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Computational Aspects of Chance-Constrained Sustained Groundwater Yield Management

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ABSTRACT

A methodology for including chance-constraints in models for maximizing regional sustained-yield groundwater extraction is presented. The procedure utilizes the spatially distributed mean and variance of saturated thickness in a confined aquifer. This data is derivable from kriging, a geostatistical technique. The technique is applied to a scenario of somewhat egalitarian agricultural development dependent on groundwater. Decision makers in this situation know that groundwater availability is insufficient to supply all potential demand. They plan to implement a sustained yield groundwater extraction strategy. They know that because of uncertain knowledge of aquifer parameters, at some point in the future, available groundwater in some locations will exceed the initially allocated amount. They also realize that some locales will probably have less available groundwater than was expected, based on initial knowledge of the aquifer. To avoid future social and political unrest, they prefer that the spatial availability of groundwater remain somewhat consistent with time. Thus, they develop a groundwater extraction strategy using a predetermined confidence level. They are then relatively certain that actual sustainable extraction does not exceed the allocated groundwater use rate for locales in which allocated pumping is at a specified upper limit.

INTRODUCTION

In the past, there has been considerable reluctance on the part of planners and design engineers to use stochastic methods in groundwater management models. However, the presence of uncertainties in estimating aquifer parameters has long been recognized. Fortunately, there have been recent developments in stochastic subsurface flow theory and its practical applications (Gelhar, 1986 and Tung, 1986). Thus, the current trend is to include the natural heterogeneity of aquifers in the governing flow equation by probabilistic in addition to deterministic approaches.

This article presents a methodology that explicitly incorporates the stochasticity of an aquifer parameter in a chance-constrained formulation of a groundwater management model that maximizes steady-state (sustainable) groundwater extraction. The methodology falls under the broad category of explicity stochastic optimization. It has two parts: (a) regional process identification, and (b) chance-constrained optimization. The regional process identification establishes and describes the random nature of the aquifer parameter. This is accomplished by a statistical procedure known as block kriging. The statistical information obtained from kriging is then utilized as input to an optimization model. This optimization model includes the finite-difference approximation of the steady-state flow equation expressed in probabilistic terms.

The method is applied to a hypothetical agricultural area administered by a governmental agency. Results show the applicability of the methodology. Computational aspects of the methodology are discussed. The practical significance of alternative formulations are also included.

PREVIOUS WORK

The need to systematically relate the hydraulic behavior of groundwater flow systems to the optimal use of water supplies has been accomplished by coupling the physical principles of groundwater flow and optimization theory. The "embedding" approach involves inclusion of flow equations as constraints in an optimization model (e.g. Gorelick, 1983 and Peralta, 1985, among many others).

Representation of the random nature of the system components has been attempted and reported in groundwater literature only recently. The need to represent the random nature of aquifer parameters has been recognized by groundwater researchers. A number of methods have been proposed in water resources literature. However, researchers have yet to agree on which method is best (Carrera and Neuman, 1985). Finding the proper representation of the random process has been posed as an identification problem. It involves finding the solution of the inverse problem. Numerous approaches to the inverse problem have been proposed. Ponzini and Lozej (1982) reported excellent results using a comparison model to compute interblock transmissivities. Dagan (1985) presented a methodology for solving the problem of determining the random distribution of transmissivity through unconditional and conditional probabilities. More recently, Carrerra and

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Neuman (1986) published methods of estimating the parameters of steady and unsteady groundwater flow by a maximum likelihood method. Gutjahr and Gelhar (1981) considered hydraulic conductivity as a spatial variable. They showed that variogram analysis (as a part of kriging) yielded consistent results with analytical approximations such as first-order analysis and covariance differential equations. This is significant because it indirectly underscores the importance of kriging as a method of describing the spatial random nature and distribution of aquifer parameters.

