Reconnaissance-Level Alternative Optimal Ground-Water Use Strategies

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Abstract: This study develops regionally optimal ground-water extraction strategies. Alternative explicit planning objectives are: (1) Maximize total pumping from the underlying aquifer while causing the evolution of a steady potentiometric surface; and (2) maintain a prespecified target potentiometric surface. Implicit objectives involve controlling stream/aquifer interflow and water flow across a state boundary, and attempting to avoid gross disruption of current cropping patterns. Models, bounds, constraints, and data are formulated. Alternative optimal strategies and the rationale for preferring one strategy are presented for a region in Arkansas. The objective of maintaining the relatively unstressed target potentiometric surface yields politically and socially unacceptable water-use strategies. The most acceptable strategy maximizes sustainable ground-water extraction, maintains recent ground-water flow to Louisiana, maintains current potentiometric surface heads at the Louisiana-Arkansas border, maintains more than minimally acceptable surface water flow to Louisiana, and approximately maintains current cropping distributions. Developed planning models utilize the embedding approach, over 300 pumping variables, and 700 total variables, indicating the utility of the embedding method for regional sustained yield (steady-state) planning.

Introduction

For any large ground-water use area there are an infinite number of feasible sustainable ground-water withdrawal strategies (spatially distributed pumping rates sustainable over the long term) (Datta and Orlob 1988). Also, there are generally several water-management objectives. To develop feasible strategies that best achieve these objectives, optimization techniques are used (Willis and Yeh 1987).

Especially when developing a regional management strategy encompassing a large area, one must deal with a number of explicit management objectives and a number of implicit objectives specified as bounds and constraints. Generally, only one management goal is expressed as a formal objective function. Bounds and constraints circumscribe the decision space and are used to insure physical (hydraulic) feasibility and managerial (social, economic, and political) feasibility.

Since constraints and bounds can be changed to reflect feasibility and preference, there are an infinite number of decision spaces and attendant *optimal* strategies for most study areas. Accordingly, there is increasing emphasis on presenting a range of alternative optimal strategies to decision makers.

This paper describes the process of developing alternate optimal sustained-yield regional pumping strategies for the Bayou Bartholomew basin in southeastern Arkansas (Fig. 1). Many organizations are interested in preserving the agricultural productivity of this region where ground-water use for irrigation has caused declines in the potentiometric surface. Water agencies want to know how much ground water is sustainably available without disrupting flow across the state border to Louisiana, and causing other undesirable consequences.

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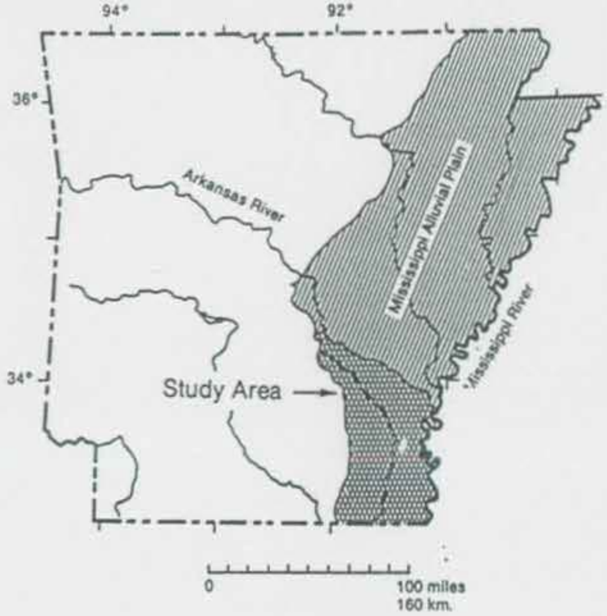

FIG. 1. Location of Study Area in Arkansas

This type of information can be obtained by utilizing a combined optimization/simulation model of the region. Performing a simple volume balance utilizing historic data is inadequate because recharge is affected by pumping (Bredehoeft et al. 1982). Anticipated recharge is unknown since stream/aquifer (S/A) interflow depends on heads resulting from pumping.

The first goal of this paper is to describe the optimization models developed and used for the planning agencies. These models incorporate ground-water flow via the "embedding" approach. The embedding approach was selected because, for steady-state conditions, if the number of pumping cells in an area is relatively large, the embedding approach will frequently require much less computer memory and processing time than will a response matrix model that yields comparable information (Azarmnia 1988).

The second goal involves presentation of tested management scenarios. (A scenario consists of a unique combination of objective function and values used as bounds and constraints.) The third goal is to present the alternative optimal pumping strategies computed for each scenario and the rationale used by the planners in preferring a particular strategy. This process differs from application of the constraint method (Cohon and Marks 1975) of multiobjective optimization as previously reported in regional ground-water management (Yazicigil and Rasheeduddin 1987; Datta and Peralta 1986a). We approach the multiobjective problem by simultaneously changing entire sets of bounds or boundary conditions. This approach is practical when a model uses a large number of bounds or constraints to represent spatially distributed implicit objectives.

The following objective functions are used in different scenarios:

Maximization of the total withdrawal from the aquifer subject to sustained yield hydraulic constraints.

Maximization of the sustainable maintenance of a specified springtime potentiometric surface.

Constraints incorporated in the optimization models include: (1) Limits on recharges into the area through boundary cells; (2) limits on recharges or discharges through stream/aquifer connections; and (3) bounds on pumping and potentiometric head in each cell.

