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A Monte Carlo Study on the Persistence of Variance with Garch

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A MONTE CARLO STUDY ON THE PERSISTENCE OF VARIANCE WITH GARCH

by

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ABSTRACT

The GARCH model is widely used to forecast volatility for economic and financial Data. There are, however, several shortcomings of using the simple GARCH estimator alone for forecasting volatility. The major issue with the use of the default GARCH model is the persistence of variance that evolves through time and the simple GARCH model fails to address. This paper looks at the GARCH(1,1) model and consistent with Lamoureux and Lastrapes (1990), finds that it overstates the persistence of variance due to model misspecification, specifically the lack of structural shifts.

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PUBLIC ABSTRACT

A Monte Carlo Study on the Persistence of Variance with GARCH models

This paper performs a Monte Carlo study to test Lamoureux and Lastrapes (1990)'s hypothesis on the persistence of variance present in the GARCH volatility model.

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Aristides A. Romero Moreno

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I. INTRODUCTION

Homoskedasticity, an assumption of the classical linear regression model that the variance of the errors is constant is improbable for financial time series data. If the opposite is true, the variance of the errors is nonconstant, also known as heteroskedasticity, the standard errors will be incorrect. Further, for time series data, the volatility tends to cluster over time. That is, there is a correlation between volatility and its level for subsequent periods. Large changes in asset prices are usually followed by large changes and small changes are usually followed by small changes, regardless of the sign. Therefore, it is recommended to assume heteroskedasticity and instead use a model that describes how the variance of the errors changes over time. The Generalized Autoregressive Conditional Heteroskedasticy (GARCH) is one of such models.

The GARCH model is widely used to forecast volatility for economic and financial Data. There are, however, several shortcomings with using the simple GARCH estimator for forecasting volatility. The major issue with the use of the default GARCH model is the persistence of variance that evolves through time, which the simple GARCH model fails to address.[1] The purpose of this paper is to use Lamoureux and Lastrapes (1990) study as an exercise on econometric methods for estimating variance and to study the shortcomings of using the simple GARCH model over long periods of time.

Lamoureux and Lastrapes (1990) used evidence from daily returns – based on the previous day closing prince – on a sample of 30 randomly selected stocks and a stock index

[1] Lauremoureux and Lastrapes (1990), Persistence of Variance, Structural Change, and the GARCH Model. Journal of Business & Economic Statistics, April 1990, Vol. 8, No. 2

(from January 1, 1963 to November 13, 1979) to show that due to deterministic structural shifts, the persistence of variance in the simple GARCH model is overstated. To support this hypothesis, they estimated the generalize variance equation and conduct a Monte Carlo simulation with two scenarios: one that accounts for the structural shifts and one that ignores it. In the generalized equation, Lamoureux and Lastrapes (1990) account for the structural shifts by including 14 dummy variables based following criteria: for observations ranging (1 + 302i) to [(1 + i)302], D_{it}(i= 1, …, 13) is 1, 0 otherwise. For the Monte Carlo study, the dummy variables are arbitrary. In each case, their study found supporting evidence that when discrete shifts in the unconditional variance are not accounted for, the estimates of the persistence of variance in the GARCH model can be misinterpreted – the estimates are overstated.

In this paper, I apply the methods and parameters (with a slight modification) of Lamoureux and Lastrapes (1990) to the daily returns – based on the previous day closing prince –of the the 30 constituents of the Dow Jones Industrial Average[2] from January 3, 2007 to December 31, 2015.[3] I estimate the generalize variance equation and conduct the Monte Carlo simulation with the same two scenarios: one scenario accounts for structural shifts and the other ignores it. For both, the generalized equation and the Monte Carlo Simulation, I rely on the *garchFit* and *ugarch* functions in the *fGarch*[4] and *rugarch*[5] packages in R to account for the structural shifts, which are commonly used in the Economics and Finance Literature.[6] These functions optimize the discrete shifts to better fit the data. I find that the results remain

