

Orbit/Attitude Determination and Control for the UMR SAT Mission

Michael W. Dancer and Jason D. Searcy

University of Missouri – Rolla, Rolla, MO 65401

Faculty Advisor: Dr. Henry J. Pernicka

As satellite missions become increasingly complex, a need for accurate determination and control systems using low cost hardware arises. This is especially true for university satellite programs such as the University of Missouri - Rolla satellite design team, or UMR SAT. With limited resources, mission success relies on creative and innovative hardware and software designs. This paper describes the development of control algorithms that will be used onboard the UMR SAT satellite pair. Using novel attitude and orbit control techniques and magnetometer-only attitude determination, the mission can be accomplished with low cost COTS hardware.

The UMR developed θ -D controller will be used to facilitate the attitude and formation control, and the θ -D filter will be used for orbit determination. The θ -D technique has been successfully applied to a wide variety of applications ranging from wing aeroelastic flutter suppression to hit-to-kill missile autopilot design to reusable launch vehicle control. The results of each application have been very promising and show the potential improvement over pre-existing control techniques offered by the θ -D method. Along with software development, this paper also provides high fidelity simulations of the determination and control system are presented to demonstrate the effectiveness of the algorithms.

I. INTRODUCTION

The UMR SAT project was an entry into the fourth University Nanosat Program, conducted by Air Force Research Lab (AFRL). The Flight Competition Review was held in March 2007 where UMR SAT was awarded 3rd place and was also named Most Improved School. The mission goal is to increase the technology readiness levels of key flight hardware, as well as demonstrate the autonomous formation flight of two satellites using low-cost readily available hardware. The UMR SAT project consists of a pair of micro satellites (MR and MRS SAT), which, upon arrival into orbit, will separate and autonomously maintain a fifty-meter formation for at least one orbit.

Formation flight is the cornerstone of the UMR SAT mission. Guidance, navigation, and control (GNC) is one of the most critical systems when attempting formation flight. Student satellite teams, however, do not typically possess significant resources from which to design the satellite system. Compensation must therefore

be made with an innovative control system utilizing low cost hardware. The GNC innovations developed by the UMR SAT team include orbit and attitude control using the θ -D technique and magnetometer-only attitude determination. Use of the θ -D control scheme will help to save very valuable fuel and power onboard the spacecraft.

The θ -D technique is a fully nonlinear, suboptimal control technique with a filter counterpart. The θ -D method was developed by researchers at the University of Missouri-Rolla. The θ -D controller/filter has been successfully applied to a wide range of applications, both aerodynamic and astrodynamic. The θ -D approach provides a closed form solution to the state dependent Riccati equation (SDRE) with a series of disturbance terms added to allow direct control the transient nature of the controller.

II. HARDWARE DESCRIPTION

Magnetometers are used for attitude determination during UMR SAT mission

operation. The magnetometers sense the Earth's magnetic field, from which two-axis attitude determination is possible. In order to obtain full attitude determination using only a magnetometer, the magnetic field rates are determined via a simple Kalman filter. The magnetic field and its rates provide the necessary information to determine the satellite attitude and rotation rates.

To provide attitude control, magnetic torque coils and a micropropulsion system utilizing refrigerant propellant will be used. Due to the slow response time of the magnetic coils, they will be used primarily during satellite detumble, when response time is not as critical. The need for the propulsion system attitude control arises during the formation flight phase of the UMR SAT mission, where time critical attitude maneuvers are needed for effective formation control.

The orbit determination is achieved using a Spacequest GPS-12 receiver. The θ -D filter will use the GPS receiver to determine the spacecraft position and velocity. This position and velocity is then used by the θ -D controller for maintaining the desired formation. In addition to attitude control, the propulsion system will be used for orbit control during formation flight phase.

III. OVERVIEW OF THE θ -D TECHNIQUE

The θ -D control technique, developed at UMR, is based off of the State Dependent Riccati Equation (SDRE) optimal, nonlinear control technique which minimizes the quadratic cost function

$$J = \frac{1}{2} \int_0^{\infty} [x^T Q x + u^T R u] dt \quad (1)$$

subject to the constraint

$$\dot{x} = F(x)x + G(x)u \quad (2)$$

The optimal feedback control law can be shown to be

$$u = -R^{-1}G^T(x)S(x)x \quad (3)$$

where S is the solution to the SDRE

$$S(x)F(x) + F^T(x)S(x) - S(x)B(x)R^{-1}B^T(x)S(x) + Q = 0 \quad (4)$$

The θ -D technique provides an approximate solution to Equation (4) by assuming a power series solution of the form

