the diurnal bulge of the Earth's atmosphere on the aerodynamic disturbance torque can be modeled as

$$\mathbf{T}_{d,a} = \bar{\mathbf{T}}_{d,a} \gamma^{\cos(\theta, t)} \quad (14)$$

where γ is 1.5.⁴

Table 1: Typical drag coefficients³

l/D	C_d
≤0.1	1.9
0.5	2.5
0.65	2.9
1.0	2.2
2.0	1.6
3.0	1.3

Solar Radiation Torques

The strength of the solar radiation torque is highly dependent on the surface properties of the satellite; the more reflective the surface, the stronger the torque. A simple equation for approximating the solar radiation torque is

$$\mathbf{T}_{d,s} = \frac{F_s}{c} A_s (1 - q_s) \cos(i_s) (\mathbf{C}_{p,s} - \mathbf{C}_g) \quad (15)$$

where F_s is the solar constant (1367 W/m²), c is the speed of light in a vacuum ($c \approx 3E8$ m/s), q_s is the reflectance factor (a constant between 0 and 1), $C_{p,s}$ is the center of solar pressure, and i_s is the angle of incidence of the sun.⁵

Although this torque will change in direction and magnitude as the lighting conditions of the satellite change, the application of the worst case scenario torque to all three axes is sufficient to test the functionality of the controller.

Magnetic Residuals Torques

Although significant effort should be applied to reducing the interactions of internal satellite components with the magnetic field of the earth, there inevitably will exist a residual magnetic field around the satellite that can be approximated as a dipole magnet. The strength of the resulting torque is

$$\mathbf{T}_{d,s} = D_r B \quad (16)$$

where D_r is the residual magnetic dipole of the satellite (in Am²) and B is the magnitude of the magnetic field of the earth, which can be approximated by

$$B \approx \frac{\sqrt{M}}{R^3} \quad (17)$$

where R is the distance from the satellite to the center of the earth and M is the magnetic moment of the earth. The value for M depends on the location but is largest at the magnetic poles where it is 7.96E15 tesla*m³.⁵

DETUMBLE

Because the magnetic field of the earth—from the perspective of the satellite—varies slowly over time, the angular rates of the spacecraft are approximately equal to the rate of change of the magnetic field vector. A simple algorithm for reducing the rates of the spacecraft is the well-known $\mathbf{B} \cdot \dot{\mathbf{B}}$ algorithm, which is

$$\mathbf{T} = \mathbf{K} \dot{\mathbf{B}} \quad (18)$$

where \mathbf{T} is the desired control torque generated by the torque coils, $\dot{\mathbf{B}}$ is the rate of change of the measured magnetic field vector, and \mathbf{K} is a constant chosen to fit the particular satellite, actuators, and orbit parameters.

Although the Bdot algorithm itself is dependable, it has been demonstrated that, if the sample rate is not high enough, the estimate of the magnetic field rates could be significantly different from the actual rates (see Figure 2).

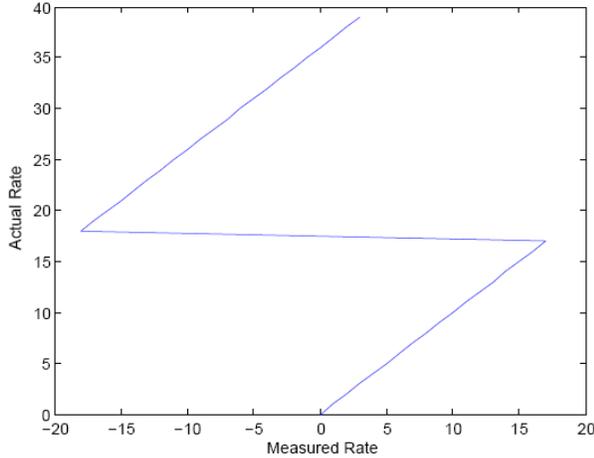


Figure 2: Measured vs actual rates using magnetometer data (measurements taken every 10 seconds)

On November 28, 2002, the first Algerian microsatellite (Alsat-1) was launched from the cosmodrome of Plesetsk in Russia. Immediately after separation from the final stage of the Kosmos launcher, the only attitude estimation device was a 3-axis magnetometer taking measurements every 10 seconds. This data was used to estimate the initial rates so that the detumble of the satellite could be accomplished using the Bdot algorithm. Unfortunately, the initial rate of the satellite was about 30°/s and the rate estimate was inaccurate. The result was that, when the detumble maneuvers were performed, the satellite rotation rate was actually increased to about 36°/s by the maneuver.⁶

The detumble of the Alsat-1 was successfully resolved by timing the deployment of a large gravity gradient boom. Since the TOROID satellite may find itself in a very similar detumble scenario, and there is no gravity gradient boom to deploy, it is essential that the magnetic field data be taken more frequently during the detumble stage. Once the initial spacecraft rates have been reduced, a slower sample rate will be sufficient.

