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CONJUNCTIVE USE/SUSTAINED GROUNDWATER YIELD DESIGN

Richard C. Peralta, A.M. ASCE[1]

ABSTRACT

Assuring the sustained availability of groundwater from all parts of an aquifer system is analagous to assuring that the potentiometric surface does not change over the long term. Such a steady-state surface is maintained by a specific spatially distributed pattern of groundwater withdrawal. The finite difference form of the linearized Boussinesq equation for steady two-dimensional flow through porous media is used in models that design optimal regional potentiometric surfaces and the conjunctive water use/sustained yield strategies that maintain them. Presented objectives of such models include minimization of unmet water needs, minimization of the regional cost of attempting to satisfy water needs and bi-objective optimization.

INTRODUCTION

"Seventy percent of the 1.8 billion people requiring new supplies of water during the International Water Decade should be provided with supplies from groundwater" (15). Assuring the sustained availability of groundwater is important for many regions of the world. In some cases, satisfying real or desirable water demand in a sustained yield scenario necessitates the coordinated (conjunctive) use of groundwater and surface water.

Assuring a sustained yield requires insuring that, on the long-term, as much water enters each part of an aquifer as leaves it. This is analagous to achieving steady-state conditions. An infinite number of sustained yield strategies are possible for any aquifer system. Assuming that diverted surface water can also be used in many "cells" of a system, the question arises as to how much groundwater and diverted river water should be used in each. Since an infinite number of sustained yield strategies are possible for any aquifer system, an infinite number of conjunctive water use/sustained groundwater withdrawal strategies also exist. Depending on regional objectives, some strategies may be more desirable than others. The purpose of this paper is to compare the results of including several different objectives in models that develop optimal regional strategies.

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Agricultural Engineering Department, University of Arkansas, Fayetteville, Arkansas, U.S.A.

THEORY AND MODEL FORMULATIONS

Governing Equations

Development of a regional steady-state set of target groundwater levels requires the use of a steady-state equation for each cell. The following has been developed for two-dimensional steady flow in a heterogeneous isotropic aquifer from both the linearized Boussinesq equation (7,14) and the Darcy equation (11):

$$q_{i,j} = -t_{i-1/2,j}\, s_{i-1,j} - t_{i+1/2,j}\, s_{i+1,j} + [t_{i-1/2,j} + t_{i+1/2,j} + t_{i,j-1/2} + t_{i,j+1/2}]\, s_{i,j} - t_{i,j-1/2}\, s_{i,j-1} - t_{i,j+1/2}\, s_{i,j+1} \quad (1)$$

where $q_{i,j}$ is the net volume flux rate of groundwater moving moving into or out of the aquifer in cell (i,j). It is positive when flow is out of the aquifer, negative when flow is into the aquifer, (L^3/T).

$s_{i,j}$ is the vertical distance between a horizontal datum located above the ground surface, and the potentiometric surface. In this paper it is a steady state drawdown, (L).

$t_{i-1/2,j}$ is the geometric average of the transmissivities of cells (i,j) and (i-1,j), (L^2/T).

To express this equation in matrix form for a groundwater system, the row-column notation is replaced with single integer identification of each cell. Thus for a groundwater flow system of n cells:

$$(Q) = [T](S) \quad (2)$$

where (Q) is an n x 1 column vector of net steady-state volume flux values, (L^3/T).

[T] is an n x n symmetric diagonal matrix of finite difference transmissivities, (L^2/T).

(S) is a column vector of steady-state drawdowns, (L).

The following equation describes the range of acceptable flux values that are in harmony with a regional aquifer volume balance.

$$(L_q) \le (Q) = [T](S) \le (U_q) \quad (3)$$

where (L_q) and (U_q) are n x 1 column vectors whose elements respectively are the lower and upper bounds on volume flux in all cells in the system, (L^3/T).

The appropriate range of potentiometric surface values is described by:

$$(L_s) \le (S_*) \le (U_s) \quad (4)$$

where (L_s) and (U_s) are m x 1 column vectors of the lower and upper bounds, respectively, on the optimal steady-state drawdowns in the m internal cells, (L).

(S_*) is an m x 1 vector of optimal drawdowns, (L).

Both Equations 3 and 4 are used as constraints within the three models (strategies) discussed below. Optimization within the models was accomplished using the QPTHOR subroutine (9).