$$S(x) = \sum_{i=0}^{\infty} T_i(x, \theta) \theta^i \quad (5)$$

Additionally, F and G are factored into constant and state dependent terms as

$$F(x) = A_0 + \frac{A(x)}{\theta} \theta \quad (6)$$

$$G(x) = B_0 + \frac{B(x)}{\theta} \theta \quad (7)$$

Substitution of Equations (5-7) into Equation (4) and matching like-orders of θ allows for the matrices, T_i , to be calculated recursively. The θ -D control also adds a series of disturbance terms, D_i , to Q in Equation (4) in the form

$$\tilde{Q} = Q + \sum_{i=1}^{\infty} D_i(x, \theta) \quad (8)$$

The matrix, T_0 , is calculated one time offline by solving the constant coefficient Riccati equation

$$0 = Q_0 + T_0 A_0 + A_0^T T_0 - T_0 B_0 R^{-1} B_0^T T_0 \quad (9)$$

The remaining matrices, T_i for $i = 1, 2, \dots$, are calculated online by the recursive equation

$$\begin{aligned} & T_n (A_0 - B_0 R^{-1} B_0^T T_0) + (A_0 - B_0 R^{-1} B_0^T T_0)^T T_n \\ & - \frac{T_{n-1} [A(x) - B(x) R^{-1} B_0^T T_0]}{\theta} \left[\frac{A(x) - B(x) R^{-1} B_0^T T_0}{\theta} \right]^T T_{n-1} \\ & + \sum_{j=1}^{n-1} \left[T_j B_0 + T_{j-1} \frac{B(x)}{\theta} \right] R^{-1} \left[T_{n-j} B_0 + T_{n-j-1} \frac{B(x)}{\theta} \right]^T - D_n \end{aligned} \quad (10)$$

which is a constant coefficient, linear Lyapunov equation. The disturbance terms are chosen to be of the form

$$D_i(x, \theta) = k_i e^{-l_i t} (RHS) \quad (11)$$

where k_i and l_i are positive constants, and RHS represents the right hand side of Equation 10. Note that when $t = 0$, the disturbance terms cancel out the effect of $A(x)$ thereby reducing large inputs that may result for large initial errors. Also, the exponential factor in Equation 11 allows the disturbance terms to diminish as time progresses.

The θ -D control technique provides a suboptimal control with performances matching that of the

SDRE control but with significantly easier online implementation.

A. Comments

Some general comments about the θ -D method are as follows:

- As can be seen, the control is written in a closed form as a function of x . Note also that θ turns out to be cancelled in the control when it multiplies, so that θ is just an intermediate variable for the convenience of power series expansion and its value does not matter in the design (equivalent to setting $\theta = 1$).
- Usually in the θ -D method, the first three terms, i.e. T_0 , T_1 and T_2 , are used to compute the necessary control. They have been found to be sufficient for a good approximation in many problems. More terms can be added if needed.
- The basic steps of applying the θ -D approach can be summarized as:
 - Rewrite the nonlinear differential equation in the linear-like structure.
 - Factor the state-dependent coefficient matrices.
 - Solve the constant Riccati equation to get T_0 .
 - Recursively solve the Lyapunov equations, to get T_i .

IV. ATTITUDE DETERMINATION

The attitude determination system consists of a magnetometer and the software to filter its measurements. The decision to use magnetometer-only determination was two-fold. One, the magnetometer is a relatively inexpensive, very reliable sensor that provides reasonable accuracy for low-cost. Second, the addition of Sun sensors or gyroscopes will complicate the system on a satellite that is already volume-constrained with a limited mass budget (~30 kg). In addition a number of studies have shown that magnetometer-only determination can be accomplished with reasonable accuracy^{2,8}.

The first challenge addressed is the sensor's inability to achieve full determination along all

three axes. The sensor measurement will always result in one axis that cannot be determined, for if the device is rotated about the magnetic field vector, identical measurements are generated. As such, another vector is needed to resolve the ambiguity in the remaining axis. A Sun sensor would suffice to provide the other vector, but another approach⁸ is to use the derivative vector of the magnetic field. This vector, along with the magnetic field vector can provide full three-axis attitude determination.

When determining the magnetic field derivative, measurement noise must be considered. The Billingsley magnetometer was selected for the UMR SAT mission and has a 3σ directional error of 3° . When attempting to determine the magnetic field derivative vector by finite differencing, as suggested in Reference 8, a noise level of three degrees can result in significant error. This problem is addressed by using a Kalman filter. The filter reduces the noise in the measurements, allowing derivative estimates of the magnetic field vector to within 7° , which is sufficient for the UMR SAT mission. The entire attitude determination process is briefly described below.