NOMINAL ATTITUDE CONTROL

The Mapping Function

In order to ensure that the torques calculated by the controller are perpendicular to the local magnetic field, a mapping function will be used. If the magnetic control torque is written

$$\mathbf{T}_c = \mathbf{I}^{-1} \mathbf{g}_m \quad (19)$$

then

$$\mathbf{g}_m = \mathbf{M} \times \mathbf{B} \quad (20)$$

and the following mapping function is suggested by Wisniewski and used again by Makovec:^{7,8,9}

$$\mathbf{M} \mapsto \tilde{\mathbf{M}} : \tilde{\mathbf{M}} = \frac{\mathbf{M} \times \mathbf{B}}{\|\mathbf{B}\|}. \quad (21)$$

This mapping function ensures that $\tilde{\mathbf{M}}$ is perpendicular to the local magnetic field, and thus the desired control torque will be a feasible control torque, given by

$$\mathbf{T} = \mathbf{I}^{-1} \mathbf{g}_m = \frac{\mathbf{I}^{-1}}{\|\mathbf{B}\|} \mathbf{B} \times \mathbf{B} \times \tilde{\mathbf{M}}. \quad (22)$$

State Space Representation

The linearized dynamics of the attitude control of a nadir-pointing satellite can be written

$$\dot{\mathbf{X}} = \mathbf{F}\mathbf{X} + \mathbf{G}(t)u + \mathbf{I}^{-1}(\mathbf{T}_d - \dot{\mathbf{H}}_w) \quad (23)$$

where \mathbf{X} is the state of the attitude (integral of the Euler angles, Euler angles, and Euler rates), and \mathbf{F} is

$$\mathbf{F} = \begin{bmatrix} \mathbf{O}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{O}_{3 \times 3} \\ \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} & \mathbf{F}_1 \end{bmatrix}. \quad (24)$$

The \mathbf{F}_1 matrix is the linearized dynamics shown earlier, and is

$$\mathbf{F}_1 = \begin{bmatrix} \mathbf{O}_{3 \times 3} & \frac{h_{w,z}}{I_{xx}} & \frac{1}{I_{xx}} [\omega_o (I_{yy} - I_{zz}) + h_{w,y}] \\ \frac{h_{w,z}}{I_{yy}} & \mathbf{O}_{3 \times 3} & \frac{h_{w,x}}{I_{yy}} \\ \frac{1}{I_{zz}} [\omega_o (I_{xx} - I_{yy}) + h_{w,y}] & \frac{h_{w,x}}{I_{zz}} & \mathbf{O}_{3 \times 3} \end{bmatrix} \quad (25)$$

and $\mathbf{G}(t)$ is the implementation of the mapping function

$$\mathbf{G}(t) = \begin{bmatrix} \mathbf{O}_{3 \times 3} \\ \mathbf{O}_{3 \times 3} \\ \mathbf{G}_1(t) \end{bmatrix} \quad (26)$$

where

$$\mathbf{G}_1(t) = \frac{\mathbf{I}^{-1}}{|\mathbf{B}|} \begin{bmatrix} -b_y^2 - b_z^2 & b_x b_y & b_x b_z \\ b_x b_y & -b_x^2 - b_z^2 & b_y b_z \\ b_x b_z & b_y b_z & -b_x^2 - b_y^2 \end{bmatrix}. \quad (27)$$