[2] Constituents of the Dow Jones Industrial Average as of January 10, 2016.
[3] An index is not included because these stocks already compose the index.
[4] Diethelm Wuertz and Yohan Chalabi. 2013. Rmetrics - Autoregressive Conditional Heteroskedastic Modelling. R package version 3010.82.
[5] Alexio Ghalanos. 2014. Univariate GARCH Models, R package version 1.3-5.

consistent. The GARCH model overestimates the persistence of variance due to time model specification, specifically the lack of structural shifts. These structural shifts confound the volatility persistence by diminishing the degree of ARCH parameters in long time series.[7]

An understanding volatility and how to estimate it correctly is pivotal for accurate forecasts of economic and financial data. Volatility, as a measure of dispersion and uncertainty, is widely used as gauge for risk. Arguably, the effects of volatility are more readily observed in the financial markets. Volatility causes wild swings in asset prices and returns; and thus, it is no surprise that volatility is one of the most important factors in asset and option pricing models. Similarly, policy makers observe closely factors that could trigger volatility in the financial markets since it can have spillover effects on the national and global economy, such as the market tumult that followed the devaluation of the Chinese yuan or renminbi on August 24th, 2015. Therefore, learning and examining common issues with one of the most common volatility estimators, GARCH, serves as a comprehensive exercise in time series econometric methods and how to address the persistence of variance that when estimating volatility over long periods of time.

To verify the robustness of these results, this paper is structured as follows: section 2 discusses a review of the literature; section 3 explains the GARCH(1,1) model specification and the restricted and unrestricted models proposed by Lamoureux and Lastrapes (1990); section 4, replicates their study using the constituents of the Dow Jones Industrial Average (DJIA). To avoid the dealing with the 2008-2009 recession this paper focuses on data from 2007 to 2015,

[7] Ibid.

half the time in Lamoureux and Lastrapes (1990) paper. Their study, however, contains data from the 60's and 70's; and thus, this study was due for an update. Section 5, presents the results of the Monte-Carlo study to test the robustness of the empirical results across the two models; the major findings are discussed in section 6; and finally section 7, summarizes the study.

II. LITERATURE REVIEW

The Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Conditional Heteroskedasticy (GARCH) models developed by Engle (1982)[8] and Bollerslev (1986)[9] have grown to be of great importance in the analysis of time series data and volatility estimation. However, this study shows that when applying the GARCH model to time series data that span a significant time period, there will be a "high persistence…. Because of the presence of deterministic shifts in the unconditional model."10 Therefore, if true, it is important for the researcher to pay special attention to these structural shifts when working with a long time series.

Many estimators have been developed that address the persistence in variance issues discussed by Lamoureux and Lastrapes (1990). For instance, Baillie and Bollerslev (1996) worked on the "Fractionally integrated generalized autoregressive conditional heteroskedasticity", in which they claim the persistence of variance decays over time due to the influence of "lagged squared innovations" Bollerslev et al. (1992). This paper assumes that the

[8] Autoregressive Conditional Heteroskedasticity (ARCH).
[9] Generalized Autoregressive Conditional Heteroskedasticity (GARCH)
[10] Lamoureux and Lastrapes: Structural Change and GARCH (1990)

persistence in variance decays over time, but it never fades out and it is also influenced by structural shifts; these results are consistent with this paper's hypothesis on the persistence of variance.

Lamoureux and Lastrapes (1990) attribute the persistence of variance to time-varying factors. To prove their hypothesis, they specify two GARCH(1,1) models: a restricted and an unrestricted model. In the restricted model, they estimate a GARCH(1,1) model without any structural shifts; then, they include 13 dummy variables among various subsamples arbitrarily to account for the structural shifts. They argue the subsamples allow for structural shifts while maintaining sensible statistical inference. Lamoureux and Lastrapes (1990) apply both models to a sample of 30 randomly selected publicly listed companies and test the persistence of variance of each of them. To prove the robustness of their results, they conduct a Monte Carlo Analysis. The results are shown in section 7. Lamoureux and Lastrapes (1990) conclude by maintaining their hypothesis of high persistence of variance for long time series stock return data. Their original paper on this topic in 1990 has been widely discussed in the literature, see Bollerslev et al. (1992); Baillie et al. (1996); Hamilton and Susmel (1994); and Lamoureux and Lastrapes (1990) for a review of the literature.

