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Formulation of a Mathematical Model for the Allocation of Colorado River Waters in Utah

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FORMULATION OF A MATHEMATICAL MODEL FOR THE ALLOCATION

OF COLORADO RIVER WATERS IN UTAH

by

Rick L. Gold

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of the requirements for the degree

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Rick L. Gold

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INTRODUCTION

Research project

This report is linked with the work done in fulfillment of the first objective of a research project now underway at the Utah Water Research Laboratory. The project, entitled "Application of Operations Research Techniques for Allocation of Colorado River Waters in Utah," is a matching fund grant by The Office of Water Resources Research of The United States Department of the Interior. The project has the following objectives: (1) formulate the mathematical model of that part of the state that can receive Colorado River water, (2) optimize the allocation model under different demand levels and study the economic effects of legal, political, and social limitations, (3) evaluate the usefulness of the operations research approach for water planning.

Scope of the report

This report will present the mathematical model which has been formulated under the research project. The model is formulated in the linear programming format and will be particularly applicable to the IBM Linear Programming Routine on the IBM 360/44 at the Utah State University Computer Center. The report will not deal with particular solutions but will give insight into model formulation and the possible uses of such models.

Much of the data used in the model in the form of demands and availabilities came from a cooperative effort with the Department of Natural Resources, Division of Water Resources, State of Utah.

DESCRIPTION OF THE REGION TO BE MODELED

Physiographic description[1]

The State of Utah is the area of the study, and the boundaries of the model conform basically to the physical boundaries of the State.

Utah is in an arid to semi-arid region of the Western United States and covers a total area of 84,916 square miles. It is divided between three major physiographic provinces: the Basin and Range, the Colorado Plateau, and the Middle Rocky Mountains.

The Basin and Range portion is made up of the western side of the state. This area contains the Great Salt Lake and the Salt Lake Desert which are the remains of ancient Lake Bonneville. The area is an interior drainage with no outlet to the ocean. It is made up of short, north-south mountain bedrock masses which form small basins with loose valley fills.

The Colorado Plateau portion lies in the south and east and is primarily made up of land within the Upper Colorado River Basin. This area is characterized by a highly dissected land surface with deep, steep-sided canyons. Major streams are often far below land surfaces.

The Middle Rocky Mountains portion is made up of the Wasatch Range and the Uinta Range. These coincide with Utah's north-east boundary lines. This mountainous region is the primary source area for runoff in the state.

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[1] The basic data in this section comes from McGuiness, 1963.

The majority of the state, being in the semi-arid classification, is utilized as grazing land and watersheds. Table 1 summarizes the land use and the water consumed by each type as a percentage of total precipitation.

Table 1. Land use and water consumption (McGuiness, 1963)

Type of land	Percent total area	water consumed percent total precipitation
Grazing land and watersheds	81.7	72.1
Arable but uncropped land, used for grazing	2.6	1.9
Dry-farmed land	1.1	1.0
Irrigated land	2.1	4.6
Cities and towns, industrial sites	.5	.2
Wasteland, national parks and monuments	9.0	6.4
Water area	3.0	9.5
	100.0	95.7
Outflow in interstate streams		4.3
		100.0

Alluvial aquifers which lie principally in the Basin and Range Province or in tracts between the mountain ranges or plateaus of the remainder of the state, mostly near the east edge of the Great Basin, provide for groundwater development.

A small portion of the state in the north-west corner drains to the Snake River Basin. This portion of the state is not included in the model.

Precipitation in the state varies over a wide range, from approximately 5 inches in the desert areas to approximately 30 inches

in the Wasatch and Uinta Ranges. Average for the whole state is about
11.5 inches (McGuiness, 1963). Runoff for the state averages 1.8
inches and varies from 0.25 inches to 20 or more inches in the high
mountains with as much as 40 inches in the highest parts (McGuiness,
1963). For a more complete description, see McGuiness (1963) and
Fenneman (1931).

Economic description

Utah's economy is based on several different industrial
sources. These sources are: agriculture, mining, construction, manu-
facturing, utilities, trade and service, and government (Nelson and
Harline, 1964). Table 2 summarizes the percentages of total personal
income from these sources for Utah and the nation in 1963.

Utah has had in recent times greater than average increases in
population, labor force, and employment. From 1940 to 1964, increase
in population was 81 percent, in labor force was 100 percent, and in
employment was 130 percent compared with national increases of 45
percent, 50 percent, and 60 percent respectively.

The change in the employment between the major economic seg-
ments shows a growth trend in the state. Personal and professional
service industries, trade, finance, and government show constant
increases while mining, manufacturing, utilities, and agriculture show
decreases due to increased utilization of capital.

Utah has experienced drastic changes in employment patterns
since 1940. Major industrial growth in the 1940's came with the con-
struction of the Geneva Steel Plant and increased development in
mineral industries. Uranium production became significant in the

Table 2. Percentage of total income from various sources

Basic physical production	Percentage of total income	
	Utah	Continental U. S.
Agriculture	3.0	4.4
Mining	4.8	1.2
Manufacturing	19.7	29.2
Utilities and transportation	8.3	7.4
Contract construction	8.8	6.4
Subtotal production	44.6	48.6
Wholesale and retail trade	19.7	19.1
Finance and insurance	4.3	5.2
Service	10.2	13.5
Government	21.1	13.2
Other Miscellaneous	0.1	0.4
Subtotal service	55.4	51.4
TOTAL	100.0	100.0

NOTE: Total personal income (millions of dollars): Utah $2,083; the nation $461,610. Does not include transfer payments, unemployment insurance, welfare, etc.

1950's; and the oil, missile, and electronics industries had tremendous growth after 1956. For particular events in this expansion, see Cluff (1964).

Because of changes in standard of living, the basic physical production industries of agriculture, mining, and manufacturing make up less than one-third of the total productive effort. On the consumer side, output of goods constitute less than 50 percent of the gross national product. This shows the increasing pyramid of economic activities based on a foundation of raw materials.

The development of the economy of Utah is probably more

dependent upon water than any other resource. According to Landsburg, Fischman, and Fisher (1963), water use in the arid West is expected to increase 50 percent for irrigation, 500 percent for manufacturing, 100 percent for thermal electric power generation, and 270 percent for municipal use. From this, the importance of water to Utah's economy can be seen.

For a more detailed description of the economy of Utah, including history and projections for the future, see Nelson (1956) and Nelson and Harline (1964).

