

Orbit Calculation and Doppler Correction Algorithm in a LEO Satellite Small Ground Terminal

El Bachir Zantou
 Centre Royale d'Etudes et de Recherche Spatiales
 Sect 21, Av Allal All Fassi, Hay Riad, Rabat, Morocco; (212) 61 44 12 31
 zantou@crers.gov.ma

Abdelhak Kherras, Adnane Addaim
 Centre de Recherche et d'Etudes Spatiales
 Av Ibn Sina – B.P. 765 - Agdal, Rabat, Morocco; 212 37 77 05 98
 kherras@emi.ac.ma, addaim@emi.ac.ma

ABSTRACT: It is an important technological challenge to ensure a LEO satellite ground terminal autonomy as well as to make it small, lightweight and power efficient along with the ability to predict the satellite visibility passes. Another important feature is to make it able to compensate for Doppler on the satellite link with an economical method. In this paper, an orbit and Doppler calculation methodology has been developed for a general orbital model. The algorithm that manages the ground terminal automatic operation ensures many advantages and is adapted to a microcontroller programming. Inputs to the algorithm are time, position and keplerian elements from NORAD. The NORAD elements are injected in the terminal memory via its serial port once before it is deployed on the operation field. Time is provided by a real time clock read and written by the terminal microcontroller. Terminal geographic position is provided by an internal integrated miniature GPS which makes the terminal free to move anywhere on the terrestrial globe and still be able to contact the satellite without any reprogramming. The orbit calculation methodology used expresses the satellite coordinates in the terminal relative topocentric (horizontal) coordinate system which easily yield the values of elevation and azimuth angles along with the slant range. This is achieved by means of satellite vector transformations through different coordinate systems. Doppler shift is obtained by deriving the slant range in time. A simple methodology for Doppler correction is also proposed in this paper and is adapted for low cost transceivers.

INTRODUCTION

Combining the use of a single LEO store-and-forward micro-satellite and small low cost ground terminals allows the developing countries to have a very economical space communication system. Missions like messaging, data collection and localization could be made accessible to large rural areas where other communication networks are not available. The store-and-forward communication concept has been used with much success in different missions including commercial, social and education applications.

The on-board hardware and software are usually kept simple to minimize failure risk as well as the overall cost, whereas the ground terminals are more complicated as they should achieve all the functions needed to establish a satellite communication link.

In opposite to a satellite constellation system, the single satellite system because of the irregular visibility passes needs to handle the transmission times. Different methods have been studied and used to manage the contact times between the satellite and the ground terminals. For example Argos terminals just don't care and transmit all the time. This method

is power consuming. Other systems use polling from satellite. In the polling method, time synchronisation is critical and the ground terminals are geographically limited.

A single microsatellite data collection system using small ground terminals faces two difficulties. On the one hand, in applications such as data collection, remote surveillance and mobile localization which need automatic operation, ground terminals should be able to predict satellite visibility passes; on the other hand they have to handle the Doppler shift that is permanently changing during the satellite pass.

This leads us to develop a software module embedded in the ground terminal which is responsible for orbit calculation and Doppler correction. In the case of satellite constellation such as Orbcom system, orbit calculation is not necessary as the satellite visibility is permanent.

An orbit and Doppler calculation algorithm implemented inside a microcontroller makes the ground terminals compact, small, lightweight, intelligent and low cost.⁶ At any time, anywhere the terminal is able to predict the satellite passes and its elevation angle relative to its current position. This feature has many advantages. First, the terminal can

operate in an automatic manner without any human operator or any Polling from the microsatellite. Second, there is no need for a PC or laptop which makes the terminal heavy and difficult to carry. Third, a power saving method is achieved: the terminal transceiver is keyed only when the satellite is in good visibility range. Fourth, since each terminal calculates the satellite elevation angle, a certain priority in the transmission protocol can be implemented for the terminals in order to enhance the system capacity. Fifth, in addition to elevation and azimuth angles the algorithm calculates also the satellite slant range. By deriving the range in time we obtain the Doppler shift that affects the transmission frequencies. This allows Doppler correction of both the transmitter and receiver frequencies.