Minimizing Unsatisfied Demand or Maximizing Groundwater Withdrawal (Strategy A)

It is assumed that diverted river water is available only in certain cells within the study area and that adequate diverted water is available to completely satisfy water needs in those cells. In cells in which no diverted surface water is available, only groundwater is used. Minimizing unsatisfied water needs for such cells is accomplished by maximizing groundwater usage in those cells. The linear objective function used to maximize groundwater pumping, p, for a group of mm cells (8) is similar to formulations used by other researchers for small systems (1,2,3):

$$\max z = \sum_{i=1}^{mm} p(i) \quad (5)$$

subject to Equations 3 and 4,

where z is the total volume of groundwater annually pumped from mm cells.

In this formulation, surface water is available in mc of the internal cells. Therefore, the number of cells without the alternative source, mm, equals m-mc.

Minimizing Regional Cost of Conjunctive Water Supply (Strategy B)

In this paper, we make use of a quadratic optimization model (10) that minimizes the total cost of attempting to satisfy regional demand from conjunctive water resources. The model uses the costs of groundwater and diverted surface water in cells in which diverted water is available. It uses the cost of groundwater and the opportunity cost of unsatisfied water needs in cells in which diverted water is unavailable. A simple statement of the model is:

$$\min y = \sum_{i=1}^{n} c_e(i)\, p(i)\, f(s(i)) + c_m(i)\, p(i) + c_a(i)\, p_a(i) \qquad (6)$$

subject to Equations 3 and 4,

where:

- y = the total annual cost of the water supply and the opportunity costs of inadequate supply, ($/yr).
- $c_e(i)$ = the pumping plant energy, repair and lubrication costs associated with raising a volume of groundwater one unit distance, ($\$/L^4$).
- $f(s(i))$ = a linear function of steady state drawdown which describes the total dynamic head at cell i, (L).
- $c_m(i)$ = the pump maintenance cost of pumping a unit volume of groundwater, ($\$/L^3$).
- $c_a(i)$ = either the cost per unit volume of river water used in cell i to which water can be diverted, or, the opportunity cost associated with each unit volume of unmet needs in that cell, ($\$/L^3$).
- $p_a(i)$ = either the annual volume of diverted water or the annual volume of unsatisfied demand in cell i, (L^3/yr).

Biobjective Optimization between Minimizing Cost and Minimizing Unsatisfied Water Needs (Strategy C)

The constraint method of multiobjective optimization is commonly used to develop the pareto optimum for the simultaneous consideration of multiple objectives (5). An application of this method, described by Killian and Peralta (unpublished manuscript) was used in this paper to simultaneously consider minimizing cost while minimizing unsatisfied water demand. To avoid having nonlinear constraints in the optimization formulation, the linear maximum pumping function (Equation 5) is used as the constrained objective and the quadratic least-cost objective function (Equation 6) is the primary function.

SAMPLE APPLICATION AND RESULTS

The physical and legal feasibility of utilizing a regional strategy of spatially distributed groundwater withdrawals to maintain a steady-state potentiometric surface has been demonstrated for Arkansas (10). Within the state water plan (11), it is proposed that such a strategy is implementable only in regions with a critical groundwater problem and where adequate divertable river water is available to satisfy existing demand that cannot be met by groundwater. The Grand Prairie of Arkansas (Figure 1), an important agricultural and aquacultural production region that has relied heavily on groundwater, is potentially such a region.

Groundwater levels in the Grand Prairie have been declining and Peralta et al (13) project further declines. Water users and managers are interested in the possibility of assuring the sustained availability of groundwater in the region. Dixon and Peralta (unpublished manuscript) have estimated that there is adequate divertable water in nearby rivers to significantly reduce reliance on groundwater. Since groundwater simulation models have been validated for the Grand Prairie (4,13), and aquifer parameters are known with reasonable confidence, the Grand Prairie is an appropriate region for demonstration of the techniques presented in this paper.

Assumptions and Constraints

The following assumptions are used in applying the models to the Arkansas Grand Prairie. Since recharge is negative in sign and there is insignificant deep percolation (recharge) to the aquifer in internal cells in the Grand Prairie, the lower bound on volume flux shown in Equation 3 is zero for those cells. For constant-head cells, the lower bound is the maximum annual recharge rate estimated using the Boussinesq equation and the springtime hydraulic gradients of 1972-83.

