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A framework for non-drastic innovation with product differentiation

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Abstract

We model non-drastic technological innovation in a duopoly model with differentiated products. We derive profit functions for both firms which depend on only one variable, the technological gap. As our model derives product demands directly from agent utility we are able to fully describe the welfare effects of innovation. We show that the welfare improvements from innovation come not only as firms accrue higher profits, by charging consumers higher prices, but also as consumers enjoy higher quality products.

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1 Introduction

The innovation and intellectual property rights literature is both vast and expansive. The quality ladder model of sequential cumulative innovation is now well known. In this model innovation forms a discrete sequence of steps along a ladder with lower rungs representing lower quality (technology) and higher rungs representing superior quality. As the quality of the product reaches the top rung through innovation the innovating firm garners monopoly profits if the innovation was of a sufficient magnitude to be patentable. This issue of breadth has been a main focus of the literature.[1] If the breadth requirement is narrow this could encourage rapid innovation in small steps, but the result could be many similar products. If the breadth requirement is too wide, then innovation may occur slowly (See Scotchmer (2004) for a summary). While the questions of when a firm should patent and when does an innovation infringe on a previous patent versus deserve a new patent are very important topics, these questions are not the focus of this research.

Our focus is on non-drastic innovation. Drastic innovation is what is commonly assumed in the traditional quality ladder models whereby the innovating firm enjoys monopoly profits. Non-drastic innovation, on the other hand, is characterized by continued competition after innovation as would occur in markets with product differentiation. Kamien and Tauman (1986) explore drastic and non-drastic innovation and the incentives to offer a license in exchange for royalties or a fixed fee. Their model contains no product differentiation with innovations being cost improvements. This is also the approach of Marjit (1990) and Wang (1998) who examine the licensing problem in a Cournot framework rather than the Stackelburg framework of Kamien and Tauman (1986). Wang (2002) extends the analysis to differentiated products in the Cournot framework. A drawback to all of these is that the role of the demand side of the economy is fairly neglected. We contend that in the case of non-drastic innovation with product differentiation, this is a major flaw. Muto (1993) derives demand for differentiated products that compete in a Bertrand style framework however innovation is modeled as cost reducing and is owned by an external patentee.

This paper extends the literature by advancing a model of sequential non-drastic innovation in the presence of differentiated products. Grossman and Helpman (1991) examine this type of differentiated innovation whereby each firm's differentiated output follows its own stochastic progression along a quality ladder bringing about quality improvements of existing products. Their main concern is the implications that such innovations have for the long run rate of growth in the economy. Our focus here is on setting up a framework which can be used to analyze various intellectual property rights regimes while shedding light on the welfare properties of innovation.

One industry that follows the pattern of non-drastic innovation (as quality improvements) with differentiated products is agricultural biotechnology. Innovation in this industry involves the creation of new plant varieties with specific traits using existing plants and genetic code as the building blocks. In this sense agricultural biotechnology innovations are cumulative. However, different farms with different weather and soil concerns demand seed with different traits. In a region where water is scarce, a seed with a drought resistant trait

[1] See Chang (1995), Green and Scotchmer (1995), O'Donoghue (1998), O'Donoghue et al. (1998), and Hopenhayn and Mitchell (2001).

is highly desirable while in regions with no water shortages resistance to herbicides and pesticides (roundup readiness) is more highly valued. Given that a farm's needs are best suited by a specific trait, seed producing companies can focus research efforts on producing seed targeted for a specific trait with innovations bringing about increases in yield (quality). When an innovation takes place that improves the yield of a specific trait, that seed may garner a larger portion of the total market share but it is unlikely that the innovation is of high enough magnitude that no farm wishes to buy a seed variety with the unimproved trait.[2] The innovation is non-drastic.

Although firm quality levels are truly the endogenous result of stochastic innovation through time, the purpose of this paper is to buid a framework for and analyze welfare effects of innovation taking it as *exogenous*.[3] This allows a focus on the trade-off between quality and firm pricing decisions. A firm with improved quality may increase the price of its product which could result in lower agent utility. This is especially true if the innovating firm is already technologically advantaged (at a higher rung on her quality ladder). If the technologically disadvantaged firm innovates this can cause increased competition in the product market and may lead to an increase in agent welfare at the expense of reduced firm profitability.