Many microsatellite builders including Surrey University have developed portable ground terminals composed of transceivers interfaced with a laptop which contains all the terminal intelligence.³ In this architecture the presence of the laptop makes the ground terminal cumbersome and heavy.

The challenge in the present work is to integrate in a small ground terminal the intelligence that works out the two problems cited above. This is achieved by developing a software module embedded in the terminal microcontroller memory. Effort has been done to care about memory usage and the speed power of the microcontroller.

ORBITAL ELEMENTS AND COORDINATES SYSTEMS

The position in space of a satellite is necessarily determined by four fundamental elements: the orbital plane orientation in space, orbit orientation in that plane, dimensions and shape of orbit and finally the

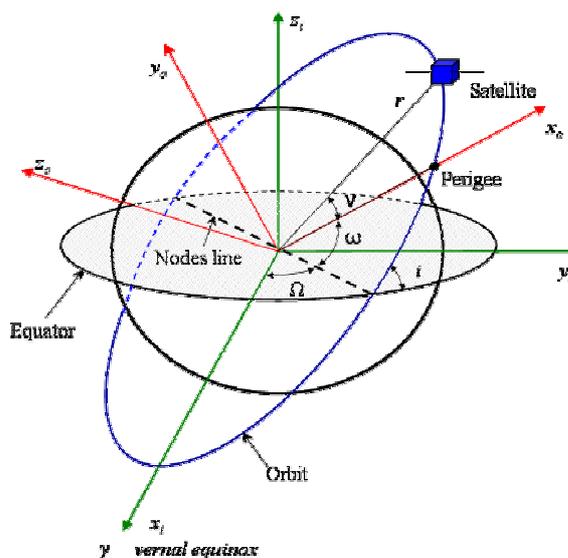


Figure 1. The orbital system (O) and inertial (I)

position of the satellite in its orbit (figure 1). These elements are defined by the orbital parameters which are: semi-major axis a , eccentricity e , inclination i , argument of perigee ω , right ascension of ascending node Ω (RAAN) and mean anomaly M .

In a satellite movement around earth, orbit calculation uses four different Cartesian coordinates:¹

- Orbital coordinates (O): determine the movement of the satellite in its own orbital plane. The x_o -axis is along the major axis of the orbit. The satellite is generally following a circle or an ellipse in the (x_o, y_o) plane.

- Inertial coordinates (I): with origin at the center of earth, the z_i -axis is along the axis of rotation and the x_i -axis pointing toward the vernal equinox γ .

- Greenwich coordinates (G): fixed to the earth and rotating with x_g -axis pointing the Greenwich meridian.

- Topocentric or horizontal (H): its origin is at the ground terminal position with (x_h, y_h) plane tangent to the terrestrial sphere.

ORBIT CALCULATION METHODOLOGY

With the terminal being on the ground at point G, localising the satellite in space at point S consists of determining the terminal-satellite vector $\vec{\rho}$. Knowing

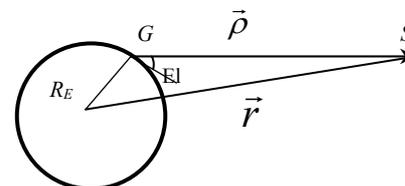


Figure 2. Terminal-satellite vector

the topocentric coordinates of that vector allows rapid computation of the elevation angle, azimuth and slant range of the satellite. By deriving the slant range in time we get the Doppler shift affecting the up and downlink frequencies.

The methodology followed in order to calculate the visibility parameters of the satellite is summarized in the following steps:

- 1- Express satellite position in the orbital plane.
- 2- Transformation to inertial coordinates.
- 3- Transformation to Greenwich coordinates.
- 4- Transformation to topocentric coordinates.
- 5- Computation of elevation, azimuth and slant range.

Satellite Orbital Coordinates

In the orbital coordinates the satellite generally has an elliptical movement around earth with a radius r and a semi-major axis a . Therefore the satellite coordinates (x_o, y_o, z_o) are given by:

$$x_o = r \cos \nu = a(\cos E - e) \quad (1)$$

$$y_o = r \sin \nu = a\sqrt{1-e^2} \sin E \quad (2)$$

$$z_o = 0 \quad (3)$$

where, ν is the true anomaly, e the orbit eccentricity, and E the eccentric anomaly.