Originally used in mining and geology, kriging is a well-established statistical technique for calculating minimum variance unbiased estimators of spatially dependent data. Most commonly, it is used to compute the best estimate of an unknown value of a parameter at a specified location. 'Punctual' kriging is used in interpolation to estimate the values of aquifer parameters at points (locations) other than those points at which field observation has been made. Marx and Thompson (1987) provide an excellent and concise discussion of the punctual kriging procedure and its practical applications. 'Block' kriging is used to estimate average values of aquifer parameters in the center of a block or cell. Burgess and Webster (1980) describe the block kriging procedure, which results in smaller estimation variance values than does punctual kriging, since it is an 'average' estimate for an area or volume.

Researchers have also scrutinized the randomness of system components other than aquifer parameters. Maddock (1974) presented a methodology for finding strategies or rules for a stream-aquifer system. He assumed that the demand for water is a random event. His work is based on the premise that the water resource system operates under stochastic water needs or demands.

Of the numerous stochastic modeling techniques that are available, chance-contrained programming includes random variation as an integral part of the constraint set of an optimization model. More importantly, specified probability limits on constraint violations may be established. From the modeling perspective, chance-constrained formulations are useful because they properly represent the random component of the system. Moreover, water resource modeling and optimal solution computation is facilitated by the ability to develop the deterministic equivalent of an originally stated chance-constrained problem.

Charnes and Cooper (1963) published the first comprehensive presentation of chance-constrained programming. Since then, the technique has been extensively implemented in surface water system studies. In groundwater literature, Tung (1986) reported the applicability of chance-constrained programming with response function groundwater modeling. He included random aquifer parameters in a compliance constraint to realistically restrain the model's performance in a probabilistic situation.

In summary, stochasticity of system parameters is of importance in groundwater modeling. Furthermore, well-established methods like kriging and chance-constrained formulations are available for adequate representation of groundwater system optimization problems.

THEORY AND MODEL FORMULATION

Governing Equation

Consider a hypothetical area that is underlain by an aquifer with large saturated thickness. Assume that the aquifer is confined, or that the change in saturated thickness, resulting from any of the pumping strategies discussed later, is small. Furthermore, assume a spatially unchanging hydraulic conductivity and a spatially correlated saturated thickness.

The aquifer in the study area is assumed to be completely surrounded by a larger area. Thus, the surrounding aquifer is a source of recharge through the boundary cells of the hypothetical area. The sole vertical discharge from the area's internal cells is groundwater pumping through wells. No other hydraulic stimuli or stresses occur at internal cells. All other recharge to or discharge from the system occurs at constant head cells along the area's boundary.

The Boussinesq equation and Darcy's law govern the aquifer recharge to or discharge from the study area. The Boussinesq equation is commonly used to describe two-dimensional flow through porous media. The equation is expressed in terms of continuous partial derivatives:

$$\frac{\partial}{\partial x}(T\frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(T\frac{\partial H}{\partial y}) - Q = S\frac{\partial H}{\partial t} \quad \ldots\ldots\ldots\ldots [1]$$

where

T = transmissivity of the aquifer material, (L^2T^{-1})
S = storage coefficient of the aquifer
H = the head, (L)
t = time, (T)
Q = net volumetric flow into (−) or out of (+) the aquifer, (L^3T^{-1}).

Under steady-state conditions the right-hand side of equation [1] vanishes. The resulting equation describes two-dimensional flow where there is no change in head with time. Equation [1] can be written in a finite-difference form to describe flow in a heterogeneous isotropic aquifer. Using block-centered two-dimensional cells to represent the system, equation [1] becomes:

$$t^i_{i,j}\,h_{i+1,j} + t^i_{i-1,j}\,h_{i-1,j} + t^j_{i,j-1}\,h_{i,j-1} + t^j_{i,j}\,h_{i,j+1}$$

$$- h_{i,j}\,t^{ij}_{i,j} = g_{i,j} + r_{i,j} \quad \ldots\ldots\ldots\ldots [2]$$

where

$g_{i,j}, r_{i,j}$ = steady groundwater pumping (+) and recharge (−) in cell i,j that will maintain $h_{i,j}$, (L^3T^{-1})
$h_{i,j}$ = steady potentiometric head that will ultimately evolve at each internal cell i,j if each is stressed by rate $g_{i,j} + r_{i,j}$, (L)
$t^i_{i,j}$ = mean transmissivity between cells i,j and i+1,j, (L^2T^{-1})
$t^j_{i,j}$ = mean transmissivity between cells i,j and i, j+1, (L^2T^{-1}).