For constant-head cells the upper bound on volume flux is a large positive number—the models need the freedom to discharge from a boundary if it will enhance regional objective attainment. For internal cells the upper bound on flux (pumping) is a demand assumed from historic acreages of aquaculture, rice and irrigated soybeans. In developing a strategy, each of the models is limited so that total water use in each cell cannot exceed that cell's assumed maximum demand. Demand not satisfied by a combination of groundwater and surface water is considered "unmet demand".

Satisfactory groundwater table elevations and saturated thicknesses are assured to result from all optimizations by appropriately bounding the steady state drawdowns via Equation 4. In each cell, the optimal water level is constrained such that it never exceeds the ground surface elevation. In addition, the optimal saturated thickness for each cell is constrained to be at least 20 feet (6 m).

In order to develop a strategy to minimize regional cost, the c_e, c_m and c_a values used in Equation 6 must first be estimated. Values of 0.18 $/ac-ft-ft (0.48 $\$/dam^3/m$) and 1.65 $/ac-ft (1.34 $\$/dam^3$) were used for c_e and c_m respectively. Values of ca, shown in Figure 2, were assumed as follows.

The value of c_a for cells in which diverted river water is

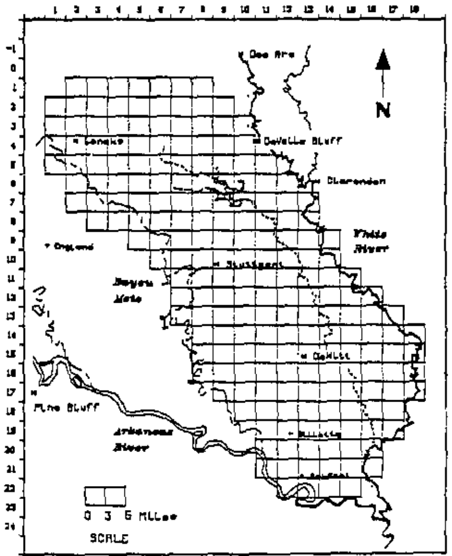

Fig. 1. The Grand Prairie Study Area

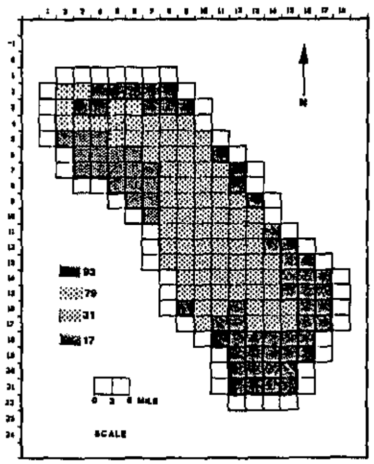

Fig. 2. Cost of Alternative Water and Opportunity Cost ($/ac-ft)

potentially available is the cost of delivering that water to fields in those cells. Reconnaissance level studies by the U. S. Army Corps of Engineers estimate costs of 14 $/ac-ft (11.3 $\$/\mathrm{dam}^3$) for diverting Arkansas River water through the Bayou Meto (16) and 28 $/ac-ft (22.7 $\$/\mathrm{dam}^3$) for distributing White River water through a canal system to cells within the area (personal communication Dwight Smith). For this paper we assume an additional 3 $/ac-ft (2.4 $\$/\mathrm{dam}^3$) expense to move the water from a waterway to the field. Figure 2 shows the resulting costs of 17 $/ac-ft and 31 $/ac-ft (13.8 and 25.1 $\$/\mathrm{dam}^3$) in those cells to which Arkansas River water and White River water may be diverted.

For cells at which no diverted surface water is available, it is possible, as a result of constraint Equations 3 and 4, that not all demand can be satisfied. If there is insufficient groundwater to satisfy the maximum demand in those cells, there is less net economic return than there would be if all demand were met. Thus an opportunity cost results from having to grow unirrigated soybeans instead of fish or an irrigated crop. Aquaculture accounts for most of the maximum water demand in certain cells in the northwestern portion of the region. Rice predominates in other parts of the region. Figure 2 shows opportunity costs of 79 and 93 $/ac-ft (64.1 and 75.4 $\$/\mathrm{dam}^3$) for those cells in which unsatisfied maximum demand will result in unirrigated soybean production instead of aquaculture or rice respectively.

__Results of Strategy Implementation__

Table I contains a summary of the three conjunctive water use/sustained groundwater yield strategies that are developed using Equations 5 and 6 and multiobjective optimization. Strategy A, using Equation 5, tries to minimize unsatisfied maximum demand in cells in which no diverted water is available. There is no unsatisfied demand in cells where surface water is available, therefore this strategy has the smallest volume of unsatisfied demand, 31,000 ac-ft (38,200 dam^3).