A. Magnetic Field Model

In order to use the magnetic field measurements to determine the spacecraft's attitude, the actual magnetic field vector must be known. Additionally, the Earth's magnetic field is a highly complex, dynamic system, so the magnetic field varies with both location and time. The UMR SAT team uses a model of the Earth's magnetic field provided by the NASA Goddard Space Flight Center. Using the orbital position vector obtained from the orbit determination process, the magnetic field vector is thereby available as a function of time and spacecraft position.

Time derivatives of the magnetic field must also be calculated for use later in the attitude determination algorithm. Finite differencing of the magnetic field model is used to calculate the needed magnetic field derivatives.

B. Magnetometer Measurement Filter

After the actual magnetic field vector (obtained from the NASA model) and its derivatives are calculated in the inertial frame, a simple Kalman

Filter⁵ is used to estimate the local magnetic field and its derivatives using the magnetometer measurements, which are expressed in the satellite body frame. Once the measurements are filtered, the attitude quaternion, representing the between inertial and satellite body coordinates, can be calculated.

The Kalman filter uses a third-order Markov process to model the magnetic field. The model is given by

$$\frac{d^3}{dt^3} \mathbf{B} = w \quad (12)$$

where \mathbf{B} is the magnetic field vector and w is white Gaussian process noise. The filter is initialized by using finite differencing on the first three magnetometer measurements. The use of a third-order, Markov process allows the filter to estimate the first and second derivatives of the magnetic field vector as well as the field itself.

C. Satellite Attitude Calculation

The method for determining the satellite attitude and rates from the magnetic field and its derivatives, is outlined in Reference 2 and 8. The method is modified slightly to allow the attitude quaternion to be calculated as opposed to the rotation matrix. This portion of the attitude determination process is nearing completion, but simulation results have not yet been completed at this time.

V. ORBIT DETERMINATION

The following sections describe the onboard algorithm, which will process the data obtained from the GPS receiver.

A. Orbit Dynamic Model

This section describes the dynamic model used by the orbit determination and formation control algorithms described in later sections. This model incorporates Earth's gravity field, with the perturbation due to the Earth's equatorial bulge, gravitational effects due to the Sun and the Moon, and atmospheric drag. Details as to the development of the dynamic model can be found in Reference 4. In this section, \mathbf{r} and \mathbf{v} are the satellite position and velocity, respectively, and r represents the satellite orbital radius.

First, for simplification, define

$$k_1 = 1 + \frac{\mu_S}{\mu_E} \left(\frac{r}{d_S} \right)^3 + \frac{\mu_M}{\mu_E} \left(\frac{r}{d_M} \right)^3 - \frac{3}{2} J_2 \left(\frac{r_E}{r} \right)^2 (3 \sin^2 \phi - 1) \quad (13)$$

$$k_2 = 3 \frac{J_2}{r} \left(\frac{r_E}{r} \right)^2 \sin \phi \quad (14)$$

$$k_3 = \frac{\rho_0}{2\beta} e^{-h/h_s} V_{rel} \quad (15)$$

where J_2 is the Earth's second zonal harmonic, μ_E , μ_S , and μ_M are the gravitational parameters for the Earth, Sun, and Moon, respectively, d_S and d_M are the mean distances between the Earth and Sun and Earth and Moon, respectively, r_E is the mean radius of Earth, ϕ is the satellite's latitude, ρ_0 is the sea level density of the Earth's atmosphere, β is the satellite's ballistic drag coefficient, h is the atmospheric scale height, and V_{rel} is the speed of the satellite relative to the Earth's atmosphere. Then, with I being the 3x3 identity matrix, the dynamic model is given as

$$\mathbf{a} = \left[-\frac{\mu_E}{r^3} (k_1 I + k_2 R_\phi) + k_3 R_\omega \right] \mathbf{r} - k_3 \mathbf{v} \quad (16)$$

where R_ϕ and R_ω are defined as

$$R_\phi = \begin{bmatrix} -z & 0 & 0 \\ 0 & -z & 0 \\ x & y & 0 \end{bmatrix} \quad (17)$$

$$R_\omega = \begin{bmatrix} 0 & -\omega_E & 0 \\ \omega_E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (18)$$

with ω_E being the angular velocity of the Earth.

B. θ -D Orbit Determination

The Orbit Subsystem has opted to use the θ -D filter technique, developed at UMR, to perform orbit determination during the UMR SAT mission. The θ -D filter, counterpart to the θ -D controller¹², is based off of the State Dependent Riccati Equation Filter⁷ (SDREF). The θ -D filter finds an approximate solution to the SDREF with disturbance terms added to provide control of the filter transient response.

