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A Rational Approach to the Estimation of Evaporation and Evapotranspiration from Climatological Data

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by

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PR 53

A contribution from the

Engineering Experiment Station

Utah State University

Logan, Utah

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at the earth's surface, both of which are affected by climatic conditions.

The problem was one of determining statistically which climate factors are significant and how they are related to the coefficient, C.

Analysis of Climatological Data

Evaporation data are more numerous than evapotranspiration data, and they are probably more reliable for statistical study. It was reasoned that if satisfactory relationships could be derived for evaporation, these relationships could be modified to apply to potential evapotranspiration. Nearly all research on evapotranspiration or consumptive use has included measurements of, or comparisons with, evaporation as measured with a Weather Bureau pan. It was decided, therefore, to begin with an analysis of evaporation data and to determine which factors have a significant effect upon the value of the coefficient, C, in equation 1. Only monthly Climatological Data as published by the Weather Bureau were used.

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given in Table 1 for each month and for each 10 degrees of latitude from the equator to 60 degrees north.

The initial study included data from only five northern Utah evaporation stations. The length of record at these stations varied from 4 to 20 years. Each month of record was considered as a separate observation. Temperature and wind data are given for each evaporation station, so these factors were considered first. It was found that they alone did not

Table 1.
Total Solar Radiation, R, Received at the Top of the Atmosphere, Expressed as Equivalent Depth of Evaporation at 20°C in Inches

Month	Latitude, Degrees North						
	0	10	20	30	40	50	60
Jan.	17.88	15.80	13.34	10.55	7.55	4.57	1.77
Feb.*	16.83	15.55	13.82	11.71	9.30	6.65	3.92
March	18.64	18.20	17.20	15.69	13.70	11.30	8.58
April	17.50	18.05	18.10	17.58	16.59	15.11	13.27
May	17.12	18.48	19.35	19.70	19.55	19.02	18.05
June	15.97	17.63	18.84	19.60	19.89	19.79	19.52
July	16.70	18.26	19.33	19.91	20.00	19.68	19.14
Aug.	17.52	18.44	18.82	18.69	18.02	16.89	15.40
Sept.	17.67	18.23	17.09	16.02	14.47	12.48	10.13
Oct.	18.38	17.34	15.79	13.78	11.38	8.66	5.73
Nov.	17.37	15.57	13.35	10.80	8.02	5.12	2.29
Dec.	17.01	15.34	12.72	9.52	7.02	3.81	1.15
TOTAL	208.59	206.89	197.75	183.55	165.49	143.08	118.95

*Feb. computed for average of 28.25 days.

Computed from table by Napier Shaw, Manual of Meteorology, Vol. II, Comparative Meteorology, Cambridge University Press, England. 2nd Ed., 1942. p 4.

Heat of vaporation of water at 20°C = 584.9 cal. per gram.

$R_{inches} = R_{cal/cm^2} \times 6.733 \times 10^{-4}$

Unfortunately, these data are reported for only a few stations in each state. For Utah, both humidity and sunshine data are available only for the Salt Lake Airport station. Humidity data are also reported for Milford and have just recently become available for Wendover. Since the five stations mentioned were all within a radius of 90 miles of Salt Lake, it was assumed that the sunshine and humidity at Salt Lake would represent that at the other stations with a reasonable degree of reliability. Other factors reported that might affect the value of the coefficient, such as elevation and latitude, did not vary sufficiently to provide a basis for statistical analysis. Later analyses indicated that both these factors are significant. The only factors considered in the determination of the coefficient C in the initial analyses were, therefore, mean monthly temperature in degrees Fahrenheit, wind in miles per day, sunshine percentage and the average of the two mean monthly humidity figures for 11 a.m. and 5 p.m.

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The value of the coefficient C in equation 1 can, therefore, be
written

$$C = C_T \; C_W \; C_S \; C_H \; C_E \; C_L \; C_M \qquad (2)$$

where the subscripts indicate the sub-coefficients for temperature, wind, sunshine percentage, humidity, elevation and latitude. The monthly coefficient, C_M, corrects for the hysteresis effect of the seasonal heat storage and release from the earth's crust.

It might appear that calculation of the coefficient C from equation 2 the value of C can be determined very simply by adding the logarithms. Likewise, the values of the logarithms of K and R can be tabulated, and the entire calculation can be performed with the aid of only an adding machine, or calculator, and a table of logarithms.

For simplicity, it was decided to relate each coefficient to the factor that it represents by a linear equation of the form

$$C_T = A + B\ T \qquad (3)$$

with values of A and B chosen so that the value of the coefficient would be unity for an arbitrary but approximate mean value of the factor. Thus, for temperature, C_T is 1.00 for a temperature of 68° F (20° C). That the coefficient is dimensionless can be shown by writing equation 3 in the form

$$C_T = \frac{T\ (°F) + K\ (°F)}{68\ (°F) + K\ (°F)} \qquad (4)$$

where K is a constant determined empirically from the analysis of the data.