Steps involved in developing the reconnaissance level sustained groundwater extraction strategies are:

1. Estimation of the historic and current extraction of ground water from the Quaternary aquifer in each cell, based on municipal and industrial records, aquacultural acreages, crop acreages and irrigation scheduling.
2. Estimation of the maximum potential demand for agricultural water use in each cell, based on soil types, climatic conditions, irrigation scheduling, and economic factors.
3. Estimation by kriging of the historic potentiometric surface elevations at the center of each cell.
4. Estimation of aquifer parameters through review of existing information and reports of past studies concerning calibration and validation of ground water flow simulation models.
5. Estimation of stream/aquifer interflow for those surface water resources hydraulically connected to the aquifer.
6. Estimation of the net recharge that has historically occurred along the study area boundaries.
7. Estimation of the annual volume of water that can be withdrawn from the Quaternary aquifer underlying each cell, so as to maximize achievement of the stated objective functions.

By following these steps, estimates of optimal sustainable ground-water extraction are obtained. The extraction strategy selected from among all computed strategies is useful in determining what feasibility studies should be conducted for planning the possible diversion of river water to agricultural lands.

Study Area

The Bayou Bartholomew region consists of portions of six counties and encompasses about 3,400 sq mi (9,400 km^2) (Fig. 1). Northern and eastern boundaries coincide with the Arkansas and Mississippi rivers, respectively. The northwestern boundary is the boundary of the Quaternary aquifer, part of the Mississippi Plain alluvial aquifer, that underlies the region. The southwestern border is not a natural boundary and leaves a part of the Quaternary

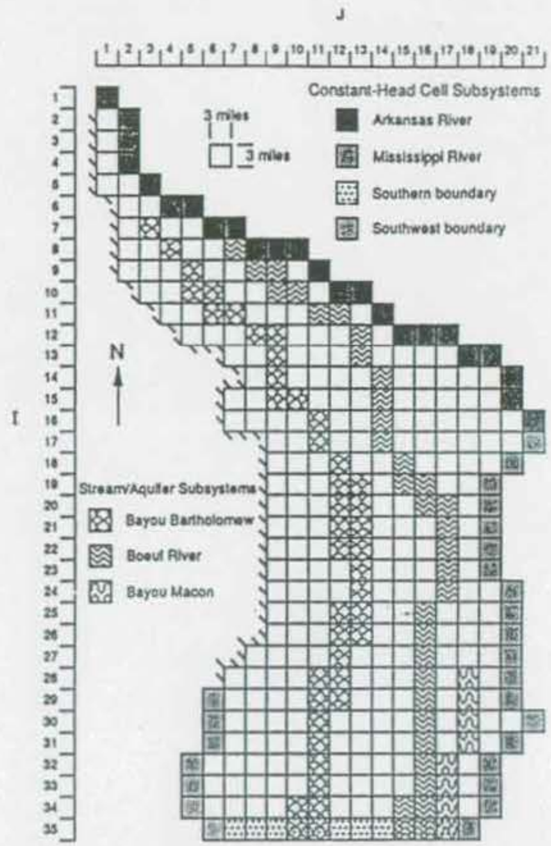

FIG. 2. Discretization, Constant-Head Subsystems and Stream/Aquifer Subsystems for Bayou Bartholomew Basin

aquifer outside the study area. It was selected so as to enclose only that portion of the aquifer where an appreciable amount of ground-water pumping has occurred. Since the small part of the aquifer that is omitted underlies forest, it is unlikely that there will be much demand for ground water in that area.

The natural surface drainage within the basin consists primarily of the Bayou Macon, Bayou Bartholomew, and Boeuf rivers, which outlet into the Ouachita River in Louisiana. Depending on the year, these rivers have historically been either net sinks or sources of water for the aquifer through stream/aquifer (S/A) interflow.

Most of the ground-water withdrawal in this area is used by agriculture for irrigation of rice, soybeans, and cotton. Other users include aquaculture, municipalities, and industry. There are thousands of wells in the area, and an undetermined number in each cell. Accurate historic pumping rates from each cell are not obtainable. Instead, estimates are made based on cropping and agricultural acreage data. Aquacultural use of ground water is estimated using hist[illegible]duction permits. As a result, there is uncertainty in the use of historic pumping as upper bounds on pumping in the models.

Assumptions concerning the aquifer were made based on previous literature and simulation model calibration, validation and sensitivity analyses (Broom and Reed 1973; Reed and Broom 1979; Solaimanian 1985). This Quaternary aquifer, part of the Mississippi Plain alluvial aquifer system, is confined by a relatively impermeable clay that limits deep percolation and makes the area well suited for rice production. It consists of unconsolidated sand and gravel with interspersed clay lenses and is underlain by a Tertiary clay. Aquifer transmissivities range from 5,600 sq ft/day at the western edge to over 35,000 sq ft/day. Average transmissivity is about 19,000 sq ft/day.

The study area was divided into 376 cells, each 3 mi square (Fig. 2). Because of the high cost of computer processing, and to insure computational feasibility of the optimization models, a finer grid spacing was not used. Gorelick (1983) and Tung and Kolterman (1986) describe numerical instabilities that result when too many finite difference flow equations are embedded in optimization models.

Study area cells are specified as either constant-head or variable-head cells. The northern, eastern, southern, and southwestern boundaries were simulated using constant-head cells. In constant-head cells, the simulated groundwater level was maintained at a constant elevation (head) during the simulation period. The weather periphery of the area was represented using variable-head cells having negligible transmissivity to the West.