Lamoureux and Lastrapes (1990) claim that the shocks to the variance using GARCH are persistent due an autoregressive moving average (ARMA) structure that happens to the residuals squared. They acknowledge Engle and Bollerslev (1986)'s paper aiming to address the high persistence of variance, so-called the integrated-GARCH (I-GARCH). Nonetheless, they dismiss it citing that it lacks theoretical motivation.

III. GARCH (1,1) MODEL SPECIFICATION

The ARCH model was first introduced by Engle in 1982 and later generalized by Bollerslev in 1986 as the GARCH model. From this point, the GARCH model has been widely used in its GARCH(1,1) form. The notation (1,1), also known as ARCH terms, indicates the number of autoregressive lags and the moving average lags, respectively.

The GARCH model considered in this paper has the following form:

$$y_t = x_t\beta + \epsilon_t, \tag{1}$$

$$(\epsilon_t|\epsilon_{t-1}, \epsilon_{t-2}, \dots) \sim N(0, h_t), \tag{2}$$

and

$$h_t = \omega + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + \cdots \alpha_p\epsilon_{t-p}^2 + \gamma_1 h_{t-1} + \cdots + \gamma_q h_{t-q}, \tag{3}$$

where x_t is an exogenous variable and ϵ_t is a random error, and p=q=1.

The GARCH(*p, q*) model in the previous three equations permits the conditional variance of the error term to depend in a linear way with previous squared errors. Lamoureux and Lastrapes (1990) argue that using the above model implies that the squared residuals are generated by an ARMA process. While there are time series methods, developed by Bollerslev

(1996) for instance, to find the optimal values of p and q, the GARCH(1,1) model has become the default for most financial times series.

Using their convention the GARCH(1,1) model becomes:

$$h_t = \omega + \lambda h_{t-1} + \alpha v_{t-1}, \tag{4}$$

$v_{t-1} \equiv \epsilon_{t-1}^2 - h_{t-1}$ is a serially uncorrelated innovation

$\lambda \equiv (\alpha + \gamma)$, measures the persistence of variance

Then by eliminating the h_{t-1} in the right side of equation (4), the expression for the conditional variance is,

$$h_t = \sigma^2 + \alpha[v_{t-1} + \lambda v_{t-2} + \lambda^2 v_{t-3} + \lambda^3 v_{t-4} + \cdots], \tag{5}$$

$\sigma^2 \equiv \omega/[1 - \lambda]$, is the unconditional variance

From this (5), the dependence of the persistence of volatility shocks (v_t) on the sum of the GARCH parameters, λ, becomes obvious. For, $\lambda = 1$, the process is said to be an I-GARCH process (Engle and Bollerslev 1986), or integrated in variance. This means that the lag polynomial of conditional variance has a unit root process. In this particular case, shocks to the variance do not decrease over time and the unconditional variance does not exist, Lamoureux and Lastrapes (1990).

According to Lamoureux and Lastrapes (1990), the sum of GARCH parameters approaches one, generally, when using daily stock data spanning long periods of time. They argue that these estimates are overstated because they do not account for structural shifts over time. This papers seeks to replicate and verify findings.

They claim there is a model misspecification when using the simple GARCH model and propose to include dummy variables for estimating the general model. Their specification follows,

$$h_t = \omega' + \delta_1 D_1 + \cdots + \delta_k D_{kt} + \lambda h_{t-1} + \alpha v_{t-1} + \alpha v_{t-1}, \tag{6}$$

$D_{it}(i = 1, \ldots., k)$ are dummy variables for periods where the GARCH is stationary.

Equation (6) specifies *k+1* periods. This paper estimates equation (4), the restricted model, and then compares it to equation (6), unrestricted model.