In the equations (1) and (2), E and a stay unknown. The eccentric anomaly E is found from Kepler's equation:

$$M = E - e \sin E \quad (4)$$

where M is the mean anomaly which is given at time t by:

$$M = n(t - t_e) + M_e \quad (5)$$

where M_e is the mean anomaly at the specified initial time (epoch) t_e and n is the mean motion.

M_e , t_e and n in addition to other parameters are extracted from the satellite ephemeris file published weekly by NORAD.⁵ Equation (4) being non linear, E is computed using Newton iterations.

Semi-Major Axis a Calculation

The semi-major axis a is not obtained from equation (6) which is only true for the simple two-body problem. To take nongravitational disturbances such as atmospheric drag and gravitational perturbations such as those due to the nonspherical earth into account equation (7) is rather used.²

$$T = 2\pi / n = 2\pi\sqrt{a^3 / \mu} \quad (6)$$

$$n = \sqrt{\mu / a^3} \left[1 + (3/2)J_2(R_E^2 / a^2)(1 - e^2)^{-3/2}(1 - (3/2)\sin^2 i) \right] \quad (7)$$

where μ is the gravitational constant ($398\,600.5 \text{ Km}^3/\text{s}^2$), J_2 is the zonal harmonic coefficient of the earth of second order, i is the orbit inclination and R_E is the equatorial radius of the earth. For the World Geodetic System gravity field model WGS84 the coefficient J_2 equals 1082.63×10^6 . Equation (7) is a nonlinear equation and is solved using a numerical iterative method (Newton).

Transformation to Inertial Coordinates (x_i, y_i, z_i)

Once the satellite orbital coordinates are obtained, the next step is a transformation to inertial coordinates. This transformation is done by means of rotations using inclination i , argument of perigee ω and RAAN Ω angles. Using the transformation matrix, the satellite inertial coordinates (x_i, y_i, z_i) are given by equation (8).¹

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{bmatrix} \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i & -\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega & \sin \Omega \sin i \\ \cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega & -\sin \omega \sin \Omega + \cos \omega \cos i \cos \Omega & -\cos \Omega \sin i \\ \sin \omega \sin i & \cos \omega \sin i & \cos i \end{bmatrix} \times \begin{pmatrix} x_o \\ y_o \\ z_o \end{pmatrix} \quad (8)$$

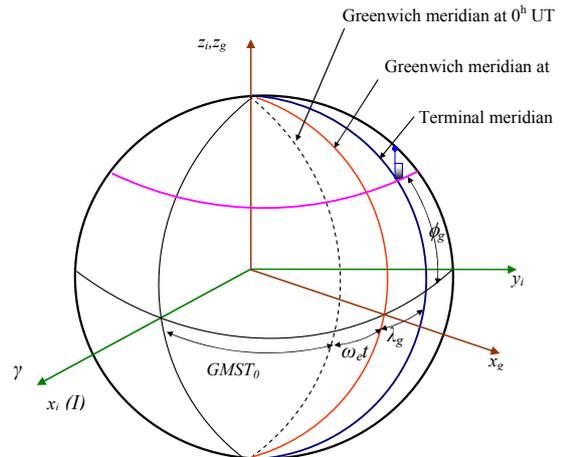


Figure 3. Geometry of the Greenwich meridian

Transformation to Greenwich Coordinates (x_g, y_g, z_g)

To obtain the Greenwich coordinates, the inertial coordinates reference frame is rotated using the GMST (Greenwich Mean Sideral Time) angle which is the angle between the vernal equinox and the Greenwich meridian (figure 3). The Greenwich coordinates (x_g, y_g, z_g) are therefore given by:

$$\begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix} = \begin{bmatrix} \cos GMST & \sin GMST & 0 \\ -\sin GMST & \cos GMST & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \quad (9)$$

At time t , the GMST angle is given by:

$$GMST = GMST_0 + \omega_e t \quad (10)$$

where ω_e is the rate of rotation of earth and $GMST_0$ is the Greenwich mean sideral time at midnight Universal Time (UT). The value of $GMST_0$ in degrees may be computed from the expression:¹

$$GMST_0 = 99,6910 + 36000,7689T + 0,0004T^2 \quad (11)$$

where T is the time elapsed since January 0, 1900, 12^h UT measured in Julian centuries of 36525 days of Universal Time. The time T is given by:

$$T = (JD - 2415020) / 36525 \quad (12)$$

where JD is the present Universal Time time t expressed in Julian Days. To convert the Universal Time to Julian day a simple algorithm is used.⁴

