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A Methodology for Estimating Instream Flow Values for Recreation

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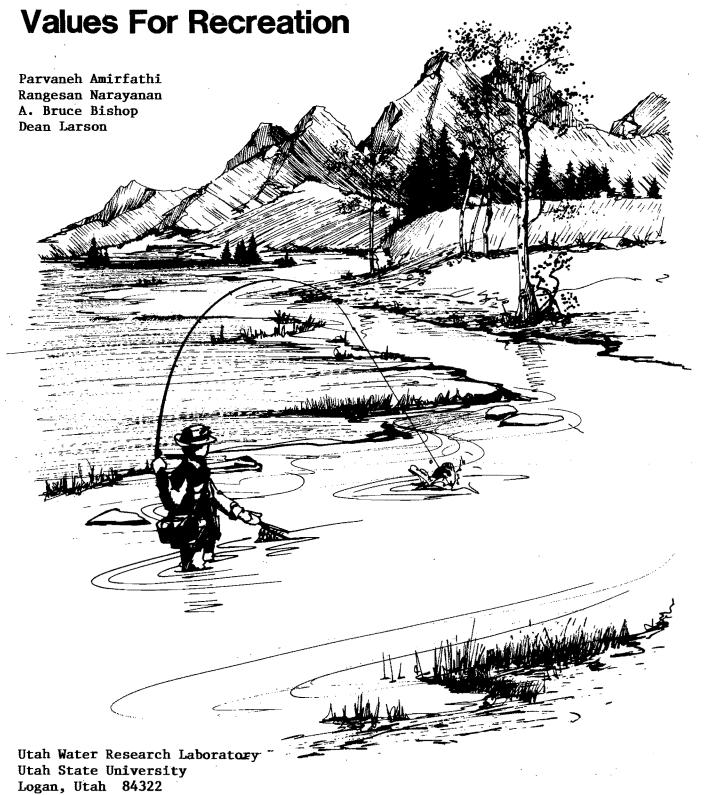
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A Methodology For Estimating Instream Flow



May 1985

WATER RESOURCES PLANNING SERIES
UWRL/P-85/01

A METHODOLOGY FOR ESTIMATING INSTREAM FLOW VALUES FOR RECREATION

Parvaneh Amirfathi Rangesan Narayanan A. Bruce Bishop Dean L. Larson

WATER RESOURCES PLANNING SERIES
UWRL/P-85/01

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Logan, Utah 84322

May 1985

ABSTRACT -

Water flowing in streams has value for various types of recreationists and is essential for fish and wildlife. Since water demands for offstream uses in the arid west have been steadily increasing, increasing instream flows to enhance the recreational experience might be in conflict with established withdrawals for uses such as agriculture, industries, and households.

Since market prices are not observable for instream flows, the estimation of economic value of instream flow would present well known difficulties. The household production function theory was used to build the theoretical model to measure economic value of instream flow.

A representative sample of 500 recreationists at three river sites were interviewed during the summer of 1982, to estimate empirical demand equation for recreational activities. In order to estimate economic value of water used in the river, it was assumed that individuals were combining goods, services, and time as input to produce recreational services. Based on this procedure, empirical estimates of multisite demands were derived. Moreover, the corresponding 'compensating variations' of consumers, from alteration of instream flow, were quantified.

ACKNOWLEDGMENTS

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INTRODUCTION

There has been a greatly increased interest in the measurement of the value of outdoor recreation, especially stream related recreation in recent years. One of the major uses of the nation's natural resources is outdoor recreation. Clawson and Knetsch (1966, p. 43) in the book "Economics of Outdoor Recreation" point out that

... visits to the national parks increased all through World War I; the Great Depression of the 1930's did hardly more than slow down growth in visits to the national park system and to the national forests. Minor variations in rates of growth occur in other years for some kinds of area, but the whole record is one of surprising uniformity in the persistence of the growth rate.

During the post-war years, the annual rate of participation in outdoor recreation in the United States has grown by an overall average of 10 percent (U.S. Department of the Interior Also all available evidences indicate that the demand for outdoor recreation will continue to increase over the next 20 years. The demand for recreation use of water resources is projected to grow 25 percent greater than other recreation activities to the year 2000 (Walsh 1980). The major factors behind the steady and rapid rise in use of outdoor recreation are: increased disposable income, 2) increased leisure time, 3) increased mobility of recreationists, and 4) a general desire for a physical outdoor activity such as outdoor recreation. As Wennergren and Fullerton (1972) argued, demand for this form of recreation is expected to almost double by the year 2000 even if individual participation does not increase above present level.

The number of participants in freshwater fishing increased by an average of 3 percent from 21.7 million in 1960 to 29.4 million in 1970 (Walsh 1980). According to the U.S. Department of Commerce, Bureau of the Census (1982-1983), fishing license sales have increased from 23.3 million in 1960 to 35.2 million in 1980, hunting license sales have increased from 18.4 million in 1960 to 27.0 million in 1980, and visits to national parks from 79.2 million in 1960 to 329.7 million in Also according to a home telephone survey, from June 1976 to June 1977, 53 percent of population, persons 12 years old and over, were fishing, 48 percent boating and 72 percent were picnicking (U.S. Department of Commerce, Bureau of the Census 1982-1983). As recreational use of water resources increases, it becomes more important that recreation must be accurately considered in allocating resources to various uses.

Competition Between Instream and Offstream Flows

As the demand for offstream water uses increase, the competition for water between instream and offstream uses intensifies. The quantity and quality of water left in streams might decrease if recreational values are not adequately incorporated in the resource allocation. Therefore, particularly where water is relatively scarce such as in the western states, this would result in

reduced recreational and aesthetic uses of the streams (U.S. Department of the Interior 1980). At the same time, more and more people are discovering and seeking recreational opportunities offered in and along rivers and the demand for water for instream uses appears to be increasing.

Activities for which instream flows are valuable include outdoor recreation, hydropower, navigation, waste transport and assimilation, fish and wildlife maintenance, and preservation of riverine ecosystems. The legal framework to govern the use of water in the western states is the prior appropriation doctrine (Hutchins 1971). According to prior appropriation doctrines, a water right could be granted to a person for "beneficial uses" of unused water. Priorities for use, then, are on a "first-in-time is first-in-right" basis. The doctrine's evolution, however, has not been hospitable to instream values with the exception of hydroelectricity generation (the actual required flow to drive the turbines). Appropriation doctrine made it virtually impossible to preserve instream values in most western states.

Historically, the lack of institutional provision of rights for instream uses could be the result of relatively abundant instream flows compared to the demand for water for offstream activi-However, with the cumulative effects of offstream development, continued availability of this abundant flow for instream values cannot be taken for granted. Furthermore, realizing benefits of instream flows make it a legitimate use of the resource. there are two main obstacles in integrating instream uses within the appropriative system. The first is the difficulty of satisfying the appropriation requirements which are: 1) a notice of intent to appropriate, 2) an actual diversion, and 3) an application to a beneficial use (Tarlock 1978). However, there is evidence that this obstacle can be overcome. States that

have statutory provisions to protect instream flow include Colorado, Montana, Oregon, Alaska, Idaho, and Washington (Bagley et al. 1983). Although this has achieved some desired results in protecting instream uses, it is still difficult to secure instream flows on heavily appropriated streams and it does not provide a balanced view of the resources, as it does not integrate instream with offstream use values. typical provision was enacted by Montana in 1973, authorizing the Board of Natural Resources and Conservation to reserve minimum streamflow (U.S. Department of the Interior 1971).

The second obstacle is the methods for determining instream flow "needs" which have not been tied to the economic Nonmarket aspects of inviewpoint. stream uses as well as the role of government in the resource allocation process present difficulty in the allocation of water between instream and offstream uses according to relative The National Conference on values. Water held in Washington, D.C., in 1975 recommended that state water law should recognize a water right for maintenance of the stream for fish and wildlife, recreation uses, and scenic beauty. The State of Utah also has a statute which requires that an application for unappropriated water be rejected when it would unreasonably affect public recreation or the natural stream environment (Utah Code Ann. §73-3-8). states have similar provisions in their statutes.

After some recognition was given to instream flows, scientists sought a reliable and practical method to determine stream flow "requirements" for aquatic environments. An easy and quick method, known as the "Montana Method," was developed for both warm water and cold water streams. The Montana Method (Tennant 1975) assures consistency from stream to stream or state to state. This method recommends an instream flow equal to at least 10 percent of the average flow with an

appropriate temperature and quality for protecting aquatic environment. There are many vastly improved methods available for determining flow requirements which are based on noneconomic criteria.

James A. Morris (1976) argued that a flow which is sufficient to support fish life may not be adequate for recreation. He further points out that water requirements will differ considerably for each activity. example, more water is required to give a satisfying experience to a white water boater than for fish within the same river segment. Therefore, instream use allocations must be integrated with allocation of offstream uses. instream values are exclusively protected by the state, or state protection and private appropriation are combined, rational allocation decisions require information on the relative benefits of instream flows.

Cost and Benefit

The supply of instream flows on the average have been decreasing over time. Since the quantity of available water is essentially fixed on the average, the measurement of the economic or monetary gain and costs of each use of water becomes important in allocating the available water among competing uses. Recently, recreation has begun to be legally recognized as a legal competing use of water. Therefore, it is essential to develop a procedure for

evaluating the benefits of instream flows for recreation. Allocation of water between instream and offstream uses to maximize overall benefits of available water resources requires estimation of cost and benefit. Instream uses and its benefits can then be compared to the opportunity cost of maintaining the flow of water in terms of foregone offstream benefits.

The growing demand for recreation is the cause of increasing value of the natural resources. Therefore, these changes will call for continuing adjustments in resource allocations to better satisfy wants and preferences of consumers. Land and water resources need to be constantly reevaluated for the value of their services.

Economic value of water for outdoor recreation could provide a means for comparing the importance of instream flows with that of other uses. value would provide a ceiling for any fees that might be charged for stream-Also, the related recreation use. estimation of instream flow benefit and cost functions will provide information with respect to efficiency in the allocation of water for outdoor recre-In determining instream flow ation. benefits one important nonmarket component is the recreational benefits. This study develops a methodology for estimating recreational benefits of instream flow when multiple sites are available.

ESTIMATION OF INSTREAM FLOW BENEFITS FOR RECREATIONISTS

Several benefit components, such as benefits from stream side and instream recreation, power generation, navigation, waste transport, aesthetics, and the aquatic ecosystem are associated with instream flow. Some of these benefits are extremely difficult to estimate. In this study an attempt is made to measure the instream flow benefits from recreation activities from data obtained from streamside survey data.

Instream flow has a public good characteristic. Given the absence of markets in public goods, there are some normarket approaches to obtaining information on demand and consequently on benefit. One of the easiest approaches is to ask individuals their willingness to pay for stated level of a public good (Walsh et al. 1980a, Walsh 1980, Walsh et al. 1981, Walsh et al. 1980b, and Vaughan and Russell 1982). For instance, in this study, the question to ask individuals would be what they would be willing to pay to avert a defined reduction in streamflow (Table This method ranging from simple interviews to sophisticated multiple questionnaires is used to determine an individual's willingness to pay (Daubert and Young 1979, 1981). The serious problem with this approach lies in the response of the individual, since individual consumers have strong incentives not to show their true preferences (Maler 1974).

The second important method to mention is the travel cost method (Clawson 1959, Clawson and Knetsch 1966, and Cesario and Knetsch 1976). This method is one of the traditional

techniques for measuring the benefits of a recreation facility. Freeman (1979) argued that there are difficulties in extending this technique to the analysis of demand, such as analysis of demand with changing quality.

The third approach is the Household Production Function method. method, the demand for recreation at several sites can be estimated by using cross-sectional household data. explained differences in estimated demand among sites could be explained by site quality differences, e.g., differences in instream flow or water quality (Saxonhouse 1977). The household production function method has been a useful approach particularly when the purpose is to evaluate benefit accruing from a change in the natural environment (Barnett 1977, Pollak and Wachter 1975, 1977). In this study, the third approach is used to estimate the multiplesite demands for instream flow recreation at three sites. Also an attempt is made to estimate flow benefits for those three sites.

A General Model of Household Behavior

The household production framework was first developed by Becker (1965), and has been expanded in a variety of ways in the recent literature (Huffman and Lange 1982, Becker and Lewiss 1973). Valuing a resource whose services contribute to the production of a final good on the basis of the value of the good is not new to economics. What is new is the application of this approach to the final good or service which is not produced or exchanged in the market

(Pajooyan 1978, Bockstael and McConnell 1981, Deyak 1978). In this approach most consumption activities are viewed as the outcome of individual or household production process, combining market goods and time.

According to conventional consumer theory, households maximize utility function subject to resource constraints:

$$\text{Max } U = U(X_1, X_2 \dots X_n)$$

S.t.
$$\sum_{i=1}^{n} P_i X_i = WT_w + N = I$$

where

X_i = goods purchased on the market
 at price P_i

I = money income

 T_w = time spent working

WTw = earnings

N = other income

According to the view, the household purchases goods on the market and combines them with time in a household production function to produce commodities. As Becker (1965) mentioned, the advantage of this approach is the systematic incorporation of nonworking time. Goods and services purchased by the consumer are not final products and will not be consumed directly. In other words, market goods and time are not desired for their own sake, but only as inputs into the production of consumption commodities. Therefore, these consumption commodities, rather than goods, are the arguments of the household utility function. For our purposes, it is sufficient to consider a rather simple variant of this model. Also, we shall assume the household maximizes a utility function expressed in terms of final service flows:

$$\max U = U(z_1, z_2, ..., z_n)$$

S.t.
$$\sum_{i=1}^{n} P_i X_i = WT_w + N = I$$

$$T = t_1 + t_2 + ... + t_n + T_w$$

where

$$Z_i = Z^i (X_i, t_i)$$

U = utility

X; = goods and services

P_i = market price of X

W = wage rate

 $Z_i = consumption commodities$

 t_i = time spent to produce Z_i

N = nonwage income

I = money income

 $T_w = working time$

T = total time available to the individual

and

$$\frac{\partial Z_i}{\partial X_i} \ge 0 \quad , \quad \frac{\partial Z_i}{\partial t_i} \ge 0$$

This approach is easily adapted to the study of nonmarket commodities. The analysis focuses on demand for consumption commodities as a function of "commodity prices" which, in turn, depend on prices of goods, wage rate, and the household's technology.

