

Decision Support for Optimal Regional Groundwater Management Strategy Modification

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ABSTRACT

AN interactive decision-support program is presented for the rapid modification of optimal regional multiobjective groundwater planning strategies. This capability is important for water managers seeking to select the most satisfactory groundwater management strategies for their areas. The program guides decision maker(s) in refining numerically optimal regional strategies into strategies that may be socially or politically more acceptable. Strategy refinements are made by informed modification of constraining conditions on regional objectives or local variables. Application is illustrated by modifying a bicriterion, sustained groundwater withdrawal strategy for minimizing the cost of meeting regional water demand on the Arkansas Grand Prairie, an important irrigated area. The strategy was developed using a model in which the finite difference form of the two-dimensional groundwater flow equation is embedded in an optimization process. Results from the formal optimization process are submitted to the interactive program for evaluation and modification. This algorithm applies the constraint method and constrained derivatives to develop noninferior solutions and tradeoff functions and to determine the influence on the regional objectives caused by repeated changes in several local decision variables. Although its application is demonstrated with only a single optimization model, the interactive program can be utilized to modify optimal strategies resulting from other models as well.

INTRODUCTION

In many areas, irrigated agricultural production is dependent on the availability of groundwater. Groundwater withdrawal by one user affects groundwater availability for other users. Hence, there is increasing emphasis on the development of appropriate strategies for managing groundwater on regional or sub-state scales.

The development of a regional groundwater management strategy often includes the application of optimization to determine the allocation plan that most

effectively satisfies a desired objective. The three major elements of many optimization formulations are the objective function, the variables and the constraints. An objective function is a statement of a desired regional goal. In a finite difference scheme, such as the one used in this paper, variables may be groundwater used or groundwater levels in each cell. The constraints in the optimization problem represent criteria which the variables must satisfy and which affect attainment of the regional objectives.

Within the complex arrangement of legislative, sociologic, and economic goals influencing water resources management, it is difficult, if not impossible, to optimize achievement of a single objective without adversely affecting other regional objectives. When conflicting objectives exist in the same problem, no single solution is available in which all goals are optimally attained. However, through the application of generating techniques (Cohon and Marks, 1975) a noninferior set of solutions can be created. This solution set is also referred to as a "nondominated" set, the "Pareto Optimum", the "transformation curve" or the "efficiency" curve.

A feasible solution is noninferior if no other feasible solution exists that will cause one objective to improve without forcing at least one other objective to degrade (Cohon 1978). At each noninferior solution, the relationship between competing goals is expressed in terms of a tradeoff function. The tradeoff function describes the amount of one objective that must be sacrificed in order to improve attainment of another objective. Consideration of tradeoff values is essential in designing strategies that best satisfy multiple regional goals.

It is also practically impossible to develop an optimal regional strategy without harming attainment of the 'local' goals of the individual cells. Regional objectives are frequently a maximization or minimization of the aggregate effects on groupings of cells within the region. This utilitarian approach provides for regional optimization at the expense of local development. As bounds on local variables change, the achievable optimum value of an objective function may also change. Dual values, LaGrange multipliers, shadow prices or constrained derivatives all describe the relative effect of changes in local decision variables on attainment of regional objectives. Knowledge of how much local changes affect optimality is important in determining how much a regionally optimal strategy should be modified in order to better achieve local goals.

An automated method for designing regional water management strategies should incorporate representation of the complex interaction between attainment of regional objectives and local satisfaction via tradeoff values and constrained derivatives. In

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addition, because several decision makers (DMs) are usually involved in selecting a water resources management strategy, the method should be as rapid and interactive as possible.

Interactive techniques have been used in the past to improve the coordination of subjective DMs with an objective numerical process (Monarchi et al., 1973; Haimes and Hall, 1974). The term 'interactive', however, reflects varying degrees of speed and efficiency.

Datta and Peralta (1986) present a interactive computer graphics-based program that aids in the selection of a water management strategy in a bi-objective problem. The program assists multiple DMs in zeroing in on the most satisfactory compromise strategy that exists within a particular Pareto Optimum. The selection of the compromise strategy is accomplished using the Surrogate Worth Trade-off (SWT) method (Haimes and Hall, 1974) through interactive query and response from a minicomputer. The shortcoming of the program is that only strategies on a predetermined Pareto Optimum can be evaluated. No new strategies that might become feasible by changing bounds or constraints on 'local' variables can be analyzed. Their program therefore cannot rapidly modify a regionally optimal strategy in order to better consider needs on the cell level. After discussing the results of using the program with a group of commissioners of a state water agency, Datta and Peralta (1986) emphasize the importance of truly interactive decision-making.

Opposing interests and ideas cannot be ignored in the development of optimal strategies for actual implementation. Truly there is a need for the ability to rapidly modify the constraining conditions on objectives or local variables and determine the resulting effect on other regional objectives. DMs need to be able to rapidly: (a) select a compromise regional strategy by facile movement through the decision space defined by multiple objectives, (b) evaluate, in map format, the spatial distribution of the consequences of strategy implementation (This is particularly important to elected decision makers that need to protect constituent interests.), and (c) modify the compromise regional strategy to reflect local concerns by changing the bounds on decision variables.

The purpose of this paper is to describe a program that provides these abilities. In order to accomplish this, we first utilize quadratic parametric programming techniques in an interactive manner to develop a noninferior solution set and tradeoff functions. (We do not discuss the manner of selecting a compromise strategy from a noninferior solution set since it has been described previously (Datta and Peralta, 1986).) Then we demonstrate how this program may be used to rapidly determine the effect on the compromise solution due to repeated changes in any number of decision variables or constraints. In addition, optional graphic products which aid the strategy design process are displayed.

As a developmental step in the Grand Prairie Water Supply Project, (Peralta et al., 1984), the interactive method is demonstrated, in this paper, through application to the bicriterion problem of developing a conjunctive use, sustained yield pumping strategy for the Grand Prairie region of Southeast Arkansas. Opposing objective functions considered in this example include a linear function to maximize regional groundwater withdrawal and a quadratic expression to minimize the

total cost of supplying regional water demand. These objective functions are simultaneously evaluated within the same framework of physical and institutional constraints.

These two regional goals are contradictory because the surface water network proposed by the Corps of Engineers does not supply surface water to all areas of the Grand Prairie. Consequently, areas not serviced by the surface water network must rely on groundwater resources alone to fulfill irrigation needs. Pumping groundwater in areas where surface water is not available, may "force" use of surface water (where available) in lieu of groundwater, even if it costs more than groundwater.