Linear Quadratic Regulation

It has been shown in various studies over the past twenty years that the use of linear quadratic (LQ) regulation can be effective for the stabilization of nadir-pointing, magnetically-actuated spacecraft.⁹⁻¹² Although LQ regulators using only proportional and derivative (PD) feedback have been shown to be sufficient for stability, it has been demonstrated that the integral control adds to the effectiveness of the system.¹¹ If the control effort is set equal to

$$\mathbf{M}(t) = -\mathbf{KX}(t) \quad (28)$$

then the control system is a time varying linear quadratic regulator system and the state space equation can be written

$$\dot{\mathbf{X}} = \mathbf{FX} + \mathbf{G}(t)\mathbf{M}(t) + \mathbf{T}_d = [\mathbf{F} - \mathbf{KG}(t)]\mathbf{X} + \mathbf{T}_d \quad (29)$$

and the ideal control torque, calculated onboard the satellite, is

$$\mathbf{T}_c = \mathbf{KG}(t)\mathbf{X}. \quad (30)$$

The resulting desired control torque will be mapped into the plane perpendicular to the local magnetic field vector.

Earth-Fixed Analysis

Since the gain scheduling will be dependent on the magnetic inclination (the inclination with respect to plane perpendicular to the magnetic north vector, see the Appendix), the controller gains themselves should be calculated and verified for a certain magnetic inclination, which changes over time as the Earth rotates.

In order to calculate the controller gains, the orbit is fixed to the earth such that the magnetic field will not change over time with respect to the orbit. This ‘‘Earth-fixed analysis’’ technique allows the performance to be evaluated for a certain magnetic inclination without the added confusion of changes in the magnetic field due to

the rotation of the Earth. The resulting gains must then be tested using a typical rotating earth analysis.

LQR Gain Calculation

Because LQ regulator gain calculation assumes a time-invariant system, the mapping function must be averaged over time in order to calculate the gain. For a controller tuned to a particular orbit, the average should be taken over a full day's worth of orbits (because the magnetic field will change throughout the day for the same position in space). When using the Earth-fixed analysis technique, only the magnetic field for a single orbit is necessary.

The gain matrix for an LQ regulator can be calculated using the equation

$$\mathbf{K} = -\mathbf{R}_r^{-1}\bar{\mathbf{G}}^T\mathbf{P} \quad (31)$$

where $\bar{\mathbf{G}}$ is the time average of the mapping function and \mathbf{P} comes from the steady-state solution to the Riccati equation

$$\dot{\mathbf{P}} + \mathbf{PF} + \mathbf{F}^T\mathbf{P} - \mathbf{P}\bar{\mathbf{G}}\mathbf{R}_r^{-1}\bar{\mathbf{G}}^T\mathbf{P} + \mathbf{Q} = 0. \quad (32)$$

The matrices \mathbf{R}_r and \mathbf{Q} are weighting matrices that are selected according to the desired response, the satellite inertia matrix, and the orbit parameters. Selecting these matrices properly is the most difficult challenge when using the LQ regulator.

Various techniques have been suggested for selecting the weighting matrices.⁹⁻¹¹ Although there is no analytical method for selecting optimal \mathbf{R}_r and \mathbf{Q} matrices, some general guidelines for selecting acceptable weighting matrices are described in later sections of this paper.

The Gain Scheduling Technique

In order to ensure a good response steady state response over a range of inclinations, controller gains were found at high and low magnetic inclinations. Earth-fixed analysis was used to verify the gain selection. Since the TOROID spacecraft will need to be in an orbit with an inclination of at least 47° (for sufficient groundstation communication time), the high and low inclination gains were calculated for magnetic inclinations of 47° and 90°.

The controller gain that is actually used at any given time is a simple linear interpolation between the two gains depending on the magnetic inclination.

In order to verify that the solution is effective over a range of magnetic inclinations, Earth-fixed analysis was used for magnetic inclinations at 5° intervals.

Finally, several simulations were run using the gain scheduling technique for a regular orbit (no longer using Earth-fixed analysis).

STABILITY AND ROBUSTNESS

Floquet Analysis

Since the system can be approximated as a periodic system it is useful to verify the stability of the system using Floquet's theorem.^{9,11,14}

Floquet's theorem states that, for a periodic system, if each of the eigenvalues of $\mathbf{F} - \mathbf{K}\mathbf{G}(t)$ have a magnitude of less than one over a full period (for all possible values of $\mathbf{G}(t)$), the system is stable. The easiest way to ensure that the Floquet criteria is met is to choose very large values for the \mathbf{R} matrix used in the Riccati equation. Unfortunately, very large values for \mathbf{R} will also result in a very slow response of the system.

Although Floquet's theory is useful for stability analysis, it does not guarantee a good response in the presence of disturbance torques. Thus, the Floquet analysis is a necessary but not sufficient condition for an acceptable response. High fidelity simulation is also necessary to ensure that the desired response can be accomplished in the presence of expected disturbances.

