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DEVELOPMENT OF OPTIMAL SUSTAINED YIELD GROUNDWATER WITHDRAWAL STRATEGIES FOR THE BOEUF-TENSAS BASIN IN ARKANSAS

by

Richard C. Peralta, Bithin Datta, Jamal Solaimanian, P.J. Killian and Amin Yazdanian.

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NOMENCLATURE

A	unit area normal to the direction of flow, (L^2)
APS_r	streambed parameter reflecting the degree of hydraulic connection between a stream and a aquifer for cell r, (L^2/T)
AR	surface area of one square finite difference cell, (L^2)
b	saturated thickness of aquifer material, (L)
CHSMIN(ics)	the lower limit on total recharge from constant-head subsystem ics, (L^3/T)
dh/dx	hydraulic gradient, (L/L)
DTR(i,j)	average transmissivity between cell (i,j) and cell (i+1,j), (L^2/T)
DTU(i,j)	average transmissivity between cell (i,j) and cell (i,j+1), (L^2/T)
G_r	reach transmissivity at reach cell r, (L^2/T)
H_r	thickness of the streambed material in cell r, (L)
h	potentiometric surface elevation, (L)
h(i,j)	the potentiometric surface elevation in finite difference cell (i,j), at time period t, (L)
i	the index in the X-direction for any cell in the study area
ic	column index for any constant-head cell in the study area

iv	column index for any variable cell in the study area
I(ics)	the total number of constant-head cells in subsystem ics
j	the index in the Y-direction for any cell in the study area
jc	row index for any constant head cell in the study area
jv	row index for any variable cell in the study
J(isa)	the total number of cells in stream/aquifer subsystem isa
k	the hydraulic conductivity of the aquifer material, (L/T);
K_r	hydraulic conductivity of streambed material in cell r, (L/T);
K	total number of inequality constraints
N	total number of cells in the study area
NCH	total number of constant-head cells
NCHSUB	total number of constant-head cell subsystems
NSUB	total number of stream/aquifer subsystems
NVAR	total number of variable-head cells
P(i,j)	groundwater pumping in cell (i,j) during simulation period, (L^3/T)
Pmax(i,j)	upper limit on groundwater pumping in cell (i,j), during simulation period, (L^3/T)
Pmin(i,j)	lower limit on groundwater pumping in cell (i,j), during simulation period, (L^3/T)
q	flux per unit area of aquifer normal to the direction of flow, (L/T)

Qr(i,j)	stream/aquifer response in cell (i,j) during simulation period, (L^3/T)
Qrmin(i,j)	minimum allowable interflow between stream and aquifer at cell (i,J) during simulation period, (L^3/T)
Q	flux through a cross section normal to the direction of flow, (L^3/T);
q_r	discharge from (+ sign) or recharge to (-sign) the aquifer for a particular cell (at reach r), (L^3/T);
R_r	an assumed constant river stage at the given cell r, for a given time step, (e.g., one month in this study), (L);
RCH(i,j)	recharge to the aquifer at cell (i,j) during simulation period, (L^3/T)
Rmax(i,j)	upper limit on recharge in constant-head cell (i,j), during simulation period, (L^3/T)
Rmin(i,j)	lower limit on recharge in constant head cell (i,j), during simulation period, (L^3/T)
RT(ics)	the total recharge in constant-head cell subsystems ics, (L^3/T)
S(i,j)	steady-state drawdown in cell (i,j), (L)
{s}	a 5 dimensional vector of drawdown values, (L)
{s'}	N dimension vector of steady-state drawdowns, (L)
Sc	storage coefficient, (L^3/L^3)
Smax(i,j)	upper limit on drawdown in cell (i,j), (L)
Smax'(i,j)	maximum drawdown in cell (i,j) such that lower limit on interflow is not violated, (L)

Smin(i,j)	lower limit on drawdown in cell (i,j), (L)
Sst(i,j)	stream drawdown at cell (i,j), (L)
ST(isa)	the total volume of flow from the aquifer to the stream in subsystem isa, (L^3/T)
ST(i,j)	the target (known) steady-state drawdown in variable-head cell number (i,j), (L)
SWMIN(isa)	the lower limit on total interflow from subsystem isa, (L^3/T)
T'_{ij}	transmissivity of aquifer, a second order tensor, (L^2/T)
t	time index, (T)
{T}	transpose of the 5 dimension vector of transmissivity values
[T]	N by N square matrix of transmissivity values
Tr(i,j)	reach transmissivity at cell (i,j), (L^2/T)
W(x,y)	excitation per unit surface area of aquifer, (L/T)
{w'}	n dimensional vector of steady-state excitation values, (L)
W_r	average width of the streambed in cell r, (L)
x	a distance in the direction of groundwater flow, (L);
X'(i,j)	slack variable associated with pumping constraint, (L^3/T)
X"(i,j)	slack variable associated with recharge constraint, (L^3/T)
Δt	the time increment, (T)
ΔV(j)	change in constrained derivative (j)

$\Delta x_{1+1/2}$	the distance between center of cell (i,j) and center of cell (i+j), (L)
$\Delta x_d(i)$	change in decision variable j
$\Delta x_s(j)$	change in state variable i
Δx_i	space increment in the x-direction for column i, (L)
$\Delta x'_d(j)$	maximum change in decision variable j due to limits on decision variable j
$\Delta x''_d(j)$	maximum change in decision variable j due to limits on state variables
$\Delta x'''_d(j)$	maximum change in decision variable j such that V(j) does not go to zero
Δy_j	space increment in the y-direction for column j, (L)
ΔZ_p	change in the principle objective function

CHAPTER I

INTRODUCTION AND SCOPE OF STUDY

By

B. Datta, R. C. Peralta and J. Solaimanian

1.1 THE STUDY AREA

The goal of this study is to develop sustained yield pumping (discharge via wells) strategies for the Boeuf-Tensas Basin area. The demarkation of the Boeuf-Tensas area is described in the Arkansas State Water Plan (Arkansas Soil and Water Conservation Commission, 1984). The Boeuf-Tensas Basin is a highly developed agricultural region located in the southeast corner of Arkansas. Hydrogeologically, it is part of the Bayou Bartholomew/Alluvial Aquifer System (Broom and Reed, 1973). Before describing the Boeuf-Tensas Basin, the Bayou Bartholomew region should be discussed.

The Bayou Bartholomew region (Figure 1.1) encompasses about 3,420 square miles (2,188,800 acres). Comprised of portions of six counties, this area has an overall length of about 105 miles in a generally north-south direction and averages about 63 miles in width. The contributions of these counties to the total area are: Ashley-495,360 acres (22.3 percent); Chicot-443,520 acres (20.3 percent); Desha-403,200 acres (18.4 percent); Drew-299,520 acres (13.7 percent); Lincoln-334,080 acres (15.3 percent); and Jefferson-213,120 acres (9.7 percent).

The total area studied in this project is identical to the one reported by Broom and Reed (1973). Its northern and eastern

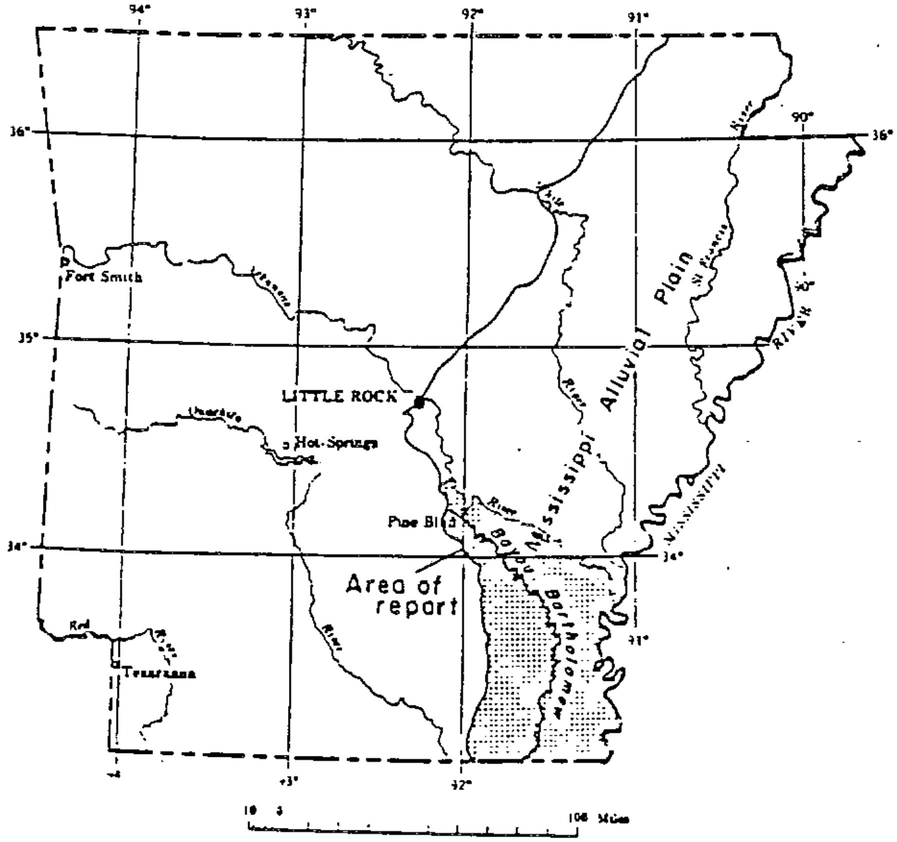

Figure 1.1 Location of The Study Area in Arkansas

boundaries coincide with a levee that protects the area from floods of the Arkansas and Mississippi Rivers. The levee extends eastward along the south bank of the Arkansas River from Pine Bluff and southward along the west bank of the Mississippi River. On the south this area is bordered by the Arkansas-Louisiana state line. The northwestern boundary is the boundary of the Quaternary aquifer that underlies the region (Broom and Reed, 1973). The southwestern boundary is not a natural boundary and leaves a part of the Quaternary aquifer outside the study area. It was selected so as to enclose only that portion of the aquifer where appreciable groundwater pumping is historically reported.

Figure 1.2 shows both the Boeuf-Tensas Basin and the Bayou Bartholomew Basin areas. The smaller area to the east of the dashed boundary line is the Boeuf-Tensas Area (Area A) which forms a part of the Bayou Bartholomew Basin area (Area B) shown in Figure 1.1. The western boundary of the Boeuf Tensas area (Area A) is the eastern divide of the Bayou Bartholomew watershed. The purpose of this study is to develop optimal sustained yield groundwater withdrawal(pumping) strategies for the Boeuf Tensas Basin (area A). However, in order to properly represent the aquifer boundary conditions, the entire Bayou Bartholomew area (area B) was included in the groundwater simulation and optimization models used for strategy development. The withdrawal strategy for Area A (Boeuf-Tensas Basin) was subsequently obtained as a subset of the withdrawal strategy developed for the entire area (area B).

The natural surface drainage within the basin consists

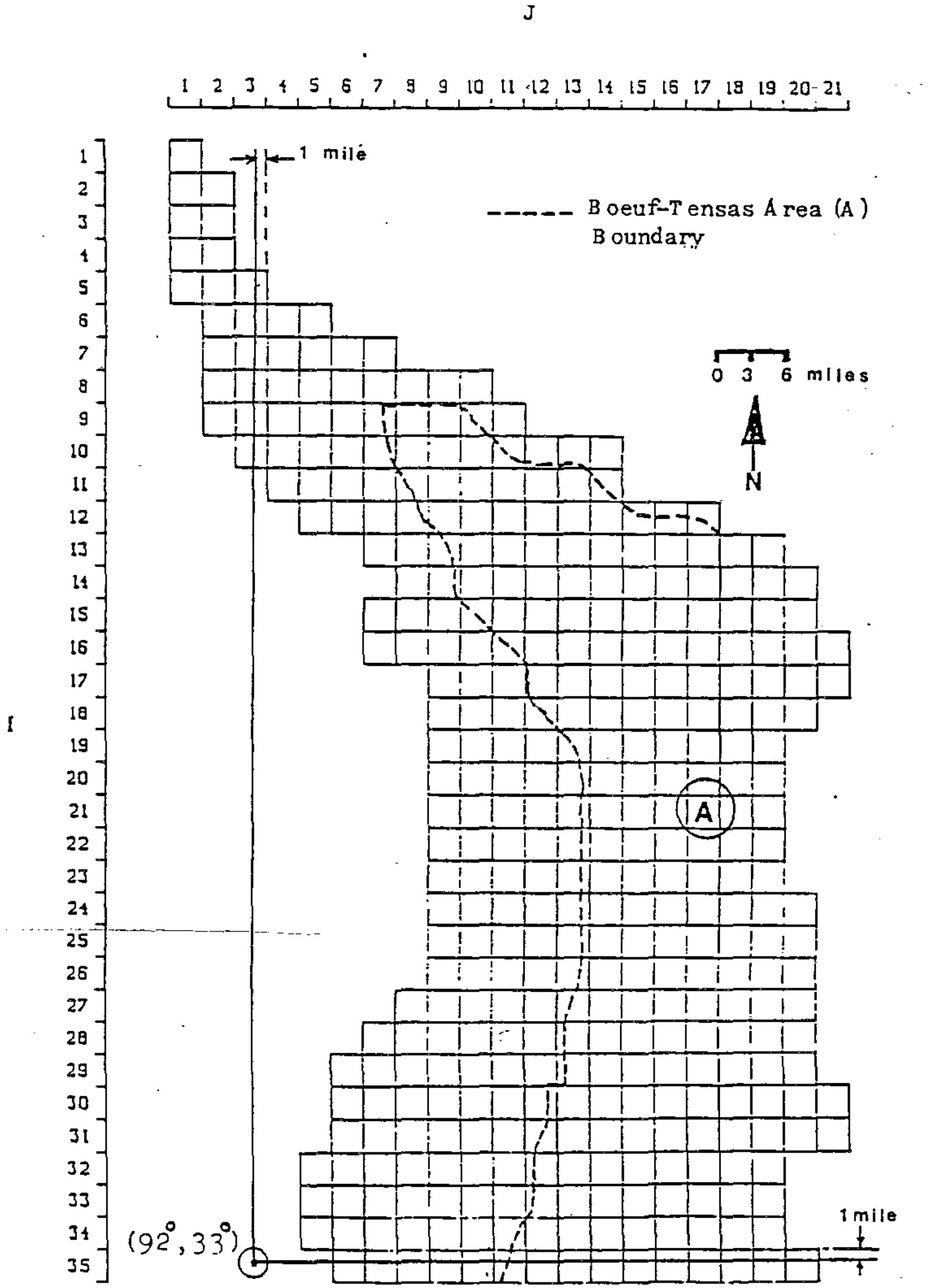

Figure 1.2 The Study Area Discretized into Finite Difference Cells

primarily of about 21 meandering streams and rivers. Excess surface water leaves the area through the Bayou Macon, Bayou Bartholomew, and Boeuf Rivers, which outlet into the Ouachita River in Louisiana. Because of hydraulic connection with the aquifer, under varying conditions these 3 rivers can cause either recharge to or discharge from the aquifer.

Most of the groundwater withdrawal in this area is used for agricultural production. Other usages include: aquacultural, municipal, and industrial. Agricultural production in this area is dependent on large quantities of groundwater to meet the irrigation demand of rice, soybean and cotton acreages.

1.2 OBJECTIVES

The objective of this study is to develop optimal sustained yield regional pumping strategies for the Boeuf-Tensas area of the Quaternary aquifer. The optimal withdrawal strategies can be based on either of the following two objectives: i) maximization of total withdrawal from the aquifer subject to sustained yield hydraulic constraints, ii) maximization of the sustainable maintenence of the current (i.e., spring 1983) potentiometric surface. The ultimate selection of one of these two objectives as the one more suitable for this region will depend on analysis of the economic and social consequences of implementing a particular optimal strategy. Since final selection is outside the scope of this study, we present a number of alternative strategies which satisfy either of the two objectives as well as plausible physical and managerial constraints.

The constraints incorporated in the optimization models include: limits on recharges into the area through the boundary cells, limits on recharges or discharges through stream/aquifer connections, upper bound on pumping at each of the finite difference internal cells and lower limit on the saturated thickness (20 ft) at every cell. The objective functions and the constraints used in this study are discussed in detail in Chapter 4.

The complete study includes the following steps:

a) estimation of the historic pumping in each cell, based on crop acreages and irrigation demands, aquacultural acreages, and recorded municipal and industrial groundwater use from the Quaternary aquifer

b) estimation of the potential demand for agricultural water use in each cell, based on maximum potential irrigation demands

c) estimation of aquifer parameters through literature review

d) estimation by geostatistical kriging of the top and base elevations of the aquifer at the center of each 3-mile by 3-mile cell

e) estimation by kriging of the water table elevations or potentiometric surface elevations at the center of each cell, for the period between 1973 and 1983

f) estimation of the degree of stream/aquifer response for those streams hydraulically connected to the aquifer

g) validation of a groundwater flow simulation model with historic data

h) estimation of the net recharge that has historically occurred: from along the study area boundaries, from unspecified stream-aquifer interaction, and from the difference between actual time-variant recharge and the assumed steady recharge.

i) estimation of the annual volume of water that can be withdrawn from the Quaternary aquifer underlying each cell, so as to maximize the total amount of annual withdrawal from the region while maintaining sustained yield conditions

j) determination of the annual volume of water that should be withdrawn from the Quaternary aquifer underlying each cell, in order to maintain the potentiometric surface approximately at current (1983) elevations.

1.3 BOUNDARY CONDITIONS OF THE AQUIFER

In order to use an optimization model that can prescribe an optimal pumping strategy for a given aquifer, the physical parameters of the aquifer need to be specified. Estimates of these parameters can be obtained by the calibration and validation of a groundwater simulation model. Implementation of both the simulation and the optimization models requires the specification of proper boundary conditions. Also, the application of a numerical model to an aquifer extending over a large area (such as the Bayou-Bartholomew Basin) requires discretization of the entire area into finite difference cells. This section describes both the boundary conditions important in the simulation model, and the discretization scheme. The precise boundary conditions used for obtaining optimal pumping strategies are discussed in detail in Chapter 5.

The aquifer is divided into 376 cells that are 3 miles by 3 miles in size (Figure 1.3). Finer grid spacing was not used for this model because of the high cost of simulation runs. The study area cells are of two types: constant-head cells or variable-head cells. Most of the area's periphery is simulated by a set of 62 constant-head cells. In each of these cells the simulated groundwater level is maintained at a constant elevation (head) during a simulation period. The rest of the study area periphery, except for 7 cells on the south-western boundary, coincides with the western edge of the aquifer. Therefore, the cells on the western boundary were assumed as variable-head cells with negligible transmissivity. All constant-head cells including those 7 cells on the south-western boundary, are shown shaded in Figure 1.3.

Some recharges to the area take place through the constant-head cells--the recharge volume being provided either from rivers penetrating to the aquifer in those cells, or water entering them from extensions of the aquifer outside the region. Analysis and study indicate that streams passing through some of the internal cells in the Boeuf-Tensas Basin are also providing recharge to the aquifer. In most of the rest of the internal cells, a relatively impermeable clay layer overlies the aquifer.

A major portion of the aquifer is confined in the springtime. However, the degree of confinement is small enough that the aquifer is probably unconfined in the vicinity of pumping wells in most of the area. Therefore, a generally applicable, linearized, two dimensional groundwater model known

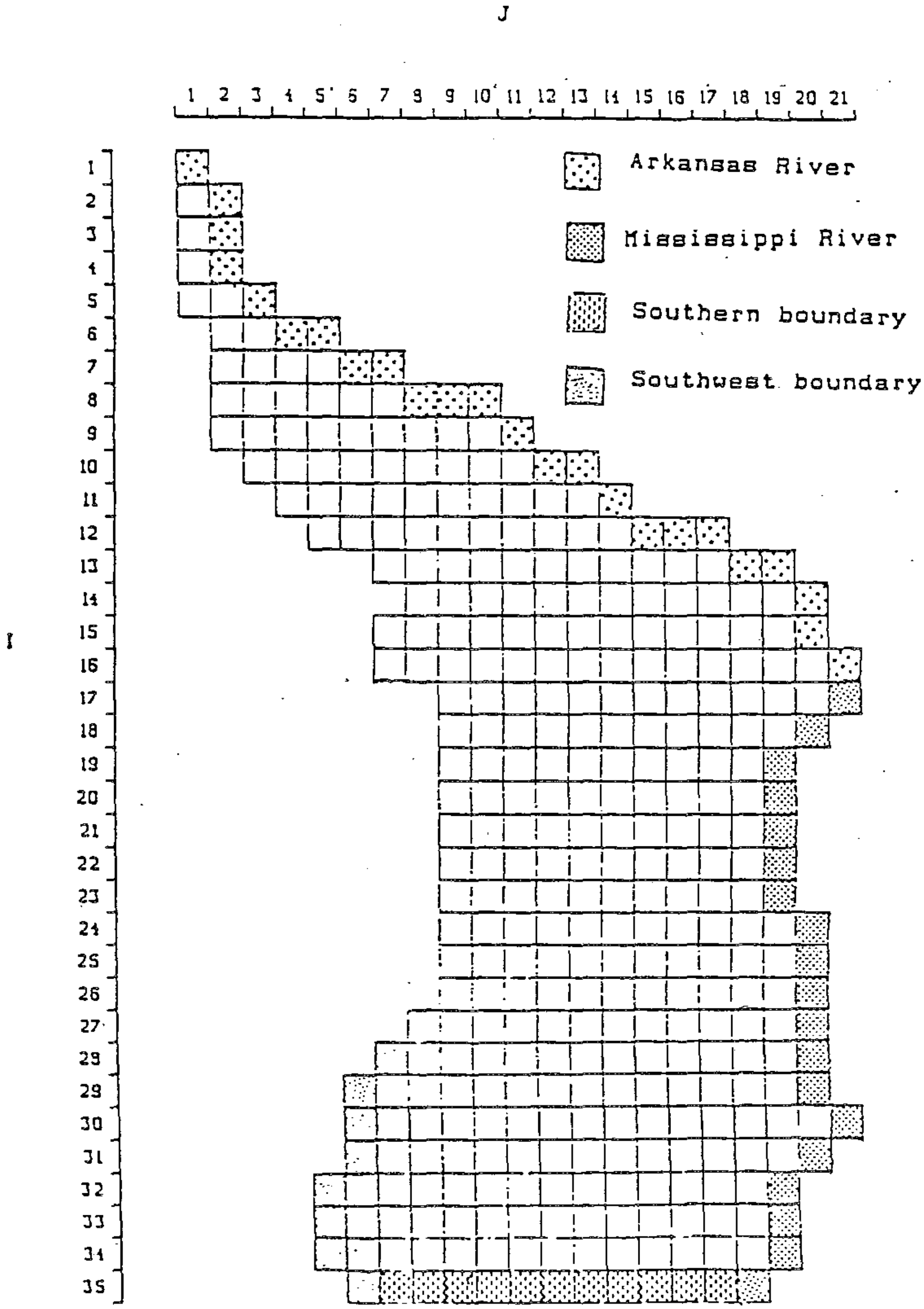

Figure 1.3 Constant-Head Cell Sub-Systems

as AQUISIM (Verdin et al, 1981) was selected for the simulation of the groundwater flow hydraulics in this study. Due to the assumptions and approximations of linearization, the model is appropriate for both confined and unconfined conditions.

1.4 SUMMARY

The main objective of this study is to estimate the volume of water that can be annually withdrawn in each cell while maintaining sustained yield conditions in the aquifer. Two different regional objectives are considered: i) maximization of total withdrawal from the aquifer, and ii) minimization of the weighted sum of deviations of optimal potentiometric surface elevations from current elevations.

Chapter 2 discusses the development of the necessary data base. Chapter 3 discusses the validation of aquifer parameters through the simulation of aquifer responses to hydraulic stresses. Chapter 4 describes the optimization theory and methodology. Finally Chapter 5 discusses the application of the optimization models to the Quaternary aquifer underlying this study area, and presents the alternative withdrawal strategies obtained as solutions of the optimization models.

CHAPTER II

DATA BANK DEVELOPMENT

By

J. Solaimanian, B. Datta and R. C. Peralta

2.1 INTRODUCTION

The basic issue in a regional groundwater management strategy is where and how much water can be withdrawn from the aquifer in order to satisfy certain objectives and constraints. The simulation and optimization of groundwater flow for an area as large as the Bayou Bartholomew/Alluvial Aquifer System, requires the use of a great deal of data. This chapter describes the development of the data that are used to determine historic recharges to and withdrawals from the Quaternary aquifer (Solaimanian, 1985). These data include crop acreages, crop water needs, aquifer top and base, reach transmissivity, river stages, and potentiometric surface elevations. The assumptions used in preparing the data are reported where appropriate in the following sections.

The historic recharges and discharges and assumed aquifer parameter values are verified by using a groundwater simulation model of the area as discussed in Chapter 3. Historic values between spring 1973 and spring 1983, termed the validation period, are used in this process.

2.2 ESTIMATING PUMPING FROM THE QUATERNARY AQUIFER

2.2.a Introduction

The major users of Quaternary groundwater in the Bayou Bartholomew Basin are agriculture, aquaculture, and industries. This section describes the procedure used to estimate the amount of pumping from the Quaternary aquifer which occurred from 1973 to 1982. Pumping from the aquifer (discharge) is considered as a positive value. Water moving into the aquifer (recharge) is considered as a negative pumping value. Pumping is not considered in constant-head cells.

2.2.b Estimating Agricultural Pumping

Agricultural pumping must be estimated since no record of actual pumping exists. The following procedure, analogous to that by Peralta et al (1983,1985), is used to estimate the amount of agricultural pumping in the study area. The 1972 Natural Resource Inventory System and Land Use Data Information System data bases contain the dominant land use of every square kilometer in the study area. (This data is reported in a series of publications by the Arkansas Department Of Local Services, 1977). Table 2.1 shows the area of total land and agricultural land in each county of the study area. The U.S. Department of Agriculture's (USDA) Economics, Statistics, and Cooperatives service reports total acreage of rice, soybean, and cotton as well as data that can be used to estimate total agricultural acreage (USDA, 1981). These acreages are shown in Tables 2.2, 2.3 and 2.4.

The total rice acreage in each county are assumed to be irrigated. The percentages of soybean and cotton acreages irrigated in each county from 1972 to 1982 are obtained from unpublished data provided by Don Von Steen, USDA, Crop Reporting Services, Little Rock, Arkansas. The yearly percentages and their 11 year averages for soybean and cotton are shown in Tables 2.5 and 2.6.

Table 2.1

Area of total land and agricultural land in each county in the study area (1972)

County	Total Area (acre)	Agricultural Area (acre)
Ashley	610,350	164,794
Chicot	467,522	336,616
Desha	542,150	288,075
Drew	582,921	162,645
Jeffer.	582,921	291,460
Lincoln	370,161	207,290

Table 2.2

Rice Acreage Harvested (1000 Acres)

Year	COUNTY					
	Ashley	Chicot	Desha	Drew	Jefferson	Lincoln
1973	8.1	11.9	17.1	5.5	21.4	10.9
1974	13.0	21.9	25.7	7.7	29.2	17.6
1975	15.6	26.7	32.5	9.7	40.3	23.3
1976	16.0	22.0	31.3	9.0	37.1	20.8
1977	13.1	16.2	22.9	8.0	28.5	16.4
1978	18.7	28.3	34.5	12.4	44.3	24.3
1979	21.4	30.0	35.0	13.7	42.5	23.4
1980	27.6	38.5	45.5	21.8	53.0	28.6
1981	27.4	50.4	52.5	21.0	67.5	36.2
1982	25.6	42.4	45.4	17.7	54.3	31.6

Table 2.3

Soybean Acreage Harvested (1000 Acres)

Year	County					
	Ashley	Chicot	Desha	Drew	Jefferson	Lincoln
1973	75.0	181.0	185.0	47.0	98.0	63.0
1974	56.0	169.0	158.0	46.0	76.0	65.0
1975	76.0	197.0	180.0	58.0	113.0	72.0
1976	61.9	194.9	157.9	45.9	95.0	59.9
1977	64.4	199.4	167.7	45.8	115.1	74.6
1978	69.0	209.6	169.5	49.5	111.8	74.5
1979	76.0	227.0	178.0	59.0	122.0	89.0
1980	66.0	185.0	160.0	45.0	118.0	71.0
1981	69.3	185.0	151.5	49.5	119.0	74.2
1982	66.0	184.0	152.0	49.0	157.0	84.0

Table 2.4

Cotton Acreage Harvested (1000 Acres)

Year	Ashley	Chicot	Desha	Drew	Jefferson	Lincoln
	County					
1973	41.3	30.5	46.3	12.6	51.3	41.0
1974	47.0	38.3	58.4	16.4	92.0	42.0
1975	37.9	16.4	35.1	17.2	68.2	26.7
1976	50.4	32.4	52.5	16.7	96.6	39.8
1977	53.7	26.6	54.4	19.5	94.6	38.4
1978	50.7	24.7	49.6	19.5	84.5	35.9
1979	49.3	21.7	44.8	19.1	66.4	24.7
1980	51.6	25.3	48.4	12.6	62.3	26.2
1981	50.6	34.0	51.0	13.7	46.0	23.0
1982	38.0	20.9	40.2	14.2	25.4	12.9

Table 2.5

Percent of soybean acreage that is irrigated in each county

County / Year	Ashley	Chicot	Desha	Drew	Jefferson	Lincoln
1972	1.4	1.9	2.1	5.2	3.2	5.8
1973	2.0	2.0	2.0	6.0	3.3	5.1
1974	1.6	1.2	1.7	2.6	2.1	7.8
1975	0.8	0.8	2.0	1.7	2.9	5.9
1976	2.2	1.6	3.2	7.4	3.0	1.0
1977	2.3	1.6	2.0	9.1	2.0	6.0
1978	1.0	5.0	4.0	7.7	5.6	2.0
1979	2.6	1.8	2.8	8.5	3.3	4.5
1980	9.1	2.7	15.6	11.1	4.2	5.2
1981	8.7	7.6	10.6	12.1	5.0	10.8
1982	9.1	5.4	14.5	16.3	11.5	5.9
Avg.	3.7	2.8	5.5	8.0	4.2	5.4

Table 2.6

Percent of cotton acreage that is irrigated in each county

County / Year	Ashley	Chicot	Desha	Drew	Jefferson	Lincoln
1972	10.3	10.6	30.5	31.8	13.1	20.8
1973	15.0	****	20.1	29.6	9.0	19.8
1974	****	****	****	****	****	****
1975	7.9	****	9.4	23.8	3.0	4.1
1976	7.4	2.6	43.6	52.4	14.8	26.1
1977	6.4	****	32.7	39.5	8.3	16.5
1978	10.4	4.0	31.6	38.5	14.9	33.4
1979	40.6	25.8	49.1	25.1	8.1	16.2
1980	20.3	11.9	28.9	30.8	14.4	21.8
1981	50.6	13.5	50.9	41.5	10.0	30.1
1982	52.6	14.3	47.3	42.9	11.8	31.0
Avg.	22.1	12.0	34.0	36.0	11.0	22.0

***** indicate that there is no record .

It is assumed that the crop acreage in each cell varies from year to year within the validation period, depending on each year's county crop acreage. Seasonal estimates of rice, soybean or cotton irrigation water needs are based on daily soil-water balance simulation and scheduling. The utilized programs were developed by Peralta and Dutram (1984) and Dutram et al., (1984). By this method annual water needs that vary depending on the year's climatological data are estimated per acre of rice, soybean, or cotton.

The daily water balance for rice is represented by the following equation:

Flood level = Initial flood level + Precipitation + Irrigation
− Evapotranspiration − Runoff − Seepage.

According to Peralta and Dutram (1984) the assumptions used in the rice water-balance are as follows. The average irrigation period extends from June 1 to Sept. 1. The initial irrigation requires 5 inches of water, one of which is needed to saturate the root zone while four remains above the soil surface. If the depth of flood drops through evapotranspiration to less than 2 inches the field is flooded to a 4-inch depth. If rainfall causes the water depth to exceed 6 inches, the levees are drained to prevent damage causesd by overflow, and the field is reflooded to a 4-inch depth on the following day. The amount of leakage through the levees is included in the estimate of seepage, and water is rarely lost at the end of the field due to overfilling. The result is an average annual pumping requirement of 23.8 inches and a requirement of 32.7 inches for 1980, a drought year.