In this study the household production function theory is used to obtain demand function for instream flow's recreation. In this formulation households are both producing units and utility maximizers. Household is

assumed to combine time and market goods to produce commodities that directly enter their utility function. These commodities will be called Z_i^j and written as

$$Z_i = f(X_i^k, T_i) \tag{1}$$

where X_i^k is a vector of market goods and T_i a vector of time inputs used in producing the commodities. Note that the partial derivatives of Z_i with respect to both X_i^k and T_i are nonnegative.

The most direct approach is to maximize the utility function subject to separate constraints on the expenditures on market goods, time, and the production functions. Since time can be converted into market goods by using less time at consumption and more at work, we could have a single constraint as:

$$\sum_{i} \sum_{k} P_{k} X_{i}^{k} + WT_{i} = I$$
 (2)

where

$$X_i^k = \sum_{j} a_{kj}^i Z_i^j$$

$$T_{i} = \sum_{j} t_{j}^{i} Z_{i}^{j}$$

 t_j^i is a vector giving the input of time per unit of Z_i^j

 a_{kj}^{i} is a vector giving the input of k market goods per unit of Z_{i}^{j}

By using the above definitions, Equation 2 can be written as

$$= I \tag{3}$$

wi th

full price of
$$Z_i = P_i^F = \sum_k P_k a_{kj}^i + Wt_i^i$$
 (4)

full income
$$\overline{I} = N + WT_w$$

The full price is the sum of direct and indirect prices. As Becker (1965) pointed out, since these direct and indirect prices are symmetrical determinants of total price, there is no analytical reason to stress one rather than the other. Therefore, the utility function can be maximized subject to full income constraint (Equation 3). In this study, it is assumed that the recreationist maximizes his total utility.

Recreation Demand Model

Clawson and Knetsch (1966) define demand for recreation activities as total attendance or use made of the facilities, which refers to the quantities taken at the prevailing recreation opportunity conditions. They also mentioned that raw attendance figures reflect demand, to be sure, but also reflect opportunity or supply as well. In practice, people use outdoor recreation opportunities to the extent to which they believe their satisfactions are exactly equal to the total costs involved. As it was mentioned before, recreationists are assumed to maximize their utility function subject to conventional linear budget constraint:

Maximize
$$U = V(q)$$

S.t.
$$p \cdot q = I$$

The solution to utility maximization is the set of Marshallian demands:

$$q_i = f_i (P, I)$$

This solution can be substituted back into utility function to get maximum attainable utility. The function is known as the indirect utility function. Since the expenditure and indirect

utility functions are inverse, the cost or expenditure function can be solved. Therefore, the derivative of the expenditure function with respect to any price gives the Hicks-compensated demand function for that good (Deaton and Muellhauer 1982).

In much of the recent study, the starting point on system of demand equation has been the specification of a function which is general enough to be a second-order approximation to any arbitrary indirect utility or a cost function. Alternatively, in the Rotterdam model a first-order approximation to the demand functions themselves are used. Deaton and Muellhauer (1980) also followed these approaches in terms of generality, but they didn't start from an arbitrary preference ordering. They start their system of demand equation from specific classes of preferences which can have an exact aggregation over consumers. These preferences, known as the PIGLOG class, are represented in a cost or expenditure function. The cost function defines the minimum expenditure necessary to have a specific utility level at a given price !. Therefore, it is a function of utility and price vector as:

Log C (U,P) =
$$\alpha_0 + \sum_{k} \alpha_k \log P_k$$

+ $1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$
k j
+ $U\beta_0 \prod_{k} P_k$
(5)

where α_i , β_i and γ_{ij}^* are parameters.

In this study, the Almost Ideal Demand System (AIDS) is chosen to derive the demand equation. The demand function can be derived directly from Equation 14 which is called AIDS cost

function. As mentioned above, the price derivatives of the cost function will be the quantities demanded:

$$W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j} + \beta_{i} U\beta_{0} \pi P_{k}^{\beta_{k}}$$

(6)

(8)

or

$$W_i = f(U, P)$$

where

$$\gamma_{ij} = 1/2 (\gamma_{ij}^* + \gamma_{ji}^*)$$

After substituting U into Equation 6 by its value, the budget shares W_i will be as a function of price and expenditure:

$$W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j}$$

$$+ \beta_{i} \log (I/P^{*}) + \epsilon_{i}$$
(7)

where P* is a price index defined as:

log
$$P^* = \alpha_0 + \sum_{k} \alpha_k \log P_k$$

 $+ 1/2 \sum_{j} \sum_{k} \gamma_{kj} \log P_k \log P_j$

 $\epsilon_{\mbox{\scriptsize i}}$ = disturbance term related to the demand function

Equation 7 is the AIDS demand function in budget share form. Price, P_j , is defined and calculated like the full price definition, and expenditure, I, is the same as the full income definition in Equation 4. Parameter β determines whether goods are luxuries or necessities. With $\beta_i > 0$, W_i will increase as I does, so that good i is luxury. Similarly, if $\beta_i < 0$, good i is a necessity. Parameter γ_{ij} measures the change in the ith budget share

¹For more detail see Appendix of Deaton and Muellhauer (1980).

following a l percent change in P_j with (I/P*) constant. Data on W_i , P_j , and I for various recreation sites will allow estimation of the parameters (Equation 7). Thus, the demand for recreation at various sites could be determined. From these demands a method to estimate instream flow benefits of recreation needs to be developed.

Determination of Instream Flow Demands

An improvement in water quality or quantity at any site will produce an increase in the demand for recreation. The area between the initial and new curve represents the benefits of improved water quantity or quality. Therefore, to measure the benefit of improved instream flow quantity, the corresponding demand curve is essential. One way to derive the new modified demand curve q_i^* is to introduce quality parameters f_i directly into the utility function:

$$U = (f_i q_i) \tag{9}$$

where f_i depends upon the observed specifications of the quality of recreation. Alternatively, one can introduce quality parameters in the cost function directly.

In this study, quality parameter fi is defined to be a function of the flow level (Fg) assuming other quality differences between sites are negligible. The cost function defined before would be modified as:

$$\log C^*(U,P) = \alpha_0 + \sum_{k} \alpha_k^* \log P_k$$

$$+ 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$$

$$+ \beta_0 U \pi P_k^{\beta_k}$$
 (10)

where

$$\alpha_k^* = f_k \alpha_k$$

$$\gamma_{kj}^* = f_k f_j \gamma_{kj}$$

$$\beta_k^* = f_k \beta_k$$

Accordingly, the modified compensated demand and Marshallian demand function, to include the effect of instream flow change as a quality measure of recreation, would be;

$$\frac{\partial \log C^*}{\partial \log P_1} = W_i^* = \alpha_i f_i + \sum_{j} \gamma_{ij} f_i f_j \log P_j$$
$$+ f_i^{\beta} i U_{\beta_0} \prod_{k}^{T} p_k^{\beta_k}$$
(11)

By substituting indirect utility function in compensated demand function (Equation 11), the modified ordinary or Marshallian demand is:

$$W_{i}^{*} = \alpha_{i} f_{i} + \sum_{j} \gamma_{ij} f_{i} f_{j} \log P_{j}$$

$$+ f_{i} \beta_{i} \log M \qquad (12)$$

where

$$W_i^* = W_i f_i$$
 where $f_j = 1$

If the quality parameters f_i are independently determined, one can first estimate Equation 7 and then using f_i s can get Equation 12. These equations represent the demand for recreation at various sites as a function of instream flows. Using these demands, changes in demand as a result of changes in instream flows at one or more sites could be determined.

Estimation of Instream Flow Benefits of Recreation

Demand function for outdoor recreation is used to make inferences about the consumer's surplus (Anderson 1981,

Burt and Bremer 1971, Cicchetti and Freeman 1971), and implicitly about the social welfare derived from particular The best estimate of recreation benefits, or the total worth of increased supply of recreation services, may be measured directly from the demand curves, since it indicates what consumers would pay for the various units of recreation output, rather than go Total area under demand without them. curves measures the total economic worth to society of the provided recreation services. Therefore, to estimate instream flow benefits, the estimated demand function could be used.

Proper ways of measuring the benefit is discussed by Bishop (1982), Russell and Vaughan (1982), Schmalensee (1972), and Schulze et al. (1981). An appropriate measure of welfare change or recreation benefit due to instream flow changes is the compensating variation CV (Houthaker 1952). This CV can be simply defined as how much compensation is needed to make the consumer as well off as before (i.e., to hold utility at $\rm U^O$) when the quality parameter (in this case an index of instream flows) changes from $\rm f_g^O$ to $\rm f_g^I$. Obviously, it is an amount equal to the change in the cost of securing $\rm U^O$ defined by

$$CV = C(f_g', P', U^o) - C(f_g^o, P^o, U^o)$$

$$= \int_{\frac{\partial C}{\partial f_1}}^{f_g} (f_1, P^0, U^0) df_1$$

The compensated variation or benefit obtained by recreationists from changing instream flow level can be defined as:

$$B_{s} = C^{*}(\hat{U}, P^{*}) - C(\hat{U}, P)$$

In this model, to be able to define B_S the following steps are taken. Let log C, the logarithm of the cost function at 1982 flow level, be Y_1 and log C^* , the logarithm of the cost function at any other flow level, be Y_2 . Then

$$ln(C^*/C) = Y_2 - Y_1$$
 (13)

taking antilog on both sides:

$$C^*/C = e^{(Y_2 - Y_1)}$$
 (14)

From Equation 14, the compensating variation of $B_{\rm S}$ can be defined as:

$$B_s = I (e^{(Y_2-Y_1)} - 1) \text{ since } C = I$$
 (15)

This equation can be used to measure benefit changes from changing instream flow levels at one or more of the recreation sites.

DATA COLLECTION FOR CASE STUDY

The Study Area

The study area includes the Blacksmith Fork and Little Bear River drainages located in the southwest portion of Cache County in northern Utah, plus the Logan River which is located in northern Utah and southern Idaho (Figure 1). Little Bear, draining an area of 339 square miles, flows roughly south to northwest to its confluence with the Bear River. The Blacksmith Fork, draining 268 square miles, flows roughly east to west to join the Logan River which later flows into the Bear River. The Logan River drains an area of about 223 square miles (Haws 1965), flows roughly northeast to southwest to join the Bear River. The headwaters of all three, Blacksmith, Logan, and Little Bear Rivers, originate in the Wasatch Mountains. Streamflows of the Little Bear, Blacksmith Fork, and Logan Rivers with the canyon areas, are primarily governed by runoff from the winter snowpack as the air temperatures increase from mid-April to mid-July.

The Logan River is joined by the Blacksmith Fork River and the Little Bear River. It finally joins Bear River, the major stream flowing through Cache Valley, and discharges into the Great Salt Lake. About 15 percent of the Little Bear drainage and 63 percent of the Blacksmith Fork drainage are in the Cache National Forest or state Approximately 32,000 acres in lands. the Little Bear drainage, and 2,000 acres in the Blacksmith Fork drainage, are irrigated. The Logan River drainage has approximately 15,000 irrigated acres in the downstream reaches. Irrigation, especially on the Blacksmith Fork and Little Bear Rivers, constitutes by far the heaviest use made of the water. Other uses include municipal, culinary, and hydroelectric water.

Farmers in the area have diverted all three rivers' streamflows for irrigation for over 50 years to irrigate corn, peas, potatoes, sugar beets, silage, hay, small grains, pasture, and orchards by the Logan River irrigation system and alfalfa full, alfalfa partial, barley, corn grain, beets, nurse crop by the Blacksmith and Little Bear Rivers irrigation systems. principal fish of the Blacksmith Fork, Little Bear, and Logan Rivers are the brown trout and mountain whitefish. addition, cutthroat trout, rainbow trout, and speckled dace are found in the Logan and Blacksmith Fork Rivers (U.S. Department of the Interior 1980). The Logan River canyon, the Blacksmith River canyon, and the Little Bear River canyon are popular recreation areas, used for fishing, camping, kayaking, etc.

The Logan River between second dam and Bridger Campground is usually dewatered during late summer, even in higher than normal flow years. In 1983, an agreement was reached between the Utah Division of Wildlife Resources and Logan City to maintain water flow in this stretch of river, which is an important area for recreationists. As a result of the agreement, the lower part of the river will be dewatered. The Blacksmith Fork is also dewatered over part of its lower reaches during the middle and late summer in years with below normal flows. Such dewatering occurred in the summer of 1981, resulting in loss of a large number of fish. A proposal by the City of Hyrum to

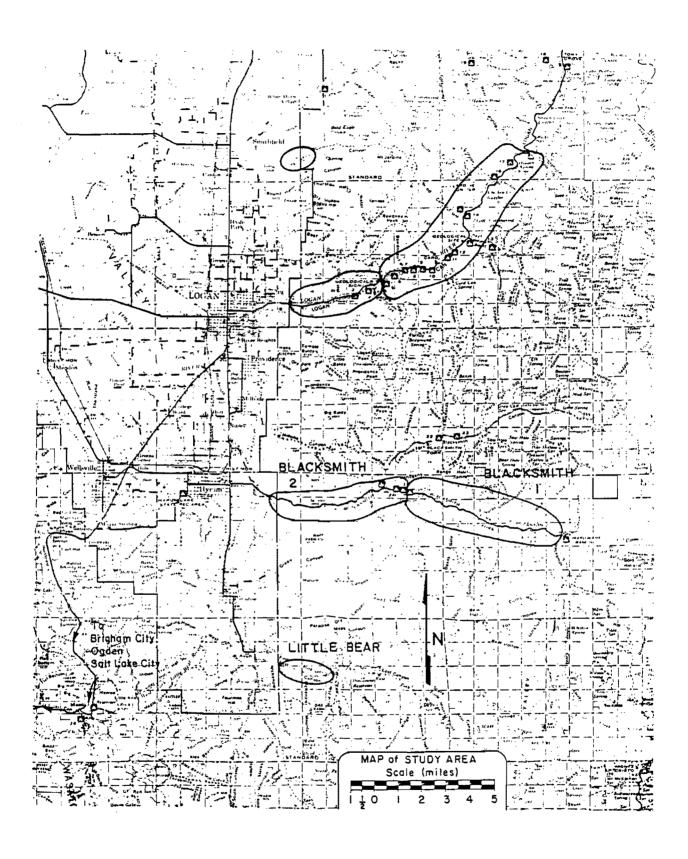


Figure 1. Map of the study area.

rehabilitate its power plant on the Blacksmith could dewater another stretch above the canyon mouth by diverting the flow into a pipe for conveyance to the downstream generation site. For flow data the Logan River has been divided into five homogeneous reaches and the Blacksmith Fork River has been divided into three uniform river reaches. divisions were determined by considering points where the amount or time distribution of streamflow changes signifi-The division points for the cantly. Logan River are:

Reach la, between 2nd and 3rd dams

Reach 1b, between 3rd dam and Right Fork tributary

Reach lc, the rest of Logan River study area which lies between Right Fork tributary upstream and end of the study area at Woodcamp Campground

Reach 2a, between 1st dam and Smithfield Canal diversion or Logan-Hyde Park

Reach 2b, between Smithfield Canal diversion and 2nd dam

The Blacksmith Fork River reaches are:

Reach la, from the mouth of the canyon to the existing reservoir structure

Reach lb, between reservoir structure point and the mouth of the left hand fork tributary

Reach 2, located from the left hand fork tributary to the end of the study area at Hardware Ranch

East Fork River or Little Bear River has only one single uniform river reach which is the whole Little Bear study area.