We model innovation as two firms move along their own respective quality ladder with each firm selling a product characterized by product differentiation. Our underlying model of the product market pulls features from both Malla and Gray (2005) and Tangerås (2009) following the tradition of Hotelling (1929). We derive profit functions for two firms in the presence of non-drastic innovation demonstrating that profit depends only on the technological gap between firms. We then compute comparative statics demonstrating that innovation increases consumer utility, firm profits, and welfare as a whole regardless of the identity of the innovating firm (technologically advantaged or disadvantaged). Welfare increases by the largest magnitude when both firms innovate.

Section 2 presents our framework for innovation and section 3 derives the welfare effects of innovation. Section 4 contains the conclusion.

2 A Framework for Innovation

Two firms produce differentiated products which they sell either directly to consumers or to other producers as intermediate goods. Firms, indexed by i and j, produce and sell their product with quality of $y^i \in \mathbb{R}^+$. Each firm simultaneously chooses the price for its product so as to maximize profits in a one stage game. Exogenous to the model, each firm may innovate as firm product quality experiences a discrete jump along that individual firms' quality ladder where each rung on the quality ladder for both firms are distance $\Gamma > 0$ apart.[4]

[2]This is exactly the type of innovation described by Malla and Gray (2005).

[3]For a detailed analyis of the endogenous choice of quality each firm faces and its relationship to the intellectual property regime, please see Jackson and Smith (2014).

[4]Innovation is exogenously treated in Kamien and Tauman (1986), Wang (1998), and Wang (2002). This is tantamount to treating R&D expenditures as an exogenous cost of conducting business.

There are a continuum of agents, with measure one, uniformly distributed over the unit interval. These agents could be consumers purchasing a final good such as cell phone services or firms purchasing an intermediate good to be used in the production of another good to be sold in the marketplace such as a farmer purchasing seed. The location of the agent identifies their preference for a specific trait in the product they purchase. Each agent demands at most one unit and each firm is situated at one end of the interval, $i = 0, 1$. Let w^i be the price firm i charges for its product and $\tau > 0$ be the marginal loss an agent bears from buying a product not at their most preferred location. Thus, an agent located at $\Psi \in [0, 1]$ gets a payoff of $y^0 - \tau\Psi - w^0$ when she buys the product of firm 0 and $y^1 - \tau(1 - \Psi) - w^1$ when she buys by the product of firm 1.

We can identify the agent who is indifferent between buying from each firm and identify her location as Ψ^*. Because each agent buys at most one unit, demand for firm zero's product is

$$\Psi^0 = \Psi^* = \frac{(y^0 - y^1) + (w^1 - w^0) + \tau}{2\tau}$$

and demand for firm one's product is

$$\Psi^1 = 1 - \Psi^* = \frac{(y^1 - y^0) + (w^0 - w^1) + \tau}{2\tau}.$$

Let T be a parameter that represents the marginal cost of marketing and reproducing a firm's product. Firms compete in a simultaneous move price setting game. Each firm sets its own price according to its best response as derived from the following maximization problem

$$\max_{w^i} \left(w^i - T\right) \Psi^i.$$

which results in the following best response function

$$w^i(w^j) = \frac{(y^i - y^j) + w^j + \tau + T}{2}.$$

The Nash equilibrium prices are then

$$w^{i*} = \frac{(y^i - y^j)}{3} + \tau + T.$$

These can then be used to identify the location of the indifferent agent, Ψ^*, as

$$\Psi^* = \frac{(y^i - y^j)}{6\tau} + \frac{1}{2}.$$

Throughout our analysis we maintain three assumptions regarding the product market. Assumption 2.1 guarantees that the entire market is served by one of the two firms.

Assumption 2.1. $\frac{y^0+y^1}{2} > \tau + T + \frac{1}{2}$

Assumption 2.2 guarantees that each firm has positive demand restricting our analysis to non-monopoly environments.

Assumption 2.2. $\mid y^i - y^j \mid < 3\tau$

Together, assumptions 2.1 and 2.2 guarantee that the two firms are actively competing against one another in the product market. This allows us to describe the profit made by each firm as it depends on the technological gap to be

$$\pi^i(y^i, y^j) = \frac{(y^i - y^j)^2 + 6\tau(y^i - y^j) + 9\tau^2}{18\tau}.$$

In this paper we restrict the magnitude of innovation to be non-drastic. Innovation is of a small enough magnitude so that firms continue to compete even after an innovation occurs. Innovation is not large enough to allow one firm to drive the other out of the market. This is captured in assumption 2.3. If this assumption is violated the market will be dominated by one firm after the technological leader successfully innovates leaving us squarely in the realm of the previous quality ladder literature with monopoly switching.