Robustness

This particular system can be expected to have a degree of robustness with respect to modeling errors. This is true because the system is a low-bandwidth time varying case and can be approximated by a time-invariant solution. Assuming that this approximation is true, the robustness properties possessed by full state feedback LQ regulators carry over to this particular system. The assumption is made more believable by the fact that the local magnetic field will be monitored continuously using a magnetometer.¹¹

WEIGHTING MATRIX SELECTION

As mentioned earlier, there is no analytical method for selecting optimal values for the \mathbf{Q} and \mathbf{R} weighting matrices used in the calculation of the controller gain; however, there are some general guidelines for selecting relative values. The process can be greatly simplified by assuming that both matrices are diagonal. Note that \mathbf{Q} is a 9x9 matrix and \mathbf{R} is a 3x3 matrix.

The \mathbf{Q} matrix is related to the desired response of the system. For the present configuration, the first three components along the diagonal of the \mathbf{Q} matrix are related to the integral response in the x, y, and z-axes of the satellite and have units of $1/(\text{rad-sec})^2$. The second three components are related to the proportional

response about the same three axes and have units of $1/\text{rad}^2$. The final three components are related to the derivative (or angular rate) response of the system and have units of sec/rad^2 .¹¹

The \mathbf{R} matrix is related to the desired control effort for the system. All three components have units of $1/(\text{amp-m}^2)^2$.¹¹

The suggested method for selecting the weighting matrices is the following

- All values for the off-diagonal terms of \mathbf{Q} and \mathbf{R} are zero
- Select the diagonal values for the \mathbf{Q} matrix according to the desired relative response, giving larger values to components where the desired response is more critical.
- The integral values of the \mathbf{Q} matrix (the first three values in the diagonal) should be several orders of magnitude smaller than the next six values.
- Give all three values of the \mathbf{R} matrix diagonal the same value. Increasing \mathbf{R} will do the following: decrease the control effort, decrease the steady state response, reduce the magnitude of the eigenvalues (for Floquet analysis), and increase the settling time.

For this type of system, it should be expected that the settling time will be very slow, which is unfortunate, but necessary to ensure that the other conditions (small eigenvalues for Floquet analysis, good steady state response, small control effort) are met.

TOROID Weighting Matrices: No Momentum Bias

The TOROID satellite is a small satellite with an inertia tensor that is approximately

$$\mathbf{I} = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 1.05 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}. \quad (33)$$

Given this configuration, it can be seen that gravity gradient will cause the z-axis to tend towards nadir pointing, which is a desirable response; however, since the magnitudes of the diagonal components are relatively close, the gravity gradient torque may not be sufficient to slow down the initial tumbling motion of the spacecraft and definitely will not be sufficient to stabilize the spacecraft to mission requirements.

In its initial configuration, the TOROID satellite will be 3-axis stabilized without internal momentum devices. The desired response is nadir pointing to within 10°, which is necessary for communication.

Weighting matrices have been selected for both high (90°) and low (39°) magnetic inclinations. The low magnetic inclination corresponds to the worst case magnetic inclination for a true inclination of approximately 47°.

Using the method described earlier, the first three values of the \mathbf{Q} matrix are related to the integral response and are selected to be 2e-3, 2e-2, and 2e-3 (for both high and low inclinations). Note that the integral response about the y axis is significantly larger, due to the fact that the magnetic field vector for low inclinations will be most closely aligned with the y-axis.

For low inclinations, the weighting matrices have been selected as

$$\mathbf{Q} = \text{diag}[2E-3 \quad 2E-2 \quad 2E-3 \quad 100 \quad 100 \quad 100 \quad 100 \quad 100 \quad 100] \quad , \quad (34)$$

$$\mathbf{R} = \text{diag}[4.34E6 \quad 4.34E6 \quad 4.34E6]$$

and the resulting controller gain, using the time average of $\mathbf{G}(t)$ over one orbit using Earth-fixed analysis and a magnetic inclination of 39°, is (showing only a few significant digits)

$$\mathbf{K} = \begin{bmatrix} -1.099E-6 & -1.099E-5 & -0.252E-5 & 0.00469 & -0.0184 \\ 6.335E-6 & 6.335E-5 & 0.202E-5 & 0.00087 & 0.1131 \\ -0.736E-6 & -0.736E-5 & 2.122E-5 & 0.00047 & -0.0056 \\ -0.013109 & 23.1636 & -9.9424 & -1.80361 \\ 0.004522 & 3.35880 & 92.8793 & 7.78148 \\ 0.039302 & -2.74780 & 2.36280 & 36.0317 \end{bmatrix} \quad (35)$$

The Floquet analysis using this controller gain over a full period yields eigenvalues with maximum values well within a unit circle.