The θ -D filter works as follows. Consider the plant and measurement given as

$$\dot{x} = F(x)x + Gu + \Gamma w \quad (19)$$

$$z = H(x)x + v \quad (20)$$

where w and v are white noise random process with $E[ww^T] = W$ and $E[vv^T] = V$ as their respective spectral densities. The estimated states are then calculated from

$$\dot{\hat{x}} = F(\hat{x})\hat{x} + Gu + K(\hat{x})[z - H(\hat{x})\hat{x}] \quad (21)$$

where $K(\hat{x}) = P(\hat{x})H^T(\hat{x})V^{-1}$ and $P(\hat{x})$ is the positive definite solution to the state dependent Riccati equation

$$\begin{aligned} \Gamma W \Gamma^T + P(\hat{x})F^T(\hat{x}) + F(\hat{x})P(\hat{x}) \\ - P(\hat{x})H^T(\hat{x})V^{-1}H(\hat{x})P(\hat{x}) = 0 \end{aligned} \quad (22)$$

A difficulty with the SDREF is that the Riccati equation, Equation 22, must be solved in real time, and this task requires a significant amount of computation time. The θ -D method works by assuming the solution as a power series in θ .

$$P(\hat{x}) = \sum_{i=0}^{\infty} T_i(\hat{x}, \theta) \theta^i \quad (23)$$

Next, the dynamic coefficient matrices are factored into constant and state dependent terms given by

$$F(\hat{x}) = A_0 + \frac{A(\hat{x})}{\theta} \quad (24)$$

$$H(\hat{x}) = C_0 + \frac{C(\hat{x})}{\theta} \quad (25)$$

Additionally, the θ -D method adds a series of disturbance terms to Equation 22 and solves the disturbed Riccati equation

$$\begin{aligned} \left[\Gamma W \Gamma^T + \sum_{j=1}^{\infty} D_j(\hat{x}, \theta) \theta^j \right] + P(\hat{x})F^T(\hat{x}) \\ + F(\hat{x})P(\hat{x}) - P(\hat{x})H^T(\hat{x})V^{-1}H(\hat{x})P(\hat{x}) = 0 \end{aligned} \quad (26)$$

where the disturbance terms, $D_j(\hat{x}, \theta)$, are design parameters and are chosen such that

$$\left\| \sum_{j=1}^{\infty} D_j(\hat{x}, \theta) \theta^j \right\| \ll \left\| \Gamma W \Gamma^T \right\| \quad (27)$$

The condition set by Equation 27 ensures that the solution to Equation 26 will closely approximate

the solution to Equation 22. Application of the θ -D approximation method to Equation 26 results in the following recursion equations

$$T_0 A_0^T + A_0 T_0 - T_0 C_0^T V^{-1} C_0 T_0 + \Gamma W \Gamma^T = 0 \quad (28.0)$$

$$\begin{aligned} T_n (A_0 - T_0 C_0^T V^{-1} C_0)^T + (A_0 - T_0 C_0^T V^{-1} C_0) T_n \\ = \frac{T_{n-1} [A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})]^T}{\theta} - \frac{[A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})] T_{n-1}}{\theta} \\ + \sum_{j=1}^{n-1} \left(C_0 T_j + \frac{C(\hat{x})}{\theta} T_{j-1} \right)^T V^{-1} \left(C_0 T_{n-j} + \frac{C(\hat{x})}{\theta} T_{n-j-1} \right) - D_n \end{aligned} \quad (28.n)$$

The disturbance terms can now be chosen to alleviate the problem of large initial observer gains by choosing

$$D_j(\hat{x}, \theta) = k_j e^{-l_j t} (RHS) \quad (29)$$

where RHS represents the entire right hand side of Equation 28.n, each $k_j > 0$ determines the canceling effect of the disturbance, and the $l_j > 0$ term allows the disturbance to die off as time progresses. It can also be shown that proper selection of the constants, k_j and l_j , will also guarantee convergence of Equation 23.

With the disturbance terms defined, Equations 28.0-n are solved, and Equation 23 then produces an approximate solution to Equation 4. From experience in using the θ -D filter on various problems, the first three terms in Equation 23 are usually sufficient to produce the desired result.

C. Staggered Filter

A continuous filter, such as the θ -D filter, has dynamics that are dependent not only on the filter state, and time in some cases, but also on the current measurement. This is to say that in general

$$\dot{x} = f(x, t) = g[x, t, z(t)] \quad (30)$$

where x is the filter state, t is time, and z is the measurement at t . A problem with implementing a continuous filter is that the measurements are not known at all t , but instead are only known at discrete times, t_{n-1} , t_n , t_{n+1} , etc. The continuous filter implementation procedure proposed involves coupling a specific numerical integration scheme with the continuous filter in

order to approximate the continuous filter in a discrete time sense.