Social and institutional description

The state of Utah has just recently passed the one million mark in population. This gives an average of 11.8 people per square mile. This figure is quite deceiving since the majority of the state's population resides along the Wasatch Front. Nearly 75 percent of the people live in this area of four counties which is only 4.5 percent of the total area of the state. The remainder of the state is sparsely populated, except for some small areas of local development. Actual population densities by counties range from 501.4 people per square mile in Salt Lake County to 0.6 people per square mile in Kane County.

A description of the resource related institutions in the state of Utah might refer to many different aspects. However, the prime consideration of this report is water allocation, and a brief description of the institutions which affect Utah's water resource development and use is included.

There are many types of institutions involved in water resources in Utah as mentioned by Webb (1967). Among the most important are the

water law as administered by the State Engineer and interpreted by the courts, the Division of Water Resources (formerly the Utah Water and Power Board), and the Committee on Water Pollution, formerly the Water Pollution Control Board, along with metropolitan water districts, water conservancy districts, irrigation districts, mutual irrigation companies, and municipal water departments.

Much could be and has been written on the water law of the state of Utah. (For a more complete description see the Utah Code, Title 73.) The basis of the law is the appropriative doctrine of water rights; but, as with many other states, the Utah law is unique.

The State Engineer has the responsibility of administering the state's water resources under the given law. The State Engineer is appointed by the Governor with Senate consent and has responsibility for supervising the measurement, apportionment, appropriation, and distribution of all waters within the state.

The Division of Water Resources of the Department of Natural Resources operates under an advisory board of seven, called the Board of Water Resources. Members are appointed from various water districts throughout the state. A director and staff carry out policies set by the Board. A major goal of the division is to achieve greater utilization of existing water supplies and development of new sources.

The Committee on Water Pollution consists of nine members appointed by the Governor with consent of the Senate. The Committee is concerned with any and all actions which may affect pollution in state waters.

The remaining institutions listed are on the local level. Their major functions are to develop, allocate, and distribute water to the

water users of the respective local areas.

Excellent coverages of duties, responsibilities, and make-up of all these institutions are given by Webb (1967), the Utah Code, Title 73, on Water and Irrigation, and Hoggan (1969).

NEED FOR AN ALLOCATION MODEL

Water available under the Upper
Colorado River Compact

In an act approved on August 19, 1921, by the Congress of the
United States of America, the states of Arizona, California, Colorado,
Nevada, New Mexico, Utah, and Wyoming entered into a compact to provide
an equitable division and apportionment of the waters of the Colorado
River System. This compact, known as the Colorado River Compact, basically
divided the waters of the Colorado River between the Upper and Lower
Basins. The compact gave each division the right to the exclusive
beneficial consumptive use of 7,500,000 acre feet of water per annum.
It also stated that in cases of deficiencies the shortages would be met
by each division in equal proportions.

In a later act passed on April 6, 1949, the states of the Upper
Division (Arizona, Colorado, New Mexico, Utah, and Wyoming) joined
together in a compact known as the Upper Colorado River Basin Compact
to further divide and apportion the waters of the Colorado River System.
This compact divided the 7,500,000 acre feet given to the Upper Division
as follows: 50,000 acre feet to Arizona, of the remaining quantity
51.75 percent to the state of Colorado, 11.25 percent to the state of
New Mexico, 23.00 percent to the state of Utah, and 14.00 percent to
the state of Wyoming.

These two compacts allocate the water to the state of Utah at
a minimum of 1,714,000 acre feet per annum. The basic premise of this
allocation scheme, however, is that the Colorado River flows in excess

of 15,000,000 acre feet per year. Various sources claim that the flow
is not this large. This gives a range of values for the actual alloca-
tion to Utah. Estimates range from the full 1,714,000 acre feet to
1,277,000 acre feet per year (Tipton and Kalmbach, 1965).

With water allocated in such a manner, the necessity for each
state to utilize its portion of the water is apparent. Benefits accrue
to the economy from the water use, and the public is benefited in many
ways. In times when use of water is such an important issue, wasteful
use and unused allocations suffer the wrath of public attack from areas
where the allocation may not be considered completely equitable. The
need, then, is established as a means of allocating the water to which
the state is entitled to its best uses.

Internal water needs and trends

The state of Utah is growing rapidly, and the demands for water
are expected to increase in proportion.

The major effort in the state at the present time in water
resource development is to transfer water which is allocated to Utah
under the Colorado Compacts from the Colorado Basin to the Great Basin.
The Great Basin is Utah's center of population and activity in industry
of all kinds. The proposed method of accomplishment of this water
transfer is the Central Utah Project. This U. S. Bureau of Reclamation
project proposes to divert waters of the Duchesne River and its tribu-
taries in northeastern Utah and to bring the water into the Central Utah
area along the Wasatch Front.

The model considered in this study is structured to include the
basic features of the Central Utah Project in the various sources of
supply and the water transfer patterns.

CURRENT STATUS OF WATER RESOURCE DEVELOPMENT

Agricultural demands

For the remainder of this report, the state of Utah will be divided into nine distinct hydrologic study areas. These areas will conform to those set forth by Utah State University and the Utah Water and Power Board (1963). See Figure 1.

Recent figures indicate that Utah has a total of approximately 5,600,000 acres of arable land. Of this, only 1,363,300 acres are being irrigated. Table 3 shows the arable land, irrigated land, and water consumption by agriculture in the nine study areas of the state.

Table 3. Land and water use by hydrologic area[a]

Hydrologic study area	Arable acres	Irr. acres	Water consumed
1	1,483,200	52,000	58,000
2	445,400	246,000	405,000
3	194,100	166,700	267,600
4	448,400	207,200	288,500
5	1,022,200	293,000	392,000
6	838,300	71,800	132,000
7	340,700	195,000	293,000
8	206,200	98,100	113,600
9	620,300	33,500	64,000
Totals	5,598,800	1,363,300	2,013,700