Streamside Recreation Sampling Procedure

To have a complete measure of instream flow value, ideally all individuals who participate in instream recreation activities should be interviewed. This is an expensive and time consuming task. Therefore, randomly selected recreationists are interviewed and inferences are made about all recreationists from that sample (Earl 1982).

The interviews were conducted for streamside recreation survey in the summer of 1982 in three river sites. In this study, only 2 percent of the people refused to fill out the survey A copy of the survey questionnaire is shown in Appendix A. questions were first tested by staff members at Utah State University for timing and ease of understanding of Then, the questionnaire the question. was tested among a couple of ordinary recreationists in each site. shortcomings of the questionnaire were corrected before the actual survey began.

The actual sample for all three sites included 500 households who participated in fishing, camping, or any shoreline and white water activities such as swimming, hiking, tubing, So each household would have the same chance of being selected, a random number of days were selected to interview over a period of six weeks, beginning in August. The interview period was chosen to ensure variations in streamflow would be observed. The higher than normal flows of 1982 required a later starting date than would have been the case in an average Interviews were made at recreation sites on four weekends and four Sampling sites for all weekdays. streams were aggregated into five reaches (Figure 1). Logan River had two From the First Dam to Second Dam was called Logan 2 and from Second Dam to Woodcamp Campground was called

Logan 1. Blacksmith Fork River also was divided in two reaches. Blacksmith 1 extended from the mouth of Blacksmith Fork Canyon upstream to Hyrum City park and Blacksmith 2 extended from the park to Rock Creek below Hardware Ranch. The last site was East Fork or Little Bear River, below Porcupine Reservoir.

The sampling procedure consisted of setting a quota for sites for each day of interviewing. The quota for each site and for each day was determined according to estimated site capacity, weekend or weekday, and whether it was earlier or later in the season. quotas were assigned for weekends. As recreation use comparatively declined later in the season, relatively lower quotas were assigned. The site interview procedure has one inherent bias, which is those who stay longer, are more available and have higher probability of being chosen for the interview.

The rate of acceptance was over 95 percent. The study especially focused on recreationists evaluation of particular streams as flows varied. This dictated that the questionnaires be administered at the recreation sites, rather than by phone, mail, or at residences. As the household was the basic sample unit, the interviewer was advised to make sure that the spokesman gave answers that represented the family.

The most difficult sample construction decision was to choose an appropriate sample size considering time, cost, and all other constraints. this study, some variables such as number of sites, number of income groups, and the number of travel distance zones plus costs of information collection were considered to set the sample size. Therefore, the decided sample size was 500 interviews and it was hoped to be enough observation for three sites in four distance zones, and Table 1 shows the three income groups. distribution of sample sizes. number of distance zone and income group classifications is arbitrary. This grouping gave 12 observations for each site which provided reasonable degree of freedom for estimation purposes.

Survey Results

The survey questionnaire is the most important factor determining the success or failure of attempt to estimate objective of survey. length of survey and the number of questions in each section of survey is important to get accurate answers. In particular, questions should not ask the individual to respond to alternatives beyond the range of his experi-In this study, the questionnaire requested information on three general topics with enough number of questions in each group to get as accurate answers as possible without making the respondents tired. These three categories 1) socio-economic, 2) recreation activities, and 3) site evaluations.

Socio-economic

Respondents were asked about composition of party, education completed, household income, and residence (Appendix A). The average size of groups were similar in five reaches and particularly between the three sites. They were 4.00 for both Logan River and Blacksmith Fork River and 3.9 for Little Bear River. Group size distribution did not follow a uniform pattern, however, a group of size 2 had the highest frequencv. There were more male recreationists than female. This conclusion is not true in every age group. The largest portion of the recreation population is under 30 years of age. At over 49 years the differences in number between male and female recreationists decrease.

The median educational attainment of respondents was high school completion. The number of recreationists with college level of education in Logan 1, Logan 2, and Blacksmith 2 was higher than with high school level of education. Also, on the average,

Table 1. Distribution of sample sizes.

		·						Site							
		Logan	1	L	ogan	2	Bla	cksmit	h 1	Blac	cksmit	h 2	Lit	tle	Bear
							Inco	ne Gro							
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Zone 1															
D_1	11	16	10	24	9 3	4	7	10	1	8	7	4	0	1	0
D_2^{-}	2	5	1	24 6	3	4 3	7 1	2	1 0	8 5	7 2	4 1	0 0	1	0
Zone 2															
D_1	6.	7	5 0	2 3	3 0	3 1	2 2	13 7	3 1	8 -	10	5 1	5 3	2 2	2
$\overline{D_2}$	2	2	0	3	0	1	2	7	1	2	0	1	3	2	1
Zone 3				•											
D_1	2	8	4	2 1	2 2	8 1	2	12 5	9 1	0 2	3 1	2 0	2	3	0
D_2^-	2	4	4	1	2	1	0	5	1	2	1	0	2 0	0	2
Zone 4															
D_1	2	5	6	3	4	3	1	8	. 4	2 1	0	4	5	4	2
$D_2^{\frac{1}{2}}$	2	4	6 2	1	1	4	0	1	0	1	0	2	5 2	4 5	1
Above 365		23			25		****	3		,	4		-	2	
Total		35			118			95			73			45	

D₁ indicates weekend

recreationists in Logan 1, Logan 2, and Blacksmith 2 reaches have higher level of education than Little Bear and Blacksmith 1 reaches. This noticeably higher level of education in those three samples could be explained by the relatively shorter distance of the sites to the university community centered in Logan, as higher level of education will indicate higher opportunity cost for recreationists.

There is a weak relationship between education level and household annual income. The high number of college students as recreationists in our sample did affect the relationship between education and income. Since these students do not earn as much as they would if they were in the work

market, the expected result, which is a relative increase in income earned as education level increases, is not shown. Distribution of household income (Table 2) is not significantly different in Logan and Blacksmith sites. median income for the Logan and Blacksmith sites is in the 20,000-24,999 range, and for Little Bear it is in the 10,000-14,999 range. If ranges above 20,000 are considered upper brackets, then almost 60 percent of the sample from Logan and Blacksmith sites are in the upper brackets and for Little Bear, the upper bracket percentage is 40.

Distance traveled from home is classified in 13 groups from less than 2 miles to almost 1000 miles (Table 3). According to our samples two groups

D₂ indicates weekday

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Table 2. Annual household income by site.

		***************************************	Site			
Annual Income	Logan 1	Logan 2	Blacksmith l	Blacksmith 2	Little Bear	Total
Less than 5,000	9	12	3	5	0	31
\$ 5,000- 9,999	10	19	5	9	6 .	50
\$10,000-14,999	12	16	9	13	12	64
\$15,000-19,999	16	12	12	13	5	59
\$20,000-24,999	25	13	27	8	8	84
\$25,000-29,999	18	10	19	4	4	56
\$30,000-34,999	17	12	12	6	2	51
\$35,000-44,999	18	17	7	11	5	59
\$45,000 or more	9	7	1	3	3	22
Total	135	118	95	72	45	476

Table 3. Travel distances by sampling site.

	*****				S	ite					A11	Sites
	Loga	an 1	Loga	an 2	Black	smith 1	Black	smith 2	Litt	le Bear	To	otal
Distance*	#	%	#	%	#	%	#	%	#	%	#	%
0-10	36	7.5	46	9.6	1	0.2	13	2.7	2	0.4	102	21.4
11-20	9	1.9	3	0.6	20	4.2	15	3.1	0	0.0	49	10.3
21-30	3	0.6	4	0.8	5	1.0	18	3.8	7	1.5	37	7.8
31-40	20	4.2	7	1.5	24	5.0	8	1.7	7	1.5	67	14.0
41-50	0	0.0	2	0.4	5	1.0	4	0.8	3	0.6	14	2.9
51-60	26	5.5	13	2.7	25	5.2	3	0.6	5	1.0	72	15.1
61-70	6	1.3	0	0.0	4	0.8	2	0.4	11	2.3	24	5.0
71-80	4	0.8	3	0.6	1	0.2	2	0.4	2	0.4	12	2.5
81-90	2	0.4	6	1.3	2	0.4	0	0.0	2	0.4	12	2.5
91-100	3	0.6	1	0.2	0	0.0	2	0.4	0	0.0	7	1.5
101-130	1	0.2	4	0.8	7	1.5	1	0.2	3	0.6	16	3.4
131-365	5	1.0	2	0.4	0	0.0	1	0.2	1	0.2	9	1.9
365-999	20	4.2	27	5.7	1	0.2	4	0.8	2	0.4	56	11.7
Total											476	100.0

^{*}Distance from home in miles.

of people mostly ended up in Logan, those living within 40 miles, especially within less than 10 miles, and those passing through Utah. But for Blacksmith and Little Bear the opposite is Although, one would generally expect that most of the visitors to a site would live in the nearest zone, as in Logan site, the survey sample for the latter two sites departs from this This could be explained by pattern. distribution pattern of population around the sites, as very few people live within 10 miles of the Little Bear and Blacksmith sites, especially Blacksmith 1. Other factors such as proximity of the site to major highways and distance between home and the nearest alternative site offering a similar recreation experience, could be mentioned to justify the results in Table 3.

Recreation activities

Table 4 presents the mean or average length of stay for each site. Logan 2 has lowest mean because of proximity of this site to the largest city in northern Utah, and average length of stay for Logan 1, Blacksmith 1, and Blacksmith 2 are exactly the Also Table 5 shows that the length of visit was not as long for a shorter travel distance. Tables 6 and 7 give a general idea of average cost of food, recreation equipment cost and cost of durable recreation equipment for each site. These two tables are the result of recreationists response to the question about the cost of food and other items which directly related to their visit (Table 6), also the cost of durable recreation equipment which they brought (Table 7). According to Table 6, there is not as much fluctuation in cost between sites as there is in Table 7. This argument can be easily explained by considering results in Tables 3, 4, and 8.

Table 8 could help us rank the different activities for each site. Fishing was the dominant recreation

activity for all five sites. For Logan site, water play has second rank, but for Blacksmith Fork and Little Bear, sleeping has second place. The result of this table might be used in deriving demand function for each recreation activity from overall recreation demand.

Site evaluation

Recreationists were asked to rate their recreation site on a scale of 1 to 10 over several site characteristics; where a rating of 10 would indicate an ideal site and a rating of 1 would indicate a least desirable site (Table 9). This table shows that three reaches, Logan 1, Logan 2 and Blacksmith 2, are close alternative sites, according to the composite site characteristic evaluation of about 7.2. The two remaining reaches have an evaluation of about 6.5

The survey year, 1982, had an unusually high instream flow. the survey was conducted in that year (Table 10), the present level of instream flow in the summer of 1982 was rated as an accepted flow level in all Table 9 shows that site three sites. characteristic evaluations by recreationists are above average for all five Nevertheless, the reaction of recreationists to no water situations is "unacceptably low" (Table 10). Furthermore, Table 11 indicates the number of people responding to a given percentage of present flow level as being "minimum acceptable." The mean levels of minimum acceptable flow in all five reaches are above 55 percent of current flow even in summer of 1982. The amount recreationists are willing to pay to maintain acceptable flow levels is shown in Table 12. These results are a strong indication of importance of required flow level for recreationists.

Tables 13 and 14 present the results of respondents answers about question of maximum number of other recreationists who would be acceptable at the site before it became too crowded

Table 4. Length of visit by site.

Hours at			Site			
Site	Logan l	Logan 2	Blacksmith l	Blacksmith 2	Little Bear	Total
<1	0	1	0	0	0	1
1-4	24	56	11	25	9	125
5-8	16	12	22	14	3	67
9-15	6	6	7	1	2	22
16-30	25	11	14	9	15	74
31-55	30	13	25	13	10	91
56 or more	34	19	16	11	6	86
Average visit	34.21	21.36	34	34	30	30.71
Total	135	118	95	72	45	466

Table 5. Length of visit by travel distance.

Distance				Hours	at Site			
Traveled	< 1	1-4	5-8	9-15	16-30	31-55	56 or more	Total
0-10	0	59	16	5	7	9	6	102
11-20	0	15	12	1	6	8	7	49
21-30	0	14	6	0	4	6	7	37
31-40	0	9	8	3	12	18	17	67
41-50	0	1	3	2	0	6	2	14
51-60	0	5	13	4	12	12	26	72
61-70	0	0	1	0	12	6 .	5	24
71~80	0	2	1	1	2	3	3	12
81-90	1	. 2	2	0	1	4	2	12
91-100	0	0	1	1	1	3	l	7
101-130	0	2	0	1	4	8	l	16
131-365	0 .	2	0	0	3	2	2	9
365 or more	0	16	5	5	11	9	10	56
Total	1	127	68	23	75	94	89	477

Table 6. Average expenditure by site.

Average			Site		
Expenditure	Logan 1	Logan 2	Blacksmith l	Blacksmith 2	Little Bear
Food	\$30.21	\$24.31	\$29.61	\$24.85	\$32.53
Equipment	8.50	8.69	13.86	5.01	19.93
Total \$	38.71	33.00	42.47	29.86	46.46

Table 7. Average cost of durable recreation equipment by site.

Equipment			Site		
Type	Logan l (\$)	Logan 2 (\$)	Blacksmith l (\$)	Blacksmith 2 (\$)	Little Bear (\$)
RV, camper, trailer	3,539.22	2,715.68	1,790	1,044	1,923
Tents and awnings	178.16	155.86	50	169	44
Sleeping bags, etc.	114.22	85.42	51	70	. 67
Food preparation					
and amenities	154.48	162.78	38	76	92
Fishing equipment	75.18	60.02	63	64	, 88
Licenses	15.00	19.80	16	10	16
Other	104.50	206.67	0	9	35
Average Total	4,180.76	3,406.23	2,009	1,442	2,266

Table 8. Average percentage of time allocated to different activities.