$$t^{ij}_{i,j} = t^i_{i,j} + t^i_{i-1,j} + t^j_{i,j} + t^j_{i,j-1} \quad \ldots\ldots\ldots\ldots [3]$$

The t^i and t^j terms in equation [2] are usually represented by either the geometric mean or the harmonic mean of the transmissivities of adjacent cells. The choice depends on the expected accuracy of the resulting transmissivities of the midpoint of adjacent cells.

Assume for this paper that, as described below, the arithmetic mean is used for the t^i and t^j terms. Assume further that hydraulic conductivity is constant for the whole area of interest and that the saturated thickness within each cell is governed by a random process. With these assumptions, equation [2] can be rewritten as:

$$0.5\,k\,[b_{i,j}\,h_{i+1,j} + b_{i+1,j}\,h_{i+1,j} + b_{i-1,j}\,h_{i-1,j}$$

$$+ b_{i,j}\,h_{i-1,j} + b_{i,j-1}\,h_{i,j-1} + b_{i,j}\,h_{i,j-1}$$

$$+ b_{i,j}\,h_{i,j+1} + b_{i,j+1}\,h_{i,j+1} - 4\,b_{i,j}\,h_{i,j}$$

$$- b_{i+1,j}\,h_{i,j} - b_{i-1,j}\,h_{i,j} - b_{i,j+1}\,h_{i,j}$$

$$- b_{i,j-1}\,h_{i,j}\,] = g_{i,j} + r_{i,j} \quad \ldots\ldots\ldots\ldots\ldots\ldots[4]$$

where

k = hydraulic conductivity that is assumed constant, (LT^{-1})

$b_{i,j}$ = saturated thickness that is governed by a random process, (L).

Equation [4] is derived from equation [2] under the simplifying assumption that the transmissivity between two adjacent cells can be adquately represented by the simple average of the cells' block-centered transmissitivies. This assumption is necessary to maintain linear terms on the left hand side of the equation. It is also consistent with the independent random distribution nature of the process that describes the saturated thickness within a cell. One should note that arithmetic averaging is appropriate for internal cells but not for cells adjacent to impermeable boundaries.

To facilitate discussion, equation [4] is rewritten in the following compact form:

$$0.5\,k\,[\Sigma\,\underline{b}_{i,j}\;\underline{h}_{i,j}\,] = g_{i,j} + r_{i,j} \quad \ldots\ldots\ldots\ldots\ldots[5]$$

where the left-hand side of equation [5] is merely an alternative notation for the sum of the terms on the left-hand side of equation [4].

Probabilistic Constraint and Its Deterministic Equivalent

The net discharge of any cell (i,j) in the aquifer system equals the sum of groundwater pumping and recharge in that cell. The **g** value on the right-hand side of equation [5] represents groundwater allocation calculated by computer management model. Because of uncertain knowledge of the aquifer, the actual sustainable (steady) groundwater yield in a cell may differ from this allocated value. If total water available in an aquifer system is insufficient to satisfy potential water demand in all cells, it might be desirable to limit allocated groundwater pumping in each cell to be less than some critical value. Let this critical value, g^c, be expressed as a fraction of potential demand. To assure, with a known probability, that the actual sustainable value does not exceed the computed (allocated) value at cells in which the allocated value equals g^c, the flow equation used in a chance-constraint expression is:

$$P\left\{0.5\,k\,[\Sigma\,\underline{b}_{i,j}\;\underline{h}_{i,j}\,] = g_{i,j} + r_{i,j} \leqslant g^c_{i,j}\right\} \geqslant 1 - \alpha \ldots[6]$$