Assumption 2.3. $\Gamma < 3\tau - \mid y^i - y^j \mid$

Equilibrium payoffs in the product market do not depend on the exact quality of each firms output; equilibrium payoffs depend only on the difference in quality across the two firms. Let k be the quality differential defined as $k = (y^0 - y^1)$. If $k > 0$ ($k < 0$) then firm 0 is said to be technologically advantaged (disadvantaged) and firm 1 is technologically disadvantaged (advantaged). With this notation product market strategies and equilibrium payoffs depend only on the variable k allowing the profit equations to be rewritten as follows.

$$\pi^0(k) = \frac{k^2 + 6\tau k + 9\tau^2}{18\tau} \tag{1}$$

and

$$\pi^1(k) = \frac{k^2 - 6\tau k + 9\tau^2}{18\tau}. \tag{2}$$

Innovation determines y^0 and y^1 which in turn determines $k = y^0 - y^1$. This framework sets up the possibility of analyzing various intellectual property rights regimes in the context of a markovian game with only one state variable, k. The movement of k across time could depend on R&D investment and equilibrium patterns of shared (licensed) intellectual property[5]. Jackson and Smith (2014) use the framework of non-drastic innovation developed here to analyze the pricing and purchase decisions of experimental use licenses with innovation being an endogenous product of shared intellectual property.

[5]Setting up a full markovian game would require consideration of drastic innovation as well. Even if the game starts where an innovation won't lead to monopolization, it is possible that over time one firm could garner a sufficient technological advantage to drive the other out of the market. We contend that this would take a long time.

3 The Welfare Effects of Innovation

The model of agents and firms as they interact in the product market set forth in the previous section allows us to explore the welfare effects of innovation and answer such questions as: Does innovation by one firm increase welfare or decrease welfare and does it matter if the innovating firm is the technological leader or follower? We answer these questions by conducting comparative static analysis on innovation. Such analysis is not trivial as innovation by one firm necessarily impacts not only the profits of both firms but also the prices paid by consumers. As R&D expenditures are external to the model developed here, they are not pertinent to welfare analysis. That R&D takes place generates a given fixed cost of market participation.

We begin the analysis by first focusing on agents (consumers) in the product market. To simplify matters we split agents into two groups. Those who buy from firm 0 and those who buy from firm 1. Total agent welfare is the total payoff for each agent group. We let TU^0 be the total payoff to agents purchasing from firm 0 and TU^1 be the total payoff to agents purchasing from firm 1 so that the total payoff to all agents can be written as $TU = TU^0 + TU^1$. Equation (3) for TU given below is derived in the appendix.

$$TU(y^0, y^1) = \left(\frac{2y^1}{3} + \frac{y^0}{3} - T - \frac{3\tau}{2}\right) + \left(\frac{(y^0 - y^1)}{3}\right)\left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2}\right) \tag{3}$$

As equilibrium product prices depend on the quality gap, which depend on innovation, the effect of innovation on total agent welfare isn't obvious. When only one firm innovates the agents who purchase the innovated product will pay a higher price but will ultimately receive a higher quality product. Proposition (1) demonstrates that total agent welfare as given in equation (3) increases when a technological innovation is made by either the technologically advantaged firm or the technologically disadvantaged firm. All of the benefits of innovation do not end up residing solely in profits to the innovating firm.

Proposition 1. *Agent welfare always increases when either firm innovates.*

Proof. See Appendix. □

We now consider the effects of innovation on firm profits. When only one firm innovates that firm is able to increase its market power resulting in a higher product price and profits. The following lemma demonstrates that an innovation by one firm is always profitable for the innovating firm.