	Site								
Activity	Logan	Blacksmith	Little Bear						
Fishing	28.2	26.45	21.2						
Eating	7.6	8.6	11.0						
Sleeping	10.8	9.05	14.2						
Water play	12.3	4.55	5.4						
Hiking	8.4	3.05	2.6						
Games	1.9	2.1	2.2						
Other	4.4	4.0	6.5						

Table 9. Site characteristics evaluation by site (10 = perfect, l = extremely poor).

			Site		
Characteristic	Logan l	Logan 2	Blacksmith l	Blacksmith 2	Little Bear
Distance	7.73		7.7	8.1	7.3
Privacy	7.43	7.04	7.9	7.3	7.5
Facilities	6.97	6.74	4.2	5.9	3.0
Landsc ape	8.37	8.67	8.5	8.3	7.3
Insects	4.58	6.03	4.0	5.5	4.9
Water	8.78	8.33	8.7	8.1	8.4
Fishing Suitability	6.71	6.54	5.8	6.9	6.8
Composite	7.22	7.32	6.7	7.2	6.5

Table 10. Average streamflow evaluations, by site (5 = unacceptably low, 1 = unacceptably high).

		÷	Site			
Flow Level	Logan l	Logan 2	Blacksmith l	Blacksmith 2	Little Bear	All Sites
2.0 x present level	1.63	1.89	1.6	1.6	1.8	1.70
1.5 x present level	2.02	2.32	1.9	2.0	2.0	2.05
Present level	3.11	3.43	3.1	3.1	3.13	3.17
0.5 x present level	4.18	4.63	4.2	4.3	4.4	4.34
No water	4.95	5.00	4.95	5.0	5.0	4.98

Table 11. Minimum acceptable flow as a percent of current flow, by site.

Percent			Site			
of Present Flow Level	Logan 1	Logan 2	Blacksmith 1	Blacksmith 2	Little Bear	All Sites
10	1	5	1	3	0	10
25	19	1	6	4	0	30
33	15	5	11	9	1	41
50	45	27	30	19	17	138
67	10	19	29	10	6	74
75	17	38	11	12	14	92
99	24	18	7	16	7	72
Mean level	57	66	58	62	67	62

Table 12. Willingness to pay to maintain acceptable flow levels, by site.

Dollars			Site			
Willing to Pay	Logan 1	Logan 2	Blacksmith l	Blacksmith 2	Little Bear	All Sites
0	32	33	15	21	5	106
1-2	40	44	60	22	22	188
3-4	35	18	14	19	11	97
5-6	. 17	9	2	7	3	38
7-10	9	9	3	3	3	27
11-19	1	2	0	1	0	4
>20	1	0	1	0	1	3

Table 13. Perceived congestion, by crowding threshold.

Number of			Crowding	Tolerance		
Others Seen	1-2	3-4	5-6	7-8	9-10	>10
Fewer than preferred	12	9	7	4	7	11
About right	60	46	57	30	40	80
More than preferred	36	12	16	11	10	21

Table 14. Crowding tolerance by group size.

			Crowding	Tolerance		4
Number in Group	1-2	3-4	5-6	7-8	9-10	>10
1	10	11	6	1	3	12
2	30	20	21	13	15	29
3	27	12	13	4	7	18
4	19	12	15	12	7	16
5-6	10	6	13	8	13	17
7-10	9	6	'8	5	9	15
>10	4	0	4	2	3	7

(respondents were asked to use their own definition of site boundaries). Most recreationists were satisfied with the number of others at the site (shown in Table 13). In summary the result of site evaluation part of the survey was as expected, that is, a high weight is given to the flow level.

The survey provided the necessary data for statistical estimation of demand equations. The estimation procedure requires an expression for determining price or money outlay per unit of recreation consumed. cost of the whole recreation experience can be used for this purpose. costs will be made up of many items, such as cost of transportation, food for that recreation experience, entrance fees, recreation equipment, and recreationists opportunity cost. These are the added expenditures which the individual must make in order to take part in the whole recreation experience.

The site interviews conducted in three instream recreation sites provided the following information required for estimating demand equation (see Appendix A for the sample questionnaire):

- l. Number of days the household spent at recreation site (part I questions #2 and #3).
- 2. Expenditures or the cost of recreation experience incurred that were specific to that trip (parts III and IV).

- Family income (part V questions #2 and #3).
- 4. Mileage driven for that specific trip (part I question #1).

Item 1 forms the basis of quantity measures for estimating the demand equation. Data obtained from these items are used to calculate budget share of good (W_i) in the demand equation (Table 15):

$$W_{ij} = \frac{P_{ij} X_{ij}}{I_{ij}}$$

where

i = 1, 2, 3 (site)

j = 1, 2, ..., 12 (group)

P_{ij} = money outlay per unit of recreation consumed which is 24 hours or a day of recreation in this study

Iij = family full income of group j
 at site i

X_{ij} = X_{ij} D_L = estimated total
 number of days recreationists
 of group j spent at site i
 per capita (Table 16)

X_{ij} = number of days recreationists spent at site i

 $D_{L} = \hat{x}_{1j} + \hat{x}_{2j}$

Table 15. Calculated budget share of good for each group by site.

		Site	
Group	Logan, W ₁	Blacksmith, W ₂	Little Bear, W3
1	$W_{11} = 0.000242$	$W_{21} = 0.000080$	W ₃₁ = 0
2	$W_{12} = 0.000128$	$W_{22} = 0.000052$	$W_{32} = 0.000010$
3	$W_{13}^{12} = 0.000026$	$W_{23} = 0.000009$	$W_{33} = 0.000001$
4	$W_{14} = 0.000007$	$W_{24} = 0.000001$	$W_{34} = 0.000001$
5	$W_{15} = 0.00012$	$W_{25} = 0.000029$	$W_{35} = 0.000005$
6	$W_{16} = 0.000096$	$W_{26} = 0.000062$	$W_{36} = 0.000007$
7	$W_{17}^{10} = 0.000032$	$W_{27}^{20} = 0.000017$	$W_{37} = 0.000011$
8	$W_{18}^{17} = 0.000005$	$W_{28}^{27} = 0.000001$	$W_{38} = 0.000002$
9	$W_{19}^{10} = 0.000069$	$W_{29}^{20} = 0.00001$	$W_{39} = 0$
10	$W_{110} = 0.000038$	$W_{210} = 0.000051$	$W_{310} = 0.000007$
11	$W_{111} = 0.000037$	$W_{211} = 0.000009$	$W_{311} = 0.000007$
12	$W_{112} = 0.000014$	$W_{212} = 0.000007$	$W_{312} = 0.0000000$

$$W_{1} = \frac{(X_{\text{Lo}_{1}} + X_{\text{Lo}_{2}})[(P_{\text{Lo}_{1}} + P_{\text{Lo}_{2}})/2]}{[(I_{\text{Lo}_{1}} + I_{\text{Lo}_{2}})/2]}$$

$$w_{2} = \frac{(x_{BL_{1}} + x_{BL_{2}})[(P_{BL_{1}} + P_{BL_{2}})/2]}{[(I_{BL_{1}} + I_{BL_{2}})/2]}$$

$$W_3 = \frac{X_{LB} \cdot P_{LB}}{I_{LB}}$$

Lo₁ = Logan 1

 Lo_2 = Logan 2

 BL_1 = Blacksmith 1

 BL_2 = Blacksmith 2

LB = Little Bear

 Lo_1 , Lo_2 = Site 1 as i = 1

 BL^1 , BL^2 = Site 2 as i = 2

LB = Site 3 as i = 3

Table 16. Number of days of recreation per capita by sites.

	Site						
Group	Logan, X ₁	Blacksmith, X ₂	Little Bear, X ₃				
1	$x_{11} = 0.043$	$x_{21} = 0.0092$	x ₃₁ = 0				
2	$x_{12}^{11} = 0.033$	$x_{22}^{-1} = 0.0123$	$x_{32} = 0.003$				
3	$x_{13}^{12} = 0.0042$	$x_{23} = 0.0022$	$x_{33} = 0.0003$				
4	$x_{14}^{13} = 0.0008$	$x_{24}^{23} = 0.00008$	$x_{34} = 0.0002$				
5	$x_{15}^{1-} = 0.0326$	$x_{25}^{24} = 0.0095$	$x_{35} = 0.0017$				
6	$x_{16}^{15} = 0.0323$	$x_{26}^{23} = 0.0215$	$x_{36} = 0.0037$				
7	$x_{17}^{10} = 0.0122$	$x_{27}^{23} = 0.0042$	$x_{37} = 0.0037$				
8	$x_{18}^{17} = 0.0015$	$x_{28}^{27} = 0.0002$	$x_{38} = 0.0006$				
9	$x_{19} = 0.029$	$x_{29}^{20} = 0.00204$	$x_{39} = 0$				
10	$x_{110} = 0.015$	$x_{210} = 0.0169$	$x_{310} = 0.0038$				
11	$x_{111} = 0.015$	$x_{211}^{210} = 0.0039$	$x_{311} = 0.0030$				
1 2	$x_{112}^{111} = 0.0051$	$x_{212}^{211} = 0.00029$	$x_{312}^{312} = 0.00009$				

$$X_1 = Lo_1 + Lo_2$$

$$X_2 = BL_1 + BL_2$$

$$X_3 = LB$$

$$\hat{x}_{1j} = \frac{(G_{1j}) \text{ (total number of week-}}{\text{ends of season)}}$$
total number of weekends of survey

$$\hat{x}_{2j} = \frac{(G_{2j}) \text{ (total number of week-}}{\text{days of season)}}$$
number of weekdays of survey

$$G_{1j} = \frac{C_k \cdot S_k^{ij}}{S_k^{i}}$$

$$G_{2j} = \frac{C_D \cdot S_D^{ij}}{S_D^i}$$

C_kⁱ = total number of cars in weekend in each site

 C_D^i = total number of cars in weekday in each site

S_k^{ij} = total number of surveys in weekend in each site for each group

$$S_D^{ij}$$
 = total number of surveys in weekday in each site for each group

S_kⁱ = total number of surveys in weekend in each site

 S_D^i = total number of surveys in weekday in each site*

The next step is to calculate full price for each site using data obtained from items 2, 3, and 4. The full price as defined before is (Table 17):

$$P_{ij} = \sum_{k} P_{k} a_{kji} + Wt_{ij} = (PA + Wt)_{ij}$$
(16)

^{*}For all of the data explained in this part refer to Appendix B.

Table 17. Full price of each group per day by site.

			S	ite		•
	Lo	gan	Blacksm	ith Fork	Little	Bear
Group	P ₁	ln P ₁	P ₂	ln P ₂	P ₃	ln P3
1	42.7	3.75	51.5	3.94	0	0
2	44.4	3.79	42.0	3.74	37.0	3.81
3	55.3	4.01	30.2	3.41	38.4	3.65
4	70.7	4.26	76.5	4.34	62.4	4.14
5	80.2	4.39	64.9	4.17	71.8	4.27
6	57.9	4.06	65.7	4.19	39.8	3.68
7	57.3	4.05	83.8	4.43	55.9	4.02
8	75.0	4.32	81.4	4.40	72.3	4.28
9	91.5	4.52	190.7	5.25	0	0
10	93.0	4.53	104.6	4.65	80.5	4.39
11	89.9	4.50	89.1	4.49	91.8	4.52
12	103.1	4.64	98.2	4.59	113.4	4.73

$$P_1 = (P_{Lo_1} + P_{Lo_2})/2$$

$$P_2 = (P_{BL_1} + P_{BL_2})/2$$

$$P_3 = P_{LB}$$

where

 $Wt_{ij} = \{[(R03/168)(1/3)] (U_2)\}_{ij}$

RO3 = monthly household salary

U₂ = V₂ - hours of nighttime at recreation site

V₂ = number of hours at recreation site

$$PA = PD + [(v_1 \cdot 2)/W06] (1.20)$$

V₁ = distance from home to site in miles

W06 = vehicle gas consumption (miles per gallon)

PD = [W08 + (W07)(0.3)]

12
+
$$(\Sigma (PE_e/t_e))(1/V6)$$

e=1

+ (W09)(F)

W08 = cost of recreation equipment for that trip (dollars)

W07 = cost of food in dollars

PE_e = cost of durable equipment used in dollars

te = life span of equipment e
 (data were obtained from
 Outdoor Recreation Center of
 Utah State University)

W09 = fee for use of that site per day in dollars

F = number of days at recreation site

V₆ = number of times the trip was

Based on this information, the full price was calculated for each sample and it was averaged for the group from each zone. The last variable to calculate is

full income which was defined as $I = N + WT_w$ and necessary data for this calculation for each site were obtained by item 3 (Table 18).

There are two issues over the role of time cost in estimation of recreation The first one is, how much of benefit. the time involved is costly and should be included in calculation of full price, and second issue is, what is the appropriate value of time spent in the recreation site. Wilman (1980) and Becker (1965) pointed out that the total time spent in an activity is costly and the appropriate value of this time is its opportunity cost; in other words the value of time in its best alternative Cesario (1976), after reviewing several studies, concludes that the appropriate value of recreation time is approximately one-third the average wage rate.

As McConnell (1975) mentioned in his discussion of the value of time, understanding and selecting appropriate opportunity cost of total time is important for accurate measurement of the economic value of outdoor recreation. In this study, after carefully considering all possible recommendations, the value of recreation time or its opportunity cost was decided to be approximately onethird of the average wage rate for the recreationist, and only day time hours of each day was considered as recreation time.

Demand curve derivation or specifically full price estimation requires determination of the fraction of the total travel distance from home to the recreation site. For the visitor living nearby (less than 120 miles), this fraction of total travel distance is actually equal to total distance between home and recreation site. For the visitor living several hundred miles away (above 120 miles) only a small fraction of total travel

distance was considered in the calculation.