The daily water balance for soybeans and cotton is represented by the following equation:

Soil moisture = Initial soil moisture + Precipitation +
Irrigation − Evapotranpiration − Runoff.

In the model by Peralta and Dutram (1984), the assumptions used in soybean water-balance simulation are as follow. The average irrigation period is from June 1 to Sept. 10. The root zone is 2.5 feet deep, and the soil is at field capacity (5 inches of available moisture) on the date of emergence (June 1). The fields are irrigated with 1.25 inches whenever evapotranspiration causes the available soil moisture to drop to 2.5 inches. Rainfall can replenish the soil moisture up to the amount of deficit in the root zone, but no more than 1.25 inches is allowed in any one day. Precipitation greater than 1.25 inches is lost as runoff. With these assumptions, the model predicts an average annual irrigation requirement of 4.3 inches and a requirement of 12.5 inches for 1980. Additional assumptions of the soybean model are that approximately 60 percent of the soybean acreage is furrow-irrigated at a system efficiency of 55 percent and that approximately 40 percent is flood-irrigated at a system efficiency of 75 percent, giving a weighted efficiency of 62 percent. Therefore the initial water requirements for soybeans are multiplied by a factor of 1.624 to estimate the volume delivered to the field. This yields an average of 9.1 inches/year and 20.3 inches for 1980.

In accordance with Dutram et al., (1984) the average irrigation period for cotton is from June 1 to September 9. For 1980 (an exceptionally dry year), cotton water requirements are estimated through the end of September. The cotton acreages are assumed to be irrigated by a furrow-irrigation system with 55 percent efficiency. Therefore, the initial estimated water needs for cotton are multiplied by a factor of 1.818 to determine the total volume delivered to the field. The result is an average of 13.6 inches/year and 27.3 inches for 1980.

The percentage of each county's total water requirement obtained by pumping from the Quaternary aquifer is derived from figures prepared by the Arkansas Geological Commission in cooperation with the U.S. Geological Survey (Halberg, 1975 ; Ludwig et al., 1980). Table 2.7 summarizes the results of this analysis. Percentage A represents the groundwater pumped from the Quaternary aquifer as a percentage of total groundwater used in a county. Percentage B is the percent of irrigation water needs satisfied by groundwater in a county. Percentage C represents irrigation needs met by groundwater from the Quaternary aquifer as a percentage of total irrigation water needs. Percentage C is the product of A and B.

Table 2.7

Significant percentages describing groundwater use

County	percentage A	percentage B	percentage C
Ashley	90%	94%	84%
Chicot	98%	48%	47%
Desha	98%	75%	73%
Drew	90%	82%	73%
Jefferson	66%	86%	57%
Lincoln	98%	82%	81%

The product of rice acreage, rice irrigation water delivered to the field for rice, and percentage of those needs coming from the Quaternary aquifer (all for a particular year and cell) is the amount of water pumped for rice from the Quaternary aquifer in that particular year and cell. This amount plus an analogous amount for soybean and cotton represents the total agricultural pumping for that cell in that particular year. A more detailed explanation of computational procedure is found in Appendix A. It should be noted that the amount of agricultural pumping varies from year to year depending on rice, soybean and irrigated cotton acreages, and climatological differences.

2.2.c Estimating Aquacultural Pumping

Estimates of aquacultural pumping are derived as follows. The location and acreage of fish ponds are obtained from Game and Fish Commission's permits for year 1979. According to these documents, the following aquacultural acreages existing within the study area were supported by the Quaternary aquifer: 395 acres in Ashley County; 60 acres in Chicot County; 1,247 acres in Desha County; none in Drew County; 900 acres in Jefferson County; and 103 acres in Lincoln county. In accordance with U.S.G.S. estimates (Halberg, 1977) an applied depth of 7 feet (7 ac-ft per acre) is assumed to compute the quantity of water pumped into the fish and minnow farms. A total of ten fish farms are located in 25 cells of the study area. The average annual pumping from the Quaternary aquifer for the cells having aquacultural use are computed to be between 535 acre-ft and 3,343 acre-ft. Most of the aquacultural pumping occurs near Pine Bluff in Jefferson County and Dumas in Desha County.

2.2.d Estimating Municipal and Industrial Pumping

Most of the municipal and industrial groundwater withdrawals in this region are obtained either from the Tertiary aquifer or from streams and rivers. The only industrial pumping from the Quaternary aquifer in the study area is by a paper company in Pine Bluff. Based on information from unpublished U.S.G.S records, the paper mill pumps 6.891 MGD (7820 acre-ft/year) from the Quaternary aquifer.

2.2.e Results From Estimating Groundwater Withdrawals

Figure 2.1 shows a representative set of cell- by-cell estimated agricultural pumping from the Quaternary aquifer, using the 1982 crop acreages and average climatic conditions. Expectedly, Chicot, Desha, and Lincoln Counties have the largest amount of agricultural pumping from the Quaternary aquifer of any counties within the study area, due to their extensive crop acreages.

The total annual pumping from the Quaternary aquifer for each cell is estimated by summing the agricultural, aquacultural and industrial pumping for the cell. Table 2.8 shows the total annual pumping for each year from 1973 to 1982. For simplicity, and because they contributed only about 14 percent of total annual pumping, aquacultural and industrial use were assumed to be constant during the validation period discussed in Chapter 3. Analysis shows that the total amount of pumping has increased with time. However, the most pumping occurred in 1980, a droughty year, and 1981 another dry year.

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0																				
2	0	0																			
3	0	0																			
4	0	0																			
5	0	0	0																		
6		0	38	0	0																
7		0	0	38	32	0	0														
8		0	9	71	71	71	83	0	0	0											
9		0	13	48	63	57	91	78	99	58	0										
10			0	9	58	58	105	93	88	84	87	0	0	0							
11				0	22	56	73	44	107	112	112	84	84	0							
12					0	0	67	78	107	84	89	101	101	71	0	0	0				
13							0	101	112	112	112	107	112	101	58	44	13	0	0		
14								0	73	101	112	95	84	112	112	112	101	105	84	0	
15							0	0	28	84	101	101	112	98	112	98	112	112	98	0	
16							0	0	39	39	90	105	112	112	112	98	112	112	112	37	0
17									0	7	32	42	78	109	112	112	93	89	27	0	0
18									0	24	52	58	52	60	98	112	93	56	0	0	
19									0	13	41	45	63	28	90	112	112	93	0		
20									0	34	17	34	41	35	90	112	112	98	0		
21									0	10	0	0	41	57	74	84	67	93	0		
22									0	21	8	17	48	89	112	112	90	44	0		
23									0	10	13	41	59	54	45	51	33	32	0		
24									0	0	10	13	52	56	42	53	48	39	8	0	
25									0	0	0	0	48	78	45	38	53	28	0	0	
26									0	0	0	0	108	52	53	45	53	53	17	0	
27								0	0	0	0	68	120	128	53	35	42	39	32	0	
28							0	46	51	0	5	92	120	128	53	46	45	45	35	0	
29						0	0	63	109	57	51	109	120	128	53	53	47	41	39	0	
30						0	51	22	40	57	51	114	109	128	53	53	53	53	44	44	0
31						0	11	0	0	0	80	128	120	128	53	53	53	46	53	0	0
32					0	17	0	0	0	5	86	92	128	126	53	53	46	44	0		
33					0	0	0.	0	0	40	103	114	128	105	44	44	44	17	0		
34					0	8	12	12	0	46	40	103	114	126	42	53	53	53	0		
35						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

Figure 2.1 Agricultural Groundwater Withdrawal From The Quaternary Aquifer For 1982 Crop Acreages And Average Climatic Conditions, (ac-ft X 10).

Table 2.8 Total Historic Pumping from the Quaternary Aquifer

Year of Estimate	Total Pumping (Ac-Ft)
1973	100,940
1974	148,032
1975	120,108
1976	152,965
1977	123,645
1978	266,147
1979	178,071
1980	352,916
1981	325,488
1982	171,252
Average	193,956

2.3 ESTIMATION OF POTENTIAL AGRICULTURAL PUMPING

An estimate of maximum potential water needs for each cell is needed as a possible upper bound on groundwater withdrawal on that cell. It is assumed that future aquacultural, municipal and industrial demands will follow historic patterns. Therefore, the maximum potential water needs is the sum of current non-agricultural water needs and maximum potential irrigation water needs. The maximum potential irrigation water needs assumed for this study are those reported by Dutram et al., (1984) for maximum potential irrigation acreages and average climatic conditions.

Figure 2.2 shows the estimated maximum potential demands for groundwater in the study area. This includes agricultural, aquacultural, municipal and industrial demands. For purpose of comparison, Figure 2.3 shows the estimated pumping values for 1982 acreages and climatic condition.

2.4 HISTORIC AQUIFER PARAMETERS (HYDRAULIC CONDUCTIVITY AND EFFECTIVE POROSITY)

As previously stated, the aquifer underlying the study area is part of an extensive aquifer system that underlies much of eastern Arkansas. It is appropriate to consider aquifer parameter estimates for other portions of the same aquifer when developing estimates for the study area. The effective porosity (specific yield) in an adjacent part of the same aquifer, the Grand Prairie, was reported by Sniegocki (1964) to be 0.30. Griffis (1972), and Peralta et al. (1985) both used this value for the Grand Prairie region. Broom and Lyford (1981) used this value for the adjacent Cache River basin.

Engler et al. (1945) reported a permeability of 1900 gallons per day per square foot (254 ft/day), and Sniegocki (1964) reported a value of 2000 gpd per square foot (267 ft/day), for the Grand Prairie. Griffis (1972) used the latter value in his work for the adjacent aquifer in the Grand Prairie region. Peralta et al., (1985) obtained best results when using a hydraulic conductivity of 270 ft/day in their simulation of the Grand Prairie region. Broom and Lyford (1981) achieved best

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21		
1	0																						
2	0	0																					
3	0	0																					
4	0	0																					
5	0	500	0																				
6		0	1,946	0	0																		
7		0	1,973	1,945	320	0	0																
8		0	2,064	2,282	718	718	636	0	0	0													
9		0	131	480	636	574	911	4,817	6,234	3,348	0												
10			0	94	566	566	1,055	2,167	7,313	7,524	4,411	0	0										
11				0	227	566	735	302	6,048	7,085	7,880	6,950	5,657	0									
12					0	0	667	789	1,481	7,112	7,446	7,906	7,559	5,036	0	0	0						
13							0	916	1,127	2,970	7,578	8,011	8,001	7,531	4,550	2,578	2,018	0	0				
14								0	735	1,011	4,606	7,088	7,573	7,373	7,746	7,642	7,355	7,652	5,449	0			
15							0	0	281	847	564	7,012	7,215	7,451	7,703	7,930	7,890	7,511	5,749	0			
16							0	0	397	397	905	1,650	7,092	7,572	7,789	7,329	7,585	7,485	5,418	376	0		
17									0	76	327	426	3,734	7,005	7,746	7,877	7,564	2,518	279	0	0		
18									0	244	525	561	204	5,110	7,338	7,903	7,542	2,145	0	0			
19									0	139	419	455	605	1,391	6,903	7,175	6,884	3,040	0				
20									0	350	175	350	419	360	8,978	6,860	7,642	5,353	0				
21									0	106	0	0	419	571	4,316	7,646	7,164	5,679	0				
22									0	211	69	175	610	756	6,165	7,525	7,659	3,663	0				
23									0	106	139	419	696	413	7,111	6,495	6,809	5,033	0				
24									0	0	106	139	559	1,754	6,904	7,153	6,321	2,949	66	0			
25									0	0	0	0	488	3,876	8,102	7,565	6,380	4,580	668	0			
26									0	0	0	0	1,085	5,753	7,438	7,251	7,314	6,414	3,696	0			
27								0	0	0	0	689	1,175	5,684	7,270	7,707	7,233	5,447	6,062	0			
28							0	461	515	0	60	922	1,396	4,762	8,003	7,296	7,596	5,051	5,395	0			
29						0	0	635	1,098	575	515	1,185	317	4,406	7,959	7,878	7,378	7,618	400	0			
30						0	1,057	1,272	401	575	515	1,192	5,582	4,521	7,544	7,633	7,482	7,149	7,525	7,526	0		
31						0	114	0	0	0	803	1,264	6,710	6,215	7,400	7,551	6,554	6,467	7,751	0			
32					0	174	0	0	0	60	862	922	6,800	6,573	7,560	7,754	6,930	6,607	0				
33					0	0	0	0	0	401	1,036	2,614	7,385	6,132	4,697	7,557	7,165	6,995	0				
34					0	65	125	125	0	461	524	6,414	6,941	5,948	3,308	7,318	6,962	6,257	0				
35						0	0	0	0	0	0	0	0	0	0	0	0	0					
	0	500	6,114	4,801	2,467	2,663	5,300	11,484	25,630	33,598	39,465	56,344	88,222	106,193	152,530	159,623	153,427	115,613	48,458	7,902	0	1,020,334	Total