Terminal-Satellite Vector $\vec{\rho}_g$ in Greenwich Coordinates

The terminal-satellite vector $\vec{\rho}_g$ coordinates in the Greenwich system (ρ_{gx} , ρ_{gy} , ρ_{gz}) are obtained by subtracting the terminal coordinates from the satellite coordinates:

$$\begin{cases} \rho_{gx} = x_g - x'_g \\ \rho_{gy} = y_g - y'_g \\ \rho_{gz} = z_g - z'_g \end{cases} \quad (13)$$

The terminal coordinates (x'_g , y'_g , z'_g) are calculated using the geocentric latitude and longitude of the terminal. In the WGS84 model, earth has an equatorial radius R_E of 6378.137 km and a polar radius R_P of 6356,755 km. If φ_{gg} , λ_g and h are respectively the geographic latitude, the longitude and the altitude of the terminal, then the geocentric latitude φ_{gc} and the earth radius r_t at the terminal position are obtained from the expressions:

$$\varphi_{gc} = \arctan\left(\frac{R_P}{R_E} \tan \varphi_{gg}\right) \quad (14)$$

$$r_t = h + R_E R_P / \sqrt{(R_E \sin \varphi_{gc})^2 + (R_P \cos \varphi_{gc})^2} \quad (15)$$

Using equations (14) and (15), the terminal Cartesian coordinates in the Greenwich system are given by:

$$\begin{cases} x'_g = r_t \cos \lambda_g \cos \varphi_{gc} \\ y'_g = r_t \sin \lambda_g \cos \varphi_{gc} \\ z'_g = r_t \sin \varphi_{gc} \end{cases} \quad (16)$$

Terminal-Satellite Vector $\vec{\rho}_h$ in Topocentric Coordinates

The next step is to transform the terminal-satellite vector to the topocentric reference frame. This is achieved by rotating the Greenwich system using the terminal geocentric latitude φ_{gc} and longitude λ_g . Thus the $\vec{\rho}_h$ vector coordinates are given by the following transformation:¹

$$\begin{pmatrix} \rho_{hx} \\ \rho_{hy} \\ \rho_{hz} \end{pmatrix} = \begin{bmatrix} \sin \varphi_{gc} \cos \lambda_g & \sin \varphi_{gc} \sin \lambda_g & -\cos \varphi_{gc} \\ -\sin \lambda_g & \cos \lambda_g & 0 \\ \cos \varphi_{gc} \cos \lambda_g & \cos \varphi_{gc} \sin \lambda_g & \sin \varphi_{gc} \end{bmatrix} \begin{pmatrix} \rho_{gx} \\ \rho_{gy} \\ \rho_{gz} \end{pmatrix} \quad (17)$$

Elevation, Azimuth and Slant Range

Finally, the elevation El , the azimuth Az and the slant range d of the satellite are computed using the terminal-satellite vector coordinates in the topocentric reference frame:

$$El = \arcsin(\rho_{hz} / d) \quad (18)$$

$$Az = \pi - \arctan(\rho_{hy} / \rho_{hx}) \quad (19)$$

$$d = \sqrt{(\rho_{hx}^2 + \rho_{hy}^2 + \rho_{hz}^2)} \quad (20)$$

ORBIT PERTURBATIONS

Gravitational perturbations due to oblateness of earth and to effects of the moon and sun and nongravitational perturbations due to atmospheric drag and to solar radiation cause the orbital parameters to change in time. In short, for Low earth orbits which are higher than 500 Km, the earth oblateness perturbation due to the zonal harmonic coefficient of the earth of second order J_2 is predominant; all the other perturbations may be disregarded.¹ This perturbation mainly causes variation of the RAAN Ω and the argument of perigee ω .

$$d\Omega/dt = -(3/2)J_2 n(R_E/a)^2(1-e^2)^{-2} \cos i \quad (21)$$

$$d\omega/dt = (3/2)J_2 n(R_E/a)^2(1-e^2)^{-2}(2-(5/2)\sin^2 i) \quad (22)$$

In conclusion these perturbations must be taken into account by calculating the actual values of RAAN Ω and argument of perigee ω :

$$\Omega = \Omega_e + (d\Omega/dt)(t-t_e) \quad (23)$$

$$\omega = \omega_e + (d\omega/dt)(t-t_e) \quad (24)$$

where Ω and ω are the RAAN and the argument of perigee at epoch time t_e .