Strategy B differs from Strategy A in that its objective is to minimize the regional expense of attempting to satisfy maximum water demand. It requires the use of significantly more groundwater and less diverted water than Strategy A. The net economic return for the aquacultural, irrigated and unirrigated acreages appropriate for Strategy B is $6,238,000. Strategy A has $271,000 less net return and 2,000 ac-ft fewer unsatisfied demand, a 135.50 $/ac-ft (109.90 $\$/\mathrm{dam}^3$) trade-off.

Table 1: Annual consequences of strategy implementation

	STRATEGIES A	B	C
WATER NEEDS (1000 AC-FT)	259	259	259
GROUNDWATER USE (1000 AC-FT)	63	92	86
SURFACE WATER USE (1000 AC-FT)	165	134	141
UNMET WATER NEEDS (1000 AC-FT)	31	33	32
DIFFERENCE IN NET ECONOMIC RETURN FROM THAT OF STRATEGY B # (1000 DOLLARS)	-271	NA	-41

Based on published crop budgets and including only specified costs.

These two strategies represent objectives that conflict over part of the range of feasible regional strategies. Choosing one of the strategies may not be as satisfactory as selecting a compromise strategy between them. Use of the constraint method of multiobjective optimization mentioned previously results in the pareto optimum shown in Figure 3. A compromise strategy lying on the pareto optimum was selected arbitrarily for purposes of this paper. Notice that the compromise strategy, Strategy C, has values lying between those of Strategies A and B for the last four rows of Table I.

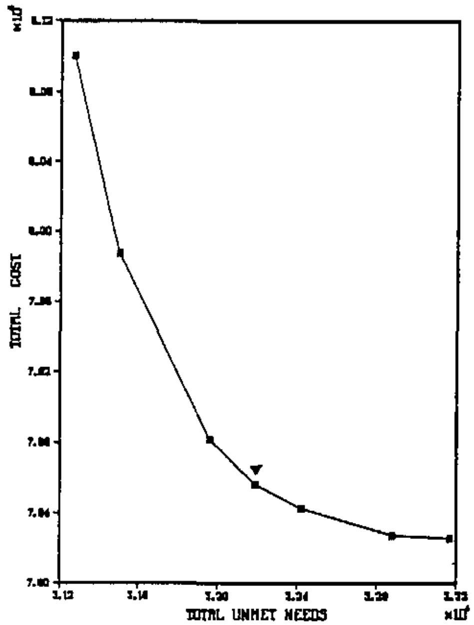

Fig. 3. Pareto Optimum of Minimizing Cost and Minimizing Unmet Needs ($/ac-ft)

SUMMARY

The formulation of finite difference optimization models for the design of alternative conjunctive water use/sustained groundwater withdrawal strategies is demonstrated. Three alternative strategies are presented for the Arkansas Grand Prairie, the aquifer of which is assumed to be bounded on all sides by constant-head cells. In the development of the strategies the sustained withdrawal of groundwater is limited to be less than the sum of the assumed sustainable recharges to the aquifer at all constant-head cells. It is assumed that historical recharge to the region will continue and that the water table elevations of the peripheral cells will be maintained. In addition, an upper limit exists for the combined volume of diverted river water and groundwater that can be used in each cell. This upper limit represents the water demand that each model attempts to satisfy and represents specific acreages of different crops in each cell.

Peralta and Killian (10) have demonstrated the gradual evolution of groundwater levels into an optimal "target" potentiometric surface once a sustained yield strategy is implemented.The steady-state potentiometric surface that will evolve from implementation of any of the strategies presented in the current paper is such that at least 20 (6 m) feet of

saturated thickness is assured in each cell.

Total feasible recharge to the area is less than maximum demand, resulting in unsatisfied demand. Since the volume of unsatisfied demand is different for each strategy, the acreages supplied with water differ for each strategy also. Strategy A, which minimizes unsatisfied demand, uses more total water than Strategy B, which minimizes the cost of attempting to satisfy demand. The model developing Strategy B considers the cost of supplying water as well as the opportunity cost of missed production from not filling demand. Strategy B has the greatest annual net economic return, $6,238,000. Strategy C is a compromise between the different regional objectives of Strategies A and B.