Figure 2-2

Numbers and columns subtotals as well as overall total from excel spreadsheet,

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```
    0.********************************************************************************************************************
    0.    0.**************************************************************************************************************
    0.    0.**************************************************************************************************************
    0.    0.**************************************************************************************************************
    0.  500.    0.********************************************************************************************************
******    0. 1946.    0.    0.************************************************************************************************
******    0. 1973. 1945.  320.    0.    0.************************************************************************************
******    0. 2064. 2282.  718.  718.  636.    0.    0.    0.******************************************************************
******    0.  131.  480.  636.  574.  911. 4817. 6234. 3348.    0.************************************************************
************    0.   94.  566.  566. 1055. 2167. 7313. 7524. 4411.    0.    0.************************************************
******************    0.  227.  566.  735.  302. 6048. 7085. 7880. 6950. 5657.    0.******************************************
************************    0.    0.  667.  789. 1481. 7112. 7446. 7906. 7559. 5936.    0.    0.    0.************************
************************************    0.  916. 1127. 2970. 7578. 8011. 8001. 7531. 4550. 2578. 2018.    0.    0.************
******************************************    0.  735. 1011. 4606. 7088. 7573. 7373. 7746. 7642. 7355. 7652. 5449.    0.******
************************************    0.    0.  281.  847.  564. 7012. 7215. 7451. 7703. 7930. 7890. 7511. 5749.    0.******
************************************    0.    0.  397.  397.  905. 1650. 7092. 7572. 7789. 7329. 7585. 7485. 5418.  376.    0.
************************************************    0.   76.  327.  426. 3734. 7005. 7746. 7877. 7564. 2518.  279.    0.    0.
************************************************    0.  244.  525.  561.  204. 5110. 7338. 7903. 7542. 2145.    0.    0.******
************************************************    0.  139.  419.  455.  600. 1391. 6903. 7175. 6884. 3840.    0.************
************************************************    0.  350.  175.  350.  419.  360. 8978. 6860. 7642. 5353.    0.************
************************************************    0.  106.    0.    0.  419.  571. 4316. 7646. 7164. 5679.    0.************
************************************************    0.  211.   69.  175.  610.  756. 6165. 7525. 7659. 3663.    0.************
************************************************    0.  106.  139.  419.  696.  413. 7111. 6495. 6809. 5033.    0.************
************************************************    0.    0.  106.  139.  559. 1754. 6904. 7153. 6321. 2949.   66.    0.******
************************************************    0.    0.    0.    0.  480. 3876. 8102. 7565. 6380. 4580.  668.    0.******
************************************************    0.    0.    0.    0. 1085. 5753. 7438. 7251. 7314. 6414. 3696.    0.******
******************************************    0.    0.    0.    0.  689. 1175. 5684. 7270. 7707. 7233. 5447. 6062.    0.******
************************************    0.  461.  515.    0.   60.  922. 1396. 4762. 8003. 7296. 7596. 5051. 5395.    0.******
******************************    0.    0.  635. 1090.  575.  515. 1185.  317. 4406. 7959. 7878. 7378. 7618.  400.    0.******
******************************    0. 1057. 1272.  401.  575.  515. 1192. 5582. 4521. 7544. 7633. 7482. 7149. 7525. 7526.    0.
******************************    0.  114.    0.    0.    0.  803. 1264. 6710. 6215. 7400. 7551. 6554. 6467. 7751.    0.******
************************    0.  174.    0.    0.    0.   60.  862.  922. 6800. 6573. 7560. 7754. 6930. 6607.    0.************
************************    0.    0.    0.    0.    0.  401. 1036. 2614. 7385. 6132. 4697. 7557. 7165. 6995.    0.************
************************    0.   65.  125.  125.    0.  461.  524. 6414. 6941. 5948. 3308. 7318. 6962. 6257.    0.************
******************************    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.******************
```

Figure 2.2 Estimated Potential Demand, (ac-ft)

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0.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
0.	49.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
0.	193.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
0.	49.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
0.	0.	0.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	0.	1946.	0.	0.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	0.	1974.	1946.	320.	0.	0.	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	0.	2064.	2282.	718.	718.	636.	0.	0.	0.	******	******	******	******	******	******	******	******	******	******	******
******	0.	131.	480.	636.	574.	911.	767.	997.	566.	0.	******	******	******	******	******	******	******	******	******	******
******	******	0.	94.	566.	566.	1055.	939.	1013.	847.	677.	0.	0.	******	******	******	******	******	******	******	******
******	******	******	0.	227.	566.	735.	450.	1074.	1127.	1127.	847.	847.	0.	******	******	******	******	******	******	******
******	******	******	******	0.	0.	677.	789.	1074.	847.	900.	1016.	1016.	716.	0.	0.	0.	******	******	******	******
******	******	******	******	******	******	0.	1016.	1127.	1127.	1127.	1452.	1122.	1016.	564.	448.	140.	0.	0.	******	******
******	******	******	******	******	******	******	0.	735.	1011.	1127.	958.	2032.	2311.	1122.	1122.	1012.	1055.	843.	0.	******
******	******	******	******	******	******	0.	0.	281.	847.	1016.	1016.	2311.	2172.	1122.	983.	1122.	1122.	983.	0.	******
******	******	******	******	******	******	0.	0.	397.	397.	985.	1055.	1122.	1122.	1122.	983.	1122.	1122.	1122.	376.	0.
******	******	******	******	******	******	******	******	0.	76.	327.	426.	780.	1098.	1122.	1122.	934.	896.	279.	0.	0.
******	******	******	******	******	******	******	******	0.	244.	525.	561.	525.	607.	983.	1122.	934.	564.	0.	0.	******
******	******	******	******	******	******	******	******	0.	139.	419.	455.	630.	267.	901.	1122.	1122.	934.	0.	******	******
******	******	******	******	******	******	******	******	0.	350.	175.	350.	419.	360.	901.	1122.	1122.	983.	0.	******	******
******	******	******	******	******	******	******	******	0.	106.	0.	0.	419.	571.	747.	843.	674.	934.	0.	******	******
******	******	******	******	******	******	******	******	0.	211.	69.	175.	488.	696.	1122.	1122.	901.	448.	0.	******	******
******	******	******	******	******	******	******	******	0.	106.	139.	419.	594.	566.	457.	514.	331.	320.	0.	******	******
******	******	******	******	******	******	******	******	0.	0.	106.	139.	525.	568.	425.	533.	466.	400.	66.	0.	******
******	******	******	******	******	******	******	******	0.	0.	0.	0.	488.	769.	457.	400.	533.	267.	0.	0.	******
******	******	******	******	******	******	******	******	0.	0.	0.	0.	1085.	523.	533.	457.	533.	533.	178.	0.	******
******	******	******	******	******	******	******	0.	0.	0.	0.	689.	1209.	1264.	533.	354.	427.	400.	320.	0.	******
******	******	******	******	******	******	0.	461.	515.	0.	60.	922.	1209.	1264.	533.	466.	457.	457.	354.	0.	******
******	******	******	******	******	0.	0.	635.	1090.	575.	515.	1090.	1209.	1264.	533.	533.	473.	414.	400.	0.	******
******	******	******	******	******	0.	1057.	1272.	401.	575.	515.	1150.	1090.	1264.	533.	533.	533.	533.	443.	443.	0.
******	******	******	******	******	0.	114.	0.	0.	0.	883.	1264.	1209.	1264.	533.	533.	533.	466.	533.	0.	******
******	******	******	******	0.	174.	0.	0.	0.	60.	862.	922.	1264.	1264.	533.	533.	466.	506.	0.	******	******
******	******	******	******	0.	0.	0.	0.	0.	401.	1036.	1150.	1264.	1052.	443.	443.	443.	178.	0.	******	******
******	******	******	******	0.	65.	125.	125.	0.	461.	401.	1319.	1150.	1264.	427.	533.	533.	533.	0.	******	******
******	******	******	******	******	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	******	******	******

Figure 2.3 Estimated Pumping for 1982, (ac-ft)

results when using a hydraulic conductivity of 270 ft/day in their simulation of the Cache Basin.

Based on these reported values an effective porosity of 0.30 and a hydraulic conductivity of 250 ft/day were considered as the intial estimates of the aquifer parameters for the Quaternary aquifer in the Bayou Bartholomew region.

It should be noted that, as is explained in section 3.3, although the aquifer is confined in much of the area during the springtime, the degree of confinement is not great. It is assumed that the aquifer behaves as if it is locally unconfined in the vicinity of high-yielding pumping wells during the water use season. For this reason, our study does not require an estimate of the storage coefficient. However, Broom and Reed (1973) reported an average value of 0.2×10^{-3} as the storage coefficient for this part of the Quaternary aquifer.

2.5 ESTIMATING AQUIFER TOP AND BASE

The elevation of the top and base of the Quaternary aquifer at the center of each 3-mile by 3-mile cell is estimated by geostatistical kriging (Sophocleous et al., 1982) from the records of construction of 328 wells in the study area. The use of this geostatistical method supplies an estimation error for each estimated elevation. This error term is a function of the semi-variograms of the observed elevations. The semi-variograms, in turn, are functions of the number and value of observations and the distance between them.

The results indicate that the top and base elevations decrease in the north-south direction (Figures 2.4 and 2.5 respectively). Figure 2.4 shows that the elevation of the aquifer top is highest in the most northwestern cell, 200 feet above sea level, and is lowest in the most southern cell, 65 feet above sea level. Figure 2.5 shows that the base elevations are highest in the most northeastern cells, 150 feet above sea level, and are lowest in the southern portion of the study area, 27 feet below sea level.

The standard deviations of probable error of estimated top and base elevations are shown in Figures 2.6 and 2.7 respectively. As seen in Figure 2.6, the standard deviation of probable error in the estimated top elevation ranges from three to twelve feet for most of the internal cells. For boundary cells and cells in the western part of Ashley and Drew counties within the study area, the standard deviation ranges from five to thirty feet, due to the scarcity of well logs in these regions.

Figure 2.7 shows that the standard deviation of probable error is much less for estimated base elevations than for estimated top elevations. For the base, the standard deviation ranges from two to six feet in most of the internal cells. The range for boundary and western cells is larger, from two to thirteen feet.

2.6 ESTIMATING HISTORICAL GROUNDWATER ELEVATIONS

Sixty-seven springtime groundwater level observations are available from U.S. Geological Survey records (Edds, 1983) for all eleven years of the validation period. From these records,

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	199																				
2	190	193																			
3	181	176																			
4	176	182																			
5	196	187	173																		
6		207	165	163	179																
7		152	166	169	181	174	163														
8		179	163	168	167	158	173	171	159	150											
9		203	191	190	167	163	163	161	154	151	151										
10			212	192	184	171	156	155	143	152	167	161	145	145							
11				204	179	167	158	152	149	149	138	133	136	142							
12					181	172	157	152	146	146	140	139	137	140	141	135	132				
13							180	152	137	148	143	131	131	137	141	138	127	121	122		
14								165	138	148	141	120	127	133	138	132	121	119	120	127	
15							184	171	138	140	137	140	133	94	132	124	119	119	122	123	
16							182	169	163	150	144	139	129	119	126	110	106	121	121	122	122
17									163	158	150	136	124	122	116	117	122	125	121	120	123
18									155	38	136	112	118	131	111	112	115	117	118	121	
19									157	148	151	104	118	114	103	97	101	105	114		
20									168	141	157	128	122	110	101	78	100	108	110		
21									160	150	151	124	118	116	109	98	109	110	110		
22									130	124	133	128	119	107	113	108	107	108	101		
23									128	118	122	120	114	109	110	80	98	96	93		
24									128	118	114	111	112	108	109	105	99	91	82	78	
25									128	123	112	103	105	108	108	106	112	84	73	72	
26									130	119	104	105	97	99	102	95	94	66	64	71	
27								133	127	119	108	105	97	92	96	86	86	73	66	75	
28							132	127	120	114	108	106	102	84	94	102	95	80	80	60	
29						122	120	117	112	105	96	97	84	92	78	95	92	87	97	92	
30						113	111	107	101	96	89	86	97	99	94	91	100	96	103	97	85
31						107	103	99	93	65	92	80	92	92	90	101	110	91	82	82	81
32					101	98	95	91	83	84	83	74	88	85	84	98	102	73	61		
33					97	95	91	79	79	81	88	76	82	86	74	95	96	91	66		
34					94	90	8	78	75	78	83	80	89	82	80	96	85	78	66		
35						86	75	75	75	80	77	65	86	86	89	98	85	79	69	64	

Figure 2.4 Kriged Elevations Of The Top Of The Aquifer, (ft).

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	151																				
2	158	144																			
3	158	149																			
4	144	150																			
5	112	110	91																		
6		75	92	80	78																
7		70	84	94	91	74	64														
8		51	69	78	81	82	75	70	74	75											
9		81	79	79	63	83	69	69	73	68	64										
10			142	109	42	55	54	66	78	73	64	79	61	62							
11				128	77	38	38	41	48	64	67	78	70	68							
12					94	67	68	45	47	63	67	77	74	67	64	62	58				
13							83	49	61	73	73	75	70	68	65	62	60	52	45		
14								56	61	76	74	76	67	59	57	56	62	52	46	44	
15							66	56	57	58	63	58	73	56	52	45	49	51	46	43	
16							67	65	65	51	62	54	64	62	53	33	57	50	45	44	44
17									67	66	54	49	58	54	18	52	42	39	39	42	43
18									67	58	45	42	54	50	29	30	27	28	35	38	
19									68	50	45	53	55	55	52	37	33	30	36		
20									72	40	45	54	57	43	48	48	52	45	38		
21									64	32	25	42	49	35	52	51	51	47	37		
22									54	38	45	46	45	42	57	48	43	38	37		
23									60	54	60	52	45	51	52	38	35	36	34		
24									67	56	56	49	35	51	49	47	30	30	33	35	
25									65	53	47	49	53	59	52	36	25	30	32	26	
26									63	49	26	16	22	55	41	31	44	37	31	27	
27								74	64	50	27	23	27	27	33	24	45	38	30	25	
28							79	74	63	51	32	33	35	48	41	23	26	29	25	20	
29						80	76	70	64	56	47	30	24	39	39	23	27	28	18	14	
30						74	72	68	62	55	45	28	28	29	27	18	20	19	16	9	4
31						71	68	64	57	47	46	15	26	38	37	13	8	4	4	2	-1
32					72	69	68	60	50	43	27	3	8	23	23	-6	0	3	7		
33					70	64	61	52	45	31	26	21	16	10	20	23	8	-1	-7		
34					65	61	58	49	45	24	9	34	51	37	10	10	0	-12	-25		
35						59	51	49	44	29	22	47	50	38	19	6	-4	-16	-27	-25	

Figure 2.5 Kriged Elevations Of The Base Of The Aquifer, (ft).

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	14.2																				
2	8.6	8.6																			
3	8.3	3.6																			
4	11.3	4.4																			
5	13.1	10.3	8.1																		
6		4.4	8.3	8.1	7.8																
7		8.0	9.1	8.1	7.3	4.0	6.8														
8		8.6	10.6	6.4	8.8	8.0	9.5	12.0	12.5	8.8											
9		10.7	9.8	6.3	7.3	8.6	6.6	11.7	12.2	10.1	8.2										
10			7.7	8.1	7.6	7.8	7.4	7.8	6.2	7.0	5.5	5.2	12.3	18.5							
11				8.8	8.6	7.0	8.8	6.6	8.5	8.8	8.8	7.5	11.8	16.4							
12					14.0	11.1	8.8	10.3	10.5	6.5	9.8	8.1	10.8	14.0	18.7	17.3	17.7				
13							8.7	7.4	7.8	10.4	11.3	10.8	8.5	11.3	12.6	12.6	13.7	12.6	6.0		
14								8.0	10.1	5.4	8.0	5.8	9.5	8.0	7.5	6.7	8.5	12.1	10.5	14.3	
15							13.8	7.0	7.3	6.3	9.6	6.3	3.3	2.8	7.3	7.1	4.8	12.6	15.5	18.3	
16							17.0	12.3	8.3	8.8	6.1	5.6	6.0	8.2	6.8	6.2	6.1	12.3	17.0	20.3	23.7
17									13.0	8.2	8.8	6.4	8.0	7.6	6.6	3.8	8.8	12.3	17.5	21.4	24.8
18									13.4	7.0	8.3	8.2	10.2	3.8	8.7	10.6	8.6	12.2	17.5	21.8	
19									12.0	8.4	6.8	7.3	8.1	6.8	7.0	11.2	11.1	8.8	17.4		
20									8.3	7.8	4.2	8.2	8.1	5.1	8.1	7.2	8.7	11.2	17.8		
21									6.1	8.8	8.3	3.3	7.2	7.7	6.8	10.1	5.2	13.8	18.1		
22									8.0	8.6	10.1	11.0	8.6	7.1	8.2	7.1	11.6	18.0	18.8		
23									11.5	11.6	8.7	12.2	11.4	8.3	5.1	3.4	11.3	15.8	18.8		
24									15.8	15.6	13.8	8.7	8.6	4.7	5.4	8.5	6.1	16.4	18.3	23.4	
25									17.6	17.0	15.0	9.0	8.6	7.8	8.7	6.8	3.6	13.3	18.4	23.7	
26									17.3	17.3	16.3	10.3	6.4	2.8	3.8	8.8	6.6	10.2	18.0	21.0	
27								11.7	15.8	16.8	16.2	12.5	4.2	4.5	8.7	7.8	7.8	8.5	14.8	18.8	
28							14.6	13.2	15.6	15.8	13.5	8.0	4.8	4.2	9.4	7.8	8.1	9.4	14.1	18.1	
29						22.4	18.4	18.8	18.6	15.0	6.0	8.7	6.6	7.1	3.1	8.3	8.6	7.4	10.8	15.2	
30						24.3	21.2	18.4	18.7	14.3	11.7	7.8	6.8	6.8	5.2	8.8	8.1	7.0	8.8	14.2	20.3
31						25.6	22.4	18.1	15.0	10.7	5.1	9.7	5.5	8.8	3.3	8.1	6.0	6.8	6.7	15.7	20.7
32					28.8	26.5	23.0	18.1	14.0	7.7	10.2	7.1	5.0	8.4	7.3	3.6	4.7	8.8	11.5		
33					30.7	27.3	23.5	18.9	14.4	10.6	4.3	8.3	6.1	7.9	7.7	7.7	10.0	4.3	11.7		
34					31.8	28.3	24.3	20.0	13.2	8.2	8.5	8.4	8.6	6.6	4.0	8.3	5.8	7.1	6.3		
35						28.7	26.5	21.5	15.6	11.1	10.7	7.1	7.6	12.8	13.7	13.5	12.4	10.5	13.0	17.8	

Figure 2.6 Standard Deviation Of The Error For Kriged Top Elevations, (ft).

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I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	6.2																				
2	4.1	3.7																			
3	4.0	1.5																			
4	4.8	1.8																			
5	5.7	4.5	3.5																		
6		1.8	4.1	3.9	3.4																
7		2.6	3.5	3.5	3.2	1.8	3.0														
8		4.2	4.6	3.8	3.8	3.5	2.8	5.2	5.4	3.0											
9		4.7	3.9	2.7	3.2	3.7	4.1	5.1	5.3	4.4	3.6										
10			3.3	3.6	3.3	3.4	3.2	3.4	3.8	3.0	2.4	2.3	5.3	6.0							
11				3.8	4.1	3.0	3.8	2.9	3.7	4.2	3.9	3.2	5.1	7.1							
12					6.1	4.8	3.0	4.5	4.6	3.7	3.8	3.5	4.7	6.1	7.2	7.8	7.7				
13							3.8	3.2	3.4	4.5	4.9	4.7	3.7	4.9	5.5	5.6	6.9	5.5	2.6		
14								3.6	4.4	2.3	3.9	2.5	3.7	3.9	3.2	2.9	3.7	5.3	4.6	6.2	
15							6.0	3.0	3.2	3.6	3.7	2.7	1.6	1.2	3.2	3.1	2.1	5.5	6.7	7.9	
16							7.4	6.6	3.6	3.0	2.8	2.4	2.8	3.8	3.0	2.7	2.8	5.4	7.4	8.8	10.3
17									5.6	3.8	3.9	3.7	3.5	3.2	2.6	1.8	3.9	5.3	7.6	9.3	10.6
18									6.8	3.1	4.0	4.0	4.4	1.8	3.8	4.6	4.1	6.3	7.6	9.5	
19									6.2	4.1	2.8	3.1	3.5	2.4	3.1	4.9	4.8	4.2	7.6		
20									2.7	3.3	1.8	3.5	4.0	2.2	3.8	3.1	3.6	4.8	7.7		
21									3.2	4.3	3.6	1.4	3.1	3.3	2.6	4.4	2.2	6.0	6.3		
22									3.9	4.1	4.4	4.8	3.7	3.1	3.6	3.1	5.0	6.6	6.7		
23									5.0	5.2	4.2	6.3	5.0	3.8	2.2	1.5	4.9	6.9	6.6		
24									8.8	6.9	6.1	4.2	3.8	2.1	2.3	3.7	2.2	6.2	8.4	10.2	
25									7.8	7.4	6.5	3.9	3.7	3.3	4.2	2.8	1.6	5.8	8.0	8.9	
26									7.6	7.6	7.1	4.6	2.8	1.2	1.7	3.7	2.8	4.4	6.9	8.1	
27								5.1	6.9	7.3	7.0	5.4	1.8	1.9	3.6	3.3	3.4	3.7	6.4	6.6	
28							6.3	8.3	6.8	6.9	5.8	3.9	2.0	1.8	3.7	3.4	4.0	3.7	6.1	7.9	
29						8.7	8.1	7.3	7.2	6.5	3.8	3.8	2.4	3.1	1.3	3.7	3.8	3.2	4.7	6.6	
30						10.5	8.2	8.1	7.2	6.2	5.1	3.4	2.5	3.0	2.2	3.8	3.5	3.0	3.8	6.2	6.6
31						11.1	9.7	8.3	6.5	4.7	2.2	3.8	2.4	4.3	1.6	4.0	3.5	3.6	3.8	6.8	8.0
32					12.8	11.5	10.0	8.3	6.1	3.3	4.4	3.1	2.2	4.1	3.1	1.7	2.0	3.0	5.0		
33					13.3	11.8	10.2	8.6	6.4	4.7	1.8	3.8	2.8	3.4	3.3	3.3	4.3	1.8	5.1		
34					13.8	12.3	10.6	8.7	6.7	3.9	3.7	4.1	3.8	2.9	1.7	4.0	2.6	3.1	2.3		
35						12.8	11.5	9.3	6.8	4.8	4.7	3.1	3.3	5.5	5.9	5.8	5.4	4.8	5.2	7.8	

Figure 2.7 Standard Deviation Of The Error For The Kriged Base Elevations, (ft).

the water levels at the center of each cell are estimated for the springs of 1973 to 1983 by kriging.

As an example, the 1983 potentiometric surface and its standard deviations of probable error are shown in Figures 2.8 and 2.9 respectively. Figure 2.8 also shows that the potentiometric surface decreases in the north-south direction, from 190 feet above sea level in the northwest to 90 feet above sea level in the southeast.

Figure 2.9 shows that the standard deviation of the probable error of estimated potentiometric surface elevations is as high as 20 ft in some cells. These are cells that are distant from an observation well. Standard deviations are much smaller in cells near observation wells.

For the internal cells, the water levels of spring 1973 are used as the initial conditions for the validation discussed in the next chapter. For each constant-head cell, the average springtime groundwater level (for 1973-82) at the center of the cell is used as the cell's constant groundwater elevation in simulations conducted for the validation period.

2.7 MODELING STREAM-AQUIFER INTERFLOW

Interflow between the aquifer and hydraulically connected streams is modeled based on Darcy's law. The program RECHARGE was written to utilize this law in estimating interflow between the aquifer and the streams. RECHARGE requires the following data: observed monthly stream stages, kriged groundwater elevations and reach transmissivities. Darcy's law and reach

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I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	194																				
2	198	194																			
3	197	196																			
4	197	196																			
5	197	195	192																		
6		194	191	187	181																
7		193	190	187	183	178	175														
8		192	190	187	183	179	175	173	173	171											
9		190	189	186	181	175	170	166	166	167	164										
10			186	185	177	171	184	157	158	157	158	158	158	158							
11				179	174	168	161	152	149	150	151	152	155	158							
12					171	167	160	152	148	146	146	147	152	156	157	154	151				
13							159	153	149	148	142	142	147	154	158	151	147	144	142		
14								154	150	148	145	142	141	142	145	144	142	140	138	137	
15							157	154	152	150	147	143	137	133	135	138	138	135	134	133	
16							157	154	153	151	147	142	138	131	128	129	130	130	130	130	130
17									152	150	147	142	135	129	123	124	125	128	128	127	126
18									149	149	146	142	135	128	124	122	123	123	123	123	
19									146	145	143	140	134	127	124	122	121	121	121		
20									141	141	139	137	133	129	125	122	120	119	118		
21									138	137	135	132	129	128	123	120	118	117	115		
22									135	135	132	128	125	122	120	117	115	114	114		
23									134	134	131	125	121	118	115	113	112	112	112		
24									133	134	129	123	118	114	111	109	108	109	110	110	
25									129	130	125	119	114	110	105	104	105	107	108	109	
26									122	121	118	113	108	103	98	99	103	105	107	108	
27								114	115	114	112	108	103	99	97	104	103	103	105	107	
28							107	108	109	108	108	105	101	97	96	99	100	101	104	107	
29						101	103	104	104	104	104	104	101	96	94	95	96	98	102	107	
30						97	100	102	102	102	102	102	100	95	93	93	94	95	101	110	112
31						97	100	103	105	103	101	99	96	93	91	92	94	97	102	109	111
32					83	98	100	103	105	102	99	88	83	91	90	91	94	98	103		
33					91	98	100	103	103	100	97	93	90	88	88	89	92	98	103		
34					90	95	99	102	100	98	95	91	88	86	87	88	89	92	102		
35						93	96	98	97	98	94	91	88	86	87	88	89	94	101	104	

Figure 2.8 Kriged Potentiometric Surface Elevations For 1983, (ft).

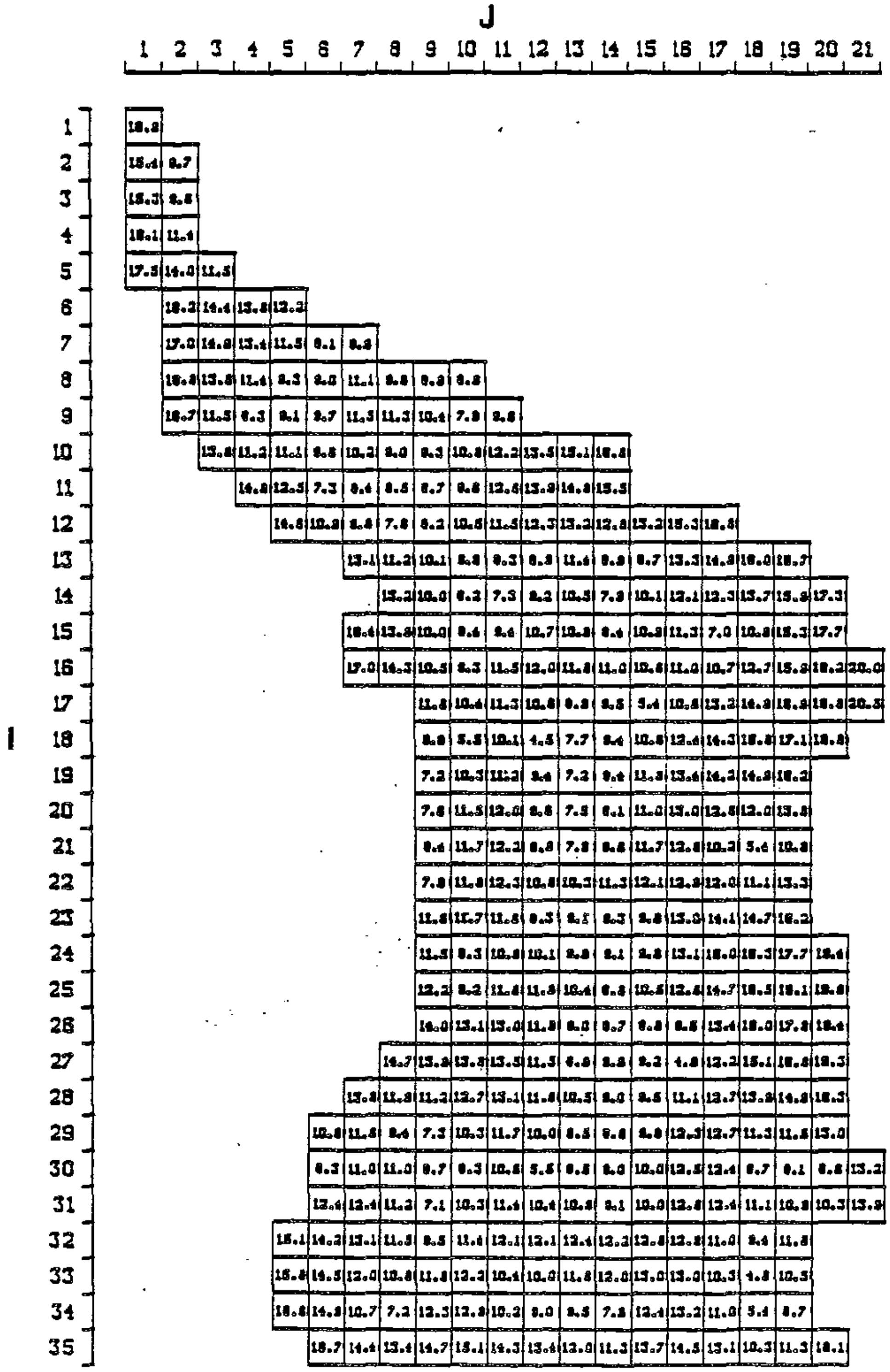

Figure 2.9 Standard Deviation Of The Error For The Kriged Potentiometric Surface Elevation Of 1983, (ft).

transmissivity are described briefly in the following sections.

2.7.a. Darcy's Law

Flow through a porous media of cross-sectional area A normal to the direction of flow, can be computed by using the following representation of Darcy's law.

$$Q = -k\ A\ (dh/dx) \qquad (2.1)$$

where,

- Q = the flux through a cross section of area A, normal to the direction of flow (L^3/T)
- k = the hydraulic conductivity of the aquifer material, (L/T);
- A = unit area normal to the direction of flow, (L^2);
- h = the potentiometric surface elevation or hydraulic head, (L);
- x = the distance in the direction of groundwater flow, (L);
- dh/dx = the hydraulic gradient, (L/L);

2.7.b Reach Transmissivity And APS Values And Their Relationship

Reach transmissivity is a measure of the ability of the streambed material to transmit water from the stream to the aquifer or from the aquifer to the stream. The reach transmissivity for a cell which is not hydraulically connected to a stream is equal to zero. Reach transmissivity can be defined by the following relationship (Morel-Seytoux, 1979):

$$q_r = G_r \cdot (h_r - R_r) \qquad (2.2)$$

where,

q_r = discharge from (+ sign) or recharge to (- sign) the aquifer for a particular cell containing reach r, (L^3/T);

h_r = potentiometric surface elevation at the given cell that contain reach r, (L);

R_r = the assumed constant river stage at the given cell that contain reach r, for a given time step, (e.g., one month in this study), (L);

G_r = reach transmissivity at reach cell r, (L^2/T);

To apply Equation 2.2, it is necessary to estimate reach transmissivity, G_r. It may be calculated analytically (Morel-Seytoux, 1979), or empirically through model calibration. A similar parameter, the APS or streambed parameter, may be similary obtained (Reed and Broom, 1979).

APS values reflect the degree of hydraulic connection between the stream and the aquifer. These values represent the hydraulic conductivity of the streambed material multiplied by the horizontal area of the streambed at a node, divided by the thickness of the streambed material. Therefore for a particular cell (Reed and Broom, 1979):

$$APS_r = \frac{K_r \cdot L_r \cdot W_r}{H_r} \qquad (2.3)$$

where,

APS_r = streambed parameter reflecting the degree of hydraulic connection between a stream and a aquifer for cell r, (L^2/T);

K_r = hydraulic conductivity of streambed material in cell r, (L/T);

L_r = length of the stream in cell r, (L);

W_r = average width of the streambed in cell r, (L);

H_r = thickness of the streambed material in cell r, (L);

For cell r containing a stream reach of length L_r, and average width W_r, and a bed of thickness H_r and hydraulic conductivity K_r, the variables in Equation 2.1 can be described as:

$$Q = q_r \qquad (2.4)$$

$$A = L_r \cdot W_r \qquad (2.5)$$

$$k = K_r \qquad (2.6)$$

$$dl = H_r \qquad (2.7)$$

$$dh = h_r - R_r \qquad (2.8)$$

Replacing these variables in Equation 2.1, it can be stated as:

$$q_r = \frac{(K_r \cdot L_r \cdot W_r)}{H_r} (h_r - R_r) \qquad (2.9)$$

Therefore, comparing Equation (2.9) and (2.2)

$$G_r = \frac{K_r \cdot L_r \cdot W_r}{H_r} \qquad (2.10)$$

The reach transmissivity G_r and the APS values are identical for the same variables on the right hand side of Equation 2.3. This identity is useful for calculating interflow between stream and aquifer (Equation 2.2) when the APS value is known.

2.7.c Stream-Aquifer Interflow

AS specified by Reed and Broom (1979), only those streams which have significant hydraulic connection with the aquifer (Mississippi River, Arkansas River, Bayou Bartholomew, Bayou Macon and Boeuf River) are considered for modeling stream-aquifer connection.

Reed and Broom (1979) reported the APS values used when validating an analogous simulation model. In some portions of the study area, their values are too large to work satisfactorily with the digitized simulation model used in this study. Therefore a different technique is used to estimate the recharges or discharges occuring in some of the cells with stream-aquifer connections. These cells (Figure 2.10) include 2 cells in the northern reach of the Boeuf River and 4 cells in its southern reach. Also included are nine cells in the region through which the Arkansas River and Mississippi River travel, which are not used as constant-head cells. The annual stream-aquifer interflow at the two northern cells on the Boeuf River is estimated by assuming that they have the same S/A interflow as adjacent S/A cells. The S/A interflow at the other 13 cells is estimated by solving for the annual volumes of steady state withdrawal which will maintain observed springtime groundwater levels.

The APS values for other internal cells with stream-aquifer connection are obtained from Reed and Broom (1979). In that study the dimensions of each cell are 7040 ft by 7040 ft. In this study the dimensions of each cell are 15840 ft by 15840 ft. Therefore, to estimate an appropriate value of reach transmissivity (G_r) for the larger cell in this study, the APS values obtained from Reed and Broom (1979) are multiplied by a factor, 2.25 (= 15840.0 / 7040.0). In this conversion the average width and thickness of the streambed are assumed to be constant. The hydraulic conductivities of streambed material used by Reed and Broom (1979) for their analog model are virtually identical to those used to model the stream-aquifer interaction in this

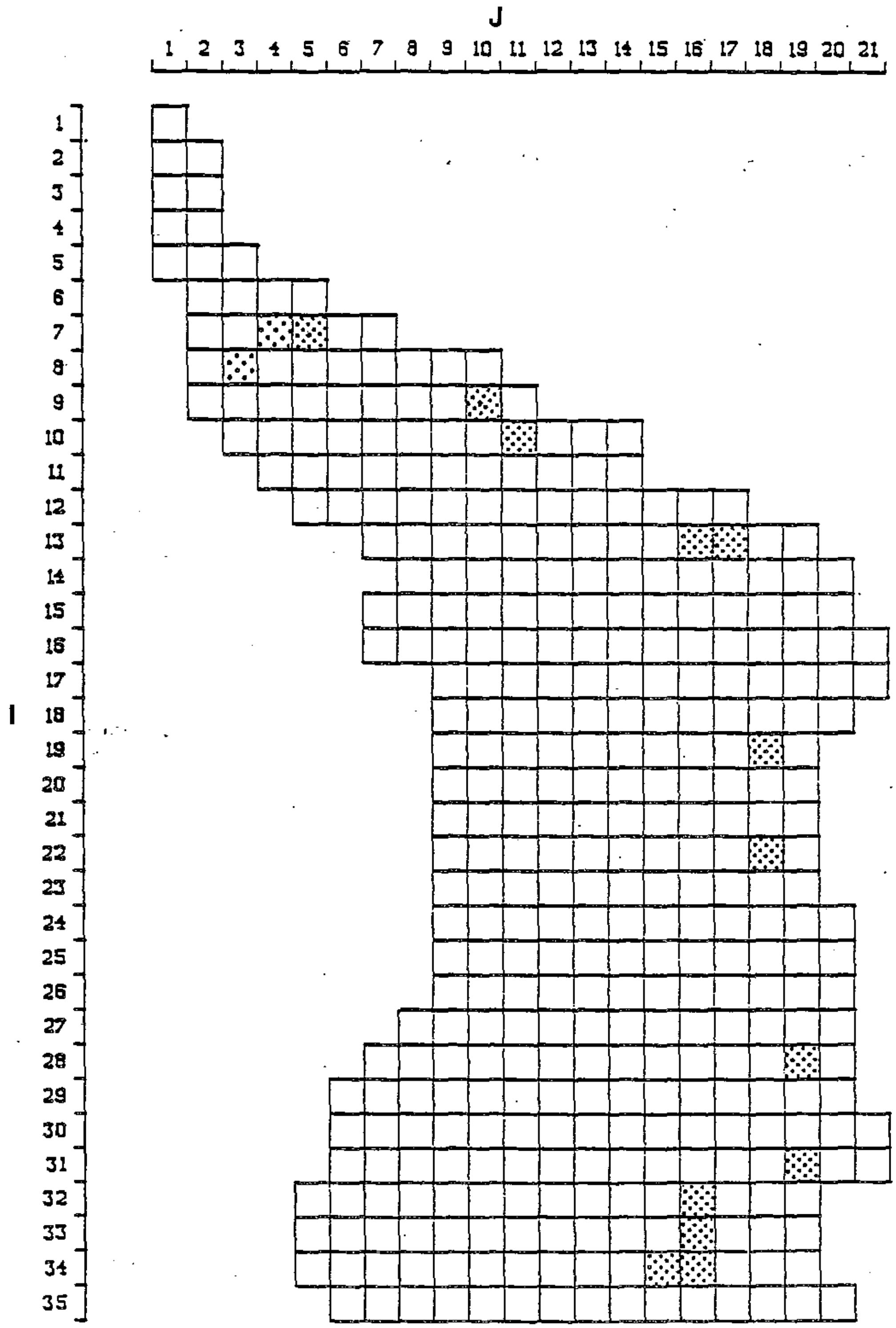

Figure 2.10 Cells For Which an Average Annual Recharge or Discharge Is Calculated Using Steady-State Equation For Two-Dimensional Flow And Kriged Springtime Groundwater Levels.

study.

Once reach transmissivities are estimated, monthly values of stream-aquifer interflow are computed based on the difference in elevation between unpublished monthly river stage records obtained from the Corps of Engineers and springtime groundwater levels in each cell with stream-aquifer connection. Monthly values of interflow are subsequently summed to estimate annual interflows. Thus, although the groundwater levels are assumed constant throughout the year, the seasonal variations in river stages are incorporated in the computation of interflow.

Where the potentiometric surface is above the stream stage, discharges occurred from the aquifer to the stream. Analysis of historic river stages and groundwater levels indicate that the Quaternary aquifer discharged to the Bayou Macon an annual average (1973-82) of 4,057 acre-ft. This analysis also indicates that Bayou Bartholomew and Boeuf River recharged the aquifer an annual average of -9,757 ac-ft and -6,678 ac-ft respectively.

In this study, the yearly cell-by-cell stream/aquifer (S/A) interflow is added to the cell-by-cell total annual pumping from the Quaternary aquifer to estimate the total annual hydraulic stimulus occurring at each cell. The use of these net stimuli in the validation process is described in next chapter.

2.8 ESTIMATING RECHARGES IN THE SYSTEM THROUGH CONSTANT-HEAD CELLS

The system can be recharged through each of the 66 constant-head cells which comprise most of the area's periphery.

The recharge comes either from rivers penetrating to the aquifer in those cells or from extensions of the aquifer outside the region. These cells include all the boundary cells through which the Arkansas and Mississippi Rivers travel as well as the south and southwestern boundary cells (Figure 1.3). The groundwater levels and pumping are fixed in constant head cells.

The average annual historic net recharge through the constant-head cells into the system is estimated using the water volume balance equation, presented in detail in the last section of the next chapter.

CHAPTER III

CALIBRATION AND VALIDATION OF PARAMETERS FOR A GROUNDWATER FLOW SIMULATION MODEL

By

B. Datta, J. Solaimanian and R. C. Peralta

3.1 INTRODUCTION

Simulation of groundwater flow hydraulics and the hydrology of an aquifer requires the validation of certain physical aquifer parameters. For a small area these parameters can be estimated through a few well tests at specific locations. However, for a large area such as the Bayou Bartholomew Basin, it is impossible to conduct enough well tests to accurately determine the spatial distribution of these parameter values. In this study, economic considerations restrict the amount of field data that can be collected. Accordingly the first objective of the validation is to verify, using existing recorded data, preliminary estimates of the effective porosity and hydraulic conductivity of the Quaternary aquifer underlying the region. The second objective is to validate estimates of vertical accretion (deep percolation through the soil profile) to the aquifer and the cell-by-cell values of reach transmissivity (and S/A interflow).

In the validation process, it is assumed that the aquifer is isotropic and homogeneous with respect to effective porosity and hydraulic conductivity. The AQUISIM Model (Verdin et al.,

1982) is used to simulate changes in the potentiometric surface within the validation period.

The AQUISIM model accomplishes the simulation process in two steps. In the first step, influence coefficients are generated. These describe the influence of a unit distributed excitation (pumping) at a given cell occurring at a given time period, on the hydraulic head at a different or the same cell during any given time period. The excitations used in this study are termed "distributed" because they are assumed to be averaged over the area of a cell. Influence coefficients are calculated for all the cells in the system. These influence coefficients are based on the effective porosity and transmissivity of the aquifer. The transmissivity depends on the hydraulic conductivity and saturated thickness.

In the second step of the simulation, the influence coefficients are multiplied by known excitations of groundwater discharges or recharges at each cell. The model sums the responses to all stimuli to simulate the hydraulic heads at the end of each time period within the time horizon of the simulation.

The ideal procedure for using a groundwater simulation model for calibration and validation of aquifer parameters can be described by the following steps (Peralta et al, 1985):

1. Use available data to determine the precise study area, select a simulation model and make the best hydrogeologic assumptions possible.

2. Use the selected assumptions, modifying them if necessary, to calibrate the model. In calibration, the model's response to pumping during a specified time period is compared with the historic observed response of the aquifer over the same period. Model response is made to be more in harmony with historic response by improving the estimates of aquifer physical characteristics used within the model. The process is continued until the model emulates historic conditions within pre-assigned approximation limits over the calibration period.

3. Test the model over a second time period, the validation period. If the model-predicted water levels again compare with historic observed levels within pre-asigned approximation limits, the model is considered sufficiently validated to be used for predictive purposes. In this step the sensitivity of the model to small changes in the assumptions is evaluated.

4. Select the best assumptions from the validation/sensitivity analysis step and use the model to predict water levels. Prediction is generally limited in time span to the same number of years as validation.

Sufficient accurate data are not always available to perform both calibration and validation for time spans of satisfactory duration. In such situations, when using a generally applicable (as opposed to site-specific) model, validation alone is adequate, as long as the hydrogeologic assumptions are not changed significantly during the validation process.

Water-use information and groundwater-level observations for the study area prior to the 1970s are not detailed and reliable enough to perform both model calibration and validation. The 10 years between 1973 and 1983 is the longest period of time for which sufficient data could be obtained. For this reason, and because previous studies on this aquifer and the adjacent Grand Prairie region were consistent in selected aquifer parameter values, the calibration period was omitted in this study.

Validation and sensitivity analysis of the parameters are accomplished for the 1973-1983 period using the common practice of history-matching and the data discussed in the preceding chapter: withdrawals from the aquifer, discharges from or recharges to the aquifer through stream aquifer interflow, aquifer top and base elevations and historic Quaternary groundwater levels for the years 1973-83.

Additional aquifer parameter values and characteristics assumed and used in the validation process include the hydraulic conductivity, effective porosity and deep percolation through the soil profile. These assumptions are described in the following paragraphs.

A value of 0.3 is used as an aquifer-wide estimate of effective porosity. An aquifer-wide estimate of 250 ft/day was selected as an appropriate hydraulic conductivity. These parameter values were selected based on the previous studies done in this area and the adjacent Grand Prairie region as discussed in Section 2.4. The hydraulic conductivity of 250 ft/day is an average of the values used by Reed and Broom (1979) for this area. In addition, 250 ft/day was considered appropriate because previous researchers reported hydraulic conductivity values of 254-270 ft/day for the adjacent Grand Prairie. In alluvial deposits, particle size usually increases with depth. Hydraulic conductivity increases with particle size. Since the aquifer in Bayou Bartholomew study area is less dewatered than that in the Grand Prairie region, one would expect the average hydraulic conductivity to be somewhat less in the Bayou Bartholomew Basin than in the Grand Prairie region.

Annual transmissivities for each cell in the study area are obtained by multiplying the annual hydraulic conductivity by the distance between the base of the aquifer and either the 1973 groundwater level or the top of the aquifer, whichever is lower at that point. These transmissivities are used in the validation process described in this section.

Broom and Reed (1973) reported a total amount of deep percolation equal to 47,000 ac-ft/year for this study area. Because, in most of the internal cells, a somewhat impermeable clay layer is assumed to overlie the aquifer, this slight amount of recharge seems reasonable. In this study, it was assumed that there is only 0.20 inches of recharge through deep percolation per year (100 ac-ft/year per cell) for all 380 cells. This totals to 38,000 ac-ft per year for the entire area. As shown in the following section, this assumption resulted in very small errors between simulated and observed storages in the aquifer at the end of the validation period.

3.2 VALIDATION OF PARAMETERS AND SENSITIVITY ANALYSIS

The preceding chapter and section described the development of our best assumptions concerning aquifer characteristics and inputs and outputs to the aquifer system. There is, however, always error associated with making aquifer-wide estimates of aquifer characteristics and in estimating pumping or recharge. In the model validation and sensitivity analysis step our aim was to determine whether we had identified the best assumptions possible for use in predicting future water levels. To accomplish model validation and sensitivity analysis, we performed a series of simulation runs. Our best assumptions were incorporated in Run 1, the validation run. In this run, a hydraulic conductivity of 250 ft/day, effective porosity of 0.3, accretion equal to 100 ac-ft/year per cell and reach transmissivities as discussed before were assumed. In order to determine the sensitivity of the model, the hydraulic conductivity, effective porosity and amount of vertical accretion were varied in seven additional runs. Table 3.1 displays the groundwater that was estimated to exist in storage in 1973 and 1983 based on observed groundwater levels and the assumed effective porosities in the 8 simulation runs. Also shown are the storage values that were simulated to exist in all eight runs, based on simulated groundwater levels and assumed effective porosities.

The criteria for estimating the errors resulting from each simulation run are the three percentage error measures described below. The observed storage in the three criteria are based on the kriged groundwater levels and effective porosity for each run.

$$PC1 = \frac{\text{simulated storage 1983} - \text{observed storage 1983}}{\text{observed storage 1983}} \times 100$$

$$PC2 = \frac{\text{simulated storage 1983} - \text{observed storage 1983}}{\text{observed storage 1973} - \text{observed storage 1983}} \times 100$$

$$PC3 = \frac{\text{simulated storage 1983} - \text{observed storage 1983}}{\text{total pumping (1973 through 1982)}} \times 100$$

Table 3.2 displays the results of the simulation runs in terms of these three error criteria. The analysis of the results is analogous to that performed by Peralta et al. (1985). The simulated results that most satisfactorily matched historic data are obtained with Runs 1 and 5. Run 1 (the validation run) underestimates groundwater storage in 1983 by 0.004 percent and the reduction in storage by 0.56 percent. The simulated storage reduction after 10 years is 0.07 percent less than the observed value (measured as a percent of the total pumping for the 10 year period).

Table 3.1 Simulated and observed groundwater storages

Run	Observed Storage 1973 (acre-ft) X 1000	Observed Storage 1983 (acre-ft) X 1000	Simulated Storage 1983 (acre-ft) X 1000
1	34101.9	33869.2	33867.9
2	39785.7	39514.0	39557.0
3	28418.4	28224.3	28175.8
4	34101.9	33869.2	33872.4
5	"	"	33870.1
6	"	"	33816.9
7	"	"	33820.9
8	"	"	33816.7

Table 3.2 Validation Results

Run	K (ft/day)	e	Deep Percolation (ac-ft)/cell	PC1 (%)	PC2 (%)	PC3* (%)
1	250	0.30	100	-0.004	-0.559	-0.067
2	250	0.35	100	0.109	15.826	2.217
3	250	0.25	100	-0.172	-24.987	-2.500
4	270	0.30	100	0.009	1.375	0.165
5	230	0.30	100	0.003	0.387	0.046
6	250	0.30	0	-0.154	-22.475	-2.700
7	270	0.30	0	-0.143	-20.756	-2.490
8	230	0.30	0	-0.155	-22.561	-2.707

* average annual pumping (1973-1982) is 193,956 ac-ft.

K is the Hydraulic Conductivity

e is the Effective Porosity

Run 5 is performed assuming an hydraulic conductivity of 230 ft/day and an effective porosity of 0.3. In addition, deep percolation equal to 100 ac-ft/year per cell is included in Runs 1 through 5. Run 5 simulated actual conditions with about the same accuracy as Run 1. In such a situation, where two runs simulate with comparable accuracy, one must determine which set of assumptions should be used for prediction of future groundwater levels. In this case, the assumptions of Run 1 are preferred for several reasons. The first reason is that 250 ft/day is more comparable than 230 ft/day to the 270 ft/day value, validated for the adjacent Grand-Prairie region of the same aquifer. The second reason is that for the ten years of validation (1973-1983), the average annual error in simulated storage compared to observed storage is smaller for Run 1 than for Run 5. The third reason is that it is safer to underestimate the available storage than overestimate. Run 1 underestimates slightly while Run 5 overestimates slightly.

Runs 2 and 3 use a hydraulic conductivity of 250 ft/day and effective porosities of 0.35 and 0.25 respectively. It is evident from Table 3.2 that these porosities result in much larger errors than those of Run 1. Assumption of an effective porosity equal to 0.3 and hydraulic conductivity of 270 ft/day as in Run 4 results in larger errors than in Run 1. Runs 6, 7, and 8 assumed no deep percolation to the aquifer. All three of these runs resulted in larger errors than did Run 1.

A comparison of Runs 1 and 6 permits an observation to be made about the sensitivity of the model to the estimated volume of deep percolation to the aquifer. The difference between Run 1 and Run 6 is that in Run 1 a deep percolation of 100 ac-ft/year per cell is assumed, while none is assumed in Run 6. From Table 3.2 it can be noted that Run 6 results in errors 40 to 50 times larger than that of Run 1. Therefore the estimated amount of vertical accretion is reasonable.

Figure 3.1 shows how accurately the best run (Run 1) predicted cell-by-cell groundwater levels for the spring of 1983. The value in each cell is the difference between simulated and observed (kriged) groundwater levels in 1983 for that cell. A negative value indicates that the simulated level is lower than the observed elevation. The standard deviation of probable error of estimated kriged potentiometric surface elevations for 1983 ranged between 4.5 and 20.5 feet in the study area (Figure 2.9). Differences between simulated and observed values that are less than the standard deviation of probable errors of estimated observed potentiometric surface elevations are considered insignificant. As a result, the difference between simulated (Run 1) and observed elevations are insignificant in all cells for 1983 groundwater levels.

In summary, after performing a literature review and judging the results of the validation and the sensitivity analysis, a hydraulic conductivity of 250 ft/day and an effective porosity (specific yield) of 0.3 were selected as being appropriate for the Quaternary aquifer underlaying the Bayou Bartholomew/Alluvial Aquifer System. In addition, an average of 0.20 inch/year of water was assumed to percolate through the soil profile to the aquifer throughout the study area. This value, equaling 100 acre-ft/year per cell, sums to 38,000 acre-ft/yr for the entire region. As described in Chapter 4 and 5 these values are used as inputs to the sustained yield groundwater withdrawal optimization models.

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1																				
2	1	1																			
3	0	-1																			
4	0	-1																			
5	0	-1	0																		
6		0	-6	2	3																
7		1	-8	-5	-1	3	3														
8		2	-8	-7	-4	-1	1	2	1	1											
9		1	-5	-5	-3	-1	1	2	0	-1	1										
10			2	-1	0	2	3	6	5	2	2	3	2	0							
11				3	1	3	4	8	6	4	3	2	1	1							
12					3	3	0	4	5	5	5	4	1	-1	0	0	0				
13							2	0	4	3	6	5	1	-2	-3	-2	0	0	0		
14								1	1	2	0	1	-1	-1	-3	-3	-2	-3	-2	-1	
15							1	0	-1	-3	-3	-2	-2	1	1	0	-1	-2	-2	0	
16							0	0	-3	-4	-4	-2	-2	0	2	2	0	-1	-1	0	0
17									0	-4	-4	-3	-2	0	4	3	2	2	2	2	1
18									0	-5	-5	-5	-2	1	2	2	2	3	4	2	
19									0	-4	-4	-4	-2	2	1	1	1	2	5		
20									3	-2	-2	-2	-2	0	-1	-1	0	1	4		
21									2	0	0	1	0	0	-1	-2	0	1	5		
22									0	0	1	1	0	-2	-3	-3	0	3	5		
23									0	-1	0	0	-1	-1	-1	0	2	3	4		
24									0	-4	-1	0	-1	-2	0	2	2	3	4	3	
25									1	-3	-1	0	-1	-2	1	4	2	2	3	3	
26									1	0	0	0	-2	-1	4	6	1	1	2	2	
27								2	0	2	1	0	-2	-2	2	-3	-1	0	1	2	
28							3	0	1	3	2	-1	-2	-2	1	1	-1	-1	0	1	
29						4	4	1	0	2	1	-3	-4	-2	1	3	1	-2	1	1	
30						4	1	0	2	2	1	-2	-4	-2	1	4	2	0	1	-3	0
31						3	3	2	1	2	0	-1	-3	-2	1	4	2	1	-1	-1	-1
32					2	2	3	2	2	1	0	1	-2	-2	1	2	1	-1	-2		
33					1	2	2	2	2	0	-1	1	-1	-1	1	2	1	-3	-5		
34					0	1	0	0	1	0	0	-1	-1	-1	1	1	1	1	-5		
35						2	1	1	1	2	3	3	2	2	1	0	-1	-2	-3	-4	

Figure 3.1 Differences Between Simulated And Observed Groundwater Levels After 10 Years Of Simulation, (ft).

3.3 Evaluating Current And Historic Aquifer Conditions Using The Validated Parameters

The program VOLCAL (Peralta et al., 1983b) was modified and used to estimate the cell-by-cell volume of groundwater stored in the aquifer in the spring of each year, as well as the change in storage from 1973 to 1983 using the assumed effective porosity. It is also used to compute the saturated elevation (the elevation of the top of the aquifer or the elevation of the potentiometric surface, whichever is lower), the saturated thickness and the degree of confinement at the center of each cell in the study area for each year of the validation period. In addition, VOLCAL calculates the changes in the potentiometric surface and saturated elevation from year to year and from 1973 to 1983.

Figure 3.2 shows the degree of aquifer confinement in the springtime of 1983 as the difference between the 1983 potentiometric surface elevation and the elevation of the top of the aquifer. Since positive values indicate confined portions of the aquifer, one sees that most of the aquifer is confined, although the degree of confinement is not great. (For representative pumping rates the aquifer can be assumed to act as if it is unconfined in the vicinity of pumping wells). Changes in groundwater storage occur only in the unconfined portions of the aquifer.

Figure 3.3 and Figure 3.4 show the change in saturated elevation and potentiometric surface elevation between 1973 and 1983 respectively. As seen from Figure 3.3, most of the cells in

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-5																				
2	6	1																			
3	16	20																			
4	21	14																			
5	1	8	19																		
6		-13	28	24	2																
7		41	24	18	2	5	13														
8		13	6	-1	16	22	1	3	14	21											
9		-13	-3	-3	15	13	7	5	11	16	13										
10			-28	-9	-6	0	8	2	12	5	-9	-5	13	13							
11				-24	-5	1	3	0	1	2	13	20	17	16							
12					-10	-5	3	0	1	1	5	9	14	16	17	19	19				
13							-1	1	11	0	-1	11	16	17	13	15	21	23	20		
14								-12	12	0	5	22	14	9	7	12	21	21	18	10	
15							-27	-17	18	10	10	3	4	39	3	11	17	17	12	11	
16							-25	-15	-10	1	4	3	7	11	2	19	24	10	9	8	7
17									-11	-6	-3	5	11	6	7	7	3	1	6	7	3
18									-6	13	10	29	16	-3	13	10	7	6	6	3	
19									-11	-3	-6	35	16	14	22	25	21	15	7		
20									-27	0	-18	9	12	19	24	45	20	11	8		
21									-22	-13	-16	8	11	11	14	22	9	7	5		
