

THREE ESSAYS IN FORWARD RATE UNBIASEDNESS HYPOTHESIS

by

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## ABSTRACT

## Three Essays in Forward Rate Unbiasedness Hypothesis

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The objective of this dissertation is to verify and explain the forward exchange rate unbiasedness hypothesis in the foreign exchange market. Since in most of the cases the unbiasedness hypothesis fails to hold, we try to provide three different explanations of this puzzling behavior in the three essays. The first essay tries to resolve the forward premium puzzle by addressing the model misspecification issue and thereby adding a time-varying risk premium term in the percentage change specification. The risk premium term is modeled using the GARCH-M representation and the model is estimated by applying a GARCH (1, 1) specification. The second essay attributes the failure of the unbiasedness hypothesis to hold to the nonstationarity of the spot and forward exchange rate. It verifies the existence of a cointegrating relationship between the spot and the forward exchange rates and thus specifies an Error Correction Model to better capture the relation between the spot and the forward rates. Further, a cointegrating or the existence of a long run relationship between the spot and forward exchange rates and the domestic

and foreign interest rates is tested. It can be viewed as a robustness check where we ensure whether the cointegrated exchange rates are still related in the long run with the inclusion of the interest rates. The objective of the third essay is to apply the generalized method of moments (GMM) to test the unbiasedness hypothesis in the foreign exchange market. Empirical evidence suggests that the spot and forward rates are nonstationary with unit roots and are cointegrated. Cointegration further suggests that the changes in the spot rate can be modeled by an Error Correction Model. The third essay explicitly derives an ECM from the levels specification and uses the GMM estimation technique to test the unbiasedness hypothesis.

(116 pages)

## DEDICATION

This dissertation is dedicated to my father (late) Mr. Hiranmoy Biswas.

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## INTRODUCTION

With an increase in trade among countries, the significance of foreign exchange market has been growing. Any form of currency trading is associated with risk due to changes in the currency exchange rate. Such risks can be lowered by the use of derivatives. The forward rate which is a derivative emerged as a result of market's reaction to the risk associated with floating exchange. "The forward exchange rate  $f_t$  observed for an exchange at time  $t+1$  is the market determined certainty equivalent of the future spot exchange rate  $s_{t+1}$ " (Fama, 1984, p. 320). The forward rate is an unbiased predictor of the future spot rate since it fully reflects available information about exchange rate expectations (Chiang, 1988).

In international asset market the relation between the forward and spot prices of foreign exchange is of concern for investors, portfolio managers, and policy makers. Any international transaction involving foreign exchange is risky due to unexpected change in exchange rates. Hence, it is important that the forward rates are efficient and rational forecasts of future spot rates.

The study of the unbiasedness hypothesis, the forward rate is an unbiased predictor of the future spot rate, is important because it answers questions such as whether a market participant can do better than accepting the forward rate as an optimal predictor of the future spot rate and whether central banks can control the exchange rate by intervention in the foreign exchange market. Another important question is that whether the gap between the forward rate and the expected future spot rate can be attributed to an exchange risk premium.

In finding a solution to the puzzle that the hypothesis has not been supported by empirical evidence, existing research has taken two main directions - the risk-premium approach and the non-rationality approach. Another explanation for this puzzling behavior lies in the use of statistical and econometric estimation techniques.

The objective of this dissertation is to verify and explain the forward exchange rate unbiasedness hypothesis. Since in most of the cases the hypothesis fails to hold, we try to provide three different explanation of this puzzling behavior in the three essays.

The rejection of the forward rate unbiasedness hypothesis can be attributed to a misspecified theoretical model. In the first essay, we consider the misspecification to be in the form of exclusion of an explanatory variable, the risk premium. This essay tries to resolve the puzzle by addressing the model misspecification issue and thereby adding a time-varying risk premium term in the percentage change specification of the test of the unbiasedness hypothesis.

The risk premium term was modeled by Domowitz and Hakkio (1985) using the ARCH representation. For the date used in this dissertation, in most of the cases, the statistical tests confirm that the error term is heteroskedastic and follows a GARCH (1, 1) process. The tests suggest the use of the GARCH model instead of the ARCH model to model the risk premium more accurately. Nieuwland, Verschoor, and Wolff (2000) estimated the risk premium by a GARCH (1, 1) process using survey data for a set of EMS currencies from 1986 to 1991.

The unique contribution of this essay lies in the use of the GARCH model applied to the exchange rates for the period January 1991 to February 2008 for U.K., Canada,

Australia, and Japan, the four advanced economies and for India, the emerging market economy, the data ranges from January 1999 to February 2008.

The second essay attributes the failure of the unbiasedness hypothesis to hold to the nonstationarity of the spot and forward exchange rates. It verifies the existence of a cointegrating relationship between the spot and the forward exchange rates and thus specifies an Error Correction Model (ECM) to better capture the relation between the spot and the forward rates. Further, a cointegrating or the existence of a long run relationship between the spot and forward exchange rates and the domestic and foreign interest rates is tested. It can be viewed as a robustness check where we ensure whether the cointegrated exchange rates are still related in the long run with the inclusion of the interest rates.

The objective of the third essay is to apply the generalized method of moments (GMM) to test the unbiasedness hypothesis. Empirical evidence suggests that the spot and forward rates are nonstationary with unit roots and are cointegrated. Cointegration further suggests that the changes in the spot rate can be modeled by an ECM. Naka and Whitney (1995) and Bakshi and Naka (1997) explicitly derives an ECM under the assumption that the spot and the forward rates are cointegrated, the first difference of forward rates is stationary, and the first order autocorrelation in the forecast error is allowed. Their results show that when tests of the unbiasedness hypothesis are conducted with an ECM using GMM, the unbiasedness hypothesis cannot be rejected. The data used in Bakshi and Naka (1997) range from January 1974 to April 1991 for Canada, U.K., Japan, France, Italy, Germany, and Switzerland. The third essay applies the above

mentioned ECM and the GMM estimation technique to test whether the same results hold for a different (more recent) time period for U.K., Canada, Australia, Japan, and India.

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## ESSAY 1: EXPLAINING FORWARD RATE UNBIASEDNESS HYPOTHESIS, THE RISK PREMIUM APPROACH

### **1. Introduction**

The origin of the modern foreign exchange market dates back to early 1970s when the Bretton Woods system collapsed and countries gradually switched from fixed to floating exchange rates. The foreign exchange market helps international trade and investment. It is the market in which international currency is traded. Its major participants are the commercial banks, corporations engaging in international trade, non bank financial institutions (asset management firms and insurance companies) and the central banks as well as individuals. Considered to be the largest financial market in the world, the significance of foreign exchange market has been growing with an increase in trade among countries. The average daily trade in the global forex and related markets currently is over US \$3 trillion (Triennial Central Bank Survey (April 2007), Bank for International Settlements).

In a floating exchange rate system a currency's value is allowed to fluctuate. Any form of currency trading is associated with risk due to changes in the currency exchange rate. Hence, with the introduction of the floating exchange rate system, uncertainty regarding the future value of the currency emerged as a concern in the area of international trade and finance. However, the market came up with an instrument, the forward rate, to deal with the risk of uncertainty. The forward rate which is a derivative emerged as a result of market's reaction to the risk associated with floating exchange rate and is thus an endogenous innovation. A derivative is a financial contract or instrument,

whose value is derived from the value of an underlying asset. The main types of derivatives are futures, forwards, options, and swaps.

The foreign exchange risk can be dealt with by engaging in a forward transaction, in which, a buyer and seller agree on an exchange rate for any date in the future, and the transaction occurs on that date, at that agreed exchange rate, regardless of what the market rates are then. The duration of the trade can be a few days, months or years. Money does not actually change hands until the agreed upon future date. On the other hand, a spot transaction represents a direct exchange between two currencies involving cash rather than a contract, the amount being paid within a couple of days. It has the shortest time frame, and interest is not included in the agreed-upon transaction.

In the “simple efficiency” specification of forward exchange markets, it is often argued that the forward rate “fully reflects” available information about the exchange rate expectations; the forward rate, thus, is usually viewed as an unbiased predictor of the future spot rate. (Chiang, 1988, p. 212).

Unbiased implies that there is no obvious alternative predictor that performs better on average. According to the efficient market hypothesis (EMH), financial markets are “informationally efficient,” that is, the price on traded assets already reflect all known information or collective beliefs of all investors about future prospects and therefore are unbiased. The foreign exchange markets are efficient in the sense that the expected rate of return to speculation in the forward exchange market will be zero. EMH holds if economic agents are risk neutral, the market is competitive and uses all available information rationally, and if taxes, transaction costs, or other frictions are ruled out. The implication of EMH is that forward rates should be unbiased forecasts of future spot rates

since forward exchange rates fully reflect available information about investor's expectations of future spot rates.

In testing market efficiency, we jointly test two null hypotheses: one is the market efficiency hypothesis and the other is the unbiasedness or rational expectations hypothesis. If we fail to reject the null hypothesis that the forward rate is an unbiased predictor of the future spot rate then we can use the forward rate available at time  $t$  as a proxy for the prediction of the spot rate at time  $t+1$ .

A large number of studies have been conducted in the past to test whether the forward rate is an unbiased predictor of the future spot rate. In the existing literature, the unbiasedness hypothesis (UH hereafter) is tested by an estimation technique which regresses the log of the current spot ( $s_t$ ) on the one-period lagged log of the forward rate ( $f_{t-1}$ ). The spot rate is defined as domestic units per foreign units. We then proceed to test the joint hypothesis that the constant term does not differ from zero, the coefficient on the one-period lagged forward rate does not significantly differ from one, and that the error term is free of serial correlation.

$$s_t = \beta_0 + \beta_1 f_{t-1} + \varepsilon_t$$

which is same as:

$$s_{t+1} = \beta_0 + \beta_1 f_t + \varepsilon_{t+1} \dots\dots\dots (1)$$

Due to the nonstationarity properties of the spot and the forward rates, tests based on a level regression of the future spot rate on the forward rate resulted in spurious regression problems. This led researchers to adopt a "difference" version of the log level regression in which the log current spot rate is subtracted from the one period future log

spot and the log forward rate. We thus consider the regression of the change in the log of the spot exchange rate on the forward discount (expressed in log form):

$$s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \varepsilon_{t+1} \dots\dots\dots (2)$$

The change in the log of the spot exchange rate,  $(s_{t+1} - s_t)$  is the expected depreciation and  $f_t - s_t$  is the forward discount.

The null hypothesis to be tested in both the level and percentage change specification is:  $H_0: \beta_0 = 0$  and  $\beta_1 = 1$  and  $E_t[\varepsilon_{t+1}] = 0$ .

In much of the research on this topic (Engel, 1996; Froot & Thaler, 1990) the sound theoretical foundations and theoretically elegant hypothesis have not been supported by the empirical evidence which has demonstrated the puzzling result that the slope coefficient is significantly less than unity and most often negative in sign. This implies that one cannot use the forward rate directly as a measure for the future spot rate. Thus the empirical evidence suggests that forward rates are neither efficient nor rational forecasts of future spot rates. This is the Forward Premium Puzzle, indicating foreign exchange market inefficiency which holds in spite of large trading volumes and low trading costs in currency markets.

The theoretical foundations and hypothesis have not been supported by empirical evidence. This might be due to the use of an inappropriate or misspecified model to test the hypothesis or some kind of empirical error (for example, not accounting for non-normality or nonstationarity in the data or improper statistical technique employed to test the hypothesis) or both.

### *1.1 Objective*

The forward rate on many occasions has been found to be a biased predictor of the future spot rate and the negative bias has been attributed to a time-varying risk premium, irrational expectations, or central bank intervention. The objective of this paper is to explain the Forward Premium Puzzle by investigating the existence of a risk premia. The literature explores alternative methodologies to measure time-varying premia. First, models were examined based on the time series properties of spot and forward exchange rates with the forecast errors being conditionally heteroskedastic. A second approach attempts to test specific theories of the risk premium. Another approach investigates the existence of time-varying risk premia by measuring expected depreciation directly using information from surveys (Nieuwland, Verschoor, & Wolff, 2000).

The rejection of the forward rate unbiasedness hypothesis (FRUH hereafter) or the existence of the forward premium puzzle can be attributed to a misspecified theoretical model. In this paper we consider the misspecification to be in the form of exclusion of an explanatory variable, the risk premium. The UH equates the forward rate with the expected future spot rate. While the former is observed with certainty, the later is an expected value. Future spot rate is unknown (random or stochastic) and the only way we can characterize it is in terms of a probability distribution. Uncertainty regarding the expected value of the future spot rate introduces an element of risk which gives rise to an extra compensation, a risk premium. Section 2 presents the role of risk premium in explaining the FRUH followed by a literature review in section 3.

In testing the FRUH we need to know how the forward rate is determined. Forward rate is a derivative, its value being derived from the value of the spot rate. In

section 4, we review the models of exchange rate determination. Provided that the spot rate and the interest rates are taken as given, the forward exchange rate can be determined by the Covered Interest Arbitrage.

In order to investigate the existence of risk premium in the foreign exchange market and its role in explaining the UH we need to incorporate the risk premium term in the empirical model used to test the UH. Our next task will be to describe an economic model of risk premium. If the probability distribution changes over time we do not have a constant risk premium but a time varying risk premium. Lucas (1982) described the nature of the risk premium in a complete dynamic general equilibrium model of interest and exchange rate determination. The spot and forward exchange rates and the risk premium are determined in the Lucas model. Since the risk premium depends on the conditional covariance of the intertemporal marginal rates of substitution (thus implicitly on the concavity of the utility function and the probability distribution function of the exogenous processes), empirical tests of such models are complex. We thus need an econometric model of risk premium, a specification which captures some major aspects of risk in a foreign exchange contract. Following Domowitz and Hakkio (1985), the risk premium is a function of the conditional variance of the forecast error (in forecasting the spot rate using the forward rate) which are assumed to follow the ARCH process. The conditional variance of the error is defined as a function of past information which includes past forecast errors. “This particular model also captures several empirical regularities noted in the estimation of exchange rate models, including conditional heteroskedasticity in the forecast errors....” (Domowitz & Hakkio, 1985, p. 49). Section 5 presents the economic model of foreign exchange risk premium followed by an

econometrically testable model in section 6. Sections 7 and 8 describe the data and the model estimation. The empirical results are presented in section 9 with the conclusions in section 10.

## 2. Role of Risk Premium in Forward Rate Unbiasedness Hypothesis

According to the UH, the forward rate represents the market's expectation of the future spot rate. UH holds under the assumptions that markets are efficient, agents are risk neutral and have rational expectations and is stated as:

$$E_t[s_{t+1}] = f_t \dots\dots\dots (3)$$

where  $f_t$  is the log forward rate at time  $t$  for the delivery of a currency at time  $t+1$ ,  $s_{t+1}$  is the corresponding log spot rate at time  $t+1$  and  $E_t[.]$  is the conditional expectation based on information available at time  $t$ .

Given the assumption of rational expectation,

$$s_{t+1} = E_t(s_{t+1}) + \varepsilon_{t+1}$$

The forward rate is then an unbiased predictor of the future spot rate,

$$s_{t+1} = f_t + \varepsilon_{t+1} \dots\dots\dots (4)$$

$\varepsilon_{t+1}$ , a random variable with  $E_t[\varepsilon_{t+1}] = 0$  is the rational-expectation forecast error.

Rejection of the null hypothesis leads to the conclusion that the forward rate is not an unbiased predictor of the future spot rate or that the forward rate does not represent the market's expectation of the future spot rate. The equilibrium relationship between the forward rate and the future spot rate is misspecified and specification errors in the model lead to biased estimates. In order to remove the bias we need to incorporate a relevant explanatory variable in the model. Inclusion of a time-varying risk premium is the most

widely accepted treatment for the misspecification. The risk premium is the extra expected return that investors demand in compensation for holding a currency that they perceive as riskier than others. Thus the relevant equation is obtained by adding a risk premium (rp) term in equation (2). The risk premium is defined as the gap between the forward discount ( $f_t - s_t$ ) and the expected depreciation ( $s_{t+1} - s_t$ ).

The forward discount is expressed as the sum of the unobserved expected depreciation in the spot rate and a risk premium<sup>1</sup>. Hence, the forward discount premium can be decomposed into expected depreciation and the risk premium:

$$fd_t = \Delta s_{t+1}^e + rp_t \dots\dots\dots (5)$$

The UH consists of two hypotheses that are tested at the same time.

1. the rational expectation hypothesis :  $\Delta s_{t+1} = \Delta s_t^e + \varepsilon_{t+1}$

(Investors predict the change in the exchange rate with a purely random error term).

2. the zero exchange risk premium hypothesis :  $fd_t - \Delta s_t^e = rp_t = 0$

The forward rate is the market determined certainty equivalent of the future spot rate which Fama splits into an expected future spot rate and a (risk) premium,

$$f_t = E_t[s_{t+1}] + rp_t \dots\dots\dots (6)$$

A number of empirical tests have lead to the conclusion that  $\beta$  is significantly less than 1 and also negative. According to Fama (1984), the negative  $\beta$  is due to the presence of a time varying risk premium ( $rp_t$ ), where

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<sup>1</sup> A forward discount is the proportion by which a country's forward exchange rate exceeds its spot rate. The forward discount is determined by the interest rate differential between the countries.

$$rp_t = f_t - E_t[s_{t+1}] \dots\dots\dots (6)$$

If we assume risk neutrality, agents would equate  $f_t$  with  $E_t[s_{t+1}]$  so that expected profits from forward market speculation would be zero. However, if  $f_t > E_t[s_{t+1}]$  then the investor incurs a premium from buying the foreign currency forward at time  $t$  relative to its expected price on the spot market at time  $t+1$ . Thus the relevant equation is obtained by adding a risk premium term in equation (2). This is the first interpretation of the forward premium puzzle.