Some recharges to the area take place through the constant-head cells—the recharge volume being provided either from a river penetrating to the aquifer in those cells, or water entering them from extensions of the aquifer outside the region. Streams passing through some of the internal cells in the region also provide recharge to the aquifer. In most of the rest of the internal cells, a relatively impermeable clay layer overlies the confined aquifer.

Estimation of the hydrogeologic and other parameters of the study area is a nontrivial portion of any regional ground-water analysis. Fortunately, much work has already been accomplished for this area. Of course, there is always some doubt about estimation accuracy (Yeh 1986).

Optimization Model

Components of optimization models include the objective function, constraints and decision variables, and their bounds. The models are summarized here before mathematical description is given. The model 1 objective function maximizes the sustained yield withdrawal (pumping) from the region (Eq. 1). Model 2 develops sustained yield withdrawal strategies that maintain ground-water elevations as close as possible to predetermined "target" elevations (Eq. 2). For sustained-yield planning in a confined aquifer, finite difference approximations of the differential equations governing steady ground-water flow (Eq. 3) are used as constraints via the "embedding" approach. Variables subject to management bounding or constraints include heads, pumping, recharge, and stream aquifer (S/A) interflow (Eqs. 4–10). More details on the methodology used here can be found in the users guide (Peralta et al. 1989) to the SSTAR Model described by Peralta et al. (1985).

The objective functions are:

$$\max \quad Z_1 = \sum_{k=1}^{n_c} Q_k \qquad (1)$$

$$\min \quad Z_2 = \sum_{k=1}^{n_c} [(H_k - H_k^t)\cdot a_k]^2 \qquad (2)$$

Subject to:

- The relationship defining steady ground-water discharge or recharge in a particular cell (i.e., the finite difference form of the linearized Boussinesq equation):

$$-Q_{k,r} + Q_k = f(k) \qquad k = 1, \ldots, n_c \qquad (3)$$

where $f(k)$ for cell number k and grid (i,j) can be expressed in terms of the finite difference transmissivities and heads such that:

$$f(k) = t_{i+1/2,j}H_{i+1,j} + t_{i-1/2,j}H_{i-1,j} + t_{i,j+1/2}H_{i,j+1} + t_{i,j-1/2}H_{i,j-1}$$
$$- (t_{i+1/2,j} + t_{i-1/2,j} + t_{i,j+1/2} + t_{i,j-1/2})H_{i,j} \qquad (4)$$

- Pumping in a particular cell must equal or exceed a given minimum value or, recharge in a boundary (constant head) cell must be greater than or equal to a given value:

$$Q_k \geq Q_{\min,k} \qquad k = 1, \ldots, n_c \qquad (5)$$

- The pumping or recharge in a particular cell is less than or equal to a given maximum value. The upper bound on pumping is determined by maximum estimated water demand for that cell.

$$Q_k \leq Q_{\max,k} \qquad k = 1, \ldots, n_c \qquad (6)$$

- The steady-state head in a particular cell equals or exceeds a specified minimum value

$$H_k \geq H_k^L \qquad k = 1, \ldots, n_c \qquad (7)$$

- The steady-state water level is below the ground surface in cell k:

$$(G_k - H_k) \geq 0 \qquad k = 1, \ldots, n_c \qquad (8)$$

- The recharge to the aquifer in a subsystem of interconnected cells with S/A connections does not exceed a specified upper bound

$$\sum_{k=1}^{K_r} Q_{k,r} \leq SQ_{r,\max} \qquad r = 1, \ldots, n_r \qquad (9)$$

The interflow between the aquifer and a stream in saturated hydraulic connection defined in terms of streambed conductance (McDonald and Harbauch 1988) is:

$$Q_{k,r} = T_k^r \cdot (H_k^s - H_k) \qquad r = 1, \ldots, n_r \qquad (10)$$

The first objective function and all the constraints are linear. Because the second power of the decision variables are used in the second objective function, that ctive is quadratic. The global optimality of solutions from the second model was verified. The target elevations in the second model, selected by planning agency, are the current springtime potentiometric surface elevations. The utilized weighting factors, a_k, are the reciprocal of the standard deviations of the estimation errors for estimating these elevations using kriging.

Streambed conductance is a measure of the ability of the streambed to transmit water to the aquifer. It represents an effective vertical transmissivity through the aquifer and streambed (clogging layer) materials. It is generally determined through model calibration (Reed and Broom 1979). The streambed conductance of a cell is equal to zero if it is not hydraulically connected to a stream or lake.

Constraints defining sustained yield hydraulic stresses are incorporated in both models. Sustained yield pumping values at each of the finite difference cells can be defined as that value of withdrawal from the aquifer averaged over a given time period, which will maintain time-averaged constant potentiometric surface elevations. Although such a situation may appear to be somewhat idealistic, it has been empirically demonstrated (Peralta and Peralta 1984; Yazdanian and Peralta 1985) that a steady-state potentiometric surface is generally maintained over the long term if the total time varying excitation during each year of a given time period equals the appropriate annual steady excitation rate. They demonstrated that occurrence for agricultural systems in which the annual pumping allotment was utilized during a six-month irrigation season. Water levels returned closely to target springtime values during each year of a 10 year simulation period.

Bredehoeft et al. (1982) discussed the convergence to a steady-state potentiometric surface caused by imposing steady stimuli. Knapp and Feinerman (1985) discussed using this phenomenon to optimize sustained groundwater yield planning using dynamic programming.