There are an infinite number of possible sustained yield strategies for the region. Certainly, the "best" conjunctive use of groundwater and surface water depends on the specific objectives of the water users and decision makers. Datta and Peralta (unpublished manuscript) describe application of the Surrogate Worth Trade-off Method (6) to assist a group of decision makers in selecting a compromise strategy. Peralta and Killian (10) and Killian and Peralta (unpublished manuscript) present procedures for refining a compromise regional strategy to better satisfy local (cell) objectives. Thus, the capability exists to tailor-make a regional conjunctive water use/sustained groundwater yield strategy.

APPENDIX I.-REFERENCES

1. Aguado, E., Remson, I., Pikul, M. F. and Thomas, W. A., "Optimal Pumping in Aquifer Dewatering", Journal of the Hydraulics Division, ASCE, Vol. 100, No. HY7, 1974, pp. 860-877.

2. Alley, W. M., Aguado, E. and Remson, I., "Aquifer Management under Transient and Steady-State Conditions", Water Resources Bulletin, Vol. 12, No. 5, 1976, pp. 963-972.

3. Elango, K., and Rouve, G. "Aquifers: Finite-Element Linear Programming Model", Journal of the Hydraulics Division, ASCE, Vol. 106, No. HY10, 1980, pp. 1641-1658.

4. Griffis, C. L., "Modelling a Groundwater Aquifer in the Grand Prairie of Arkansas", Transactions, American Society of Agricultural Engineers, Vol. 15, No. 2., 1972, pp. 261-263.

5. Haimes, Y. Y., "Integrated System Identification and Optimization", Advances in Control Systems Theory and Applications, C. T. Leondes, Ed., Vol. 10, Academic Press, New York, 1973, pp. 435-518.

6. Haimes, Y. Y., and Hall, W. A., "Multiobjectives in Water Resources Systems Analysis: the Surrogate Worth Trade-off Method", Water Resources Research, Vol. 10., No.4, 1974, pp. 615-624.

7. Illangasekare, T., Morel-Seytoux, H. J., and Verdin, K., "A Technique of Reinitialization for Efficient Simulation of Large Aquifers Using the Discrete Kernel Approach", Water Resources Research, Vol. 20, No., 11, 1984, pp. 1733-1742.

8. Killian, P. J., "Optimization of Sustained Yield Groundwater Withdrawal", thesis presented to the University of Arkansas, at Fayetteville, in 1985, in partial fulfillment of the requirements for the degree of Master of Science in Agricultural Engineering.

9. Lieffson, T., Morel-Seytoux, H. J., and Jonch-Clausen, T., "User's Manual for QPTHOR: A FORTRAN IV Quadratic Programming Routine", HYDROWAR Program, Colorado State University, Fort Colins, Colorado, 1981.

10. Peralta, R. C. and Killian, P. J., "Optimal Regional Potentiometric Surface Design: Least-Cost Conjunctive Use/Sustained Groundwater Yield", In review for publication in Transactions, American Society of Agricultural Engineers.

11. Peralta, R. C. and Peralta, A. W., "Arkansas Groundwater Management Via Target Levels", Transactions, American Society of Agricultural Engineers, Vol. 27, No. 6, 1984a, pp. 1696-1703.

12. Peralta, R. C. and Peralta, A. W., "Using Target Levels to Develop a Sustained Yield Pumping Strategy in Arkansas, a Riparian Rights State", Arkansas State Water Plan Special Report, Arkansas Soil and Water Conservation Commission, Little Rock, Arkansas, 1984b.

13. Peralta, R. C., Yazdanian, A., Killian, P. and Shulstad, R. N., "Future Quaternary Groundwater Accessibility in the Arkansas Grand Prairie 1993", Bulletin No. 877, Agricultural Experiment Station, University of Arkansas, Fayetteville, Arkansas, 1985.

14. Pinder, G. F. and Bredehoeft, J. D., "Application of the Digital Computer for Aquifer Evaluation", Water Resources Research, Vol. 4, No. 5, 1968, pp. 1069-1093.

15. Troise, F., ed.,The Groundwater Newsletter, Water Information Center, Inc., Vol. 11, No. 7, 16 April 1982, pp. 1.

16. U. S. Army Corps of Engineers, Little Rock District, "Reconnaissance Report, Interbasin Transfer Arkansas River to Bayou Meto and Lower White Basin", Arkansas State Water Plan Special Report, Arkansas Soil and Water Conservation Commission, Little Rock, Arkansas, 1984.