### 3. Literature Review

There exists a large literature testing the joint hypothesis of market efficiency and time-invariant risk premia in foreign exchange markets. The tests on the validity of the market efficiency may be classified into two groups, one testing UH and the other the EMH. As observed by several authors, the results of these studies are amazingly varied. Geweke and Feige (1979) attributed the inefficiency in foreign exchange markets to market participants' risk-averse behavior combined with the existence of transaction costs. According to Frankel and Poonawala (2006), 30 years ago researchers found that the forward rate is a biased predictor of the future spot rate. "[I]n a regression of the future change in the spot rate against the forward discount, the exchange rate was found on average to move in precisely the opposite direction from what was predicted" (p. 2).

The first tests included Hansen and Hodrick (1980) who rejected the EMH from the 1970s and the 1920s. Longworth (1981) has rejected the joint null hypothesis of an efficient exchange market and no risk premium for the period ending in October 1976. The market efficiency hypothesis have been rejected by Fama (1984), Hakkio and Rush

(1989), and Sephton and Larsen (1991). This rejection has been credited to factors such as presence of risk premiums in forward rates, the (negative) correlation between the forward risk premiums and expected future spot rates, empirical irregularities in regression tests, and the use of inappropriate econometric techniques. The conflicting results found in the literature depend upon the particular econometric specification and estimation technique, differences in the time period of estimation and currencies.

According to a version of the UH, other than a constant risk premium, the difference between forward rates and realized spot rates is unpredictable. In the existing literature, we find extensive use of two econometric specifications to test the UH. The first is a “level” specification in which the realized spot rate is regressed on the one-period forward rate. The other is the “percent change” specification in which the percent change in the realized spot rate relative to the current spot rate is regressed against the forward premium or the difference between the forward and spot rates. Some of the tests of unbiasedness suffer from specification error (misspecification). “The time series properties of spot and forward exchange rate data rule out certain econometric specifications used to test for the UFRH” (Barnhart & Szakmary, 1991, p. 246).

Gregory and McCurdy (1984) have addressed the misspecification issue while Chiang (1988) has taken a stochastic coefficient approach. Since the foreign exchange markets are subject to common external shocks, Chiang uses Zellner's (1962) seemingly unrelated regression (SUR) technique to estimate the related equations. This is more efficient and consistent with an efficient market analysis than single equation estimation techniques.

The regression analysis shows that the parameters  $\alpha_t$  and  $\beta_t$  in the “simple efficiency” specification are sensitive to the newly available information and vary throughout all the sub sample periods. Our research suggests that the time-series properties of the parameters should be exploited effectively and incorporated into the exchange rate predictions. (Chiang, 1988, p. 230-231).

Froot and Frankel (1989) relax the joint assumptions of risk neutrality and rational expectation and attribute (empirically) the bias to the systematic forecast error, which is negatively correlated with the forward premium, rather than to a forward market risk premium.

Engel (1996) states that the UH represents the equilibrium condition when markets are efficient, agents are risk-neutral and have rational expectations. It does not rely on assumptions about the environment of agents, the nature of preferences or of technology. “However, agents value returns in real terms. The real return on a financial asset will depend on the environment and preferences of the risk-neutral agent” (Engel, 1996, p. 132). The market efficiency condition (no real profit opportunities) for a domestic agent is given by

$$E_t \left[ \frac{F_t - S_{t+1}}{P_{t+1}} \right] = 0 \dots\dots\dots (7)$$

where the variables are in levels and  $P_{t+1}$  is the domestic price level. Assuming that all variables are conditionally log-normally distributed, this expression is written as

$$E_t[s_{t+1}] = f_t - 0.5\text{Var}_t(s_{t+1}) + \text{Cov}_t(s_{t+1}, p_{t+1}) \dots\dots\dots (8)$$

where  $p_{t+1}$  is the logarithm of the domestic price level. Here, the forward rate is not an unbiased predictor because of the presence of the two additional terms  $0.5\text{Var}_t(s_{t+1}) + \text{Cov}_t(s_{t+1}, p_{t+1})$  commonly referred to as the Jensen’s inequality terms

(JIT). Many authors have tested the hypothesis that the JIT significantly affect the estimation results (Backus, Gregory, & Telmer, 1993; Engel, 1984; McCulloch, 1975). Their results show that including the JIT do not change the bias of the slope coefficient.

The empirical tests on the UH are inconclusive and conflicting. The UH is supported by Cornell (1977) and Kohlhagen (1979), but Levich (1979), Bilson (1981), Gregory and McCurdy (1984), Hodrick and Srivastava (1986), among others, have rejected the UH. Similarly, other studies (e.g., Barnhardt & Szakmary, 1991; Domowitz & Hakkio, 1985; Edwards, 1982; Lin & Chen, 1998) have also provided mixed results for the UH. Such uncertainty is attributed to the limitations of the statistical procedures used. Cornell (1989) interprets the bias due to two measurement errors in the data used to test the UH. The first is because the data used are bid or ask rate or average of them, thus neglecting transaction cost information embodied in the bid-ask spread. The second is timing problem between the forward rate delivery day and the corresponding spot rate. However, Bekaert and Hodrick (1993) show that Cornell's interpretations do not significantly change the slope coefficient. Breuer and Wohar (1996) find that the sampling problems account for some, but not all of the bias in the coefficient on the forward premium.

A large number of recent papers empirically testing the UH adopt some advanced econometric techniques in order to overcome the problems faced in past due to inadequate statistical techniques. Hsu and Kugler (1997) analyze the UH using a nonlinear impulse-response function approach. Estimating an exponential GARCH-in-mean model, they reject the UH and find that the forward premium has a nonlinear influence on the spot rate. According to Baillie and Bollerslev (2000), the forward

premium puzzle is due to small sample sizes and persistent autocorrelation of the forward premium. They estimate a fractionally integrated GARCH-in-mean (FIGARCH-M) model for the German mark / US dollar parity. Roll and Yan (2000) argue that the forward rate is an unbiased predictor of the future spot rate and suggest that the puzzle arises because the forward rate, the spot rate and the forward premium follow nearly nonstationary time series processes.

It has been empirically verified that the slope coefficients are significantly different from unity which can be viewed as a sign of the presence of a risk premium ( $rp_t$ ). “The equilibrium relationship between the forward rate and the  $k$ -period ahead spot rate is misspecified. Inclusion of a time-varying risk premium is the most widely accepted treatment for the misspecification” (Breuer, 2000, p. 211). The existence of a time-varying risk premia in the foreign exchange market has been documented in the literature by Hansen and Hodrick (1980), Hodrick (1981), Frankel (1982), Fama (1984), Hodrick and Srivastava (1984), Domowitz and Hakkio (1985), Canova and Ito (1991), Breuer (2000), Backus et al. (2002), and Verdelhan (2006).

Hodrick (1981) modifies the asset pricing model to allow time-varying expected returns on assets, eliminates the assumption of a risk-free real asset and derives the characteristics of the risk premia in the forward market as well as the equilibrium yield relationships among the equities and riskless nominal bonds of all countries.

Hodrick and Srivastava (1984) use a statistical model that examines the determination of risk premiums in foreign exchange markets based on a theoretical model of asset pricing. In examining the robustness of these tests to time variation in parameters and to the presence of heteroskedasticity, they find evidence for heteroskedasticity and

that the conditional expectation of the risk premium is a nonlinear function of the forward premium. Accounting for this nonlinearity, the specification appears to be time invariant.

Canova and Ito confirmed the presence of a risk premium using VAR model. “The conditional variance of the risk premium changed over time, but its unconditional distribution seems stable across sub samples. Despite these features, the volatility of the series was substantial and varied considerably throughout the sample” (Canova and Ito, 1991, p. 140). Breuer (2000) explains how “[T]he length of the forward contract, may, in part, contribute to the time-varying risk premium and its effect on the bias in the coefficient on the forward premium” (p. 218) Verdelhan (2006) talks about habit based explanation of the exchange rate risk premium. The model reproduces the uncovered interest rate parity puzzle, based on a time-varying business-cycle related risk premium where agents have preferences with external habits.

Thus, we can state that in spite of a huge body of literature and empirical studies in this area there is still a scope of further research in terms of coming up with an improved econometric technique that is capable of testing the joint null hypothesis of efficiency and unbiasedness for the foreign exchange market. For example, some studies while testing the UH have taken into consideration nonstationarity but have not corrected the statistical techniques for non-normality in the data. With the help of better statistical procedures and longer time periods we should be able to eliminate ambiguity in our empirical results.

Along with a continuous improvement in the estimation techniques, the null hypothesis that the forward rate is an unbiased predictor of the future spot rate has been questioned time and again. One reason for the rejection of the null hypothesis is due to

the misspecification of the model which can be corrected by including a time-varying risk premium. We aim at testing the UH by modeling the risk premia using the GARCH-M representation. We measure the exchange risk premia conditional on the assumption of rational forecasts of exchange rates using data for Australia, Canada, India, Japan, and U.K.

#### **4. Exchange Rate Determination**

Foreign exchange is a financial asset and exchange rate is an asset price. Asset is a form of wealth, a way of transferring purchasing power from present to the future. Current asset price is related to the purchasing power that buyers expect it to yield in future. Current dollar/pound exchange rate is related to peoples' expectations about the future level of that rate. The demand for a currency as a financial asset relative to the demands of other currencies determines the price of a currency. This demand is based on the utility this currency provides in terms of a medium of exchange, store of value and unit of account. The demand for an asset depends on its return and risk relative to a market index but demand for foreign exchange is based on a broader range of arguments. The foreign exchange market cannot be explained by the capital market theory alone (which is used to determine asset price), hence it should be combined with macroeconomic theory to explain the exchange rate movements. Exchange rates are relative prices between two currencies which are determined by the desire of residents to hold domestic and foreign financial assets. Some of the models of exchange rate determination discussed are discussed below.

#### *4.1 Purchasing power parity*

Purchasing power parity (PPP) states that in the long run, the exchange rate between two countries' currencies equals the ratio of the countries' price level.

$$E_{\$/\pounds} = P_{US} / P_{UK}$$

PPP follows from the law of one price, which states that in competitive markets, identical goods will sell for identical prices when valued in the same currency. It relates to an individual product and its generalization is the absolute version of PPP. Relative PPP relates to changes in prices and exchange rates, rather than on absolute price levels. It states that change in exchange rates is proportional to the change in the ratio of the two nations' price levels, structural relationships remaining unchanged. The assumptions for PPP to hold are that goods are identical, all goods are tradable, there are no transportation costs, information gaps, taxes, tariffs, or restrictions of trade, and exchange rates are influenced only by relative inflation rates. Due to these restrictive assumptions and empirical violation of the law of one price which is the building block of PPP, monetary models of exchange rate determination was adopted.

#### *4.2 Monetary model of exchange rate determination*

Since currencies are considered assets, exchange rates are asset prices that adjust to equilibrate international trade in financial assets. Like other asset prices, exchange rates are determined by expectations about the future. Since currencies are treated as assets this approach is called the asset approach.

The exchange rate determination is influenced only by money demand and money supply factors and thus it is known as the monetary approach. According to the monetary

approach to exchange rate determination, exchange rates are determined through the process of matching the total supply of and the total demand for the domestic money in each country. It is a long run theory since it allows price to adjust instantaneously to maintain full employment and PPP. It is assumed that in the long run the foreign exchange market sets the rate such that PPP holds or there are no market rigidities which prevent the exchange rate and other prices from immediately adjusting to the levels consistent with full employment. It also assumes that capital is fully mobile across national borders, and that domestic and foreign assets are perfect substitutes.

In the simplified version, the monetary approach combines the PPP theory with the quantity theory of money - increases or decreases in the money supply lead to proportionate increases or decreases in the price level over time, without any permanent effects on output or interest rates.

The domestic money market is in equilibrium when the demand for money equals the supply of money. Hence domestic price levels in US and UK can be expressed in terms of domestic money demands and supplies.

$$P_{US} = M_{US}^S / L(R_{\$}, Y_{US}) \dots\dots\dots (9a)$$

$$P_{UK} = M_{UK}^S / L(R_{\pounds}, Y_{UK}) \dots\dots\dots (9b)$$

where  $M^S$  is the money supply which can be controlled by the nation's monetary authorities and  $L(R, Y)$  is the aggregate real money demand which is negatively related to the interest rate,  $R$ , and positively related to the real output,  $Y$ .

The monetary approach states that the exchange rate which is the relative price of the two currencies, is fully determined in the long run by the relative money supply and

money demand. Any change in the interest rates and output affect the exchange rate only through their influence on money demand. Thus we can define the exchange rate as:

$$E_{\$/\pounds} = [M_{US}^S / L(R_{\$}, Y_{US})] / [M_{UK}^S / L(R_{\pounds}, Y_{UK})] \dots\dots\dots (10)$$

The monetary model can be extended to allow individuals or firms to hold a portfolio of two or more currencies and substitute among currencies. However demand for assets beyond currency is not considered. Modeling real disturbances within a monetary framework may not be efficient. These problems are resolved in the portfolio balance approach which specifies asset demand functions and provides an explicit role for the current account.

The portfolio balance model is an asset market model of short-run exchange rate determination based on the relative price of assets, specifically with the relationship between the relative price of domestic and foreign bonds and the exchange rate. Demand for currencies in the foreign exchange market is derived from demand for financial assets. It is assumed that supply and demand is affected by changes in monetary and/or fiscal conditions. Empirical tests of the portfolio balance model have not met with great success (Levich, 1983).

#### *4.3 Foreign exchange market equilibrium*

The condition that deposits of all currencies offer the same expected rate of return when measured in the same currency is called the interest parity condition. The implication is that holders of foreign currency deposits consider them all to be equally desirable assets.

The foreign exchange market is in equilibrium only when the interest parity condition holds, that is, there exists no excess supply of some type of deposit and no excess demand for another. The interest parity condition holds when expected rates of return are equal and only then is the foreign exchange market in equilibrium.

$$R_{\$} = R_{\pounds} + (E_{\$/\pounds}^e - E_{\$/\pounds}) / E_{\$/\pounds} \dots\dots\dots (11)$$

where  $E_{\$/\pounds}^e$  and  $E_{\$/\pounds}$  is the expected and current dollar pound exchange rate and  $R_{\$}$  and  $R_{\pounds}$  are the US and UK interest rates, respectively.

#### 4.4 Covered interest rate parity

Under the assumption that the interest parity condition always holds, a forward exchange rate equals the spot exchange rate expected to prevail on the forward contract's value date. The covered interest parity condition defines the foreign exchange market equilibrium involving the forward exchange rate rather than the expected future spot exchange rate as in the uncovered interest parity condition. This condition states that the rates of return on dollar deposit and covered foreign deposit must be the same. The covered return on pound deposit is:

$$[F_{\$/\pounds}(1 + R_{\pounds}) - E_{\$/\pounds}] / E_{\$/\pounds}$$

which approximately equals

$$R_{\pounds} + [F_{\$/\pounds} - E_{\$/\pounds}] / E_{\$/\pounds}$$

when the product  $R_{\pounds} * [F_{\$/\pounds} - E_{\$/\pounds}] / E_{\$/\pounds}$  is a small number. The covered interest parity condition is then stated as:

$$R_{\$} = R_{\pounds} + [F_{\$/\pounds} - E_{\$/\pounds}] / E_{\$/\pounds} \dots\dots\dots (12)$$

$[F_{\$/\pounds} - E_{\$/\pounds}] / E_{\$/\pounds}$  is the forward premium on pound against dollar or the forward discount on dollar against pound.

The covered interest parity condition is thus stated as: the interest rate on dollar deposits equals the interest rate on pound deposits plus the forward premium on pound against dollar.

The uncovered interest parity condition which is:

$$R_{\$} = R_{\pounds} + (E_{\$/\pounds}^e - E_{\$/\pounds}) / E_{\$/\pounds}$$

equals the covered interest parity condition when the forward rate quoted today equals the expected future spot exchange rate, that is,  $F_{\$/\pounds} = E_{\$/\pounds}^e$ . Hence the forward rate is an unbiased predictor of the future spot rate.

## 5. Economic Model of Foreign Exchange Risk Premium

The uncovered interest parity (UIP) states that when domestic interest rate is higher than the foreign interest rate the domestic currency is expected to depreciate by an amount approximately equal to the interest rate differential.

$$s_{t+k}^e - s_t = R_{\$} - R_{\pounds}$$

If foreign exchange market participants are risk averse, they would demand a higher rate of return than the interest differential in return for the risk of holding foreign currency. Then the uncovered interest parity condition maybe distorted by a risk premium which may be time varying and correlated with interest differential. The foreign exchange risk might arise due to the covariance of monetary shocks with real output shocks.

The forward premium includes a risk-premium term besides the traders' prediction for the exchange rate depreciation which arises because trade using forward rate hedges against the risk of currency fluctuations. The risk premium is the extra expected return that investors demand in compensation for holding a currency that they perceive as riskier than others.

The economic model for determining foreign exchange risk premium is developed in Lucas (1982) and is extended by Hodrick and Srivastava (1984). Domowitz and Hakkio (1985) present an example that yields an exchange rate equation with a time-varying risk premium that is a function of the conditional variance of domestic and foreign money. Also, the error term is heteroskedastic. We present a brief summary of the Lucas model and follow Hodrick and Srivastava to extend it to come up with an expression of the risk premium.