Groundwater flow is simulated by applying the finite difference form of the two-dimensional steady-state groundwater flow equation, (Pinder and Bredehoeft, 1968) as part of the constraining conditions in the optimization model. This technique of including simulation equations within an optimization model is called the embedding method (Gorelick, 1983). Utilization of steady-state equations is a quasi-black box approach which relies on the premise that implementation of approximately steady stimuli (pumping and recharge) will ultimately cause a stable response in the groundwater potentiometric surface. While not as sophisticated as response matrix methods of optimizing groundwater management, the approach does have the advantages of causing the evolution of stable water levels and of requiring less computer storage. It is also more practical than using the embedding approach with unsteady flow equations. Gorelick (1983), Tung and Koltermann (1985) and Casola et al (1986) report either numeric difficulties or unwieldiness with embedding unsteady flow equations as constraints.

In the illustrative example, local variables subject to management constraint include drawdown, pumping, and recharge in each finite difference cell. (Some considerations for limiting these variables are listed by Bear (1979).) In this paper drawdown is defined as the difference in elevation between a horizontal datum, located above the ground, and the potentiometric surface. Groundwater pumping is the volume of groundwater removed from the system by wells penetrating the aquifer. Recharge is the volume of water entering the groundwater system from outside the region. Because of an impermeable layer, recharge at internal cells is insignificant. The net sum of pumping and recharge in each cell is referred to as excitation.

OBJECTIVES FOR THE GRAND PRAIRIE

The quadratic objective function applied in the example, estimates the cost of maintaining a sustained yield by minimizing the cost of both groundwater and surface water required to satisfy regional demand. A complete derivation of this objective function and the factors involved is presented by Peralta and Killian (1985). For the purposes of this paper the following general representation is satisfactory.

$$\text{minimize } z_1 = \sum_{i=1}^N c_c(i) p(i) f(s(i)) + c_m(i) p(i) + c_a(i) p_a(i) \dots \dots \dots [1]$$

where:

- z_1 = the total annual cost of water supply, \$/year
- N = The total number of finite difference cells in which drawdown and pumping are variable
- $c_e(i)$ = the cost associated with raising a unit volume of groundwater one unit distance, \$/L⁴
- $p(i)$ = the positive valued annual volume of groundwater pumped from cell i , L³/year
- $f(s(i))$ = a linear function of drawdown which describes the total dynamic head at cell i , L
- $c_m(i)$ = the cost associated with a unit volume of groundwater pumped, \$/L³
- $c_a(i)$ = the cost per unit volume of alternative water supplied in cell i , \$/L³
- $p_a(i)$ = the annual volume of alternative water use at cell i , L³/year.

Because water requirements of each cell are satisfied by the conjunctive use of groundwater and an alternative water source, the following relationship is used to replace $p_a(i)$ in equation [1].

$$p_a(i) = w(i) - p(i) \quad \text{for } i=1, N \dots \dots \dots [2]$$

where:

- $w(i)$ = the annual water requirements in cell i , L³/year.

The linear objective function used to maximize regional groundwater pumping is similar to the formulation used by Aguado and others (1974), Alley and others (1976), and Elango and Rouve (1980). This is described as follows:

$$\text{maximize } z_2 = \sum_{i=1}^N p(i) \dots \dots \dots [3]$$

where:

- z_2 = the total volume of groundwater annually withdrawn from the region, L³/year.

The problem consisting of both objective functions is a two dimensional vector within a solution space of dimension $2N + M$, where M is the total number of constant head cells. The following notation is used to describe this situation:

$$\text{optimize } z = \{ z_1, z_2 \} \dots \dots \dots [4]$$

Because it is not possible to maximize or minimize this problem without either prior knowledge or numerical representation of management preference, the term "optimize", as it appears in equation [4], refers to defining the set of noninferior solutions.

The regional goals expressed by the objective functions are dependent on the drawdown, pumping, and recharge in each finite difference cell. Each of these local variables is limited by an upper and lower bound. The bounds on these variables delineate the feasible region, or solution space. The feasible region for the bicriterion example problem is defined by the following constraints.

$$p(i) = \sum_{j=1}^K (-t(i,j) s(j)) \quad \text{for } i=1, N \dots \dots \dots [5]$$

$$r(m) = \sum_{j=1}^K (-t(m,j) s(j)) \quad \text{for } m=1, M \dots \dots \dots [6]$$

$$s_{\min}(i) \leq s(i) \leq s_{\max}(i) \quad \text{for } i=1, N \dots \dots \dots [7]$$

$$P_{\min}(i) \leq p(i) \leq P_{\max}(i) \quad \text{for } i=1, N \dots \dots \dots [8]$$

$$r_{\min}(m) \leq r(m) \leq r_{\max}(m) \quad \text{for } m=1, M \dots \dots \dots [9]$$

where

$$t(i,i) = \sum_{j=1, j \neq i}^K (-t(i,j))$$

- $t(i, j)$ = The geometric mean transmissivity between finite difference cell i and cell j , for $i \neq j$, L²/year
- $s(j)$ = the drawdown in finite difference cell j , L
- K = the total number of cells in the study area, also the total number of inequality constraints, $K = N + M$
- M = the total number of constant head cells in the region
- $s_{\min}(i)$ = the lower limit on drawdown in cell i , L
- $s_{\max}(i)$ = the upper limit on drawdown in cell i , L
- $P_{\min}(i)$ = the lower limit on annual groundwater pumping in cell i , L³/year
- $P_{\max}(i)$ = the upper limit on annual groundwater pumping in cell i , L³/year
- $r(m)$ = the annual recharge at constant head cell m , (positive implies discharge, negative means recharge) L³/year
- $r_{\min}(m)$ = the lower limit on annual recharge in constant head cell m , L³/year
- $r_{\max}(m)$ = the upper limit annual recharge in constant head cell m , L³/year.

Equality constraint [6] describes the 'recharge', necessary to achieve mass balance, which occurs in the constant head cells. The lower bound used at a particular constant head cell, $r_{\min}(m)$, represents the maximum recharge volume that can physically occur at that cell without causing the assumed constant head elevation to drop. Using the bound implies that as long as the recharge is less negative than $r_{\min}(m)$, one can validly treat cell m as a constant head cell. The upper bound, $r_{\max}(m)$, if non-positive assures that no groundwater will leave the region at this point. In application of the management model, the upper limit on r was typically set equal to a positive value of large magnitude such that there was no restriction on the annual volume of water which could leave the system at constant head cells, under steady-state conditions.

Equality constraints [5] and [6] are substituted into the objective functions and constraints [8] and [9] such that the only explicitly defined variable is drawdown. Pumping and recharge are defined in terms of the slack variables associated with constraints [8] and [9], respectively.