Equation [6] assures, with a probability equalling or exceeding $(1 - \alpha)$, that the net sustainable discharge from each cell is less than a prespecified critical value. Assuming that the saturated thickness in a cell, $b_{i,j}$, is sufficiently described by a normally distributed process, with mean $m_{i,j}$ and variance $v_{i,j}$, the probabilisitic constraint (equation [6]) can be rewritten as:

$$0.5\,k\,[\,(\Sigma\,\underline{m}_{i,j}\;\underline{h}_{i,j}\,) + F^{-1}(\alpha)\sqrt{(\Sigma \underline{v}_{i,j}\;\underline{h}^2_{i,j})}\,]$$

$$= g_{i,j} + r_{i,j} \leqslant g^c_{i,j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots[7]$$

where $F^{-1}(\alpha)$ denotes a standard normal deviate corresponding to the normal cumulative distribution function of α. (Stability aspects as a consequence of the conversion from equation [6] to equation [7] are mathematically analyzed by Dupacova (1984)). All other notations are consistent with those of equation [6]. For the stated assumptions, the deterministic constraint (equation [7]) can replace the probabilistic constraint (equation [6]). As written, equation [7] assumes spatial independence and is a conservative estimate if spatial dependency occurs. If written to assume spatial dependence (using covariance functions) the top line of equation [7] would be smaller in magnitude for a given set of heads. Thus, more pumping would be possible than is computed using equation [7].

Problem Formulation

Consider a sustained groundwater yield (steady-state) management problem in which the objective is to maximize total groundwater pumping while satisfying constraints on heads, recharges, and pumping. The amount of groundwater pumping in each cell is required to be less than the cell's water demand. In addition, the probability that actual net sustainable discharge in each cell does not exceed prespecified critical values is set to $(1 - \alpha)$. This problem is applicable to the scenario described below.

A planning agency for a developing country wishes to compute an optimal sustained groundwater yield pumping strategy for an area. The area is to be an important region for irrigated agricultural production. Naturally, the agency wishes to maximize sustainable groundwater pumping. The agency also recognizes that knowledge of spatially variable saturated thickness is uncertain. Furthermore, agricultural reform policies make the agency desire to spread irrigated acreage out in the area, rather than concentrate it in a few cells.

Possibly, the agency could compute maximum pumping strategies subject to chance-constraints on drawdown, and absolute upper and lower limits on pumping. Setting a lower limit of zero pumping is always easy. Setting a higher value for a lower limit may be infeasible if the aquifer cannot provide enough water.

Setting firm upper limits is easy and will not adversely affect identifying feasible solutions. However, as stated, the agency wants to achieve a somewhat egalitarian distribution of pumping.

Decision makers (DMs) know that since knowledge of the aquifer is uncertain, the actual sustainable groundwater pumping in a cell will differ from the sustained yield computed by a groundwater model—in some cells more water will be sustainably available, in some cells less. They known that this difference will not be obvious immediately, but that it will eventually be discernable from potentiometric surface response to pumping. For example, annual pumping in compliance with the rates allocated by the management model should ultimately cause the evolution of a relatively steady potentiometric surface. This surface consists of the potentiometric heads in the steady-state flow equation. If head in a cell remains higher than expected, one might surmise that physically feasible sustainable groundwater extraction at that cell is larger than the existing withdrawal rates.

For social and political reasons, the DMs hope to avoid having to significantly adjust annual groundwater allocations in the future. Realizing that all optimal heads and allocated pumping rates are computed simultaneously by management model, DMs might wish to use chance-constrained upper bounds on pumping for all cells.

Decision makers (DMs) in this study choose to develop a range of maximum pumping strategies. Each strategy is subject to the constraints (equation [7]) that the DMs must be x% sure that actual sustainable pumping in each cell does not exceed prespecified critical values. The confidence level is varied systematically. This approach incorporates uncertain knowledge of aquifer saturated thickness in the upper bound on groundwater pumping allocation.