Lemma 1. *When one firm innovates, profits for the innovating firm increase.*

Proof. See Appendix. □

We now combine firm profits and total agent payoffs to get a measure of total welfare in the product market. It is straightforward to show that

$$\pi^0(k) + \pi^1(k) = \frac{(y^0 - y^1)^2}{9\tau} + \tau.$$

Therefore, total welfare, $TW = TU + \pi^0(k) + \pi^1(k)$, can be written as

$$TW = \left(\frac{2y^1}{3} + \frac{y^0}{3} - T - \frac{\tau}{2}\right) + \left(\frac{(y^0 - y^1)}{3}\right)\left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2}\right) + \frac{(y^0 - y^1)^2}{9\tau}$$

Intuition suggests that if an innovation is both profitable and increases agent welfare it should also have a positive impact on total welfare in the economy. The only way this could not be true would be if the non-innovating firm suffered a decrease in profits greater than the gains to the innovator and agents. That total welfare increases is demonstrated in proposition (2).

Proposition 2. *Total welfare increases when either the technologically advantaged or technologically disadvantaged firm innovates.*

Proof. See Appendix. □

Likewise, if an innovation by one firm increases profits and agent welfare then it is also likely to be the case that total welfare increases whenever both firms simultaneously innovate. Such an innovation produces no change in the technological advantage of either firm[6] but does increase the quality of all of the products purchased by agents. Proposition (3) establishes that a simultaneous innovation by both firms will increase welfare. All potential profits from the innovation are competed away and the benefits of simultaneous innovation accrue exclusively to agents as they purchase a higher quality product at the same price.

Proposition 3. *Total welfare increases when both the technologically advantaged and disadvantaged firms simultaneously innovate.*

Proof. See Appendix. □

A final pertinent question that needs to be answered is whether total welfare increases more when innovation happens one firm at a time or when both firms innovate simultaneously. Proposition 4 provides proof that total welfare increases more when both firms simultaneously innovate than when only one firm innovates. Innovation by one firm only creates a profit opportunity in which the innovating firm captures some of the new found surplus generated for consumers and steals some of the other firms market share. Likewise, innovation by one firm only increases the quality of product consumed by a subset of the agents. However, when both firms innovate simultaneously there is no competitive edge gained or increase/decrease in profits for either firm and all agents enjoy an increase in product quality with no increase in price paid.

Proposition 4. *Welfare increases by a larger magnitude when both firms innovate rather than innovation by only one firm.*

[6] Both firms advance to the next rung on their respective quality ladder producing no change in differential quality.

Proof. See Appendix. □

4 Conclusion

In this paper we derived a model of duopoly competition in the presence of product differentiation. Previous literature on non-drastic innovation did not model consumers nor derive demand in the product market. We derived our model from primitives on consumer preferences which allow us to analyze the welfare effects of non-drastic innovation. We show that non-drastic innovation always produces an increase in total welfare but welfare increases are the largest when both firms simultaneously innovate. This can have policy implications for the construction of intellectual property rights surrounding innovation. An intellectual property rights regime that encourages the sharing of intellectual property in R&D, such as one granting an experimental use exemption, may welfare dominate a regime with licensing requirements.

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A Welfare Proofs

Proof. Proof of Proposition 1

Calculate the partial derivatives of $TU(y^0, y^1)$ as

$$\frac{\partial TU}{\partial y^0} = \frac{1}{2} + \frac{(y^0 - y^1)}{9\tau}$$

$$\frac{\partial TU}{\partial y^1} = \frac{1}{2} - \frac{(y^0 - y^1)}{9\tau}.$$

If $y^0 > y^1$ then $\frac{\partial TU}{\partial y^0} > 0$. If $y^0 < y^1$ then $\frac{\partial TU}{\partial y^1} > 0$. So that agent welfare increases when the leader innovates.

Let $y^i > y^j$. Then $\frac{\partial TU}{\partial y^j} > 0$ if $y^j - y^i > -\frac{9\tau}{2}$ which can be rewritten as $| y^j - y^i | < \frac{9\tau}{2}$ which is true by assumption 2.2. Thus welfare always increases when the follower innovates. □

Proof. Proof of Lemma 1

Inspection of the functional form of $\pi^0(k)$ and $\pi^1(k)$ reveal that they are each a parabolic in shape with a minimum occurring at $k = -3\tau$ and $k = 3\tau$, respectively. Each firm's profits are the smallest when they are technologically behind by a magnitude of 3τ. The assumption 2.2 guarantees that an innovation will increase profits. □

Proof. Proof of Proposition 2

Suppose $y^0 > y^1$.