One of the most important points to address is how to choose the timing of the structural shifts. Given the available econometric tools available, this paper used a method that optimizes the number of dummy variables. The packages *fGarch*[11] and *rugarch*[12] in R provide an algorithm for these purposes. Therefore, this paper departs slightly from Lamoureux and Lastrapes (1990) and instead of building an algorithm from scratch, this paper relies on the *garchFit* and *ugarch* functions in the *fGarch* and *rugarch* packages in R. While dummy variables are not reported, the function accounts for these time-varying factors.

Test statistics are calculated taking into consideration the non-normality of the standard errors. Additionally, robust standard errors are used for every estimate of the parameters.

[11] Diethelm Wuertz and Yohan Chalabi. 2013. Rmetrics - Autoregressive Conditional Heteroskedastic Modelling. R package version 3010.82.
[12] Alexio Ghalanos. 2014. Univariate GARCH Models, R package version 1.3-5.

The test conducted determines whether or not the persistence of variance, λ, in both the restricted and unrestricted models is equal against the alternative that there is less persistence of variance in the unrestricted model. The immediate test that comes to mind to test the hypothesis is a likelihood ratio test. Nonetheless, the R functions used limit the linear hypothesis testing.

IV. GARCH (1,1): APPLICATION TO DOW JONES (DJIA) CONSTITUENTS

Table 1 shows the components of the stocks in the Dow Jones Industrial Average. The information presented from these 30 stocks was obtained from the Center for Research in Security Prices (CRISP) at the Wharton Research Data Services data base. The information in table 1 contains the permanent number assigned in CRISP, the stock symbol (ticker), and CUSIP.

For each stock this paper estimates equation (4), the restricted model, and compares it with equation (6), the unrestricted model. The returns used in the models are the arithmetic returns obtained from CRISP from January 03, 2007 to December 15, 2015.

Table 1. Companies in the DJIA[13], used in estimation of the GARCH model

PERMNO	Ticker	Company Name	CUSIP
10107	MSFT	MICROSOFT CORP	59491810
11308	KO	COCA COLA CO	19121610
11703	DD	DU PONT E I DE NEMOURS & CO	26353410
11850	XOM	EXXON MOBIL CORP	30231G10
12060	GE	GENERAL ELECTRIC CO	36960410
12490	IBM	INTERNATIONAL BUSINESS MACHS C(	45920010

[13] Dow Jones Industrial Average (DJIA), stocks that are component of the Index as of January 24, 2016.

PERMNO	Ticker	Company Name	CUSIP
14541	CVX	CHEVRON CORP NEW	16676410
14593	AAPL	APPLE INC	3783310
17830	UTX	UNITED TECHNOLOGIES CORP	91301710
18163	PG	PROCTER & GAMBLE CO	74271810
18542	CAT	CATERPILLAR INC	14912310
19561	BA	BOEING CO	9702310
21936	PFE	PFIZER INC	71708110
22111	JNJ	JOHNSON & JOHNSON	47816010
22592	MMM	3M CO	88579Y10
22752	MRK	MERCK & CO INC NEW	58933Y10
26403	DIS	DISNEY WALT CO	25468710
43449	MCD	MCDONALDS CORP	58013510
47896	JPM	JPMORGAN CHASE & CO	46625H10
55976	WMT	WAL MART STORES INC	93114210
57665	NKE	NIKE INC	65410610
59176	AXP	AMERICAN EXPRESS CO	2581610
59328	INTC	INTEL CORP	45814010
59459	TRV	TRAVELERS COMPANIES INC	8.9417E14
65875	VZ	VERIZON COMMUNICATIONS INC	92343V10
66181	HD	HOME DEPOT INC	43707610
70519	C	CITIGROUP INC	17296742
76076	CSCO	CISCO SYSTEMS INC	17275R10
86868	GS	GOLDMAN SACHS GROUP INC	38141G10
92611	V	VISA INC	92826C83
92655	UNH	UNITEDHEALTH GROUP INC	91324P10

The parameters α and γ are optimized using the package *rugarch* in R[14]. The *ugarch* function in the *rugarch* package is widely used in the Economics and Finance literature to maximize the likelihood function. To allow for a direct comparison, Figure 1 presents Lamoureux and Lastrapes (1990) results. The results in table 2 are consistent with Lamoureux and Lastrapes (1990), see figure 1. The estimated models show an average persistence of variance of 0.991, as measured by $\lambda = \alpha + \gamma$. The half life (HL) of the volatility of shocks, which depends only on λ, is computed to provide some perspective on the persistence of

[14] Alexio Ghalanos. 2014. Univariate GARCH Models, R package version 1.3-5

variance and measures the number of days a shock to volatility decays to half its original size, $HL = 1 - [\frac{\log(2)}{\log(\lambda)}]$.