The Runge-Kutta, 4th order (RK4) integrator is a very commonly used integrator for its balance between numerical accuracy and required computations. As a result, it poses as a prime candidate for the implementation of a continuous time filter in a discrete time sense. As will be seen shortly, however, some care must be taken when using this integrator.

When using the RK4 integrator, the state at t_n is propagated to the time t_{n+1} via the algebraic equations

$$k_1 = hf(x_n, t_n) \quad (31)$$

$$k_2 = hf(x_n + k_1/2, t_n + h/2) \quad (32)$$

$$k_3 = hf(x_n + k_2/2, t_n + h/2) \quad (33)$$

$$k_4 = hf(x_n + k_3, t_n + h) \quad (34)$$

$$x_{n+1} = x_n + (k_1 + 2k_2 + 2k_3 + k_4)/6 \quad (35)$$

where x_n and x_{n+1} are the filter states at t_n and t_{n+1} , respectively, and $h = t_{n+1} - t_n$. Now consider the result when the RK4 integrator is used to propagate state estimates between the discrete measurement times.

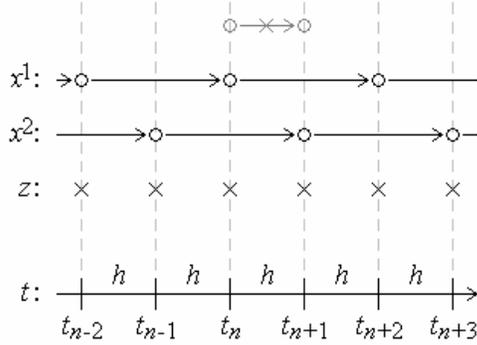


Figure 1 Staggered Filter Concept

In Figure 1, the available measurements are marked by X's and the state estimates are marked by O's. Consider the case, indicated by the grey arrow, when the state estimate at t_n is known and the RK4 integrator is used to propagate the state to t_{n+1} . In this case a measurement is required at $t_{n+1/2} = t_n + h/2$, but a measurement at this time is unavailable. Now consider the line labeled x^1 where the RK4 is

used to propagate the state to t_{n+2} , instead of t_{n+1} . In this case all required measurements are available for the integration, but unfortunately the state at t_{n+1} is not estimated. This problem is corrected by introducing the concept of a staggered filter.

Consider the case when two separate filters, indicated by lines x^1 and x^2 , are used to propagate each state estimate between two measurement intervals. If the two filters are operated in a staggered fashion, as depicted in Figure 1, a state estimate is obtained at each measurement time, albeit from alternating filters. In the staggered filter concept, two identical filters are propagated independently of each other, but the filters share the available measurements. Using this staggered filter concept, it is possible to implement a continuous filter in a discrete time sense.

VI. ATTITUDE AND ORBIT CONTROL

The attitude and orbit control system for the UMR SAT mission uses the θ -D technique to provide a servo-mechanism based control architecture. The desired attitude of the satellite has the side panel, Panel 1 (containing the space-to-ground communications antennas), continually directed toward Earth while the satellite's top panel is maintained parallel to the orbit plane. The satellite performs a single rotation per orbit in order to maintain Panel 1 in the proper orientation. This configuration allows for uninterrupted communication with the ground and will direct the thrusters onboard the vehicle appropriately to allow for adequate formation control.

The attitude control architecture was developed using Euler's equations for rigid body motion in quaternion form as

$$\dot{q} = \frac{1}{2} q \omega \quad (36)$$

$$I \dot{\omega}_b = \tau - \omega_b \times I \omega_b \quad (37)$$

where $q = q_0 + q_1i + q_2j + q_3k$ is the quaternion representation of the satellite's attitude, ω is the quaternion representation of the satellite's angular velocity, ω_b , and I is the inertia tensor of the satellite. The propulsion system will provide a torque, τ , to control the satellite's attitude.

In order to make use of the θ -D technique, Equations 36 and 37 must be cast into a linear-like structure. There are an infinite number of ways for which this can be done, however, certain guidelines are followed³. The SDRE control, and thus θ -D as well, is susceptible to local uncontrollability due to the nature of the linear-like form. Even though a nonlinear system may be controllable, if the controllability matrix, $C = [B(x) A(x)B(x) \dots A^{n-1}(x)B(x)]$ is rank deficient, then the SDRE will not possess a positive definite solution. Following techniques outlined in Reference 3, the linear-like form for Equations 36 and 37 is given by