THEORY

Generation Techniques

The method used in this paper to generate the noninferior solution set is referred to by Cohon and

Marks (1975) as the constraint method. Under the constraint method, all but one objective become additional constraints. The single, or principal objective is optimized by conventional methods while the constrained objectives are limited by a chosen value. The selection of a principal objective does not indicate management preference.

To construct the noninferior solution set, the limiting value for a particular unconstrained objective is varied and the principal objective optimized at each new point. This is generally defined by the following formulation.

$$\min/\max z_p = f(x) \dots \dots \dots [10]$$

subject to:

$$z_h \geq L_h \quad \text{For } h=1, H \dots \dots \dots [11]$$

where:

- z_p = value of the principal objective function
- z_h = value of objective constraint h
- L_h = the limiting value of objective constraint h
- H = total number of objective constraints.

For example, the linear objective function, equation [3], becomes an objective constraint and the problem description is represented in the operational form:

$$\text{minimize } z_1 = g(s) \dots \dots \dots [12]$$

Subject to the conditions of the feasible region as previously defined by equations [5-9], and the following additional condition.

$$z_2 \geq L_2 \dots \dots \dots [13]$$

where:

- $g(s)$ = equation [1] expressed in terms of drawdown alone;
- L_2 = the minimum allowable total groundwater annually withdrawn from the aquifer underlying the region, L^3/year .

At the value of L_2 , a new value of z_1 is computed. Within the feasible region of the solution space, the objective constraint will be binding. Therefore, a noninferior solution exists as a set of N drawdown values, at which z_2 is equal to L_2 .

The values of L_2 represent the minimum allowable regional pumping imposed by a management decision. The range of L_2 for which the objectives will be conflicting and the corresponding range of regional cost values are defined by the following limits.

$$z_2 \text{ at } \min z_1 \leq L_2 \leq \max z_2 \dots \dots \dots [14]$$

for:

$$\min z_1 \leq z_1 \leq z_1 \text{ at } \max z_2$$

for values of L_2 less than z_2 at $\min z_1$, the constrained objective and the principal objective are not in opposition, the objective constraint is not binding and the value of z_1 resulting from the optimization is equal to $\min z_1$.

A systematic approach to developing the noninferior solution set varies the value of L_2 from one extreme to the

other, covering the entire range in a predetermined number of steps. By using a controlled interactive method, only areas of the solution set which are of particular interest to the decision makers need be examined. Thus, by ignoring areas of the region which are of little concern, such as the extreme ends of the feasible range, each decision maker can accurately pinpoint his best-compromise solution with minimal computational effort. By using a differential algorithm in this interactive procedure, tradeoff functions for each regional objective and each local decision variable are readily available.

General Differential Algorithm

The General Differential Algorithm, developed by Wilde and Beightler (1967) and discussed in detail by Morel-Seytoux (1972), is a direct climbing method of locating the optimal solution through a systematic gradient search routine. The interactive technique presented in this paper uses an extension of the General Differential Algorithm to evaluate the change in the value of the principal objective function and the system response resulting from a change in the optimal solution set.

To aid in the explanation of the General Differential Algorithm consider the minimization of a quadratic objective function with N variables subject to K inequality constraints. During any iteration in the search process, the problem will consist of K equations and $N+K$ variables, (K of these variables are slack variables introduced to transform the inequality constraints into equality conditions). The constraining equations are linear and K variables can be expressed as a function of N independent variables. N independent variables are initially referred to as decision variables while K dependent variables are referred to as solution or state variables. The specific separation of variables into state variables and decision variables is known as the partition of the system.

The functional equivalents of the state variables are directly substituted into the objective function such that the objective function is an unconstrained expression of N decision variables and no state variables. During each iteration in the optimization process, one decision variable is changed to improve the value of the objective function. A change in any decision variable will cause every state variable related by the K equality conditions to change.

Because the objective function is expressed in terms of drawdown alone in the example problem, a decision variable is either a drawdown variable, or a slack variable corresponding to one of the inequality conditions described by constraints [8], [9], and [13]. At the optimum, all decision variables that are limited by a binding constraint are associated with a non-zero constrained derivative. Assuming a minimization process, if a decision variable is against an upper limit, the related constrained derivative must be negative. A decision variable has a positive constrained derivative associated with it if the lower limit is binding. If the value of a decision variable is not equal to a limiting condition, the corresponding constrained derivative is zero and any change in the decision variable does not improve the value of the objective function. This is simply a non-dogmatic explanation of the Kuhn-Tucker conditions.

Constrained Derivatives

The change in the value of the unconstrained form of the principal objective function, for a given change in a particular decision variable, is expressed in terms of the gradient of the unconstrained objective function. The gradient of the objective function is the vector of first partial derivatives with respect to the decision variables. Each first partial derivative is referred to as a constrained derivative. ("Constrained" derivative implies that the constraining conditions have been substituted into the objective function.) The constrained derivative describes the direction and magnitude of a change in the value of the objective function for an instantaneous change in the value of the decision variable.

Because the objective function described in this application is a quadratic expression, each constrained derivative of the objective function is a linear function of decision variables. Thus, for a change in the value of a single decision variable, the values of all related constrained derivatives also change. The change in the value of each constrained derivative is determined by evaluating the vector of second partial derivatives of the objective function with respect to the decision variables. For a quadratic objective function, this is a vector of constant terms. The change in the constrained derivatives of the principal objective function for a change in decision variable *i* is described in terms of the second partial derivatives as follows.

$$\Delta v(j) = b(j,i) \Delta x_d(i) \quad \text{for } j = 1, N \dots \dots \dots [15]$$

and *i* = 1, N

where:

- $\Delta v(j)$ = the change in the value of the constrained derivative of *j* on the objective function
- $\Delta x_d(i)$ = the specific change in the decision variable *i*, or the difference between $x'_d(i)$ and $x_d(i)$
- $b(j, i)$ = the second partial derivative of z_p , taken first with respect to decision variable *j* and again with respect to decision variable *i*
- $x'_d(i)$ = the new value of decision variable *i*
- $x_d(i)$ = the value of decision variable *i*, prior to increasing or decreasing the value.

Utilizing equation [15], the change in the value of the objective function for a change in one decision variable can be expressed in terms of both the first order and second order partial derivatives.

$$d z_p / d x_d(i) = v(i) + b(i,i) \Delta x_d(i) \dots \dots \dots [16]$$

for *i*=1, N

where:

- $v(i)$ = the constrained derivative of z_p with respect to decision variable $x_d(i)$;
- $b(i, i)$ = the second partial derivative of z_p with respect to decision variable $x_d(i)$.