[a]Data for this table come from the Utah Division of Water Resources and Wilson, Hutchings, and Shafer (1968).
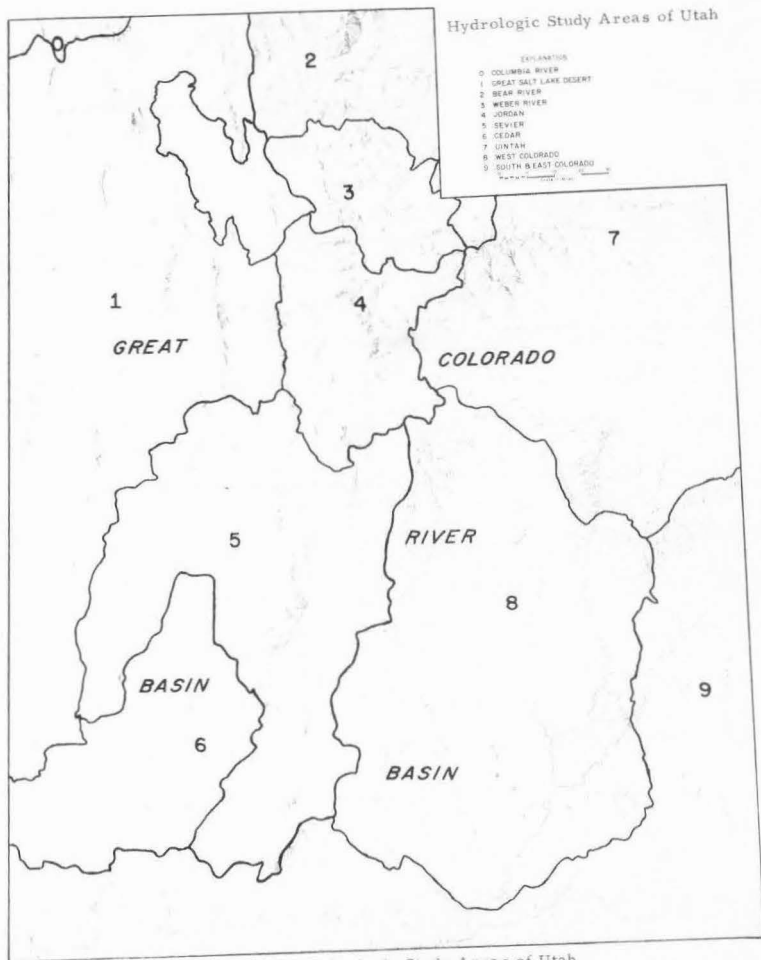
Figure 1. Hydrologic Study Areas of Utah

Municipal and industrial demands

The location of the population centers of the state of Utah gives rise to demands for municipal and industrial water which are very non-homogeneous over the nine study areas of the state. Table 4 shows the approximate population according to 1960 census and water demands for each area according to the Division of Water Resources.

For the purpose of this report, the municipal water is not separated from the industrial water demands. The allocation of such water seems to follow the same general pattern (i.e., large municipal demand occurs with large industrial demand).

Table 4. Population and municipal and industrial demand

Hydrologic study area	Population	Municipal and industrial water demand (ac-ft/yr)
1	23,000	3,000
2	69,800	14,000
3	214,000	21,000
4	567,000	84,000
5	33,000	9,000
6	15,800	4,000
7	20,000	3,000
8	26,000	5,000
9	28,200	6,000
Totals	996,800	

Groundwater availability

The state of Utah has some areas of high potential groundwater development. Some areas of the state are well explored and capacity and yields of these areas are known. In much of the state, however, there

are few data available on the groundwater resource. Table 5 shows the estimated availability in each of the areas on a perennial yield basis. This technical information comes from a review of publications of the State Engineer's office and water supply papers of the U. S. Geological Survey which deal with areas where groundwater studies have been made.

Table 5. Groundwater availability

Hydrologic study area	Groundwater availability in acre feet/year
1	46,000
2	295,000
3	75,000
4	402,000
5	286,300
6	138,800
7	40,000
8	a
9	a

[a]Too small for consideration

Local surface water availability

Most of the important streams within the state are fairly well gaged, and the surface water availabilities are well defined. In some cases the small ungaged tributaries may give rise to differences in accepted figures.

Table 6 lists the availabilities to be used in the study as provided by the Division of Water Resources.

Table 6. Local surface water availability

Hydrologic study area	Local surface water availability acre feet/year
1	800,000
2	970,000
3	800,000
4	1,000,000
5	800,000
6	210,000
7	1,750,000[a]
8	650,000[a]
9	690,000[a]

[a]This water considered as available for transfer.

Surface waters available from the Colorado River

As was mentioned before, the amount of water available for Utah under compact agreement is subject to some controversy. The allocation of this water will include all of the Colorado River water used within the state of Utah, whether in the Colorado Basin or in the Great Basin. For this reason, the water used in areas 7, 8, and 9 is treated as depletion from allotment in the same manner as the water transferred to the Great Basin. For this report, a figure of 1,440,000 acre feet per year will be used for Utah's allotment from the Colorado River.

THE MATHEMATICAL MODEL

General description

This mathematical model fits into the category of the general linear-programming problem. According to Gass (1964, p. 45), "The general linear-programming problem is to find a vector $(x_1, x_2, \ldots, x_j, \ldots, x_n)$ which minimizes the linear form (i.e., the objective function) $c_1 x_1 + c_2 x_2 + \ldots + c_j x_j + \ldots + c_n x_n$ subject to the linear constraints $x_j \geq 0$ $j = 1, 2, \ldots, n$ and

$$a_{11} x_1 + a_{12} x_2 + \ldots + a_{1j} x_j + \ldots + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + \ldots + a_{2j} x_j + \ldots + a_{2n} x_n = b_2$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{ij} x_j + \ldots + a_{in} x_n = b_i$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mj} x_j + \ldots + a_{mn} x_n = b_m$$

where the a_{ij}, b_i, and c_j are given constants and $m < n$."

In the case at hand, the objective function, or mathematical relation to be minimized, is an expression for the total cost in dollars of allocating the water resources of Utah. The vector $(x_1, x_2, \ldots, x_j, \ldots, x_n)$ is made up of the various n alternatives of allocation which may combine to form the solution to the problem, in this case the minimum cost. The jth element in this X_j vector represents a quantity

of water to be allocated to the jth alternative in acre feet per year. Each element in the vector has an associated c_j coefficient which reflects the cost of allocating one acre foot per year to the jth alternative or activity. When the cost coefficient, c_j, is multiplied by the quantity, x_j, and the result is summed over all alternatives ($j=1, \ldots, n$), the result is a total cost in dollars per year.

The linear constraints are of two general types. The first type is of the $x_j \geq 0$ form. This constraint simply makes negative quantities, sometimes referred to as activities, impossible. In other words, no alternative can have a negative quantity. This type of constraint is common to all linear-programming problems and is therefore not an obvious part of the following model since it is an automatic, or built-in, constraint for most computer routines.