Alternative optimal sustained yield strategies are obtained by incorporating different sets of physical and managerial constraints. Solutions of both models, with those different sets of constraints, represent alternative optimal sustained yield pumping strategies for the different scenarios. Tested scenarios represent plausible conditions that may have to be satisfied based on institutional and managerial considerations. Presentation of the alternative strategies aids in the informed selection of a single optimal sustained yield pumping strategy for the study area. Because of simplifying assumptions made in the models and the lack of detailed data, computed strategies are useful for reconnaissance evaluation and are not detailed recommendations.

The following section describes the different scenarios which were tested. One of the scenarios tested for model 1 was selected as being most appropriate for implementation. The constraints for this scenario, and a slight variation of it, were then used to pose scenarios and develop strategies using model 2.

General Description of Alternative Scenarios

The scenarios differ because of the assumptions made in the constraining equations. Each set of assumptions and the scenarios to which they apply are discussed here. The following three categories of information are utilized for the solution of the optimization model.

Boundary Conditions

Fig. 2 shows those cells along the study area boundary treated, at least in some scenario, as constant-head cells or as constituents of a constant-head cell subsystem. Those boundary cells not shown as constant-head cells abut an impermeable boundary of the aquifer. Cells along the western boundary above $I = 28$ were treated as no-recharge, no-pumping, variable-head cells. In all scenarios, those boundary cells containing the Arkansas or Mississippi rivers were assumed to be constant-head cells.

It was assumed that a minimum accretion of 100 acre-ft per year occurs at every cell in the region. This is based on water balance simulation and the low vertical permeability of the confining layer (Broom and Reed 1973; Solaimanian 1985). Some cells receive more than 100 acre-ft/yr of vertical recharge, via assumed constant interflow from surface water resources. As suggested by the Corps of Engineers, Vicksburg District (Fred Hoffman, personal communication), no upper limit on recharge was imposed on the boundary cells having stream-aquifer connection with the Mississippi River. For the subsystem of boundary cells having S/A connection with the Arkansas River, the maximum legally permissible recharge was assumed to be 7,240,000 acre-ft/yr. This value is the difference between the average annual flow at Murray Dam gauging station and the minimum annual flow volume required to meet stream water quality criterion (Dixon and Peralta 1984). The recharge in this subsystem, required to implement any one of the optimal strategies, was only a fraction of this value.

An upper bound on recharge of 500 acre-ft per year, including 100 acre-ft vertical accretion, was used for each of the southwestern constant-head boundary cells (Fig. 2). This value is based on historic flow and the fact that although the aquifer extends to the west beyond that artificial boundary, it is unlikely that water need will increase there. Boundary conditions for the cells along the southern boundary of the study area were changed in different scenarios, and are presented later.

The *general boundary conditions*, that are unchanged in all scenarios, can be summarized as:

1. Impermeable boundary along the western periphery of the Bayou Bartholomew basin, above $I = 28$ (Fig. 2).
2. A minimum constant vertical accretion of 100 acre-ft/yr in each cell.
3. No upper bound on recharge for those boundary cells having S/A connection with the Mississippi River.
4. Maximum permissible recharge of 7,240,000 acre-ft/yr for the subsystem of boundary cells having S/A connection with the Arkansas River.
5. Upper bound on recharge of 500 acre-ft/yr for each southwestern boundary cell including and south of $I = 28$.

In order to insure political and physical feasibility, constraints on groundwater flow across the *southern border* (the Arkansas/Louisiana border) were also specified. Four alternative boundary conditions were used for that border. When combined with the *general boundary conditions* they yield the *four types* of boundary conditions that were tested in this study. *Southern boundary specifications* for Types 1–4 boundary conditions are as follow:

- Type 1: In each of the 11 southern boundary cells, up to 500 acre-ft/yr of recharge from Louisiana is allowed per cell. All those cells are treated as c[illegible]-head cells including those with S/A connection: (35, 10), (35, 11), (35, 15), (35, 16), and (35, 17).

- Type 2: Six of the southern boundary cells without S/A connection are treated as a constant-head cell subsystem. The total net recharge for this subsystem is bounded to be less than 600 acre-ft/yr (6 × 100 acre-ft/yr per cell of vertical accretion). Recharge in each of the other five cells is bounded to be less than 500 acre-ft/yr.
- Type 3: All 11 cells on the southern boundary are treated as a constant-head cell subsystem. A total of at least 3,000 acre-ft/yr discharge is forced to occur through this subsystem to Louisiana.
- Type 4: All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 acre-ft/yr on recharge through each cell.

Bounds of Stream/Aquifer Interflow for Internal Cells

The stream/aquifer cells for the three internal rivers (Bayou Bartholomew, Boeuf River, and Bayou Macon) were assumed to be in three different subsystems (Fig. 2). In different scenarios the upper limit on recharges to the aquifer from these rivers were varied to satisfy potential institutional goals, while assuring coarse physical realism. Average interflow (1973–1983) between the aquifer and the Boeuf River, Bayou Bartholomew, and Bayou Macon are −6,700, −9,800 and +4,000 acre-ft/yr, respectively (negative value means recharge to aquifer from stream). Flow to the aquifer increased in the more recent years. Maximum observed interflows were 37,900, 25,800, and 14,000 acre-ft/yr, respectively.