For a specific change in a decision variable the above equation is integrated over $\Delta x_d(i)$ to yield

$$\Delta z_p = \left\{ v(i) + 0.5 b(i,i) (\Delta x_d(i)) \right\} (\Delta x_d(i)) \dots \dots [17a]$$

for *i*=1, N

where:

- Δz_p = the change in the value of the principal objective function;

For a specific change in the decision variable associated with an objective constraint, equation [17b] describes the tradeoff function.

$$\Delta z_p = \left\{ v(h) + 0.5 b(h,h) (\Delta x_d(h)) \right\} (\Delta x_d(h)) \dots [17b]$$

for *h*=1, H

Equations [15], [16], [17a] and [17b] are valid when the change in the decision variable does not cause a repartitioning of system variables. This limitation is discussed in detail in a subsequent section.

The change in all system variables in response to a change in the value of a single decision variable is referred to as the system response. Because all decision variables are independent, a change to one decision variable will not affect the value of the remaining decision variables. Every state variable however, is expressed as a function of decision variables and is therefore affected. By evaluating the gradients of the state variables, the change to the state variables in response to a change in the value of a single decision variable is determined.

In the example, the constraints are linear and the resultant state gradients are vectors of constants. Therefore, the first partial of a state variable with respect to each decision variable is valid for any arbitrary change in a single decision variable, not merely an incremental change. The system response to a change in the value of a single decision variable is represented by the following formulation.

$$\Delta x_s(k) = d(k,i) \Delta x_d(i) \quad \text{for } k=1, K \dots \dots \dots [18]$$

- $\Delta x_s(k)$ = the change in state variable *k*
- $d(k, i)$ = the first partial derivative of state variable *k* with respect to decision variable *i*.

The partial derivatives of the state variables, $d(k, i)$, are revised each time the system variables are re-partitioned.

The concepts described indicate how the value of the principal objective function and the system variables change for a given change in a single decision variable. These methods are applied in the development of the interactive procedure.

THE INTERACTIVE PROCEDURE

The bicriterion examples problem is formulated as it appears in equation [12] and [13] with L_2 set equal to any feasible value of total regional pumping. This problem is initially solved by a quadratic programming procedure written by Leifsson and others (1981) which uses the General Differential Algorithm to determine the optimal solution. The optimal set of N drawdown values, N pumping values, and M recharge values that result from the initial optimization represent one noninferior solution. These values, along with the values of the first and second order partial derivatives are transferred to a separate program for interactive evaluation.

In constrained optimization, decision variables are usually tight variables with non-zero constrained derivatives. To modify the original noninferior solution, any decision variable may be changed by modifying its upper or lower bound to expand or reduce the size of the solution space. To some extent, changing the bound forces the decision variable to assume the value of the new bound when the problem is optimized under the revised conditions.

Moving Through the Noninferior Solution Set

To generate the set of noninferior solutions, several changes to the binding limit, L_2 , of the objective constraint are input, consecutively, to the interactive program. This modifies the value of the slack variable associated with constraint [13]. The system response to each change is determined by equation [18] and the new value of the principal objective function is determined by equation [17b]. The values of the constrained derivatives are revised by equation [15] and the system is checked for optimality. If the solution is not numerically optimal, the interactive program performs the interactions necessary to make the solution noninferior.

At any point in the noninferior solution set, the relationship between regional objectives is described by the constrained derivative of the principal objective function with respect to the decision variable associated with each objective constraint. Once a favorable relationship is achieved and a compromise solution agreed upon, the resulting values of all local variables may be examined.

In examining the local variables, a group of decision makers may identify areas at which the variable values of drawdown, pumping, or recharge are socially or otherwise unsatisfactory. To refine the compromise strategy and address local concerns, the interactive program is utilized as explained in the following section.

Local Influence on Regional Objectives

At a noninferior solution, each local variable is either a state variable, or a decision variable. The constrained derivative of the principal objective function with respect to a state variable is zero, indicating the independence between the principal objective function and the state variables. A change to a local condition represented by a state variable may be made by changing a decision variable, (or several decision variables), such that the desired effect on the particular state variable, described by equation [18], is achieved. To change the value of a decision variable representing drawdown, pumping or recharge, the binding limit is appropriately changed.

A change in the bound on a local decision variable changes the feasible region of the solution space common to both the principal objective and the objective constraints. Depending on the extent of the change, the noninferior solution that exists prior to changing a local bound is not necessarily optimal after the bound has been re-established. In other words, the solution may become inferior. At an inferior solution, one objective can be changed without adversely affecting the other objectives. Using the interactive procedure, the decision makers may choose the regional dimension in which to move such that the solution becomes noninferior. That is, the decision must be made as to what regional objective to improve.

Equation [16a] is used to determine the change in the principal objective function resulting from a specific change in the value of a decision variable. In making this change the objective constraints remain fixed and a new solution set results. At the new solution, the change in the value of an objective constraint, needed to insure that the principal objective retains its original value, may be calculated by solving equation [16b] for $\Delta x_d(h)$. This value is then used as input to the interactive program such that the original value of the objective function is obtained.

Conditions Under Which the Procedure may be Utilized

To change the value of a decision variable, the limiting bound is replaced with a value that either expands or reduces the size of the solution space. This effectively creates a new problem. Depending on the extent of the change to the bound, the new problem may require subsequent iterations to achieve optimality.

The solution that exists prior to changing the bound (the old optimal solution) is the starting point for the new problem and must be feasible within the new solution space. If a change in a bound increases the size of the solution space (if the upper limit is increased or the lower limit is decreased) the old solution is always a feasible starting point. If however, the solution space is reduced (a lower bound is increased or an upper bound is decreased) the extent of the change to the bound on a decision variable is limited by feasibility criteria. A reduction in the size of the solution space that causes the old optimal solution to be infeasible within the new solution space is not permitted with the interactive procedure.

The magnitude of the feasible change is determined by the constraints imposed on the variables involved. A decision variable is allowed to increase or decrease until it, or another variable, encounters a limiting condition. Since the bound on the decision variable itself is dictated by the user, the feasible positive and negative deviation is controlled by the first state variable to reach its upper or lower limit. The value of the feasible deviation is found by solving equation [18] for Δx_d with $\Delta x_d(i)$ defined as the difference between the state variable and its approaching bound.

If the change in the bound on a decision variable is within, or equal to the feasible deviation, the corresponding change in the value of the decision variable is equal to the change in the bound. The constraint remains tight, and the system response is feasible, though not necessarily optimal.