$$\begin{bmatrix} \dot{q} \\ \dot{\omega}_b \end{bmatrix} = \begin{bmatrix} (1-a_1)\Omega & a_1\bar{Q} \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} q \\ \omega_b \end{bmatrix} + \begin{bmatrix} 0 \\ I^{-1} \end{bmatrix} \tau \quad (38)$$

where

$$\Omega = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \quad (39)$$

$$\bar{Q} = \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \quad (40)$$

$$\Phi = I^{-1} \left[(1-a_2)[I\omega^\times] - a_2[\omega^\times]I \right] \quad (41)$$

and $[I\omega^\times]$ and $[\omega^\times]$ are the matrix representations used for cross products. The parameters, a_1 and a_2 , are designer choice values and have been set to $a_1 = a_2 = 0.5$ in the UMR SAT implementation. Using the linear-like structure of Equation 38, the θ -D method is readily applied to obtain the required control torque, τ , to maintain the desired satellite attitude.

To provide formation control of the satellite, the θ -D control technique is applied to the orbit model used by the orbit determination algorithm, Equation 16. The desired position and velocity is calculated by the formation guidance algorithm described in the following section.

A. Formation Guidance

The onboard software uses the information provided by the orbit determination algorithms to produce commands, which are in turn sent to the

propulsion system onboard MR SAT. Before these commands can be determined, however, the onboard computer must first determine a position to which the satellite should steer. A relatively simple guidance algorithm is used for this target position determination.

From the position and velocity estimates of MRS SAT, provided by the orbit determination software running onboard, the six Keplerian orbital elements – semi-major axis (a), eccentricity, inclination, right ascension of the ascending node, argument of periapsis, and true anomaly (ν) – are first computed. Then the orbital elements for MR SAT are found by simply subtracting an angular amount from ν . This angular amount is given by D/a where D is the desired formation distance. Finally MR SAT's orbital elements are converted into a target position and velocity. The methods for conversion between position and velocity and Keplerian orbital elements can be found in Reference 11.

B. Linear Programming Solution.

The appropriate throttle settings for each of the eight thrusters are determined using linear programming⁶. While the thrusters cannot actually be throttled, the thrusters can be used in a pulse-width modulation (PWM) type scheme to obtain a desired throttle setting. This procedure is discussed in the next section. This method allows the calculation of the optimal throttle settings that minimize the cost function $J = \sum \theta_i$ ($i = 1, 2, \dots, 8$), where $0 \leq \theta_i \leq 1$ is the throttle setting for the i^{th} thruster. This cost function effectively minimizes the net fuel consumption rate for the entire thruster control system. In the linear programming solution, the throttle settings are constrained by $F = \sum F_i \theta_i$ and $\tau = \sum \tau_i \theta_i$, which ensures that the force and torque needed by the orbit and attitude control systems are achieved.

C. Throttle Filter.

Because the thrusters for MR SAT can not be throttled, a method is needed to determine whether the thrusters should be activated during a fixed time interval, Δt . The method used is to compare a simple filter estimate of the current throttle setting, $\hat{\theta}$, with the desired throttle setting calculated from the linear programming thruster solution. If the estimated throttle setting

is higher than the desired throttle setting, then the thruster does not fire during the next Δt time interval. If the estimated throttle setting is lower than the desired throttle setting, then the thruster is fired during the next Δt . Thus the thruster logic is given by

$$u = \begin{cases} 0 & \theta < \hat{\theta} \\ 1 & \theta > \hat{\theta} \end{cases} \quad (42)$$

where $u = 1$ indicates the thruster will fire and $u = 0$ means the thruster will not fire.

Once the u is determined for each thruster, the throttle estimates are updated using the following filter equation.

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K(u - \hat{\theta}_k) \quad (43)$$

where K is a filter gain. Using the discrete on/off values for each thruster, the appropriate thrusters are turned on during the next time interval, Δt , after which Equation 42 is used to determine which thrusters are activated during the next time interval. During attitude control testing, an appropriate value for the filter gain, K , was found to be between 1/3 and 1/4, although further parametric studies are underway.

VII. SIMULATIONS

The dynamic model used in the formation flight simulation utilized a six-degree of freedom (6 DOF) model for both MR SAT and MRS SAT. For both satellites the translational degrees of freedom were modeled using Newtonian mechanics with the following orbital perturbations included:

- GRACE gravity model for the Earth with degree and order 12
- Atmospheric drag with Harris-Priester atmospheric density model
- Solar and lunar third-body gravitational effects with locations determined from JPL's DE405 ephemeris
- Solar radiation pressure

The attitude of each satellite was modeled using quaternion dynamics, and angular rates were propagated using Euler's equations with products of inertia included. Lastly, the mass of MR SAT was modeled with a constant mass flow rate for each thruster.

The parameters used in the simulation are outlined in Table 1.