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Table 2. Equations for the Coefficients and the Mean Value of the Product K C_E C_L for the Five Northern Utah Stations

Coefficient for	Equation	Value of the Factor for which C = 1.00
Temperature	$C_T = -0.095 + 0.0161\ T$	68° F
Wind	$C_W = 0.650 + 0.00583\ W$	60 miles/day
Sunshine	$C_S = 0.560 + 0.575\ S$	0.765 (76.5%)
Relative Humidity	$C_H = 1.106 - 0.340\ H$	0.312 (31.2%)

Table 3. Mean Values of the Monthly Coefficients, C_M, for the Seven Months for which Data are Available

Month	C_M	Month	C_M
April	0.940	August	1.046
May	0.929	September	1.092
June	0.963	October	1.116
July	0.988	Season	1.000

A comparison of the actual evaporation, and that calculated from the above relationships, for the five northern Utah stations is given in Table 4.

Table 4. Comparison of Actual and Calculated Evaporation at the Five Northern Utah Evaporation Stations

Month	Actual or Calc.	Provo Radio KOVO 1951-59	Utah Lake, Lehi 1940-59	Saltair Salt Plant 1956-59	Bear River Refuge 1940-59	Logan Agricultural Exp. Station 1951-59
April	A	4.82	5.68	6.37	4.75	4.31
	C	4.79	5.44	6.33	5.01	4.25
May	A	6.38	8.08	9.42	7.50	6.17

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April	A	4.82	5.68	6.37	4.75	4.31
	C	4.79	5.44	6.33	5.01	4.25
May	A	6.38	8.08	9.42	7.50	6.17
	C	6.35	7.93	9.89	7.61	5.99
June	A	8.00	9.66	13.00	8.91	7.45
	C	7.64	9.57	13.38	9.05	7.45
July	A	8.55	10.71	14.91	10.91	8.88
	C	8.92	10.47	15.46	10.99	8.57
August	A	7.61	9.48	13.36	9.89	8.21
	C	7.79	9.64	13.45	9.62	8.07
September	A	5.61	6.72	9.23	6.49	5.58
	C	5.86	6.84	8.82	6.33	5.51

It will be seen that the computed evaporation is within 6 percent of the measured evaporation in all but one instance. The mean error, or difference in the actual and computed evaporation, was only 2.7 percent.

A comparison of the results obtained with this procedure and from the Blaney-Criddle and the Hargreaves formulas is given in Table 5.

Table 5.
Comparison of the Ratios of Computed to Actual Evaporation for the Blaney-Criddle, Hargreaves and the Rational Formulas for the Five Northern Utah Stations

Month	Formula	B-C Coef. k*	Provo KOVO	Utah Lake, Lehi	Saltair Salt Plant	Bear River Refuge	Logan Agricultural Exp. Station
April	Blaney-Criddle	1.20	1.11	0.91	0.81	1.11	1.16
	Hargreaves		0.93	0.70	0.68	0.89	0.88
	Rational		0.99	0.96	0.99	1.05	0.99
May	Blaney-Criddle	1.30	1.16	0.91	0.82	1.05	1.17
	Hargreaves		1.12	0.89	0.88	1.11	1.11
	Rational		1.00	0.98	1.05	1.01	0.97
June	Blaney-Criddle	1.41	1.11	0.94	0.77	1.08	1.22
	Hargreaves		1.23	1.06	1.01	1.28	1.41
	Rational		0.96	0.99	1.03	1.02	1.00
April	Blaney-Criddle	1.20	1.11	0.91	0.81	1.11	1.16
	Hargreaves		0.93	0.70	0.68	0.89	0.88
	Rational		0.99	0.96	0.99	1.05	0.99
May	Blaney-Criddle	1.30	1.16	0.91	0.82	1.05	1.17
	Hargreaves		1.12	0.89	0.88	1.11	1.11
	Rational		1.00	0.98	1.05	1.01	0.97
June	Blaney-Criddle	1.41	1.11	0.94	0.77	1.08	1.22
	Hargreaves		1.23	1.06	1.01	1.28	1.41
	Rational		0.96	0.99	1.03	1.02	1.00
July	Blaney-Criddle	1.43	1.23	0.98	0.75	1.02	1.18
	Hargreaves		1.64	1.30	1.11	1.41	1.56
	Rational		1.04	0.98	1.04	1.01	0.97
August	Blaney-Criddle	1.41	1.25	1.00	0.74	1.01	1.16
	Hargreaves		1.59	1.30	1.02	1.37	1.49
	Rational		1.02	1.02	1.01	0.97	0.98
September	Blaney-Criddle	1.28	1.18	0.98	0.74	1.07	1.17
	Hargreaves		1.49	1.19	0.97	1.37	1.45
	Rational		1.04	1.02	0.96	0.98	0.99
October	Blaney-Criddle	1.01	1.15	1.01	0.69	1.18	1.20
	Hargreaves		1.20	0.99	0.85	1.25	1.20