22									5	10	-1	1	6	16	6	8	8	6	13		
23									7	16	9	6	8	9	5	33	13	15	19		
24									5	17	14	12	6	6	2	4	9	18	28	33	
25									3	7	13	16	8	2	-3	-2	-7	23	35	37	
26									-8	2	14	8	11	4	-4	4	9	39	42	37	
27								-19	-12	-4	4	3	6	7	1	18	16	30	39	33	
28							-24	-19	-12	-6	-1	-1	-1	12	2	-4	5	21	24	27	
29						-22	-18	-14	-6	-2	8	7	17	4	16	0	5	10	5	15	
30						-16	-11	-4	1	6	13	16	3	-4	-2	2	-6	-3	-2	13	26
31						-10	-3	5	12	18	9	19	4	0	1	-9	-16	6	21	27	30
32					-8	-2	5	13	21	19	18	22	5	5	6	-5	-8	25	42		
33					-5	1	10	24	23	19	9	18	8	2	14	-6	-3	7	37		
34					-4	5	13	28	25	20	12	12	-1	3	7	-8	5	14	36		
35						6	21	23	22	16	16	26	2	0	-2	-10	-6	15	32	39	

Figure 3.2 Degree Of Confinement in 1983, (ft).

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-2																				
2	0	0																			
3	0	0																			
4	0	0																			
5	0	0	0																		
6		0	0	0	0																
7		0	0	0	0	0	0														
8		0	0	-1	0	0	0	0	0	0											
9		-1	-2	-2	0	0	0	0	0	0	0										
10			-2	-3	-4	0	0	0	0	0	-5	-3	0	0							
11				-4	-5	0	0	0	0	0	0	0	0	0							
12					-4	-4	0	0	0	0	0	0	0	0	0	0	0				
13							-1	0	0	0	-1	0	0	0	0	0	0	0	0		
14								-3	0	0	0	0	0	0	0	0	0	0	0	0	
15							-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	
16							-1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
17									-1	0	1	0	0	0	0	0	0	0	0	0	0
18									0	0	0	0	0	-2	0	0	0	0	0	0	
19									0	0	1	0	0	0	0	0	0	0	0		
20									-4	0	0	0	0	0	0	0	0	0	0		
21									-3	-2	-1	0	0	0	0	0	0	0	0		
22									0	0	-1	0	0	0	0	0	0	0	0		
23									0	0	0	0	0	0	0	0	0	0	0		
24									0	0	0	0	0	0	0	0	0	0	0	0	
25									0	0	0	0	0	0	-1	-2	-2	0	0	0	
26									-2	0	0	0	0	0	-3	0	0	0	0	0	
27								-3	-3	-2	0	0	0	0	0	0	0	0	0	0	
28							-5	-4	-3	-3	-1	-1	-1	0	0	-1	0	0	0	0	
29						-7	-6	-5	-3	-2	0	0	0	0	0	0	0	0	0	0	
30						-7	-5	-4	0	0	0	0	0	-2	-1	0	-3	-3	-2	0	0
31						-5	-3	0	0	0	0	0	0	0	0	-4	-6	0	0	0	0
32					-3	-2	0	0	0	0	0	0	0	0	0	-5	-8	0	0		
33					-2	0	0	0	0	0	0	0	0	0	0	-5	-3	0	0		
34					0	0	0	0	0	0	0	0	-1	0	0	-3	0	0	0		
35						0	0	0	0	0	0	0	0	0	-2	-3	-3	0	0	0	

Figure 3.3 Change in Saturated Elevation Between 1973 And 1983, (ft).

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-2																				
2	0	-1																			
3	1	1																			
4	1	1																			
5	1	0	-1																		
6		0	-1	-2	-3																
7		0	-1	-1	-2	-3	-4														
8		-1	-1	-1	-1	-1	-3	-4	-4	-2											
9		-1	-2	-2	-2	-3	-4	-5	-5	-4	-2										
10			-2	-3	-4	-7	-7	-8	-6	-6	-5	-3	-2	0							
11				-4	-5	-7	-7	-8	-7	-8	-8	-7	-4	-2							
12					-4	-4	-5	-6	-7	-8	-10	-10	-6	-2	0	0	0				
13							-4	-4	-4	-8	-12	-13	-8	-3	0	-1	-2	-2	-1		
14								-3	-1	-1	-5	-8	-9	-8	-5	-4	-4	-3	-3	-1	
15							-2	-2	-2	-2	-1	-4	-7	-9	-8	-7	-6	-5	-4	-3	
16							-1	-2	-2	-2	-1	-1	-3	-6	-7	-7	-7	-8	-5	-4	-2
17									-1	0	1	1	1	-3	-7	-8	-7	-7	-6	-5	-4
18									0	0	1	3	2	-2	-5	-7	-8	-7	-7	-6	
19									0	0	1	2	-1	-4	-5	-6	-7	-8	-8		
20									-4	-2	0	1	-1	-5	-5	-5	-6	-8	-9		
21									-3	-2	-1	0	-1	-3	-4	-4	-4	-7	-10		
22									0	-1	-2	-2	-2	-2	-2	-3	-4	-6	-8		
23									0	-1	-2	-3	-2	0	0	-1	-3	-5	-6		
24									-1	-1	-2	-2	-1	1	1	-1	-3	-4	-5	-5	
25									-1	-1	-1	-1	0	0	-1	-2	-2	-3	-4	-4	
26									-2	-2	-1	-1	0	0	-3	-3	-2	-2	-3	-3	
27								-3	-3	-2	-2	-1	0	-1	-2	0	-1	-2	-2	-2	
28							-5	-4	-3	-3	-4	-4	-3	-2	-1	-1	-1	-2	-2	-2	
29						-7	-6	-5	-3	-3	-3	-7	-6	-3	-1	-1	-2	-2	-2	-1	
30						-7	-5	-4	-3	-3	-5	-8	-7	-2	-1	-2	-3	-3	-2	0	0
31						-5	-4	-4	-3	-4	-5	-6	-5	-2	-1	-4	-6	-5	-2	0	0
32					-3	-3	-3	-3	-3	-4	-4	-5	-4	-3	-3	-5	-8	-7	-1		
33					-2	-2	-2	-2	-3	-3	-4	-4	-3	-3	-4	-5	-5	2	4		
34					0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-1	4	3		
35						0	-1	-1	-2	-2	-3	-3	-3	-3	-3	-3	-3	-5	-7	-4	

Figure 3.4 Change in Potentiometric Surface Elevation Between 1973 and 1983, (ft).

the area have no change in saturated elevation. This is due to the fact that a large portion of the aquifer is confined in the springtime. Figure 3.4 shows that the changes in the potentiometric surface elevation are fairly small. The average change in potentiometric surface elevation over the entire area is about 4 feet, with larger changes occurring in the eastern portion of the area, where pumping was greatest.

The saturated thicknesses in spring 1983 are shown in Figure 3.5. The saturated thickness is the distance between the potentiometric surface elevation and the base of the aquifer. According to Peralta et al., (1985) the minimum desirable saturated thickness for 1980 climatic conditions (the most severe recent drought year) is about 25 feet for the adjacent Grand Prairie region of the same aquifer. The average saturated thickness over the entire Bayou Bartholomew Basin is about 80 feet, which is adequate. The lowest saturated thicknesses are observed in southwestern cells and range from 20 to 40 feet. Three of these cells have saturated thickness less than 25 feet.

The storage and the change in storage for each year of the validation period (1973-1983) are shown in Table 3.3. Table 3.3 also shows the change in groundwater storage as a percentage of 1973 storage and previous year pumping for each year of the observation period. Because the aquifer is confined in most of the study area, the change in storage is very small, despite the fact that potentiometric surface elevations are widely declining. The total decrease in groundwater storage from 1973 to 1983 is 232,700 acre-ft, which is only 0.68 percent of the 1973 storage.

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	43																				
2	34	49																			
3	25	27																			
4	32	32																			
5	84	77	82																		
6		119	73	73	103																
7		83	82	76	90	99	98														
8		128	114	111	86	75	98	101	85	75											
9		109	110	107	103	79	94	92	81	83	87										
10			44	73	135	116	102	88	65	80	94	79	85	83							
11				51	87	131	124	111	102	85	71	54	68	76							
12					77	100	91	107	101	84	73	62	63	73	77	73	74				
13							76	103	76	73	68	58	61	71	76	74	67	70	77		
14								88	77	71	67	44	60	74	81	76	80	67	74	83	
15							91	88	79	82	74	84	60	39	80	80	70	68	76	79	
16							90	89	88	99	82	85	65	57	74	76	46	70	77	78	78
17									85	84	93	88	68	68	98	66	80	86	81	78	81
18									83	80	91	70	84	79	81	82	86	89	83	83	
19									77	95	98	51	83	58	51	80	69	78	78		
20									70	101	95	74	65	67	53	28	48	63	72		
21									73	105	110	62	68	61	57	46	58	63	73		
22									76	86	87	79	75	64	57	62	64	69	64		
23									67	64	61	69	69	58	59	45	64	61	59		
24									61	62	58	61	77	57	60	58	69	61	46	42	
25									60	70	65	54	52	48	53	67	60	54	41	47	
26									58	70	76	89	74	45	57	64	50	23	33	44	
27								40	50	64	61	92	70	65	63	62	41	35	36	50	
28							28	34	46	37	76	72	88	38	53	75	67	52	54	60	
29						21	27	34	40	46	48	68	60	53	39	66	65	60	76	76	
30						23	28	35	40	41	44	58	68	66	65	73	74	76	85	88	81
31						26	32	35	35	38	45	65	67	54	53	79	86	87	78	80	83
32					31	27	29	31	34	41	56	71	81	62	62	98	94	70	54		
33					21	31	30	27	34	50	63	55	66	76	55	67	65	93	73		
34					25	29	28	27	31	54	74	46	37	45	70	77	85	91	92		
35						27	24	28	31	51	58	17	37	48	67	82	94	95	98	91	

Figure 3.5 Saturated Thickness in 1983, (ft).

The total change in groundwater storage between 1973 and 1983 is 12 percent of total groundwater pumping (1,939,560 acre-ft) for that period. This percentage, the mining percentage, indicates the proportion of groundwater pumped during the observation period that is not replaced by recharge.

Table 3.3

Storage and Change in Storage

Year	Storage (ac-ft) X 1000	Change in Storage (acre-ft)	Change in Storage as a percent of 1973 Storage (%)	Change in Storage as a percent of previous year Pumping (%)
1973	34101.9			
		-23125.9	-0.07	-23
1974	34078.9			
		8700.5	0.03	6
1975	34087.5			
		-43125.6	-0.13	-36
1976	34044.3			
		-47464.7	-0.14	-31
1977	33996.9			
		-59382.6	-0.17	-48
1978	33937.5			
		1567.3	0.01	1
1979	33939.1			
		33910.1	0.10	19
1980	33973.0			
		-51943.6	-0.15	-15
1981	33921.0			
		-95841.0	-0.28	-29
1982	33825.2			
		43087.6	0.13	25
1983	33869.2			
avg.	33979.5	-23270.0	-0.07	-13

3.4 Result From Estimating a Volume Balance For The Area

The average annual historic net recharge from causes other than those previosly specified can be determined by conducting a water volume balance analysis. In applying the following water volume balance equations, all discharges from the aquifer are positive values and all recharges to the aquifer are negative values.

$$\text{Regional Discharge} + \text{Regional Recharge} + \text{Change In Storage} = 0 \qquad (3.1)$$

or,

$$(\text{Pumping} + \text{S/A Discharge}) + (\text{S/A Recharge} + \text{Deep Percolation} + \text{Net Recharge From All Other Sources}) + \text{Change In Storage} = 0 \qquad (3.2)$$

The average annual components of the water balance for 1973-83 estimated as described in the preceding sections are:

Pumping = +193,956 ac-ft

S/A Discharge = +4,057 ac-ft

S/A Recharge = -16,432 ac-ft

Deep Percolation = -38,000 ac-ft

Change In Storage = -23,270 ac-ft

Therefore, the average annual Net Recharge From All Other Sources is:

$$193{,}956 + 4{,}057 - 16{,}432 - 38{,}000 - 23{,}270 = 120{,}311 \text{ ac-ft}$$

Conventionally, this value is negative in sign. It includes recharge entering through peripheral constant-head cells, recharges entering at internal stream/aquifer cells that were not so designated in the models, and the difference between steady recharge rates based on springtime gradients and the actual time-variant recharge.

CHAPTER IV

METHODOLOGY

By

P. J. Killian, R. C. Peralta and A. Yazdanian

Components of any optimization problem include the objective function, the involved variables and constraints. Two objective functions are applied in the management model presented in this report. The first seeks to maximize the sustained yield withdrawal from a given region. The second develops sustained yield withdrawal strategies that maintain groundwater elevations as close as possible to predetermined 'target' elevations.

The variables subject to management bounding or constraint include drawdown, pumping, and recharge. In order to assure that the models properly simulate groundwater flow, the finite difference approximation to the differential equation of steady-state groundwater flow is used as part of the constraining conditions in the management model. This technique of linking the simulation to the optimization is referred to as the embedding method (Gorelick, 1983). The embedding method is used to express both the constraining equations and the objective functions in terms of only a single type of variable, static drawdown. This is done so that other objective functions can be applied without modifying the constraint set. Much of this chapter is dedicated to developing the constraining equations used in the model.

The optimal solution is found through application of operations research theory. The optimization algorithm employed in this management model is QPTHOR, a linear and quadratic programing subroutine written by Leifsson and others (1981). QPTHOR uses the General Differential Algorithm, a direct climbing method of locating the optimal solution through a systematic gradient search routine.

4.1 FINITE DIFFERENCE APPROXIMATION OF THE TWO-DIMENSIONAL FLOW EQUATION

The following linearized Boussinesq equation describes groundwater flow in two dimensions (Konikow and Bredehoeft, 1975).

$$W - \partial/\partial x_i \; (T'_{ij} \; (\partial h/\partial x_j)) + Sc \; (\partial h/\partial t) = 0 \qquad i,j = 1,2 \tag{4-1}$$

where

W	= the excitation or volumetrix flux of recharge or withdrawal per unit surface area of aquifer, (L/T);
T'_{ij}	= the transmissivity tensor such that $T' = k\,b$, (L^2/T);
k	= the hydraulic conductivity of the aquifer material, (L/T);
b	= the saturated thickness of the aquifer material, (L);
h	= potentiometric head, (L);
Sc	= the storage coefficient, (dimensionless).

The saturated thickness, (b), is assumed constant as it appears in Equation (4-1). This assumption is valid only in the case of a confined aquifer. In unconfined situations, a change in potentiometric head results in an equal change in saturated thickness. If the saturated thickness of an unconfined aquifer is very large compared to the change in head, then Equation (4-1) is relatively accurate. If however, the saturated thickness is only slightly larger than the change in potentiometric head, Equation (4-1) is an inaccurate representation of unconfined groundwater flow. This problem is addressed when using the management model by sequential re-initialization in a manner which is presented in a subsequent section.

Konikow and Grove (1977) provide a summary of the assumptions considered in the development of Equation (4-1):

1) The porous medium can only deform vertically.
2) Isothermal conditions prevail.
3) The volume of individual grains remains constant during the deformation of the medium.
4) Fluid density is a linear combination of pressure.
5) The permeability is independent of pressure and temperature.
6) Hydraulic head gradients are the only significant driving mechanism.
7) Homogeneous fluid density and viscosity.
8) Two dimensional flow.

To approximate the differential equation describing groundwater flow, a block centered cell system is used. The study area is subdivided into a number of square blocks or cells in which the aquifer properties are assumed uniform. The continuous derivatives in Equation (4-1) are replaced by finite difference approximations at the center of each cell to yield:

$$1/\Delta x_i \, [(T'_{xx} (\partial h/\partial x)_i)_{i+1/2,j} - (T'_{xx} (\partial h/\partial x)_i)_{i-1/2,j}] +$$

$$1/\Delta y_j \, [(T'_{yy} (\partial h/\partial x)_j)_{i,j+1/2} - (T'_{yy} (\partial h/\partial x)_j)_{i,j-1/2}] =$$

$$S(i,j)/\Delta t \, (h(i,j) - h(i,j,t-1)) + W(i,j,t) \tag{4-2}$$

where:

Δx_i = the space increment in the x-direction for column i, (L);

Δy_j = the space increment in the y-direction for row j, (L);

Δt = the time increment, (T);

i = the index in the x-direction for any cell in the study area;

j = the index in the y-direction for any cell in the study area;

t = the time index.

Further approximation yields:

$$
\begin{aligned}
&1/\Delta x_i \{[DTR(i,j)\ (h(i+1,j) - h(i,j))/\Delta x_{i+1/2}] - \\
&\quad [DTR(i-1,j)\ (h(i,j) - h(i-1,j))/\Delta x_{i-1/2}]\} + \\
&1/\Delta y_j \{[DTU(i,j)\ (h(i,j+1) - h(i,j))/\Delta y_{j+1/2}] - \\
&\quad [DTU(i,j-1)\ (h(i,j) - h(i,j-1))/\Delta y_{j-1/2}]\} = \\
&S(i,j)/\Delta t\ (h(i,j) - h(i,j,t-1)) + W(i,j,t)
\end{aligned}
\tag{4-3}
$$

where:

h(i,j) = the potentiometric surface elevation in finite difference cell (i,j) at time period t, (L);

DTR(i,j) = the transmissivity between cell (i,j) and cell (i+1,j), (L^2/T);

DTU(i,j) = the transmissivity between cell (i,j) and cell (i,j+1), (L2/T);

$\Delta x_{i+1/2}$ = the distance between center of cell (i,j) and center of cell (i+1,j), (L).

The size and dimension of each finite difference cell depends on the anisotropy of the aquifer characteristics and the capabilities of available computational resources. The use of smaller dimensions in defining the cells will more closely approximate Equation (4-1), but at the same time increase the number of unknowns in the solution set.

The management model applies a square cell system to the study area. Consequently, all the space increments in Equation (4-3) are equal and some simplification occurs. Multiplying each side of Equation (4-3) by the area of one square cell yields

$$
\begin{aligned}
& DTR(i,j)\ (h(i+1,j)-h(i,j)) - DTR(i-1,j)\ (h(i,j)-h(i-1,j)) \\
& + DTU(i,j)\ (h(i,j+1)-h(i,j)) - DTU(i,j-1)\ (h(i,j)-h(i,j-1)) \\
& = S(i,j)\ (A)/\Delta t\ (h(i,j)-h(i,j)) + W(i,j)\ AR
\end{aligned}
\tag{4-4}
$$

where:

AR = the surface area of one square finite difference cell, (L^2).

4.2 STEADY-STATE GROUNDWATER FLOW

Steady-state groundwater flow is simulated in order to approximate a sustained yield condition. Steady-state excitation rates are those values of pumping and recharge which, when applied to the system, continuously maintain constant potentiometric surface elevations. For a given set of potentiometric surface elevations, there exists a corresponding set of steady-state pumping values.

This idealistic description of a steady-state system is not representative of a natural system of groundwater recharge and withdrawal. Seasonal precipitation and pumping for irrigation are time-varient. In an agricultural area, more groundwater is removed from the aquifer during the growing season and more recharge is available during the spring. Fortunately, Peralta and Peralta (1984) and Yazdanian and Peralta (1985) have shown that a steady-state potentiometric surface is generally maintained over the long term if the total transient excitations for a given time period equal the appropriate total steady excitation rate over the same time period.

Under steady-state conditions, Equation (4-4) becomes:

$$DTR(i,j)\ (h(i+1,j)-h(i,j)) - DTR(i-1,j)\ (h(i,j)-h(i-1,j)) + DTU(i,j)\ (h(i,j+1)-h(i,j)) - DTU(i,j-1)\ (h(i,j)-h(i,j-1)) = W(i,j)\ A \quad (4\text{-}5)$$

The potentiometric surface elevations are replaced by static drawdown values for computational efficiency. Drawdown is defined as the difference between the elevation of a horizontal datum located above the ground surface and the elevation of the potentiometric surface. With this substitution and the distribution of transmissivity terms, the following equation results.

$$-DTR(i,j)\ S(i+1,j) - DTR(i-1,j)\ S(i-1,j) + T(i,j)\ S(i,j) - DTU(i,j)\ S(i,j+1) - DTU(i,j-1)\ S(i,j-1) = W(i,j)\ A \quad (4\text{-}6)$$

where:

$S(i,j)$ = the steady-state drawdown in cell i,j during the time period of simulation, (L);

$$T(i,j) = DTR(i,j) + DTR(i-1,j) + DTU(i,j) + DTU(i,j-1) .$$

This relationship is simplified by writing it in vector notation as

$$\{T\}\{s\} = W(i,j)\ A \qquad (4\text{-}7)$$

where:

$\{T\}$ = the transpose of the 5-dimension vector of transmissivity values;

$\{s\}$ = a 5 dimensional vector of drawdown values.

The expansion of Equation (4-7) over all cells of the study area is written in matrix form as

$$[T]\{s'\} = \{w'\} \qquad (4\text{-}8)$$

where:

$[T]$ = an N by N square matrix of transmissivity values, having a maximum of 5 nonzero elements in each row;

$\{s'\}$ = an N dimensional vector of drawdown values;

$\{w'\}$ = an N dimensional vector of excitation values;

N = the total number of cells in the study region.

Solution of Equation (4-8) for $\{s'\}$ when $\{w'\}$ is known is accomplished by simultaneous evaluation of N equations and N unknowns.

4.3 SEPARATION OF VOLUMETRIC FLUX

The right hand side of Equation (4-6) represents the rate of groundwater entering or leaving the aquifer at a particular finite difference cell. This term is positive if water is leaving the system, and negative if water is entering from outside the system. To add more flexibility to the model, the volumetric flux is separated into three components including groundwater pumping, stream/aquifer flux (referred to as stream/aquifer response or interflow), and recharge such that

$$W(i,j)\ A = P(i,j) + Qr(i,j) + RCH(i,j) \tag{4-9}$$

where:

$P(i,j)$ = the steady-state groundwater pumping in cell (i,j) during the simulation period, (L^3/T);

$Qr(i,j)$ = the stream/aquifer response in cell (i,j) during the simulation period, (L^3/T);

$RCH(i,j)$ = the recharge to the aquifer at cell (i,j) during simulation period, (L^3/T);

The sign convention and above definitions imply that values of groundwater pumping, P(i,j), are typically positive, to indicate the volume withdrawn. A negative pumping value is interpreted as caused by an injection well.

The recharge values, RCH(i,j), are negative when water enters the aquifer from outside the system. A positive recharge value occurs at a particular cell, when water is leaving the system at that location. Some examples of negative recharge include infiltration and flow from adjacent study areas. Evapotranspiration is one example of positive recharge.

The boundary of the study area is treated as a no flow boundary. Consequently, the transmissivity at the periphery is zero. Flow into the aquifer from outside the system is simulated by applying recharge in the peripheral cells where conditions indicate the existence of such a condition.

Substituting Equation (4-9) into Equation (4-7) and writing the relationship in matrix form, results in the following expression.

$$\{T\}\{s\} = P(i,j) + Qr(i,j) + RCH(i,j) \qquad (4\text{-}10)$$

The interflow between the aquifer and a stream in hydraulic connection is represented by the term, $Qr(i,j)$, of Equation (4-10). This response is greater than zero if the direction of flow is from the aquifer to the stream, and less than zero if the stream is recharging the aquifer. No stream aquifer response is present in cells which are not hydraulically connected to a surface water source. Assuming that the river or stream is penetrating the aquifer such that the medium between the streambed and the aquifer material is saturated, the interflow is determined by

$$Qr(i,j) = Tr(i,j)\ (Sst(i,j) - S(i,j)) \qquad (4\text{-}11)$$

where:

$Tr(i,j)$ = positive valued reach transmissivity of cell (i,j), (L^2/T);

$Sst(i,j)$ = static stream drawdown or the difference between the elevation of the datum and the elevation of the water in the stream at cell (i,j), (L).

Reach transmissivity, $Tr(i,j)$, is a measure of the ability of the streambed to transmit water to the aquifer. The reach transmissivity for a cell which is not hydraulically connected to a stream or lake is equal to zero. The value of reach transmissivity can be calculated analytically (Morel-Seytoux 1979), or empirically through model calibration. By assuming a constant value of reach transmissivity, and a constant stream stage for the period of simulation, stream/aquifer response becomes a function of groundwater drawdown alone. Thus, interflow between a stream and the underlying aquifer can be controlled in a particular cell, by limiting the drawdown.

The expression for stream/aquifer response is incorporated into Equation (4-10) to obtain

$$\{T\}\{s\} + Tr(i,j)\ S(i,j) = P(i,j) + RCH(i,j) + Tr(i,j)\ Sst(i,j)\ . \tag{4-12}$$

Equation (4-12) is the finite difference form of the equation of groundwater flow used as a controlling condition in the management model. The expansion of Equation (4-12), as applied to a single finite difference cell, is written as

$$\begin{aligned} &(T(i,j) + Tr(i,j))\ S(i,j) - DTR(i,j)S(i+1,j) - DTR(i-1,j)S(i-1,j) \\ &- DTU(i,j)S(i,j+1) - DTU(i,j-1)S(i,j-1) - P(i,j) - RCH(i,j) \\ &= Tr(i,j)Sst(i,j) \end{aligned} \tag{4-13}$$

The drawdown, pumping and recharge terms are either constant or variable depending on the type of cell being modeled.

4.4 TYPES AND CHARACTERISTICS OF CELLS

To model a region using the management method introduced in this report, the area must be defined by a set of finite difference cells. Each cell is identified according to the variable nature of the drawdown, pumping, and recharge at that location. It is not within the scope of this section to comment on techniques used to determine the hydrologic boundaries and characteristics of this system of cells, only on the application of the information.

For each cell there are three factors to consider. These factors, found in Equation (4-12), include drawdown, groundwater pumping, and recharge. The different types of cells are the result of the various combinations of these three factors considering each as a constant or a variable. For each variable value there is a corresponding upper and lower limit defining the range of feasibility.

A variable cell is any cell which has a variable drawdown, a variable value of groundwater pumping, and a constant value of recharge. For every variable cell there is an upper and lower limit on drawdown, and an upper and lower limit on groundwater pumping. Variable cells exist in areas where the steady-state drawdown and the corresponding pumping value is unknown while the vertical infiltration is assumed constant. This type of cell will usually comprise the major portion of the study area.

A special type of a variable cell is a constant-flux cell. A constant-flux cell has a variable drawdown, a constant value of groundwater pumping, and a constant recharge value. The constant sum of pumping and recharge may represent actual estimates of system conditions, or design withdrawal rates which the water manager considers necessary to achieve. A variable cell becomes a constant-flux cell when the upper and lower limit on groundwater pumping are equal.

A constant-head cell represents a cell in which the drawdown and pumping values remain constant, but recharge may vary. Typically, constant-head cells will be along the periphery, although this is not a necessary condition. Because the drawdown and the pumping values are constant, to reduce computational requirements, the objective function is not applied to constant head cells.

Any variable cell may be further characterized as a stream/aquifer cell if investigations indicate the existence of a hydraulic connection between surface water and groundwater in that cell. For every stream/aquifer cell, the reach transmissivity and stream stage are estimated and applied as outlined previously. The interflow in a stream/aquifer cell is subject to conditions imposed by an upper limit on the volume transferred as indicated by

$$Qr(iv,jv) \geq Qrmin(iv,jv) \qquad (4\text{-}14)$$

where:

Qrmin(iv,jv) = the minimum allowable interflow between the stream and the aquifer during the time period of simulation, (L^3/T);

iv = the column index for any variable cell in the system;

jv = the row index for any variable cell in the system.

If Qrmin(iv,jv) is positive, the interflow at that cell is strictly return flow from the aquifer to the stream. If Qrmin(iv,jv) is negative, recharge from the stream to the aquifer is limited. Qrmin(iv,jv) is expressed as a function of cell drawdown by re-arranging Equation (4-11) to yield:

$$Smax'(iv,jv) = Sst(iv,jv) - Qrmin(iv,jv)/Tr(iv,jv) \tag{4-15}$$

where:

Smax'(iv,jv) = the maximum drawdown allowed in cell (iv,jv) such that the lower limit on interflow is not violated, (L).

This capability is included to provide a means by which surface water supplies can be protected.

4.5 VARIABLES

The variables involved in the management model include drawdown, pumping, and recharge at constant-head cells. The restrictions and limitations imposed on these variables are expressed as constraints. These constraints represent conditions which must be met in order for the variable values to be considered as a feasible solution. The constraints imposed indicate physical conditions or the implementation of management decisions.

The primary constraining condition is the equality condition expressed by Equation (4-12). This constraint represents the physical relationship between the variables by maintaining the conditions of steady-state groundwater flow. The remaining constraints are formulated as bounds on drawdown, pumping and recharge in constant-head cells.

Limits on the drawdown in variable cells define the range in which water levels can rise or fall. Because of the relationship between drawdown and water elevation, the upper limit on drawdown corresponds to the lower limit on potentiometric surface elevation, while the lower limit on drawdown relates to the upper limit on elevation. A natural upper limit on drawdown is the physical bottom of the aquifer. If additional saturated thickness is desired for economic reasons or for drought protection, the upper limit is decreased accordingly. The lower limit on drawdown is provided to prevent the flooding of foundations of construction sites. Several other considerations for determining limitations on drawdowns are listed by Bear (1979).

The feasible range of values for cell drawdowns is summarized by the general formulation:

$$Smin(iv,jv) \geq S(iv,jv) \geq Smax(iv,jv) \qquad (4\text{-}16)$$

where:

Smin(iv,jv) = the lower limit on drawdown in cell (iv,jv), (L);

Smax(iv,jv) = the upper limit on drawdowm in cell (iv,jv), (L);

and:

$$Smax'(iv,jv) \geq Smax(iv,jv) .$$

Recall that Smax'(iv,jv) is the maximum drawdown in the cell such that the lower limit on interflow is not violated. Thus, the final condition maintains the limit on stream/aquifer response.

The groundwater pumping in cells other than constant-head cells is a variable and as such is bounded by an upper and a lower limit. The lower limit on groundwater pumping is established in terms of variable drawdown values by utilizing the relationship expressed in Equation (4-12), recalling that in all variable cells the value of recharge is a constant and, if the variable cell is a constant-flux cell, the pumping is a constant as well. The following relationship applies to every variable cell.

$$P(iv,jv) = \{T\}\{s\} + Tr(iv,jv)\ S(iv,jv) - RCH(iv,jv) - Tr(iv,jv)Sst(i,j) \geq Pmin(iv,jv) \qquad (4\text{-}17)$$

where:

Pmin(iv,jv) = the minimum value of groundwater pumping at cell (iv,jv) during the simulation period, (L^3/T).

Based on the adopted sign convention, a negative value of groundwater pumping signifies water going into the system. This internal injection is prevented by setting Pmin(iv,jv) equal to zero. A lower limit on groundwater pumping which is greater than zero reflects the design requirements of the water manager. For example, if a particular cell must have no less than a certain amount of groundwater available due to strict quality requirements, then the lower limit on pumping would indicate this necessity.

The recharge in constant-head cells is described in terms of drawdown by again rearranging Equation (4-12). Because both drawdown and pumping in constant-head cells are fixed, the lower limit on recharge is expressed as:

$$RCH(ic,jc) = \{T\}\{s\} + Tr(ic,jc)\, S(ic,jc) - P(ic,jc) - Tr(ic,jc) Sst(ic,jc) \geq Rmin(ic,jc) \quad (4-18)$$

where:

$Rmin(ic,jc)$ = the minimum allowable recharge in cell (ic,jc) which can occur during the simulation period, (L^3/T);

ic = the column index for any constant-head cell in the study area;

jc = the row index for any constant-head cell in the study area;

The lower limit on recharge refers to the greatest possible amount of water which can enter the constant-head cell from outside the system. The lower limit on recharge is typically less than zero unless it is desired to model a condition in which water can only leave the system.

By using Equation (4-12), a constraining condition, to define the pumping and recharge terms, it is not necessary to include Equation (4-12) as a distinct equality constraint. For a study area with NVAR variable cells and NCH constant-head cells, there are at least (NVAR + NCH) inequality constraints. NVAR constraints are defined by (4-17) and NCH constraints are expressed by (4-18).

4.6 UPPER LIMIT ON PUMPING AND RECHARGE

When an inequality constraint is input to the General Differential Algorithm, one of the initial steps is the common practice of transforming the inequality constraints into equality conditions. This is accomplished by adding what is referred to as a slack variable. A slack variable describes the difference between the left and the right hand side of the constraint, or how "close" a constraining condition is to its limit. If a slack variable is zero, the value of the left hand side of the constraint is equal to the right hand limit and the constraint is said to be "tight".

The introduction of a slack variable to constraint (4-17) yields the following equation.

$$\{T\}\{s\} + Tr(iv,jv)\,S(iv,jv) - RCH(iv,jv) - Tr(iv,jv)\,Sst(iv,jv) - Pmin(iv,jv) = X'(iv,jv) \tag{4-19}$$

where:

$X'(iv,jv)$ = the slack variable associated with the groundwater pumping constraint, (L^3/T).

This relationship can be simplified to

$$P(iv,jv) - Pmin(iv,jv) = X'(iv,jv) \tag{4-20}$$

or

$$P(iv,jv) = X'(iv,jv) + Pmin(iv,jv)\ . \tag{4-21}$$

The upper bound on groundwater pumping is applied such that

$$Pmax(iv,jv) \geq P(iv,jv) = X'(iv,jv) + Pmin(iv,jv) \geq Pmin(iv,jv) \tag{4-22}$$

where:

$Pmax(iv,jv)$ = the maximum allowable pumping in cell (iv,jv) during the simulation period, (L^3/T).

Equation (4-22) is reduced such that the limits on the slack variable are defined.

$$\text{Pmax}(iv,jv) - \text{Pmin}(iv,jv) \geq X'(iv,jv) \geq 0 \qquad (4\text{-}23)$$

By applying a lower limit of zero and an upper limit, equal to the difference between Pmax(iv,jv) and Pmin(iv,jv), on the slack variable of the pumping constraint, the groundwater pumping in every variable cell is bounded by:

$$\text{Pmax}(iv,jv) \geq P(iv,jv) \geq \text{Pmin}(iv,jv) \qquad (4\text{-}24)$$

A similiar procedure applied to recharge constraint (4-18) yields

$$\text{Rmax}(ic,jc) - \text{Rmin}(ic,jc) \geq X''(ic,jc) \geq 0 \qquad (4\text{-}25)$$

such that:

$$\text{Rmax}(ic,jc) \geq \text{RCH}(ic,jc) \geq \text{Rmin}(ic,jc) \qquad (4\text{-}26)$$

where:

Rmax(ic,jc) = the maximum allowable recharge at constant-head cell (ic,jc) during the simulation period, (L^3/T);

X"(ic,jc) = the slack variable associated with the recharge constraint at constant-head cell (ic,jc), (L^3/T);

An upper limit on recharge which is less than zero describes a situation in which water can only enter the system and is restricted from leaving the system at that cell. Because a steady-state condition is modeled, the total flux into the system is equivalent to the total flux out of the system. A negative upper limit on recharge may prevent the maximum utilization of available recharge at other constant-head cells. For this reason, it is suggested that a large positive value of Rmax(ic,jc) be used for the initial optimization.

4.7 REACH CONSTRAINTS

The preceding section discusses constraints imposed on recharge values in a particular constant-head cell. In addition to these constraints, it is possible to constrain the total recharge which occurs in a given subsystem or reach of constant-head cells. This capability is utilized to simulate a system where the constant-head cells represent a stream or lake from which the total recharge is limited. This constraint is formulated by applying Equation (4-18) to all constant-head cells in the designated subsystem. The following relationship represents this summation.

$$RT(ics) = \sum_{i=1}^{I(ics)} RCH(i) \geq CHSMIN(ics) \qquad (4\text{-}27)$$

for ics = 1, NCHSUB

where:

RT(ics) = the total recharge in constant-head subsystem ics, (L^3/T);

I(ics) = the total number of constant-head cells in subsystem ics;

$CHSMIN(ics)$ = the lower limit on total recharge from constant-head subsystem ics, (L^3/T);

NCHSUB = the total number of constant-head cell subsystems.

A constant-head cell cannot belong to more than one constant-head reach at a time. However, any constant-head cell in a subsystem can have an additional constraint limiting the amount of recharge in that particular cell, (see relationship (4-18)).

In addition to constraining a reach of constant-head cells, it is also possible to constrain a reach of stream/aquifer variable cells. The stream/aquifer subsystem constraints are formulated by applying Equation (4-11) to all the cells in the stream/aquifer subsystem. The following expression represents this summation.

$$ST(isa) = \sum_{j=1}^{J(isa)} Qr(j) > SWMIN(isa) \qquad (4\text{-}28)$$

for isa=1,NSUB

where:

$ST(isa)$ = the total volume of flow from the aquifer to the stream in subsystem isa, (L^3/T);

$J(isa)$ = the total number of cells in stream/aquifer subsystem isa;

$SWMIN(isa)$ = the lower limit on total interflow from subsystem isa;

NSUB = the total number of stream/aquifer subsystems.

A variable cell cannot belong to more than one stream/aquifer subsystem at a time. However, in addition to the reach constraint, any variable cell can also be constrained such that the minimum allowable interflow in that cell is not violated. (see Equation (4-15)).

Considering the reach constraints discussed in this section, the total number of inequality constraints applied to the study area is described as

$$K = NVAR + NCH + NCHSUB + NSUB \tag{4-29}$$

where:

K = the total number of inequality constraints;

NVAR = the total number of variable cells;

NCH = the total number of constant-head cells;

The total number of variables, including slack variables, is equal to K + NVAR. As described previously, each variable has an upper and lower limit imposed upon it. The slack variables representative of the reach constraints have an upper bound set artificially high.

4.8 OBJECTIVE FUNCTION FOR DEVELOPMENT OF MAXIMUM SUSTAINED YIELD STRATEGY

One possible regional policy is to maximize annual sustainable groundwater pumping. A linear expression to describe maximization of the total volume of groundwater withdrawn from a region during a specific time period is formed by summing Equation (4-17) for all variable-head cells:

$$\text{Maximize } Z_1(s) = \sum_{iv=1}^{I} \sum_{jv=1}^{J} \{T\}\{s\} + Tr(iv,jv)S(iv,jv) - RCH(iv,jv) - Tr(iv,jv)\,Sst(iv,jv) \tag{4-30}$$

subject to constraints (4-16), (4-24), (4-26), (4-27) and (4-28).

The objective function (4-30) is similiar to those used by Aguado and others (1974), Alley and others (1976), and Elango and Rouve 1980.

4.9 OBJECTIVE FUNCTION FOR DEVELOPMENT OF SUSTAINED YIELD STRATEGIES THAT APPROXIMATELY MAINTAIN 'TARGET' LEVELS

Another possible regional objective is to maintain water levels as close as possible to some pre-determined 'target' elevations. This is the Target Level Approach (TLA) proposed by Peralta and Peralta (1984) and described by Peralta and others (1985). As a technical problem, this can be re-stated as 'developing the pumping strategy that will cause the evolution of a steady-state potentiometric surface that is as close to target elevations as possible'.

The objective function employed is an application of goal-programming (Cohon, 1978) to the design of sustained yield groundwater withdrawal strategies (Yazdanian and Peralta, 1985). The approach seeks to minimize the sum of deviations of a set of regionally optimized groundwater elevations from their corresponding targets.

The objective function is:

$$\text{Minimize} \quad Z_2(s) = \sum_{iv=1}^{I} \sum_{jv=1}^{J} \{(S(iv,jv) - St(iv,jv)) \times w(iv,jv)\}^2 \qquad (4\text{-}31)$$

subject to constraints (4-16), (4-24), (4-26), (4-27) and (4-28); where:

St(iv,jv) is the target (known) steady-state drawdown in variable-head cell number (iv,jv), (L);

w(iv,jv) is a weighting factor assigned to achievement of the target drawdown in cell number (iv,jv).

Attainment of the target elevations are usually required with different degrees of importance in different parts (cells) of a region. There are occasions when the exact targets, and associated gradients, need to be achieved, for example, to provide a minimum saturated thickness for drought protection, or to control groundwater contaminant movement. In contrast, there are cells where attainment of the exact target elevations are less critical. The weighting factors in Equation (4-31) make it possible to emphasize achievement of target elevations more or less in different parts of the region according to management requirements. Further explanation of weighting factors is provided by Yazdanian and Peralta (1985). An application case is also presented by Peralta and others (1985).

4.10 OPTIMIZATION METHOD

A direct climbing method of locating the optimal solution through a systematic gradient search, known as the General Differential Algorithm (Wilde and Beightler, 1967; Morel-Seytoux ,1972), is employed for optimization. To aid in the explanation of the General Differential Algorithm consider the minimization of a quadratic objective function with N variables subject to K inequality constraints. During any iteration in the search process, the problem will consist of K equations and N+K variables. (K of these variables are slack variables introduced to transform the inequality constraints into equality conditions). The constraining equations are separable and as such, K variables are expressed as a function of N independent variables. N independent variables are initially referred to as decision variables while K dependent variables are referred to as solution or state variables. The specific separation of variables into state variables and decision variables is known as the partition of the system.

The functional equivalents of the state variables are directly substituted into the objective function such that the objective function is an unconstrained expression of N decision variables and no state variables. During each iteration in the optimization process, one decision variable is changed to improve the value of the objective function. In the model presented here, a decision variable is either a drawdown variable, or a slack variable corresponding to one of the inequality conditions described previously. A change in any decision variable will cause every state variable related by the K equality conditions to change.

The change in the value of the unconstrained form of the principal objective function, for a given change in a particular decision variable, is expressed in terms of the gradient of the unconstrained objective function. The gradient of the objective function is the vector of first partial derivatives with respect to the decision variables. Each first partial derivative is referred to as a constrained derivative. ("Constrained" derivative implies that the constraining conditions have been substituted into the objective function.) The constrained derivative describes the direction and magnitude of a change in the value of the objective function for an instantaneous change in the value of the decision variable. For the linear objective function (section 4.8), each constrained derivative of the objective function is a constant and is independent of the other variables. For the quadratic objective function (section 4.9) the constrained derivatives are linear functions of the decision variables. Therefore, the vector of second partial derivatives of the unconstrained objective function is a vector of constants. These constants identify the change in the value of a constrained derivative for a change in the value of any decision variable. Any change in the value of a decision variable will change the value of all related constrained derivatives.

The General Differential Algorithm searches for the decision variable for which the absolute value of the constrained derivative is the largest. This variable is referred to as $x_d(jmax)$. This decision variable is changed to improve the value of the objective function. The sign on the constrained derivative indicates the direction in which to change the variable in order to improve the value of the objective function. Considering a minimization process, if a constrained derivative is positive, the correponding decision variable is decreased to improve the value of the objective function. Similiarly, if the constrained derivative is negative, the decision variable is increased during the optimization process.

The change in the value of the objective function for a specific change in one decision variable is expressed in terms of the constrained derivatives as

$$\Delta z = v(jmax)\ \Delta x_d(jmax) + (1/2)b(jmax,jmax)(\Delta x_d(jmax))^2 \tag{4-32}$$

where:

Δz is the change in the value of the objective function;

$\Delta x_d(jmax)$ is the specific change in the decision variable i;

v(jmax) is the first partial derivative (constrained derivative) of z with respect to $x_d(jmax)$;

b(jmax,jmax) is the second partial derivative of z with respect to $x_d(jmax)$.

The change in a constrained derivative resulting from a change in a single decision variable due to the non-linearity of the objective function is expressed as follows.

$$\Delta v(j) = b(j,jmax)\ x_d(jmax) \tag{4-33}$$

for j=1,N

where $\Delta v(j)$ is the change in the constrained derivative of the objective function with respect to decision variable j.

Equation (4-32) is valid when the change in the decision variable does not cause a repartitioning of system variables. This limitation is subsequently discussed.

The change in all system variables in response to a change in the value of a single decision variable is referred to as the system response. Because all decision variables are independent, a change to one decision variable will not effect the value of the remaining decision variables. Every state variable, however, is expressed as a function of decision variables and is therefore affected. By evaluating the gradients of the state variables, the change to the state variables in response to a change in the value of a single decision variable is determined.

In this model, the constraints are linear and the resultant state gradients are column vectors of constants. Therefore, the first partial of a state variable with respect to each decision variable is valid for any arbitrary change in a single decision variable. The system response to a change in the value of a single decision variable is represented by the following formulation.

$$\Delta x_s(i) = d(i,jmax)\,\Delta x_d(jmax) \qquad \text{for } i=1,K \tag{4-34}$$

where:

$\Delta x_s(i)$ is the change in state variable i;

d(i,jmax) is the first partial derivative of state variable k with respect to decision variable i.

The partial derivatives of the state variables, d(i,jmax), are revised each time the system variables are repartitioned.

Having determined which decision variable to change, and the direction in which to change it, the next step is to determine how much change is possible. There are three factors controlling the maximum change to a decision variable. The decision variable is changed until 1) the decision variable reaches its upper or lower limit, 2) a state variable reaches its upper or lower limit, or 3) the constrained derivative coresspponding to the decision variable becomes zero. The smallest change in $x_d(jmax)$ that satisfies the above conditions is the maximum amount $x_d(jmax)$ can be changed.

The first restriction is the difference between the current value of the decision variable and the bound it is approaching. This deviation is described by

$$\triangle x'_d(jmax) = x_d(jmax)_{bound} - x_d(jmax) \qquad (4\text{-}35)$$

where:

$\triangle x'_d(jmax)$ = the maximum change in $x_d(jmax)$ due to limits on decision variable;

$x_d(jmax)_{bound}$ = the bound that $x_d(jmax)$ is approaching;

$x_d(jmax)$ = the current value of decision variable with the largest constrained derivative.

For a minimization process, the bound approached by the decision variable is the upper limit on the decision variable if the corresponding constrained derivative is negative. If the constrained derivative is positive, the approaching bound is the lower limit on the decision variable.

The state variables, as functions of decision variables, are subject to change as each decision variable changes. This change is described by Equation (4-34). The gradient of each state variable is applied to define the change in a decision variable which causes that state variable to reach a limiting condition.

$$\Delta x''_d(jmax) = \min (x_s(i)_{bound} - x_s(i))/ d(i,jmax) \qquad (4\text{-}36)$$

for i=1,K

where:

$\Delta x''_d(jmax)$ = the maximum change in $x_d(jmax)$ due to limits on state variables;

$x_s(i)_{bound}$ = the bound approached by state variable i;

$x_s(i)$ = the current value of state variable i.

The bound approached by a state variable depends on both the sign of the constrained derivative of the objective function, v(jmax), and the sign of the constrained derivative of the state variable, d(i,jmax). In a minimization process, if v(jmax) is positive, the decision variable $x_d(jmax)$ is decreased. If, in addition, d(i,jmax) is positive, state variable $x_s(i)$ also decreases and the bound approached is the lower limit on $x_s(i)$.

The third restriction must be considered in the case of a quadratic objective function. If the objective function were linear, Equations (4-35) and (4-36) would be sufficient in determining the maximum change in a decision variable.

The gradient of a quadratic objective function is a linear function of the decision variables. As a single decision variable changes, the vector of constrained derivatives is also affected as described by Equation (4-33). The initial sign of the constrained derivative indicates the direction in which the decision variable must be changed to improve the value of the objective function. For example, in a minimization process, if the constrained derivative is positive, the decision variable must be decreased in order to decrease the value of the objective function. If the sign of b(jmax,jmax) is negative, a decrease in x_d(jmax) causes constrained derivative v(jmax) to increase in accordance with Equation (4-33). It is possible to decrease the decision variable such that the constrained derivative changes sign (goes from positive to negative). Any further decrease in the decision variable will increase the value of the objective function, an undesirable effect.