A large number of people, unlike a single individual, will have a predictable and measurable reaction to an outdoor recreation opportunity. If we can measure the demand curve for a large group of people, then it is probable that another large group, chosen with more or less similar characteristics to the first group, will respond in a similar fashion to costs and other characteristics of the recreation experience. This assumption is basic to demand curve analysis in this study. Since one single individual cannot be observed at the same time in different sites, therefore, a group of recreationists with similar characteristics were interviewed at different sites, at the same time in estimating multisite demand The data used in this evalufunction. ation were gathered by the survey which was conducted on site for 12 days in summer of 1982. These 12 days included four weekdays and eight weekend days. The total recreation season was estimated to be 93 days of which 67 days were weekdays and 26 days were weekends. The number of groups surveyed on the four weekdays and on the eight weekends for each reach were recorded (Table B-2, Appendix B). This information plus the number of cars at each site were used to estimate total visits for the season adjusted for weekdays, weekends and unsampled visitors on survey days (Tables B-3, B-4, B-5, and B-6, Appendix B). The samples were grouped using four zones and three income classifications. The four zones classification based on average distances of 20, 40, 60 and over 60 miles from the site were defined in such a way that population could be estimated using census district maps. A statistical computer package (SPSS) was used to analyze the data obtained from survey for developing a recreation multi-site demand function. The demand estimation procedure is discussed in the next section.

Table 18. Full income of each group by site.

		Site	
Group	Logan, I ₁	Blacksmith Fork, I ₂	Little Bear, I3
1	7,574.32	9,399.04	0
2	11,406.25	10,000.0	10,625.0
3	8,854.17	7,524.75	10,000.0
4	8,131.25	8,750.0	11,666.67
5	22,648.81	21,388.89	22,500.0
6	19,444.45	22,625.0	20,833.33
7	22,083.33	20,274.51	19,166.67
8	23,833.33	20,833.33	23,611.11
9	38,538.96	39,000.0	0
10	36,250.0	35,000.0	43,333.33
11	36,770.83	40,000.0	40,000.0
12	38,080.36	36,937.50	39,166.67

 $I_1 = (I_{Lo_1} + I_{Lo_2})/2$

 $I_2 = (I_{BL_1} + I_{BL_2})/2$

 $I_3 = I_{LB}$

ECONOMETRIC ESTIMATION AND MODEL RESULTS

In this section, recreationists demand equation which was developed before is estimated. The objective is to estimate the structural demand for three recreation sites (Morey 1981) from the cross-sectional household data. The next step will be to estimate consumer surplus corresponding to various levels of instream flow. The AIDS (Almost Ideal Demand System) cost function is used to derive a demand function which is in the semilog form. Selection of an appropriate functional form is very important. As Ziemer et al. (1980) pointed out, different functional forms can produce dramatically different consumer surplus estimates. He also carefully tested the specification problem involving the selection of an appropriate functional form. compared three kinds of functional forms namely, linear, quadratic, and semilog. The conclusion was that semilog specification is the appropriate functional form for warm-water fishing in Georgia. Even though this conclusion might be different for Utah recreation sites, the semilog form was attempted as an appropriate functional form for this study. Deaton and Muellhauer (1982) discussed different models of demand function and their specifications in a whole chapter of their "Economics and Consumer Behavior" book. They identified a new model of demand as Almost Ideal Demand System (AIDS) which preserves the generality of both Rotterdam and Translog models. Also, they added that an important feature of this function from an econometric viewpoint is that it is close to being linear. These models can be estimated equation by equation using ordinary least squares, since P* (the price index) is defined as a linearly homogeneous

function of the individual prices. Thus P^* would be approximately proportional to appropriately defined price index, such as the one used by Stone, the logarithm of which is given by Σ W_k

 $\log P_k$ (Deaton and Muellhauer 1980). This index was calculated directly before estimation, so that Equation 7 becomes straightforward to estimate. Estimation procedure started by applying ordinary least square (OLS) to each equation of the form:

$$W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j}$$

$$+ \beta_{i} \log M + \epsilon_{i} \qquad (17)$$

where

 $M = I/P^*$ (Table 19) and ϵ_i are disturbances with usual properties.

Applying OLS method to estimate multiple-side demand parameter (Equation 17) might encounter some econometric problems since assumptions of non-autocorrelation might be violated. To avoid these econometric problems, the three demand equations were estimated using Generalized Least Square (GLS) method. Since variance-covariance matrix of disturbances are not known, the estimation is done in a two-stage procedure based on Zellner's SUR technique.

First stage: In order to define the variance-covariance matrix of disturbances, estimated value of the disturbance terms were obtained by applying OLS on Equation 7. The estimated form of this equation is:

Table 19. Estimated Mi for each group by site.

	Site					
Group	Logan, M ₁	Blacksmith Fork, M ₂	Little Bear, Mg			
1	7,565	9,387	0			
2	11,397	9,992	10,617			
3	8,852	7,523	9,998			
4	8,130	8,749	11,666			
5	22,633	21,374	22,484			
6	19,431	22,609	20,819			
7	22,077	20,269	19,161			
8	23,832	20,832	23,610			
9	38,524	38,985	0			
10	36,234	34,984	43,314			
11	36,762	39,990	39,990			
12	38,077	36,934	39,163			

$$\hat{\mathbf{W}}_{\mathbf{i}} = \hat{\alpha}_{\mathbf{i}} + \sum_{\mathbf{j}} \hat{\gamma}_{\mathbf{i}\mathbf{j}} \log P_{\mathbf{j}} + \hat{\beta}_{\mathbf{i}} \log M$$
(18)

where

$$i = 1, 2, 3$$

The empirical forms of above equations are:

$$\hat{W}_{1} = 38.83 - 9.13 \log P_{1}$$

$$(1.32) \quad (0.82)$$

$$- 6.57 \log P_{2} - 2.85 \log P_{3}$$

$$(0.90) \quad (2.12)^{*}$$

$$+ 4.52 \log M_{1}$$

$$(0.91)$$

$$R^{2} = 0.639$$
F-statistic = 3.10

$$\hat{W}_{2} = 12.18 - 8.08 \log P_{1}$$

$$(0.93) (1.66)^{*}$$

$$- 2.23 \log P_{2} - 0.47 \log P_{3}$$

$$(0.67) (0.81)$$

$$+ 3.64 \log M_{2}$$

$$(1.62)^{*}$$

$$R^{2} = 0.55$$

$$F-statistic = 2.16$$

$$\hat{W}_{3} = 1.59 - 1.22 \log P_{1}$$

$$(0.99) (1.96)^{*}$$

$$- 0.75 \log P_{2} - 0.61 \log P_{3}$$

$$(2.21)^{*} (1.20)$$

$$+ 0.35 \log M_{3}$$

$$(1.69)^{*}$$

$$R^{2} = 0.703$$

$$F-statistic = 4.14^{**}$$

^{*}Indicates that the estimated parameters are significant at 10 percent level of significance.

^{**}Indicates that the estimated vector of the parameters are significant at 5 percent level of significance.

The numbers inside the parentheses indicate t-statistic for the relevant parameters.

The residuals can be estimated for each observation group as:

$$W_i - \hat{W}_i = \hat{\varepsilon}_i$$

A Fortran program was developed for estimating the contemporaneous variance-covariance matrix of the disturbance terms across equations based on Zellner's SUR technique.

Stage two: The next step is to apply ordinary least-squares on Equation 22 with a premultiplied observation matrix. Equation 7 in matrix notation with transformed observation would be written as:

$$PW_{i} = P X \beta + P\varepsilon$$
 (22)

where

$$X = [1 log P log M]$$

The three estimated demand equations with GLS estimators are:

$$\hat{W}_{1} = 1.62 - 21.17 \log P_{1}$$

$$(0.81) (3.86)^{*}$$

$$- 2.41 \log P_{2} - 0.92 \log P_{3}$$

$$(0.79) (1.31)$$

$$+ 10.89 \log M$$

$$(3.85)^{*}$$

$$R^{2} = 0.87$$

$$F-\text{statistic} = 11.31^{**}$$

$$\hat{W}_{2} = 0.73 - 12.92 \log P_{1}$$

$$(0.81) (6.72)^{*}$$

$$- 3.93 \log P_{2} - 0.38 \log P_{3}$$

$$(2.58)^{*} (1.12)$$

$$+ 7.56 \log M$$

$$(6.95)^{*}$$

$$R^{2} = 0.91$$

F-statistic =
$$18.42^{**}$$

 $\hat{W}_3 = 0.12 - 0.54 \log P_1$
 $(1.26) (3.9)^*$
 $-0.45 \log P_2 - 0.75 \log P_3$
 $(3.82)^* (6.42)^*$
 $+0.38 \log M_3$
 $(7.86)^*$
 $R^2 = 0.93$
F-statistic = 24.27^{**}

The numbers in parentheses indicate t-statistic for the relevant parameters. The resulting vector of estimated parameters from three different econometric methods of demand estimation is shown in Table 20. Column 2 in this table shows the value of parameters when OLS is applied. The result of using Zellner's procedure without restriction on seemingly unrelated regression equations is shown in column 3 and the 4th column shows the parameters when Zellner's SUR technique with imposing symmetric condition was used. In the case of applying Zellner's SUR technique with imposing symmetric condition the value of $R^2 = 0.83$ and F-statistic = 9.95.

If $\beta_i > 0$, good i is a luxury good. Since in all three methods $\beta_l > 0$, $\beta_2 > 0$, $\beta_3 > 0$ the implication is that recreation is a luxury good. Since $\gamma_{12} < 0$ and $\gamma_{13} < 0$ in all three methods of estimation (Table 20), sites 2 and 3 (Blacksmith Fork and Little Bear) are not good alternate sites for Logan or site 1. On the contrary, Blacksmith Fork and Little Bear (sites 2 and 3)

^{*}Indicates that the estimated parameters are significant at 10 percent level of significance.

^{**}Indicates that the estimated vector of the parameters are significant at 5 percent level of significance.

Table 20. Comparison of the estimated parameters using different estimation methods.

Parameters	Estimated Parameters Using OLS	Estimated Parameters Using GLS Unrestricted	Estimated Parameters Using GLS Restricted
	1	. 2	3
α1	38.83	1.62	1.54
α ₂	12.18	0.73	0.22
α 3	1.59	0.12	0.62
Υ ₁₁	-9.13	-21.17	-21.93
⁷ 12	-6.57	-2.41	-4.06
Ϋ 13	-2.85	-0.92	-0.96
^Y 21	-8.08	-12.92	-4.06
Y 22	-2.23	-3.93	-0.52
Υ 23	-0.47	-0.38	0.62
Υ 31	-1.22	-0.54	-0.96
^Y 32	0.75	0.45	0.62
Υ3 3	-0.61	-0.75	-0.62
β_1	4.53	10.89	11.97
β 2	3.64	7.56	1.52
β 3	0.35	0.38	0.42

are good alternate sites for each other, because $\gamma_{32}>0$ and in the third method of estimation γ_{23} is also positive.

To check differences in estimated demand due to site quality or a stream characteristic such as water quality, which are not explained by the model or by the estimators, Table 21 was arranged using the data obtained from the survey. According to Table 21, site characteristic evaluations are not significantly different in three sites in this study area. A composite of site characteristics (Table 21) range from 6.5 to 7.28, and the only item in the table which makes this small difference is the evaluation of site facilities. The site

characteristics on demand function can be balanced by considering the entrance fee paid by users. The argument is that, as Little Bear has a lower facility evaluation score than Logan site, it has a lower or no user fee. Therefore, in summary, higher fee with higher evaluation of facilities score is as attractive as a lower fee with lower evaluation score. Thus, except for flow level there was no significant site characteristic differences between the three sites in the study area.

Any change in flow level affects visitation rate and consequently the demand function (Sutherland 1982). Table 11 indicates the change of

Table 21. Site characteristic evaluation.

•	Site						
Characteristic	Logan	Blacksmith Fork	Little Bear				
Distance	7.82	7.90	7.3				
Privacy	7.24	7.60	7.5				
Facilities	6.86	5.05	3.0				
Land sc ape	8.52	8.40	7.3				
Insects	5.31	4.75	4.9				
Water .	8.56	8.40	8.4				
Fishing Suitability	6.63	6.35	6.8				
Composite	7.28	6.92	6.5				

10 = perfect

1 = poor

visitation as a function of flow level variation. For instance, this table shows the number of recreationists who would not visit the sites when the flow levels drop to less than 50 percent of flows in the summer of 1982. This information was used to derive the modified estimated demand functions at each flow level, by quality parameter fi, as a function of flow and, therefore, incorporating the effect of site quality changes in terms of flow levels.

Instream Flow Effects on Visitation

In order to measure the effects of hypothetical changes in instream flows on visitation, a quality parameter f_i for site i was defined and estimated as a function of instream flows. Defining

$$f_i = \frac{v_g^i}{v_8^i} = f(Fg) = \frac{1}{1 + e^{-(a + bF_g)}}$$
(26)

where f(Fg) is between 0 and 1. Therefore, the function f(Fg) reduces the visitation rate as Fg becomes smaller. Moreover, f(Fg) = 1, as Fg corresponds to 100 percent of 1982 flow for which data were collected. For F_1 , the instream flow is zero, and $f(F_1) = 0$

which implies no visitation. survey for demand estimation, the visitors were asked to indicate the percent of current flow below which they would not visit the site. These data are used to obtain hypothetical visitation at various Fg's which were compiled for two zones in each site. The plot of these data indicates that the visitation rate increased from Fg = O at an increasing rate up to about 50 percent of 1982 flows and it increased at an almost decreasing rate from 50 percent and up. Therefore, a logistic function, Equation 26 appeared to provide the best fit.

The classification of visitation rate at various flow levels specified two zones based on average distances of 40 and over 40 miles from the site. This classification was used to estimate the effect of hypothetical changes in instream flow on visitation rates. In the question of indicating the percentage of current flow below which the visitors would not visit the site, the percentages given as options were 0, 10, 25, 33, 50, 67, 75, and 100. Since 1982 had a much higher flow level than average flows (Table 22), the maximum flow was limited to the present flow level (100 percent). Table 23 shows the estimated number of visitation

Table 22. Streamflow volumes at different probabilities of occurrence in acre-feet.

			Site	e		
	Loga	n	Blacksm:	th	Little 1	Bear
State	Probability of Occurrence	Seasonal Total	Probability of Occurrence	Seasonal Total	Probability of Occurrence	Seasonal Total
1	0.037	100,000	0.037	50,000	0.074	20,000
2	0.259	150,000	0.259	80,000	0.148	30,000
3	0.074	170,000	0.222	100,000	0.148	40,000
4	0.296	190,000	0.259	120,000	0.111	50,000
5	0.074	210,000	0.074	140,000	0.333	70,000
6	0.185	250,000	0.037	160,000	0.111	90,000
7	0.074	300,000	0.074	180,000	0.074	100,000

Table 23. Data for estimating f(Fg) function.