Optimality is affected if a single decision variable is changed such that application of equation [16] causes one of the constrained derivatives to change signs. The maximum absolute change in the value of a decision variable such that none of the non-zero constrained derivatives change sign is referred to as the optimal deviation. To change sign, a constrained derivative must first change from a positive or negative value, to zero. The optimal deviation is determined by applying equation [15] with $\Delta v(j)$ as the difference between the value of the constrained derivative and zero. If the change in the bound on a decision variable is within both the optimal deviation and the feasible deviation, the change in the value of the decision variable is equal to the change in the bound and the resulting strategy is optimal.

The bound on a decision variable can be changed in excess of the feasible and optimal deviation if the change increases the size of the feasible region. In such a case, a state variable reaches its bound and the initial change in the decision variable is less than the input change in the bound. A re-partitioning of the variables is performed such that the state variable becomes a decision variable and the decision becomes a state variable. Additional iterations may be necessary to make the feasible solution optimal as well.

In summary: (a) the interactive process may be used to modify an existing strategy when a change in the limiting

bound on any decision variable decreases the size of the solution space, if the change to the bound is within the feasible deviation determined through the use of the constrained derivatives; (b) the interactive modifications method may not be used to change a bound in excess of the feasible deviation if the change decreases the size of the solution space; (c) the method can analyze any arbitrary change in the limiting bound on a decision variable if the change increases the size of the solution space. When the change in the solution space exceeds the optimal deviation, additional iterations are necessary if the optimal result is desired. These iterations are performed by the interactive program by utilizing the same subroutines developed for the optimization process; (d) the procedure is also applicable to other optimization models as long as they have linear or quadratic objective functions and linear constraints.

APPLICATION AND DISCUSSION

Site Description

The quadratic and linear objective functions for minimizing total cost and maximizing total regional groundwater withdrawal are applied in the multiobjective format to the Grand Prairie of eastern Arkansas. Fig. 1 shows the Grand Prairie subdivided into 204 finite difference cells. Of the 204 total cells, 52 are constant head cells used to simulate conditions along the periphery of the study area. Of the 204 inequality constraints, 152 are pumping constraints (equation [5]) and 52 are recharge constraints applied to the constant head cells (equation [6]). The total number of variables,

including slack variables is 356: 152 decision variables and 204 state variables.

The Grand Prairie is an extensively cultivated and irrigated agricultural area and one of the prime rice producing regions of the country (Griffis, 1972). A heavy layer of clay underlies the topsoil and inhibits infiltration from recharging the aquifer. The only apparent sources of recharge are the rivers which border the area and extensions of the aquifer outside the study area. Extensive pumping and limited recharge has resulted in a declining water table and water shortages in this Quaternary aquifer.

Aquifer characteristics utilized for simulation are those used by Peralta and others (1985). These data include elevation of the potentiometric surface and top and base of the aquifer, (used in determining the saturated thickness), and a hydraulic conductivity of 82 m/day, (270 ft/day). The potentiometric surface is depressed in the central Grand Prairie. Recharge enters the area through its periphery. Because of this obvious stress, it is assumed that the maximum physically feasible recharge rates for the peripheral cells (equation [9]), have been observed and quantified (using Darcy's Law).

The drawdown and pumping in the non-constant head cells are bounded by an upper and a lower limit. The lower limit on drawdown represents the average elevation of the ground surface in each cell. The upper limit on drawdown is such that 6 m (20 ft) of saturated thickness is guaranteed in each cell. The lower limit on pumping is zero (to prevent physically unrealistic internal recharge from being computed) and the upper limit on pumping is equal to the current average annual groundwater withdrawals. The variable recharge in constant head cells is limited such that maximum annual observed recharge from outside the system is never exceeded.

Cost coefficients used in the quadratic objective function are estimated from information received from the U.S. Army Corps of Engineers, (personal communication with Joe Clements, Dwight Smith, and Stony Burke). In areas where no surface water is available for use as an alternative source, the opportunity cost associated with reduced production is used as the alternative water cost.

The matrix of second partial derivatives in the least-cost objective function, equation [1], consists of groundwater cost coefficients and transmissivity values. Before optimization, this Hessian matrix was examined and found to be positive-definite, thus insuring that the resulting solution is the global optimum.

Noninferior Solution Set

Fig. 2 displays the resulting set of noninferior solutions interactively generated as outlined previously. Shown with every exact noninferior solution is the corresponding tradeoff function expressed by the first order partial derivatives in units of dollars per cubic decameter. Although the total range defined by equation [14] is presented in Fig. 2, in actual practice it is not necessary to produce the entire set of solutions.

From the noninferior solution set, the best-compromise solution may be determined by implementing the surrogate worth tradeoff method introduced by Haimes and Hall (1974). For illustrative purposes, solution set A is chosen as a compromise

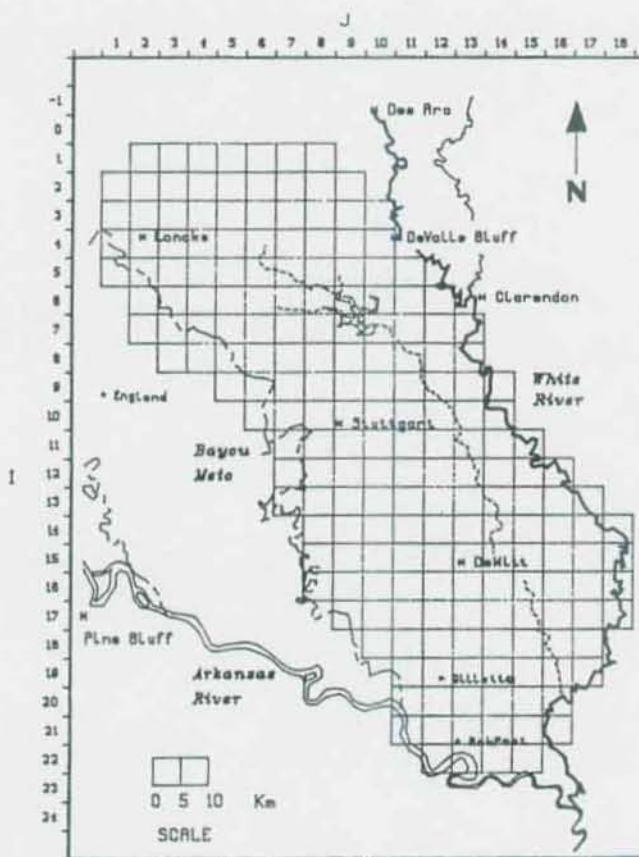


Fig. 1—The Grand Prairie study area subdivided into finite difference cells.