The change in the value of a decision variable is limited such that the constrained derivative does not change signs. The magnitude of this change is determined by rearranging Equation (4-33).

$$\Delta x'''_d(\text{jmax}) = (0 - v(\text{jmax}))/b(\text{jmax},\text{jmax}) \qquad (4\text{-}37)$$

where:

$\Delta x'''_d$(jmax) is the maximum change in x_d(jmax) such that v(jmax) does not go to zero.

In summary, the systematic optimization process first locates the decision variable with the largest constrained derivative (absolute value) and determines the direction in which to change the decision variable. Equations (4-35), (4-36) and (4-37) are used to calculate the $\Delta x'_d$, $\Delta x''_d$ and $\Delta x'''_d$. The smallest of these three values is the maximum change in the decision variable. After this change is made, the values of the state variables are revised in accordance to Equation (4-34). If the change causes a state variable to become tight, the system variables are re-partitioned with the tight state varaible becoming a decision variable. The value of the objective function is then updated as shown by Equation (4-32), thus completing a single iteration. The process continues until the optimal solution has been reached.

At the optimum, all decision variables that are limited by a binding constraint are associated with a non-zero constrained derivative. Assuming a minimization process, if a decision variable is against an upper limit, the related constrained derivative must be negative. A decision variable has a positive constrained derivative associated with it if the lower limit is binding. If the value of a decision variable is not equal to a limiting condition, the corresponding constrained derivative is zero and any change in the decision variable does not improve the value of the objective function. This is simply a non-dogmatic explanation of achievement of the Kuhn-Tucker conditions.

4.11 INITIAL FEASIBLE SOLUTION

The gradient search technique used to optimize the objective function must be given a starting point from which to proceed. This initial feasible solution consists of a set of drawdown values which satisfy all constraining and limiting conditions. When a large number of variables are considered, a trial and error method is time consuming and evasive. Because this problem is formulated under steady-state conditions, the initial system drawdowns, a product of transient phenomenon, also fail to provide a feasible solution to the problem.

To compute an initial feasible solution, the set of equations described by Equation (4-8) is solved for the variable drawdowns with the vector of excitation, {w'}, set equal to the vector of lower limits on excitation. The Gauss-Siedel iterative technique is employed to solve the set of simultaneous equations.

The recharge values at constant-head cells are calculated using the initial feasible set of drawdowns and Equation (4-18). If the lower limit on recharge has been violated in any constant-head cell, no optimization can be performed. The least amount of feasible groundwater withdrawal cannot be supported by the maximum allowable recharge. In this case, the computer program is set up to issue a warning message stating which recharge constraint(s) are violated and by what amount.

Since the constraint set is the same for both objective functions, the optimal solution from one objective function can be used as initial feasible solution for the other.

4.12 RE-INITIALIZATION

The transmissivity values, used in the formulation of the objective functions and constraints, are calculated as the product of hydraulic conductivity and saturated thickness. If an unconfined aquifer is modeled, the saturated thickness changes as drawdown increases or decreases. This causes a nonlinearity in the groundwater flow equation. As a result, transmissivity values initially determined are not representative of optimal conditions and the accuracy of the results is reduced.

If the difference between the initial drawdowns and the optimal drawdowns is small, relative to the saturated thickness, the effect on the transmissivity values is negligible. If, however, the change in drawdown is large, such that transmissivity values are significantly altered, a re-initialization procedure is performed.

To improve the accuracy of the results without introducing nonlinear constraints, the drawdowns resulting from one optimization process are used as the initial conditions for a second optimization. If necessary, a third optimization is performed using the results of the second optimization, to calculate transmissivity values. Each additional re-initialization brings initial conditions closer to optimal conditions such that the saturated thickness more accurately corresponds to the resulting drawdown, pumping, and recharge values. Subsequent optimizations are continued until a predetermined convergence criterion is satisfied.

4.13 SUMMARY

This chapter describes the methodology used for obtaining optimal sustained yield pumping strategies. Two regional policy objectives are considered. The first objective seeks to maximize the total annual sustainable volume of groundwater withdrawal. The second objective is set to develop sustained yield pumping strategies that maintain an optimal potentiometric surface as close as possible to a predetermined 'target' surface.

Constraints used in the optimization model include limits on recharges or discharges into the area through the boundary cells, limits on recharges or discharges through cells with stream/ aquifer connection, upper and lower bounds on pumping at each variable-head cell, and lower limit on saturated thickness in every cell. The application of this methodology to the Boeuf-Tensas area is described in the next chapter.

CHAPTER V

DEVELOPMENT OF OPTIMAL SUSTAINED YIELD REGIONAL PUMPING STRATEGY

By

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5.1 INTRODUCTION

This chapter describes the application of alternative water management policies as constraints in developing optimal sustained yield pumping strategies for the Boeuf Tensas area of Arkansas. The optimal strategies are obtained from the solution of the SSTAR5 model, described in Chapter 4. The data required to construct this model for this area is based on the aquifer parameter values and estimates of historical conditions as discussed in Chapters 2 and 3. It should be mentioned that the process of reinitialization (Section 4.12) was not used in developing the strategies because: the aquifer is in general initially confined, saturated thicknesses are large, and this is a reconnaisance level study.

The two objective functions, which represent two different optimization models, are: i) maximize total withdrawal from the aquifer, and ii) minimize the total deviation of optimal water table elevations (or potentimetric surface elevations) at the center of each variable head cell, from target (or current) elevations. As discussed in Appendix B, either of these two objective functions can be used in SSTAR5 by assigning appropriate values to the index ISUS. In this study the target elevations in the second model are the current (1983) water table elevations. The weighting factors W_k are the inverse of the standard deviations of the estimation errors for estimating these elevations by kriging. Other weighting factors or other techniques of computing these standard deviations can be used also.

Constraints defining sustained yield hydraulic stresses are incorporated in both models. However, alternative optimal sustained yield pumping strategies are obtained by incorporating different sets of physical and managerial constraints. Solutions of the optimization model with these different sets of constraints represent different optimal sustained yield pumping strategies for different scenarios. The scenarios tested in our study represent plausible conditions which may have to be satisfied based on other economic, social, and political considerations. Presentation of the alternative strategies should aid in the selection of a single optimal sustained yield pumping strategy for the Boeuf-Tensas area, from a set of alternatives.

The following section describes the different scenarios which were tested. Amongst the scenarios tested for Model 1 (the maximize pumping objective function) one was selected as being most appropriate for implementation. The constraints for this scenario and a slight variation of it were then used develop strategies using Model 2.

5.2 DESCRIPTIONS OF SCENARIOS WITH MAXIMUM PUMPING OBJECTIVE FUNCTION

The scenarios differ because of the assumptions made in the constraining equations. Each set of assumptions and the scenarios to which they apply are discussed here.

5.2a General Description Of Assumptions and Constraints

Formulation of the constraining equations of an optimization model requires estimates of physical parameters of the aquifer. Estimation of these parameters was described in Chapters 2 and 3. Other important assumptions required to formulate the constraints are the physical boundary conditions of the aquifer underlying this area. Finally, bounds and constraints imposed on the decision variables should reflect both physical and institutional feasibility. Therefore a number of variations of these assumptions were utilized for obtaining alternative management strategies. For the sake of systematic presentation we will separate these assumptions into the following three categories. It should be noted however, that the partitioning between these categories are very artificial. It is no doubt possible to argue that there is some overlap.

i) Boundary Conditions:

a) assumptions regarding the hydraulic states of peripheral cells (treatment as constant-head or variable-head cells)

b) specified steady values of hydraulic variables at given cells which include i) constant recharges at a boundary cell or, constant recharge to an internal cell through stream/aquifer (S/A) interaction; ii) constant vertical accretions (deep percolation)

c) bounds on recharges through a single cell or a subsystem of boundary cells

ii) Bounds on recharges or discharges through S/A interactions for a single cell or a sub-system of cells

iii) Bounds on pumping at internal cells

Boundary Conditions

Figure 1.3 shows those cells along the study area boundary which were identified as constant-head cell or as constituents of a constant-head cell sub-system. Those boundary cells not shown as constant-head cells constitute the impermeable boundary of the aquifer. Cells along the western boundary above I=28 were treated as no-recharge, no-pumping, variable-head cells. In all the scenarios, those boundary cells which contained the Arkansas or the Mississippi Rivers were assumed to be constant-head cells.

For all scenarios, constant recharges due to stream/aquifer interflow were assumed for some internal cells. These recharges were calculated based on springtime gradients between 1973 and 1983, and the 2-dimensional Boussinesq equation for steady flow. The cells treated in this manner included all those cells where reliable estimates of APS (reach transmissivities obtained from the U.S.G.S.) values were not available. The estimated average annual recharge for the period between 1973 and 1983 were used as constant recharges.

The assumption that a vertical accretion of 100 ac-ft per year occurs uniformly in the region (at every cell) is based on water balance simulation and the low vertical permeability of the soil above the aquifer. As discussed in Chapter 3, this value is close to the value estimated by Broom and Reed (1973).

No maximum recharge constraints were imposed on the boundary cells having stream-aquifer connection with the Mississippi River (personal communication, Corps of Engineers, Vicksburg District). For the sub-system of boundary cells having S/A connection with the Arkansas River, the maximum legally permissible recharge was assumed to be 7240 thousand ac-ft/year. This value is the difference between the average annual flow at Murray Dam gaging station and the minimum annual flow volume required to meet stream water quality criterion according to Dixon and Perälta, (1984). It was used only as an upper bound, and the actual recharge in this sub-system, required to implement any one of the optimal strategies, was only a fraction of this value.

An upper bound on recharge of 500 ac-ft per year, including 100-ac-ft vertical accretion was used for each of the southwestern constant-head boundary cells (Figure 1.3). This value is based on the fact that although the aquifer extends to the west beyond that artificial boundary, the aquifer is relatively untapped by wells beyond that line.

The preceding boundary conditions can be stated as the general boundary conditions for the aquifer, since they remain unchanged in all the scenarios. Boundary conditions for the cells along the southern boundary of the study area were changed in different scenarios, and are presented later. The general boundary conditions can be restated as:

i) impermeable boundary along the western periphery of the Bayou Bartholomew basin, above I=28 (Figure 1.3);

ii) vertical accretion of 100 ac-ft/year in each cell;

iii) constant recharges based on the solution of 2-dimensional Boussinesq equation, for some internal S/A cells where reliable estimates of APS values were not available;

iv) no upper bound on recharge for those boundary cells having S/A connection with the Mississippi river;

v) maximum permissible recharge of 7240 thousand ac-ft/year for the sub-system of boundary cells having S/A connection with the Arkansas River;

vi) upper bound on recharge for 500 ac-ft/year for each south western boundary cell including and below I=28 (Figure 28).

The boundary conditions for the cells along the southern boundary were treated in four different ways. In order to assure that historic discharges of groundwater flowing into Louisiana are maintained in an optimal strategy, an upper limit was placed on the groundwater entering the region through the southern boundary. The different assumptions regarding the boundary conditions were grouped into the following four categories. These four categories differ only with respect to the treatment of the southern boundary of the aquifer underlying the Bayou Bartholomew basin.

Type 1 Boundary Conditions:

a) General boundary conditions

b) In each of the 11 southern boundary cells, up to 500 ac-ft/year of recharge from Louisiana is allowed per cell. All the 11 cells are treated as constant-head cells including those with S/A connection: (35,10),(35,11),(35,15),(35,16), and (35,17).

Type 2 Boundary Conditions:

a) General boundary conditions

b) Six of the southern boundary cells without S/A connection are treated as a constant-head cell sub-system. The total net recharge for this sub-system is bounded to be less than 600 ac-ft/year (6 x 100 ac-ft/yr per cell of vertical accretion). Recharge in each of the other 5 cells is bounded to be less than 500 ac-ft/year.

Type 3 Boundary Conditions:

a) General boundary condition

b) All 11 cells on the southern boundary are treated as a constant-head cell sub-system. A total of at least 3000 ac-ft/year discharge is forced to occur through this sub-system to Louisiana.

Type 4 Boundary Conditions:

a) General boundary conditions

b) All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 ac-ft/year on recharge through each cell.

<u>Bounds on Stream/Aquifer Interflow</u>

The stream/aquifer cells for the three internal rivers (Bayou Bartholomew, Bouef River, and Bayou Macon) were also assumed as three different sub-systems (Figure 5.1). In different scenarios the upper limit on recharges to the aquifer from these rivers were varied to satisfy potential institutional goals, while assuring physical realism. Table 5.1 shows the estimated historic S/A responses.

As seen from Figure 5.2, there has been substantial decline in groundwater levels in the cells along the Bayou Bartholomew River (as much as 7 feet) in the last ten years. This has caused an increase in recharge from the stream to the aquifer for the last few years of the simulation period. Therefore, in some of the scenarios, the maximum estimated recharge through S/A connection computed for any year between 1973-1983 was

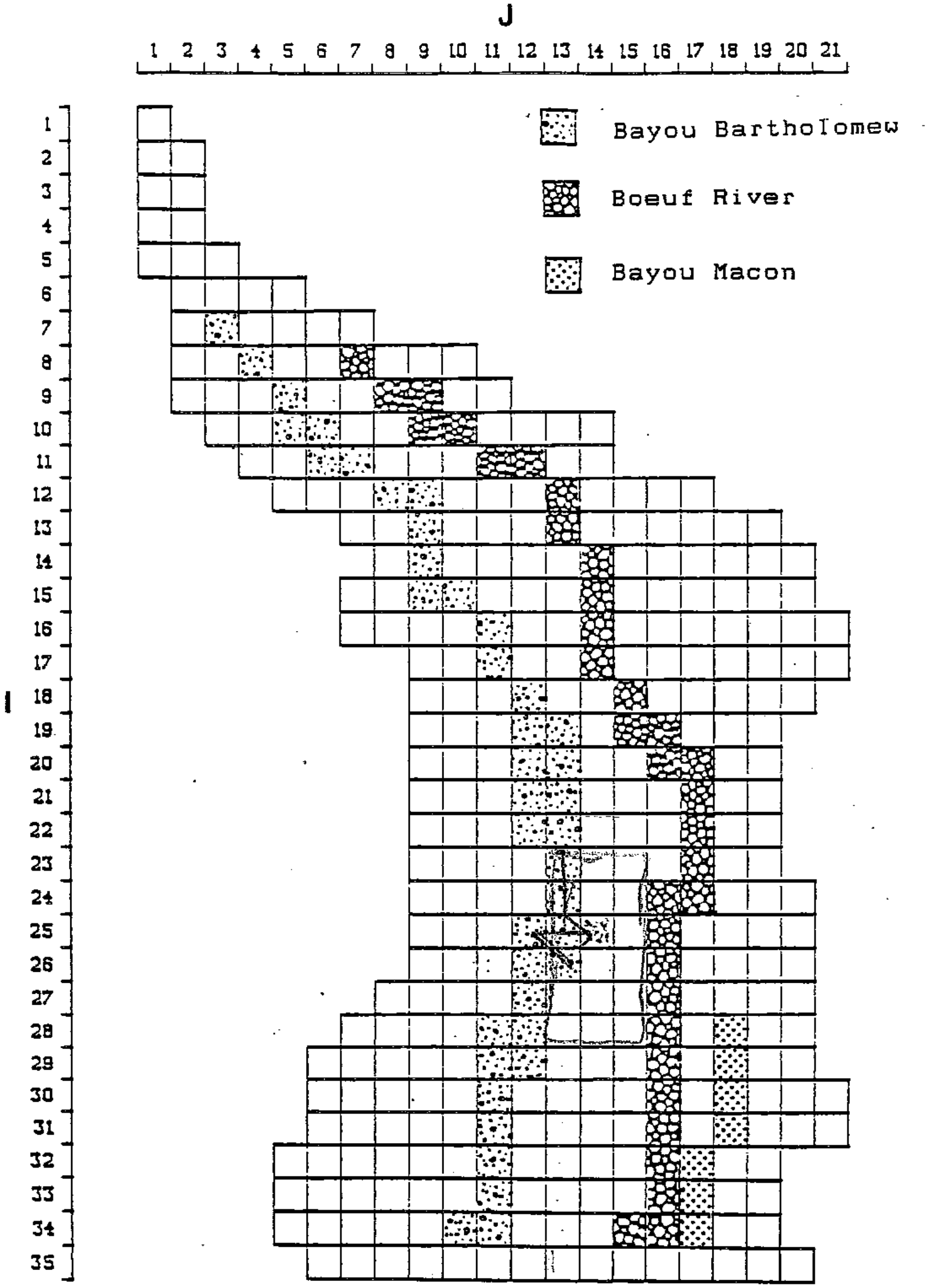

Figure 5.1 Stream-Aquifer Subsystems

Table 5.1 Historic Stream/Aquifer Interflow*

S/A Interflow	Maximum Recharge (ac-ft/year)	Average Recharge (ac-ft/year)
Boeuf River	-37,900	- 6,700
Bayou Bartholomew	-25,800	-9,800
Bayou Macon	-14,000	+4,000

* Based on data from 1973-1983. Negative value means recharge to aquifer from stream

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-2																				
2	0	-1																			
3	1	1																			
4	1	1																			
5	1	0	-1																		
6		0	-1	-2	-3																
7		0	-1	-1	-2	-3	-4														
8		-1	-1	-1	-1	-1	-3	-4	-4	-2											
9		-1	-2	-2	-2	-3	-4	-5	-5	-4	-2										
10			-2	-3	-4	-7	-7	-6	-6	-6	-5	-3	-2	0							
11				-4	-5	-7	-7	-6	-7	-8	-8	-7	-4	-2							
12					-4	-4	-5	-6	-7	-8	-10	-10	-6	-2	0	0	0				
13							-4	-4	-4	-6	-12	-13	-8	-3	0	-1	-2	-2	-1		
14								-3	-1	-1	-5	-8	-9	-8	-5	-4	-4	-3	-3	-1	
15							-2	-2	-2	-2	-1	-4	-7	-9	-9	-7	-6	-5	-4	-3	
16							-1	-2	-2	-2	-1	-1	-3	-6	-7	-7	-7	-6	-5	-4	-2
17									-1	0	1	1	1	-3	-7	-8	-7	-7	-6	-5	-4
18									0	0	1	3	2	-2	-5	-7	-8	-7	-7	-6	
19									0	0	1	2	-1	-4	-5	-6	-7	-8	-9		
20									-4	-2	0	1	-1	-5	-5	-5	-6	-8	-9		
21									-3	-2	-1	0	-1	-3	-4	-4	-4	-7	-10		
22									0	-1	-2	-2	-2	-2	-2	-3	-4	-6	-8		
23									0	-1	-2	-3	-2	0	0	-1	-3	-5	-6		
24									-1	-1	-2	-2	-1	1	1	-1	-3	-4	-5	-5	
25									-1	-1	-1	-1	0	0	-1	-2	-2	-3	-4	-4	
26									-2	-2	-1	-1	0	0	-3	-3	-2	-2	-3	-3	
27								-3	-3	-2	-2	-1	0	-1	-2	0	-1	-2	-2	-2	
28							-5	-4	-3	-3	-4	-4	-3	-2	-1	-1	-1	-2	-2	-2	
29						-7	-6	-5	-3	-3	-5	-7	-6	-3	-1	-1	-2	-2	-2	-1	
30						-7	-5	-4	-3	-3	-5	-8	-7	-2	-1	-2	-3	-3	-2	0	0
31						-5	-4	-4	-3	-4	-5	-6	-5	-2	-1	-4	-6	-5	-2	0	0
32					-3	-3	-3	-3	-3	-4	-4	-5	-4	-3	-3	-5	-8	-7	-1		
33					-2	-2	-2	-2	-3	-3	-4	-4	-3	-3	-4	-5	-5	2	4		
34					0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-1	4	3		
35						0	-1	-1	-2	-2	-3	-3	-3	-3	-3	-3	-3	-5	-7	-4	

Figure 5.2 Change in Observed Potentiometric Elevations (1973-1983) (ft)

imposed as the upper bound on S/A recharge.

For other scenarios, the average annual S/A interflow was used as an upper bound on recharge from S/A interflow at internal cells. However, the use of this constant may be overly conservative. This is pointed out by the fact that, although the average S/A interflow for the Bayou Macon river is a discharge from the aquifer, in some years, a substantial amount of recharge was estimated to occur.

Bounds on Pumping

The maximum allowable pumping in each internal cell was constrained to be less than one of the following three values:

i) estimated annual pumping based on 1982 acreage and average climatic conditions (Figure 2.1)

ii) estimated annual pumping in a drought year (1980) for 1980 acreage and climatic conditions

iii) estimated annual maximum potential pumping as discussed in Chapter 2. (Figure 2.2)

The sum of these upper bounds on pumping are shown in Table 5.2. The minimum allowable value of pumping in each internal cell was assumed equal to zero.

5.2b Description Of Scenarios For Model 1

The different scenarios used for obtaining the alternative strategies are discussed in this sub-section. These scenarios differ on the basis of the assumed boundary conditions, and the bounds on S/A interflow and pumping. A summary is presented in tabular form in Table 5.3.

Scenario 1.

Table 5.2 Historic Groundwater Withdrawal in Bayou-Bartholomew Basin

	Average (1973-1983) Pumping) (ac-ft/yr)	1980 pumping (ac-ft/yr)	1982 pumping (ac-ft/yr)
Regional Pumping	194,000	353,000 (277,400)	171,300 (140,300)

Values inside parentheses represent total pumping in the Boeuf-Tensas area.

1.1. The southern boundary consists of 11 constant head (35,16), and (35,17), with S/A connection. In each of these 11 constant-head cells, up to 500 ac-ft/year of recharge is allowed per cell.

1.2. A vertical accretion of 100 ac-ft/year is assumed to occur in each (boundary and internal) cell.

1.3. The Boeuf River, Bayou Bartholomew, and Bayou Macon are considered as three different stream/aquifer sub-systems (Figure 5.1).

1.4. The maximum recharges to the aquifer from each of the three stream/aquifer sub-sytems are constrained not to exceed the maximum observed annual values for 1973-1983.

1.5. All the southwest boundary cells (Figure 1.3) are treated as constant-head cells with a maximum allowable recharge of 500 ac-ft/year per cell.

1.7. Maximum potential irrigation demand is used as the upper bound on pumping in each internal cell.

Scenario 2

2.1. Same as Scenario 1, except that the 1982 base pumping values are used as the upper bound on pumping in each of the internal cells.

Scenario 3

3.1. Same as Scenario 1, except that the 1980 (a drought year) pumping values are used as the upper bound on pumping in each internal cell.

Scenario 4

4.1. Same as Scenario 1, except that the total net recharge in the southern boundary constant-head cell sub-system (excluding the 5 S/A connection cells) is bounded to be less than 600 ac-ft/year (vertical accretion of 100 ac-ft/yr per cell). Recharge in each of the 5 S/A cells bounded to be less than 500 ac-ft/year. This implies no net groundwater movement from Louisiana to Arkansas through these cells.

Scenario 5

5.1. Same as Scenario 4, except that the 1982 base pumping values are used as the upper bound on pumping in each internal cell.

Scenario 6

6.1. Same as Scenario 4, except that the 1980 pumping values are used as the upper bound on pumping in each internal cell.

Scenario 7

7.1. Same as Scenario 1, except that the maximum allowable recharge through stream/aquifer connections for the three sub-systems, are the average annual values for the period between 1973 and 1983.

Scenario 8

8.1. Same as Scenario 7, except that the 1982 base pumping values are used as the upper bound on pumping in each internal cell.

Scenario 9

9.1. Same as Scenario 7, except that the 1980 pumping values are used as the upper bound on pumping in each internal cell.

Scenario 10

10.1. Same as Scenario 4, except that the maximum allowable recharge to the aquifer through stream/aquifer connection for the three sub-systems are the average annual values for the period between 1973 and 1983.

Scenario 11

11.1. Same as Scenario 10, except that the 1982 base pumping values are used as the upper bound on pumping in each internal cell.

Scenario 12

12.1. Same as Scenario 10, except that the 1980 pumping values are used as the upper bound on pumping in each internal cell.

Scenario 13

13.1. Same as Scenario 1, except that the discharge to the Louisiana portion of the aquifer through the southern boundary cell sub-system, including S/A cells, is constrained to be not less than 3000. ac-ft/yr.

Scenario 14

14.1. Same as Scenario 13, except that the 1982 base pumping values are used as the upper bound on pumping in each internal cell.

Scenario 15

15.1. Same as Scenario 13, except that the 1980 pumping values are used as the upper bound on pumping in each internal cell.

Scenario 16

16.1. Same as Scenario 13, except that the maximum allowable recharge through stream/aquifer connections for the three sub-systems, are the average annual value for the the period between 1973 and 1983.

Scenario 17

17.1. Same as Scenario 16, except that the 1982 base pumping values are used as the upper bound on pumping in each internal cell.

Scenario 18

18.1. Same as Scenario 16, except that the 1980 pumping values are used as the upper bound on pumping in each internal cell.

Scenario 19

19.1. Same as Scenario 7, except that the southern boundary cells, including 5 S/A cells, are assumed to be variable-head cells, with an upper bound of 500 ac-ft/year on recharge through each of the 11 constant-head cells.

5.2c Description of Scenarios for Model 2

Scenario 20 represents the use of Model 2 with Type 3 southern boundary conditions; 1982 base pumping values as upper limits on each cell by cell pumping; and maximum annual S/A recharge as the upper limit on recharge from internal S/A sub-systems. These assumptions, identical to those of Scenario 14, were selected as being most realistic and acceptable for future management purposes. Scenario 21 represents the same constraints as in Scenario 20 except that the upper limit on pumping in each cell is the potential demand for groundwater in those cells. Because 1983 water table elevations were the most recent data available during the development of our data base for this study, these elevations were considered as the current (or target) elevations in Scenarios 20 and 21. In summary the following scenarios were used for developing strategies using Model 2.

Scenario 20

20.1. Constraints same as in Scenario 14, and 1983 groundwater table elevations are the target elevations.

Scenario 21

21.1. Constraints same as in Scenario 13, and 1983 groundwater table elevations are the target elevations.

5.3 DISCUSSION OF ASSUMPTIONS AND RESULTS

The afore mentioned scenarios are used for obtaining alternative sustained yield pumping strategies for the Bayou Bartholomew Basin. The total values of pumping, recharge, and S/A interflows (for study area B), obtained as solutions of Model 1 for different scenarios, are shown in Tables 5.3 to 5.6. The regional sustainable values of pumping, recharge and S/A interflows, obtained as solutions of Model 2 for Scenarios 20 and 21 are shown in Table 5.7 to 5.9. The total annual sustainable pumping values for a selected number of scenarios are also computed for the Boeuf-Tensas basin area. These pumping values are shown in parentheses in Tables 5.4 and 5.7. The Boeuf-Tensas basin constitutes that portion of the Bayou Bartholomew basin area which is to the east of the eastern divide of the Bayou Bartholomew watershed. This boundary of the Boeuf-Tensas basin (as shown in Figure 1.2) partitions the cells lying on this boundary. The fraction of the total area (9 square miles) of each

Table 5.3 Scenario Numbering System for Model 1

S/A Upper Bound	Pumping Upper Bound	Type 1 Boundary Conditions*	Type 2 Boundary Conditions*	Type 3 Boundary Conditions*	Type 4 Boundary Conditions*
		Strategy Number			
Max. S/A Rech.	Poten. Need	1	4	13	
	1982 Pumping	2	5	14	
	1980 Pumping	3	6	15	
Avg. S/A Rech.	Poten. Need	7	10	16	19
	1982 Pumping	8	11	17	
	1980 Pumping	9	12	18	

*

Type 1 Boundary Conditions (for southern boundary):
In each of the 11 southern boundary cells, up to 500 ac-ft/year of recharge from Louisiana is allowed per cell. All the 11 cells are treated as constant-head cells including those with S/A connection: (35,10),(35,11),(35,15),(35,16), and (35,17).

*
Type 2 Boundary Conditions (for southern boundary):
Six of the southern boundary cells without S/A connection are treated as a constant-head cell sub-system. The total net recharge for this sub-system is bounded to be less than 600 ac-ft/year (6 x 100 ac-ft/yr per cell of vertical accretion). Recharge in each of the other 5 cells is bounded to be less than 500 ac-ft/year per cell.

*
Type 3 Boundary Conditions (for southern boundary):
All 11 cells on the southern boundary are treated as a constant-head cell sub-system. A total of at least 3000 ac-ft/year discharge is forced to occur through this sub-system to Louisiana.

*
Type 4 Boundary Conditions (for southern boundary):
All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 ac-ft/year on recharge through each cell.

Table 5.4 Total Regional Maximum Pumping (Solutions of Model 1)

S/A Upper Bound	Pumping Upper Bound	* Type 1 Boundary Conditions	* Type 2 Boundary Conditions	* Type 3 Boundary Conditions	* Type 4 Boundary Conditions
		Total Pumping (ac-ft/year)			
Max. S/A Rech.	Poten. Need	344,500	344,500	336,200	·
	1982 Pumping	156,000	155,700	147,200 (116,000)	
	1980 Pumping	208,700	208,200	201,600 (172,800)	
Avg. S/A Rech.	Poten. Need	148,400	148,400	144,300	176,400
	1982 Pumping	88,900	88,900	86,900 (55,800)	
	1980 Pumping	109,600	109,600	106,200	

*
Type 1 Boundary Conditions (for southern boundary):
In each of the 11 southern boundary cells, up to 500 ac-ft/year of recharge from Louisiana is allowed per cell. All the 11 cells are treated as constant-head cells including those with S/A connection: (35,10),(35,11),(35,15),(35,16), and (35,17).

*
Type 2 Boundary Conditions (for southern boundary):
Six of the southern boundary cells without S/A connection are treated as a constant-head cell sub-system. The total net recharge for this sub-system is bounded to be less than 600 ac-ft/yr (6x100 ac-ft/yr per cell vertical accretion). Recharge in each of the other 5 cells is bounded to be less than 500 ac-ft/year per cell.

*
Type 3 Boundary Conditions (for southern boundary):
All 11 cells on the southern boundary are treated as a constant-head cell sub-system. A total of at least 3000 ac-ft/year discharge is forced to occur through this sub-system to Louisiana.

*
Type 4 Boundary Conditions (for southern boundary):
All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 ac-ft/year on recharge through each cell.

Values inside parentheses are for the Boeuf-Tensas area.

Table 5.5 Total Net Recharge From Boundaries Including Recharge Through Deep Percolation (Accretion)

S/A Upper Bound	Pumping Upper Bound	Type 1 Boundary Conditions *	Type 2 Boundary Conditions *	Type 3 Boundary Conditions *	Type 4 Boundary Conditions *
		Total Recharge (ac-ft/year)			
Max. S/A Rech.	Poten. Need	-276,700	-276,900	-269,000	
	1982 Pumping	-117,200	-117,100	-103,600	
	1980 Pumping	-165,800	-165,200	-158,800	
Avg. S/A Rech.	Poten. Need	-143,100	-143,100	-141,900	-164,300
	1982 Pumping	-98,300	-98,300	-95,500	
	1980 Pumping	-116,900	-116,900	-113,500	

*
Type 1 Boundary Conditions (for southern boundary):
In each of the 11 southern boundary cells, up to 500 ac-ft/year of recharge from Louisiana is allowed per cell. All the 11 cells are treated as constant-head cells including those with S/A connection: (35,10),(35,11),(35,15),(35,16), and (35,17).

*
Type 2 Boundary Conditions (for southern boundary):
Six of the southern boundary cells without S/A connection are treated as a constant-head cell sub-system. The total net recharge for this sub-system is bounded to be less than 600 ac-ft/yr (6x100 ac-ft/yr per cell vertical accretion). Recharge in each of the other 5 cells is bounded to be less than 500 ac-ft/year per cell.

*
Type 3 Boundary Conditions (for southern boundary):
All 11 cells on the southern boundary are treated as a constant-head cell sub-system. A total of at least 3000 ac-ft/year discharge is forced to occur through this sub-system to Louisiana.

*
Type 4 Boundary Conditions (for southern boundary):
All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 ac-ft/year on recharge through each cell.

Table 5.6 Total Stream Aquifer Interflow

		Type 1 Boundary Conditions*	Type 2 Boundary Conditions*	Type 3 Boundary Conditions*	Type 4 Boundary Conditions*
S/A Upper Bound	Pumping Upper Bound	S/A Response (ac-ft/yr)	S/A Response (ac-ft/yr)	S/A Response (ac-ft/yr)	S/A Response (ac-ft/yr)
Max. S/A Rech.	Poten. Need	-67,800	-67,800	-67,100	
	1982 Pumping	-38,900	-38,900	-43,600	
	1980 Pumping	-43,000	-43,000	-42,900	
Avg. S/A Rech.	Poten. Need	-5,400	-5,400	-2,400	-12,200
	1982 Pumping	+8,300	+8,300	+8,600	
	1980 Pumping	+7,300	+7,300	+7,300	

*

Type 1 Boundary Conditions (for southern boundary):
In each of the 11 southern boundary cells, up to 500 ac-ft/year of recharge from Louisiana is allowed per cell. All the 11 cells are treated as constant-head cells including those with S/A connection: (35,10),(35,11),(35,15),(35,16), and (35,17).

*

Type 2 Boundary Conditions (for southern boundary):
Six of the southern boundary cells without S/A connection are treated as a constant-head cell sub-system. The total net recharge for this sub-system is bounded to be less than 600 ac-ft/yr (6x100 ac-ft/yr per cell vertical accretion). Recharge in each of the other 5 cells is bounded to be less than 500 ac-ft/year per cell.

*

Type 3 Boundary Conditions (for southern boundary):
All 11 cells on the southern boundary are treated as a constant-head cell sub-system. A total of at least 3000 ac-ft/year discharge is forced to occur through this sub-system to Louisiana.

*

Type 4 Boundary Conditions (for southern boundary):
All 11 southern boundary cells are treated as variable head cells, with an upper bound of 500 ac-ft/year on recharge through each cell.

Table 5.7 Total Regional Pumping to Maintain Current Groundwater Levels (Solutions of Model 2)

		Type 3 Boundary Conditions	
S/A Recharge Upper Bound	Pumping Upper Bound		Total Pumping (ac-ft/year)
Maximum S / A Recharge	1982 Pumping	Strategy No. 20	52,800 (50,400)
	Potential Need	Strategy No. 21	55,300 (54,700)

Values inside parentheses represent total pumping in the Boeuf-Tensas area bordered by the eastern divide of the Bayou Bartholomew River, on the west.

Table 5.8 Total Net Recharge From Boundaries Including Recharge Through Deep Percolation (Solutions of Model 2)

		Type 3 Boundary Conditions	
S/A Recharge Upper Bound	Pumping Upper Bound		Total Recharge (ac-ft/year)
Maximum S / A Recharge	1982 Pumping	Strategy No. 20	-52,800
	Potential Need	Strategy No. 21	-50,800

Table 5.9 Total Stream/Aquifer Interflow
(Solutions of Model 2)

		Type 3 Boundary Conditions	
S/A Recharge Upper Bound	Pumping Upper Bound		Total S/A Response (ac-ft/year)
Maximum S / A Recharge	1982 Pumping	Strategy No. 20	+ 29.0
	Potential Need	Strategy No. 21	-4,600

partitioned cell which falls within the Boeuf-Tensas side of this boundary, and the corresponding I,J coordinates of the cells, are shown in Table C.1 (Appendix C).

The most important constraints imposed in the two models can be separated into the following broad categories.

i) maximum allowable pumping at each cell is constrained not to exceed the maximum potential demand for groundwater in that cell (Scenarios 1,4,13,7,10,16,19,21);

ii) maximum allowable pumping at each cell is constrained not to exceed the estimated pumping for droughty climatic conditions and irrigated acreage of 1980 (Scenarios 3,6,15,9,12,18);

iii) maximum allowable pumping in each cell is constrained not to exceed the estimated pumping for 1982 irrigated acreage and climatic conditions (Scenarios 2,5,14,8,11,17,20);

iv) the maximum possible recharge that can enter the region through stream/aquifer connections is limited not to exceed the maximum annual values estimated to occur between 1973 and 1983 (Scenarios 1,2,3,4,5,6,13,14,15,20,21);

v) the maximum possible recharge that can enter the region through stream/aquifer connections is limited not to exceed the average annual values estimated to occur between 1973 and 1983 (Scenarios 7,8,9,10,11,12,16,17,18,19);

vi) no net recharge is allowed to enter the aquifer from the Louisiana side, through the southern boundary cells

(Scenarios 4,5,6,10,11,12);

vi) discharge to the Lousiana side of the aquifer through the southern boundary cells, is constrained to be at least 3000 ac-ft/year (Scenarios 13,14,15,16,17,18, 20,21);

The four types of bundary conditions differ in how much recharge is permitted to enter the study area from Louisiana thru the southern boundary cells and whether these cells are treated as variable-head cells, individual constant-head cells, or as a part of a constant-head cell subsystem. The physical interaction that exists between the aquifer and rivers in five of the southern boundary cells was not modelled directly. In each of Types 1-3, all southern boundary cells are treated as Constant-head cells.

Type 1 boundary conditions permit a net recharge of 5500 ac-ft/year (11 x 500 ac-ft/year per cell) from the aquifer underlying Louisiana. This represents the assumption of maximum recharge to our area through the southern boundary.

In Type 2 boundary conditions, those southern boundary cells with S/A connections were excluded from the sub-system of constant-head cells along this boundary. Type 1 and Type 2 boundary conditions differ only in the restrictions imposed on the six southern boundary cells without any S/A connection. In Type 2 boundary conditions, we assume that 500 ac-ft/year per cell is reasonable recharge from S/A interflow in southern boundary cells with S/A connection. Therefore, the constraining of each of those cells without S/A connection to less than 100 ac-ft/year (the estimated annual deep percolation value) in Type 2 conditions assures no net movement of groundwater thru the aquifer from Louisiana. It was observed that even with Type 1 boundary conditions the resulting optimal strategies did not require any recharge from the Louisiana side of the aquifer. Therefore, Type 1 and Type 2 boundary conditions produced virtually identical optimal strategies. (The small differences between some of the optimal values for scenarios using Type 1 and Type 2 boundary conditions are due to the use of convergence criteria in specifying when the optimization algorithm should terminate.)

Type 3 boundary conditions included a constraint to ensure a discharge of at least 3000 ac-ft/year to the Louisiana part of the aquifer. This quantity is based on the average value of estimated historical net recharge/discharge (1972-1982) through the southern boundary cells, obtained by solving the 2-dimensional Boussinesq equation for observed springtime elevations. This 3000 ac-ft/year discharge is the sum of the discharge through interflow, and recharge to these cells through vertical accretion. Since the S/A cells are treated the same as other southern boundary cells this 3000 ac-ft/year bound does not incorporate any separate S/A responses.

Type 3 boundary conditions represent the most restrictive of the four conditions. It can be noted from Table 5.4 that Type 3 boundary conditions permits less sustainable pumping than any other type. For example, Scenario 13 differs from Scenarios 4 and 1 only in southern boundary constraints, yet its sustainable pumping is less.

When using Type 4 boundary conditions, the group of cells along the southern boundary were assumed to be variable-head cells. However, this variation was not rigorously tested because such a relaxation of the constant head conditions along the boundary may lead to large declines in water table elevations along the Louisiana boundary. Such an alternative is politically undesirable.

The average value of the net annual groundwater pumping in the Bayou Bartholomew basin during 1973-1982 was estimated to be 194,000. ac-ft/year (Table 5.2). Scrutiny of the total pumping values for Scenarios 16-18 in Table 5.4 indicates that the use of "average" historic recharge (through boundary cells and from S/A interflow) rates as constraints does not permit this much sustainable annual pumping. This is expected since groundwater levels become stable in a sustained yield scenario, whereas the historically observed withdrawals caused declines in the groundwater levels.

In a number of scenarios the recharge constraints were changed to allow greater recharges than those estimated as historic averages, along the boundary, and/or through S/A cells. For example, Scenarios 1-6 allowed greater recharges in the boundary cells and the three S/A sub-systems than the historic averages; Scenarios 7-12 allowed greater recharges through the boundary cells only, and Scenarios 13-15 and 20-21 allowed greater recharges through S/A sub-systems, and all boundary cells except those along the southern boundary; while Scenarios 16-18 allowed greater recharges through all boundary cells except those along the southern boundary. Scenario 19 represents a special case in which the southern boundary was treated as a variable-head cell boundary and permitted more recharges through the boundary cells than was estimated to be the average hisoric values.

The historic annual pumping estimates that were used as upper bounds on pumping in each cell were either 1982 values (Scenarios 2,5,14,8,11,17,20) or 1980 values (Scenarios 3,6,15,9,12,18). Estimated groundwater withdrawal values for 1982 acreages and average climatic conditions are considered more realistic because they reflect the most recent information available when this study was initiated. The 1980 values are significantly greater than the 1982 values because 1980 was a drought year. Therefore, the use of 1980 instead of 1982 withdrawal (pumping) values represented a relaxation of the upper bounds on pumping at each cell, and resulted in an increase in sustainable pumping (Table 5.4). However, this increase in total regional withdrawal was accomplished by sacrificing the more uniform regional distribution of optimal cell-by-cell pumping obtained when using 1982 values as bounds.

The maximum potential demand for groundwater at each cell was used as the upper bound on allowable pumping in Scenarios 1, 4, 7, 10, 13, 16, 19 and 21. As seen in Table 5.4, this resulted in an increase of total sustainable withdrawal from the region compared to that obtained from scenarios which used historic pumping values as the upper limit. However, the resulting optimal pumping was very much concentrated in a small fraction of the entire area. This strategy of permitting groundwater withdrawals according to potential needs (where physically feasible) diminishes the spatial equity in the distribution of pumpings. Such a strategy is socially unrealistic since it would require the shift in irrigated acreages, from current locations to other locations nearer to recharge sources. Therefore, using historic pumping as an upper bound on pumping at each cell is a more desirable alternative.

Some scenarios (Scenarios 7-12, 16-19) used average estimated S/A recharge (1972-83) as the upper bound on recharge to the aquifer from the Boeuf River, Bayou Macon, and Bayou Bartholomew. The rest of the scenarios (Scenarios 1-6, 13-15, 20,21) used maximum estimated annual S/A recharge (1972-83) as the upper bound on the recharge to the aquifer from the three internal rivers. As seen from Table 5.3, if all other constraints remain the same, the use of maximum S/A recharge as the upper bound on S/A recharge to the aquifer substantially increased the sustainable amount of pumping. Use of these values as upper bounds is not physically unrealistic since the implementation of a desirable sustained yield strategy will probably cause initial declines in the groundwater table elevations along the streams. This would result in increased amounts of recharges from these streams into the aquifer.

Scenario 14 was chosen as the most appropriate scenario for Model 1, because:

a) use of estimated pumping in 1982, as the upper bound on cell-by-cell pumping, is most efficient in maintaining the historic spatial distribution of pumping;

b) use of maximum estimated annual S/A recharge as the upper bound on recharge to the aquifer from the three internal rivers is realistic; and

c) use of Type 3 boundary conditions preserves the estimated historic groundwater flow into Louisiana.

The use of Model 1 and results in a total withdrawal of 147,200 ac-ft/yr (116,000 ac-ft/yr from area A, the Boeuf-Tensas basin) from the Bayou Bartholomew Basin Quaternary aquifer. In contrast, Scenario 15 (which is identical to Scenario 14 except that the upper bounds on pumping in each cell is the 1980 pumping value) results in 201,600 ac-ft/yr (173,000 ac-ft/year from area A) in the Bayou Bartholomew basin. This increase in sustainable pumping from the aquifer is achieved at the cost of spatial equity in pumping, i.e., some cells are allowed to pump more than that specified for Scenario 14, while other cells are allowed to pump much less or not at all. Thus, assuming that our estimates of cell-by-cell water demands are accurate, a pumping strategy based on Scenario 15 may be undesirable. Figures 5.3 and 5.4 show these characteristics of Scenarios 14 and 15. The optimal sustainable potentiometric surface elevations for these scenarios are shown in Figures 5.5 and 5.6 respectively.

Scenario 17 is also similar to Scenario 14 except that the upper bounds on recharges in the form of S/A responses are the average values for 1972-83. This reduction in allowable recharge from the streams to the aquifer results in a large reduction in the total sustainable pumping from the aquifer. Only 86,900 ac-ft/year can be pumped from the whole area (55,800 from the Boeuf-Tensas portion) if Scenario 17 is used. This value is far less

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    0.************************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    1.    0.************************************************************************************************************
******    2. 1945.    0.    0.************************************************************************************************
******    4. 1973. 1945.  319.    0.    0.************************************************************************************
******    6. 2064. 2282.  718.  718.  635.    0.    0.    0.******************************************************************
******    3.  131.  480.  636.  574.  911.  767.  996.  565.    0.************************************************************
************    2.   94.  566.  566. 1055.  938. 1013.  847.  677.    0.    0.************************************************
******************    2.  228.  566.  735.  450. 1074. 1127. 1127.  846.  846.    0.******************************************
************************    3.    3.  677.  788. 1074.  847.  900. 1016. 1016.  715.    0.    0.    0.************************
************************************    3. 1016. 1127. 1127. 1127. 1452. 1122. 1016.  563.  448.  139.    0.    0.************
******************************************    5.  735. 1011. 1127.  958. 2032. 2311. 1122. 1122. 1011. 1055.  842.    0.******
************************************    6.    5.  281.  847. 1016. 1016. 2311. 2172. 1122.  983. 1122. 1122.  982.    0.******
************************************    3.    2.  397.  397.  985.    0. 1122. 1122. 1122.  983. 1122. 1122. 1122.  375.    0.
************************************************    3.   76.  327.    0.  780.  910. 1122. 1122.  934.  896.  279.    1.    0.
************************************************    3.  244.  525.  561.  524.  607.  983. 1122.  935.  564.    1.    0.******
************************************************    2.  139.  419.    0.    0.  267.  991. 1122. 1122.  934.    0.************
************************************************    2.  350.  175.    0.    0.  359.  991. 1122. 1122.  982.    0.************
************************************************    2.  106.    2.    1.    0.    0.  746.  843.  674.  934.    0.************
************************************************    2.  211.   69.    0.    0.  441. 1122. 1122.  901.  448.    0.************
************************************************    2.  106.  139.  419.    0.  566.  457.  514.  331.  320.    0.************
************************************************    1.    1.    0.  139.  525.  568.  425.  532.  466.  400.   66.    0.******
************************************************    1.    1.    1.    0.  488.  769.  457.  400.  533.  267.    0.    0.******
************************************************    1.    1.    1.    0. 1085.  523.  532.  457.  533.  532.  178.    0.******
******************************************    0.    1.    0.    0.    0. 1209. 1264.  533.  354.  427.  400.  320.    0.******
************************************    0.  461.    0.    0.    0.    0. 1209. 1264.  532.  466.  457.  455.  354.    0.******
******************************    0.    0.    0.    0.    0.    0.    0.    0. 1264.  532.  530.  473.  411.  400.    0.******
******************************    0.  697.    0.    0.    0.    0.    1.    0.    0.  533.  530.  532.  530.  443.  443.    0.
******************************    0.    0.    0.    0.    0.    0.    1.    0.    0.  533.  530.  532.  466.  532.    0.******
************************    0.    0.    0.    0.    0.    0.    0.    1.    0.    0.    0.  530.  466.  586.    0.************
************************    0.    0.    0.    0.    0.    0.    0.    1.    0.    0.    0.  441.    0.  178.    0.************
************************    0.    0.    0.    0.    0.    0.    1.    1.    0.    0.    3.    2.    0.    1.    0.************
******************************    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.******************
```