We consider the world consisting of two countries, A and B and two goods  $x$  and  $y$ . At time  $t$ , consumers in country A receive an endowment  $\xi_t$  of good  $x$  and nothing of good  $y$ ; consumers in country B receive nothing of good  $x$  and an endowment of  $\eta_t$  of good  $y$ . Agents have identical preferences and each agent of country  $i$  maximizes utility

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(x_{it}, y_{it}) \right\}, 0 < \beta < 1 \dots\dots\dots (13)$$

where  $x_{it}$  and  $y_{it}$  are the consumption of good  $x$  and  $y$  in country  $i$ . The utility function is bounded, continuously differentiable, increasing in both arguments and strictly concave. The utility function and the discount factor  $\beta$  are same for both countries. The current state of the system is given by  $s_t = (\xi_t, \eta_t)$  and the transition function is

$$F(s_{t+1}, s_t) = F(\xi_{t+1}, \eta_{t+1}, \xi_t, \eta_t).$$

Knowledge of the equilibrium price and transition function implies knowledge of the probability distribution of all future prices or rational expectations. In equilibrium each representative agent consumes half of the endowment of each country and the relative price of y in terms of x is given by the ratio of the marginal utility of y to the marginal utility of x.

$$p_y(s_t) = U_y(1/2\xi_t, 1/2\eta_t) / U_x(1/2\xi_t, 1/2\eta_t) \dots\dots\dots (14)$$

Lucas (1982) derives the equilibrium in a barter exchange economy with no money and then extends his model by introducing money. He considers a single world currency and then presents a flexible exchange rate version with two national currencies. Endowment of country i can be purchases by currency of country i only. At the time of trade in securities and goods, there exists no uncertainty about the state of the economy and the finance constraint holds for all. The equilibrium nominal goods prices are

$$p_x(s_t, M_t) = M_t / \xi_t \dots\dots\dots (15a)$$

$$p_y(s_t, N_t) = N_t / \eta_t \dots\dots\dots (15b)$$

where M and N are monies of country A and B, respectively. The transition function for the two monies follow a known, exogenous Markov process,  $K(w_{t+1}, w_t, s_{t+1}, s_t)$  where

$$w_t = (w_{At}, w_{Bt})$$

is the vector of the stochastic growth rates for the two monies between period t and t+1.

The equilibrium exchange rate depends on the relative currency supply and real endowments and is given by the PPP theory as

$$e(s_t, M_t, N_t) = [ p_x(s_t, M_t) / p_y(s_t, N_t) ] p_y(s_t) = [ M_t \eta_t / N_t \xi_t ] p_y(s_t) \dots\dots\dots (16)$$

Money in this model is demanded only for transaction motive; it is needed to purchase current period goods. An individual's portfolio consists of current goods and next period's (future) goods. The nominal prices of these goods depend on the money supply and endowments. There exists no uncertainty regarding the current period price or spot price of the goods but price in the next period is uncertain since the money supply and endowments follow a Markov process. In order to form an expectation regarding the future price of the goods, we need to consider both the current and future period's money supply and endowments.

In intertemporal asset pricing models, the equilibrium asset price is determined at a point where the marginal utility foregone by purchasing the asset is equal to the conditional expectation of the marginal utility of the return from holding the asset. The determination of the risk premium in the foreign exchange market depends on the exchange rate which being the relative price of two currencies is influenced by the intertemporal rates of substitution of the currencies.

As discussed in Hodrick and Srivastava (1984), let  $b(s_t, w_t)$  be the dollar price of an amount invested in period  $t$  which gives a return of dollar one in period  $t+1$  which is equivalent to  $1/p_x(s_{t+1}, M_{t+1}) = \pi_{t+1}^N$  units of  $x$  in period  $t+1$ .  $\pi_{t+1}^M$  is the purchasing power of dollar in period  $t+1$ . The marginal utility lost in paying for the investment in period  $t$  must equal the marginal utility gained from the return in period  $t+1$ . The loss in marginal utility is given by the product of the dollar price of an amount invested in period  $t$ , the marginal utility of  $x$  in period  $t$  and the purchasing power of dollar in period  $t$ . The gain in marginal utility in period  $t+1$  is given by the product of the marginal utility of  $x$  in

period t+1, the purchasing power of dollar in period t+1 and the discount factor  $\beta$ . Thus the dollar price of the amount invested in period t is given by

$$b_x(s_t, w_t) = E_t \left[ \frac{\beta U_x(s_{t+1}) \pi_{t+1}^M}{U_x(s_t) \pi_t^M} \right] \dots \dots \dots (17a)$$

which is the conditional expectation of the intertemporal marginal rate of substitution of dollar. Similarly for pound we have

$$b_y(s_t, w_t) = E_t \left[ \frac{\beta U_y(s_{t+1}) \pi_{t+1}^N}{U_y(s_t) \pi_t^N} \right] \dots \dots \dots (17b)$$

The intertemporal rates of substitution of the two currencies are defined as:

$$Q_{t+1}^M = \beta U_x(s_{t+1}) \pi_{t+1}^M / U_x(s_t) \pi_t^M \dots \dots \dots (18a)$$

and

$$Q_{t+1}^N = \beta U_y(s_{t+1}) \pi_{t+1}^N / U_y(s_t) \pi_t^N \dots \dots \dots (18b)$$

which are important determinants of the risk premium in the forward market since the exchange rate is the relative price of the two currencies.

In order to find an expression for the risk premium, we need to derive the forward price of foreign exchange. In a risk free situation, an investor will be indifferent between investing in the sure dollar denominated asset which yields a return of  $1/b_x(s_t, w_t)$  per dollar invested and the covered interest arbitrage strategy in which dollar is converted into pounds, the amount is invested in the sure pound denominated asset and the proceeds is sold in the forward market at a price of  $f(s_t, w_t, M_t, N_t)$  dollars per pound. This strategy yields  $[1/e(s_t, M_t, N_t)][1/b_y(s_t, w_t)][f(s_t, w_t, M_t, N_t)]$  per dollar invested.

Equating the return from both the strategies we derive an expression of the forward rate as:

$$f(s_t, w_t, M_t, N_t) = e(s_t, M_t, N_t) [b_y(s_t, w_t) / b_x(s_t, w_t)] \dots \dots \dots (19)$$

The risk premium is defined as the difference between the expected future spot rate and the forward rate. After substitutions we derive an expression for the risk premium which depends on the intertemporal rates of substitution of the two currencies:

$$[E_t(e_{t+1}) - f_t] / e_t = E_t [Q_{t+1}^N / Q_{t+1}^M] - E_t [Q_{t+1}^N] / E_t [Q_{t+1}^M] \dots \dots \dots (20)$$

Given the stochastic nature of the endowments and money supply, both real and monetary uncertainty enters the determination of the risk premium. Thus “[I]n a regression of the rate of depreciation on the forward premium there will be a time-varying risk premium and the error term of the regression will exhibit conditional heteroskedasticity” Domowitz and Hakkio (1985).

If participants in foreign exchange markets are risk averse, they might require greater return from foreign assets thus creating a foreign exchange risk premium. However, investors should not be rewarded for holding foreign assets simply because of some foreign exchange risk.

As Frankel emphasizes, most foreign exchange risk is diversifiable. Investors should not be rewarded for taking on unnecessary risk. In modern models of returns on financial assets, a risk premium is awarded only when the return on an asset covaries with some benchmark (such as the return on the market portfolio, or the aggregate marginal rate of substitution in consumption) that makes risk undiversifiable. The foreign exchange risk premium depends on the relative riskiness of domestic and foreign nominal assets. (Engel, 1996, p. 148).

## 6. Econometric Model of Foreign Exchange Risk Premium

Before we discuss the econometric model of foreign exchange risk premium we review the ARCH in mean (ARCH-M) model because the risk premium is a function of the conditional variance of the forecast error which are assumed to follow the ARCH process.

### 6.1 ARCH – M Model

The fundamental feature of the ARCH in mean (ARCH-M) model is that the standard deviation of each observation affects the mean of that observation. The ARCH–M model is used in financial applications where expected return on an asset is related to the expected asset risk. The basic insight of this model is that risk averse agents will require compensation for holding a risky asset. Given that an asset’s riskiness can be measured by the variance of returns, the risk premium will be an increasing function of the conditional variance of returns.

Excess return from holding a risky asset is given by:

$$y_t = u_t + e_t$$

$$E_{t-1}[y_t] = u_t \dots\dots\dots (21)$$

where  $u_t$  is the risk premium and  $e_t$  is the unforecastable shock,  $u_t$  is an increasing function of the conditional variance of  $e_t$  (greater the conditional variance of returns, greater will be the compensation needed to induce agents to hold the asset).

$$u_t = b + dh_t, \quad d > 0 \dots\dots\dots (22)$$

$h_t$  is the conditional variance of  $e_t$  and  $h_t$  is ARCH (q) process :

$$h_t = a_0 + \sum_{i=1}^q a_i e_{t-i}^2 \dots\dots\dots (23)$$

The ARCH-M model is used in financial economics to account for the variation in asset return. Asset markets are characterized by periods of turbulence and tranquility which foster uncertainty in investment. As risk averse investors demand greater compensation in periods of above-average uncertainty risk, premiums increase with an increase in the conditional variance. This behavior is captured by the ARCH-M model since it allows conditional volatility to directly influence the conditional mean.

### *6.2 Modeling Risk Premium*

The literature investigating foreign exchange market efficiency propose that there exists strong evidence confirming the view that forward rates are biased predictors of future spot rates, the biasedness being attributed to the existence of a time varying risk premia. Hodrick and Srivastava (1984) and Domowitz and Hakkio (1985) developed theoretical models which generate risk premia in foreign exchange market. There have been several attempts to model risk premia, however, according to Bollerslev, Chou, and Kroner (1992), a suitable model for the time varying risk premium in the forward foreign exchange market has not been formulated yet.

As discussed by Nieuwland et al. (2000), Domowitz and Hakkio (1985) model time varying risk premia conditional on the hypothesis that the foreign market is efficient or rational. They present an intertemporal asset pricing model (based on the utility optimizing models of Lucas, 1982) in which the risk premium is a function of the conditional variances of the domestic and foreign money supplies. They are of the opinion that it is difficult to model a time varying risk premium due to the lack of an

empirically tractable economic model. The Domowitz and Hakkio model is given by the following equations:

$$E_t S_{t+k} - S_t = RP_t^k + \beta_1 (F_{t+k} - S_t) + \varepsilon_t \dots\dots\dots (24)$$

$$RP_t^k = \beta_0 + \theta h_t \dots\dots\dots (25)$$

$$\varepsilon_t | I_{t-1} \sim N(0, h_t^2)$$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \dots\dots\dots (26)$$

where  $I_t$  represents the set of available information at time  $t$ . The risk premium,  $RP_t^k$ , depends directly on the conditional variance of  $\varepsilon_t$  which is denoted by  $h_t^2$ . Given information available at time  $t$ , the conditional variance of the expected rate of depreciation is assumed to depend on the realizations of the squared error terms in the previous months.  $\beta_1$  should equal 1 and  $\varepsilon_t$  should be white noise.  $\theta \neq 0$  implies that the conditional variance plays a role in determining the deviation of the forward rate from the expected future spot rate.  $\beta_0 = 0$  and  $\theta = 0$  implies a zero risk premium,  $\beta_0 \neq 0$  and  $\theta = 0$  implies a nonzero constant risk premium, and both not equal to zero implies a time-varying risk premium.

Risk premium is a function of the conditional variance of the serially correlated market forecast errors which are assumed to follow the ARCH process and also the macroeconomic variables influencing the premium are AR(1) stochastic processes. Thus the risk premium depends on the conditional variance of the exogenous variables and the error term is heteroskedastic. Any movement in the conditional variance brings about movement in the risk premium which can be positive or negative and can switch signs. Thus it can be stated that the risk premium is volatile in nature.

In the ARCH model, the errors are related through the second moment. The conditional variance acts like an autoregressive process resulting in conditionally heteroskedastic errors. It is a special case of GARCH specification in which there is no lagged forecast variance in the conditional variance equation, i.e. GARCH (0, 1). The GARCH model which is an extension of the ARCH model allows for both autoregressive and moving average components in the heteroskedastic variance. Hence the conditional variance acts like an autoregressive and moving average process. The last period's forecast variance is the GARCH term. A high order ARCH model may have a more parsimonious GARCH representation that is easier to identify and estimate. The more parsimonious model will entail fewer coefficient restrictions (Enders, 2003). A GARCH (1, 1) specification is the most popular form of conditional volatility especially for financial data where volatility shocks are persistent.

A generalization of the ARCH model proposed by Bollerslev (1986) is the GARCH model. The key feature of the GARCH model is that the conditional variance of the disturbance constitutes an ARMA (autoregressive moving average) process. For the first-order GARCH-in-mean model, the conditional variance becomes:

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 \dots\dots\dots (27)$$

Nonnegativity constraints are imposed on the parameters of the conditional variance whose magnitude determine the degree of persistence in variance. A GARCH (1, 1) specification is a parsimonious representation of conditional variance that adequately fits many economic and financial time series (Bollerslev, 1986).

## **7. Data**

The data for the forward and spot exchange rates are for the period January 1991 to February 2008 for U.K., Canada, Australia, and Japan, the four advanced economies and for India, the emerging market economy, the data ranges from January 1999 to February 2008. We have used the forward and spot exchange rates from the last working day of each month. All the rates are quoted in foreign currency units per U.S. dollar. The data were obtained from Bloomberg.

## **8. Model Estimation**

As mentioned earlier, Domowitz and Hakkio (1985) model a time-varying risk premium using the autoregressive conditional heteroskedasticity (ARCH) framework. Nieuwland et al. (2000) modifies the analysis of Domowitz and Hakkio (1985) to be applied to a survey data set of exchange rate expectations covering a wide range of EMS currencies from 1986 to 1991. In the analysis of Nieuwland et al. (2000), several currencies display significant ARCH effects. They specify a GARCH model rather than a general equilibrium model since the true structure of the covariance matrix and variables to which it is related is not known. "A (G)ARCH model is an acceptable alternative because it can be interpreted as a reduced form of a more complicated dynamic structure for the time-varying conditional second order moments"(Nieuwland et al., 2000, p. 355). In this essay we thus model the risk premium by extending the Domowitz and Hakkio (1985) ARCH framework and by applying a GARCH (1, 1) specification following the analysis of Nieuwland et al. (2000).

The model can be expressed as:

$$s_{t+1} - s_t = RP_t + \beta_1(f_t - s_t) + \varepsilon_t \dots\dots\dots (28)$$

$$RP_t = \beta_0 + \theta h_t \dots\dots\dots (29)$$

Risk premium,  $RP_t$ , depends directly on the conditional variance of  $\varepsilon_t$ , denoted by  $h_t^2$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 \dots\dots\dots (30)$$

$h_t^2$  is the conditional variance for the first-order GARCH-in-mean model.

Thus we can write:

$$s_{t+1} - s_t = \beta_0 + \theta h_t + \beta_1(f_t - s_t) + \varepsilon_t \dots\dots\dots (31)$$

$$\varepsilon_t | I_{t-1} \sim N(0, h_t^2)$$

where  $I_t$  represents the set of available information at time  $t$ .

## 9. Empirical Results

This section provides results and discusses their implications for the test of the UH, test for heteroskedastic ordinary least squares (OLS) residuals, the ARCH-M model and the GARCH-M model for all the five countries. All estimations were performed using the software package SAS.

### 9.1 Test of the unbiasedness hypothesis

We first test the FRUH using the level and percentage change specification:

$$s_{t+1} = \beta_0 + \beta_1 f_t + \varepsilon_t \dots\dots\dots (1)$$

and

$$s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \varepsilon_t \dots\dots\dots (2)$$

The null hypothesis is  $\beta_0 = 0$  and  $\beta_1 = 1$ .

We first test both these specifications using OLS. The results for all the countries are reported in table 1.1 below.

Table 1.1

*Test of the Unbiasedness Hypothesis*

Countries	Estimates and t-statistic	Level specification		Percent change specification	
		$\beta_0$	$\beta_1$	$\beta_0$	$\beta_1$
Australia	estimates	-0.001	0.999	-0.001	0.194
	t-statistic	-0.21	-0.149	0.5	-2.48**
Canada	estimates	-0.003	1.008	-0.0006	0.964
	t-statistic	-0.93	0.807	-0.49	-0.0962
India	estimates	-0.056	1.014	0.0004	-0.727
	t-statistic	-0.6	0.580	0.31	-5.89***
Japan	estimates	0.229	0.952	0.0002	0.781
	t-statistic	2.54**	-2.53***	0.09	-0.395
U.K.	estimates	0.017	0.968	0.003	2.272
	t-statistic	1.81*	-1.68**	1.36	1.107

\*, \*\*, \*\*\* denotes significance at 10%, 5%, and 1% level, respectively.

In the level specification we fail to reject the null hypothesis of  $\beta_0 = 0$  for all the countries except for Japan and U.K., while in the percentage change specification we fail to reject the null hypothesis of  $\beta_0 = 0$  for all the countries. We reject the null hypothesis of  $\beta_1 = 1$  in the level specification for Japan and U.K., while in the percentage change specification we reject the null hypothesis of  $\beta_1 = 1$  for Australia and India.

## 9.2 Tests for heteroskedastic OLS residuals

In order to test for the presence of heteroskedasticity in the OLS residuals, we conduct the Q and LM Tests for ARCH Disturbances. The results are reported in table 1.2 below.

Table 1.2

### *Q and LM Tests for ARCH Disturbances*

Countries	Order	Q	Pr > Q	LM	Pr > LM
Australia	1	8.36	0.0038	8.46	0.0036
	6	19.08	0.004	18.18	0.0058
	12	24.45	0.0177	24.01	0.0203
Canada	1	8.35	0.0039	8.38	0.0038
	6	28.06	<.0001	28.07	<.0001
	12	40.75	<.0001	41.31	<.0001
India	1	5.60	0.018	5.57	0.0183
	6	7.27	0.2965	6.61	0.3587
	12	12.82	0.3821	12.24	0.4268
Japan	1	0.79	0.375	0.78	0.3777
	6	6.52	0.3672	7.27	0.2963
	12	8.08	0.7787	8.58	0.7383
U.K.	1	21.88	<.0001	21.65	<.0001
	6	37.82	<.0001	31.58	<.0001
	12	47.50	<.0001	38.41	0.0001

The Q statistics test and the LM test confirm that the error term is heteroskedastic and follows a GARCH (1, 1) process for Australia, Canada, and U.K. The Q statistics test for changes in variance across time using lag windows ranging from 1 through 12 and strongly indicate heteroskedasticity for all lag windows for Australia, Canada, and U.K. This indicates that a very high-order ARCH model is needed to model the heteroskedasticity. The Lagrange multiplier (LM) test also indicates heteroskedasticity. The basic ARCH( $q$ ) model ( $p = 0$ ) is a short memory process in that only the most recent

$q$  squared residuals are used to estimate the changing variance. The GARCH model ( $p > 0$ ) allows long memory processes, which use all the past squared residuals to estimate the current variance. The LM test suggests the use of the GARCH model ( $p > 0$ ) instead of the ARCH model. GARCH (1, 1) conditional variance model is used. The error term is thus found to be heteroskedastic and follows a GARCH (1, 1) process.