ALGORITHM IMPLEMENTATION

The ground terminal incorporates a 16-bit microcontroller, an OEM (Original Equipment Manufacturer) GPS board and an RTC (Real Time Clock).⁶ Prior to releasing the terminal on the field the satellite ephemeris for the specified orbit at epoch time are loaded via the serial port. The integrated GPS allows provision of the terminal geographic coordinates, thus making it free to move on the whole terrestrial globe without reprogramming its position for local satellite visibility prediction. In case of fixed ground terminal the GPS is optional and the terminal position is loaded via its external serial port.

The time and date input to the terminal are provided by the RTC which is read by the microcontroller. The

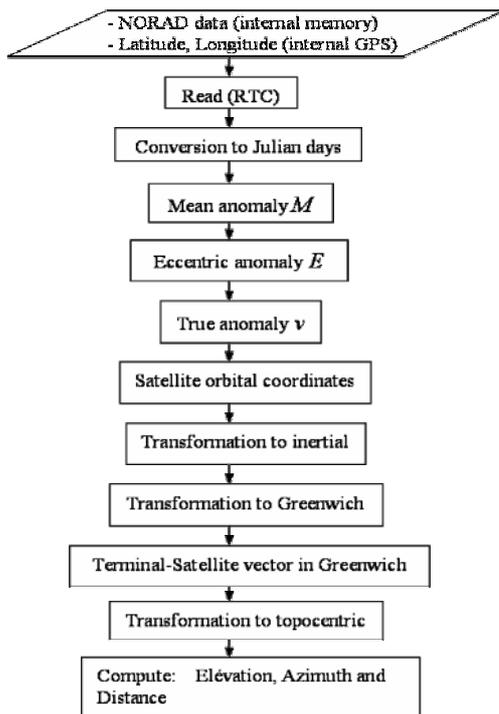


Figure 4. Orbit calculation algorithm

RTC is adjusted using the GPS time each time a position fix is made. Going through the steps of the algorithm illustrated in figure 4, the microcontroller computes the satellite elevation, azimuth and slant range each 10 seconds interval time. Then it switch to communication mode with the satellite when the elevation becomes positive or higher than the preset mask angle to take into account the geographical environment.

Doppler shift is calculated by deriving in time the slant range. The frequencies Doppler correction is operated before each data transmission or reception.

In order to minimise power consumption a cyclic sleep mode is implemented. Each 10 seconds, the microcontroller comes out of the sleep mode and computes the satellite elevation and starts a communication session when the satellite is visible. Otherwise it goes back to sleep mode if no other task like data sampling is scheduled.

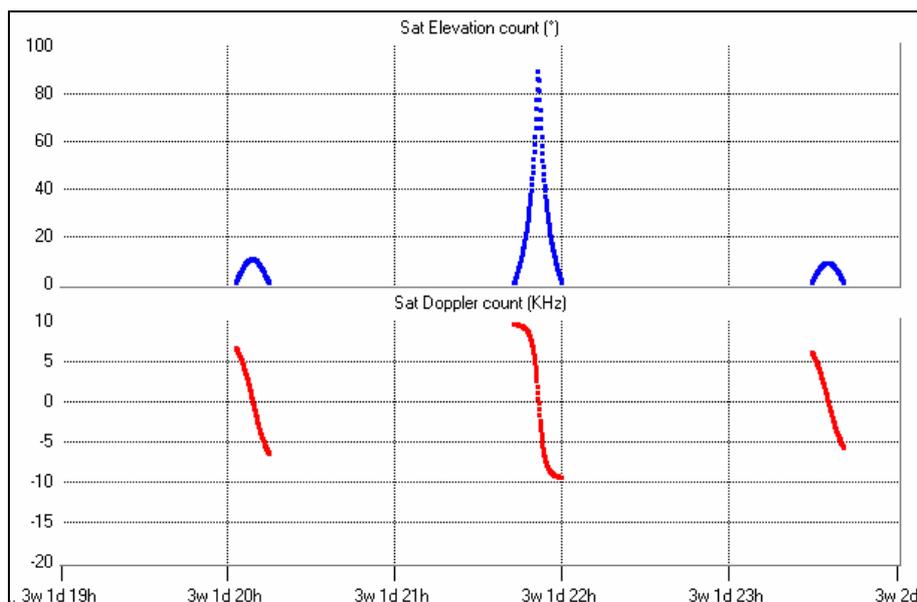
The fact that each terminal in the ground network knows its relative satellite elevation angle allows optimising the network capacity by setting up a certain priority in the satellite channel access. The priority protocol which is elevation based could help solving out the network congestion due to multiple packet collisions on board the satellite.