Unlike Lamoureux and Lastrapes (1990) this paper does not perform a bootstrap estimation due to computing limitation. The purpose of this study is to understand volatility and how to estimate and test it using Monte Carlo simulations. It is now widely accepted in the literature that structural breaks are needed in the GARCH model. Therefore, since the results of this study show the same statistical results than Lamoureux Lamoureux and Lastrapes (1990), it relies on their bootstrap to claim statistical difference.

For every stock in the Dow Jones Industrial Average (DJIA) the persistence of variance is overstated in the restricted model. All but 6 parameters for alpha are not significant at the 5% level, which would theoretically lower the estimated persistence of value calculation. Thus, the persistence of variance is not the same regardless of structural shifts. These results are consistent with Lamoureux and Lastrapes (1990). Due to the lack of time-varying structural shifts, the restricted GARCH(1,1) model overstates the persistence of variance.

V. MONTE CARLO STUDY

To check the robustness of our results, a Monte Carlo study is conducted. The objective is to replicate the results obtained by Lamoureux and Lastrapes on the nature of the misspecification bias by controlling for extraneous factors.

I simulate the GARCH(1,1) process with the following parameters:

$\beta_0 = -0.15, \beta_1 = 0.01, \alpha = 0.15, \gamma = 0.45, \epsilon_0 = 0$. The dummy variables are applied automatically by the *fGarch* function in R, thus, no specification was used (to Lamoureux and Lastrapes specified the dummies as $\delta_1 = 20, \delta_2 = -10$ in the first environment and $\delta_1 = -10, \delta_2 = -15$ in the second environment). Since the dummy variables are applied automatically, only one environment is considered, as opposed to Lamoureux and Lastrapes (1990) who considered two environments.

To simulate equations (4), (the restricted model) and (6), (the unrestricted model) , one thousand simulations are performed in one environment.

Figure 1. Lamoureux and Lastrapes (1990): Persistence in Variance for 30 stocks: Restricted and Unrestricted GARCH(1,1) Specifications

Company	Restricted model α	γ	λ	HL	5% CV	Unrestricted model α	γ	λ	HL
1	.087	.884	.971	24.55	.942	.123	.654	.777	3.75
2	.053	.935	.988	58.41	.959	.055	.884	.939	12.01
3	.042	.950	.992	87.30	.963	.071	.814	.885	6.67
4	.050	.922	.972	25.41	.943	.063	.817	.880	6.42
5	.035	.950	.985	46.86	.956	.105	.570	.675	2.76
6	.067	.904	.971	24.55	.942	.084	.785	.869	5.94
7	.063	.917	.980	35.31	.951	.084	.817	.901	7.65
8	.047	.944	.991	77.67	.962	.063	.809	.872	6.06
9	.071	.908	.979	33.66	.950	.086	.811	.897	7.38
10	.034	.963	.997	231.70	.968	.138	.550	.688	2.85
11	.025	.970	.995	139.28	.966	.036	.887	.923	9.65
12	.042	.952	.994	116.18	.965	.050	.905	.955	16.05
13	.050	.944	.994	116.18	.965	.080	.691	.771	3.67
14	.093	.897	.990	69.97	.961	.200	.521	.721	3.12
15	.061	.926	.987	53.97	.958	.126	.664	.790	3.94
16	.076	.862	.938	11.83	.909	.147	.494	.641	2.56
17	.059	.936	.995	139.28	.966	.126	.681	.807	4.23
18	.056	.908	.964	19.91	.935	.098	.489	.587	2.30
19	.047	.945	.992	87.30	.963	.103	.775	.878	6.33
20	.114	.898	1.012	—[a]	.983	.234	.453	.687	2.85
21	.081	.881	.962	18.89	.933	.121	.719	.840	4.98
22	.050	.945	.995	139.28	.966	.078	.809	.887	6.78
23	.025	.967	.992	87.30	.963	.022	.959	.981	37.13
24	.037	.954	.991	77.67	.962	.041	.926	.967	21.66
25	.056	.922	.978	32.16	.949	.119	.601	.720	3.11
26	.062	.934	.996	173.94	.967	.082	.847	.929	10.41
27	.152	.740	.892	7.06	.863	.169	.582	.751	3.42
28	.077	.907	.984	43.97	.955	.120	.664	.784	3.85
29	.073	.899	.972	25.41	.943	.111	.670	.781	3.80
30	.127	.742	.869	5.94	.840	.152	.572	.724	3.15
Mean	.064	.914	.978	32.16[b]		.103	.714	.817	4.43[b]
Index	.082	.907	.989	63.67	.960	.080	.878	.958	17.15