In the case of Japan, the Q statistics test and the LM test confirm that the error term is homoskedastic while for India the two tests confirm that the error term is heteroskedastic for order of lags 1 and 2 only. Hence for Japan we do not use either ARCH or GARCH.

### 9.3 ARCH-M model

We now test the UH using the ARCH-M model. We test the ARCH-M model for all the countries except for Japan and test the hypothesis,  $H_0: \beta_0 = \alpha_0 = \alpha_1 = \theta = 0$  and  $\beta_1 = 1$ . The results are reported below in table 1.3 below.

Table 1.3

#### ARCH-M Model

Countries	Estimates and t-statistic	$\beta_0$	$\beta_1$	$\alpha_0$	$\alpha_1$	$\theta$
Australia	estimate	0.019	0.111	0.000	0.114	-27.624
	t-Statistic	0.940	-2.510**	6.84***	1.240	-0.980
Canada	estimate	0.662	0.650	0.000	0.004	-2261
	t-Statistic	0.050	-1.010	9.660***	0.050	-0.050
India	estimate	0.001	-0.350	0.000	0.340	-6.162
	t-Statistic	0.450	-4.45***	10.54***	2.240**	-0.290
U.K.	estimate	0.004	1.321	0.000	0.2460	-4.272
	t-Statistic	0.730	0.394	7.68***	2.10**	-0.430

\*, \*\*, \*\*\* denotes significance at 10%, 5%, and 1% level, respectively.

We fail to reject the hypothesis that  $\beta_0 = 0$ ,  $\theta = 0$  and reject the hypothesis that  $\alpha_0 = 0$  for all the countries. However, the estimates of  $\alpha_0$  are very close to zero. We fail to reject the hypothesis  $\beta_1 = 1$  for Canada and U.K. only. The estimated  $\alpha_1$  coefficients are statistically significant for India and U.K. only thus supporting the ARCH specification. Thus in the test of the UH using the ARCH model we fail to find evidence of a risk premium.

#### 9.4 GARCH-M model

The Maximum Likelihood estimation technique is used to estimate the first-order GARCH-M model specified below.

$$s_{t+1} - s_t = \beta_0 + \theta h_t + \beta_1 (f_t - s_t) + \varepsilon_t$$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2$$

In order to test whether the conditional variance matters in determining the deviation of the forward rate from the expected future spot rate we test the hypothesis that  $\theta = 0$ . With  $\beta_1 = 1$  and  $\varepsilon_{t+1}$  white noise,  $\beta_0 = 0$  and  $\theta = 0$  implies a zero risk premium,  $\beta_0 \neq 0$  and  $\theta = 0$  implies a nonzero constant risk premium and  $\beta_0 \neq 0$  and  $\theta \neq 0$  implies a time-varying risk premium.

We next test the UH using the GARCH-M model. The results are reported in table 1.4 on the next page.

We test the GARCH-M model for all the countries except for Japan and test the hypothesis,  $H_0: \beta_0 = \alpha_0 = \alpha_1 = \gamma_1 = \theta = 0$  and  $\beta_1 = 1$ .

Table 1.4

*GARCH-M Model*

Countries	Estimates and t-statistic	$\beta_0$	$\beta_1$	$\alpha_0$	$\alpha_1$	$\gamma_1$	$\theta$
Australia	estimate	0.004	0.240	0.000	0.106	0.829	-6.932
	t-Statistic	0.540	-1.890*	0.880	1.670*	7.550***	-0.710
Canada	estimate	0.009	1.030	0.000	0.054	0.946	-30.950
	t-Statistic	2.980***	0.093	0.520	2.170**	34.440***	-2.840***
India	estimate	0.001	-0.350	0.000	0.340	0.000	-6.162
	t-Statistic	0.450	-4.45***	10.540***	2.240**	0.000	-0.290
U.K.	estimate	0.007	1.577	0.000	0.178	0.680	-9.090
	t-Statistic	1.560*	0.566	1.360	2.050**	4.370***	-0.970

\*, \*\*, \*\*\* denotes significance at 10%, 5%, and 1% level, respectively.

We reject the hypothesis that  $\beta_0 = 0$  for Canada and U.K. and  $\alpha_0 = 0$  for India. However, in all the three cases, the estimated coefficients are very close to zero. We fail to reject the hypothesis  $\beta_1 = 1$  for Canada and U.K. only. The estimated  $\alpha_1$  and  $\gamma_1$  coefficients are statistically significant in all cases except for India, thus supporting the GARCH specification. The estimated  $\alpha_1$  coefficient is statistically significant for India which supports the ARCH specification. In the case of Canada, the  $\theta$  coefficient is statistically significant which suggests that the risk premia follows GARCH (1, 1) process. With  $\beta_1 = 1$ , in case of Canada,  $\beta_0 \neq 0$  and  $\theta \neq 0$  implies a time-varying risk premia while  $\beta_0 \neq 0$  and  $\theta = 0$  implies a nonzero constant risk premium in case of U.K.

In case of Australia and India,  $\beta_0 = 0$  and  $\theta = 0$  implies a zero risk premium. Thus we fail to find evidence of a time varying risk premium although the UH is rejected and the error is heteroskedastic. For India, the Q statistics test and the LM test confirm that

the error term is heteroskedastic for order of lags 1 and 2 only. The heteroskedastic error follows ARCH (1) framework while for Australia, the heteroskedastic error follows GARCH framework.

In case of Japan since the Q statistics test and the LM test confirm that the error term is homoskedastic, we do not estimate the percentage change specification using ARCH or GARCH. Regressions results in the percentage change specification, using OLS suggests that we fail to reject the null hypothesis of  $\beta_0 = 0$  and  $\beta_1 = 1$  both at 1% level of significance. Thus we fail to reject the UH.

The GARCH specification is supported by Australia, Canada, and U.K. while the ARCH specification is supported by India and neither is supported by Japan.

## **10. Conclusions**

Since emerging market currencies are riskier to hold than major currencies; the risk premium would be larger and more variable than for major currencies. Also, these currencies are more prone to bouts of high inflation and other sources of medium-term trends, so it would be easier to forecast the direction of movement of their spot rate than for major currencies, where the exchange rate is closer to a random walk. If the bias is greater for emerging market currencies, the risk premium interpretation holds, if less, the other interpretation is valid (Frankel & Poonawalla, 2006). For our data set, in case of India, the later interpretation seems to hold.

In this paper we have examined the existence of a risk premium in the foreign exchange market based on the conditional variance of market forecast errors. The Q statistics test and the LM test confirm that the error term is heteroskedastic and follows a

GARCH (1, 1) process for Australia, Canada, and U.K. and an ARCH process for India. In the case of Japan, the tests confirm that the error term is homoskedastic. In case of Japan we fail to reject the UH when using the percentage change specification while we reject the UH when using the levels specification.

For all the countries except Japan, we have modeled the conditional variance directly using the GARCH specification and estimated model coefficients using maximum likelihood techniques. We find evidence of a time varying risk premium for Canada and a nonzero constant risk premium in case of U.K. and in both cases we fail to reject the UH. In case of Australia and India, we fail to find evidence of a time varying risk premium although the UH is rejected and the error is heteroskedastic.

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## ESSAY 2: EXPLAINING FORWARD RATE UNBIASEDNESS HYPOTHESIS, STATIONARITY, COINTEGRATION AND ERROR CORRECTION

### 1. Introduction

The foreign exchange market in which international currency is traded is characterized by risk due to currency fluctuations. In reducing the foreign exchange market risk, the use of forward contracts can be very effective. A trader can use a forward rate and engage in a forward transaction, in which, a buyer and seller agree on an exchange rate for any date in the future, and the transaction occurs on that date, at that agreed exchange rate, regardless of what the market rates are then. Forward rates which lock in a rate in the current period for a future date are calculated by using the current exchange rate for the currency pair and the interest rates for the two currencies. A forward rate can be interpreted as the sum of a premium and the expected future spot rate, “The forward exchange rate  $f_t$  observed for an exchange at time  $t+1$  is the market determined certainty equivalent of the future spot exchange rate  $s_{t+1}$ ” (Fama, 1984, p. 320). The FRUH states that the forward rate is an unbiased predictor of the future spot rate. Let the price of a particular foreign currency on the spot market be  $s_t$  dollars and the price for delivery one period into the future be  $f_t$  dollars. A speculator purchases forward currency at the price  $f_t$  dollars per unit. At the beginning of period  $t+1$  he receives the currency and pays  $f_t$  dollars per unit received. In period  $t+1$  the spot foreign exchange can be sold at  $s_{t+1}$  and the speculator can earn a profit or loss of  $s_{t+1} - f_t$  per unit transacted. According to FRUH the expected profit from such speculative behavior should be zero.

In international asset market the relation between the forward and spot prices of foreign exchange is of concern for investors, portfolio managers, and policy makers. Any international transaction involving foreign exchange is risky due to unexpected change in exchange rates. Hence it is important that the forward rates are efficient and rational forecasts of future spot rates. The relation between spot and forward exchange rate is not only important from an economic perspective but also because deviations from efficiency and rationality are observed in foreign exchange market even with large trading volumes and low trading costs. However, it has been empirically verified that forward rates are neither efficient nor rational forecasts of future spot rates.

### *1.1 Forward Premium Puzzle*

Under the assumptions of risk neutrality and rational expectations in an efficient foreign exchange market, the forward rate will be an unbiased predictor of the future spot rate. In a large number of existing literatures it has been empirically verified that the slope coefficient is significantly less than unity and most often negative in sign. This puzzling observation is termed as the Forward Premium Puzzle. This indicates foreign exchange market inefficiency.

In finding a solution of this puzzle the large volume of research has taken two main directions - the risk-premium approach and the non-rationality approach. Since currency trading is associated with risk due to changes in the currency exchange rate, traders use forward rate to hedge against this risk of currency fluctuations. Thus the forward premium includes a risk-premium term along with the expected exchange rate depreciation. However, the empirical evidence in favor of the risk premium term is rare.

If the agents are not rational in forming their expectations regarding the future spot exchange rate then there exists an error in the forward premium which generates a biased regression coefficient. Another explanation for this puzzling behavior lies in the use of statistical and econometric estimation techniques. For example, in testing the UH, before we estimate the regression equation of the spot rate on the forward rate, we should first verify whether any one or both of the time series exhibit nonstationarity.

There is considerable evidence that many financial time series such as the foreign exchange rates are nonstationary which has important implications for modeling exchange rates. The spot and forward rates are nonstationary with unit roots but determining exchange rate under rational expectations requires the assumption of stationarity (Baillie & Bollerslev, 1989). If we regress one nonstationary series against another, we end up with spurious results (Granger & Newbold, 1974) in which the conventional significance tests will indicate a relation between the variables when in fact none exists. Regressing spot rates on forward rates in the presence of a unit root in both the series will result in a downwardly biased slope estimate. In such case, a conventional significance test would lead towards the rejection of null hypothesis of unbiased forecasts (Granger & Newbold, 1974).

According to economic theory there might exist a long-run equilibrium relationship among nonstationary variables. If first differencing of these variables makes them stationary these variables are said to be integrated of order one,  $I(1)$ . The variables have unit roots and in such cases the cointegration technique can be used to model the long-run relations.

The time paths of cointegrated variables are influenced by the extent of any deviation from long run equilibrium. In order to restore long run equilibrium, movement in some of the variables must correspond to the extent of disequilibrium. The short run dynamics is thus influenced by deviation from long run equilibrium. This dynamic model is termed as an error correction model (ECM). In a two variable model, for an error correction representation it is essential that the two variables be integrated of the same order and hence cointegrated. In this case each variable contains a single unit root and the linear combination of the variables is stationary.

### *1.2 Objective*

In the existing literature it has been established that cointegration with CI (1, 1) implies unbiasedness. The objective of this paper is to analyze the UH from the perspective of time series properties of the exchange rates.

This paper is presented in 10 sections. We start with a brief literature review on cointegration and UH in section 2. The relation between unbiasedness and cointegration and the FRUH and cointegration is explored in sections 3 and 4 respectively. The data is described in section 5. We conduct stationarity test on the spot and forward exchange rates of five countries, Australia, Canada, India, Japan, and U.K. in section 6 and find that they are nonstationary but first differencing of these variables makes them stationary. Hence, these variables have unit roots and are integrated of order one,  $I(1)$ . The cointegration test in section 7 confirms the existence of a long-run relationship between the spot and forward exchange rates. Since the spot and forward rates are cointegrated of order CI (1, 1), an ECM based on the percent change specification is used to model these

dynamic relationships in section 8. We regress the change in the spot rate on the lagged forward-spot differential and lagged changes in spot and forward rates. Empirical evidence suggests that the ECM is a proper specification of the relationship between spot and forward rates. We next include the domestic and the foreign interest rates in our analysis in section 9 and conduct cointegration test among all four variables followed by a conclusion in section 10.

In section 9 we empirically test whether a long run relationship exists between the spot and forward exchange rates and the domestic and foreign interest rates. In a way we are also conducting a robustness check or a sensitivity analysis where we ensure whether the two cointegrated variables, the spot and forward exchange rates, are still related in the long run with the inclusion of two more variables, the domestic and foreign interest rates. In most of the cases we are able to confirm the existence of a long run relationship between the spot and forward exchange rates and the domestic and foreign interest rates.

The economic rationale for using the ECM can be explained by the covered interest arbitrage and the interest parity condition. In a covered interest arbitrage investment strategy, an investor buys a foreign currency denominated financial instrument and hedges his foreign exchange risk by using the forward contract. An investor buys the foreign currency in the spot market, invests in the foreign financial instrument and sells the payoff from the foreign investment in the forward market. The decision to invest in the domestic or in the foreign market depends on whichever gives higher returns and hence on the interest rates. Thus if the foreign interest rate is higher than the domestic rate, investing in the foreign financial instrument is more lucrative. The investor will buy the foreign currency in the spot market and sell it in the forward market.

This will raise the spot rate and lower the forward rate of the foreign currency. If transaction costs are low, such arbitrage will continue until the two interest rates are equal. This result is the interest rate parity condition which states that the spot and the forward price of a currency incorporate any interest rate differentials between the two currencies. Thus equilibrium is restored by adjustments in the spot and the forward rates.

## **2. Literature review**

In the literature, the test of the FRUH is conducted by using two econometric specifications. The first is a level specification in which the realized spot rate is regressed on the one-period forward rate and the other is the percent change specification in which the percent change in the realized spot rate relative to the current spot rate is regressed against the difference between the forward and spot rates, the forward premium. Since spot and forward exchange rates have unit roots and are co-integrated, the level specification is inappropriate and the other is incomplete. The time series properties of spot and forward exchange rate data rule out certain econometric specifications used to test the UFRH. Barnhart and Szakmary (1991) use an alternative error correction specification in which the variables used in the regression meet conditions necessary for stationarity. They find that the FRUH is rejected for the full modern floating exchange rate era. “We confirm the finding that the coefficients in our test of the UFRH are unstable; however, our results indicate that the evolution of the estimated parameters is becoming increasingly inconsistent with the UFRH with the passage of time” (Barnhart & Szakmary, 1991, p. 246).

If  $s_t$  and  $f_t$  are cointegrated, a correct specification of the vector auto regression (VAR) requires adding error-correction terms. If the cointegrating vector is known to be (1, -1), then lags of the forward discount must be included in the VAR (Baillie, 1989). This approach is followed in Hakkio and Rush (1989), Baillie (1989), and Bekaert (1995), and in all cases, the authors reject the UH.

Baillie and Bollerslev (1989) find evidence for the presence of a unit root in the time-series representation for seven daily spot and forward exchange rate series all of which are cointegrated. One common unit root, or stochastic trend, is detectable in the multivariate time-series models for the seven spot and forward rates, respectively. The seven exchange rates possess one long-run relationship and the disequilibrium error around that relationship partly accounts for subsequent movements in the exchange rates.

Naka and Whitney (1995) examines the FRUH using an ECM, the advantage being that the parameters of the level specification appear in the error correction term, and hence the ECM provides a direct link to the levels specification. Both specifications yield the same result when the ECM is explicitly derived from levels specification and the levels parameters are estimated simultaneously with other parameters of the ECM.

Hai, Mark, and Wu (1997) find spot and forward exchange rates to be cointegrated with cointegration vector (1, -1), and the slope coefficient in regressions of the future depreciation on the forward premium is negative and significantly less than 1. They regress the k-period-ahead spot rate on the k-period-forward rate, for  $k = 1, 3$  and induce stationarity. The forward premium predicts future changes in the spot rate but with the “wrong” sign, which is due to the error term being correlated with the forward premium.

Roll and Yan (2000) argue that the forward rate is an unbiased predictor of future spot rate and suggest that the puzzle arises because the forward rate, the spot rate and the forward premium follow nearly nonstationary time series processes.

Zivot (2000) shows that the cointegrated model for  $s_{t+1}$  and  $f_t$  derived from the VECM for  $s_t$  and  $f_t$  is not a simple finite order VECM and that estimating a first order VECM for  $s_{t+1}$  and  $f_t$  can lead to mistaken inferences concerning the exogeneity of the spot rates and the unbiasedness of the forward rates.