		Logan	Rla	Site cksmith	T :+	tle Bear
Fg	Zone l (40 miles) f(Fg)xl00	Zone 2 (over 40 miles) f(Fg)xl00	Zone 1 (40 miles) f(Fg)x100	Zone 2 (over 40 miles) f(Fg)x100	Zone 1 (40 miles) f(Fg)x100	Zone 2 (over 40 miles) f(Fg)x100
0	4.62	0.90	3.06	0.064	0	0
10	14.43	5.80	6.63	3.07	0.05	0
25	35.83	14.4	9.66	8.64	0.05	0
33	55.98	28.9	21.14	16.79	40.71	7.09
50	69.86	61.4	65.55	58.41	60.87	65.2
67	82.78	76.5	80.2	75.91	78.0	84.2
75	87.98	81.7	82.90	80.35	84.6	89.9
100	100.0	100.0	100.0	100.0	100.0	100.0

days for various flow levels as a percentage of the number of visitation days at 100 percent of the flow for the two defined zones.

To estimate the logistic function defined in Equation 26, data from Tables 23 and 24 were used. Moreover, for estimating purposes, this function was rewritten in stochastic form as:

$$\log \frac{f(Fg)}{1 - f(Fg)} = a + bFg + \varepsilon \qquad (27)$$

where the stochastic disturbance, ϵ is assumed to be random normal with zero mean and constant variance.

$$\varepsilon \sim N(0, \sigma^2)$$

The three estimated equations for each site are,

$$\log \frac{f_1(Fg)}{1 - f_1(Fg)} = -2.96 + 0.03 Fg$$

$$(9.56) \quad (11.16)$$

$$(28)$$

$$R^2 = 0.899 \qquad F = 124.5$$

$$\log \frac{f_2(Fg)}{1 - f_2(Fg)} = -4.19 + 0.06 Fg$$
(9.21) + (9.05)

$$R^2 = 0.86$$
 F = 81.9

$$\log \frac{f_3(Fg)}{1 - f_3(Fg)} = -2.91 + 0.06 Fg$$
(2.7) (3.07)

(30)

$$R^2 = 0.40$$
 F = 9.45

The values in parentheses are the corresponding t values. The F ratio and the \mathbb{R}^2 for Equations 28, 29, and 30 are written under each equation.

Benefit Estimation

The benefit equation (Equation 15) was used to compute compensating variation, CV, for each site under different conditions. The quality parameters f_i which depends upon the observed specification (flow level) were estimated by Equations 28 to 30. These parameters

Table 24. Average monthly flows in cfs (Fg) for the season.

	Site						
Percent of	Log	gan	Blacl	ksmith-	Little Bear		
1982 Flows	F*	F	F*	F	F*	F	
0 (1)	0	0	0	0	0	0	
10	23.06	22.02	13.0	12.67	9.75	8.08	
25 ·	57.65	55.05	32.5	31.68	24.38	20.20	
33	76.1	72.67	42.9	41.82	32.18	26.66	
50	115.3	110.10	65.0	63.36	48.76	40.40	
67	149.89	147.53	87.1	84.90	65.33	54.14	
75	172.95	165.15	97.5	95.04	73.13	60.6	
100 (8)	230.6	220.2	130.0	126.72	97.51	80.8	

F* = Flow data for water year of 1982 (from State Engineer's Office).

F = Flow data for 3 months of summer 1982.

were used to modify the cost function and the demand functions. The results obtained from estimating multiple-site demand functions were used in Equation 15. The benefit equation was estimated for various instream flows for one site at a time (holding flows at other sites at 100 percent of 1982 levels) expressed as percentage of 1982 flows using the data from Table 24. Loss of benefits

is shown in Table 25 for different percents of current flow level. Using the estimated demand equations, changes in benefits as a result of changes in instream flows at more than one site could also be estimated. For nine different selected strategies, the estimated loss of benefits is shown in Table 26. Both totals and marginal benefits are shown.

Table 25. Estimated total benefit changes of instream flows at different flow levels (dollars).

		Reduced Flow Level	
Site	50 Percent of	25 Percent of	20 Percent of
	Current Flow	Current Flow	Current Flow
Logan	0	39,395	151,472
Blacksmith Fork	0	41,242	151,138
Little Bear	0	36,506	134,819

Table 26. Total benefits of instream flows in dollars.

	Site							
Strategy	Logan	Blacksmith	Little Bear					
1	2,806 (0.29)		_					
2	-	36,772 (6.97)	-					
3	-	_	106,059 (8.9)					
4	128,802 (12.9)	. -	<u> </u>					
5		97,412 (11.5)	-					
6	-	- -	152,747 (15.98)					
7	805,414 (69.27)		_					
8	-	487,614 (74)	_					
9	-	- -	242,416 (30.96)					

Strategy 1 = 35 percent flow for Logan and 50 percent flow for others.

Strategy 2 = 35 percent flow for Blacksmith River and about 50 percent flow for others.

Strategy 3 = 35 percent flow for Little Bear, 35 percent for Logan, and 50 percent for Blacksmith River.

Strategy 4 = 30 percent flow for Logan and 50 percent flow for others.

Strategy 5 = 30 percent flow for Blacksmith River and the rest as above.

Strategy 6 = 30 percent flow for Little Bear River and the rest as above.

Strategy 7 = 25 percent flow for Logan River and almost 50 percent flow for others.

Strategy 8 = 25 percent flow for Blacksmith River and the rest as above.

Strategy 9 = 25 percent flow for Little Bear River and the rest as above.

Values in parentheses are corresponding marginal benefits (in \$/AF).

SUMMARY AND CONCLUSIONS

Major economic conflicts exist between withdrawal and instream flow water use. Until recently, most western government agencies encouraged water diversions and related development projects as a source of new income and economic growth. However, recently increased attention has focused on studies to include instream flow in the water allocation policy. Increases in mobility, leisure time, income and population cause water-based recreation demand to assume a greater importance. Therefore, to achieve efficient allocation of water return and instream and offstream uses, estimates of cost and benefits from recreational use of instream flow are needed.

Economists usually rely on the private market system to reveal appropriate economic values. However, most water allocation decisions are made outside the market place. To aid such decisions in the absence of market prices, a methodology is needed to estimate instream flow values in achieving efficiency in allocation. The theoretical model developed in this study to estimate recreationists demand function is based on Becker's (1965) approach to the consumer behavior, since it is best suited to estimate a multiple site demand system. In this approach, which is known as the household production function theory, unlike the conventional consumer theory, consumption activities are viewed as the outcome of individual or household production process, combining market goods and time. Almost Ideal Demand System (AIDS) was chosen to derive the multi-site demand equations. The AIDS leads to a semilog form of demand function which has been shown to be an appropriate

functional form for economic evaluation of warm-water recreation activities (Ziemer et al. 1980). The data used in this evaluation were gathered by a survey conducted on three sites, Logan River, Blacksmith Fork River, and Little Bear River, during the summer of 1982. The full price and full income are defined and calculated according to household production theory.

The structural demands for three recreation sites are estimated using Zellner's SUR technique. Applying Ordinary Least Square (OLS) method to estimate multiple-site demand parameters causes econometric problems. assumption of homoscedasticity and nonautocorrelation of random disturbances inherent in the OLS may not be met in multiple-site demand estimation. The estimated demand function for all three sites and the results of Table 21 indicate that there is not a significant site characteristic effect on demand functions. The positive sign of coefficients, β , leads to the conclusion that recreation is a luxury good. Since $\gamma_{12} < 0$ and $\gamma_{13} < 0$, sites 2 and 3 are not good alternative sites for site 1; but $\gamma_{23} > 0$ means sites 2 and 3 are relatively good alternative sites for each other. According to Table 21, site characteristic evaluations are not significantly different in the three sites in the study area, because composite of site characteristics range from 6.5 (Little Bear) to 7.28 (Logan) in the scale of 1 (poor) to 10 (ex-This information indicates cellent). that, at a given flow level, each of these recreation sites is as attractive as any other. But, the flow level has an important weight on attractiveness of the sites as can be concluded from Table

11. This table indicates how drastically the visitation rate will reduce as flow level decreases.

To test the instream flow effect on visitation rate and estimating compensated variation, CV, of altering instream flow level, the quality parameter f; (a function of flow) was defined and estimated on the basis of observed values. The necessary data for this estimation were obtained through a conducted survey in summer This quality parameter was used 1982. to modify the ordinary demand function and the corresponding cost function. Then, the CV was measured by differences between original cost function and modified cost function at different instream flow levels. From Table 25, the benefit obtained from altering instream flow above 50 percent of average flow is negligible. On the contrary, reduction of instream flow below 30 percent of average will involve substantially large losses of benefits to society.

The following specific conclusions are drawn from this study:

- 1. The obtained result indicates no significant site characteristic effect on demand function. Alteration of flow will have an effect on visitation rate.
 - 2. Recreation is a luxury good.
- 3. Blacksmith Fork and Little Bear Rivers are not good alternative sites for Logan River recreation site. However, Blacksmith Fork and Little Bear Rivers are good alternatives for each other.
- 4. Instream flow level above 50 percent of average flow does not significantly add to economic value of recreation in the case study areas.
- 5. Reduction of instream flow level below 30 percent of average flow will adversely affect potential recreation.

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Appendix A

Water-Related Recreation Survey

UTAH WATER RESEARCH LABORATORY UTAH STATE UNIVERSITY LOGAN, UTAH

WATER-RELATED RECREATION SURVEY

NUMBER:	
DATE:	
SITE:	
INTERVIEV	<i>T</i> ER:
Introduct	cion
a study of we need to recreation this info the purpo opportunion opportuni	Utah Water Research Laboratory at Utah State University is conducting on the value of water for recreation. In order to determine these values, to get some information from people who come to enjoy the streamside on opportunities in this area. We would appreciate your helping us to get ormation by taking 15 to 20 minutes to answer some questions. In general, use of the questions is to help us estimate the value of the recreation ities from the actual expenses that recreationists incur to enjoy those ities. You need not answer any questions you would prefer not to, and of your answers will be kept confidential.
	first 12 questions are designed to give us some background and de- ption of your visit to this site.
1.	Where do you live? (Locate on map on last page if home is in map area. Otherwise give place name.)
2.	How long have you been at this site? (Locate site on map on last page.)
3.	How much longer do you plan to stay?
4.	How many people did you come with? (Total in vehicle and group.)
5.	What is the age and sex of those in your party? (Place "M" or "F" beside the appropriate age group.)
	0 to 9 yrs. 40 to 49 yrs. 50 to 59 yrs. 60 to 69 yrs. 30 to 39 yrs. 70 yrs or more

Elementary 1 2 3 4 5 6 Secondary 7 8 9 10 11 12 College 13 14 15 16+

6. Circle the highest year of education you have completed.

7.	How do you plan to spend your time he in each activity below. (Includes rein party.)	
	eating	games
	fishing	sleeping
	hiking	other (specify)
	water play	
8.	How often do you go on this kind of	recreation outing?
	1-2 times/yr. 1-2 times/mo.	1/wk. more than 1/wk.
9.	Where do you usually go on such outin visits at each site. Refer to map or	
	Smithfield % Logan 1 % Logan 2 % Blacksmith 1 %	Blacksmith 2 % Little Bear % Other % (specify)
10.	Compared to your idea of a perfect reate this site on the characteristics use a scale of 1 to 10, where a "10" "1" means the site is extremely poor	below? (For each characteristic means the site is perfect, and a
	distance privacy/uncrowded facilities vegetation/landscape	fishing suitability and probable success other important site
	insects/pests water	characteristics (specify)
11.	For your recreation purposes, would y	
	a. more than you would preb. fewer than you prefer? c. about the right number?	
12.	What is the maximum number of other that you would tolerate before decide	
	1-2 3-4 5-6	7-8 9-10 more than 10 (give
	**************************************	number range)

II.	Now we would 1	ike you to	imagine w	hat the	stream	would be	like a	t different
	flow levels, a	nd indicate	how these	e change	s would	affect	your ev	aluation of
	this site for	recreation.						

1.	For	each	of	the	alte	ernative	stream	conditions	below	indicate	the	response
	you	feel	to	be n	nost	appropr	iate.					

		would look	Higher than ideal but acceptable	About right (or indif-ferent)	Lower than ideal but accept-able	So low I would look for an-other site
а.	Present level	***************************************		**************************************		
Ь.	Twice the pres- ent level					
с.	1 1/2 times the present level				-	· .
d.	Half the pres- ent level					-
e.	No water	ACCOMPANIES AND	**************************************	WINTER	***************************************	

(Answer 2 only for "so low" responses.)

2. As a percent of the present flow, approximately what is the minimum amount of water acceptable for your purposes?

0 10 25 33 50 67 75 100

One effect of some water resource developments is to deplete stream flow over certain stretches of a river. The next question asks how you might react if a development were proposed that would deplete the flow in this portion of the river.

3. If the flow at this site went below your minimum acceptable level, where would you probably go as an alternative?

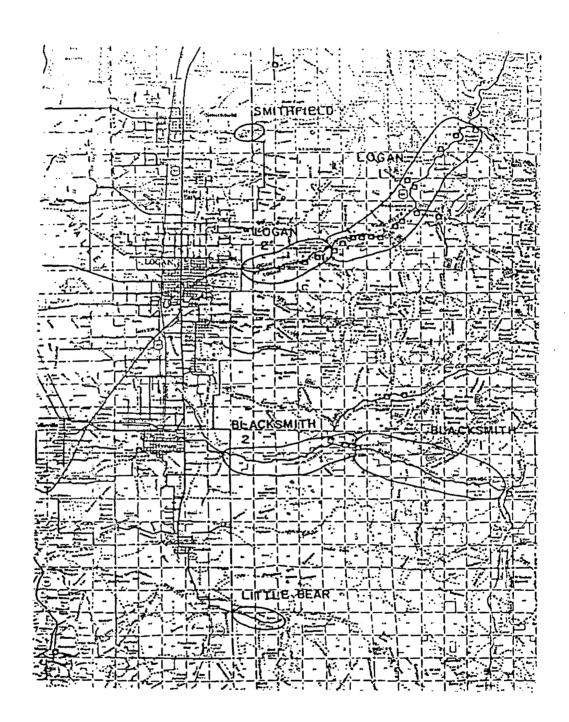
4. If the only practical way to preserve the flow was to establish a system of user fees to cover the costs of keeping water in the river, how much would you be willing to pay per visit to maintain the flow level you desire?