Analysis of Data for the Western States

The results of these analyses seemed sufficiently gratifying to warrant further study. The first step was to check the equations by computing the evaporation at 47 additional stations in the western states and Texas, ranging in latitude from 26.15 to 48.50 degrees north and in elevation from 9 to 6007 feet above sea level. Only one year of record, usually 1959, was taken for this study. The agreement between the computed and actual evaporation was remarkably good. Then it was decided to use all of the additional data to further improve the equations for general application and to determine the coefficients for elevation and latitude. Later, ten-year means were substituted for the one year of record for six of the stations and an additional station was added, making a total of 53 stations in the analysis. These stations were chosen from those available on the basis of their proximity to locations for which sunshine and humidity data are available, or because it seemed reasonable to assume that sunshine and humidity conditions would be similar to that at these locations.

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On the basis of this analysis, the coefficients given in Tables 2 and 3 have been modified to give a better over-all fit. Tentative equations, and the value of the constant K, based on this study are given in Table 6. Tentative mean values of monthly coefficients, C_M, are given in Table 7.

Table 6.
Tentative Equations for the Coefficients and the Value of the Constant K Based on a Study of Climatological Data for 53 Stations in the Western States and Texas

Coefficient for	Equation	Value of Factor for which C = 1.000
Temperature	$C_T = 0.0147\ T$	68° F
Wind	$C_W = 0.676 + 0.0054\ W$	60 miles/day
Sunshine	$C_S = 0.560 + 0.550\ S$	0.80 (80%)
Humidity	$C_H = 1.288 - 0.720\ H$	0.40 (40%)
Elevation	$C_E = 0.925 + 0.000015\ E$	5000 feet
Latitude	$C_L = 1.520 - 0.013\ L$	40 degrees
Constant	$K = 0.490$	

Table 7.
Tentative Mean Values of the Monthly Coefficients, C_M, for the Seven Months Included in the Analysis

Elevation	$C_E = 0.925 + 0.000015\ E$	5000 feet
Latitude	$C_L = 1.520 - 0.013\ L$	40 degrees
Constant	$K = 0.490$	

Table 7.
Tentative Mean Values of the Monthly Coefficients, C_M, for the Seven Months Included in the Analysis

Month	C_M	Month	C_M
April	0.933	August	1.063
May	0.943	September	1.081
June	0.962	October	1.044
July	0.991	Season	1.000

Discussion

The equations, constants and coefficients given in Tables 6 and 7 are tentative. Of the 53 stations, 40 included records for only one year. Obviously a simple linear equation cannot be expected to be valid for a wide range of latitudes. Unfortunately, there are not enough suitable evaporation stations at latitudes less than 30 degrees or more than 45 degrees north to derive statistically an empirical relationship that would be valid for all latitudes. A different approach to this problem is planned.

The study is continuing as a Master's Thesis Project. It is planned to include at least 10 years of record for as many stations as possible. To date, most of the computations have been made on an electric calculator. It is planned, however, to program the analysis so that the computations can be made with an electronic computer.

The reliability of the tentative coefficients and the constant has been checked by determining the average difference between the actual and computed evaporation for all of the months of record. This average differ-

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The reliability of the tentative coefficients and the constant has been checked by determining the average difference between the actual and computed evaporation for all of the months of record. This average differ-
ence ranged from 6.1 percent for June to 10.5 percent for October, with a mean of 8.3 percent for the season. Slightly better results were obtained when a second-degree equation was used to obtain the latitude coefficient.

Deficiencies in the Data

As mentioned previously, only the Weather Bureau Climatological Data were used in the analyses. Not all of these data are completely satisfactory for this purpose, and this undoubtedly accounts for some of the discrepancies noted. Temperature data are satisfactory and mean tempera-

For other locations, wind velocities corresponding to that measured at evaporation stations would have to be estimated from that at nearby locations.

Sunshine data, although available at only a limited number of stations in each state, are probably adequate as sunshine percentages are reasonably uniform over a considerable area. Humidity data, as now reported, are not satisfactory. Mean monthly values of relative humidity are now reported four simultaneous times each day at all stations. They are, therefore, reported for different clock hours in the different time zones and are not comparable. Dry and wet bulb temperatures, dew-point temperatures and relative humidity are reported in supplements published separately for each of a limited number of stations in each state; but these supplements are not readily available in most libraries. Analyses were made to determine if the difference between maximum and minimum temperatures could be used as an index of humidity; but this proved unsatisfactory for some stations, especially those in the more arid loca-

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Application to Evapotranspiration

This same statistical approach has been tried for evapotranspiration data reported by Pruitt (11 and 12). The results were very satisfactory, but additional data must be analyzed before coefficients can be deterto be considered. Pruitt (11) and others have shown that wind, for example, does not have nearly as much effect on evapotranspiration as it does on evaporation.

Conclusions

The results of the analyses reported here suggest that this approach to the problem can lead to fairly reliable estimates of evaporation and evapotranspiration for locations where other climatic data are available.

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