Ho (2002) re-examines the FRUH by panel cointegration developed by Kao and Chiang's (1999) dynamic ordinary least square (OLS) to examine the panel of 17 OECD countries. Their results suggest that panel cointegration is strongly confirmed. Lin et al. (2002) have used a logarithmic change specification which is transformed into a variable mean response model estimated by a four-step generalized least squares procedure. Baghli (2005) has employed a version of the Breitug nonparametric cointegration approach. The advantage of this approach is that it does not impose any parametric specifications on the relationship.

Sekioua (2006) tests for a unit root in the forward premium allowing for nonlinearity in the data by employing bootstrap methods based on threshold autoregressive (TAR) models. The null hypotheses of linearity and nonstationarity are rejected. Furthermore, large deviations of the forward premium from its equilibrium have faster speed of mean reversion than the strongly persistent small deviations. The forward premium exhibits mean reversion in a manner not captured by the usual linear tests.

Kellard (2006) examines the assumption that a small unit root or fractionally integrated component is dominated in finite samples by a large stationary component.

They employ Monte Carlo techniques which demonstrate that “[W]ith typical sample sizes and variable magnitudes, the Engle–Granger test overwhelmingly finds spurious cointegration. Conversely, under certain conditions, Johansen tests are shown to be relatively robust to differences in variable magnitudes.” Kellard (2006).

Villanueva (2007) allows for structural endogenous breaks in spot-forward cointegration regressions which helps explain the persistence of the forward-premium, and provides more favorable evidence for long-run and short-run unbiasedness.

Delcoure, Barkoulasb, Baumd, and Chakraborty (2003) state that the empirical evidence on the existence of cointegration between the spot and lagged forward exchange rates is mixed. Baillie and Bollerslev (1989) and Hai et al. (1997) find that the spot and lagged forward exchange rates form a cointegrated system with a unitary cointegrating vector. Alexakis and Apergis (1996) and Evans and Lewis (1993) fail to even find a long-run relationship between forward and corresponding future spot rates. Evans and Lewis (1995) and Luintel and Paudyal (1998) find evidence of cointegration between the spot and lagged forward exchange rates but they reject the null of a unitary cointegrating vector. According to Guerra (2003) cointegration tests do not provide a clear-cut answer, while Hakkio and Rush (1989) reject the hypothesis of no cointegration using the Engle and Granger (1987) test, Evans and Lewis (1993) find that the spot and lagged forward exchange rates are not cointegrated when Johansen (1988) tests are applied. Ngama (1992) obtains mixed results depending on the forward rate horizon using the Phillips and Hansen (1990) methods.

The reason for this mixed evidence is well summarized in the survey of the literature by Engel (1996):

Some have found  $s_{t-1}$  and  $f_t$  are cointegrated with cointegrating vector (1, -1); some have found they are cointegrated but not with cointegrating vector (1, -1); and some have found that they are not cointegrated. These conflicting results hold on tests for the same set of currencies. To some extent these conflicts may arise from different sampling periods, but more likely they result from different properties of the various test statistics employed. (p. 141).

### 3. Unbiasedness and Cointegration

In testing the efficiency hypothesis in the foreign exchange market, three conditions must be satisfied: spot and forward rates must be cointegrated, the cointegrating factor must be one and the forecast error must be a white noise process, which is a special case of a stationary process. If we find that the spot and forward rates are cointegrated it implies evidence in favor of the weak form of market efficiency. The strong form of market efficiency requires that the forward rate must be an unbiased predictor of future spot rate which is achieved only if the spot and forward rates are cointegrated with a vector (1, 1).

Spot and forward rates are cointegrated if they are nonstationary in levels, are stationary in first differences and there exists a linear combination in levels where

$$u_t = s_t - \beta_1 f_{t-1} \dots\dots\dots (1)$$

is stationary.

Under the hypothesis of rational expectations and risk neutrality, the UH is

$$f_{t-1} = E_{t-1}(s_t) \dots\dots\dots (2)$$

where  $f_t$  and  $s_t$  are the logarithms of the spot and forward rates at time  $t$  and  $E_t(\cdot)$  is the expectations operator conditional on information available at time  $t$ . This equation is expressed as the levels relationship by

$$s_t = f_{t-1} + \varepsilon_t \dots\dots\dots (3)$$

where  $\varepsilon_t$  is a random, zero mean variable.

Given the level specification of the UH and taking into consideration that spot and forward rates are generally found to be nonstationary, the existence of cointegration between the spot and lagged forward rate is a necessary condition for market efficiency.

The cointegrating regression can be specified as

$$s_t = \beta_0 + \beta_1 f_{t-1} + u_t \dots\dots\dots (4)$$

which is the same as the level specification of the test of the UH. Empirical tests for cointegration generally confirm that the spot and lagged forward rates are cointegrated, CI (1, 1). In order for the UH to hold we require  $\beta_0 = 0$ ,  $\beta_1 = 1$  and that  $u_t$  is not serially correlated.

As an equivalent approach for testing the UH, let us express the residual term in the level specification as

$$\varepsilon_t = s_t - f_{t-1} = (s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) \dots\dots\dots (5)$$

Given that the spot return  $(s_t - s_{t-1})$  is stationary, the lagged forward premium  $(f_{t-1} - s_{t-1})$  determines the order of integration of the forecast error  $(s_t - f_{t-1})$ . It has been empirically verified that the forward premium has a unit root, which imply that the forecast error is nonstationary. Due to persistence in the forecast error, we would be able to predict the forecast error from past values. Since the UH requires that the forecast error be a white noise process, this provides a rejection of the UH. Thus we can state that the cointegration between  $f_t$  and  $s_{t+1}$  with a unitary cointegrating vector is a necessary condition for the UH.

Alternatively we can show that if  $s_{t+1}$  and  $f_t$  are cointegrated with a unitary cointegrating vector then the condition that  $\beta_1=1$  must hold. In other words, cointegration implies unbiasedness.

We have discussed that  $u_t = s_t - \beta_1 f_{t-1}$  is stationary if  $s_t$  and  $f_{t-1}$  are cointegrated with the cointegrating parameter  $\beta$ . We add and subtract  $s_{t-1}$  and  $f_{t-1}$  to  $(s_t - \beta_1 f_{t-1})$  and can write

$$f_{t-1} - s_{t-1} = (1 - \beta) f_{t-1} + (s_{t-1} - s_t) + u_t \dots\dots\dots (6)$$

We know that  $(s_{t-1} - s_t)$  and  $u_t$  are stationary while  $f_{t-1}$  is not stationary. Hence, for  $(f_{t-1} - s_{t-1})$  to be stationary, the term  $(1 - \beta) f_{t-1}$  must not exist which is possible only if  $\beta=1$ .

As discussed by Delcoure et al. (2003), given that the spot and the lagged forward exchange rates are first order integrated or I(1) processes, the FRUH requires that these variables be cointegrated with the cointegrating vector (1, -1). Under these restrictions, the forward rate correctly predicts the future spot rate; the forward rate is an unbiased predictor of the corresponding future spot rate. In order for the FRUH to be empirically supported, spot and lagged forward exchange rates should share one common stochastic trend and the realized forecast error, should be an I(0) process, more strictly a white-noise process, a stronger condition than covariance stationarity. Hence the cointegration of spot and lagged forward exchange rates with a unitary cointegrating vector is a necessary condition for the FRUH.

#### 4. Forward Rate Unbiasedness Hypothesis and Cointegration

According to the FRUH, the forward rate represents the market's expectation of the future spot rate. This holds under the assumptions that markets are efficient, agents are risk neutral and have rational expectations and is stated as:

$$E_t(s_{t+1}) = f_t \dots\dots\dots (2)$$

where  $E_t[\cdot]$  is the conditional expectation based on information available at time  $t$ ,  $s_{t+1}$  and  $f_t$  are the spot and forward exchange rates. The forward rate is then an unbiased predictor of the future spot rate,

$$s_{t+1} = f_t + \varepsilon_{t+1} \dots\dots\dots (3)$$

$\varepsilon_{t+1}$ , a random variable with  $E_t[\varepsilon_{t+1}] = 0$  is the rational-expectation forecast error. This is the level specification of the UH in which the realized spot rate is regressed on the one-period forward rate. The regression equation of the "level" specification is

$$s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1} \dots\dots\dots (4)$$

In order to test the null hypothesis that the UH is true, we impose the restrictions  $\alpha = 0$ ,  $\beta = 1$  and  $E_t[\varepsilon_{t+1}] = 0$ .

As discussed in Zivot (2000), according to Barnhart and Szakmary (1991), Liu and Maddala (1992), Naka and Whitney (1995) and Hai et al. (1997), testing  $\alpha = 0$ ,  $\beta = 1$  is same as testing the FRUH. If the UH is not rejected then testing the condition  $E_t[\varepsilon_{t+1}] = 0$ , is referred to as testing forward market efficiency under rational expectations and risk neutrality.

If  $s_t$  and  $f_t$  have unit roots, i.e., these variables are  $I(1)$ , then the UH requires that  $s_{t+1}$  and  $f_t$  be cointegrated with cointegrating vector  $(1, -1)$  and that the cointegrating

residual,  $\varepsilon_{t+1}$ , is stationary, i.e.,  $I(0)$ , and  $E_t [\varepsilon_{t+1}] = 0$ . Thus testing the UH corresponds to testing for cointegration between  $s_{t+1}$  and  $f_t$  with the cointegrating vector being  $(1, -1)$  while testing that the forecast error  $(s_{t+1} - f_t)$  has conditional mean zero is same as testing forward market efficiency.

Since  $s_t$  and  $f_t$  have unit roots the estimation of the level regression equation results in spurious regression problems. We now discuss the second econometric specification, the “percent change” specification.

$$\Delta s_{t+1} = \alpha + \beta (f_t - s_t) + \varepsilon_{t+1} \dots \dots \dots (7)$$

The difference between the forward and spot rates  $(f_t - s_t)$  is the forward premium and  $\Delta s_{t+1}$  or  $(s_{t+1} - s_t)$  is the percent change in the realized spot rate relative to the current spot rate. The null hypothesis testing the UH is the same as in the level specification.

For a balanced regression, all variables in the regression equation must be integrated of the same order.  $s_t$  and  $f_t$  have unit roots, i.e., they are  $I(1)$  and  $\Delta s_{t+1}$  is stationary i.e.,  $I(0)$ . Thus the forward premium  $(f_t - s_t)$  must be stationary i.e., they are  $I(0)$  and  $s_t$  and  $f_t$  be cointegrated with cointegrating vector  $(1, -1)$ . The “percent change” specification can be viewed as an ECM for  $f_t$  and  $s_t$ .

In the level specification, the spot and forward markets are in long run equilibrium when  $\varepsilon_{t+1} = 0$ . If  $f_t$  and  $s_{t+1}$  differ from each other there must be some adjustment which restores equilibrium in the next period. The adjustment process can be

$$s_{t+2} = s_{t+1} - \alpha (s_{t+1} - f_t) + \varepsilon_{st+2} \dots \dots \dots (8a)$$

$$f_{t+1} = f_t + \beta (s_{t+1} - f_t) + \varepsilon_{ft+1} \dots\dots\dots (8b)$$

$\alpha$  and  $\beta$  are positive and the mean value of  $\varepsilon_{st+2}$  and  $\varepsilon_{ft+1}$  are zero.

The short-run adjustment towards the long-run equilibrium can be represented by an error correction mechanism. The movement of the variables in any period is related to the previous period's gap from long run equilibrium; hence this dynamic model is an ECM. If  $s_{t+1}$  equals  $f_t$  the spot and forward rates are expected to remain unchanged but if there exists a positive gap between the spot and forward rates ( $s_{t+1} - f_t$ ) is positive, the spot rate will fall and the forward rate will rise.

Barnhart and Szakmary(1991) estimates an error correction model for the percent change specification which takes the form

$$S_t - S_{t-1} = \alpha + \beta (F_{t-1} - S_{t-1}) + \delta \text{ lagged } (S_t - S_{t-1}) + \gamma \text{ lagged } (F_t - F_{t-1}) + e_t \dots\dots (9)$$

A general model of cointegration and error correction of the spot and forward rates is formulated by introducing the lagged changes of each variable into both equations.

$$\Delta s_{t+1} = \mu_s + \alpha_s (s_t - \beta f_t) + \delta_s \Delta s_t + \gamma_f \Delta f_{t-1} + \varepsilon_{st+1} \dots\dots\dots (10a)$$

$$\Delta f_t = \mu_f + \alpha_f (s_t - \beta f_t) + \delta_s \Delta s_t + \gamma_f \Delta f_{t-1} + \varepsilon_{ft} \dots\dots\dots (10b)$$

The two variable ECM is a bivariate VAR in first differences improved by the error correction terms  $\alpha_s (s_t - \beta f_t)$  and  $\alpha_f (s_t - \beta f_t)$ .  $\alpha_s$  and  $\alpha_f$  are the speed of adjustment parameters whose value when large enables the system to return to its long run equilibrium at a faster rate. If both the parameter values are equal to zero, the long run equilibrium relationship does not exist and the model does not represent cointegration or error correction.

## 5. Data

The data for the forward and spot exchange rates are the same as used in the first essay. For the interest rate data, we use the end of month, 3 month T bill rate for Australia, Canada, and U.K. and for Japan we use the end of month prime rate. We use both the interest rate measures for USA, the T bill rate when using with Australia, Canada, U.K., and for Japan we use the prime rate. The data for Australia is from January 1995 to April 2007, for Canada the data is from January 1998 to February 2008, for Japan the data is from January 1991 to June 2006 and for U.K. the data is from March 1997 to February 2008. The data were obtained from EconStats ([www.econstats.com](http://www.econstats.com)).

## 6. Stationarity test

An error correction representation of the spot and forward rates requires the two variables to be cointegrated of order CI (1, 1). Thus in the cointegration modeling, we first need to test for stationarity. In order to test whether a variable is nonstationary we apply the unit root test. The most common is the Augmented Dickey-Fuller (ADF) test in which the null hypothesis is the existence of a unit root.

The Dicky Fuller (DF) test: Consider a series generated by a first order process

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t \dots\dots\dots (11)$$

where  $\{ \varepsilon_t \}$  is a white noise process. Now subtract  $y_{t-1}$  from both sides to get

$$\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t \dots\dots\dots (12)$$

where  $\gamma = a_1 - 1$

The above equation represents a random walk model with a drift or intercept term. In order to test for the presence of a unit root we test the hypothesis  $a_1=1$  or  $\gamma =0$ .

The ADF test adds extra lags in order to make sure that the error term is white noise. The null hypothesis is the same as in the DF test.

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^p \hat{\beta}_i \Delta y_{t-i+1} + \varepsilon_t \dots\dots\dots (13)$$

where,  $\gamma = \sum_{i=2}^p \alpha_i - 1$  and  $\beta_i = - \sum_{j=1}^p \alpha_j$

If the unit root hypothesis is not rejected, we conclude that the series is nonstationary. We conduct the ADF test first in level followed by the test in first difference. The test results show that the spot and forward exchange rates are nonstationary but the first difference of both the time series variables are stationary. Hence both the variables have unit roots, are integrated of order one, i.e. I(1).

The number of observations included in the ADF test is 216 for Canada, Australia, and Japan, 206 for U.K. and 109 for India. For the inclusion of an intercept term and not a trend term, the ADF test critical values are -3.50, -2.89, and -2.58 for 1%, 5%, and 10% level of significance, respectively. For all the countries the ADF test statistic and the p value are reported in table 2.1 on the next page.

Given that the value of the t statistic of the unit root test of the spot and forward rates in level for all the countries are less than the critical values, we conclude that we fail to reject the null hypothesis of a unit root. When we take the first difference of the variables and conduct the unit root test, we find that the values of the t-statistic are greater than the critical values. We thus reject the null hypothesis of a unit root. The first difference of the nonstationary spot and forward rates are stationary. Hence, these variables are I(1).

Table 2.1

*Stationarity Test for Spot and Forward Rates*

Countries	Variable	Test in level		Test in first difference	
		t-statistic	p value	t-statistic	p value
Australia	Spot rate	-0.729	0.836	-13.893	0.000
	Forward rate	-0.786	0.821	-13.915	0.000
Canada	Spot rate	0.156	0.969	-13.992	0.000
	Forward rate	0.069	0.963	-13.565	0.000
India	Spot rate	-0.895	0.786	-8.175	0.000
	Forward rate	-0.904	0.784	-8.117	0.000
Japan	Spot rate	-2.448	0.13	-13.929	0.000
	Forward rate	-2.437	0.133	-14.133	0.000
U.K.	Spot rate	-1.855	0.353	-13.132	0.000
	Forward rate	-1.837	0.362	-13.139	0.000

We also conduct stationarity test for the change in the spot rate ( $s_{t+1} - s_t$ ) and the forward premium ( $f_t - s_t$ ). The number of observations included in the ADF test for all the countries and the critical values are same as above. The ADF test statistic and the p value for the unit root test in level are reported in table 2.2 and 2.3 on the next page.

The value of the t statistic of the unit root test for the change in the spot rate and the forward premium in level for all the countries except for Indian forward premium, are greater than the critical values. Thus we reject the null hypothesis of a unit root and the change in the spot rate ( $s_{t+1} - s_t$ ) and the forward premium ( $f_t - s_t$ ) are stationary, I (0). However, for India and U.K., the forward premium is not stationary. The first difference of the nonstationary forward premium for India and U.K. are stationary with the t-statistic and the p value being -10.37, 0.00 and -6.12, 0.00 respectively. Hence the forward premium for India and U.K. are I (1).

Table 2.2

*Stationarity Test for Change in Spot Rate*

Counties	Australia	Canada	Japan	India	U.K.
t-statistic	-16.636	-17.003	-21.458	-15.053	-19.011
p value	0.000	0.000	0.000	0.000	0.000

Table 2.3

*Stationarity Test for the Forward Premium*

Counties	Australia	Canada	Japan	India	U.K.
t-statistic	-12.652	-12.913	-6.816	-1.838	-2.563
p value	0.000	0.000	0.000	0.360	0.102

The change in the spot rate and the forward premium both meet necessary conditions for stationarity. Thus in testing the UH, the parameter estimates obtained from the level specification (the realized spot rate is regressed on the one-period forward rate) encounter problems related to stationarity but the estimates obtained using the percent change specification (the percent change in the realized spot rate relative to the current spot rate is regressed against the forward premium or the difference between the forward and spot rates) should not be subject to these problems.