In some scenarios, the maximum estimated recharge through S/A connection was used as the upper bound on S/A recharge. (This is justifiable since recharge has increased in recent years as the potentiometric surface in the aquifer has dropped.) For other scenarios, the average annual S/A interflow was used as an upper bound on recharge at internal cells. This latter approach may be overly conservative.

Bounds on Pumping

The maximum allowable pumping in each internal cell was constrained to be less than one of the following three values:

1. Estimated annual pumping based on current spatially distributed acreage and average climatic conditions (for the entire area this sums to 171,300 acre-ft/yr).
2. Estimated annual pumping in a drought year (1980) for 1980 acreage and climatic conditions. (For the entire area this sums to 353,000 acre-ft/yr).
3. Estimated average annual maximum potential pumping based upon soil types, likely crops, and irrigation scheduling for average climatic conditions. (This is a maximum production situation of interest to the Corps of Engineers.)

Scenarios for Model 1

The scenarios used to obtain the alternative strategies are discussed here. These scenarios differ on the basis of the variations in assumed *boundary conditions*, bounds on *pumping*, and *S/A interflow*. Table 1 summarizes the different scenarios in terms of these three categories of assumptions. It can be noted from Table 1 that the rest of the 19 scenarios are simply variations of scenario 1. To illustrate, in *scenario 1:*

TABLE 1. Scenario Numbering System for Model 1

S/A upper bound (1)	Pumping upper bound (2)	Strategy Number			
		Type 1 boundary conditions (3)	Type 2 boundary conditions (4)	Type 3 boundary conditions (5)	Type 4 boundary conditions (6)
Maximum S/A Recharge	Potential need	1	4	13	—
	Current pumping	2	5	14	—
	1980 pumping	3	6	15	—
Average S/A Recharge	Potential need	7	10	16	19
	Current pumping	8	11	17	—
	1980 pumping	9	12	18	—

1. Type 1 boundary conditions are used.
2. A vertical accretion of at least 100 acre-ft/yr is assumed to occur in each (boundary and internal) cell.
3. The Boeuf River, Bayou Bartholomew, and Bayou Macon are considered as three different stream/aquifer subsystems (Fig. 2).
4. The maximum recharges to the aquifer from each of the three stream/aquifer subsystems are constrained not to exceed the maximum observed annual values.
5. All the southwest boundary cells (Fig. 2) are treated as constant-head cells with a maximum allowable recharge of 500 acre-ft/yr per cell.
6. Maximum potential irrigation demand is used as the upper bound on pumping in each internal cell.

Scenario 2 is the same as scenario 1, except that in each internal cell, that cell's estimated pumping for current acreage and average climatic conditions is used as the upper bound on pumping. Scenario 3 is the same as scenario 1, except that the 1980 pumping value is used as the upper bound on pumping in each internal cell.

Scenarios for Model 2

Scenario 20 represents the use of model 2 with the same assumptions as those of scenario 14. Scenario 21 utilizes the same constraints as scenario 20, except that the upper limit on pumping in each cell is the maximum potential demand for ground water in those cells (i.e., constraints are the same as in scenario 13). At the request of the Corps of Engineers, 1983 water table elevations were used as the target elevations in scenarios 20 and 21.

Results

The described scenarios were used for obtaining alternative sustained yield pumping strategies for the Bayou Bartholomew Basin. The scenario numbering system and total values of pumping, recharge, and S/A interflow, obtained as solutions of model 1 for different scenarios, are shown in Tables 1–4. Comparable information, obtained as solutions of model 2 for Scenarios 20 and 21, is presented in the text. The following discussion describes

TABL… Total Regional Maximum Pumping (Solutions of Model 1)

S/A upper bound (1)	Pumping upper bound (2)	Strategy Number			
		Type 1 boundary conditions (3)	Type 2 boundary conditions (4)	Type 3 boundary conditions (5)	Type 4 boundary conditions (6)
Maximum S/A Recharge	Potential need	344,500	344,500	335,200	—
	Current pumping	156,000	155,700	147,200	—
	1980 pumping	208,700	208,200	201,600	—
Average S/A Recharge	Potential need	148,000	148,000	144,300	175,400
	Current pumping	88,900	88,900	86,900	—
	1980 pumping	109,600	109,600	106,200	—

the process of selecting the most appropriate strategy. First let us consider the effect of southern boundary constraints.

The four types differ in how much recharge is permitted to enter the study area from Louisiana and whether cells on that boundary are treated as variable-head cells, individual constant-head cells, or as parts of a constant-head cell subsystem.

TABLE 3. Total Net Recharge from Boundaries Including Recharge Through Deep Percolation (Accretion)

S/A upper bound (1)	Pumping upper bound (2)	Total Recharge (acre-ft/yr)			
		Type 1 boundary conditions (3)	Type 2 boundary conditions (4)	Type 3 boundary conditions (5)	Type 4 boundary conditions (6)
Maximum S/A Recharge	Potential need	−276,700	−276,900	−269,000	—
	Current pumping	−117,200	−117,100	−103,600	—
	1980 pumping	−165,800	−165,200	−158,800	—
Average S/A Recharge	Potential need	−143,100	−143,100	−141,900	−165,300
	Current pumping	−98,300	−98,300	−95,500	—
	1980 pumping	−116,900	−116,900	−113,500	—