Table 2. Simulation Parameters

Description	Value	Units
Moments of Inertia	I_{xx}	0.9090
	I_{yy}	0.8660
	I_{zz}	0.8778
Products of Inertia	I_{xy}	0.0109
	I_{xz}	0.0042
	I_{yz}	0.0040
Satellite Mass	30	kg
Reflectivity Coefficient	1	
Reference Area	0.2864	m ²
Drag Coefficient	2	
Maximum Thrust (per thruster)	24.856	mN
Mass Flow Rate (per thruster)	0.0611	g/s

A. Initial Conditions

The initial conditions used in the simulation correspond to the instant immediately following the separation of the two satellites. The initial location of the two satellites was determined from the following Keplerian orbital elements.

- Semimajor axis, $a = 6778$ km (altitude 400 km)
- Eccentricity, $e = 0.01$
- Inclination, $i = 51.6$ degrees
- Right ascension of the ascending node, $\Omega = 20$ degrees
- Argument of periapsis, $\omega = 35$ degrees
- True anomaly, $\nu = -10$ degrees

The initial velocities of the two satellites were calculated so that the center of mass velocity corresponds to the orbital elements given above with the relative velocity between the satellites as 0.5 m/s. The relative velocity was directed in the general direction of the center of mass velocity, but angled approximately 6.5 degrees below the horizon. This ejection configuration would, under ideal conditions, allow MR SAT to drift exactly 50 meters behind MRS SAT. Finally, a satellite mass ratio of $\mu = 1/3$ was used to distribute the relative velocity between the satellites as

$$V_1 = V_{cm} - \mu V_{rel} \quad (44)$$

$$V_2 = V_{cm} + (1 - \mu) V_{rel} \quad (45)$$

The initial attitude of the two satellites was set with the top of each satellite directed along the relative velocity direction and the x -axis pointing towards the Earth. The initial angular velocities of the two satellites were set so that both satellites were initially fixed with respect to the local vertical/local horizontal (LVLH) reference frame.

A simulation was also developed for the filtering of magnetometer data using the MATLAB language and an arbitrary orbit generated using the UMR SAT orbit model. Once the orbit position is known at one-second intervals, the magnetic field model is used to obtain “actual” magnetic field vectors to which white Gaussian noise is then added thus simulating magnetometer measurements. At this point, the measurement filter is applied to the simulated measurements. The filter removes some of the noise in the measurement and numerically calculates the magnetic field derivative.

B. Results

The formation flight phase simulation described in the previous sections was run for a simulation period of thirty minutes. The simulation step size was set a 0.1 seconds and data were saved every second. Prior to each integration time step, the onboard Orbit/ADAC software was used to determine which of the eight thrusters would be firing during the next time step. The number of thrusters firing at each time step was totaled in order to determine the total amount of propellant used during the simulation.

Figure 2 shows the relative separation distance between MR SAT and MRS SAT. The initial conditions were set so that MR SAT would drift to nearly 50 meters behind MRS SAT, but without any corrective maneuvers, MR SAT would continue to drift well beyond communication range. As is seen in Figure 2, the formation control software successfully manages to correct the drift rate induced by the separation of the two satellites, but MR SAT actually drifts to approximately 85 meters before returning to the desired formation. This is attributed to the necessity of MR SAT to reorient itself so that the satellite is in a proper orientation for the thrusters to produce the needed corrective force.

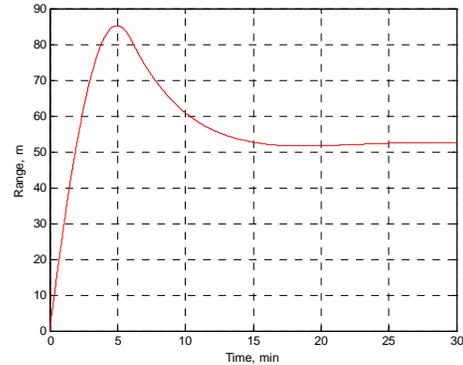


Figure 2 Relative Separation Distance

The reorientation of MR SAT is seen in Figure 3, which shows the attitude error for MR SAT during the 30 minute simulation. The nominal attitude is defined with the top of MR SAT parallel to the satellite’s orbit plane and the x -axis directed towards the Earth. This attitude is required for the communication purposes as well as to allow the eight thrusters to produce needed in-plane correction maneuvers. From Figure 3 it is clear that MR SAT experiences several attitude oscillations prior to settling at the nominal attitude. It is also clear from Figures 2 and 3 that the formation control software has a significant effect on the performance of the attitude control software and vice versa.