## 7. Test for Cointegration

Granger and Newbold (1986) prove that in the presence of nonstationary variables, standard regression results in spurious estimation. The estimators are biased and inconsistent. Under such circumstances, it is appropriate to apply the cointegration technique. Over time an economic system converges to a long-run equilibrium which is

imitated by cointegration. If two or more series are themselves nonstationary, but a linear combination of them is stationary, then the series are said to be cointegrated. Cointegration is an econometric property of time series variables. It has been empirically verified that the spot and forward rates are cointegrated. Cointegration implies that the changes in the spot rate can be modeled by an error correction model.

Now we proceed to test whether the I(1) spot and forward rates for the currencies are cointegrated. The two main methods for testing for cointegration are the Engle-Granger two-step method and the Johansen procedure.

### *7.1 The Engle-Granger method*

In the first step, we pretest the variables for their order of integration because cointegration requires the variables to be integrated of the same order. Using the ADF test we found that the spot and the forward rates are both I(1). In the second step, we estimate the long run equilibrium relationship in the form

$$s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1} \dots\dots\dots (4)$$

which is the regression equation of the “level” specification used to test the UH. The residual sequence is denoted by  $\{\hat{\varepsilon}_{t+1}\}$  which is the estimated value of the deviations from the long run relationship. The I(1) spot and forward rates are cointegrated of order CI (1, 1) if the residual series are stationary. In order to test the residual series for their order of integration we conduct the ADF test.

The number of observation included in the ADF test for the residual series for all the countries and the critical values are the same as before. The ADF test statistic and the p values for the test in level are reported below in table 2.4 on the next page.

Table 2.4

*Stationarity Test for the Residual Series*

Counties	Australia	Canada	India	Japan	U.K.
t-statistic	-13.318	-13.923	-6.945	-13.726	-12.987
p value	0.000	0.000	0.000	0.000	0.000

Given that the value of the t-statistic of the unit root test of the residual series in the level for all the countries are greater than the critical values, we conclude that we reject the null hypothesis of a unit root. Thus the residual series are stationary and the spot and forward rates are cointegrated of order CI (1, 1).

We next test for cointegration between the contemporaneous spot and forward rates in which the regression equation is of the form

$$s_t = \alpha + \beta f_t + \varepsilon_t \dots\dots\dots (14)$$

The stationarity test is conducted on the residual sequence denoted by  $\{\hat{\varepsilon}_t\}$ . The number of observation included in the ADF test and the critical values are same as before. The ADF test statistic and the p values for the test in level are reported in table 2.5 below.

Table 2.5

*Stationarity Test for the Residual Series (Comtemporaneous Rates)*

Counties	Australia	Canada	India	Japan	U.K.
t-statistic	-13.287	-13.269	-2.234	-6.864	-2.310
p value	0.000	0.000	0.196	0.000	0.170

Since the values of the t-statistic of the unit root test except for India and U.K. are greater than the critical values, we reject the null hypothesis of a unit root. Thus the residual series are stationary and the contemporaneous spot and forward rates are cointegrated of order CI (1, 1), except for India and U.K. However, the first difference of the residual series for India and U.K. is stationary with the t-statistic and the p value being -10.407 and -17.465 0.0000 and the p value is zero in both the cases. Hence the residual series for Indian and U.K. is I(1).

### *7.2 The Johansen procedure*

The test for cointegration using the Engle and Granger procedure requires placing of one of the variables as the dependent variable and the rest of the variables as regressors. If all the variables are cointegrated, the residual series is stationary. We should get the same result irrespective of which variable is placed on the left hand side, i.e., the choice of the variable selected for normalization. Stationarity test on the residual series in a two variable case often yields the same result for a large sample irrespective of which variable is placed on the left hand side. When we aim at testing for cointegration between more than two variables, stationarity test on the residual series may not yield the same result but depend on the choice of the left hand side variable. Also, there might be more than one cointegrating vector. In order to avoid these problems, one can use the Johansen procedure. The Johansen (1988) and Stock and Watson (1988) procedure use the maximum likelihood estimation technique to test for the presence of multiple cointegrating vectors.

The Johansen's maximum likelihood estimator (MLE) is set up on a multivariate approach whose starting point is a k-order VAR model.

$$x_t = A_0 + A_1x_{t-1} + A_2x_{t-2} + \dots + A_px_{t-p} + \varepsilon_t \dots \dots \dots (15)$$

where  $x_t$  is an n-vector of I(1) variables and  $\varepsilon_t$  are Gaussian errors.

First differencing the VAR model we get a vector error correction model (VECM).

$$\Delta x_t = A_0 + \pi x_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta x_{t-i} + u_t \dots \dots \dots (16)$$

where  $\pi = -I + \sum_{i=1}^p A_i$  and  $\pi_i = - \sum_{j=i+1}^p A_j$

The key point to note here is that the rank of the  $\pi$  matrix is equal to the number of independent cointegrating vectors. If the rank of  $\pi$  is zero, it is a null matrix and the VECM is the VAR in first difference. If the rank of  $\pi$  is n, it is of full rank and the vector process is stationary. When the rank of  $\pi$  is 1, there is a single cointegrating vector with  $\pi x_{t-1}$  being the error connection term and if the rank of  $\pi$  is greater than 1 but less than n, there are multiple cointegrating vectors.

The rank of a matrix is equal to the number of its non zero characteristic roots. The number of significant characteristic roots of  $\pi$  equals the number of distinct cointegrating vectors. If the variables are not cointegrated, the rank of the  $\pi$  matrix is zero and all the characteristic roots of  $\pi$  equals zero.

Since all the variables except  $x_{t-1}$  are stationary, I(0) for the VECM to be consistent,  $\pi$  should not be of full rank. Let r be the rank of  $\pi$ , the properties of  $\alpha$  and  $\gamma$  are such that

$$\pi = \alpha\gamma'$$

$\alpha$  is an  $n \times r$  matrix of adjustment coefficients and can be viewed as the matrix of the speed of adjustment parameters.  $\gamma'$  is an  $r \times n$  matrix of cointegrating parameters, coefficients of the  $r$  cointegrating vectors.

There are two tests of cointegration derived from Johansen's method. One is the trace test ( $\lambda_{trace}$ ) and the other is the maximum eigenvalue test ( $\lambda_{max}$ ). The null hypothesis of the trace test is  $H_0$ : there are at most  $r$  cointegrating vector. The test statistic takes the form

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i) \dots \dots \dots (17)$$

where the  $\hat{\lambda}_i$  are the estimated values of the characteristic roots and  $T$  is the total number of observations. The maximum eigenvalue statistic tests the null hypothesis of  $r$  cointegrating vectors against the alternative of  $(r+1)$  cointegrating vectors:

$$\lambda_{max}(r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1}) \dots \dots \dots (18)$$

The critical values of the  $\lambda_{trace}$  and the  $\lambda_{max}$  tests are given by Osterwald – lenum (1992).

The statistics distribution depends on the deterministic components of the model; three cases are considered: a) no drift in the VECM (i.e. no constant term ( $A_0 = 0$ )); b) the VECM include a constant term ( $\Delta x_t$  have a non zero mean, hence the components of  $x_t$  drift); c) a constant term in the cointegrating vectors.

### 7.3 Test results

We first conduct cointegration test between  $s_{t+1}$  and  $f_t$  by the Johansen procedure. The number of observations used for Australia, Canada, and Japan are 212, for U.K. we have 200 observations and for India 105 observations. We have used the linear deterministic trend assumption for the unrestricted cointegration rank test. The first columns in the results indicate the hypothesized number of cointegrating equations denoted by CE. The results for the trace test are reported in table 2.6 below and the maximum eigenvalue test results are reported in table 2.7 on the next page.

Table 2.6

#### *Trace Test*

Country	CE	Eigenvalue	Trace statistic	0.05 critical value	p value
Australia	None*	0.189	45.342	15.495	0.000
	At most 1	0.004	0.913	3.841	0.339
Canada	None*	0.139	31.895	15.495	0.000
	At most 1	0.000	0.005	3.841	0.944
India	None	0.114	15.426	15.495	0.051
	At most 1	0.026	2.744	3.841	0.098
Japan	None*	0.113	33.861	15.495	0.000
	At most 1 *	0.038	8.301	3.841	0.004
U.K.	None	0.027	6.196	15.495	0.672
	At most 1	0.004	0.789	3.841	0.374

\* denotes rejection of the hypothesis at the 0.05 level.

Table 2.7

*Maximum Eigenvalue Test*

Country	CE	Eigenvalue	Max -Eigen statistics	0.05 critical value	p value
Australia	None*	0.189	44.429	14.265	0.000
	At most 1	0.004	0.913	3.841	0.339
Canada	None*	0.139	31.895	14.265	0.000
	At most 1	0.000	0.005	3.841	0.944
India	None	0.114	12.682	14.265	0.088
	At most 1	0.026	2.744	3.841	0.098
Japan	None*	0.113	25.561	14.265	0.000
	At most 1*	0.038	8.301	3.841	0.004
U.K.	None	0.027	5.407	14.265	0.690
	At most 1	0.004	0.789	3.841	0.374

\* denotes rejection of the hypothesis at the 0.05 level.

Both the tests indicate one cointegrating equation at the 0.05 level for Australia and Canada and no cointegrating equation at the 0.05 level for India and U.K. In the case of Japan there are two cointegrating equation at the 0.05 level. For India there are two cointegrating equation at the 0.10 level for both the tests while for U.K. there is no cointegrating equation at the 0.10 level for any of the tests. The results for the trace test and the maximum eigenvalue test for India at the 10% level are reported in table 2.8 below and those for U.K. at the 10% level are reported in table 2.9 on the next page.

Table 2.8

*Trace Test (India & U.K.)*

Country	CE	Eigenvalue	Trace statistic	0.01 critical value	p value
India	None *	0.113773	15.42592	13.42878	0.0512
	At most 1*	0.025793	2.743814	2.705545	0.0976
U.K.	None	0.026675	6.196434	13.42878	0.6723
	At most 1	0.003937	0.788982	2.705545	0.3744

\* denotes rejection of the hypothesis at the 0.05 level.

Table 2.9

*Maximum Eigenvalue Test (India & U.K.)*

Country	CE	Eigenvalue	Max -Eigen statistics	0.01 critical value	p value
India	None *	0.114	12.682	12.296	0.088
	At most 1 *	0.026	2.744	2.705	0.098
U.K.	None	0.027	5.407	12.296	0.690
	At most 1	0.004	0.789	2.705	0.374

\* denotes rejection of the hypothesis at the 0.05 level.

We observe that the cointegration test results using the Engle-Granger two step methods and the Johansen procedure are the same except for India and U.K. The former test suggest cointegration between  $s_{t+1}$  and  $f_t$  for all the countries while the later test indicates cointegration between  $s_{t+1}$  and  $f_t$  for Australia, Canada, and Japan while no cointegration between  $s_{t+1}$  and  $f_t$  for U.K. at 5% and 10% levels of significance. However, for India, there are two cointegrating equation at the 0.10 level for both the tests in the Johansen procedure.

Next we conduct cointegration test between  $s_t$  and  $f_t$  by the Johansen procedure. The number of observations used for Australia, Canada, and Japan are 213, for U.K. we have 201 observations and for India 106 observations. We have used the linear deterministic trend assumption for the unrestricted cointegration rank test. The results for the trace test are reported in table 2.10 and the maximum eigenvalue test results are reported in table 2.11 on the next page.

Table 2.10

*Trace Test (Comtemporaneous Rates)*

Country	CE	Eigenvalue	Trace statistic	0.05 critical value	p value
Australia	None*	0.148	34.758	15.495	0.000
	At most 1	0.002	0.507	3.841	0.476
Canada	None*	0.116	26.361	15.495	0.001
	At most 1	0.0002	0.061	3.841	0.804
India	None *	0.114	17.268	15.495	0.027
	At most 1*	0.041	4.443	3.841	0.035
Japan	None*	0.096	31.028	15.495	0.0001
	At most 1 *	0.044	9.624	3.841	0.002
U.K.	None	0.030	7.545	15.495	0.515
	At most 1	0.007	1.410	3.841	0.235

\* denotes rejection of the hypothesis at the 0.05 level.

Table 2.11

*Maximum Eigenvalue Test (Comtemporaneous Rates)*

Country	CE	Eigenvalue	Max -Eigen statistics	0.05 critical value	p value
Australia	None*	0.148	34.250	14.265	0.000
	At most 1	0.002	0.507	3.841	0.476
Canada	None*	0.116	26.230	14.265	0.0004
	At most 1	0.0002	0.061	3.841	0.804
India	None	0.114	12.825	14.265	0.083
	At most 1*	0.041	4.443	3.841	0.035
Japan	None*	0.096	21.404	14.265	0.003
	At most 1*	0.044	9.624	3.841	0.002
U.K.	None	0.030	6.135	14.265	0.596
	At most 1	0.007	1.410	3.841	0.235

\* denotes rejection of the hypothesis at the 0.05 level.

The test results are exactly the same as with the cointegration test between  $s_{t+1}$  and  $f_t$  for all the countries except for India, where the trace test indicates two cointegration equations the 0.05 level while the maximum eigenvalue test indicates no cointegration at

the 0.05 level. For India there are two cointegrating equation at the 0.10 level for both the tests while for U.K. there is no cointegrating equation at the 0.10 level for any of the tests. The results for the trace test and the maximum eigenvalue test for India and U.K. at the 10% level are reported in table 2.12 and 2.13 below.

Table 2.12

*Trace Test (India & U.K., Contemporaneous Rates)*

Country	CE	Eigenvalue	Trace statistic	0.01 critical value	p value
India	None *	0.114	17.268	13.429	0.027
	At most 1 *	0.041	4.443	2.705	0.035
U.K.	None	0.030	7.545	13.429	0.515
	At most 1	0.007	1.410	2.705	0.235

\* denotes rejection of the hypothesis at the 0.05 level.

Table 2.13

*Maximum Eigenvalue Test (India & U.K., Contemporaneous Rates)*

Country	CE	Eigenvalue	Max -Eigen statistics	0.01 critical value	p value
India	None *	0.114	12.825	12.296	0.083
	At most 1 *	0.041	4.443	2.705	0.035
U.K.	None	0.030	6.135	12.296	0.596
	At most 1	0.007	1.410	2.705	0.235

\* denotes rejection of the hypothesis at the 0.05 level.

*7.4 Implications of the Test Results*

Barnhart and Szakmary (1991) state that the cointegration test results have important implications for empirical tests of the FRUH. This hypothesis is not often

rejected when using the level specification because the realized spot and forward rates are cointegrated of order CI (1, 1). The UH holds in the long run with considerable departures from the long run relationship in the short run. Only the long run relationship is examined by the level specification with the short run dynamics being ignored. This leads us to the wrong conclusion that the long run relation is the true one and that the UH holds.

Since the contemporaneous spot and forward rates are cointegrated of order CI (1, 1) an error correction model (ECM) based on the percent change specification should be used to model these dynamic relationships. In the ECM, we regress the change in the spot rate on the lagged forward spot differential and lagged changes in spot and forward rates.

Consequently, neither the level nor the percent change specifications are appropriate representations of the spot or forward relation. The level specification is simply a long run equilibrium relation that ignores the short run dynamics altogether, while the percent change specification is a misspecified ECM that incorporates the cointegrating relation between contemporaneous spot and forward rates, but ignores the short run dynamics by leaving out the lagged differences in spot and forward rates. Theoretically then, only the ECM is a proper specification of the relationship between spot and forward rates. (Barnhart & Szakmary, 1991, p. 253).

## 8. Error Correction Model

Having found that the I(1) spot and forward rates are cointegrated of order CI (1, 1), we proceed to estimate an error correction representation of the spot and forward rates. The ECM for all the countries takes the following form:

$$\Delta s_{t+1} = c_s + \alpha_s (s_{t-1} - \beta f_{t-1} - \mu) + \delta_{s1} \Delta s_{t-1} + \delta_{s2} \Delta s_{t-2} + \gamma_{s1} \Delta f_{t-1} + \gamma_{s2} \Delta f_{t-2} + \varepsilon_{st+1} \dots \dots \quad (19a)$$

$$\Delta f_t = c_f + \alpha_f (s_{t-1} - \beta f_{t-1} - \mu) + \delta_{f1} \Delta s_{t-1} + \delta_{f2} \Delta s_{t-2} + \gamma_{f1} \Delta f_{t-1} + \gamma_{f2} \Delta f_{t-2} + \varepsilon_{ft} \dots \dots \quad (19b)$$

We estimate the ECM for all the countries and briefly discuss the results. For both the equations of the ECM, the parameter estimates and the t-values for all the countries are reported in table 2.14 on the next page.