The second type of constraint is of the $a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{in}x_n = b_i$ form. In this model this constraint is used in connection with both availabilities and demands. The X_j vector is the same as in the objective function. These constraints show the relationship between the elements of the vector and the total amount of water available or demanded. The vector $(b_1, b_2, \ldots, b_j, \ldots, b_m)$ gives a figure known as a right hand side for each of the constraints. The element b_j is the total availability for the jth alternative source. The sum of the $a_{ij}x_j$ must equal the b_j right hand side.

For this model the constraints are not strictly equalities. In constraints defining the availability of the water resource, the total quantity diverted must be less than or equal to the total availability so $\leq$ replaces the $=$ sign. In constraints describing the water demands

in each of the hydrologic study areas, the quantity diverted must be greater than or equal to the demand so $\geq$ replaces the = sign.

Explanation of the variable names

The c_j cost coefficients as well as the x_j variable names in the objective function and the constraints are made up of a group of letters and numerals which identify that represented element. The first letter of each element will be either $\underline{C}$ or $\underline{Q}$. If the letter is $\underline{C}$, the term is a cost coefficient which is further identified by the letters or numerals following. If the letter is $\underline{Q}$, the term is a variable quantity or activity and is further identified by the letters or numerals following it.

The system of identifying letters used is as follows:

BU   Colorado River water via Bonneville Unit

UI   Colorado River water via Ute Indian Unit

SA   Colorado River water to Sevier Area

LSW   Local surface water

GW   Groundwater

WW   Waste water from municipal and industrial systems

AG   Agricultural use

MI   Municipal and industrial use

R   Recharge to groundwater basins

T   Transfer between basins

S   Storage

B   Boosting to allow gravity feeds

D   Distribution to users

P   Pumping

H   Chlorination

TR Treatment by rapid sand filtration and chlorination

RC Reclamation of sewage water

K   Percentage of M&I water recharged to groundwater basin

L   Percentage of AG water returned to local surface water

M   Percentage of water coming through storage

N   Percentage of AG water returned to groundwater basin

The numerals 1-9 following these letters indicate the number of the hydrologic study area in which activity takes place. Two numerals in succession indicates a transfer from one study area to the other. For example: (CBU + CD)QBUAG4 would represent in word the cost of Bonneville Unit water plus the cost of distribution times the quantity of Bonneville Unit water used for agriculture in Study Area 4.

Objective function and cost coefficients

    The objective function is made up of the summation of the alternative quantities of supply times the corresponding cost of that allocation and is expressed in dollars. Written mathematically and with the above system of identification, the objective function is as follows:

Total Cost = (CBU + CD)QBUAG4 + (CBU + CD)QBUAG5 + (CBU + CD)QBUAG6 + (CBU + CR4)QBUR4 + (CBU + CR5)QBUR5 + (CBU + CR6)QBUR6 + (CBU + CTR + CB)QBUMI4 + (CBU + CTR + CB)QBUMI5 + (CBU + CTR + CB)QBUMI6 + (CUI + CD) QUIAG3 + (CUI + CD)QUIAG4 + (CUI + CD)QUIAG5 + (CUI + CD)QUIAG6 + (CUI + CR3)QUIR3 + (CUI + CR4)QUIR4 + (CUI + CR5)QUIR5 + (CUI + CR6) QUIR6 + (CUI + CTR + CB)QUIMI3 + (CUI + CTR + CB)QUIMI4 + (CUI + CTR + CB) QUIMI5 + (CUI + CTR + CB)QUIMI6 + (CSA + CD)QSAAG4 + (CSA + CD)QSAAG5 + (CSA + CD)QSAAG6 + (CSA + CR4)QSAR4 + (CSA + CR5)QSAR5 + (CSA + CR6)

QSAR6 + (CSA + CTR + CB)QSAMI4 + (CSA + CTR + CB)QSAMI5 + (CSA + CTR +

CB)QSAMI6 + (CD)QLSW1AG1 + (CR1)QLSW1R1 + (CTR + CB)QLSW1MI1 + (CD)

QLSW2AG2 + (CR2)QLSW2R2 + (CTR + CB)QLSW2MI2 + (CD)QLSW3AG3 + (CR3)

QLSW3R3 + (CTR + CB)QLSW3MI3 + (CD)QLSW4AG4 + (CR4)QLSW4R4 + (CTR + CB)

QLSW4MI4 + (CD)QLSW5AG5 + (CR5)QLSW5R5 + (CTR + CB)QLSW5MI5 + (CD)

QLSW6AG6 + (CR6)QLSW6R6 + (CTR + CB)QLSW6MI6 + (CD)QLSW7AG7 + (CR7)

QLSW7R7 + (CTR + CB)QLSW7MI7 + (CD)QLSW8AG8 + (CTR + CB)QLSW8MI8 +

(CD)QLSW9AG9 + (CTR + CB)QLSW9MI9 + (CP1 + CD)QGW1AG1 + (CP2 + CD)

QGW2AG2 + (CP3 + CD)QGW3AG3 + (CP4 + CD)QGW4AG4 + (CP5 + CD)QGW5AG5 +

(CP6 + CD)QGW6AG6 + (CP7 + CD)QGW7AG7 + (CP1 + CH + CB)QGW1MI1 +

(CP2 + CH + CB)QGW2MI2 + (CP3 + CH + CB)QGW3MI3 + (CP4 + CH + CB)

QGW4MI4 + (CP5 + CH + CB)QGW5MI5 + (CP6 + CH + CB)QGW6MI6 + (CP7 +

CH + CB)QGW7MI7 + (CRC + CR1)QWW1R1 + (CRC + CR2)QWW2R2 + (CRC + CR3)

QWW3R3 + (CRC + CR4)QWW4R4 + (CRC + CR5)QWW5R5 + (CRC + CR6)QWW6R6 +

(CRC + CR7)QWW7R7 + (CT23 + CD)QLSW2AG3 + (CT23 + CR3)QLSW2R3 +

(CT23 + CTR + CB)QLSW2MI3 + (CT31 + CD)QLSW3AG1 + (CT31 + CR1)

QLSW3R1 + (CT31 + CTR + CB)QLSW3MI1 + (CT34 + CD)QLSW3AG4 + (CT34 +

CR4)QLSW3R4 + (CT34 + CTR + CB)QLSW3MI4 + CT45 + CD)QLSW4AG5 +

(CT45 + CR5)QLSW4R5 + (CT45 + CTR + CB)QLSW4MI5 + (CT56 + CD)QLSW5AG6 +

(CT56 + CR6)QLSW5R6 +(CT56 + CTR + CB)QLSW5MI6 + (CS4)QBUS4 + (CS5)