Figure 5.3 Optimal Groundwater Withdrawal for Scenario 14 (ac-ft/year)

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0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	1.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	2.	2287.	0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	4.	1973.	2287.	606.	0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	6.	0.	2467.	1361.	1360.	1204.	0.	0.	0.	*	*	*	*	*	*	*	*	*	*	*
*	3.	0.	0.	0.	1088.	1726.	1453.	1820.	1033.	0.	*	*	*	*	*	*	*	*	*	*
*	*	2.	0.	0.	1034.	1926.	1714.	1741.	1546.	1236.	0.	0.	*	*	*	*	*	*	*	*
*	*	*	2.	7.	7.	1343.	822.	1962.	2059.	2059.	1546.	1546.	0.	*	*	*	*	*	*	*
*	*	*	*	3.	3.	1237.	1440.	1962.	1546.	1644.	1856.	1856.	1307.	0.	0.	0.	*	*	*	*
*	*	*	*	*	*	3.	1856.	2059.	2059.	2059.	2340.	2345.	2124.	1177.	936.	291.	0.	0.	*	*
*	*	*	*	*	*	*	5.	1343.	1847.	2059.	1750.	2950.	3534.	2345.	2345.	2114.	2204.	1761.	0.	*
*	*	*	*	*	*	6.	5.	513.	1032.	2.	1.	3534.	3242.	2345.	2053.	2345.	2345.	2053.	0.	*
*	*	*	*	*	*	3.	2.	0.	0.	0.	0.	0.	2345.	2345.	2053.	2345.	2345.	2345.	784.	0.
*	*	*	*	*	*	*	*	3.	0.	0.	0.	0.	217.	2345.	2345.	1953.	1872.	584.	1.	0.
*	*	*	*	*	*	*	*	2.	0.	0.	0.	1.	1496.	2053.	2345.	1953.	1178.	1.	0.	*
*	*	*	*	*	*	*	*	2.	0.	0.	0.	0.	0.	1882.	2345.	2345.	1952.	0.	*	*
*	*	*	*	*	*	*	*	2.	0.	0.	0.	0.	0.	1882.	2345.	2345.	2053.	0.	*	*
*	*	*	*	*	*	*	*	2.	0.	1.	1.	0.	0.	0.	1761.	1409.	1952.	0.	*	*
*	*	*	*	*	*	*	*	2.	0.	0.	0.	0.	0.	0.	2345.	1882.	936.	0.	*	*
*	*	*	*	*	*	*	*	1.	0.	0.	0.	0.	0.	0.	897.	578.	558.	0.	*	*
*	*	*	*	*	*	*	*	1.	1.	0.	0.	0.	0.	0.	928.	813.	697.	115.	0.	*
*	*	*	*	*	*	*	*	1.	1.	0.	0.	0.	0.	0.	697.	929.	466.	0.	0.	*
*	*	*	*	*	*	*	*	1.	0.	0.	0.	0.	0.	0.	797.	928.	929.	311.	0.	*
*	*	*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	618.	745.	697.	558.	0.	*
*	*	*	*	*	*	0.	876.	0.	0.	0.	0.	0.	0.	0.	813.	797.	795.	618.	0.	*
*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	876.	825.	719.	697.	0.	*
*	*	*	*	*	0.	1561.	0.	0.	0.	0.	1.	0.	0.	0.	3.	929.	926.	773.	772.	0.
*	*	*	*	*	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	3.	929.	813.	928.	0.	*
*	*	*	*	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	3.	0.	836.	0.	*	*
*	*	*	*	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	3.	0.	1.	0.	*	*
*	*	*	*	0.	0.	0.	0.	0.	0.	1.	1.	0.	0.	2.	2.	0.	1.	0.	*	*
*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	*	*	*