Table 2.14

*Parameter Estimates of the ECM*

Parameters	Counties	Australia	Canada	Japan	India	U.K.
$c_s$	estimates	-0.001	-0.001	-0.002	-0.001	0.000
	t-statistic	-0.723	-0.464	-0.654	-0.987	-0.080
$c_f$	estimates	-0.001	-0.001	-0.001	-0.0002	0.000
	t-statistic	-1.875*	-2.655**	-3.632***	-0.866	1.875*
$\alpha_s$	estimates	0.435	-0.314	-0.450	0.903	-1.295
	t-statistic	0.971	-0.560	-0.061	2.318**	-1.169
$\alpha_f$	estimates	0.873	0.790	0.560	0.369	0.025
	t-statistic	9.461***	8.374***	6.589***	4.150***	1.931*
$\beta$	estimates	-1.009	-1.007	-0.997	-1.030	-0.996
	t-statistic	-314.417***	-447.326***	-238.879***	-85.846***	-135.434***
$\mu$	estimates	0.005	0.0023	-0.014	0.117	-0.004
$\delta_{S1}$	estimates	-0.368	0.376	0.074	-0.741	1.363
	t-statistic	-0.809	0.673	0.101	-1.738*	1.236
$\delta_{f1}$	estimates	0.101	0.211	0.432	0.561	0.971
	t-statistic	1.077	2.245**	5.118***	5.758***	74.018***
$\delta_{S2}$	estimates	-0.071	0.856	-0.717	-0.077	1.282
	t-statistic	-0.208	2.040**	-1.265	-0.184	0.218
$\delta_{f2}$	estimates	0.022	0.161	0.311	0.222	0.239
	t-statistic	0.316	2.272**	4.736***	2.341**	3.418***
$\gamma_{S1}$	estimates	-0.080	-0.880	0.775	0.031	-1.391
	t-statistic	-0.236	-2.063**	1.362	0.076	-0.235
$\gamma_{f1}$	estimates	-0.210	-0.135	-0.325	-0.206	-0.242
	t-statistic	-0.302	-1.887*	-4.929***	-2.213**	-3.440***
$\gamma_{S2}$	estimates	0.084	-0.049	0.073	0.038	0.067
	t-statistic	1.219	-0.696	1.050	0.386	0.956
$\gamma_{f2}$	estimates	-0.000	-0.013	-0.002	-0.017	-0.001
	t-statistic	-0.258	-1.074	-0.240	-0.755	-1.337

\*, \*\*, \*\*\* denotes significance at 10%, 5%, and 1% level respectively.

Next, the estimated ECM is written for each country containing only those variables which are significant at least at 10% level of significance. The ECM contains

error correction terms  $\alpha_s (s_{t-1} - \beta f_{t-1} - \mu)$  and  $\alpha_f (s_{t-1} - \beta f_{t-1} - \mu)$  and lagged spot forward exchange rates.  $\alpha_s$  and  $\alpha_f$  are the speed of adjustment parameters/coefficients.

The estimated errors are  $\hat{\varepsilon}_{st+1}$  and  $\hat{\varepsilon}_{ft}$ .

Australia:

$$\Delta f_t = -0.001 + 0.873(s_{t-1} - 1.009 f_{t-1} + 0.005) + \hat{\varepsilon}_{ft}$$

Canada:

$$\Delta s_{t+1} = 0.856 \Delta s_{t-2} - 0.880 \Delta f_{t-1} + \hat{\varepsilon}_{st+1}$$

$$\Delta f_t = -0.001 + 0.790(s_{t-1} - 1.007 f_{t-1} + 0.0023) + 0.211 \Delta s_{t-1} + 0.161 \Delta s_{t-2} - 0.135 \Delta f_{t-1} + \hat{\varepsilon}_{ft}$$

Japan:

$$\Delta f_t = -0.001 + 0.560(s_{t-1} - 0.997 f_{t-1} - 0.268) + 0.422 \Delta s_{t-1} + 0.311 \Delta s_{t-2} - 0.325 \Delta f_{t-1} + \hat{\varepsilon}_{ft}$$

India:

$$\Delta s_{t+1} = 0.903(s_{t-1} - 1.030 f_{t-1} + 0.117) - 0.741 \Delta s_{t-1} + \hat{\varepsilon}_{st+1}$$

$$\Delta f_t = 0.369(s_{t-1} - 1.030 f_{t-1} + 0.117) + 0.561 \Delta s_{t-1} + 0.222 \Delta s_{t-2} - 0.206 \Delta f_{t-1} + \hat{\varepsilon}_{ft}$$

U.K.:

$$\Delta f_t = 4.28E-05 + 0.025(s_{t-1} - 0.996 f_{t-1} - 0.004) + 0.971 \Delta s_{t-1} + 0.239 \Delta s_{t-2} - 0.242 \Delta f_{t-1} + \hat{\varepsilon}_{ft}$$

In all the cases the parameter  $\alpha_f$  is significant while  $\alpha_s$  is significant for India only. In those equations where the error correction term is not significant, it represents a VAR. In the first equation of the ECM, the sign of the speed of adjustment coefficient  $\alpha_s$  should be negative while in the second equation of the ECM, the sign of the speed of adjustment coefficient  $\alpha_f$  should be positive to be in agreement with convergence towards the long run equilibrium. Consider the error correction term  $(s_{t-1} - \beta f_{t-1} - \mu)$ ; if the spot rate exceeds the forward rate,  $(s_{t-1} - f_{t-1} > 0)$ , the adjustment in the next period should be such that the spot rate decreases while the forward rate increases. Only then will the system converge towards the long run equilibrium. Hence the coefficient  $\alpha_s$  should be negative and  $\alpha_f$  positive. The larger the value of the speed of adjustment coefficient, the faster is the adjustment process; larger is the value of  $\Delta s_{t+1}$  and  $\Delta f_t$ . If  $\alpha_s$  (or  $\alpha_f$ ) is zero, the spot exchange rate (or the forward rate) does not respond to any deviations from the long run equilibrium. These variables should not be equal to zero if the spot rate and the forward rate are cointegrated. If they are both zero, there is no error correction and the ECM is a VAR. For all the countries,  $\alpha_f$  is positive and significant while  $\alpha_s$  is positive and significant for India, positive but not significant for Australia, negative but not significant for Canada, Japan, and U.K. Since  $\alpha_s$  is not significantly different from zero, the spot exchange rate does not respond to any deviations from the long run equilibrium for all the countries except India. Since in the Indian context,  $\alpha_s$  has a wrong sign, the system does not converge towards the long run equilibrium. Moreover, the first equation of the ECM is not significant for Australia, Japan, and U.K. while in case of Canada it represents a VAR. The estimated value of  $\beta$  is negative, approximately equal to one and significant at all levels for all the countries.

## **9. Are Exchange Rates and Interest Rates Cointegrated?**

Exchange rates, the relative prices between two currencies are determined by the desire of residents to hold domestic and foreign financial assets. According to the monetary approach to exchange rate determination, exchange rates are determined through the process of matching the total supply of and the total demand for the domestic money in each country. The money supply can be controlled by the nation's monetary authorities and the aggregate real money demand is negatively related to the interest rate and positively related to the real output. Any change in the interest rates and output affect the exchange rate through their influence on money demand.

Domestic interest rate influences the decision of foreigners to purchase currency in order to invest in domestic asset. Higher interest rates attract capital from abroad which results in an increase in the money base and thus the domestic currency appreciates. On the other hand, an increase in the domestic interest rate leads to a decrease in investments, higher interest rates contract bank loans taken by firms to finance the wage bill which reduces employment and output and higher interest rates raise the government's fiscal burden resulting in higher expected inflation. These effects tend to depreciate the currency. A reduction in interest rates abroad would have the same effects. Interest rates thus have an important impact on exchange rate but the exchange rate response depends on the size of the interest rate increase and on the initial level of the interest rate.

Due to financial market arbitrage, the interest rate differential is an unbiased predictor of change in future spot rate. According to the Interest Rate Parity Theory the Covered Interest Arbitrage profits does not last long, and in equilibrium they are zero.

Higher interest rates on a currency are offset by forward discounts while lower interest rates are offset by forward premiums.

The interest rate parity, a non-arbitrage condition is the identity that relates interest rates and exchange rates. Exchange rate is the rate at which domestic goods are traded for foreign goods and interest rate is the rate at which goods today are traded for future goods. The condition that deposits of all currencies offer the same expected rate of return when measured in the same currency is called the interest parity condition. The implication is that holders of foreign currency deposits consider them all to be equally desirable assets.

The foreign exchange market is in equilibrium only when the interest parity condition holds, that is, there exists no excess supply of some type of deposit and no excess demand of another. The interest rate parity states that the spot and future prices for currency trades incorporate any interest rate differentials between the two currencies. The two versions of the interest rate parity are covered interest rate parity and uncovered interest rate parity.

The covered interest parity condition defines the foreign exchange market equilibrium involving the forward exchange rate while the uncovered interest parity condition uses the expected future spot exchange rate. The uncovered interest parity condition equals the covered interest parity condition when the forward rate quoted today equals the expected future spot exchange rate. Under such situation the forward rate is an unbiased predictor of the expected future spot exchange rate and thus the UH holds. The covered interest parity condition states that the rates of return on domestic deposit and covered foreign deposit must be the same, thus the interest rate on domestic deposits

equals the interest rate on foreign deposits plus the forward premium. The forward exchange premium equals the difference between nominal interest rates in the two countries

$$\text{Forward premium} = f_t - s_t = R_{dt} - R_{ft}$$

where  $f_t$  and  $s_t$  are the natural logarithms of the forward and spot exchange rates observed at time  $t$  respectively and  $R_{dt}$  and  $R_{ft}$  are the nominal interest rates observed at time  $t$  in the domestic and foreign country respectively.

Given the close connection between the exchange rates and the interest rates, it will be interesting to test empirically whether there exists a long run relation between these variables. In other words, are exchange rates and interest rates cointegrated? We have already empirically verified that the nonstationary spot and forward exchange rates are  $I(1)$ , i.e., their first difference is stationary and they are  $CI(1, 1)$  which confirms that a long run relationship exists between them. Now we proceed to empirically test whether a long run relationship exists between the spot and forward exchange rates and the domestic and foreign interest rates. Thus the test involves testing for cointegration between the four variables.

Here we are not only testing for a long run relationship between these four variables but also conducting a robustness check or a sensitivity analysis where we ensure whether the two cointegrated variables, the spot and forward exchange rates, are still related in the long run with the inclusion of two more variables, the domestic and foreign interest rates.

We first conduct stationarity test on the interest rates followed by cointegration test among all four variables for the four advanced economies.

### *9.1 Stationarity Test for Interest Rates*

We now conduct the stationarity test for interest rates in level and in first difference. The number of observations included in the ADF test is 147 for Australia, 199 for Canada, 184 for Japan, 129 for U.K., 182 for the USA prime rate and 202 for the USA T bill rate. For the inclusion of an intercept term and not a trend term, the ADF test critical values are -3.47, -2.89, and -2.58 for 1%, 5%, and 10% level of significance, respectively. For all the countries the ADF test statistic and the p values are reported in table 2.15 below.

Table 2.15

#### *Stationarity Test for Interest Rates*

Countries	Test in level		Test in first difference	
	t-statistic	p value	t-statistic	p value
Australia	-3.374	0.013	-11.631	0.000
Canada	-1.878	0.342	-6.399	0.000
Japan	0.209	0.973	-8.763	0.000
U.K.	-2.004	0.285	-4.646	0.000
USA(T-bill)	-1.681	0.439	-5.937	0.000
USA(prime)	-2.074	0.256	-3.897	0.002

The value of the t-statistic of the unit root test in level for all the countries are less than the critical values except for Australia where it is less than the critical value at 10% level of significance. Hence, we fail to reject the null hypothesis of a unit root. The

values of the t-statistic of the unit root test conducted on the first difference of the interest rates are greater than the critical values. We thus reject the null hypothesis of a unit root. The first difference of the nonstationary interest rates is stationary.

## 9.2 Cointegration Test

Given that the exchange rates and the interest rates are all integrated of the same order, i.e. they are all I(1), we conduct the cointegration test. The Johansen cointegration test for all the countries has the linear deterministic trend assumption. The number of observations used after adjustments are 143 for Australia, 116 for Canada, 180 for Japan and 126 for U.K. The results for the unrestricted cointegration rank test for all the countries are reported in table 2.16 below and table 2.17 on the next page.

Table 2.16

### *Trace Test (Exchange Rates and Interest Rates)*

Country	CE	Eigen value	Trace statistic	0.05 critical value	p value
Australia	None *	0.218	69.245	47.856	0.000
	At most 1 *	0.123	34.056	29.797	0.015
	At most 2	0.082	15.297	15.495	0.053
	At most 3	0.020	2.970	3.841	0.085
Canada	None *	0.273	53.240	47.856	0.014
	At most 1	0.077	16.302	29.797	0.691
	At most 2	0.048	6.989	15.495	0.579
	At most 3	0.011	1.265	3.841	0.261
Japan	None *	0.159	56.574	47.856	0.006
	At most 1	0.100	25.407	29.797	0.147
	At most 2	0.035	6.438	15.495	0.644
	At most 3	0.000	0.001	3.841	0.978
U.K.	None	0.114	34.282	47.856	0.486
	At most 1	0.089	19.048	29.797	0.489
	At most 2	0.052	7.396	15.495	0.532
	At most 3	0.005	0.605	3.841	0.437

\* denotes rejection of the hypothesis at the 0.05 level.

Table 2.17

*Maximum Eigen Value Test (Exchange Rates and Interest Rates)*

Country	CE	Eigen value	Max -Eigen statistics	0.05 critical value	p value
Australia	None *	0.218	35.189	27.584	0.004
	At most 1	0.123	18.759	21.132	0.104
	At most 2	0.082	12.327	14.265	0.099
	At most 3	0.020	2.970	3.841	0.085
Canada	None *	0.273	36.938	27.584	0.002
	At most 1	0.077	9.313	21.132	0.806
	At most 2	0.048	5.723	14.265	0.649
	At most 3	0.011	1.265	3.841	0.261
Japan	None *	0.159	31.167	27.584	0.017
	At most 1	0.100	18.969	21.132	0.098
	At most 2	0.035	6.437	14.265	0.558
	At most 3	0.000	0.001	3.841	0.978
U.K.	None	0.114	15.234	27.584	0.729
	At most 1	0.089	11.652	21.132	0.582
	At most 2	0.052	6.791	14.265	0.514
	At most 3	0.005	0.605	3.841	0.437

\* denotes rejection of the hypothesis at the 0.05 level.

*9.3 Implication of the Test Results*

The test results indicate that there exists a long run relation between the spot exchange rate, the forward exchange rate, the domestic interest rate and the foreign interest rate for Australia, Canada, and Japan while there is no cointegration between these variables in the case of U.K. This does not come as a surprise because for U.K. the cointegration test between the spot and the forward exchange rate indicated no cointegrating relationship when the Johansen procedure was applied.

**10. Conclusions**

In this paper we analyzed the UH from the perspective of time series properties of the exchange rates. Stationarity test on the spot and forward exchange rates of the five

countries confirms that they are nonstationary but first differencing of these variables makes them stationary. Hence, they are  $I(1)$ , are integrated of the same order, a precondition for cointegration. Further, the cointegration test confirms the existence of a long-run relationship between the spot and forward exchange rates in most of the cases. Since the spot and forward rates are cointegrated of order  $CI(1, 1)$ , an ECM based on the percent change specification is used to model these dynamic relationships.

We next include the domestic and the foreign interest rates in our analysis and conduct cointegration test among all four variables. In all of the cases except for U.K. we are able to confirm the existence of a long run relationship between the spot and forward exchange rates and the domestic and foreign interest rates.

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ESSAY 3: EXPLAINING FORWARD RATE UNBIASEDNESS HYPOTHESIS,  
ERROR CORRECTION MODEL AND GENERALIZED METHOD OF MOMENTS

### 1. Introduction

Forward rates should be unbiased forecasts of future spot rates since it fully reflects available information about investor's expectations of future spot rates. However, empirical evidence suggests that forward rates are neither efficient nor rational forecasts of future spot rates. This might be due to the use of an inappropriate or misspecified model to test the hypothesis or some kind of empirical error (for example, not accounting for non-normality or nonstationarity in the data or improper statistical technique employed to test the hypothesis) or both. In a large literature, the FRUH have not been supported by the empirical evidence which has demonstrated the puzzling result that the slope coefficient ( $\beta_1$ , from equations 1 and 2 in essay 1) is significantly less than unity and most often negative in sign. As discussed in essay 2, this is the Forward Premium Puzzle, indicating foreign exchange market inefficiency. This implies that one cannot use the forward rate directly as a measure of the future spot rate.

From the results of the test of the UH in essay 1, we observe an anomaly between estimates of  $\beta_1$  obtained from the level specification, equation (1) and estimates obtained from the percent change specification, equation (2). It has been empirically verified in the past that the estimates of  $\beta_1$  in the level specification are close to one while in the percent change specification the estimates are less than one and most often negative in sign. The explanation for this contradiction can be found in the literature on cointegration which was the main objective of essay 2.

Neither specification appropriately represents the relationship between the spot and forward exchange rates. The forward rate correctly forecasts the realized spot rate in the long run with considerable departures in the short run. The level specification examines the long run relationship while ignoring the short run dynamics. Since the realized spot and forward rates are co-integrated with cointegrating parameter close to one, the estimates of  $\beta_1$  in the level specification are close to one. Hence, we cannot reject the UH when using the level specification. On the other hand, the percent change specification is a misspecified ECM which ignores the short run dynamics by leaving out the lagged differences in the spot and forward rates. Therefore, only the ECM is a proper specification of the relationship between the spot and forward rates.

Empirical evidence suggests that the spot and forward rates are nonstationary with unit roots and are cointegrated, CI (1, 1). Cointegration further suggests that the changes in the spot rate can be modeled by an ECM in which we regress the change in the spot rate on the lagged forward-spot differential and lagged changes in spot and forward rates. Naka and Whitney (1995) show that the paradox disappears when an ECM is explicitly derived from levels specification, both linked by the error correction term, of the ECM.

### *1.1 Objective*

The objective of essay 2 was to analyze one of the interpretations for the rejection of the FRUH, the nonstationarity of the spot and forward rates. As mentioned in essay 2, nonstationarity implies that the estimators of the slope coefficient ( $\beta_1$ ) are biased and inconsistent. If we regress one nonstationary series against another, we end up with spurious results (Granger & Newbold, 1974) in which the conventional significance tests

will indicate a relation between the variables when in fact none exists. For the data set used in this dissertation, essay 2 confirms that the spot and forward rates are nonstationary with unit roots and are cointegrated of order CI (1, 1). Hence, an ECM was used to model these dynamic relationships.