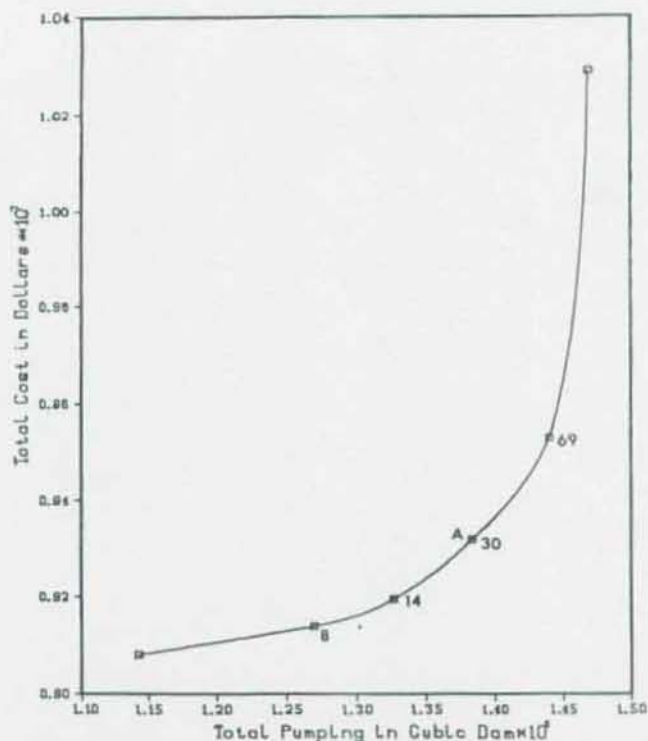


Fig. 2—The noninferior solution set and tradeoff functions in dollars per cubic dekameter.

solution, though not necessarily the best compromise solution. For solution A, the total annual regional groundwater pumping is maintained at 138,000 dam³ (112,000 acre ft). The total regional cost of the conjunctive use strategy is 9.3 million dollars, including opportunity cost and cost of groundwater and diverted river water.

Local Change

At the compromise solution, the local groundwater pumping in cell (3,4), (see Fig. 1 for row, column location coordinates), is equal to its lower limit, which is 0.0. In other words, for the benefit of the region as a whole, no groundwater withdrawal is permitted at this cell and in fact, no water needs are satisfied. Assuming that a group of decision makers wish to improve the equity of the compromise solution to groundwater users in cell (3,4), the lower limit on groundwater pumping in cell (3,4) is increased, and the regional effect analyzed.

The constrained derivative for the pumping in cell (3,4) is \$32/dam³ (\$40/acre ft). For every cubic dekameter increase in groundwater pumping in cell (3,4), the regional cost increases by \$32. Because the second partial derivative of the objective function with respect to the pumping is a positive \$0.008/dam³/dam³ (\$0.012/acre ft/acre ft) the constrained derivative, (\$32/dam³), will increase as the local pumping increases.

The most that pumping can be increased in cell (3,4) and still maintain feasibility is 237 dam³, (192 acre ft), at which point the pumping in cell (5,5) reaches its lower limit. Because the change will reduce the size of the solution space, the limit of 237 dam³ must be recognized. If the desired increase in the pumping at cell (3,4) is greater than 237 dam³, the original problem must be reformulated and submitted for execution using a standard optimization procedure.

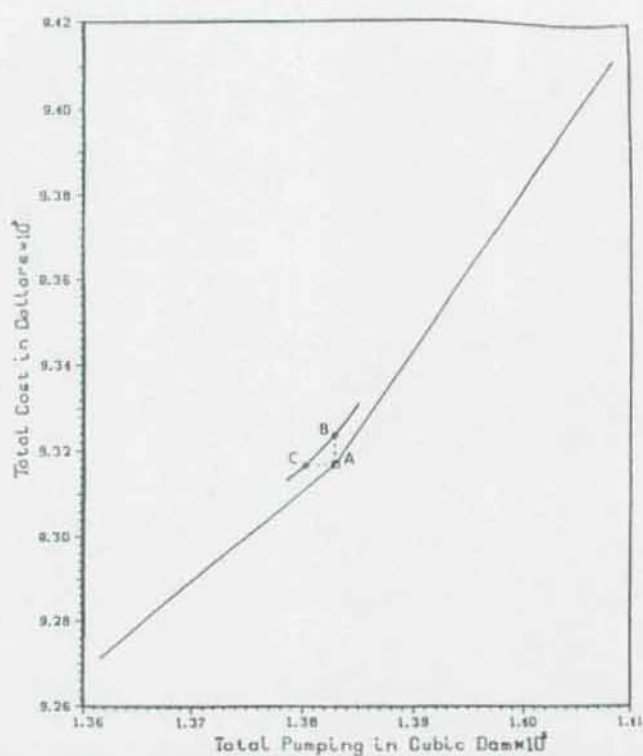


Fig. 3—The noninferior solution set in the vicinity of the compromise solution.

Assume that the decision makers agree to increase pumping in cell (3,4) by 227 dam³ (184 acre ft). In accordance with equation [17a], the modification causes the total regional cost to increase by \$7,730. The change of 227 dam³ also causes the values of some of the constrained derivatives to change sign, thus making the solution inferior. The interactive program requires five subsequent iterations to calculate the optimal solution. At the revised optimum, the increase in total regional cost is \$7,400 and the pumping in cell (3,4) is 227 dam³.

This new noninferior solution is point B on Fig. 3, an enlarged section of Fig. 2 in the vicinity of the compromise solution. At point B, the total regional pumping is still 138,00 dam³ but the cost is \$7,400 greater than the cost of solution point A.

The decision makers may also want to know how the total regional pumping of strategy A is affected by a local increase of 227 dam³ in cell (3,4), if the total cost remains constant. At point B, the constrained derivative of the principal objective with respect to the constrained objective, (the instantaneous tradeoff function), is \$30/dam³ (\$37 dollars/acre ft), and the corresponding second partial derivative is \$0.002/dam³/dam³. (\$0.003/acre ft/acre ft). Solving equation (17b) for Δx_d with Δz_p equal to -7,400 dollars results in a reduction in total regional pumping of 250 cubic dekameters (202 acre ft). Because this increase in the size of the feasible region is less than the maximum feasible deviation, the first and second partial derivatives remain valid. This means that in order to increase groundwater availability at cell (3,4) from 0 to 227 dam³, while maintaining total regional cost at 9.3 million dollars, a total of 477 dam³ of groundwater must be forsaken in all remaining cells. Implementing this change results in the noninferior solution indicated by point C in Fig. 3.

At point C, the total cost is the original 9.3 million

ENTER VARIABLE YOU WANT TO CHANGE

.p 3 4

THE CHANGE IN THE VALUE OF THE OBJECTIVE FUNCTION FOR AN INSTANTANEOUS CHANGE IN THIS DECISION VARIABLE IS...