For high inclinations, where a somewhat better response can be expected, the weighting matrices were selected to be

$$\mathbf{Q} = \text{diag}[2E-3 \quad 2E-2 \quad 2E-3 \quad 150 \quad 150 \quad 150 \quad 100 \quad 100 \quad 100] \quad . \quad (36)$$

$$\mathbf{R} = \text{diag}[1.45E6 \quad 1.45E6 \quad 1.45E6]$$

Again using the time average of $\mathbf{G}(t)$, this time over a period of one orbit using Earth-fixed analysis, the controller gain for high inclination orbits without

momentum bias is (again, showing only a few significant digits)

$$\mathbf{K} = \begin{bmatrix} -0.067E-5 & -0.067E-4 & -0.76E-5 & 9.93E-3 & -0.0062 \\ 1.105E-5 & 1.105E-4 & -0.58E-5 & 0.28E-3 & 0.1603 \\ 0.166E-5 & 0.166E-4 & 3.58E-5 & 2.15E-3 & 0.0088 \\ -0.01901 & 25.00438 & 0.36893 & -5.23756 \\ -0.01581 & 2.65131 & 106.9653 & -17.2423 \\ 0.06608 & -1.55533 & -7.6527 & 62.0432 \end{bmatrix} \quad (37)$$

The gain that will actually be used at any given time will be a linear interpolation between these two gains, depending on the actual magnetic inclination.

TOROID Weighting Matrices: With Momentum Bias

After initial attitude has been acquired using magnetic torquers, the ground station will establish communication with the TOROID satellite and run pre-science diagnostics. The main science instrument, shown in in Figure 3, is deployed and begins to rotate.

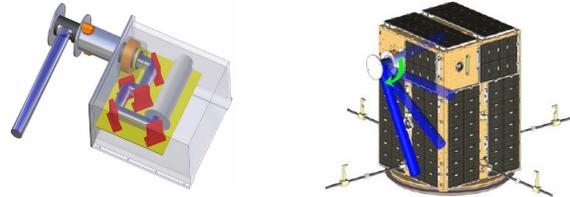


Figure 3: The TOROID science instrument

The rotation of the science instrument provides a small amount of momentum bias (on the order of about 0.02 Nms) about the y-axis of the satellite which, ideally, coincides with the orbital momentum vector.

Although the same controller gains used previously could be used for this scenario as well and the spacecraft would still stabilize, the response can be significantly improved by using controller gains that are specially suited to the scenario.

For low magnetic inclinations (39°) and a small amount of internal momentum bias (0.02 Nms), the selected weighting matrices are

$$\mathbf{Q} = \text{diag}[5E-6 \quad 8E-6 \quad 2E-3 \quad 100000 \quad 5 \quad 100000 \quad 100000 \quad 5 \quad 100000] \quad (38)$$

$$\mathbf{R} = \text{diag}[3.2E6 \quad 3.2E6 \quad 3.2E6]$$

As can be seen, the y axis still has the largest integral weighting value in the \mathbf{Q} matrix. The proportional and derivative weightings have been significantly increased in the x and z-axes to take advantage of the internal

momentum about the y-axis. The resulting controller gain is

$$\mathbf{K} = \begin{bmatrix} 1.279\text{E-}8 & 0.204\text{E-}5 & -0.274\text{E-}6 & 0.1709 & 0.0071 \\ 9.690\text{E-}8 & 1.550\text{E-}5 & -0.117\text{E-}6 & -0.0271 & 0.0427 \\ 1.224\text{E-}8 & 0.195\text{E-}5 & 1.213\text{E-}6 & 0.0359 & 0.0058 \\ -0.0389 & 134.29 & 11.5864 & 4.0684 & \\ -0.0166 & -4.145 & 58.7065 & 0.1503 & \\ 0.1722 & 16.293 & 7.8892 & 77.661 & \end{bmatrix} \quad (39)$$