Lastly, the propellant consumption is shown in Figure 4. There are two distinct phases seen in Figure 4: formation stabilization and formationkeeping. Immediately following separation, MR SAT must perform a corrective maneuver to drive the relative velocity between the satellites to near zero. Then once MR SAT is in a stable formation 50 meters behind MRS SAT, small corrective maneuvers are used to maintain the formation. The formation stabilization phase is marked by the steep portion (hence high fuel consumption rate) of Figure 4 and the formationkeeping phase is marked by the noticeably reduced fuel consumption rate (shallower portion).

Another important aspect of Figure 4 is that approximately 60 grams of fuel have been consumed in roughly twelve minutes. With the current pressure restriction on the fuel tank (100 psi), MR SAT would only have about 60 grams of fuel available. Thus MR SAT would deplete its propellant resource before the formation has even been fully stabilized. However, if the pressure restriction is adjusted to 200 psi, MR

SAT would have nearly 130 grams of available propellant, and thus the formation could be adequately stabilized with a considerable amount of time for formationkeeping.

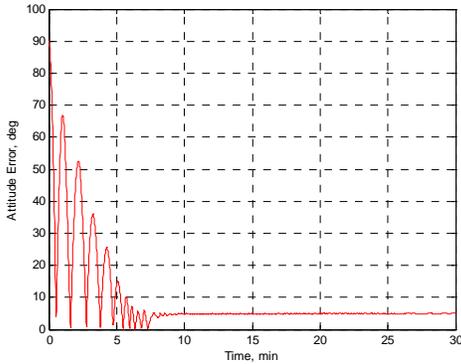


Figure 3 Attitude Error

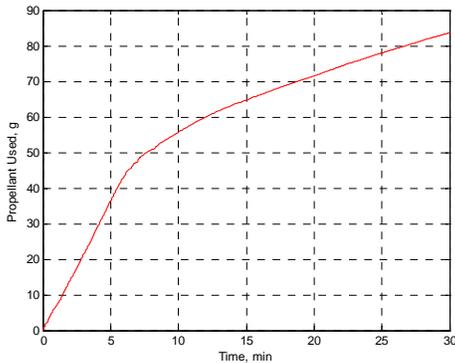


Figure 4 Propellant Consumed

Figure 5 and Figure 6 show the angular error between the filtered magnetic field vector, its derivative, and their actual counterparts.

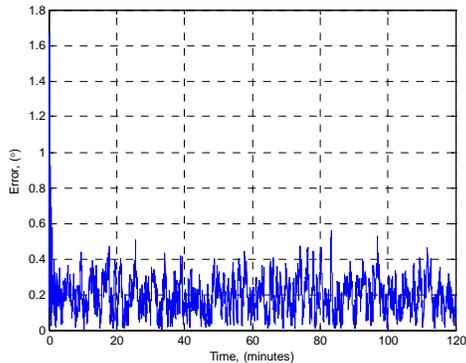


Figure 5 Magnetic Field Vector Error

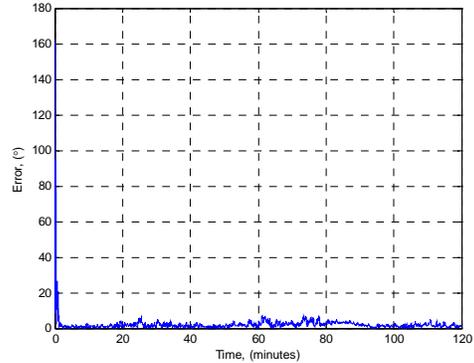


Figure 6 Magnetic Field Vector Derivative Error

As can be seen, the filter was able to reduce the noise substantially and produce reasonably accurate estimates of the magnetic field and its derivative. The initial error for the magnetic field derivative calculation is caused by the finite differencing used to find the initial estimate. The noise in the sensor measurements kept the finite differencing from accurately predicting the derivative, but, as seen in 6, the error decreases to under five degrees in less than two minutes with the use of measurement filtering.

VIII. CONCLUSION

As future satellite missions become increasingly more complex, a need for accurate determination and control systems using low cost hardware arises. This need is very apparent in university satellite design teams such as the UMR SAT project. To that end, this paper has presented new techniques for determination and control of a satellite's orbit and attitude.

The novel θ -D controller/filter, developed at UMR, has been selected for use in the orbit determination, using a single GPS receiver, and both formation and attitude control, using a micro propulsion system with refrigerant propellant. Additionally, magnetometer-only attitude determination allows accurate attitude and rotation rate estimates with low cost COTS hardware.

The flight software presented herein, coupled with low cost, reliable sensors and actuators, produces an effective, yet affordable, flight control system. Such a system will allow greater design flexibility for future satellite applications.

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