The problem is mathematically formulated as:

$$\text{Maximize} \sum_{i \epsilon I} \sum_{j \epsilon J} g_{i,j} \quad \text{[8]}$$

$$h^L_{i,j} \leqslant h_{i,j} \leqslant h^U_{i,j} \quad \text{for } i\epsilon I, j\epsilon J \quad \text{[9]}$$

$$r^L_{i,j} \leqslant r_{i,j} \leqslant r^U_{i,j} \quad \text{for } i\epsilon I, j\epsilon J \quad \text{[10]}$$

$$g^L_{i,j} \leqslant g_{i,j} \leqslant g^U_{i,j} \quad \text{for } i\epsilon I, j\epsilon J \quad \text{[11]}$$

$$g_{i,j} \leqslant w_{i,j} \quad \text{for } i\epsilon I, j\epsilon J \quad \text{[12]}$$

and the two probability constraints of equation [6], where

$h^L_{i,j}$ and $h^U_{i,j}$ = known lower and upper limits on the potentiometric head in cell (i,j), (L)

$r^L_{i,j}$, $r^U_{i,j}$, $g^L_{i,j}$ and $g^U_{i,j}$ = known lower and upper limits on groundwater recharge and pumping, respectively, (L^3T^{-1})

$w_{i,j}$ = potential water demand based on soil and likely crops, (L^3).

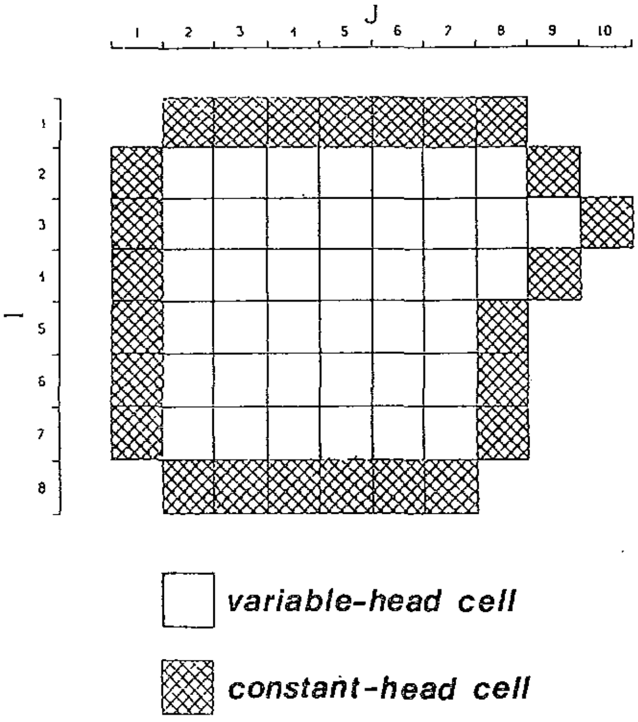

Fig. 1—Hypothetical study area.

All other notations have been defined previously. Equations [8], [9], [10], [11], [12] and [6] consist of a chance-constrained problem where the decision variables are $g_{i,j}$, $r_{i,j}$ and $h_{i,j}$, that is, groundwater pumping, recharge and potentiometric head, respectively. The formulation implies that the distribution process that governs the random aquifer parameter (in this paper, the saturated thickness) is also known. Assuming that this is the case and that the distribution is normal or can be converted to a normal distribution, equation [7] can substitute for equation [6].

The programming problem that includes equations [8], [9], [10], [11], [12] and [7] (Model A) is a nonlinear programming problem due to the nonlinear terms introduced by equation [7]. Nonlinear programming algorithms are currently available to solve programming problems of this structure. The GAMS/MINOS software package was selected for this paper. It consists of a General Algebraic Modeling System (GAMS) developed by the World Bank (Kendrick and Meeraus, 1985) and a Modular In-Core Nonlinear Optimization System (Murtagh and Saunders, 1983). Because of the nonlinear constraints, global optimality of the computed strategies cannot be assured.