TABLE 4. Total Stream Aquifer Interflow

S/A upper bound (1)	Pumping upper bound (2)	Total Recharge (acre-ft/yr)			
		Type 1 boundary conditions (3)	Type 2 boundary conditions (4)	Type 3 boundary conditions (5)	Type 4 boundary conditions (6)
Maximum S/A Recharge	Potential need	−67,800	−67,800	−67,100	—
	Current pumping	−38,900	−38,900	−43,600	—
	1980 pumping	−43,000	−43,000	−42,900	—
Average S/A Recharge	Potential need	−5,400	−5,400	−2,400	−12,200
	Current pumping	+8,300	+8,300	+8,600	—
	1980 pumping	+7,300	+7,300	+7,300	—

Type 1 boundary conditions permit maximum recharge to the study area through the southern boundary. Type 2 conditions assure no net movement of ground water through the aquifer from Louisiana. Even with type 1 boundary conditions, the resulting optimal strategies did not require any recharge from the Louisiana side of the aquifer. Therefore, type 1 and type 2 boundary conditions produced virtually identical optimal strategies. (The small differences between some of the optimal values for scenarios using type 1 and type 2 boundary conditions are due to the use of convergence criteria for specifying when the optimization algorithm should terminate). Neither type permitted historic ground-water flow rates to Louisiana to continue.

In type 4 boundary conditions, the group of cells along the southern boundary were assumed to be variable-head cells. However, this variation was not rigorously tested because such a relaxation of the constant head conditions along the boundary might lead to large changes in water table elevations along the Louisiana boundary. Such an alternative is politically undesirable and hydrologically disruptive.

Type 3 boundary conditions ensured the continuation of historic discharge to the Louisiana part of the aquifer. Table 2 illustrates that type 3 boundary conditions permit less sustainable pumping than any other type. For example, scenario 13 differs from scenarios 4 and 1 only in southern boundary constraints, yet its sustainable pumping is less. However, since only type 3 conditions maintain historic boundary heads and discharges to Louisiana, only scenarios using this type are considered further.

Next, we review the effect of constraints dealing with S/A interflow. It is reasonable to hope that a sustained yield strategy can provide a total volume comparable to current withdrawals. Current withdrawals are 171,300 acre-ft/yr. Table 2 indicates that none of the strategies using average S/A recharge rates permit nearly this much extraction. (Scenario 17 permits a maximum sustained extraction of 86,900 acre-ft/yr.) For this reason, and because historic S/A interflow has increased with time, only strategies permitting maximum S/A interflow are considered further. Use of these values as upper bounds is not unrealistic since the implementation of an optimal sustained-yield strategy might cause initial decline in the water table elevations along the streams. This would result in increased recharge to the aquifers. (We prefer that a selected strategy not utilize all of the 77,700 acre-ft/yr of total maximum S/A interflow that might be permitted by the model.)

Lastly, we consider the effect of different upper bounds on pumping in each cell (Table 2). The historic annual pumping estimates that were used as upper bounds on pumping in each cell were either current or 1980 values (scenarios 14 and 15, respectively). Maximum sustainable ground-water withdrawal values using current acreages and average climatic conditions as bounds total 147,000 acre-ft/yr (86% of extracted current pumping). The spatial distribution of this optimal pumping reflects current cropping patterns. The 1980 pumping values were significantly greater than the current values because 1980 was a drought year. Therefore, the use of 1980 instead of current withdrawal (pumping) values represented a relaxation of the upper bounds on pumping at each cell. That resulted in an increase in sustainable pumping to 201,600 acre-ft/yr (118% of current pumping). However, this increase in total regional withdrawal was accomplished by sacrificing the more uniform regional distribution of optimal cell-by-cell pumping obtained when using current values as bounds.

The maximum potential demand for ground water at each cell was used as the upper bound on allowable pumping in some scenarios. As seen in Table 2, this resulted in an increase of total sustainable withdrawal from the region compared to that obtained from scenarios which used historic pumping values as the upper limit. However, the resulting optimal pumping was very much concentrated in a small fraction of the entire area. This strategy of permitting ground-water withdrawals according to potential needs diminishes the spatial equity in the distribution of pumping. Such a strategy is socially unrealistic since it would require a massive shift in irrigated acreages from current locations to other locations nearer to recharge sources. Therefore, using historic pumping as an upper bound on pumping at each cell is a more desirable alternative.

In summary, the constraints and boundary conditions of scenario 14 are acceptable if maximization of sustained ground-water withdrawal is the management objective. Salient features include:

1. Use of type 3 boundary conditions preserves the estimated historic ground-water flow into Louisiana.

2. Use of maximum estimated annual S/A recharge as the upper bound on recharge to the aquifer from the three internal rivers is realistic. (When steady state conditions are achieved, S/A recharge will be only 56% of the total upper bound.)

3. Use of estimated current pumping, as the upper bound on cell-by-cell pumping, most effectively maintains the historic spatial distribution of pumping and cropping areas.

Of course, total pumping and spatial equity can probably both be improved by developing an intermediate set of bounds on pumping through the creative use of marginals, shadow prices, or constrained derivatives.

The sensitivity of the optimal strategy of scenario 14 to assumed aquifer parameters was tested. Transmissivities or streambed conductances were changed globally to be either 140 or 60% of the generally assumed values. If transmissivity increased by 40%, total pumping increased 1.7%. If transmissivity decreased by 40%, total pumping increased 1.7%. Proportionally comparable charges in streambed conductance caused a decrease in 3.4% or an increase in 2.5%, respectively. It seems that the optimal strategies are fairly stable with respect to these parameters.