A floating point unit containing arithmetic and trigonometric operations has also been developed using the standard 32-bits IEEE format numbers (1 sign bit, 8 bits power and 23 bits mantissa). The whole algorithm is coded in assembler providing efficient programming in terms of code size and time.

DOPPLER SHIFT AND COMPENSATION

In the LEO satellite systems, VHF bands are affected with Doppler shift of about 3 KHz whereas it is 10 KHz in the UHF bands. In general it is not necessary to compensate for Doppler in the VHF band although it is better to do it in order to improve the signal to noise ratio. In the UHF band Doppler correction is mandatory when there is no AFC (automatic frequency control) in the transceiver.

In this paragraph, we show a method to compensate

Figure 5. Doppler shift for three consecutive satellite passes. $f = 440\text{MHz}$, $h = 1000\text{Km}$

for Doppler when the transceiver used doesn't integrate AFC which makes it more expensive.

DopplerShift Computation

Doppler shift between a ground terminal and a LEO satellite may be expressed by the simple formula:

$$\Delta f = V_r \cdot f / c \quad (25)$$

during the same satellite pass which is called Doppler slope. Doppler slope is expressed by the first derivative of the Doppler shift and is given with unit Hz/s.⁷

Figure 6 illustrates Doppler slope for two consecutive satellite passes with max elevations of 55 and 18 degrees. The respective maximums of the Doppler slope are 60 and 32 Hz/s. For a satellite pass with elevation reaching 90 degrees, the maximum Doppler

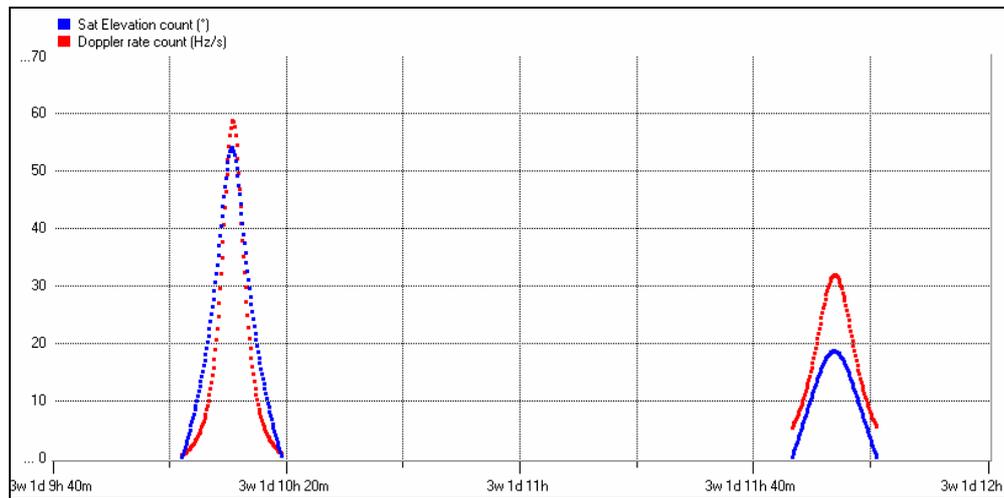


Figure 6. Doppler slope for two consecutive satellite passes. $f = 440\text{MHz}$, $h = 1000\text{Km}$

where, Δf is the Doppler shift, f is the link frequency, c is the speed of light and V_r is the relative velocity of the satellite with respect to the terminal. For simplification the ground terminal speed and the earth rotation are disregarded.