NUMERICAL EXPERIMENTS

The formulated problem has been applied to a hypothetical area (Fig. 1). The same area was used previously by Peralta and Kowalski (1986). The area consists of 65 cells, 40 of which are internal cells. Head is constant in all peripheral cells. A spatially constant hydraulic conductivity value of 82 m/day (270 ft/day) is assumed. The hypothetical area, a small portion of the Bayou Bartholomew Basin in Arkansas, is shown as the irregularly shaped area in Fig. 2. The relative position of the hypothetical area is shown as area XYZD in Fig. 2.

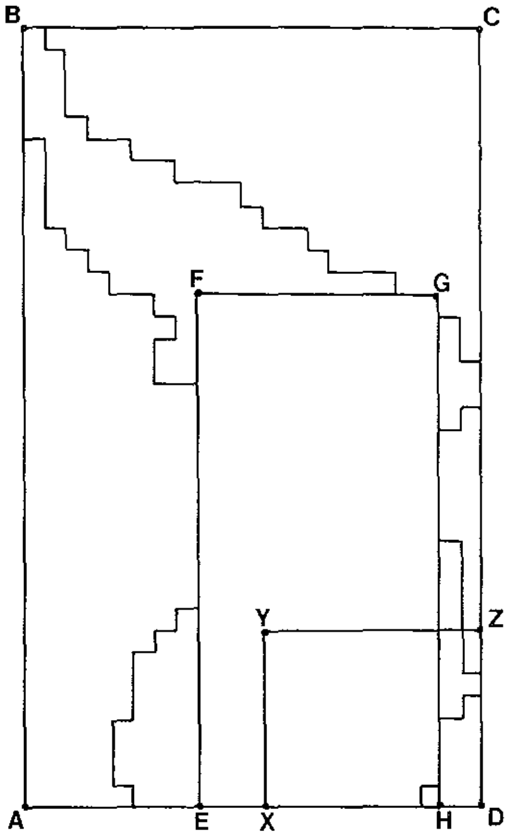

Fig. 2—Block kriging areas.

A standard block kriging procedure was implemented to compute the standard error of the estimated saturated thickness in each cell. Comparative results of the kriging study are shown in Table 1. Results show that the larger the area the greater the variance calculated by block kriging. Results for area ABCD (see Fig. 2) differs from that of area EFGH by about 50%. Note that area ABCD includes cells that are outside the basin. Results show that smaller variance and mean estimates are obtained when the rectangular area used in kriging includes only cells within the basin. Block kriging estimates are applicable to the center of the rectangular study area. More accurate results are possible when block kriging is applied to each of the cells of the hypothetical area. Block kriging done on a cell by cell basis is similar to application of punctual kriging. When a regionalized mean and variance of the aquifer parameter for the entire study area can be justified, block kriging defines the random process just as well as punctual kriging.

Optimal solutions for Model A as formulated above have been systematically calculated for different confidence levels, specified as $(1 - \alpha)$. Critical values in the deterministic equivalent of the chance-constraint are

TABLE 1. Comparison of Block Kriging Results

Area	Mean (ft)	Standard Deviation (ft)
ABCD	148.03	17.91
EFGH	105.62	13.73
XYZD	97.91	3.51

TABLE 2. Chance-Constrained Modeling Results

	FRAC=0.5	FRAC=0.9
$1-\alpha$	OP*	OP*
0.95	18.44	20.18
0.90	18.03	19.74
0.80	17.52	19.21
0.60	16.84	18.50

$$*\text{OP} = \frac{\text{Optimal Total Groundwater Pumping}}{\text{Total Water Need}} \times 100$$

also varied to provide a comparative study of the methodology's application. In the numerical experiments, the critical value $g^c_{i,j}$ is computed as a fraction (FRAC) of the cell's water demand volume. Table 2 shows results from two groups of computer runs. These are results when (a) FRAC = 0.5, and (b) FRAC = 0.9 at varying levels of confidence.