For high inclinations with momentum bias, the same \mathbf{Q} and \mathbf{R} matrices that were used for low inclinations (Equation 37) are used again. The high inclination gain using these weighting matrices is

$$\mathbf{K} = \begin{bmatrix} 0.371\text{E-}8 & 0.059\text{E-}5 & -0.1504\text{E-}6 & 0.1753 & 0.0018 \\ 9.721\text{E-}8 & 1.555\text{E-}5 & 0.2007\text{E-}6 & -0.0031 & 0.0414 \\ 1.548\text{E-}8 & 0.247\text{E-}5 & 1.2245\text{E-}6 & 0.0220 & -0.0079 \\ -0.0213 & 103.38 & 2.5566 & 10.802 & \\ 0.0283 & 2.063 & 54.938 & 1.6466 & \\ 0.1738 & 11.803 & -11.937 & 102.531 & \end{bmatrix} \quad (40)$$

Note that, although the same weighting matrices were used, several components of the actual gain matrix are significantly different when using the time average of $\mathbf{G}(t)$ for high inclinations rather than for low inclinations.

Once again the Floquet analysis using these controller gains over a full period yields eigenvalues with maximum values well within a unit circle, for all magnetic inclinations.

RESULTS

In order to ensure that the results were valid over a range of inclinations, Earth-fixed analysis was used to test the response of the system for magnetic inclinations at 5 degree increments.

On average, 5-7 simulations were run at each magnetic inclination without momentum bias. For each run initial conditions and disturbance torques were varied with disturbance torques near the expected maximum values about all three axes. Different runs also considered large initial pointing errors (greater than 90°) and initial rates not greater than 1°/s.

A similar method was used to verify performance with momentum bias except that initial pointing errors were assumed to be less than 10° (because the initial stabilization will be accomplished without momentum bias), and initial rates were assumed to be less than 0.01°/s.

Besides the Earth-fixed analysis verification, several orbits were selected and the controller was tested using a high fidelity simulation (similar to the Earth-fixed simulation, except that the orbit is no longer fixed to the earth).

Three-axis, nadir pointing attitude control was achieved to within 5° (worst case) without momentum bias and 0.6° (worst case) with momentum bias using the Earth-fixed analysis. The performance in the case of the rotating earth (and thus a slowly changing controller gain) was unchanged.

Limitations

The most significant limitations of the use of LQ regulation as presented in this paper are the long settling times and the sensitivity of the system to disturbance torques. The time required for the torque coils to stabilize the spacecraft using LQ regulation can vary significantly depending on the initial conditions. Table 2 shows the worst case settling times for the limited number of runs that were carried out for this particular study.

Table 2: Worst case settling times

i_{mag} (deg)	Worst Case Settling Time (hours)	
	With 0.02 Nms	No Momentum Bias
39	55.6	22.2
45	56.9	19.4
50	48.6	25.0
55	83.3	25.0
60	61.1	26.4
65	59.7	26.4
70	55.6	26.4
75	90.3	33.3
80	66.7	26.4
85	72.3	25.3
90	64.2	25.7

The average settling time will be somewhat less than the values shown in Table 2; however, it is obvious that this particular control scheme is not useful for satellites with short missions or that will be changing pointing targets regularly. It may be possible to improve the settling time using further gain scheduling.

The second restriction for this particular type of control system is the level of disturbance torques that can be tolerated. Table 3 shows an estimate of the worst disturbance torques that can be applied to the satellite without leading to instability. The values in the tables

are estimates based on a limited number of runs for each magnetic inclination.

Table 3: Worst allowable disturbance torques

i_{mag} (deg)	Worst Allowable Disturbance Torques (N-m)	
	With 0.02 Nms	No Momentum Bias
39	1.7E-8	0.8E-8
45	1.1E-8	0.8E-8
50	1.1E-8	0.8E-8
55	1.1E-8	1.1E-8
60	1.0E-8	1.1E-8
65	1.2E-8	1.1E-8
70	1.4E-8	1.1E-8
75	1.8E-8	1.1E-8
80	1.8E-8	1.1E-8
85	2.5E-8	1.1E-8
90	2.5E-8	1.1E-8

Note that the altitude of the orbit and the residual magnetic dipole of the satellite will have a significant role in ensuring that the disturbance torques will not be greater than is specified in Table 3.