0 \$2 \$4 \$6 \$8 \$10 \$12 \$14 \$16 \$18 \$20

- 5. If you answered "0," was it because
 - a. reduced flow levels, or a dry stream, would not adversely affect your use of this site?
 - b. user fees on this site are already as high or higher than they should be? (Applicable only on developed sites.)
 - c. you think stream flows should be maintained, but do not believe recreation users should have to pay to maintain them?
- III. The next four questions concern your expenses for this visit.
 - What mileage does the vehicle you came in get? (Specify vehicle type and mileage whether vehicle belongs to respondent or to another in party.)
 - 2. About how much did you spend for food for this visit?
 - 3. About how much did you spend for recreation equipment (fishing, swimming, etc.) for this visit?
 - 4. Did you pay a fee for use of this site? How much?
- IV. This group of questions concerns the value of the equipment you are using. The list below is intended as a fairly comprehensive checklist of the kinds of things you might have brought with you. We have three questions we would like you to answer concerning the items on the list. First, we would like you to tell us the cost of those items you have with you. Second, we would like to know how old those items are. Finally, we would like you to tell us how much you plan to spend on new equipment.

Equipment category	1. Cost	2. Age	3. New purchases
RV, camper, trailer Tents, awnings Sleeping bags, pads Stoves, grills, heaters Cooking utensils Furniture Ice chests Fishing rods & reels Other fishing equip. Special apparel Licenses Other (specify)			

٠.	rue	rinar set or questions has to do with	your occupation and income.
	1.	What is your occupation?	
	2.	In what interval does your total annu	al household income fall?
		less than \$5,000 \$5,000 to \$9,999 \$10,000 to \$14,999 \$15,000 to \$19,999	\$20,000 to \$24,999 \$25,000 to \$29,999 \$30,000 to \$34,999 \$35,000 to \$44,999 \$45,000 or more
	3.	In what interval does your monthly ho	-
		less than \$500 \$500 to \$999	\$2,000 to \$2,499 \$2,500 to \$2,999
		\$1,000 to \$1,499	\$3,000 or more
		\$1,500 to \$1,999	



Appendix B

Table B-1. Number of surveys for each site and group.

		<u>.</u>	Site		
Group	Lol	Lo ₂	BL1	BL ₂	LB
1	13	30	. 0	8	0
2	8	4	8	13	6
3	4	3	5	4	3
4	5	10	2	3	7
5	21	12	3	4	0
6	. 9	3	15	11	4
7	12	3	16	6	4
8	17	16	24	3	9
9	11	7	0	3	0
10	5	4	1	8	0
11	8	9	7	1	5
12	16	15	12	7	5
Total	1 29	116	93	71	43

This table does not show transit recreationists (recreationists who are passing through and stop for a short period of time).

Table B-2. Number of surveys for each site, group, and days.

					Sit					
		$_{ m Lo_1}$		Lo ₂		$^{\mathtt{BL}_1}$		BL ₂		L
Group	D ₁ *	D ₂ **	$\overline{D_1}$	D ₂	$\overline{D_1}$	D ₂	$\overline{D_1}$	D ₂	$\overline{D_1}$	D ₂
1	11	2	24	6	7	1	8	5	0	0
2 .	6	. 2	1	3	2	2	8	2	5	3
3	2	· 2	2	1	2	0	0	2	2	0
4	2	2	3	1	1	0	1	1	5	1
5	16	5	9	3	10	2	7	2	1	1
6	7	2	3	0	13	7	10	0	2	1
7	8	4	2	1	12	5	3	0	3	0
8	5	4	4	1	8	1	0	0	4	5
9	10	1	4	3	1	0	4	1	0	0
10	5	0	3	1	3	1	5 .	1	2	1
11	4	4	8	1	9	1	2	0	0	2
12	6	2	3	4	4	0	4	1	2	1
Total	82	30	66	25	72	20	52	15	26	15

This tables does not show transit recreationists (recreationists who are coming from above 365 miles to sites).

 $[\]begin{tabular}{lll} *D_1 indicates weekend \\ *D_2 indicates weekdays \\ \end{tabular}$

Table B-3. Number of cars in each site.

	Da	ıys	
Site	Weekend	We ekd ay	Total
Lol	507	115	622
Lo ₂	371	100	471
BL_1^2	90	26	116
\mathtt{BL}_2	101	29	130
LB ~	48	19	67
Total	1,117	289	1,406

Table B-4. Estimated G_1 and G_2 for each group.

.*					Site					
	L	71	Lo	2	BL	BL ₁		2	LB	
Group	G_1	G ₂	G_1	G ₂	G_1	G ₂	G ₁	G ₂	Gl	G ₂
1	67.19	5.0	109.93	17.14	8.63	1.30	14.96	8.53	0	0
2	36.65	5.0	4.58	8.57	2.47	2.60	14.96	3.41	8.57	3.80
3	12.22	5.0	9.16	2.86	2.47	0	0	3.41	3.43	0
4	12.22	5.0	13.74	2.86	1.23	0	1.87	1.71	8.57	1.27
5	97.73	12.50	41.22	8.57	12.33	2.60	13.09	3.41	1.71	1.27
6	42.76	5.0	13.74	0	16.03	9.10	18.70	0	3.43	1.27
7	48.87	10.0	9.16	2.86	14.79	6.50	5.61	0	5.14	0
8	30.54	10.0	18.32	2.86	9.86	1.30	0	0	6.86	6.33
9	61.08	2.50	18.32	8.57	1.23	0	7.48	1.71	0	0
10	30.54	0	13.74	2.86	3.70	1.30	9.35	1.71	3.43	1.27
11	24.43	10.0	36.64	2.86	11.10	1.30	3.74	0	0	2.53
12	36.65	5.0	13.74	11.43	4.93	0	7.48	1.71	3.43	1.27

Lo₁ = Upper Logan River Lo₂ = Lower Logan River BL₁ = Upper Blacksmith Fork River BL₂ = Lower Blacksmith Fork River LB = Little Bear River

Table B-5. Estimated \hat{x}_1 and \hat{x}_2 for each group.

					Sit	е				
	Lo	1	Lo ₂ B		\mathbf{L}_{1}		L ₂	I	В	
Group	\hat{x}_1	\hat{x}_2	\hat{x}_1	$\hat{\mathbf{x}}_{2}$	\hat{x}_1	Ŷ2	\hat{x}_1	\hat{x}_2	\hat{x}_1	Ŷ2
1	218.37	83.75	357.27	287.10	28.05	21.78	48.62	142.88	0	0
2	119.11	83.75	14.89	143.55	8.03	43.55	48.62	57.12	27.85	63.6
3	39.72	83.75	29.77	47.91	8.03	0	0	57.12	11.15	0
4 .	39.72	83.75	44.66	47.91	4.0	0	6.08	28.64	27.85	21.2
5	317.62	209.38	133.97	143.55	40.07	43.55	42.54	57.12	5.56	21.2
6	138.97	83.75	44.66	0	52.10	152.43	60.78	0	12.58	31.2
7	158.83	167.5	29.77	47.91	48.07	108.88	93.97	0	86.10	0
8	99.26	167.5	59.54	47.91	32.05	21.78	0	0	114.91	106.03
9	198.51	41.88	59.54	143.55	4.0	0	24.31	28.64	0	0
10	99.26	. 0	103.43	77.91	12.03	21.78	30.39	28.64	11.15	21.2
11	79.40	167.5	119.08	47.91	36.08	21.78	12.16	0	0	42.3
12	119.11	83.75	44.66	191.45	16.02	0	24.31	28.64	11.15	21.2

 $[\]hat{\boldsymbol{x}}_1$ indicates weekends $\hat{\boldsymbol{x}}_2$ indicates weekdays

Table B-6. Estimated D for each group.

Group	Site							
	Lol	Lo ₂	BL ₁	BL ₂	LB			
1	D ₁₁ = 302.12	D ₂₁ = 644.37	D ₃₁ = 49.83	D ₄₁ = 191.50	D ₅₁ = 0			
2	$D_{12} = 202.86$	$D_{22}^{21} = 158.44$	$D_{32} = 51.58$	$D_{42} = 105.74$	$D_{52} = 91.50$			
3	$D_{13} = 123.47$	$D_{23}^{-2} = 77.68$	$D_{33} = 8.03$	$D_{43} = 57.12$	$D_{53} = 11.15$			
4	$D_{14} = 123.47$	$D_{24} = 92.57$	$D_{34} = 4.0$	$D_{44} = 34.72$	$D_{54} = 49.12$			
5	$D_{15} = 527.00$	$D_{25}^{-1} = 277.52$	$D_{35} = 83.62$	$D_{45} = 99.66$	$D_{55} = 26.83$			
6	$D_{16} = 222.72$	$D_{26}^{25} = 44.66$	$D_{36} = 204.53$	$D_{46} = 60.78$	$D_{56} = 43.85$			
7	$D_{17} = 326.33$	$D_{27}^{-3} = 77.68$	$D_{37} = 156.95$	$D_{47} = 93.97$	$D_{57} = 86.10$			
8	$D_{18} = 266.76$	$D_{28} = 107.45$	$D_{38} = 53.83$	$D_{48} = 0$	$D_{58} = 220.92$			
9	$D_{19} = 240.39$	$D_{29}^{-3} = 203.09$	$D_{39} = 4.0$	$D_{49} = 52.95$	$D_{59} = 0$			
10	$D_{110} = 99.26$	$D_{210} = 181.34$	$D_{310} = 33.81$	$D_{410} = 59.03$	$D_{510} = 32.42$			
11	$D_{111} = 246.90$	$D_{211} = 166.99$	$D_{311} = 57.86$	$D_{411} = 12.16$	$D_{511} = 42.38$			
12	$D_{112} = 202.86$	$D_{212}^{-11} = 236.11$	$D_{312} = 16.02$	$D_{412} = 52.95$	$D_{512} = 32.42$			

Table B-7. Distribution of income in Utah.

Income	# of Families	Total	Percentage
0- 2,499	7,731		
2,500- 4,999	11,415		
5,000- 7,499	19,063		
7,500- 9,999	22,584		
10,000-12,499	28,656		
12,500-14,999	27,228		
•	-	116,677.0	32.94
15,000-17,499	31,330		
17,500-19,999	28,774		
20,000-22,499	32,492		
22,500-24,999	25,198		
25,000-27,499	23,935		
27,500-29,999	17,257		
	***************************************	158,986.0	44.89
30,000-34,999	28,626		•
35,000-39,999	17,118		
40,000-49,999	17,563		
50,000-74,999	10,952		
75,000	4,253		
•	-	78,512.0	22.17
			-
		354,175.0	100.00

Table B-8. Population by zone.

•	Zone	Population 1*	Population 2**
	1	46,895	42,864
	2	50,225	45,815
	3	243,462	156,638
	4	1,052,924	1,199,464

^{*}Population for Logan site.

**Population for Blacksmith Fork and Little Bear sites.

Table B-9. Population in each income group and each zone.

Site	Zone	Income Group One 32.94%	Income Group Two 44.89%	Income Group Three 22.17%
Logan	1	15,447.54	21,051.61	10,396.84
J	2	16,544.12	22,546.00	11,134.88
	3	80,196.38	109,290.09	53,975.53
	4	346,833.17	472,657.58	233,433.25
Blacksmith	1	14,119.40	19,241.65	9,502.95
Fork and	2	15,091.46	20,566.35	10,157.19
Little Bear	3	51,596.56	70,314.80	34,726.64
	4	395,103.44	538,439.39	265,921.17

Table B-10. Variables for each group, Logan.

Site	Group		Variables				
		X	PA	WT	I		
Lo ₁ Lo ₂	1	$x_{11} = 1.6$ $x_{21} = 0.3$	$PA_{11} = 25.0$ $PA_{21} = 6.42$	$WT_{11} = 28.2$ $WT_{21} = 9.41$	$I_{11} = 7,315.31$ $I_{21} = 7,833.3$		
Lo ₁ Lo ₂	2	$X_{12} = 2.031$ $X_{22} = 1.31$	$PA_{12} = 39.0$ $PA_{22} = 24.2$	$WT_{12} = 41.5$ $WT_{22} = 40.4$	$I_{12} = 10,312.5$ $I_{22} = 12,500.0$		
Lo ₁ Lo ₂	3	$x_{13} = 2.31$ $x_{23} = 0.2$	$PA_{13} = 22.53$ $PA_{23} = 7.80$	$WT_{13} = 53.6$ $WT_{23} = 7.85$	$I_{13} = 9,375.0$ $I_{23} = 8,333.3$		
Lo ₁ Lo ₂	4	$X_{14} = 1.42$ $X_{24} = 1.31$	$PA_{14} = 37.1$ $PA_{24} = 48.2$	$WT_{14} = 46.1$ $WT_{24} = 59.7$	$I_{14} = 8,137.5$ $I_{24} = 8,125.0$		
Lo ₁ Lo ₂	5	$X_{15} = 1.2$ $X_{25} = 0.195$	$PA_{15} = 30.0$ $PA_{25} = 7.49$	$WT_{15} = 47.5$ $WT_{25} = 11.31$	$I_{15} = 23,214.3$ $I_{25} = 22,083.3$		
Lo ₁ Lo ₂	6	$x_{16} = 3.3$ $x_{26} = 1.2$	$PA_{16} = 49.5$ $PA_{26} = 16.6$	$WT_{16} = 74.9$ $WT_{26} = 72.1$	$I_{16} = 19,722.0$ $I_{26} = 19,166.7$		
Lo _l Lo ₂	7	$ \begin{array}{r} x_{17} & = 3.1 \\ x_{27} & = 3.1 \end{array} $	$PA_{17} = 73.3$ $PA_{27} = 48.8$	$WT_{17} = 131.2$ $WT_{27} = 101.2$	$I_{17} = 23,333.3$ $I_{27} = 20,833.3$		
Lo ₁ Lo ₂	8	$x_{18} = 1.92$ $x_{28} = 2.24$	$PA_{18} = 51.7$ $PA_{28} = 80.1$	$WT_{18} = 87.5$ $WT_{28} = 94.2$	$I_{18} = 24,166.7$ $I_{28} = 23,500.0$		
Lo ₁ Lo ₂	9	$x_{19} = 0.7$ $x_{29} = 0.5$	$PA_{19} = 23.98$ $PA_{29} = 16.7$	$WT_{19} = 52.8$ $WT_{29} = 16.3$	$I_{19} = 38,863.6$ $I_{29} = 38,214.3$		
Lo ₁ Lo ₂	10	$x_{110} = 1.35$ $x_{210} = 0.31$	$PA_{110} = 34.38$ $PA_{210} = 10.61$	$WT_{110} = 82.1$ $WT_{210} = 20.0$	$I_{110} = 35,000.0$ $I_{210} = 37,500.0$		
Lo ₁ Lo ₂	11	$x_{111} = 1.61$ $x_{211} = 2.8$	$PA_{111} = 33.1$ $PA_{211} = 82.79$	$WT_{111} = 106.2$ $WT_{211} = 178.92$	$I_{111} = 36,875.0$ $I_{211} = 36,666.7$		
Lo ₁ Lo ₂	12	$X_{112} = 1.72$ $X_{212} = 4.1$	$PA_{112} = 43.3$ $PA_{212} = 87.0$	$WT_{112} = 111.9$ $WT_{212} = 384.6$	$I_{112} = 36,875.0$ $I_{212} = 39,285.7$		

Table B-11. Variables for each group, Blacksmith Fork.