QBUS5 + (CS6)QBUS6 + (CS3)QUIS3 + (CS4)QUIS4 + (CS5)QUIS5 + (CS6)

QUIS6 + (CS4)QSAS4 + (CS5)QSAS5 + (CS6)QSAS6 + (CS1)QLSW1S1 +

(CS2)QLSW2S2 + (CS3)QLSW3S3 + (CS4)QLSW4S4 + (CS5)QLSW5S5 + (CS6)

QLSW6S6 + (CS7)QLSW7S7 + (CS8)QLSW8S8 + (CS9)QLSW9S9 + (CS3)QLSW2S3 +

(CS1)QLSW3S1 + (CS4)QLSW3S4 + (CS5)QLSW4S5 + (CS6)QLSW5S6

    The cost coefficients for the objective function are an

important part of the model. The accuracy with which they are determined

may be the foundation of a particular allocation pattern. For best results, the costs should be determined for the specific area under study. As is the case in many areas, cost data for Utah are not readily available. The cost figures for this model have come from many sources, and in the absence of source material, estimates were made. In post optimal solution procedures, however, the sensitivity of the particular allocation to a change in cost can be checked and further refinement may be done on those costs which, if changed slightly, would affect the solution.

One of the major difficulties in the current research work is not being able to directly determine the cost of importing water to the Great Basin from the Colorado River Basin. The costs are buried in unidentified subsidies and proposed charge rates. In this instance, in lieu of better data, the suggested charge rates for Central Utah Project water will be used. This cost, approximately $4.00/ac. ft., will be used for CBU, CUI, and CSA.

The cost of distribution of water to the water users is referred to as $\underline{CD}$ in the objective function. With detailed study, a cost might be derived for each study area. In this model, one cost ($4.00 per acre foot) is used for the whole state. This figure corresponds to the range given by Milligan (1969) for a part of the Sevier drainage.

Recharge costs, $\underline{CR}$, for the state are somewhat limited since the practice of artificial recharge is seldom used in Utah. A figure of $15.00 per acre foot is used for this model. This figure comes from some California averages given by Todd (1965).

The pumping costs for the model are determined from a curve given by Milligan (1969) which relates cost in dollars per acre foot

to pumping lift in feet. As an average for the whole state, a pumping
list of 125 feet was assumed. This figure is a starting assumption;
research may give a different depth for each study area. Using this
pumping lift, the cost, CP, is $3.25 per acre foot.

For boosting water to a pressure head for municipal and
industrial use, a pumping lift of 140 feet was selected. This lift
corresponds to a pressure of 60 pounds per square inch. This gives a
cost, CB, of $3.80 per acre foot.

The cost of storage may be extremely variable with each
particular area and reservoir site. Again as with many of the other
costs, much more accuracy and detail could be gained through extensively
researching the costs in each area. For this model, the figure of
$6.00 per acre foot will be used for all storage costs. This cost
falls within the range given by Milligan (1969).

The cost of interbasin transfers of local surface water is one
with little supporting data. For this model, the figure of $4.00 per
acre foot will be used. This figure is the same as that for Central
Utah Project water.

The treatment costs for the model come from Dracup (1966). For
complete reclamation of sewage water, CRC, the cost is $20.80 per acre
foot. For rapid sand filtration and chlorination, CTR, the cost is
$7.70 per acre foot. For chlorination only, CH, the cost is $2.70
per acre foot.

Constraints and constants

The system of constraints will be discussed in the framework
of the topic with which they are concerned. The first constraint places

an upper bound on the total amount of water diverted from the Colorado
River. The constraint requires that the summation of all the alter-
natives for diversion be less than or equal to the amount of Colorado
River water to which Utah is entitled. This is accomplished by summing
the water supplied by natural flow and that supplied from storage
by subtracting the return flows, and setting the sum less than or equal
to the total allotment. This gives a depletion from the Colorado River
which is considered as use by the state. Mathematically the constraint
is written as follows: $(1-M4)QBUAG4 + (1-M5)QBUAG5 + (1-M6)QBUAG6 +$
$QBUR4 + QBUR5 + QBUR6 + (1-M4)QBUMI4 + (1-M5)QBUMI5 + (1-M6)QBUMI6 +$
$QBUS4 + QBUS5 + QBUS6 + (1-M3)QUIAG3 + (1-M4)QUIAG4 + (1-M5)QUIAG5 +$
$(1-M6)QUIAG6 + QUIR3 + QUIR4 + QUIR5 + QUIR6 + QUIS3 + QUIS4 + QUIS5 +$
$QUIS6 + (1-M3)QUIMI3 + (1-M4)QUIMI4 + (1-M5)QUIMI5 + (1-M6)QUIMI6 +$
$(1-M4)QSAAG4 + (1-M5)QSAAG5 + (1-M6)QSAAG6 + QSAR4 + QSAR5 + QSAR6 +$
$QSAS4 + QSAS5 + QSAS6 + (1-M4)QSAMI4 + (1-M5)QSAMI5 + (1-M6)QSAMI6 +$
$(1-M7)QLSW7AG7 + QLSW7R7 + (1-M7)QLSW7MI7 + QLSW7S7 - (L7)(QLSW7AG7 +$
$QGW7AG7) + (1-M8)QLSW8AG8 + (1-M8)QLSW8MI8 + QLSW8S8 - (L8)(QLSW8AG8) +$
$(1-M9)QLSW9AG9 + (1-M9)QLSW9MI9 + QLSW9S9 - (L9)(QLSW9AG9) \leq 1,440,000$

The next series of constraints deals with the amount of local
surface water which may physically be diverted for use in each area.
The treatment is the same for all areas, and the format is identical
to the above constraint. In other words, the water coming from natural
flow and the water coming through storage for each use are summed. Then
return flows are subtracted, and the result is set less than or equal
to the total availability of local surface water, as previously defined.
This method is used for each study area. The constraints are written

as follows:

Area 1 $(1-M1)QLSW1AG1 + QLSW1R1 + (1-M1)QLSW1MI1 + QLSW1S1 - (L1)$
$(QLSW1AG1 + QGW1AG1 + QLSW3AG1) \leq 800,000$

Area 2 $(1-M2)QLSW2AG2 + QLSW2R2 + (1-M2)QLSW2MI2 + QLSW2S2 + (1-M3)$
$QLSW2AG3 + QLSW2R3 + (1-M3)QLSW2MI3 + QLSW2S3 - (L2)(QLSW2AG2$
$+ QGW2AG2) \leq 970,000$

Area 3 $(1-M3)QLSW3AG3 + QLSW3R3 + (1-M3)QLSW3MI3 + QLSW3S3 + (1-M4)$
$QLSW3AG4 + QLSW3R4 + (1-M4)QLSW3MI4 + QLSW3S4 - (L3)(QLSW3AG3$
$+ QLSW2AG3 + QUIAG3 + QGW3AG3) \leq 800,000$

Area 4 $(1-M4)QLSW4AG4 + QLSW4R5 + (1-M4)QLSW4MI4 + QLSW4S4 + (1-M5)$
$QLSW4AG5 + QLSW4R5 + (1-M5)QLSW4MI5 + QLSW4S5 - (L4)$
$(QLSW4AG4 + QLSW3AG4 + QBUAG4 + QUIAG4 + QSAAG4 + QGW4AG4)$
$\leq 1,000,000$

Area 5 $(1-M5)QLSW5AG5 + QLSW5R5 + (1-M5)QLSW5MI5 + QLSW5S5 + (1-M6)$
$QLSW5AG6 + QLSW5R6 + (1-M6)QLSW5MI6 + QLSW5S6 - (L5)(QLSW5AG5$
$+ QLSW4AG5 + QBUAG5 + QUIAG5 + QSAAG5 + QGW5AG5) \leq 800,000$

Area 6 $(1-M6)QLSW6AG6 + QLSW6R6 + (1-M6)QLSW6MI6 + QLSW6S6 - (L6)$
$(QLSW6AG6 + QLSW5AG6 + QBUAG6 + QUIAG6 + QSAAG6 + QGW6AG6)$
$\leq 210,000$

The coefficients L1 through L6 reflect the percent return flow to local surface water from use to availability in each area. The coefficients M1 through M6 reflect the amount of use coming through storage. The percentage (1-M) is then the amount coming from natural flow. The (1-M) quantities plus the storage quantities for the area include all the water used. The numbers 1 through 6 refer to study areas involved. The right hand side for these constraints comes from Table 6.

The series of constraints relating to groundwater availability

have the same structure as do the local surface water constraints. The only change is that the recharge quantities appear as added availabilities. The constraints are as follows:

Area 1 QGW1AG1 + QGW1MI1 - QLSW1R1 - QLSW3R1 - QWW1R1 - (N1)(QLSW1AG1 + QGW1AG1 + QLSW3AG1) $\leq$ 46,000

Area 2 QGW2AG2 + QGW2MI2 - QLSW2R2 - QWW2R2 - (N2)(QLSW2AG2 + QGW2AG2) $\leq$ 295,000

Area 3 QGW3AG3 + QGW3MI3 - QUIR3 - QLSW3R3 - QLSW2R3 - QWW3R3 - (N3)(QLSW3AG3 + QGW3AG3 + QLSW2AG3 + QUIAG3) $\leq$ 75,000

Area 4 QGW4AG4 + QGW4MI4 - QBUR4 - QUIR4 - QSAR4 - QLSW4R4 - QLSW3R4 - QWW4R4 - (N4)(QLSW4AG4 + QLSW3AG4 + QBUAG4 + QUIAG4 + QSAAG4 + QGW4AG4) $\leq$ 402,000

Area 5 QGW5AG5 + QGW5MI5 - QBUR5 - QUIR5 - QSAR5 - QLSW5R5 - QLSW4R5 - QWW5R5 - (N5)(QLSW5AG5 + QLSW4AG5 + QBUAG5 + QUIAG5 + QSAAG5 + QGW5AG5) $\leq$ 286,300

Area 6 QGW6AG6 + QGW6MI6 - QBUR6 - QUIR6 - QSAR6 - QLSW6R6 - QLSW5R6 - QWW6R6 - (N6)(QLSW6AG6 + QLSW5AG6 + QBUAG6 + QUIAG6 + QSAAG6 + QGW6AG6) $\leq$ 138,800

Area 7 QGW7AG7 + QGW7MI7 - QLSW7R7 - QWW7R7 - (N7)(QLSW7AG7 + QGW7AG7) $\leq$ 40,000

Areas 8 and 9 have insufficient groundwater to allocate. The coefficients N1 through N7 reflect the percent of agricultural use which returns to groundwater as return flows. The numbers 1 through 7 refer to study areas, and the right hand sides come from Table 5.

Demand constraints are treated much like availability constraints. The summation of all the alternative sources of supply must be greater than or equal to the demand level in each area. The municipal and

industrial requirements are met as follows:

Area 1   QLSW1MI1 + QGW1MI1 + QLSW3MI1 $\geq$ 3,000

Area 2   QLSW2MI2 + QGW2MI2 $\geq$ 14,000

Area 3   QUIMI3 + QLSW3MI3 + QLSW2MI3 + QGW3MI3 $\geq$ 21,000

Area 4   QBUMI4 + QUIMI4 + QSAMI4 + QLSW4MI4 + QLSW3MI4 + QGW4MI4 $\geq$ 84,000

Area 5   QBUMI5 + QUIMI5 + QSAMI5 + QLSW5MI5 + QLSW4MI5 + QGW5MI5 $\geq$ 9,000

Area 6   QBUMI6 + QUIMI6 + QSAMI6 + QLSW5MI6 + QLSW6MI6 + QGW6MI6 $\geq$ 4,000

Area 7   QLSW7MI7 + QGW7MI7 $\geq$ 3,000

Area 8   QLSW8MI8 $\geq$ 5,000

Area 9   QLSW8MI8 $\geq$ 6,000

The right hand side values come from demands in Table 4.