Figure 5.4 Optimal Groundwater Withdrawal for Secnario 15 (ac-ft/year) .

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195.																				
195.	195.																			
196.	197.																			
195.	197.																			
192.	190.	192.																		
	181.	180.	187.	182.																
	174.	171.	175.	175.	179.	176.														
	169.	166.	165.	166.	168.	169.	174.	173.	172.											
	167.	165.	163.	161.	160.	160.	161.	161.	162.	164.										
		163.	160.	158.	155.	153.	151.	150.	152.	155.	158.	159.								
			158.	155.	152.	149.	145.	142.	142.	143.	146.	150.	150.							
				153.	150.	145.	141.	138.	135.	134.	135.	140.	148.	158.	155.	151.				
						141.	137.	133.	129.	127.	127.	131.	140.	145.	143.	142.	145.	142.		
							133.	131.	128.	126.	123.	121.	124.	127.	127.	128.	130.	133.	137.	
						132.	132.	130.	128.	126.	126.	119.	118.	119.	118.	118.	121.	126.	134.	
						131.	131.	129.	128.	128.	134.	126.	123.	117.	114.	113.	115.	121.	127.	130.
								128.	128.	129.	135.	132.	137.	119.	112.	111.	114.	119.	124.	127.
								127.	127.	127.	130.	132.	126.	117.	110.	110.	113.	119.	124.	
								125.	125.	125.	129.	129.	123.	114.	108.	108.	114.	121.		
								123.	123.	123.	124.	125.	122.	111.	105.	106.	110.	118.		
								121.	121.	121.	121.	120.	119.	109.	103.	105.	109.	116.		
								119.	119.	118.	117.	116.	113.	105.	102.	104.	109.	114.		
								117.	116.	115.	113.	112.	107.	104.	103.	105.	108.	113.		
								115.	114.	112.	110.	107.	104.	103.	105.	105.	106.	109.	111.	
								113.	112.	110.	107.	104.	102.	101.	103.	102.	104.	107.	110.	
								110.	110.	108.	105.	100.	98.	98.	101.	100.	100.	105.	109.	
							108.	109.	108.	106.	103.	97.	94.	96.	100.	98.	98.	103.	108.	
						108.	106.	107.	107.	105.	102.	96.	93.	97.	101.	98.	95.	102.	108.	
					101.	104.	105.	106.	105.	104.	102.	99.	95.	100.	110.	99.	93.	101.	108.	
					97.	99.	103.	104.	104.	103.	102.	100.	99.	100.	106.	98.	92.	100.	107.	112.
					97.	100.	102.	103.	103.	102.	101.	99.	99.	99.	102.	98.	97.	103.	109.	
				93.	97.	100.	101.	102.	101.	100.	99.	98.	97.	97.	98.	96.	98.	104.		
				92.	96.	99.	100.	100.	99.	97.	97.	95.	94.	93.	93.	95.	98.	103.		
				90.	95.	98.	99.	99.	97.	93.	92.	92.	90.	88.	85.	92.	97.	103.		
					93.	97.	99.	98.	96.	94.	91.	88.	87.	87.	88.	90.	94.			

Figure 5.5 Optimal Potentiometric Level for Scenario 14 (ft)

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195.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
195.	195.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
196.	197.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
195.	197.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
192.	190.	192.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	182.	180.	187.	182.	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	175.	172.	174.	174.	179.	176.	******	******	******	******	******	******	******	******	******	******	******	******	******	******
******	171.	169.	165.	164.	164.	165.	174.	173.	172.	******	******	******	******	******	******	******	******	******	******	******
******	169.	166.	163.	158.	154.	152.	154.	154.	157.	164.	******	******	******	******	******	******	******	******	******	******
******	******	163.	158.	153.	147.	141.	139.	138.	141.	148.	158.	159.	******	******	******	******	******	******	******	******
******	******	******	153.	149.	144.	136.	131.	126.	125.	129.	136.	143.	158.	******	******	******	******	******	******	******
******	******	******	******	145.	140.	131.	125.	119.	116.	115.	117.	126.	140.	158.	155.	151.	******	******	******	******
******	******	******	******	******	******	126.	120.	115.	110.	107.	106.	111.	122.	131.	131.	135.	145.	142.	******	******
******	******	******	******	******	******	******	120.	117.	113.	111.	106.	100.	102.	105.	106.	109.	117.	125.	137.	******
******	******	******	******	******	******	122.	121.	120.	119.	120.	118.	104.	97.	94.	92.	93.	101.	114.	134.	******
******	******	******	******	******	******	122.	122.	123.	124.	127.	132.	120.	109.	94.	87.	85.	93.	106.	122.	130.
******	******	******	******	******	******	******	******	126.	127.	130.	135.	131.	137.	101.	87.	85.	93.	107.	120.	127.
******	******	******	******	******	******	******	******	127.	128.	130.	132.	132.	118.	99.	86.	86.	96.	112.	124.	******
******	******	******	******	******	******	******	******	127.	128.	128.	130.	127.	116.	97.	86.	86.	99.	121.	******	******
******	******	******	******	******	******	******	******	127.	127.	126.	126.	125.	118.	99.	86.	87.	98.	118.	******	******
******	******	******	******	******	******	******	******	125.	125.	124.	123.	120.	117.	104.	98.	91.	99.	116.	******	******
******	******	******	******	******	******	******	******	124.	123.	122.	121.	117.	114.	104.	92.	93.	101.	114.	******	******
******	******	******	******	******	******	******	******	122.	122.	120.	118.	115.	112.	105.	98.	98.	102.	113.	******	******
******	******	******	******	******	******	******	******	120.	119.	118.	116.	114.	111.	106.	103.	100.	100.	107.	111.	******
******	******	******	******	******	******	******	******	117.	117.	116.	114.	112.	110.	106.	102.	97.	97.	103.	110.	******
******	******	******	******	******	******	******	******	115.	114.	113.	112.	110.	108.	104.	100.	92.	90.	98.	109.	******
******	******	******	******	******	******	******	109.	112.	112.	111.	110.	109.	107.	103.	98.	87.	80.	92.	108.	******
******	******	******	******	******	******	108.	105.	109.	110.	110.	109.	108.	106.	103.	98.	82.	61.	87.	108.	******
******	******	******	******	******	101.	103.	105.	107.	108.	108.	107.	106.	106.	106.	109.	85.	66.	88.	108.	******
******	******	******	******	******	97.	95.	102.	105.	106.	105.	104.	104.	104.	105.	106.	88.	73.	90.	104.	112.
******	******	******	******	******	97.	99.	102.	103.	103.	103.	101.	102.	102.	102.	102.	92.	88.	97.	109.	******
******	******	******	******	93.	97.	99.	101.	102.	101.	100.	99.	99.	98.	98.	98.	94.	94.	104.	******	******
******	******	******	******	92.	96.	99.	100.	100.	99.	97.	97.	96.	94.	93.	93.	94.	97.	103.	******	******
******	******	******	******	90.	95.	98.	99.	99.	97.	93.	93.	92.	90.	88.	85.	91.	97.	103.	******	******
******	******	******	******	******	93.	97.	99.	98.	96.	94.	91.	88.	87.	87.	88.	90.	94.	******	******	******

Figure 5.6 Optimal Potentiometric Level for Scenario 15 (ft)

than the estimated average annual pumping from this aquifer and is not very acceptable.

In summary, the constraints and boundary conditions of Scenario 14, are the most acceptable if maximization of sustained groundwater withdrawal is the management objective. If the maintenance of average historical interflow to the internal rivers is also an important consideration, Scenario 17 is the one that should be used with Model 1.

The same constraints used in Scenario 14 for Model 1 were used for Model 2. The resulting scenario, number 20, used 1982 pumping as the upper bound on cell-by-cell pumping, Type 3 boundary conditions, and 1983 potentiometric surface elevations as the target elevations. The total sustainable pumping with Scenario 20 was only about 30 percent of the average historic pumping. Therefore, in Scenario 21, the upper bound on cell-by-cell pumping was increased to the potential need in each cell to verify what additional amount of total sustainable pumping can be obtained with this relaxation of pumping upper bounds. Scenario 21 represents constraints identical to Scenario 13, and differs from Scenario 20 only in the pumping upper bounds. Even with this relaxation of pumping upper bounds, the total sustainable pumping increased only by about 3000 ac-ft/year. Constraints similar to those of Scenario 15 were not used with Model 2, because, the 1980 pumping values are in between 1982 values and the potential needs. Thus the use of constraints similar to Scenario 15 would result in sustainable total withdrawal less than that for Scenario 21.

As evident from Table 5.7, Scenario 20 specifies an optimal regional pumping value of 52,800 acre-ft/yr (50,400 ac-ft/yr for the Boeuf-Tensas area). This amount can be pumped from the study area while maintaining the potentiometric surface near that of 1983. If the maximum allowable pumping at each cell is the potential demand for that cell (Scenario 21), the optimal regional value of pumping is slightly greater, 55,900 ac-ft/year (54,700 ac-ft/year from the Boeuf-Tensas area). The cell-by-cell regional optimal pumping values for Scenarios 20 and 21 and the corresponding potentiometric surface elevations that those strategies will maintain are shown in Figures 5.7 to 5.10 respectively.

The total optimal pumping values obtained by using Model 2 for the two most realistic scenarios are far less than those historically observed. A major reason is that the historic withdrawals caused a continuous decline in the potentiometric elevations (Figure 5.1). Therefore, sustenance of current potentiometric surface elevations will permit much less pumping than has been historically observed.

5.4 SUMMARY AND CONCLUSIONS

Solutions of an optimization model useful for developing a regional groundwater management strategy are dependent on the specified boundary conditions. Two types of considerations are necessary while selecting these boundary conditions: i) physical feasibility based on hydraulic conditions, and ii) managerial feasibility based on social, economic and political considerations. These two criteria were considered in developing optimal sustained yield pumping strategies for the following two

J

```
    0.************************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    0.******************************************************************************************************************
    0.    1.    0.************************************************************************************************************
******    2.    1.    0.    0.******************************************************************************************************
******    4.    3.    2.    1.    0.    0.************************************************************************************************
******    6.    5.    4.    3.    2.    1.    0.    0.    0.************************************************************************
******    3.    5.    5.    5.    4.    3.  767.  996.    1.    0.******************************************************************
************    2.    5.    7.    7.    5.  939. 1013.  847.  677.    0.    0.************************************************************
******************    2.    7.    7.  735.  450. 1074. 1127. 1127.    1.    0.    0.******************************************************
************************    3.    3.  424.  788. 1074.  847.  900. 1016.    1.    0.    0.    0.    0.************************
************************************    3.    5.    3.    0. 1127. 1452.  382.    0.    0.    0.    0.    0.    0.************
******************************************    5.    4.    3.    2.    1.    1. 1982.    0.    0.    0.    0.    0.    0.******
************************************    6.    5.    4.    3.    2.    1.    0. 2172. 1122.    0.    0.    0.    0.    0.******
************************************    3.    2.    4.    3.    1.    0.    0.  722. 1122.  983.  650.    0.    0.    0.    0.
************************************************    3.    2.    1.    0.    0.    0. 1122. 1122.  644.    0.    0.    0.    0.
************************************************    3.    2.    1.    0.    0.    0.  983. 1122.  934.    0.    0.    0.******
************************************************    2.    2.    1.    1.    0.    0.    0. 1122.    0.    0.    0.************
************************************************    2.    2.    1.    1.    0.    0.    0.    0.    0.    0.    0.************
************************************************    2.    2.    2.    1.    0.    0.    0.    0.    0.  934.    0.************
************************************************    2.    2.    1.    1.    0.    0.    0.    0.    0.    0.    0.************
************************************************    2.    1.    1.    0.    0.    0.    0.    0.    0.    0.    0.************
************************************************    1.    1.    1.    0.    0.    0.    0.    0.  466.    0.    0.    0.******
************************************************    1.    1.    1.    0.    0.    0.  457.  400.  533.  267.    0.    0.******
************************************************    1.    1.    1.    0.    0.    0.  533.  457.  532.  533.    0.    0.******
******************************************    0.    1.    1.    0.    0.    0. 1264.  533.  354.  427.    0.    0.    0.******
************************************    0.    0.    0.    0.    0.    0.    0. 1264.  532.  466.    0.    1.    0.    0.******
******************************    0.    0.    0.  499.    0.    0.    0.    0. 1264.  532.  532.  473.    1.    0.    0.******
******************************    0.    0.    0.    0.    0.    0.    0.    0. 1264.  533.  532.  532.    1.    0.    0.    0.
******************************    0.    0.    0.    0.    0.    0.    0.    0. 1264.  533.    1.  533.    0.    0.    0.******
************************    0.    0.    0.    0.    0.    0.    0.    0.    0.   98.  533.    1.    0.    0.    0.************
************************    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    1.    0.    0.    0.************
************************    0.    0.    0.    0.    0.    0.    0.    1.    0.    0.    1.    1.    0.    0.    0.************
******************************    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.******************
```

Figure 5.7 Optimal Groundwater Withdrawal for Scenario 20 (ac-ft/year) . . . .

J

0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.	1.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	2.	1.	0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	4.	3.	2.	1.	0.	0.	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	6.	5.	4.	3.	2.	1.	0.	0.	0.	*	*	*	*	*	*	*	*	*	*	*
*	3.	5.	6.	5.	4.	3.	2.	1.	1.	0.	*	*	*	*	*	*	*	*	*	*
*	*	2.	5.	7.	7.	5.	2000.	866.	2.	1.	0.	0.	*	*	*	*	*	*	*	*
*	*	*	2.	7.	7.	6.	5.	5807.	2.	847.	1.	0.	0.	*	*	*	*	*	*	*
*	*	*	*	3.	3.	5.	114.	1481.	3.	1627.	1.	1.	0.	0.	0.	0.	*	*	*	*
*	*	*	*	*	*	3.	5.	3.	0.	2.	3385.	1.	1.	0.	0.	0.	0.	0.	*	*
*	*	*	*	*	*	*	5.	4.	3.	2.	1.	1.	1397.	0.	0.	0.	0.	0.	0.	*
*	*	*	*	*	*	6.	5.	4.	3.	2.	1.	1.	2797.	0.	0.	0.	0.	0.	0.	*
*	*	*	*	*	*	3.	2.	3.	3.	1.	0.	0.	0.	0.	3100.	0.	0.	0.	0.	0.
*	*	*	*	*	*	*	*	3.	2.	1.	0.	0.	0.	4987.	0.	0.	0.	0.	0.	0.
*	*	*	*	*	*	*	*	3.	2.	2.	0.	0.	0.	0.	1507.	719.	0.	0.	0.	*
*	*	*	*	*	*	*	*	3.	2.	1.	1.	0.	0.	0.	0.	921.	0.	0.	*	*
*	*	*	*	*	*	*	*	2.	2.	2.	1.	0.	0.	0.	0.	0.	0.	0.	*	*
*	*	*	*	*	*	*	*	2.	2.	2.	1.	0.	0.	0.	0.	768.	0.	0.	*	*
*	*	*	*	*	*	*	*	2.	2.	1.	1.	0.	0.	0.	0.	0.	0.	0.	*	*
*	*	*	*	*	*	*	*	2.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	*	*
*	*	*	*	*	*	*	*	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	*
*	*	*	*	*	*	*	*	1.	1.	1.	0.	0.	0.	0.	0.	1163.	0.	0.	0.	*
*	*	*	*	*	*	*	*	1.	1.	1.	0.	0.	0.	3194.	0.	1518.	0.	0.	0.	*
*	*	*	*	*	*	*	0.	1.	0.	0.	0.	0.	0.	298.	0.	0.	0.	0.	0.	*
*	*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	1492.	883.	0.	0.	1.	0.	0.	*
*	*	*	*	*	0.	0.	0.	410.	0.	0.	0.	0.	0.	3074.	1.	2300.	1.	0.	0.	*
*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	3043.	1.	0.	1.	0.	0.	0.
*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	190.	2199.	1.	3126.	0.	0.	0.	*
*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	*	*
*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	*	*
*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	0.	3.	0.	*	*
*	*	*	*	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	*	*	*

Figure 5.8 Optimal Groundwater Withdrawal for Scenario 21 (ac-ft/year)

J

195.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
196.	195.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
197.	197.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
196.	197.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
195.	193.	192.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	191.	190.	187.	182.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	189.	189.	189.	183.	179.	176.	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	187.	187.	185.	181.	177.	175.	174.	173.	172.	*	*	*	*	*	*	*	*	*	*	*
*	186.	185.	182.	178.	174.	171.	167.	165.	166.	164.	*	*	*	*	*	*	*	*	*	*
*	*	182.	178.	175.	171.	167.	162.	159.	158.	158.	158.	159.	*	*	*	*	*	*	*	*
*	*	*	175.	172.	169.	164.	159.	154.	152.	152.	154.	156.	158.	*	*	*	*	*	*	*
*	*	*	*	170.	167.	161.	156.	152.	149.	147.	148.	152.	156.	158.	155.	151.	*	*	*	*
*	*	*	*	*	*	158.	155.	153.	149.	145.	144.	149.	155.	156.	152.	148.	145.	142.	*	*
*	*	*	*	*	*	*	153.	151.	149.	147.	145.	144.	142.	144.	144.	142.	140.	139.	137.	*
*	*	*	*	*	*	151.	151.	149.	148.	146.	144.	141.	134.	134.	136.	136.	136.	135.	134.	*
*	*	*	*	*	*	151.	150.	147.	146.	144.	143.	140.	134.	130.	128.	129.	131.	131.	131.	130.
*	*	*	*	*	*	*	*	144.	144.	142.	141.	139.	137.	128.	124.	125.	127.	128.	127.	127.
*	*	*	*	*	*	*	*	142.	141.	140.	139.	137.	133.	126.	122.	123.	125.	124.	124.	*
*	*	*	*	*	*	*	*	139.	138.	137.	137.	135.	131.	127.	122.	123.	124.	121.	*	*
*	*	*	*	*	*	*	*	136.	135.	134.	133.	131.	130.	126.	123.	121.	120.	118.	*	*
*	*	*	*	*	*	*	*	133.	132.	131.	129.	126.	126.	124.	121.	119.	116.	116.	*	*
*	*	*	*	*	*	*	*	130.	129.	128.	126.	122.	121.	121.	119.	117.	116.	114.	*	*
*	*	*	*	*	*	*	*	127.	126.	125.	122.	119.	119.	117.	116.	114.	114.	113.	*	*
*	*	*	*	*	*	*	*	124.	123.	121.	119.	117.	115.	114.	112.	110.	111.	111.	111.	*
*	*	*	*	*	*	*	*	120.	119.	118.	116.	113.	111.	109.	108.	107.	108.	109.	110.	*
*	*	*	*	*	*	*	*	117.	116.	114.	112.	110.	107.	104.	105.	103.	104.	107.	109.	*
*	*	*	*	*	*	*	112.	114.	113.	112.	109.	106.	101.	101.	103.	101.	102.	105.	108.	*
*	*	*	*	*	*	108.	110.	111.	110.	109.	107.	103.	97.	100.	103.	100.	96.	104.	108.	*
*	*	*	*	*	101.	105.	107.	107.	108.	107.	104.	101.	96.	100.	110.	100.	94.	102.	108.	*
*	*	*	*	*	97.	103.	105.	106.	106.	104.	102.	99.	95.	99.	106.	99.	93.	102.	108.	112.
*	*	*	*	*	97.	102.	104.	104.	104.	102.	100.	98.	94.	97.	102.	99.	99.	105.	109.	*
*	*	*	*	93.	97.	101.	102.	102.	101.	100.	99.	97.	94.	95.	98.	98.	100.	104.	*	*
*	*	*	*	92.	96.	99.	101.	100.	99.	97.	96.	95.	93.	92.	93.	96.	99.	103.	*	*
*	*	*	*	90.	95.	98.	99.	99.	97.	93.	92.	91.	90.	88.	85.	92.	97.	103.	*	*
*	*	*	*	*	93.	97.	99.	98.	96.	94.	91.	88.	87.	87.	88.	90.	94.	*	*	*

Figure 5.9 Optimal Potentiometric Level for Scenario 20 (ft)

J

195.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
196.	195.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
197.	197.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
196.	197.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
195.	193.	192.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	191.	190.	187.	182.	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	189.	189.	189.	183.	179.	176.	*	*	*	*	*	*	*	*	*	*	*	*	*	*
*	187.	187.	185.	181.	177.	175.	174.	173.	172.	*	*	*	*	*	*	*	*	*	*	*
*	186.	185.	182.	178.	175.	171.	168.	167.	166.	164.	*	*	*	*	*	*	*	*	*	*
*	*	182.	179.	175.	172.	167.	160.	157.	159.	160.	158.	159.	*	*	*	*	*	*	*	*
*	*	*	176.	172.	169.	164.	157.	148.	153.	153.	155.	156.	158.	*	*	*	*	*	*	*
*	*	*	*	170.	167.	162.	156.	150.	150.	147.	149.	153.	156.	158.	155.	151.	*	*	*	*
*	*	*	*	*	*	159.	155.	152.	150.	147.	141.	149.	156.	156.	152.	148.	145.	142.	*	*
*	*	*	*	*	*	*	153.	151.	149.	147.	144.	145.	144.	145.	144.	142.	141.	139.	137.	*
*	*	*	*	*	*	151.	151.	149.	148.	146.	144.	141.	134.	137.	136.	136.	136.	135.	134.	*
*	*	*	*	*	*	151.	150.	148.	146.	144.	143.	140.	136.	130.	126.	130.	131.	132.	132.	130.
*	*	*	*	*	*	*	*	144.	144.	143.	141.	139.	137.	123.	124.	126.	128.	128.	128.	127.
*	*	*	*	*	*	*	*	142.	141.	140.	139.	137.	133.	127.	123.	123.	125.	125.	124.	*
*	*	*	*	*	*	*	*	139.	138.	137.	137.	135.	132.	127.	124.	122.	124.	121.	*	*
*	*	*	*	*	*	*	*	136.	135.	134.	133.	131.	130.	127.	124.	121.	120.	118.	*	*
*	*	*	*	*	*	*	*	133.	132.	131.	129.	126.	126.	124.	121.	118.	118.	116.	*	*
*	*	*	*	*	*	*	*	130.	130.	128.	126.	122.	121.	121.	119.	117.	117.	114.	*	*
*	*	*	*	*	*	*	*	127.	126.	125.	122.	119.	118.	117.	116.	114.	114.	113.	*	*
*	*	*	*	*	*	*	*	124.	123.	121.	119.	116.	115.	113.	112.	111.	111.	111.	111.	*
*	*	*	*	*	*	*	*	120.	119.	118.	116.	113.	111.	108.	108.	105.	108.	109.	110.	*
*	*	*	*	*	*	*	*	117.	116.	114.	112.	110.	105.	98.	104.	101.	105.	108.	109.	*
*	*	*	*	*	*	*	113.	114.	113.	112.	110.	107.	103.	100.	103.	101.	102.	106.	108.	*
*	*	*	*	*	*	108.	110.	111.	111.	109.	107.	104.	97.	97.	102.	99.	96.	104.	108.	*
*	*	*	*	*	101.	106.	108.	107.	108.	107.	105.	102.	97.	92.	110.	96.	94.	102.	108.	*
*	*	*	*	*	97.	103.	106.	106.	106.	105.	103.	100.	96.	92.	106.	97.	92.	102.	108.	112.
*	*	*	*	*	97.	102.	104.	104.	104.	102.	101.	98.	95.	92.	101.	94.	97.	104.	109.	*
*	*	*	*	93.	97.	101.	102.	102.	101.	100.	99.	97.	95.	95.	98.	96.	99.	104.	*	*
*	*	*	*	92.	96.	99.	101.	101.	99.	97.	97.	95.	93.	92.	93.	95.	99.	103.	*	*
*	*	*	*	90.	95.	98.	100.	99.	97.	93.	92.	91.	90.	88.	85.	92.	97.	103.	*	*
*	*	*	*	*	93.	97.	99.	98.	96.	94.	91.	88.	87.	87.	88.	90.	94.	*	*	*

Figure 5.10 Optimal Potentiometric Level for Scenario 21 (ft)

objectives:

i) Maximizing sustained yield groundwater pumping (Model 1);

ii) Maximizing maintenance of current potentiometric surface elevations (Model 2).