NOTE: HL is the half-life of variance; HL = 1 − [(log 2)/log λ]. The bootstrap 5% critical value is denoted by 5% CV.
[a] Because λ exceeded 1 in this case, HL cannot be interpreted. Note that HL approaches infinity as λ approaches 1, so persistence should be considered large for this company.
[b] The mean HL is obtained by applying the half-life formula to the mean of λ.

Table 2. Persistence of Variance for DJIA Stocks, GARCH (1,1) Specifications

Ticker	Restricted Model α	γ	λ	HL	Unrestricted Model α	γ	λ	HL
MSFT	0.0065**	0.9925**	0.999	692.79	0.0491**	0.9331**	0.9821	38.4014
KO	0.000**	0.999**	0.999	692.79	0.1003**	0.8646**	0.9649	19.4142
DD	0.0040	0.9949**	0.9989	678.42	0.1458**	0.8264**	0.9723	24.6503
XOM	0.0214**	0.9775**	0.999	692.77	0.0886**	0.8946**	0.9834	41.0651
GE	0.0062**	0.9927**	0.9989	668.38	0.1133**	0.8770**	0.9903	71.0160
IBM	0.0067**	0.9923**	0.9989	673.52	0.1189**	0.8272**	0.9461	12.5147

CVX	0.0094**	0.9895**	0.9989	687.99	0.0940**	0.8945**	0.9886	60.3837
AAPL	0.0002**	0.9990**	0.9989	682.04	0.0869**	0.8848**	0.9717	24.1705
UTX	0.0006	0.9989**	0.9989	687.33	0.0622**	0.9231**	0.9853	46.6983
PG	0.0109**	0.9880**	0.9989	692.60	0.1344**	0.7885**	0.9198	8.2972
CAT	0.0018	0.9971**	0.9989	690.59	0.0641**	0.9258**	0.9899	68.3394
BA	0.0036**	0.9953**	0.9989	688.77	0.073**	0.9114**	0.9850	45.8952
PFE	0.000	0.999**	0.999	692.79	0.0642	0.9239**	0.9881	58.0231
JNJ	0.0029	0.9961**	0.9989	692.65	0.1472**	0.8030**	0.9502	13.5829
MMM	0.0054**	0.9936**	0.9989	692.73	0.0455*	0.9400**	0.9855	47.5918
MRK	0.0084**	0.9906**	0.9989	692.67	0.1115**	0.8584**	0.9699	22.6782
DIS	0.0001**	0.9989**	0.9989	691.08	0.0891**	0.8770**	0.9661	20.1290
MCD	0.0208**	0.9781	0.9989	692.71	0.0427	0.9469**	0.9896	66.2615
JPM	0.0042**	0.9947**	0.9989	664.07	0.1161**	0.8766**	0.9927	95.4391
WMT	0.0050**	0.9939**	0.9989	692.70	0.0396	0.9532**	0.9928	96.1595
NKE	0.0151**	0.9838	0.9989	691.63	0.0836**	0.8711**	0.9548	14.9734
AXP	0.0053**	0.9937**	0.9989	684.96	0.0804*	0.9118**	0.9922	88.7583
INTC	0.0131**	0.9859**	0.9989	692.60	0.0509**	0.9309**	0.9819	37.8702
TRV	0.0103**	0.9887**	0.9989	691.64	0.0925*	0.8967**	0.9892	64.1202
VZ	0.0092**	0.9897**	0.9989	689.44	0.0820**	0.8933**	0.9753	27.7615
HD	0.0176**	0.9814	0.9989	691.26	0.1111	0.8656**	0.9767	29.4520
CSCO	0.0173**	0.9816**	0.9989	672.69	0.0778**	0.8558**	0.9336	10.0996
GS	0.0099**	0.9890**	0.9989	689.98	0.0793	0.9170**	0.9963	189.634
V	0.0072**	0.9917**	0.9989	642.52	0.1526**	0.8137**	0.9664	20.2818
UNH	0.0098**	0.9891**	0.9989	688.36	0.0582	0.9347**	0.9929	97.2206
Index	0.0111	0.9877*	0.9988	590.29	0.1193**	0.8656**	0.9848	45.4866