The agricultural requirements are met as follows:

Area 1   QLSW1AG1 + QLSW3AG1 + QGW1AG1 $\geq$ 58,000

Area 2   QLSW2AG2 + QGW2AG2 $\geq$ 405,000

Area 3   QUIAG3 + QLSW3AG3 + QLSW2AG3 + QGW3AG3 $\geq$ 268,000

Area 4   QBUAG4 + QUIAG4 + QSAAG 4 + QLSW4AG4 + QLSW3AG4 + QGW4AG4 $\geq$ 289,000

Area 5   QBUAG5 + QUIAG5 + QSAAG5 + QLSW5AG5 + QLSW4AG5 + QGW5AG5 $\geq$ 392,000

Area 6   QBUAG6 + QUIAG6 + QSAAG6 + QLSW6AG6 + QLSW5AG6 + QGW6AG6 $\geq$ 132,000

Area 7   QLSW7AG7 + QGW7AG7 $\geq$ 293,000

Area 8   QLSW8AG8 $\geq$ 114,000

Area 9   QLSW9AG9 $\geq$ 64,000

The right hand side values come from Table 3.

The final series of constraints perform a transfer function. In the first group it is required that the waste water for recharge in any area be less than or equal to some percentage of the municipal and industrial use in that area. These constraints appear as follows:

Area 1 $QWW1R1 \leq K1(QLSW1MI1 + QLSW3MI1 + QGW1MI1)$

Area 2 $QWW2R2 \leq K2(QLSW2MI2 + QGW2MI2)$

Area 3 $QWW3R3 \leq K3(QUIMI3 + QLSW3MI3 + QLSW2MI3 + QGW3MI3)$

Area 4 $QWW4R4 \leq K4(QBUMI4 + QUIMI4 + QSAMI4 + QLSW4MI4 + QLSW3MI4 + QGW4MI4)$

Area 5 $QWW5R5 \leq K5(QBUMI5 + QUIMI5 + QSAMI5 + QLSW5MI5 + QLSW4MI5 + QGW5MI5)$

Area 6 $QWW6R6 \leq K6(QBUMI6 + QUTMI6 + QSAMI6 + QLSW6MI6 + QLSW5MI6 + QGW6MI6)$

Area 7 $QWW7R7 \leq K7(QLSW7MI7 + QGW7MI7)$

The second group requires that $\underline{M}$ percent of the use come from storage. These constraints are written as follows:

Area 1 $M1(QLSW1AG1) + M1(QLSW1MI1) = QLSW1S1$

        $M1(QLSW3AG1) + M1(QLSW3MI1) = QLSW3S1$

Area 2 $M2(QLSW2AG2) + M2(QLSW2MI2) = QLSW2S2$

Area 3 $M3(QLSW3AG3) + M3(QLSW3MI3) = QLSW3S3$

        $M3(QLSW2AG3) + M3(QLSW2MI3) = QLSW2S3$

        $M3(QUIAG3) + M3(QUIMI3) = QUI2S3$

Area 4 $M4(QLSW4AG4) + M4(QLSW4MI4) = QLSW4S4$

        $M4(QLSW3AG4) + M4(QLSW3MI4) = QLSW3S4$

        $M4(QBUAG4) + M4(QBUMI4) = QBUS4$

        $M4(QUIAG4) + M4(QUIMI4) = QUIS4$

        $M4(QSAAG4) + M4(QSAMI4) = QSAS4$

Area 5 $M5(QLSW5AG5) + M5(QLSW5MI5) = QLSW5S5$

        $M5(QLSW4AG5) + M5(QLSW4MI5) = QLSW4S5$

        $M5(QBUAG5) + M5(QBUMI5) = QBUS5$

        $M5(QUIAG5) + M5(QUIMI5) = QUIS5$

$$M5(QSAAG5) + M5(QSAMI5) = QSAS5$$

Area 6 $M6(QLSW6AG6) + M6(QLSW6MI6) = QLSW6S6$

$$M6(QLSW5AG6) + M6(QLSW5MI6) = QLSW5S6$$

$$M6(QBUAG6) + M6(QBUMI6) = QBUS6$$

$$M6(QUIAG6) + My(QUIMI6) = QUIS6$$

$$M6(QSAAG6) + M6(QSAMI6) = QSAS6$$

Area 7 $M7(QLSW7AG7) + M7(QLSW7MI7) = QLSW7S7$

Area 8 $M8(QLSW8AG8) + M8(QLSW8MI8) = QLSW8S8$

Area 9 $M9(QLSW9AG9) + M9(QLSW9MI9) = QLSW9S9$

There are many constants associated with the system of con-
straints. The first series of constants is M1 through M9. These con-
stants reflect the amounts of water, on a yearly basis, that must be
supplied by storage for use in areas 1 through 9. Then (1-M) is the
amount that is supplied by natural flow. These constants are deter-
mined by studying the flow hydrographs in connection with the demand
patterns and determining volumes of water supplied by each method. More
extensive study may give a factor for each area; however, in this model
one figure is used for the whole state. On an average the determina-
tions show that approximately 30 percent of the use comes from storage.
Figure 2 shows the method used in determination of this constant.

The next series of constants is the L1 through L9 group. These
constants reflect the percentage of water used for a particular use
which reappears in the local surface water system as a return flow in
each of the nine areas. These constants are determined by examination
of water budgets and by finding differences between diversion and con-
sumptive use. This amount is return flow; however, part of this water
returns to local surface water and part to groundwater. In arriving at
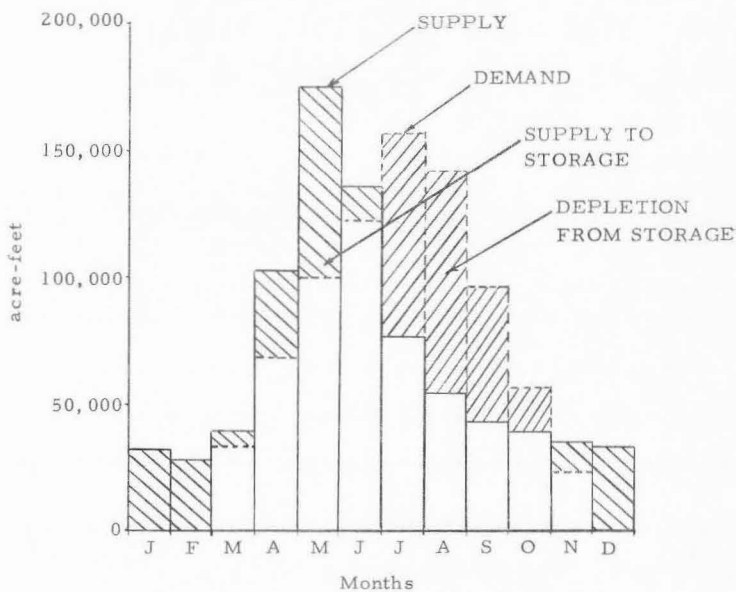
$$M = \frac{\text{DEPLETION FROM STORAGE}}{\text{TOTAL DEMAND}} = \frac{240,000}{800,000} = 0.30$$

Figure 2. Determination of Storage Constants

these constants it was assumed that 1/6 of the return flow returns to
local surface water while 5/6 returns to groundwater.