The anomaly between estimates of  $\beta_1$  obtained from the level and the percentage change specification has been explained by cointegration. According to Naka and Whitney (1995) and Bakshi and Naka (1997), this inconsistency disappears when an ECM is explicitly derived from the cointegration of the forward and the future spot rate. The results of Bakshi and Naka (1997) are that when test of the UH is conducted with an ECM using the generalized method of moments (GMM), the UH cannot be rejected. Their data ranges from January 1974 to April 1991 for Canada, U.K., Japan, France, Italy, Germany, and Switzerland.

The objective of this essay is to apply the ECM derived by Naka and Whitney (1995) and Bakshi and Naka (1997) and use GMM estimation technique to test whether the same results hold for a different (more recent) time period for four developed economies (U.K., Canada, Australia, and Japan) and an emerging market economy, India.

We start by briefly describing the GMM in section 2 followed by the derivation of the Naka and Whitney (1995) and Bakshi and Naka (1997) error correction model in section 3. The empirical results are in section 5 and section 6 concludes this essay.

## 2. The Generalized Method of Moments

Ordinary least squares (OLS) regression requires that the errors be serially uncorrelated. In the test of the FRUH, estimation is complicated due to the presence of moving average error. Autocorrelation requires the use of generalized least squares (GLS) instead of OLS. In case of time series data, GLS requires the independent variable to be uncorrelated with the error term. In the test of FRUH, the contemporaneous forward exchange rate, past forecast errors, or past rates of return to speculation is included as independent variables which violates the assumption of exogeneity of the independent variable. If the independent variables are not strictly exogenous, GLS estimation filters the data, distorts the orthogonality conditions and renders the estimator inconsistent (Hansen & Hodrick, 1980). In such a situation the GMM estimation technique will produce a consistent estimator.

GMM estimators do not require parametric assumptions. Unlike maximum likelihood estimation (MLE), GMM does not require complete knowledge of the distribution of the data. GMM estimation requires specified moments derived from an underlying model. This technique is called method of moments because a population moment can be estimated using the corresponding moment of a sample (e.g., the mean, variance, or skewness). GMM estimates sample statistics of the system parameters that have the same property in the sample as population parameters have in the population. Also, the GMM estimation provides the option to use a non-parametric or semi-parametric approach.

The advantage of GMM over other estimation techniques is that it requires specification of only certain moment conditions (e.g., the mean, variance, or skewness)

for the parameters to be estimated, rather than the full distribution function. Many other estimation methods are special cases of GMM, e.g., OLS is a special case in which the regression residuals are uncorrelated with the explanatory variables and the system is just identified. GMM has the advantage of being consistent even in the presence of arbitrary heteroskedasticity. GMM can correct for heteroskedasticity when an appropriate weighting scheme for generalized least-squares estimation cannot be determined and that it yields the White estimator for heteroskedastic disturbances.

### **3. The Error Correction Model**

Bakshi and Naka (1997) derive an ECM under the assumption that the spot and the forward rates are cointegrated, the first difference of forward rates is stationary, and the first order autocorrelation in the forecast error is allowed. The coefficient of the error correction term is linked with the presence of serial correlation in the residuals of the levels specification. “Hence this ECM offers the opportunity to test hypotheses concerning the parameters and serial correlation of the levels specification” (Naka & Whitney, 1995, p. 859).

In Hakkio and Rush (1989) and Barnhart and Szakmary (1991), the FRUH is tested using an ECM which is not explicitly derived from the levels specification. Hence, they fail to examine the link between the parameter of ECM and that of levels specification. Naka and Whitney (1995) differ from them since they identify parameters in the ECM that correspond to the parameters of the levels specification. They estimate the ECM using GMM and fail to reject the UH.

The Naka and Whitney (1995) ECM is derived below. They begin with equation (1) of essay 1 and allow for first order serial correlation in the forecast errors. Equation (1) of the first essay (the levels specification) can be written as:

$$s_t - \beta f_{t-1}^f = \alpha + \varepsilon_t \dots\dots\dots (1)$$

The forecast errors are assumed to be serially correlated.

$$\varepsilon_t = \rho \varepsilon_{t-1} + v_t \dots\dots\dots (2)$$

where  $\rho$  is the autoregression coefficient of the forecast error and  $v_t$  is a white noise error term. We have  $\varepsilon_{t-1} = s_{t-1} - \beta f_{t-2}^f - \alpha$  and substituting equation (2) into equation (1)

$$s_t - \beta f_{t-1}^f = (1 - \rho)\alpha + \rho(s_{t-1} - \beta f_{t-2}^f) + v_t \dots\dots\dots (3)$$

Existing empirical evidence suggests that the market for forward exchange rates is efficient (Bakshi & Naka, 1997). Thus the forward exchange rate is generated by the following process,

$$f_{t-1} = f_{t-2} + e_{t-1} \dots\dots\dots (4)$$

where  $e_{t-1}$  is a white noise error term. If the error terms in equation (3) and (4) are i.i.d, and uncorrelated (past changes in the forward premium do not contain information about future spot rates) then equation (3) and (4) form a triangular system of the type described by Phillips (1991) and Phillips and Loretane (1991). Subtracting  $s_{t-1}$  from both sides of equation (3) and rearranging terms we obtain an ECM of the following form:

$$s_t - s_{t-1} = (1 - \rho)\alpha + (1 - \rho)(\beta f_{t-2}^f - s_{t-1}) + \beta(f_{t-1} - f_{t-2}) + v_t \dots\dots\dots (5)$$

The most important difference between Naka and Whitney (1995) ECM and other ECMs, for example, Hakkio and Rush (1989) and Barnhart and Szakmary (1991) is that the coefficient on the error correction term, the second term of equation (5), has an

autocorrelation term embedded and is affected by the presence of serial correlation in the residuals of the level specification.

We can test the FRUH using equation (5) by imposing the restrictions that  $\alpha = 0$  and that  $\beta = 1$ . The hypotheses tests regarding  $\alpha$  and  $\beta$  (the cointegrating parameters) using equation (5) may not be conducted with the standard asymptotic chi-squared statistics. Equations (3) and (4) forming a triangular system can be estimated by a single equation using Non-linear Least Squares (NLS), and the standard chi-squared statistics can be used in hypothesis tests. Equation (5) is a nonlinear equation based on the cointegrated system of equations which is estimated by Bakshi and Naka (1997) by the GMM estimation technique. GMM directly test hypotheses concerning the parameters of interest.

..... in a rigorous econometric setting, the model provides a convenient framework to estimate the unknown parameters and to test the statistical adequacy of the economic theory. Second, the GMM is a more general method that satisfies the first-order condition for minimizing or maximizing the objective function, i.e., nonlinear least squares and maximum likelihood is special cases of the GMM (Hansen, 1982). Third, the GMM estimators and standard errors are consistent even if the disturbances,  $u(t)$ , are conditionally heteroskedastic (Hodrick, 1991). (Bakshi & Naka, 1997, p. 150-151).

Next, to apply GMM to the ECM and to test the FRUH, rewrite equation (5) as:

$$E \{s_t - s_{t-1} - (1 - \rho)\alpha - (1 - \rho)(\beta f_{t-2} - s_{t-1}) - \beta(f_{t-1} - f_{t-2})\} = 0 \dots\dots\dots (6)$$

$$E\{v_t | Z_t\} = 0$$

$Z_t$  consists of the variables contained in current information. Under the null hypothesis that the restrictions implied by the economic theory are true, the expected value of the error term,  $E\{v_t | Z_t\} = 0$ . The GMM estimations are based upon minimizing the quadratic

form,  $J_t = g_t' W_t g_t$  where  $g_t$  is the sample counterpart of the process  $\{v_t | Z_t\}$  and  $W_t$  is a positive-definite symmetric weighting matrix. The minimized value of the quadratic form, called the  $J(df)$  statistic, is  $\chi^2$  distributed under the null hypothesis that the model is true with degrees of freedom,  $dh$  equal to the number of orthogonality conditions net of the number of parameters to be estimated. The  $J(df)$  statistic provides a goodness-of-fit test for the model, and a high value implies that the model is misspecified. (Note: in a GMM context, when there are more moment conditions than parameters to be estimated, a chi-square test can be used to test the overidentifying restrictions. The test statistic can be called the  $J$  statistic.)

In Bakshi and Naka (1997) the first set of instruments,  $Z1$ , consists of a constant and one lag each of logarithmic change in the spot exchange rate and the forward rate. The instrument set,  $Z2$ , consists of a constant and two lags each of logarithmic change in the spot exchange rate and the logarithmic change in the forward rate. They conduct the test for the seven exchange rates: the British pound, Canadian dollar, Deutsche mark, French franc, Italian lira, Japanese yen, and Swiss franc from January 1974 through April 1991. Their results suggest that if the effect of cointegration of the forward and the future spot rate is taken into consideration through an appropriate ECM, the results are consistent with the FRUH. "A central message of the paper is that when tests of the unbiasedness hypothesis are conducted with an error correction model using the generalized methods of moments, the unbiasedness hypothesis cannot be rejected" (Bakshi & Naka, 1997, p. 159).

#### 4. Data

The data for this essay is the same as used in essays 1 and 2.

#### 5. Empirical results

The UH test results in essay 1 (table 1.1) indicate that the values of  $\beta_1$  estimated from the levels specification, equation (1) are all positive and approximately equal to one. In contrast, the values of  $\beta_1$  estimated from the percentage change specification, equation (2) are not close to one. Therefore, the unbiasedness of the forward rates is rejected for all currencies by equation (2). As mentioned earlier, Naka and Whitney (1995) show that this contradiction disappears when an ECM is explicitly derived from levels specification, both linked by the error correction term, of the ECM.

In order to apply the ECM to test the FRUH, the forward and spot rates need to be cointegrated and in order to be cointegrated they have to be integrated of the same order. The stationarity (Augmented Dickey-Fuller) test results in essay 2 (table 2.1) shows that the spot and forward exchange rates are nonstationary but the first difference of both the time series variables are stationary. Hence both the variables have unit roots, are integrated of order one, i.e. I(1).

The Engle-Granger two-step method and the Johansen procedure were used to test whether the I(1) spot and forward rates for the currencies are cointegrated. The results of the two tests from essay 2 are given below.

##### *4.1 The Engle-Granger Method*

Stationarity test results for the realized spot and forward rates are reported in table 2.4. Given that the value of the t-statistic of the unit root test of the residual series in the

level for all the countries are greater than the critical values, we conclude that we reject the null hypothesis of a unit root. Thus the residual series are stationary and the spot and forward rates are cointegrated of order CI (1, 1).

Stationarity test results for the contemporaneous spot and forward rates are reported in table 2.5. Since the values of the t-statistic of the unit root test except for India and U.K. are greater than the critical values, we reject the null hypothesis of a unit root. Thus the residual series are stationary and the contemporaneous spot and forward rates are cointegrated of order CI (1, 1), except for India and U.K.

#### *4.2 The Johansen Procedure*

We conduct cointegration test between the realized spot and forward rates ( $s_{t+1}$  and  $f_t$ ) by the trace test (table 2.6) and the maximum eigenvalue test (table 2.7). Both the tests indicate one cointegrating equation at the 0.05 level for Australia and Canada and no cointegrating equation at the 0.05 level for India and U.K. In the case of Japan there are two cointegrating equation at the 0.05 level. For India there is one cointegrating equation at the 0.10 level for the trace test only while for U.K. there is no cointegrating equation at the 0.10 level for any of the tests.

We next conduct the cointegration test between the contemporaneous spot and forward rates ( $s_t$  and  $f_t$ ) by the trace test (table 2.10) and the maximum eigenvalue test (table 2.11). The test results are exactly the same as with the cointegration test between  $s_{t+1}$  and  $f_t$  for all the countries except for India, where the trace test indicates two cointegration equations the 0.05 level while the maximum eigenvalue test indicates no cointegration at the 0.05 level. For India there are two cointegrating equation at the 0.10

level for both the tests while for U.K. there is no cointegrating equation at the 0.10 level for any of the tests.

In most of the cases, the spot and forward rates are nonstationary with unit roots and are cointegrated of order CI (1, 1) which provide justification for applying the ECM to test for the unbiasedness of the forward rates.

Next we estimate the ECM as given by equation (6) using the GMM estimation technique. Prior to the estimation, our concerns are the choice of information instruments to be contained in  $Z(t)$ . In the first set of instruments,  $Z1$ , we include a constant and one lag each of the change in the spot exchange rate and the forward rate. In the second set of instruments,  $Z2$ , we include a constant and two lags each of the change in the spot and the forward rate. We also tried other instrument sets which included a constant and one lag each of the logarithmic change in the spot and the forward rate and a constant and two lags each of the logarithmic change in the spot and the forward rate. However the GMM estimation involving these instruments sets resulted in a singular matrix in most of the cases and failed to provide parameter estimates. The estimation is conducted in EVIEWS and the results are reported in table 3.1 on the next page.

For the model derived, a test of the UH, using equation (8) is accomplished by testing the restriction that  $\beta = 1$ . The hypothesis  $\alpha = 0$  is rejected in all the cases except for Canada when using instrument set  $Z2$ . We fail to reject the hypothesis that  $\beta = 1$  for all the cases except Australia and Japan when using instrument set  $Z1$ . However, estimates of  $\beta$  is positive and approximately equals one in case of Canada and U.K. only. Estimates of  $\beta$  approximately equals one but is negative in case of Australia and

Japan when using instrument set Z2. In the case of India, the estimate of  $\beta$  is positive when using instrument set Z1 and negative when using instrument set Z2 but in both the case they are not close to one. The estimates of  $\rho$  are significant and positive in case of Australia, India, and Japan. In case of Canada and U.K.  $\rho = 0$  when using instrument set Z1. The estimates of  $\rho$  are significant but negative in case of Canada and U.K. when using instrument set Z2.

Table 3.1

*GMM Estimation*

Countries	Instruments	Estimates and t-statistic	$\alpha$	$\beta$	$\rho$
Australia	Z1	estimates	1.288	0.104	0.943
		t-statistic	11.594***	1.515	59.750***
	Z2	estimates	2.732	-0.944	0.984
		t-statistic	7.872***	-9.427 ***	80.804 ***
Canada	Z1	estimates	0.029	0.978	0.075
		t-statistic	1.782*	79.352***	0.980
	Z2	estimates	-0.009	1.006	-1.037
		t-statistic	-0.638	95.289***	-12.567***
India	Z1	estimates	25.261	0.436	0.883
		t-statistic	3.331***	2.633***	13.807***
	Z2	estimates	78.035	-0.780	0.980
		t-statistic	6.643***	-4.180***	47.947***
Japan	Z1	estimates	104.218	0.099	0.856
		t-statistic	8.415***	0.942	29.841***
	Z2	estimates	224.307	-0.968	0.947
		t-statistic	14.215***	-8.712***	42.280***
U.K.	Z1	estimates	0.212	0.874	0.169
		t-statistic	2.202**	14.744***	1.233
	Z2	estimates	0.056	0.968	-0.970
		t-statistic	1.667*	45.281***	-6.961***

\*, \*\*, \*\*\* denotes significance at the 10%, 5%, and 1% level, respectively.

## 6. Conclusions

Since the spot and forward rates for the data set used in this dissertation are nonstationary with unit roots and are cointegrated of order CI (1, 1) we use an ECM to model these dynamic relationships. In this essay we use the ECM derived by Naka and Whitney (1995) and Bakshi and Naka (1997) and use GMM to test the UH. We have two set of instruments, Z1 which has a constant and one lag each of the change in the spot and forward rate and Z2 which has a constant and two lags each of the change in the spot and forward rate. The null hypothesis of  $\alpha = 0$  is rejected in nine out of ten cases, estimates of  $\beta$  is positive and approximately equals one in case of Canada and U.K. only and the estimates of  $\rho$  are significant and positive in case of Australia, India, and Japan.

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## CONCLUSIONS

In this dissertation we have tried to verify and explain the forward exchange rate unbiasedness hypothesis. Since in most of the cases the UH fails to hold, we have provided three different explanation of this puzzling behavior in the three essays.

In the first essay we tried to resolve the forward premium puzzle by addressing the model misspecification issue and thereby adding a time-varying risk premium term in the percentage change specification. The risk premium term was modeled using the GARCH-M representation, the model being estimated by applying a GARCH (1, 1) specification. We find evidence of a time varying risk premium for Canada and a nonzero constant risk premium in case of U.K. and in both cases we fail to reject the unbiasedness hypothesis. In case of Australia and India, we fail to find evidence of a time varying risk premium although the unbiased hypothesis is rejected and the error is heteroskedastic. In the case of Japan, we cannot apply GARCH model because the error term is homoskedastic.

In the second essay we have analyzed the time series properties of the exchange rates. It attributes the failure of the UH to hold to the nonstationarity of the spot and forward exchange rate and verified the existence of a cointegrating relationship between them. Thus an Error Correction Model (ECM) was used to better capture the relation between the spot and the forward rates. Further, a cointegrating or the existence of a long run relationship between the spot and forward exchange rates and the domestic and foreign interest rates was tested. As a robustness check, we ensure whether the cointegrated exchange rates are still related in the long run with the inclusion of the

interest rates. In all of the cases except for U.K. we are able to confirm the existence of a long run relationship between the four variables.

The objective of the third essay was to apply the generalized method of moments (GMM) to test the FRUH in the foreign exchange market. As in Bakshi and Naka (1997), the third essay derives an ECM and uses the GMM estimation technique to test the UH. Since the spot and forward rates for the data set used in this dissertation are nonstationary with unit roots and are cointegrated of order CI (1, 1) we can use an ECM to model these dynamic relationships. In this essay we have used the ECM derived by Naka and Whitney (1995) and Bakshi and Naka (1997) and applied GMM to test the UH. The null hypothesis of  $\alpha = 0$  was rejected in nine out of ten cases, estimates of  $\beta$  was found to be positive and approximately equal to one in case of Canada and U.K. only.

## CURRICULUM VITAE

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