V_r is calculated by the terminal microcontroller by differentiating the slant range d in time:

$$V_r = [d(t + \Delta t) - d(t)] / \Delta t \quad (26)$$

Figure 5 below shows Doppler shift calculated by the ground terminal using 10 seconds steps for three consecutive satellite passes. A maximum of 10 KHz in UHF is denoted. The Doppler is always zero at the max elevation point where the satellite is at its closest approach to the terminal, whereas it is maximum at the zero elevation points.

Doppler Slope Characterisation

Another important feature which impacts the performance of the terminal transceiver with AFC is the ability to follow the variation of Doppler shift

slope is approximately 72 Hz/s.

Doppler Correction Method

Transceivers with ability to lock on the satellite carrier for Doppler correction are more expensive. The simplest way is to calculate the Doppler shift and by means of control signals the microcontroller monitors the transceiver PLL. This method allows using very cheap on the shelf transceivers for ground terminals which are numerous for an operational data collection network. This method also simplifies the satellite payload hardware.

Whenever the terminal wants to transmit or receive a packet, it calculates the Doppler shift and corrects the link frequency. This allows following the Doppler variation during the satellite pass.

Uplink and Downlink Frequencies Correction

In the first half of the visibility pass (from rise time to

Table 1. Doppler correction for the terminal transmit and receive frequencies.

f_t : Tx frequency, f_r : Rx frequency, f_{sat} : sat frequency

	Satellite approaching ($El=0^\circ \rightarrow Emax$) ($\Delta f > 0$)	Max elevation point ($El=Emax$) ($\Delta f = 0$)	Satellite fading ($El=Emax \rightarrow 0^\circ$) ($\Delta f < 0$)
f_e	$f_{sat} - \Delta f $	f_{sat}	$f_{sat} + \Delta f $
f_r	$f_{sat} + \Delta f $	f_{sat}	$f_{sat} - \Delta f $

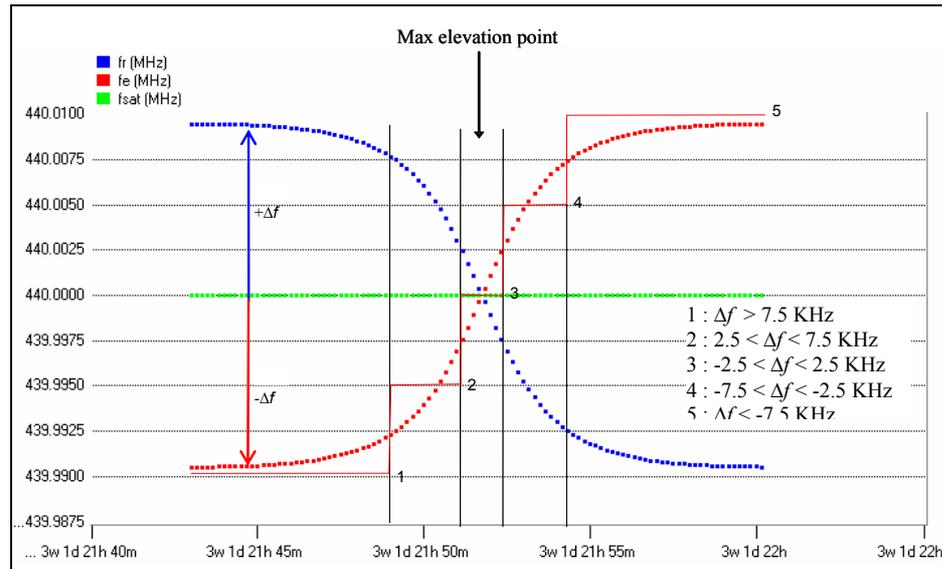


Figure 7. Corrected transmit and receive frequencies during one satellite pass. $f_{sat}=440$ MHz; $\Delta c =5$ KHz

max elevation), the satellite is approaching and Doppler is positive whereas it is negative during the second half. Therefore Doppler correction for the uplink frequency is done by subtracting the Doppler shift Δf during the first half and adding it during the second half. The opposite way is done for the downlink frequency. Thus when using the same frequency for uplink and downlink, the difference between the transmit and the receive frequency is always equal to $2(\Delta f)$. Table 1 summarises the

corrections of the terminal frequencies and figure 7 shows the frequencies variations that the terminal should normally operate during an example satellite pass.