The same constraints used in scenario 14 for model 1 were used for model 2 in scenario 20. The resulting sustainable annual pumping, total net recharge and total stream/aquifer interflow are 52,800, −52,800, and 29 acre-ft/yr, respectively. This is only 31% of that needed for current acreages and average climatic conditions. Therefore, the upper bound on cell-by-cell pumping was increased in scenario 21, to the maximum potential need. The results demonstrate what additional amount of total sustainable pumping can be obtained by relaxing pumping upper bounds. In this scenario (identical to scenario 13), total pumping, net recharge, and S/A interflow are 55,300, −50,800, and −4,500 acre-ft/yr, respectively. Even with this relaxation, total sustainable pumping increased only by about 3,000 acre-ft/yr. Constraints similar to those of scenario 15 were not used with model 2 because the magnitudes of the 1980 pumping values are between the current values and the potential needs. The use of constraints similar to scenario 15 would result in less total sustainable withdrawal than that for scenario 21.

The total optimal pumping values obtained by using model 2 for the two most realistic scenarios are much less than those historically observed. Unless a target potentiometric surface is very stressed and induces much of available recharge, the model 2 objective function should probably not be used if achieving large yield is desired. It should not be used in planning for the Bayou Bartholomew region.

Summary and Conclusions

Estimates of optimal ground-water use and the need for imported surface water are dependent on the specified boundary conditions. Two considerations important in selecting these boundary conditions include: (1) Physical feasibility (based on hydraulic conditions); and (2) managerial feasibility. These criteria were considered in developing alternative optimal sustained-yield pumping strategies for the objectives of: (1) Maximizing sustained yield ground-water extraction (model 1); and (2) maximizing maintenance of current potentiometric surface elevations (model 2).

The choice of a single strategy from a set of alternative strategies requires analysis of institutional and economic consequences. If the goal is to maximize sustainable ground-water pumping, a sustainable pumping strategy for this area should be based on the solution of model 1. However, if the dominant criterion is the maintenance of the potentiometric elevations as close as possible to target levels, then the regional withdrawal policy might be based on the solution of model 2. Of course, any computed optimal regional strategy can be modified and enhanced by changing bounds and constraints or weights in an objective function. Marginal values can be a useful guide in such a process.

This paper uses only two possible objective functions from a variety of objectives and multiple objectives that can be suitable for regional management of a ground-water system. In addition, ground-water quality objectives were not included. Methods of modifying a regional pumping strategy (based solely on quantity considerations), to accommodate quality constraints at specified locations have been developed (Datta and Peralta 1986b). The explicit incorporation of uncertainties and errors in the estimation of aquifer parameters and boundary conditions will certainly modify the results presented here. It must be emphasized that uncertainties due to spatial heterogeneity, nonuniformity, or measurement errors are important issues that must eventually be addressed in optimization models.

Generally, all optimization models include some simplifying assumptions to ensure computational feasibility and data availability. The most significant limitations of this study are: (1) Uncertainties in the estimation of the aquifer parameters were not incorporated explicitly; (2) S/A interactions are modeled assuming steady river stages, although the upper bounds on recharge through S/A interactions were established by using monthly river stage records and then summed to estimate annual interflow; and (3) the objective functions do not explicitly incorporate economic considerations such as *spatially variable* pumping costs and benefits. A detailed evaluation of a regional ground-water management strategy must address these limitations.

This study establishes the fact that even when ignoring the uncertainties in parameter estimations, a large number of management strategies can be designated as "optimal" within the scope of the formulated model. In this respect, the evolved alternative strategies are only a subset of a complete set of "alternative optimal" strategies. The absolute optimality of any of the developed strategies and others incorporating uncertainties and explicit economic consideration is difficult to establish. This paper only demonstrates the alternative optimal solutions of the optimization models, with variations in some institutional, physical, and managerial constraints. The specific solutions are useful for reconnaissance level evaluation.

Our case study clearly demonstrates the relevancy and significance of imposed physical and institutional constraints for obtaining alternative regional pumping strategies that optimize specified objective functions. This study also depicts somewhat the degree of detail that can be incorporated in the management models, given the typical limitations imposed by sparsity of data related to physical parameters of the aquifer, and difficulties in estimating spatially distributed historically pumped quantities of ground water.

In addition, this study demonstrates the utility of the embedding approach for computing optimal sustained yield (steady-state) ground-water management strategies for realistic and reasonable large aquifer systems.

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Appendix I. References

Azarmnia, H. (1989). "Selected tools for optimal regional groundwater management," thesis presented to the University of Arkansas, at Fayetteville, Ark., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Bredehoeft, J. D., Papadopulos, S. S., and Cooper, H. H., Jr. (1982). "Groundwater: The water budget myth." *Scientific basis of water resources management*. National Academy Press, Washington, D.C., 51–57.

Broom, M. E., and Reed, J. E. (1973). "Hydrology of the Bayou Bartholomew alluvial aquifer stream system, Arkansas." *Geological survey, progress report*, U.S. Dept. of the Interior, Washington, D.C.

Cohon, J. L., and Marks, D. M. (1975). "A review and evaluation of multiobjective programming techniques." *Water Resour. Res.*, 11(2), 208–220.

Datta, B., and Orlob, G. T. (1988). "Elements of groundwater management: The saturated zone." *Proc., 16th Biennial Conf. on Groundwater, Report No. 66 ISSN 0575-4968*, California Water Resources Center, Univ. of California, 3–21, J. DeVries (Ed).