Site	Group Variables				
		X	PA	WT	I
$^{\mathrm{BL}}_{\mathrm{1}}$	1	$x_{31} = 0.33$ $x_{41} = 0.59$	$PA_{31} = 6.39$ $PA_{41} = 18.4$	$WT_{31} = 8.7$ $WT_{41} = 15.6$	$I_{31} = 9,375.0$ $I_{41} = 9,423.1$
$^{\mathtt{BL}_1}_{\mathtt{BL}_2}$	2	$x_{32} = 2.42$ $x_{42} = 0.650$	$PA_{32} = 29.19$ $PA_{42} = 16.26$	$WT_{32} = 53.1$ $WT_{42} = 16.1$	$I_{32} = 11,250.0$ $I_{42} = 8,750.0$
$^{\mathtt{BL}_1}_{\mathtt{BL}_2}$	3	$X_{33} = 1.5$ $X_{43} = 1.9$	$PA_{33} = 20.8$ $PA_{43} = 26.91$	$WT_{33} = 26.79$ $WT_{43} = 26.79$	$1_{33} = 8,750.0$ $1_{43} = 6,299.50$
$^{\mathrm{BL}}_{\mathrm{BL}_2}$	4	$x_{34} = 3.0$ $x_{44} = 0.61$	$PA_{34} = 35.77$ $PA_{44} = 54.86$	$WT_{34} = 53.57$ $WT_{44} = 19.59$	$I_{34} = 7,500.0$ $I_{44} = 10,000.0$
$^{\mathtt{BL}_1}_{\mathtt{BL}_2}$	5	$x_{35} = 0.81$ $x_{45} = 1.083$	$PA_{35} = 19.4$ $PA_{45} = 37.52$	$WT_{35} = 28.42$ $WT_{45} = 38.64$	$I_{35} = 22,500.0$ $I_{45} = 20,277.8$
$^{\mathrm{BL}}_{\mathrm{1}}$	6	$X_{36} = 1.82$ $X_{46} = 1.17$	$PA_{36} = 49.1$ $PA_{46} = 36.31$	$WT_{36} = 64.77$ $WT_{46} = 44.29$	$I_{36} = 24,250.0$ $I_{46} = 21,000.0$
$^{\mathtt{BL}_1}_{\mathtt{BL}_2}$	7	$X_{37} = 0.998$ $X_{47} = 1.5$	$PA_{37} = 56.738$ $PA_{47} = 32.1$	$WT_{37} = 44.26$ $WT_{47} = 67.46$	$I_{37} = 23,382.4$ $I_{47} = 19,166.7$
$^{\mathrm{BL}_1}_{\mathrm{BL}_2}$	8	$x_{38} = 1.74$ $x_{48} = -$	$PA_{38} = 71.04$ $PA_{48} = -$	$WT_{38} = 70.41$ $WT_{48} = -$	$1_{38} = 20,833.3$ $1_{48} = -$
BL ₁ BL ₂	9	$X_{39} = 0.104$ $X_{49} = 0.321$	$PA_{39} = 4.07$ $PA_{49} = 39.96$	$WT_{39} = 13.64$ $WT_{49} = 27.78$	$I_{39} = 40,000.0$ $I_{49} = 38,000.0$
$\frac{\mathtt{BL}_1}{\mathtt{BL}_2}$	10	$x_{310} = 0.885$ $x_{410} = 2.44$	$PA_{310} = 59.88$ $PA_{410} = 49.9$	$WT_{310} = 55.93$ $WT_{410} = 141.12$	$I_{310} = 32,500.0$ $I_{410} = 37,500.0$
$^{\mathrm{BL}}_{\mathrm{1}}$	11	$x_{311} = 1.996$ $x_{411} = 1.56$	$PA_{311} = 45.63$ $PA_{411} = 36.62$	$WT_{311} = 126.74$ $WT_{411} = 107.14$	$I_{311} = 37,500.0$ $I_{411} = 42,500.0$
$rac{ ext{BL}_1}{ ext{BL}_2}$	12	$X_{312} = 1.44$ $X_{412} = 1.008$	$PA_{312} = 59.73$ $PA_{412} = 39.92$	$WT_{312} = 55.31$ $WT_{412} = 77.28$	$I_{312} = 34,375.0$ $I_{412} = 39,500.0$

Table B-12. Variables for each group, Little Bear.

Group		Variables						
	X	PA	WT	I				
1	x ₅₁ = -	PA ₅₁ = -	WT ₅₁ = -	I ₅₁ = -				
2	$x_{52} = 1.817$	$PA_{52} = 51.04$	$WT_{52} = 16.18$	$I_{52} = 10,625.0$				
3	$x_{53} = 1.21$	$PA_{53} = 22.09$	$WT_{53} = 24.31$	$I_{53} = 10,000.0$				
4	$x_{54} = 1.33$	$PA_{54} = 54.025$	$WT_{54} = 29.1$	I ₅₄ = 11,666.7				
5	$x_{55} = 1.25$	$PA_{55} = 21.24$	$WT_{55} = 68.45$	$I_{55} = 22,500.0$				
6	$x_{56} = 1.74$	$PA_{56} = 39.47$	$WT_{56} = 29.52$	$I_{56} = 20,833.3$				
7	$x_{57} = 3.06$	$PA_{57} = 81.54$	$WT_{57} = 89.29$	$I_{57} = 19,166.7$				
8	$x_{58} = 1.41$	$PA_{58} = 42.38$	$WT_{58} = 59.3$	I ₅₈ = 23,611.1				
9	x ₅₉ = -	PA ₅₉ = -	WT ₅₉ = -	1 ₅₉ = -				
10	$x_{510} = 1.194$	$PA_{510} = 16.66$	$WT_{510} = 79.37$	$I_{510} = 43,333.3$				
11	$x_{511} = 2.5$	$PA_{511} = 23.95$	$WT_{511} = 205.36$	$I_{511} = 40,000.0$				
12	$x_{512} = 0.896$	$PA_{512} = 38.77$	$WT_{512} = 62.75$	$I_{512} = 39,166.67$				

Table B-13. Number of total days of recreation for season.

	Site					
Group	Lol	Lo ₂	BL_1	BL ₂	LB	
1	495.17	192.667	16.593	112.985	0	
2	412.01	208.032	124.669	66.781	50.508	
3	285.59	15.614	12.045	107.10	13.469	
4	174.96	120.804	12.0	20.971	65.477	
5	684.16	54.116	67.230	107.932	33.538	
6	733.19	50.868	373.063	71.173	76.12	
7	1,006.08	241.662	156.636	140.955	263.12	
8	513.780	240.903	93.557	0	310.863	
9	165.629	93.624	0.416	16.997	0	
10	134.001	55.67	29.922	143.915	38.709	
11	398.003	498.240	115.489	19.006	105.950	
12	349.122	960.023	23.037	53.374	29.048	

Table B-14. Number of days of recreation per capita.

	Site				
Group	Lol	Lo ₂	BL ₁	BL ₂	LB
1	0.03	0.013	0,0012	0.008	0
2	0.02	0.013	0.0083	0.004	0.003
3	0.004	0.0002	0.0002	0.002	0.0003
4	0.0005	0.0003	0.00003	0.00005	0.0002
5	0.03	0.0026	0.0035	0.006	0.0017
6	0.03	0.0023	0.018	0.0035	0.0037
7	0.01	0.0022	0.0022	0.002	0.0037
8	0.001	0.0005	0.0002	0	0.0006
9	0.02	0.009	0.00004	0.002	0
10	0.01	0.005	0.0029	0.014	0.0038
11	0.007	0.008	0.0033	0.0006	0.0030
12	0.001	0.0041	0.00009	0.0002	0.00009

Table B-15. Full price* of each group per day** by site.

	Site				
Group	Lol	Lo ₂	BLl	BL ₂	LB
1	32.5	52.8	45.4	57.6	0
2	39.6	49.2	34.1	49.8	36.984
3	32.9	77.6	31.7	28.6	38.4
4	58.7	82.7	29.8	123.3	62.4
5	64.0	96.4	59.5	70.3	71.8
6	37.8	77.9	62.5	68.8	39.8
7	66.4	48.2	101.2	66.4	55.9
8	72.3	77.7	81.4	0	72.3
9	111.5	71.6	170.2	211.2	0
10	86.3	99.7	130.8	78.3	80.5
11	86.4	93.3	86.3	91.9	91.8
12	90.2	116.0	79.97	116.3	113.4

^{*}Full price = PA + WT = P
**Full price per day = P/X

Table B-16. Calculated budget share* of good for each group by site.

	Site							
Group	Lol	Lo ₂	BL ₁	BL ₂	LB			
1	0.00013	0.000088	0.000006	0.000049	0			
2	0.00008	0.000051	0.000025	0.000023	0.00001			
3	0.000014	0.000002	0.000001	0.000009	0.000001			
4	0.000004	0.000003	0.0000001	0.000001	0.000001			
5	0.000083	0.000011	0.000009	0.000021	0.000005			
6	0.000057	0.000009	0.000046	0.000011	0.000007			
7	0.000028	0.000005	0.00001	0.000007	0.000011			
8	0.000003	0.000002	0.000001	0 .	0.000002			
9	0.000057	0.000017	0.000002	0.000011	0			
10	0.000025	0.000013	0.000012	0.000029	0.000007			
11	0.000016	0.000020	0.000008	0.000001	0.000007			
12	0.000002	0.000012	0.0000002	0.000001	0.0000003			

$$*_{W_{i}} = \frac{P_{i} X_{i}}{I_{i}}$$

Appendix C

Derivation of AIDS demand function from the PIGLOG class of preferences

These preferences are represented via the cost or expenditure function:

$$Log C(U, P) = (1-U) log{a(P)} + U log{b(P)}$$
 (C-1)

where a(P) and b(P) are linear homogeneous concave functions, and defined as:

$$Log a(P) = \alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$$
(C-2)

and

Log b(P) = Log a(P) +
$$\beta_0 = \pi P_k^{\beta_k}$$

So

$$Log b(P) = \alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j + \beta_0 \pi P_k^{\beta_k}$$
(C-3)

Substituting for log a(P) and log b(P) in Equation C-1 will give us the AIDS flexible cost function.

Log C(U,P) =
$$(1-U)(\alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j)$$

+ $(U)(\alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j)$
+ $\beta_0 \prod_{k} P_k^{\beta_k}$
= $\alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$
- $U\alpha_0 - U \sum_{k} \alpha_k \log P_k - 1/2 U \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$
+ $U\alpha_0 + U \sum_{k} \alpha_k \log P_k + 1/2 U \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$
+ $\beta_0 U \prod_{k} P_k^{\beta_k}$

Then

$$\log C (U,P) = \alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{j} \sum_{k} \gamma_{kj}^* \log P_k \log P_j + k \qquad k \qquad j$$

$$\beta_0 U \pi P_k^{\beta_k} \qquad (C-4)$$

where α_i , β_i , γ_{ij} * are parameters

C = cost or expenditure

P = price

U = utility

Hicks-compensated demand function can be derived directly from expenditure function. The price derivatives of cost function will be the quantities demanded:

$$\frac{\partial C(U,P)}{\partial P_i} = q_i \tag{C-5}$$

Multiply both sides of Equation C-5 by $P_i/C(U,P)$:

$$\frac{\partial C(U,P)}{\partial P_{i}} \cdot \frac{P_{i}}{C(U,P)} = \frac{q_{i} P_{i}}{C(U,P)} \tag{C-6}$$

Equation C-6 can be written as:

$$\frac{\partial \log C(U,P)}{\partial \log P_i} = \frac{q_i P_i}{C(U,P)} = W_i$$

where W_i = the budget share of good i. Therefore, logarithmic differentiation of Equation C-4 will give us W_i as a function of price and utility.

$$\frac{\partial \log C(U,P)}{\partial \log P_{i}} = W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j} + \beta_{i} U \beta_{0} \pi P_{k}^{\beta_{k}}$$
 (C-7)

where

$$\gamma_{ij} = 1/2 \left(\gamma_{ij}^* + \gamma_{ji}^*\right) \tag{C-8}$$

For a utility maximizing consumer, total expenditure X is equal to cost function. This equality can be inverted to get indirect utility function as a function of price and expenditure as:

log C (U,P) = log I =
$$\alpha_0$$
 + $\sum_{k} \alpha_k \log P_k$ + 1/2 $\sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j$ + β_0 U $\prod_{k} P_k^{\beta_k}$

then

$$U = (-\alpha_0 - \sum_{k} \alpha_k \log P_k - 1/2 \sum_{k} \sum_{j} \gamma_{kj}^* \log P_k \log P_j + \log I)/\beta_0 \prod_{k} P_k^{\beta_k}$$
 (C-9)

Substituting Equation C-9 in Equation C-7:

$$W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j} + \beta_{i}\beta_{0} \pi P_{k}^{\beta_{k}}(-\alpha_{0} - \sum_{k} \alpha_{k} \log P_{k} - 1/2 \sum_{k} \sum_{k} i)$$

$$\log P_{k} \log P_{j} + \log I)/\beta_{0} \pi P_{k}^{\beta_{k}} \qquad (C-10)$$

Then we have budget shares as a function of price and X.

$$W_{i} = \alpha_{i} + \sum_{j} \gamma_{ij} \log P_{j} + \beta_{i} \log\{I/P^{*}\}$$
(C-11)

where

P* is price index which is defined by:

$$\log P = \alpha_0 + \sum_{k} \alpha_k \log P_k + 1/2 \sum_{k} \sum_{j} \gamma_{kj} \log P_k \log P_j$$
 (C-12)