IMPLEMENTATION

A significant implementation issue for this control system, particularly for the case of 3-axis control without momentum bias, is its sensitivity to disturbance torques. In 2003, a study was performed to verify control algorithms that are able to provide 3-axis stabilization using only a magnetometer for attitude determination and magnetic torquers for attitude control. These algorithms were tested onboard the Israeli Gurwin-TechSAT. One of the algorithms tested was an LQR routine very similar to that of the TOROID satellite. Although some valuable results were found, the control in the absence of momentum bias was not successful. It was suspected that the cause of the failure was magnetic disturbance torques due to the currents in the momentum wheel coil.¹⁵

Since the magnetic disturbance torques are subject to the same restrictions as the magnetic control torques (orthogonal to the local magnetic field vector), the magnetic residuals torques, theoretically, can always be canceled out. In order to accomplish magnetic residuals cancellation, the residual magnetic dipole of the satellite must be known very accurately. As components onboard the satellite are turned on and off, it is likely that the residual magnetic dipole of the satellite will change over time.

At least three possible solutions exist: one, improve the control algorithm such that higher disturbances are acceptable; two, design an estimator capable of tracking the changes in the residual magnetic dipole; and three, test each component to find the strength and direction of the magnetic field generated by each component. The most likely solution will be found by working on all three areas.

Since it is possible for the atmospheric density to vary significantly (1-2 orders of magnitude) over time at a given altitude (depending on the solar activity), it will be necessary to monitor the density of the atmosphere at the desired altitude and time of the launch in order to ensure that the density is below an acceptable level. The acceptable level is defined by the values in Table 3 and the physical characteristics of the spacecraft. As long as the altitude is high enough that the aerodynamic disturbance torques are less than the values indicated, and less than around 1000 km (further testing may show that this value can be increased), the control system can be expected to be effective.

CONCLUSIONS

A simple control system capable of 3-axis attitude control using only magnetic actuators has been presented. The system is capable of attitude control to better than 5° (3σ) at inclinations between 47° and 90° and altitudes between about 350 km and 1000 km. The addition of a small amount of internal momentum bias about the axis parallel to the orbital momentum vector can significantly increase the accuracy of the control. For the TOROID satellite, the accuracy increased to 0.5° (3σ) with the inclusion of 0.02 Nms internal momentum bias about the satellite y-axis.

Future work should address the implementation concerns, specifically: the performance of the system in the presence of larger disturbance torques; and the reduction or estimation of the residual magnetic dipole such that the magnetic torques will be within the expected range. Future work may also consider the use of further gain scheduling to reduce the settling time.

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APPENDIX

The Mean Magnetic North Coordinate System

The Mean Magnetic North Coordinate system is the coordinate system in which the magnetic inclination discussed in this paper is defined. The z-axis of the MMN coordinate system is the vector from the center of the Earth to the average location of the magnetic north pole; which is the location in the northern hemisphere where the magnetic field lines are perpendicular to the surface of the earth.

In this paper, the mean location of magnetic north for the year 2005 will be used unless otherwise stated. The x-axis is located 90 south from the magnetic north pole along the longitudinal line that passes through the magnetic north pole. The y-axis completes a right-handed orthogonal set.

Transforming from ECF to MMN

The ECF to MMN coordinate system transformation consists of two simple rotations. The first is a rotation about the z-axis of the ECF coordinate system to align the x-axis with the longitudinal line that passes through the magnetic north pole. The second is a rotation about the y-axis to align the z-axis with the magnetic north pole. If l_m and l_o_m are the North latitude and West longitude of the magnetic north pole, then the transformation from ECF to MMN is

$$T_{MMN}^{ECF} = \begin{bmatrix} \cos(l_o_m)\cos(l_a_m) & \sin(l_o_m) & -\cos(l_o_m)\sin(l_a_m) \\ -\sin(l_o_m)\cos(l_a_m) & \cos(l_o_m) & \sin(l_o_m)\sin(l_a_m) \\ \sin(l_a_m) & 0 & \cos(l_a_m) \end{bmatrix} \quad (26)$$

The mean latitude and longitude of magnetic north for the years 2001 to 2005 can be seen in Table 4.

Table 4: Location of Magnetic North, 2001-2005¹⁶

Year	Latitude (° North)	Longitude (° West)
2001	81.3	110.8
2002	81.6	111.6
2003	82.0	112.4
2004	82.3	113.4
2005	82.7	114.4