Notes: **, * indicate 0.01 (t-stat >1.960), 0.05 (t-stat> 2.576) significance levels, respectively. *t*-statistics correspond to robust standard errors computed yet omitted due to space limitations. Please see code in the Appendix for replication. HL is the half life of the variance: HL = 1 – [(log 2)/log λ. The half life was computed using the *halflife* function in R. Rows may not add up due to rounding.

I found that, generally, the results are consistent with Lamoureux and Lastrapes (1990), see figure 2. The persistence of variance measured by $\alpha + \gamma = 0.581$, unrestricted model, against the true value $\lambda = 0.6$. Accounting for structural shifts in the unrestricted model prevents overstating the variance of stock returns in long time series. These results contrast sharply with the restricted model.

In the restricted model λ is largely overstated with a value of roughly 0.683. In each parameter in the simulation, in the restricted model overstates the true value, particularly α and γ

as a measure of persistence of variance. These results are consistent with the hypothesis that including structural shifts in the model helps in not overstating the persistence of variance.

Figure 2. Lamoureux and Lastrapes (1990): Monte Carlo Simulation Results

	Unrestricted model		Restricted model		
	Mean	Standard deviation	Mean	Standard deviation	True value
			Environment 1		
β_0	−.162	.224	−.163	.229	−.15
	(.218)	(.009)	(.221)	(.010)	
β_1	.010	.039	.009	.036	.01
	(.038)	(.001)	(.039)	(.002)	
ω'	21.346	8.541	1.615	.674	20.00
	(8.388)	(4.520)	(1.890)	(.537)	
α	.146	.049	.102	.022	.15
	(.050)	(.008)	(.051)	(.008)	
γ	.422	.187	.872	.026	.45
	(.182)	(.090)	(.078)	(.013)	
δ_1	21.601	10.005	—	—	20.00
	(9.128)	(5.278)			
δ_2	−10.629	4.905	—	—	−10.00
	(4.877)	(2.472)			
			Environment 2		
β_0	−.148	.158	−.150	.161	−.15
	(.156)	(.006)	(.160)	(.007)	
β_1	.089	.039	.009	.035	−.01
	(.039)	(.001)	(.040)	(.002)	
ω'	21.909	8.873	.649	.280	20.00
	(8.939)	(3.949)	(.964)	(.289)	
α	.149	.050	.086	.019	.15
	(.049)	(.008)	(.054)	(.009)	
γ	.409	.195	.893	.023	.45
	(.194)	(.082)	(.082)	(.014)	
δ_1	−10.831	5.084	—	—	−10.00
	(5.121)	(2.125)			
δ_2	−16.404	6.885	—	—	−15.00
	(6.950)	(3.067)			

NOTE: Means and standard deviations are calculated across 1,000 simulations. The numbers in parentheses are the summary statistics calculated for the estimated coefficient standard deviations.