$$\frac{\partial TW}{\partial y^0} = \frac{1}{2} + \frac{2(y^0 - y^1)}{18\tau} + \frac{2(y^0 - y^1)}{9\tau} = \frac{1}{2} + \frac{(y^0 - y^1)}{6\tau}$$

$$\frac{\partial TW}{\partial y^1} = \frac{1}{2} - \frac{2(y^0 - y^1)}{18\tau} - \frac{2(y^0 - y^1)}{9\tau} = \frac{1}{2} - \frac{(y^0 - y^1)}{6\tau}$$

Since $y^0 > y^1$ then $\frac{\partial TW}{\partial y^0} > 0$. Combined with assumption 2.2 we also have $\frac{\partial TW}{\partial y^1} > 0$. □

Proof. Proof of Proposition 3 From the equation for TW we see that if both y^0 and y^1 increase by a magnitude of Γ due to a simultaneous innovation by both firms. Then total welfare increases by the size of the innovation, $\Gamma > 0$.

□

Proof. Proof of Proposition 4

Since both partial derivatives of total welfare are positive but less than one, as a result of assumption 2.2, the welfare increase when both firms innovate, $\Gamma > 0$, is larger.

$$\Gamma > \Gamma \frac{\partial TW}{\partial y^i}$$

$\square$

A.1 Derivation of Equation 3

$TU = TU^0 + TU^1$. Each of these pieces can be computed as an integral.

$$TU^0 = \int_0^{\Psi^*} \left(y^0 - \tau\Psi^0 - w^0\right) d\Psi^0$$

Substituting for equilibrium price w^{0*} from above we get

$$TU^0 = \int_0^{\Psi^*} \left(y^0 - \tau\Psi^0 - \frac{y^0 - y^1}{3} - \tau - T\right) d\Psi^0.$$

We then compute the integral to get

$$TU^0 = \left[\left(y^0 - \frac{(y^0 - y^1)}{3} - \tau - T\right) \Psi^0 - \frac{\tau {\Psi^0}^2}{2}\right]\Bigg|_{\Psi^0=0}^{\Psi^0=\Psi^*}$$

which reduces to

$$TU^0 = \left(y^0 - \frac{(y^0 - y^1)}{3} - \tau - T\right) \Psi^* - \frac{\tau {\Psi^*}^2}{2}.$$

Then substitute Ψ^* and rearrange terms to get

$$TU^0 = \left(\frac{2y^0}{3} + \frac{y^1}{3} - \tau - T\right)\left(\frac{(y^0 - y^1}{6\tau} + \frac{1}{2}\right) - \frac{\tau}{2}\left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2}\right)^2$$

Likewise

$$TU^1 = \int_{\Psi^*}^{1} \left(y^1 - \tau\Psi^1 - w^1\right) d\Psi^1$$

Substituting for equilibrium price w^{1*} from above we get

$$TU^1 = \int_{\Psi^*}^{1} \left(y^1 - \tau\Psi^1 - \frac{y^1 - y^0}{3} - \tau - T\right) d\Psi^1.$$

We then compute the integral to get

$$TU^1 = \left[\left(y^1 - \frac{(y^1 - y^0)}{3} - \tau - T \right) \Psi^1 - \frac{\tau {\Psi^1}^2}{2} \right] \Bigg|_{\Psi^1 = \Psi^*}^{\Psi^1 = 1}$$

which reduces to

$$TU^1 = \left(y^1 - \frac{(y^1 - y^0)}{3} - \tau - T - \frac{\tau}{2} \right) - \left(y^1 - \frac{(y^1 - y^0)}{3} - \tau - T \right) \Psi^* + \frac{\tau {\Psi^*}^2}{2}.$$

Then substitute Ψ^* and rearrange terms to get

$$TU^1 = \left(\frac{2y^1}{3} + \frac{y^0}{3} - T - \frac{3\tau}{2} \right) - \left(\frac{2y^1}{3} + \frac{y^0}{3} - \tau - T \right) \left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2} \right) + \frac{\tau}{2} \left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2} \right)^2$$

These can be combined to give an expression for the total payoff to all agents in the market as

$$TU(y^0, y^1) = \left(\frac{2y^1}{3} + \frac{y^0}{3} - T - \frac{3\tau}{2} \right) + \left(\frac{(y^0 - y^1)}{3} \right) \left(\frac{(y^0 - y^1)}{6\tau} + \frac{1}{2} \right).$$