As the transceiver used for the terminal doesn't have AFC, it can't follow exactly all the instantaneous values of the curves in figure 7, but rather do it in an incremental way. The value of the increment should be equal to the channel step (Δc) of the radio.

The curves in figure 7 are drawn choosing an example

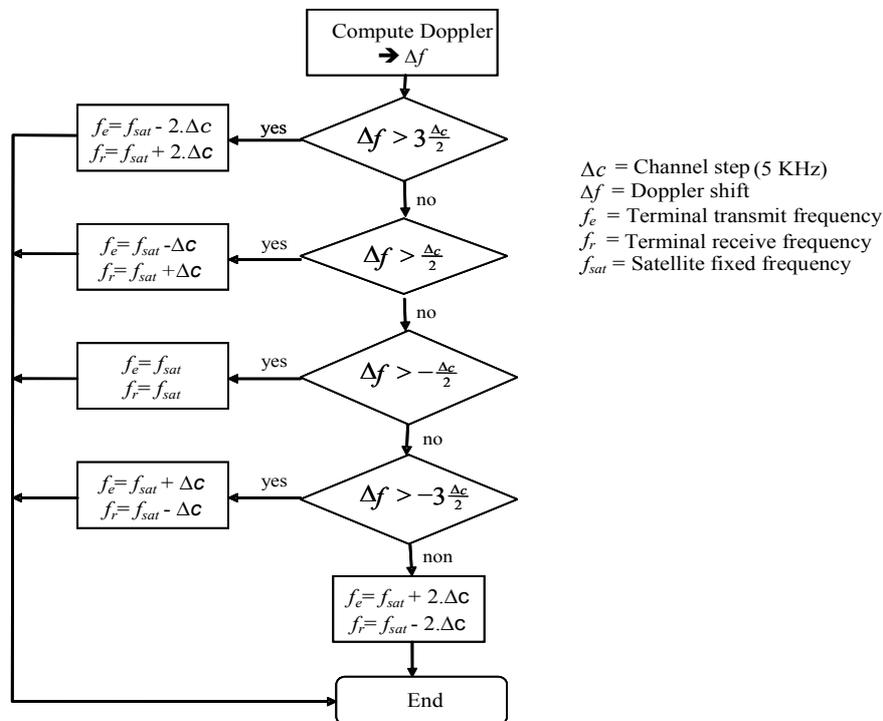


Figure 8. Algorithm for Doppler correction

of satellite frequency equal to 440 MHz for better illustration. However for any frequency the curves may be normalised (divide by 440 and multiply by the new value) as we have the following expression:

$$f = f_{sat} + \Delta f = f_{sat} (1 + V_r / c) \quad (27)$$

$(1 + V_r/c)$ is the normalisation coefficient and depends on the specified orbit, the satellite pass (max elevation reached) and the time or the position of the satellite on the orbit during the specified pass.

Doppler Correction Algorithm Implementation

When the satellite comes into range, the ground terminal starts a communication session to send its waiting packets. Before each transmission or reception the microcontroller computes the Doppler shift and corrects the transmit and the receive frequencies. The correction is made by successive comparisons to multiples of the transceiver channel step as illustrated in figure 8.

CONCLUSION

In opposite to a satellite constellation system, the ground terminals in a single satellite system because of the irregular visibility passes need to handle the transmission times. An orbit calculation and Doppler correction algorithm for mobile or fixed LEO satellite ground terminals has been developed and tested. Embedding a software module for orbit calculation in a ground terminal microcontroller makes it intelligent and low cost along with many other advantages. The terminal without any external PC is lightweight and can operate automatically which is a feature that suits many automatic data collection applications. It is also power efficient as the transceiver is only keyed on when the satellite is in good visibility range and the output power level can be adjusted as the slant range changes. Another important feature is that a certain network capacity optimisation could be achieved using an elevation based channel access protocol. By deriving the slant range in time, Doppler shift is computed and the transceiver frequencies are compensated for by software instead of electronic AFC which makes the transceiver more expensive.

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