32.30 \$/Cu Dm

THE CURRENT VALUE OF THIS VARIABLE IS LIMITED BY ITS LOWER BOUND. TO IMPROVE THE VALUE OF THE O.F. YOU SHOULD DECREASE THIS BOUND.

THE CURRENT PUMPING IN THIS CELL IS 0.0 Cu Dm

THE VALUE OF THE SECOND PARTIAL DERIVATIVE IS...

0.0076 \$/Cu Dm /Cu Dm

DO YOU WANT TO CHANGE THIS VARIABLE?

.y

DO YOU WANT TO (I)INCREASE OR (D)DECREASE THIS VARIABLE?

.i

VALUE OF OBJECTIVE FUNCTION WILL INCREASE. THE MOST THIS VARIABLE WILL INCREASE IS 237.15 Cu Dm

ENTER THE AMOUNT OF CHANGE IN THE BOUND.

.220.0

SYSTEM RESPONSE TO THIS CHANGE IS BEING DETERMINED. RESULTS ARE FEASIBLE BUT NOT OPTIMAL. DO YOU WANT TO OPTIMIZE THE NEW SITUATION? (Y)ES OR (N)O.

.y

RESULTS ARE OPTIMAL AS WELL AS FEASIBLE. NUMBER OF ITERATIONS= 5

TOTAL COST = .0324515E+07

ENTER VARIABLE YOU WANT TO CHANGE

.n 1

THE CHANGE IN THE VALUE OF THE OBJECTIVE FUNCTION FOR AN INSTANTANEOUS CHANGE IN THIS DECISION VARIABLE IS...

29.84 \$/Cu Dm

THE CURRENT VALUE OF THIS VARIABLE IS LIMITED BY ITS LOWER BOUND. TO IMPROVE THE VALUE OF THE O.F. YOU SHOULD DECREASE THIS BOUND.

THE CURRENT VALUE IS 112971. Cu Dm

THE VALUE OF THE SECOND PARTIAL DERIVATIVE IS...

0.0010 \$/Cu Dm /Cu Dm

DO YOU WANT TO CHANGE THIS VARIABLE?

.y

DO YOU WANT TO (I)INCREASE OR (D)DECREASE THIS VARIABLE?

.d

VALUE OF OBJECTIVE FUNCTION WILL DECREASE. THE MOST THIS VARIABLE WILL DECREASE IS -840.30 Cu Dm

ENTER THE AMOUNT OF CHANGE IN THE BOUND.

-.250.0

SYSTEM RESPONSE TO THIS CHANGE IS BEING DETERMINED. RESULTS ARE OPTIMAL AS WELL AS FEASIBLE. NUMBER OF ITERATIONS= 1

TOTAL COST = .031711E+07

Fig. 4—Output from a sample interactive session.

dollars, but the total regional pumping has decreased by 250 dam³. The curve connecting points B and C indicates a portion of the set of noninferior solutions for the new solution space. At the point on the revised curve, the minimum amount of groundwater pumping at cell (3,4) is 227 dam³. Fig. 4 is a copy of the output from the interactive session used to locate points B and C on Fig. 3.

The extension of the noninferior solution set in a local dimension is possible at any compromise solution with any decision variable. Therefore, for the 152 decision variables in this example, the total number of possible decision directions, including the two regional dimensions, is 154.

Decision makers can review a variety of information in gridded (planar map) form to aid decision-making while using the interactive program. This includes spatially distributed information on water levels, aquifer

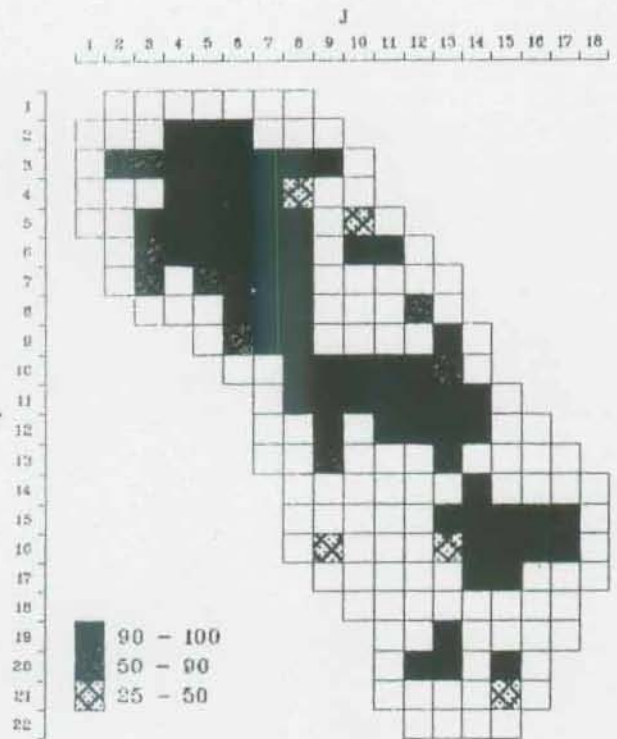


Fig. 5—Percentage of water needs that are unsatisfied in a particular strategy.

saturated thickness, water needs, supplied water, unsatisfied needs and recharge. For example, Figs. 5 and 6, enhanced images of a color graphics display, show data useful for refining an optimal strategy. To date, most data is reviewed in gridded numeric form not requiring graphic processing to get the most rapid response possible.

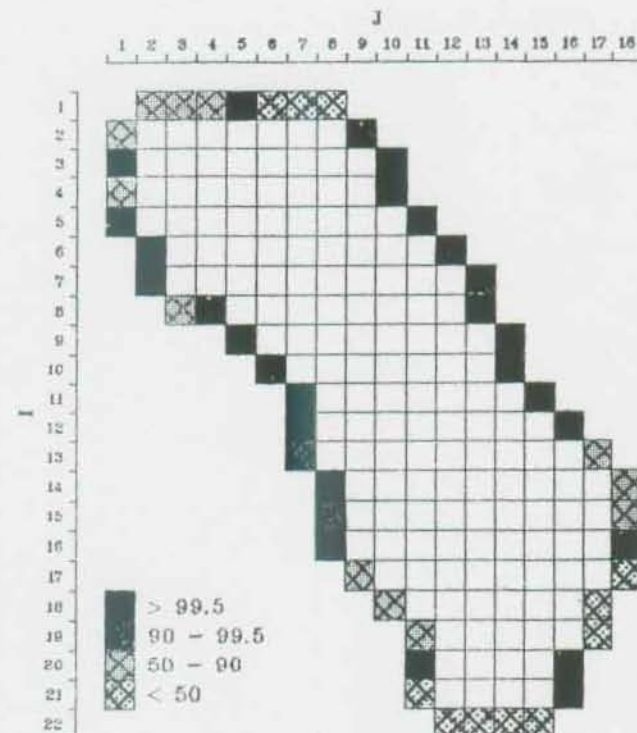


Fig. 6—Percentage of maximum physically feasible recharge that is induced under steady-state conditions for a particular strategy.