Datta, B., and Peralta, R. C. (1986a). "Interactive computer graphics-based multiobjective decision-making for regional groundwater management." *Agric. Water Mgmt.*, 11, 91–116.

Datta, B., and Peralta, R. C. (1986b). "Optimal potentiometric surface design for groundwater contamination containment." *Trans.*, American Society of Agricultural Engineers, 29(6), 1611–1623.

Dixon, W. D., and Peralta, R. C. (1986). "Potential Arkansas and White Rivers water available for diversion to the Grand Prairie." *Arkansas state water plan special report*, Arkansas Soil and Water Conservation Commission, Little Roc[illegible]

Gorelick, S. M. (1983). "A review of distributed parameter groundwater management modeling methods." *Water Resour. Res.*, 19(2), 305–319.

Knapp, K. C., and Feinerman, E. (1985). "The optimal steady-state in groundwater management." *Water Resour. Bull.*, 21(6), 967–975.

McDonald, M. G., and Harbaugh, A. W. (1988). "A modular 3-dimensional finite difference ground water flow model." *Techniques of water resources investigations of the U.S.G.S. (6)*, U.S. Government Printing Office, Washington, D.C.

Morel-Seytoux, J. J., Peters, G., and Illangasekare, T. (1979). "Field verification of the concept of reach transmissivity." *Proc. Int. Symp. on Hydrology of Areas of Low Precipitation*, International Association of Scientific Hydrology, Canberra, Australia.

Peralta, R. C., and Killian, P. (1985). "Optimal regional potentiometric surface design: Least-cost water supply/sustained groundwater yield." *Trans.*, American Society of Agricultural Engineers, 28(4), 1098–1107.

Peralta, R. C., and Peralta, A. W. (1984). "Arkansas groundwater management via target level." *Trans.*, American Society of Agricultural Engineers, 27(6), 1696–1703.

Peralta, R. C., et al. (1985). "Optimal sustained yield groundwater withdrawal strategies for the Boef-Tensas Basin in Arkansas." *Miscellaneous Publication No. 29*, Arkansas Water Resour. Res. Ctr., Univ. of Arkansas, Fayetteville, Ark.

Peralta, R. C., et al. (1989). SSTAR users manual, *Report IIC-89/2*, Int. Irrig. Ctr., Dept. of Agric. and Irrig. Engrg., Utah State Univ., Logan, Utah.

Reed, J. E., and Broom, M. D. (1978). "Digital model of the Bayou Bartholomew alluvial aquifer system, Arkansas." Geological Survey, U.S. Dept. of the Interior, Washington, D.C.

Solaimanian, J. (1985). "Quaternary groundwater assessment and consumptive water use in the Bayou Bartholomew/Alluvial aquifer system, Arkansas," thesis presented to the University of Arkansas, at Fayetteville, Ark., in partial fulfillment of the requirements for the degree of Master of Science.

Tung, Y. K., and Kolterman, C. E. (1986). "Some computational experiences using embedding technique for groundwater management." *Groundwater*, 23(4), 455–464.

Willis, R., and Yeh, W. W-G. (1987). *Groundwater systems planning and management*. 1st Ed., Prentice Hall Inc., New York, N.Y.

Yazdanian, A., and Peralta, R. C. (1986). "Sustained-yield ground water planning by goal-programming." *Groundwater*, 24(2), 157–165.

Yazicigil, M., and Rasheeduddin, M. (1987). "Optimization model for groundwater management in multi-aquifer systems." *J. Water Resour. Plan. Mgmt. Div.*, ASCE, 113(2), 257–23.

Yeh, W. W-G. (1986). "Review of parameter estimation procedures in groundwater hydrology: The inverse problem." *Water Resour. Res.*, 22(2), 95–108.

Appendix II. Notation

The following symbols are used in this paper:

a_k = weighting factor assigned to achievement of target drawdown in cell k (dimensionless);
$f(\cdot)$ = function of;
G_k = ground surface elevation in cell k (L);
$H_{i,j}$ = steady-state head in grid (i,j) (cell k);
H_k = optimal steady-state head in cell k (L);
H_k^L = minimum allowable steady-state head permitted in cell k;
H_k^s = elevation of water surface in stream in cell k (L);
H_k^t = target (specified) steady-state head in cell k (L);
K_r = total number of cells belonging to S/A subsystem r;

n_c = total number of cells in system (study area);
n_r = total number of S/A subsystems in study area;
Q_k = withdrawal or recharge in cell k (coordinates i, j), except for that due to steam-aquifer interflow (L^3/T);
$Q_{k,r}$ = stream-aquifer interflow for cell k, belonging to stream aquifer (S/A) subsystem r (L^3/T);
$Q_{max,k}$ = maximum allowable pumping (withdrawal) or recharge in cell k, determined by total water supply demand in cell k (L^3/T);
$Q_{min,k}$ = minimum allowable pumping or recharge in cell k (L^3/T);
$SQ_{r,max}$ = maximum allowable total recharge to aquifer for cells in stream aquifer subsystem r (L^3/T);
T'_k = positive valued streambed conductance of cell k (L^2/T); and
$t_{i,j}$ = transmissivity at center of grid (i,j) (cell k).

Appendix III. Conversion to SI Units

To convert	To	Multiply by
acre	m^2	4.047×10^3
acre-ft	m^3	1.23×10^3
mi	m	1.609×10^3
ft/sec	m/s	0.304