This leads to the constants N1 through N7 which are the per-
centages of return flows reappearing in groundwater systems as return
flows from each area. Table 7 gives constants $\underline{L}$ and $\underline{N}$ for each area
along with the percentage of the diversion which is not consumptively
used.

Table 7. Return flow constants

Hydrologic study area	Percent of ag. water not consumptively used	L	N
1	53.2	.089	.443
2	49.3	.082	.441
3	56.3	.094	.469
4	63.7	.106	.531
5	60.6	.101	.505
6	56.0	.093	.467
7	60.7	.101	.506
8	62.2	.104	.518
9	45.8	.076	.382

The final constant considered is the K1 through K7 group. This
constant reflects the percentage of municipal and industrial water
which is reclaimed and recharged to the groundwater basin in each area.
This constant not only reflects the amount of water which remains after
municipal and industrial use and reclamation but also the amount of this
water which can be recharged. In the state of Utah not much emphasis
is placed on artificial recharge; however, the potential exists.

Dracup (1966) indicates that about 35 percent of the municipal and industrial use could be recharged. This is the value of K1 through K7 in the model. Further refinement by area would increase the accuracy of the constraint should it be a critical activity in the solution.

APPLICATIONS OF THE MODEL

Optimal solutions

The main use of this allocation model is to arrive at an optimal allocation of resources. The optimal solution to this model would be the least cost method of allocation which would satisfy all of the demand requirements and mathematical assumptions made during formulation.

The computer printout of the optimal solution will give the name of the variable which is in the solution and the level of its activity. In other words, the solution would tell which sources to develop to satisfy the demands and how much water should be developed from each source.

The validity of the solution is completely based upon the input data in the form of cost coefficients, demand levels, amounts in availability, and constants. The solution is correct only to the extent that all the data are correct.

The optimal solution may give much valuable information about which parts of the resource should be developed and which should not. Even in the absence of absolute figures because of questionable input data, the relative magnitude of allocation patterns may be helpful.

An optimal solution also gives a range of costs and activities over which the variables in the solution are unchanged. If the cost data are reliable within the ranges given by the solution, the same variables would appear in the solution if the costs were exactly correct

down to the last penny. This fact gives a certain amount of flexibility to the determination and manipulation of the model.

For this model, in its preliminary stages, an optimal solution was obtained. This verifies the logic of the procedure used. The structure of the model is sound, and modifications and refinements will give more exact solutions.

Post optimal analysis

Perhaps the most valuable part of the linear programming technique is the use of post optimal analysis. As the name implies, there are a number of information gathering procedures which may be applied after the initial optimal solution is found.

Through a procedure known as parametric analysis the solution can be observed as parts of the model are changed systematically. Both the right hand side values and the cost coefficients can be parameterized either independently or simultaneously.

In this particular study, performing parametric analysis on right hand side values is of worthwhile significance. This procedure gives the opportunity to simulate the effect of time on the model. This is accomplished by systematically increasing the demand for water, both for agriculture and for municipalities and industry. By doing this and using demand projections for a certain future year, the effect of time can be simulated.

This parametric technique was applied to a preliminary model during the study. The system reacts logically to the changes by allocating more and more water as demand levels rise. Finally as demand levels became extremely high, the program was terminated because all

constraints could not be satisfied. In other words, the model gave a
demand level at which time there would be no additional water to allocate.

By parameterizing different right hand sides in the model, many
other things can be studied. For instance, if the model remained un-
changed except that all the groundwater availabilities were allowed to
double or triple their current levels, this in effect would allow a
study of the effect of relaxing or removing the groundwater laws pro-
hibiting the mining of groundwater. Many other types of changes like
this one would allow a comparison of the total cost of allocation under
various circumstances.

Critique of the model

In attempting to visualize the value of a model like the one
just formulated, the reader may feel that many areas of uncertainty
exist in the data. This fact is not as critical in the linear pro-
gramming approach as with many other analytical techniques. By formu-
lating the model using the best data at hand, not only is the logic of
the model tested, but also the sensitivity analysis of early solutions
will point out those parts of the model where changes in basic data
would affect the solution. This is an efficient approach since research
on all the data may not be needed.

There are several distinct advantages in using the linear pro-
gramming approach. One feature is the necessity for good descriptive
data on the region under study. In using this approach, the planner is
scientifically and numerically oriented. This insures more complete
study, and all available data are likely to be used. This scientific
approach is less likely to be used in other, more political approaches

to planning.

A major advantage of the linear programming approach over techniques like dynamic programming is the number of variables and constraints which can be handled. The model just formulated contains some 115 variables and 64 constraints. Many more could be handled with larger computer facilities.

The main attribute of computer solution is the simultaneous solution of the given set of constraints. In other words, the computer looks at all the possible alternatives at once, an extremely difficult task for manual computations.

SUMMARY AND CONCLUSIONS

A mathematical model for an allocation problem of this type is relatively simple to conceive, but difficult to formulate. The first part of the model is the objective function. In this model the objective function is composed of the costs of allocations for each possible alternative. The quantities of water allocated multiplied by the unit costs of allocations are summed for each possible alternative.

The second part of the model is an extensive series of equations or inequalities which describes the relationships between variables, requires demands to be met, assures that no more water will be allocated than is available, eliminates the possibility of negative flows, and in general describes the physical limitations of the system.

The computer then searches for the alternatives which will give the least cost allocation while satisfying all the other requirements of the model simultaneously.

In this study, an allocation model has been formulated. The logic of the approach has been proven with preliminary solutions. The results of early solutions pointed out areas where refinement was needed. The model was refined and expanded and again solutions were obtained. The sensitivity analysis of these solutions will now aid in determining which basic data may need further research.

In the future, the model will undoubtedly be further refined by appropriate changes in the objective function and constraints, basic data will be updated, and many more solutions will be obtained.

Parametric analysis will allow exploration of the effects that future physical changes might have on the allocation patterns.

Other studies in this area which may be worthwhile would be the expansion of the present model to include benefits. A study of this nature would then maximize net benefits rather than minimize total cost as this study has done.

An allocation model formulated by the linear programming approach is another valuable tool to help water planners find the best possible resource development pattern.

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