Fig. 5 shows the percentage of unsatisfied water needs that would exist in each cell of the Grand Prairie under a particular optimal strategy. From such mapped output a program user can identify cells at which strategy modification is desirable. Review of the constrained derivatives showing the effect of changing the strategy on other cells and the region is helpful in deciding what change to make.

Fig. 6 shows the percentage of the assumed maximum physically feasible recharge that is being induced at each boundary cell for an optimal strategy. This information, combined with knowledge of constrained derivatives, is useful in determining where recharge basins should be placed or where additional hydrologic data should be gathered (in order to possibly justify relaxing the recharge constraints).

The described procedure was implemented in two computer programs. Unpublished instructions and documentation were prepared for each. QPSTEP, developed on an Amdahl 470, modified optimal quadratic strategies. LPSTEP, developed on a Portable Graphics Mainframe (PGM) with 640 K RAM and a 23 MB hard disk, modified strategies having a linear objective function.

It should be mentioned that the presented optimal strategies are the result of deterministic modeling. To achieve an understanding of the effect of uncertain knowledge of aquifer parameters on the likelihood of achieving desired goals after implementing an optimal strategy, the stochastic nature of aquifer parameters need to be considered. Current research is addressing this topic.

SUMMARY

An interactive computer program is presented which enables decision makers (DMs) to effectively and efficiently design a regional water management strategy. With the program, users can evaluate several conflicting groundwater management objectives. They can interactively investigate any area of the feasible solution space and utilize both regional and local tradeoff functions in the design process. They are provided with information in gridded map format that allows consideration of the local consequences of regional strategy implementation. In short, DMs are provided with the information necessary to rapidly modify a numerically optimal management strategy to better satisfy regional and local concerns.

Regional changes are made by moving through the set of noninferior solutions to locate a compromise solution and regional tradeoff functions. Local changes, or modifications in finite difference variables, are accomplished by changing the constraining conditions on the local decision variables. Constrained derivatives are readily available for evaluating the response of regional objectives to repeated changes in local decision variables.

In the example, the procedure is used to locate and modify a compromise solution to a regional conjunctive use, sustained groundwater withdrawal strategy. The strategy is initially obtained from a management model that minimizes the cost of meeting water needs from the conjunctive use of groundwater and surface water while maintaining a sustained yield. The optimization process uses the finite difference form of a two dimensional

groundwater flow equation as part of the constraining conditions. For multiobjective analysis, a second objective function that maximizes the total regional groundwater withdrawal under sustained yield conditions is included in the original problem as an additional constraint. The results of the formal optimization include local values representing the drawdown, pumping, and recharge in each finite difference cell. The initial results also include the value of a decision variable that represents the total regional groundwater withdrawal under the optimum strategy.

The results of the formal optimization are used as input to an interactive computer program and the set of noninferior solutions is generated. At any feasible solution, the tradeoff function between competing objectives is given to aid in locating a compromise solution. The procedure also provides information on the response of the regional objectives to a change in any local decision variable. This information is used for modifying the compromise solution with respect to local concerns.

The interactive modification method may be applied for any change in a bound on a decision variable, when the change increases the size of the feasible region. For the given example of 152 decision variables and 204 inequality constraints, if a change in the bound on a decision variable is less than the maximum feasible deviation, the optimal solution is calculated with a few additional iterations. If the change in the bound causes a re-partitioning of the system variables, it may take more than a hundred iterations and considerably more processing time to arrive at an optimum.

When a change in a bound decreases the size of the feasible region, the change is limited by the feasible deviation determined by utilizing constrained derivatives. The interactive procedure is not appropriate if a desired change decreases the size of the feasible region in excess of the feasible deviation. In such a case the problem must be re-submitted and solved by a standard optimization process.

In conclusion, although the decision support program is demonstrated by application with a particular optimization model, it can be used to refine strategies developed by other models having quadratic or linear functions and linear constraints.

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$c_e(i)$ the cost associated with raising a unit volume of groundwater one unit distance, $\$/L^4$

$d(k,i)$ the first partial derivative of state variable k with respect to decision variable i

$f(s(i))$ a linear function of drawdown describing the total dynamic head at cell i , L

H the total number of objective constraints

h an index defining a specific objective constraint

i a general index

j a general index

K the total number of finite difference cells in the region, also the total number of inequality constraints

k an index defining a specific state variable

L_h the limiting value of objective constraint h

L_2 the minimum allowable total groundwater annually withdrawn from the aquifer underlying the region, $L^3/year$

M the total number of constant head cells in the region

m an index defining a specific constant head cell

N the total number of finite difference cells in which pumping and drawdown are variable

$p(i)$ the annual volume of groundwater pumped from cell i , $L^3/year$

$p_a(i)$ the annual volume of alternative water use at cell i , $L^3/year$

$p_{min}(i)$ the lower limit on annual groundwater pumping in cell i , $L^3/year$

$p_{max}(i)$ the upper limit on annual groundwater pumping in cell i , $L^3/year$

$r(m)$ the annual recharge in constant head cell m , $L^3/year$

$r_{min}(m)$ the lower limit on annual recharge in constant head cell m , $L^3/year$

$r_{max}(m)$ the upper limit on annual recharge in constant head cell m , $L^3/year$

$s_{min}(i)$ the lower limit on drawdown in cell i , L

$s_{max}(i)$ the upper limit on drawdown in cell i , L

$t(i, j)$ the average transmissivity between finite difference cell i and cell j , for $i = j$, $L^2/year$

$v(i)$ the first partial derivative of the unconstrained objective function with respect to variable j

$w(i)$ the annual water requirements in cell i , $L^3/year$

$x_d(i)$ the old value of decision variable i

$x'_d(i)$ the new value of decision variable i

z_h the value of objective constraint h

z_p the value of the principal objective function

z_1 the total annual cost of water supply, $\$/year$

z_2 the total volume of groundwater annually withdrawn from the region, $L^3/year$

$\Delta v(j)$ the change in the value of constrained derivative j .

$\Delta x_d(i)$ the change in decision variable i

$\Delta x_s(k)$ the change in state variable k

Δz_p the change in the value of the principal objective function.

NOMENCLATURE

$b(i,j)$ the second partial derivative of the unconstrained objective function with respect to variable i and variable j

$c_a(i)$ the cost per unit volume of alternative water supplied in cell i , $\$/L^3$

$c_m(i)$ the cost associated with a unit volume of groundwater pumped, $\$/L^3$