The slight trend observed in Table 2 is that as the confidence level $(1 - \alpha)$ decreases, total optimal allocated pumping also decreases. This results because F^{-1} in Equation [7] decreases from 1.645, for $(1 - \alpha) = 0.95$, to 0.255 for $(1 - \alpha) = 0.60$.

For FRAC = 0.5, as reliability and total allocated pumping decreases, there is a slight decrease in the number of cells having optimal pumping values that are tight at their upper bounds. There is insignificant change in this respect for FRAC = 0.9. On the other hand, one expects the number of cells in which actual sustainable pumping exceeds $g^c_{i,j}$ to increase with decreasing $(1 - \alpha)$.

It is also important to point out that total allocated pumping increases slightly for FRAC = 0.9 as opposed to FRAC = 0.5. In this case, when constraining the probability that fluxes not exceed a critical value, increasing the critical value causes a small increase in total pumping.

Computer runs for the small hypothetical area are accomplished using the University of Arkansas IBM/370 in the CMS environment. A typical run required about 5 s of CPU time while using the GAMS 2.04 nonlinear optimization package option. It is important to point out that providing reasonable initial values for the decision variables results in shorter CPU times. Computer runs may terminate before finding the optimal solution. In these cases, changing the initial values is necessary. There were also cases where the initial values supplied resulted in infeasibility. Multiple optimal solutions existed in some cases. Based on these observations, application of the methodology to areas with a large number of cells may pose problems due to system size and nonlinearity of the deterministic equivalent of the chance-constrained formulation.

ALTERNATIVE FORMULATIONS

Although the problem formulated as Model A in this paper may have definite practical importance in situations of water scarcity, two alternative formulations are worthy of mention. Changing equation [6] to:

$$P\left\{ 0.5\,k\,(\Sigma\, \underline{b}_{i,j}\, \underline{h}_{i,j}) \geqslant g^c_{i,j} \right\} \geqslant 1 - \alpha \quad \ldots\ldots\ldots [13]$$

is appropriate for an entirely different management problem. This formulation, Model B, is more applicable in situations where available resources are not as limiting. The model seeks to guarantee at least the critical amount at a particular level of reliability. However, there is no assurance that all problems of model B structure would result in feasible strategies. As Peralta et al. (1985) determined in developing an egalitarian groundwater allocation strategy for the correlative rights doctrine based on historic water use, the system may be physically unable to provide the prespecified critical value due to its hydraulics and physical properties. Another formulation improves this weakness. Permit the critical level in each cell to vary by stating the chance-constraint as:

$$P \left\{ 0.5\, k\, (\Sigma\, \underline{b}_{i,j}\, \underline{h}_{i,j}) \geq f_{i,j}\, w_{i,j} \right\} \geq 1 - \alpha \quad \ldots\ldots [14]$$

where $f_{i,j}$ is a decision variable. It is the fraction of the water demand that can at least be satisfied at confidence level of $(1 - \alpha)$ at each cell. Equation [14] is a chance-constraint similar in purpose to that described by Peralta et al. (1985). The range of possible values for $f_{i,j}$ is:

$$0.0 \leq f_{i,j} \leq 1.0 \quad \ldots\ldots [15]$$

The following constraint is also added:

$$f_{i,j} \geq d \quad \ldots\ldots [16]$$

Equation [16] restricts the cell by cell fractional levels to be greater than a particular dummy variable **d**. Now, changing the objective function to equation [17] completes Model C.

$$\text{Maximize} \quad d \quad \ldots\ldots [17]$$

Model C, a model that consists of equations [17], [9] through [12] and [14] through [16], is in the form of a max min problem. The problem seeks the best possible set of fractional levels that will provide water needs at a prespecified level of certainty.

SUMMARY AND CONCLUSIONS

Results of numerical experiments show that chance-constrained formulation is possible and useful in developing groundwater sustained yield extraction strategies. Computational aspects of the methodology and its practical implication are also discussed. Alternative formulations for several management scenarios are presented. Applicability of the presented models depends on validity of assumptions. Being able to quantitatively describe the random process is crucial to converting the chance-constraint to its deterministic equivalent.

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