Table 3. Monte Carlo Simulation for Garch(1,1) process

Restricted Model			Unrestricted Model		
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	**True Value**
β_0	-0.189	0.147	-0.1867	0.14	-0.15
β_1	0.062	0.257	0.021	0.158	0.01
γ	0.527	0.021	0.429	0.065	0.45
ω'	25.42	5.848	17.657	0.257	20
α	0.156	0.047	0.152	0.026	0.15

Means and Standard Deviations are computed from 1,000 simulations. Dummies are not reported, but are included in the computation using the *fGarch* package and *garchFit* function in R.

VI. DISCUSSION OF THE RESULTS

Given the results obtained in section 3 and 4, the initial hypothesis is maintained. The persistence of variance is overstated if time-varying structural shifts are not included in the model. In table 2, demonstrates that the results obtained by Lamoureux and Lastrapes in 1990 still hold today. Date form the 30 components of the Dow Jones show a very large persistence of variance, which this paper argues is overstated. The Monte Carlo simulation supports this hypothesis.

Although a slightly different approach was used to account for dummy variables, the results are essentially the same. This approach, due to the size of our data set, is more appropriate today. Furthermore, the results from the Monte Carlo simulation show that even after controlling for extraneous factors the results remain robust. The simulation parameters in the unrestricted model closely follow the true values, while the parameters in the restricted model are overstated.

VII. CONCLUSION

This study reaffirms the hypothesis tested by Lamoureux and Lastrapes (1992). When dealing with time series data for stock prices spanning for a long time, the persistence of variance estimated by the regular GARCH(1,1) model is overstated due to the lack of structural shifts misspecification. This paper used market data and performed a Monte Carlo analysis to show consistency with the hypothesis.

This study served as an exercise that aimed at using advanced econometric models and the use of Monte Carlo simulation methods to estimate the persistence of variance in stock prices. The results were successful, not only in replicating Lamoureux and Lastrapes (1990) study, but also applying today's method for GARCH estimation and obtaining similar results. Further research should focus on reviewing and applying recent GARCH estimators that address the structural misspecification and its practical use for forecasting economic and financial data.

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APPENDIX

Title: R CODE FOR MONTECARLO SIMULATIONS

MODEL SPECIFICATION FOR DOW JONES INDUSTRIAL AVERAGE COMPONENTS DATA

```
#Set Directory
#setwd("~/Documents/FALL 2015/Independent Research")

#Unrestricted Model
require(fGarch)
## Read CSV file (header assumed), then put that into "csv.data" data object (any name is ok).
xData = read.csv("masterData.csv")

#Select Stock from DataSet
#returns= subset(xData, ticker=="V")
rets.vec = as.numeric(data.matrix(returns[,4], rownames.force = NA))

#Include dummy interation
rets.timeSeries = dummyDailySeries(matrix(rets.vec), units = "GARCH11")

require(rugarch)
#### Specify a standard GARCH(1,1) model with mean-equation being a constant.
specU = ugarchspec(variance.model=list(model="sGARCH"),
        mean.model=list(armaOrder=c(0,0)))
### Estimmation command
fitU=ugarchfit(spec=specU, data=rets.timeSeries)
```

```
show(fitU) #Print the model

persistence(fitU) #Compute the persistence of variance

halflife(fitU) #Compute the half-life

coef(fitU) #Print coefficients
```

MODEL SPECIFICATION FOR MONTE CARLO STUDY IN R

Package: *fGarch*

```
# A numeric Vector from default GARCH(1,1)

N = 1000

x.vec = as.vector(garchSim(garchSpec(model=list(mu=-0.15, omega=.45, alpha=0.15,
    gamma=.45, beta=0.01, cond.dist=c("norm"))), n = N)[,1])

garchFit(~ garch(1,1), data = x.vec, trace = TRUE)


# An univariate timeSeries object with dummy dates:

x.timeSeries = dummyDailySeries(matrix(x.vec), units = "GARCH11")

garchFit(~ garch(1,1), data = x.timeSeries, trace = FALSE)
```