For Model 1, the most acceptable strategy (Scenario 14) assumed that:

a) the estimated average annual groundwater flow to the Louisiana portion of the aquifer should be maintained.

b) the upper bounds on pumping in each internal cell should be 1982 values. (The spatial distribution of the resulting optimal pumping strategy closely conforms to the historically observed values.)

c) the maximum annual values of recharge to the aquifer from the internal streams that were estimated for 1973-1983 should be used as upper bounds on stream/aquifer recharge.

The sustainable pumping from the Quaternary aquifer under lying the Boeuf-Tensas basin for this scenario is 116,000 ac-ft/year. This value is within 18 percent of the estimated pumping in 1982 in this region, 140,300 ac-ft.

A greater, and quantitatively satisfactory, sustained yield can be obtained if one uses the same constraints as those of Scenario 14, with the exception that the upper bounds on pumping in each internal cell are the 1980 pumping values. The result of this scenario, number 15, is an annual sustained yield of 172,800 ac-ft/year for the Boeuf-Tensas basin. This is about 110 percent of this area's estimated average annual pumping. Unfortunately, to achieve this yield, pumping is concentrated near recharge sources. Assuming that our estimate of the spatial distribution of historic pumping reasonably accurate, the redistribution of pumping locations that would be required by adopting this scenario is probably not politically desirable.

The other most acceptable strategy for Model 1 (Scenario 17) had the same constraints as those of Scenario 14 except that: the average estimated stream/aquifer recharges for 1973-83 were used as the upper bounds for the three internal rivers. The sustainable pumping for Scenario 17, 55,800 ac-ft/year for the Boeuf-Tensas basin region, is naturally lower than that for Scenario 14. Scenario 17 may represent overly restrictive conditions, since groundwater levels have been declining, and hence recharge from the streams is likely to increase with time.

The same assumptions as were used for Scenario 14 in Model 1 were applied to Model 2, Scenario 20. The resulting sustainable annual pumping for this strategy is 50,400 ac-ft/year. Because this value of sustainable pumping is only about 30 percent of the average historic (1972-82) pumping, in Scenario 21 the upper bound on cell-by-cell pumping was increased to potential need in each cell, while retaining all other constraints the same as those of Scenario 20. The total value of sustainable pumping for Scenario 21, 54,600 ac-ft/year, is only slightly different from that of Scenario 20. For both of these scenarios used for Model 2, the allowable total pumpings for the Boeuf Tensas Basin are only about 30 percent of the estimated annual pumping in 1982. Therefore, it might be extremely difficult to implement a pumping strategy based on the objective of Model 2.

No doubt, the choice of a single strategy from a set of alternative strategies requires analysis of social, political and economic consequences. However, if the sole criterion for implementing an optimal strategy is the maintainance of the potentiometric elevations as close as possible to the current levels, then the regional withdrawal policy must be based on the solution of Model 2. On the other hand, if the goal for implementing a regional pumping strategy is to maximize sustainable groundwater pumping, a sustainable yield pumping strategy for this area should be based on the solution of Model 1, preferably with the constraints of Scenario 14.

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APPENDIX A

Procedure to Estimate 1981
Agricultural Pumping in Cell M, County A
(Peralta et. al., 1983)

ACRE (M) = the agricultural acreage in cell M in 1977 (ac)

TAGAC (A) = the total agricultural acreage in county A within the study area in 1977 (ac)

RAGA (A,81) = the rice acreage in county A within the study area in year 1981 (ac)

SAGA (A,81) = the soybean acreage in county A within the study area in year 1981 (ac)

CAGA (A,81) = the cotton acreage in county A within the study area in year 1981 (ac)

RIR (81) = irrigation water used for rice irrigation in 1981 (ft)

SIR (81) = irrigation water used for soybean irrigation in 1981 (ft)

CIR (81) = irrigation water used for cotton irrigation in 1981 (ft)

QUAT (A) = the percent of the county A's irrigation water which is drawn from the Quaternary aquifer

Z (A,81) = RAGA (A,81) * RIR (81) + SAGA (A,81) * SIR(81) + CAGA (A,81) * CIR (81)

Z (A,81) = total water used for rice, soybean and cotton irrigation in county A in year 1981 (ac-ft)

AGPUMP (M,81)= Z (A,81) * (ACRE(M) / TAGAC(A)) * QUAT (A)

AGPUMP (M,81) = the volume of the water used for rice, soybean and cotton irrigation in cell M year 1981 (ac-ft) which is pumped from the Quaternary aquifer

APPENDIX B

SUBROUTINES USED IN SSTARS

Subroutine MAIN

The main function of this subroutine is to direct the execution to other subroutines. This subroutine also initializes the input unit, IN, and the two output units, ITOUT and IMAP. This subroutine directly calls subroutines READRO, TSAVG, COEF3, COEF1, SWCON, LPMIN, TARGET, INFOUT, SENSE and CHECK. Additional subroutines describing other objective functions may be called from subroutine MAIN any time after the call to subroutine COEF1.

Subroutine READRO

This subroutine reads in data from the main data file from unit IN. Some variables and arrays are initialized for subsequent subroutines including QPTHOR. From the input data, this subroutine calculates the initial saturated thickness, the initial drawdowns, and the midpoint transmissivities. If IW=1, the following information is output to unit IMAP: the cell numbering system, the initial potentiometric surface elevations, the initial drawdowns, the upper bound on drawdown, the initial saturated thickness, the minimum allowable recharge, and the upper and lower limits on groundwater pumping. The subroutines directly called from subroutine READRO include NUMBER and MAP.

Subroutine NUMBER

This subroutine assigns a one-dimensional integral value to identify each finite difference cell located by (I,J) coordinates. The set of cells is defined by a cartesian coordinate system with the origin located at I=Ø, J=Ø. An integer designation is assigned separately to each variable-head cell and each constant-head cell. The variable-head cells are numbered beginning with the cell in column one which is in the row with the smallest J-value. Sequential numbering continues in the vertical direction until reaching the last row in column one. The next number is assigned to the cell in column two which is in the row with the smallest J-value. This pattern continues until all NVAR variable cells have been numbered. The constant-head cells are similarly numbered beginning with one and ending with NCH.

Subroutine TSAVG

From the midpoint transmissivity values determined by subroutine READRO, this subroutine calculates the five-point finite difference transmissivities. A geometric averaging method is used to determine the average transmissivity between each finite difference cell and the cells immediately adjacent to it in the positive I-direction and the positive J-direction. The transmissivity values are in units of (square feet per year)*E-Ø6. Values are truncated one place after the decimal point.

Subroutine MAP

This subroutine takes the data in the array which is passed to it, and outputs this data on unit IMAP in a map format. The data in the argument array must be identified by (I,J) coordinates.

Subroutine COEF3

This subroutine formulates the constraints imposed on recharge in every constant-head cell in which ISW is equal to zero. This set of KCH constraints limit the recharge in constant-head cells such that it is greater than ACCRN. The upper limit on recharge is imposed in subroutine LPMIN or TARGET (whichever is called) in the form of an upper bound on the slack variable associated with inequality constraints (NVAR+1) to (NVAR+KCH). Subroutine COEF3 also formulates a constraint for every constant-head cell subsystem. There are NCHSUB additional constraints. The lower limit on total recharge in each constant-head subsystem is set equal to CHSMIN. The upper limit is imposed in subroutine LPMIN (or TARGET) in the form of an upper bound on the slack variable associated with inequality constraints (NVAR+KCH+1) to (NVAR+KCH+NCHSUB).

Subroutine COEF1

The finite difference transmissivity values are utilized in this subroutine to determine the coefficients and the right hand side of the linear constraints on pumping in all variable cells. The constraint limiting groundwater pumping to be greater than PMIN is formulated. The upper limit on groundwater pumping is imposed in subroutine LPMIN (or TARGET) in the form of an upper bound on the slack variable associated with the first NVAR inequality constraints.

<u>Subroutine SWCON</u>

This subroutine formulates the final NSUB inequality constraints. A constraint for each stream/aquifer subsystem is developed limiting the sum of the stream/aquifer response to be greater than or equal to SWMIN and less than or equal to SWMAX. The index ISA indicates to which subsystem a stream/aquifer cell belongs.

<u>Subroutine LPMIN</u>

In this subroutine, the linear objective function to maximize total regional groundwater pumping is formulated and submitted to QPTHOR. Upper and lower limits on drawdown, are applied to the first NVAR variables. Limits on the slack variable associated with the pumping constraints are imposed on the next NVAR variables. The upper and lower limit on the slack variables associated with the recharge constraints are applied to the next KCH variables. The limits on the recharge in the constant-head subsystems are applied to the next NCHSUB variables. Finally, the limits on stream/aquifer interaction in each stream/aquifer subsystem are applied to the last NSUB variables. This subroutine is directly called from the main program when the index ISUS = Ø. The subroutines directly called from subroutine LPMIN include subroutine GSIMEQ, and QPTHOR.

Subroutine GSIMEQ

The function of this subroutine is to develop an initial feasible solution from which the optimization process will begin. A Gauss-Sidel iterative method is used to solve the set of simultaneous equations to compute the drawdown values for which the groundwater pumping in all cells is equal to PMIN. If the lower limit on pumping in all cells is zero, this strategy represents an unstressed aquifer condition. From the initial set of drawdown values, the recharge necessary to support minimum groundwater reguirements is computed and compared to the imposed recharge constraints. If the maximum amount of recharge at any constant-head cell is exceeded, a message is output to unit IOUT.

Subroutine QPTHOR

This subroutine optimizes the objective function formulated by subroutine LPMIN (or TARGET) under the constraints defined by subroutines COEF1 and COEF3. It has the capability of optimizing both the linear and quadratic objective functions. This subroutine is a slightly modified version of the QPTHOR written by Leifsson and others (1981). A user's manual of the unmodified program may be purchased from H. J. Morel-Seytoux, Civil Eng. Dept., Colorado State University, Ft. Collins, CO 80523. QPTHOR is not to be extracted from SSTAR5 and used for other purposes without the permission of H. J. Morel-Seytoux.

Subroutine INFOUT

This subroutine outputs the results from subroutine QPTHOR in a map format on output unit IMAP. The information output from this subroutine includes: drawdowns, elevations, groundwater pumping, recharge, total excitation, the percent of maximum recharge used, stream/aquifer response, and optimal saturated thickness.

Subroutine SENSE

This subroutine writes out the constrained derivatives associated with each decision variable to output unit ITOUT. The constrained derivatives indicate the change in the value of the objective function due to a change in the value of a single decision variable. The information from this subroutine is useful in determining the effect of relaxing a constraining condition on drawdown, pumping, or recharge.

Subroutine TARGET

This subroutine formulates a quadratic goal-programming objective function to create a set of optimized potentiometric levels so that the deviations of the latter set from the input elevations (current or any given set of 'target' elevations), are regionally at a minimum. Limits are imposed on variables in the same way as in subroutine LPMIN. Subroutine QPTHOR is called for optimization. Subroutine INFOUT is called subsequently, to calculate the steady-state pumping values that correspond to this optimized set i.e., sustained yield strategy for the optimized elevations.

If this subroutine is being called, an input data file containing the weighting factors (as described under methodology) must be provided. The description of this input file is given in the next section.

This subroutine is called directly from the main program if the value of the index ISUS on card C is set to one of its values other than Ø (i.e., 1, 2 or 3). In other words, if ISUS equals Ø, this subroutine is not executed (see Input Data to SSTAR5). Under specific options, calls are also made by this subroutine to subroutines LPMIN, GSIMEQ and DETERM.

Subroutine CHECK

This subroutine is included to print out, on unit ITOUT, the optimum value of all variables, including original and slack variables, and their corresponding lower and upper bounds. The combination of feasibility and optimality conditions dictates that all the optimal values must be within, or at one of, their bounds. This print out is provided, however, to indicate any remotely probable instance when the condition may be violated due computational inaccuracy encountered in extermely large size problems. In such a case, the message 'Violated' is printed on the same line as the violating variable and its bounds.

The output from this subroutine is also provided for a practical purpose. That is, a computer file containing the optimal values may also be saved if future use of these values as initial feasible solution to a subsequent optimization is envisaged.

The optional call to this subroutine is made from the main program only if the index ICHK = 1, on card C.

Subroutine DETERM

Subroutine QPTHOR is designed for minimization of convex quadratic functions. In other words, global optimality of a solution is assured only when the function is convex (concave in the case of maximization). The necessary and sufficient condition for convexity is that the coefficient matrix of quadratic terms, called the Hessian matrix, must be positive definite. A matrix is positive definite if the determinants of all minors are greater than zero. The coefficient matrix of the objective function introduced here is a diagonal matrix whose diagonal elements are reciprocals of squares of the weighting factors. Therefore they are all nonzero and positive. This guarantees the positive definiteness of this matrix. However, the present subroutine is included as a capability for the management model to check the condition of any Hessian matrix that may arise by using different quadratic objective functions. The subroutine first transforms the Hessian matrix to a lower triangular matrix. It then examines whether all diagonal elements are positive. The result of this test is printed on unit ITOUT as a message stating whether the coefficient matrix is or is not positive definite.

INPUT DATA FOR SSTAR5

These are the definitions and format for the input data to SSTAR5.

Card A. Format (F10.8,F10.6)

Field 1: Hydraulic conductivity in feet per year ~~*10~~ ?

Field 2: Computational accuracy criterion (ACC). The suggested value for ACC for optimization of both objective functions is 0.002.

Card B. Format(4I5)

Field 1: ISTART: Minimum I-coordinate associated with a finite-difference cell.
Field 2: IMAX: Maximum I-coordinate associated with a finite-difference cell.
Field 3: JMAX: Maximum J-coordinate associated with a finite-difference cell.
Field 4: ICELL: Total number of cells in the study area.

Card C. Format (5I5)

Field 1: IRCH: Index defining whether recharge will be constrained. If IRCH = 0 no recharge constraints are recognized. If IRCH = 1 there are recharge constraints.

Field 2: ITER: Index defining whether optimal results will be used as initial conditions for purposes of sequential optimization If ITER = 0 initial conditions are read from main data file and results are not saved on FT23F001. If ITER = 1 initial conditions are read from main data file and results saved on FT23F001. If ITER = 2, input is read from FT23F001 and results are not saved on FT23F001. If ITER = 3 input is read from FT23F001 and results saved on FT23F001.

Field 3: ISUS: Index defining whether the subroutine TARGET is to be called. If ISUS = 0 no call is made. If ISUS equals 1, 2 or 3 this subroutine is called. Whenever this subroutine is called, an input file containing the cell co-ordinates and the weighting factors for all cells in the region must be provided under FORTRAN UNIT 11. The input format for this file is (2I5, F10.3). The three fields correspond to the cell co-ordinates I,J and the reciprocals of the weighting factors respectively. The reciprocals are needed because subroutine TARGET has been developed to use the inverse of the standard deviations from kriging as the weighting factors and these standard deviations were directly input to the program. Therefore, if any other weighting factor is to be used, its reciprocal must be input.

The three values of index ISUS determine the source of the initial feasible solution (IFS) to be used by Subroutine QPTHOR for optimization:

If ISUS = 1, subroutine LPMIN is called by TARGET, and the solution found by that subroutine is transferred as an IFS to QPTHOR.

If ISUS = 2, an IFS must be provided as an input file on unit 25. The file must have the initial feasible values for each of the original variables (variables in the objective function) in continuous sequence from variable number one to number N, according to the variable numbering system explained in a following section of this report. The format for this file is (15X,F15.7).

If ISUS = 3, subroutine GSIMEQ is called to generate the IFS.

For a large size aquifer system, the numerical difficulties inherent in optimization of quadratic objective functions require the fairly careful selection of an IFS and an accuracy criterion. It has been observed that for the Boeuf-Tensas Basin, which is a fairly unstressed system, an IFS generated by subroutine GSIMEQ (ISUS = 3) and an ACC = Ø.ØØ2 are most appropriate. Our experience has been that when using subroutine TARGET an IFS that is close to target levels will cause a more rapid convergence and a satisfactory set of water levels than an IFS that is far from the target elevations.

For large size problems, subroutine TARGET may take a considerable amount of computer time even for a fairly small number of iterations. For the Boeuf-Tensas Basin, a problem with 66Ø variables and 346 constraints, a computer run required 1661 seconds of CPU time to perform 68 iterations. The most important single factor determining the computer run time is, of course, the size of the problem. Run time increases exponentially with increase in size. Accuracy is another factor that affects execution time. More accurate solution is normally associated with more iterations and, thus, with more CPU time. Therefore, the model user must decide whether he can afford more iterations required by a smaller ACC in order to achieve a more satisfactory solution.

Field 4: ICHK; Index defining whether subroutine CHECK is called. If ICHK = 1, the subroutine is called; if it is zero no call is made.

Field 5: IPDM; Index defining whether subroutine DETERM is called. The subroutine is called for IPDM = 1; for IPDM = Ø, no call is made. If ISUS = Ø, IPDM should be zero too.

Card D. Format (3I5)

Field 1: ISP; Index defining whether sensitivity analysis on the optimization results is performed. If ISP = Ø no sensitivity analysis is performed. If ISP = 1 sensitivity analysis of optimal solution is performed by subroutine SENSE.

Field 2: IWP; Index defining whether results of optimization are output in map format. If IWP = Ø no map output of optimal solution is provided. If IWP = 1 results are output in map format.

Field 3: IW; Index defining whether any input data is output in map format. If IW = Ø, no input data is output in map format. If IW = 1, selected input data is output in map format.

Card E. Format(I5)

Field 1: NSUB; Total number of stream/aquifer subsystems. If NSUB = Ø skip to Card G.

Card(s) F. Format(F1Ø.8) (optional)

Field 1: SWMIN(1): The lower limit on interflow, in acre feet per year, from the stream in stream/aquifer subsystem 1 to the aquifer.

Continue with the next F-Card for stream/aquifer subsystem 2. The total number of F-Cards is equal to NSUB, (Card E, Field 1).

Card(s) G. Format (2I5)

Field 1: JSTART(1): The smallest, lower-most J-coordinate in the left-most column, column 1.

Field 2: JEND(1): The largest, upper-most J-coordinate in column 1.

Continue with the next G-Card for I=2 (column 2) with the smallest and largest J-coordinate in column 2. Then a G-Card for column 3.... The total number of G-Cards should equal IMAX (Card B, Field 2).

Card H. Format(2I5)

Field 1: NCH; The total number of constant-head cells. If NCH = Ø then skip to Card K.

Field 2: NCHSUB; The total number of constant-head cell subsystems.

Card(s) I. Format(3I5) (optional)

Field 1: ICH(1); The I-coordinate of the first constant-head cell.

Field 2: JCH(1); The J-coordinate of the first constant-head cell.

Field 3: ICF(1); An index defining whether a constant-head cell is part of a constant-head cell subsystem. If ICF = Ø, the cell is constrained alone. If ICF is greater than zero, ICF indicates the constant-head subsystem to which the constant-head cell belongs. The largest value of ICF is NCHSUB.

Continue with I-Cards until all constant-head cells have been located. The total number of I-Cards equals NCH (Card H, Field 1).

If NCHSUB = Ø, (Card H, Field 2), skip to Card K.

Card(s) J. Format (F1Ø.8) (optional)

Field 1: CHSMIN(1); The lower limit on total recharge in constant-head subsystem 1.

Continue with the next J-Card for constant-head subsystem 2. The total number of J-Cards is equal to NCHSUB (Card H, Field 2)

Card K. Format (I5)

Field 1: NSA; Total number of stream/aquifer cells. If NSA = Ø then skip to Card M.

Card(s) L. Format (3I5,F1Ø.2,E12.6,F1Ø.2) (optional)

Field 1: I2; I-coordinate of first stream/aquifer cell.

Field 2: J2; J-coordinate of first Stream/Aquifer cell.

Field 3: ISA(1); An index defining whether cell (I2,J2) is in a stream/aquifer subsystem. The integral value of ISA indicates the subsystem to which the stream/aquifer cell belongs. The largest value of ISA is NSUB. If ISA is equal to Ø, cell (I2,J2) does not belong to a stream/aquifer subsystem and is constrained by STMAX, (Field 6).

Field 4: XSST; The elevation of the water in the stream at cell (I2,J2).

Field 5: XAPS; The reach transmissivity at cell (I2,J2) in units of square feet per year.

Field 6: STMAX: The minimum allowable interflow between the stream and the aquifer at cell (I2,J2) in acre-feet per year. Use appropriate sign convention. If STMAX is equal to Ø, the cell is constrained only as part of a stream/aquifer subsystem. If STMAX is not equal to zero, the drawdown in cell (I2,J2) is limited such that STMAX is not exceeded.

Continue with L-Cards until all stream/aquifer cells have been located. The total number of L-Cards should equal NSA (Card K, Field 1).

Card(s) M. Format(1X,I2,1X,2I2,8F9.2)

Field 1: I2; The I-coordinate of any cell in the study area.

Field 2: J2; The corresponding J-coordinate of the cell partially identified by field 1.

Field 3: ISW; An index defining which cells will be used in the maximization of selected groundwater withdrawal. If ISW = 1, the cell will be included in the linear optimization. If ISW = Ø, the cell will be excluded from the linear optimization. If ISW = Ø for a constant-head cell, the constant-head cell is constrained by XACN and XACX (Field 8 and Field 9) If ISW = 1 for a constant-head cell, the constant-head cell is constrained only as part of a constant-head cell subsystem.

Field 4: XELEV; The initial (current or target) elevation of the potentiometric surface at cell (I2,J2) in feet.

Field 5: XTOP; The elevation of the top of the aquifer at cell (I2,J2) in feet.

Field 6: XBOT; The elevation of the base of the aquifer at cell (I2,J2) in feet.

Field 7: XSATM; The minimum acceptable saturated thickness in cell (I2,J2) in feet.

Field 8: XACN; The minimum acceptable recharge (maximum flux from outside the system) in cell (I2,J2) in acre-feet per year. If cell (I2,J2) is a variable-head cell, this value is considered a constant.

Field 9: XACX; The maximum allowable recharge in cell (I2,J2)in acre-feet per year. If cell (I2,J2) is a variable-head cell, this value is ignored.

Field 1Ø: XPMIN; The minimum acceptable groudwater pumping in cell (I2,J2) in acre-feet per year. If cell (I2,J2) is a constant-head cell, this value is considered a constant.

Field 11: XPMAX: The maximum allowable groundwater pumping in cell (I2,J2) in acre-feet per year. If cell (I2,J2) is a constant-head cell, this value is ignored.

Continue with M-Cards until all cells have been assigned these characteristics. The total number of M-Cards must equal ICELL (Card B, Field 4). The order is not important.

MATRIX DIMENSIONS

To modify SSTAR5 to execute for any given study area, the following matrix dimensions must be changed based on the characteristics of the region. The matrix modifications listed for QPTHOR are from the QPTHOR User's Manual (Leifsson and others, 1981).

```
 COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(IMAX),JEND(IMAX)

 COMMON/BUNCH 3/NCELL(IMAX,JMAX),NCHN(IMAX,JMAX),ICH(NCH),JCH(NCH),
*ICF(NCH),ISW(ICELL)

 COMMON/CHUNK 1/DTR(IMAX,JMAX),DTU(IMAX,JMAX),T(IMAX,JMAX)

 COMMON/CHUNK 2/ACCRX(IMAX,JMAX),ACCRN(IMAX,JMAX),PMAX(IMAX,JMAX),
*PMIN(IMAX,JMAX),XM(IMAX,JMAX),SI(IMAX,JMAX)

 COMMON/BLOCK 1/CA(NVAR+K),AA(K,NVAR+K),R(K),B(K,K),D(K,NVAR),
*V(NVAR)

 COMMON/BLOCK 2/X(NVAR+K),ITYPE(NVAR+K),XO(NVAR+K),XU(NVAR+K),XL(NVAR+K)

 COMMON/BLOCK 3/NS(K),ND(NVAR),NN(NVAR+K)

 COMMON/OPOUT/QG(IMAX,JMAX),SSOP(IMAX,JMAX)

 COMMON/SWC 1/NSUB,SWMIN(NSUB),SWMAX(NSUB),CHSMIN(NCHSUB),CHSMAX(NCHSUB)

 COMMON/AQUIF/TOP(ICELL),BOT(ICELL)

 COMMON/STAQ/SST(IMAX,JMAX),APS(IMAX,JMAX),ISA(ICELL)

 COMMON/QUA 1/Q(NVAR,NVAR)

 COMMON/AAA/TAR(IMAX,JMAX)

 COMMON/YAZ/SD(IMAX,JMAX), S2(ICELL)

 DIMENSION DATA(IMAX,JMAX)

 DIMENSION SCR(IMAX,JMAX)

 DIMENSION CH(IMAX,JMAX)

 DIMENSION FACTOR(NVAR)

 DIMENSION TV(NVAR),SCR(NVAR),SCR2(VAR)
```

```
DIMENSION SS(NVAR),SCR(IMAX,JMAX),SCR2(IMAX,JMAX),SCR3(IMAX,JMAX)

DIMENSION RT(NSUB+NCHSUB)

DIMENSION Z(NVAR+K)
```

The following definitions apply to the above variable dimensions.

IMAX = the maximum number of columns in the finite difference cell system.

JMAX = the maximum number of rows in the finite difference cell system.

NCH = the total number of constant-head cells in the study area.

NVAR = the total number of variable-head cells in the study area.

ICELL = the total number of finite difference cells in the study area. ICELL = NCH + NVAR.

K = the total number of constraints in the problem: K = NVAR + KCH + NCHSUB + NSUB.

KCH = the total number of individually constrained constant-head cells.

NCHSUB = the total number of constant-head subsystems.

NSUB = the total number of stream/aquifer subsystems.

DEVICE INPUT UNITS FOR SSTAR5

The following is a listing of the computing device units used by the SSTAR5 water management model to read in data:

Unit IN

The main input, described under INPUT DATA FORMAT FOR SSTAR5, should be provided on this unit. IN is currently assigned to unit 4.

Unit 11

If the index ISUS (Card C, Field 3) is either 1, 2 or 3, the input file containing the weighting factors must be provided on this unit. Format and order for this input file are given under Field 3 of Card C (page B-9).

Unit 23

This unit is used if values of ITER (Card C, Field 2) are other than zero. The unit may be either an input or an output unit or both, depending on the value assigned to ITER. The output file created on this unit is 80 column and the same file is used as input in a subsequent run. Therefore, the format ~~an~~ order need not concern the user.

Unit 25

If ISUS = 2, the file containing initial feasible solution must be provided on this unit. Format and order for this file are given under Field 3 of Card C for the option ISUS = 2 (page B-10).

OUTPUT FROM SSTAR5

The output from the SSTAR5 water management model appears on three separate output files. General output information, including any sensitivity analysis, is less than 80 columns in width and is directed to unit ITOUT. Output and results from the optimization subroutine QPTHOR are found on Unit 6, where 132 column record length is needed. Any requested map output is directed to unit IMAP, also of 132 column record length. Each of these output files are described in detail for a typical problem.

Unit ITOUT

This output listing indicates the number of individually constrained constant head cells, KCH. The number of constant-head subsystems, NCHSUB, and stream/aquifer subsystems, NSUB, are also noted in this file. The total number of constraints, K, is equal to NVAR + KCH + NCHSUB + NSUB, where NVAR is the total number of variable-head cells.

If IPDM = 1 and ISUS is not equal zero, there will be a two line message at this point in the output file that indicates whether the coefficient matrix of the quadratic terms is positive definite or not.

If ISUS is not zero, the next line in the output file indicates the constant value of weighted sum of squares of the initially input (current or 'target') drawdowns in variable-head cells . This numerical value is the constant term in the quadratic objective function (Model 2) described in chapter four.

If ISUS is not 2, the following part of the output file is a note indicating the number of iterations used in subroutine GSIMEQ, and whether the total number of iterations, NIT, has been exceeded. Subroutine GSIMEQ uses the Gauss-Sidell iterative process to calculate the drawdown values which correspond to the lower limit on groundwater pumping in all cells. (Further description of this subroutine appears in the Description of Subroutines used in SSTARS.) Because this is an iterative method of simultaneous equation solution, an accuracy criteria is employed for determining when the process should be stopped.

The accuracy criteria used in subroutine GSIMEQ is the same criteria used in QPTHOR for determining whether any constraints have been violated by the initial feasible solution. Therefore, a solution created by GSIMEQ within the maximum number of iterations, is automatically accepted by QPTHOR. However, if the maximum number of iterations is exceeded before the accuracy criteria is satisfied, the initial feasible solution may be rejected by QPTHOR.

The accuracy criteria in both GSIMEQ and QPTHOR is a function of ACC (Card A, Field 2). The values of ACC in the range 0.001 - 0.002 have been satisfactory for most of the developmental simulations. The maximum number of iterations in GSIMEQ is defined by the integer NIT and has been internally set to 100. If this is exceeded and the solution rejected by QPTHOR it may be necessary to increase NIT. Under certain input conditions, such as a large value of reach transmissivity (above the order of E+09 ft^2/year), a degenerate situation may arise in which convergence in subroutine GSIMEQ is not guaranteed. If such a situation occurs, the accuracy term, ACC may have to be increased.

After a solution has been calculated in subroutine GSIMEQ, the corresponding recharge in the constant-head cells is calculated. This calculated recharge is that which supports the initial feasible solution. If the initial recharge in a constant-head cell is less than the input lower limit on recharge in that cell, the lower limit is decreased and set equal to the initial recharge. When this is necessary, a message is output to unit ITOUT indicating the magnitude of the change. When a recharge constraint is tight at the initial feasible solution, and the initial feasible solution represents minimum allowable groundwater pumping, further optimization is severely restricted.

In the specific case when ISUS = 2 (initial feasible solution supplied by the user through an input file), no call is made to subroutine GSIMEQ. In this case obviously no output from that subroutine described above, will be produced.

If ICHK = 1, the next portion of the output file is a print out of the final optimal values for all variables (including slack variables) and their bounds. If ICHK equals Ø, this output is not produced.

If no sensitivity analysis is performed (ISP = Ø), no additional output to unit ITOUT exists. If a sensitivity analysis is requested (ISP = 1), the results are directed to unit ITOUT. The results of the sensitivity analysis indicate the relationship of each decision variable to the value of the objective function. This is useful in determining the effect on the objective function due to a change in a limiting constraint.

The output listing contains three lines for each decision variable. A decision variable is any variable which is tight against either its lower bound or its upper bound at the optimal solution. The total number of decision variables is equal to NVAR.

The first line indicates the type of variable and its current value. The cell numbering system is that as discussed previously. The variable may represent drawdown or pumping in a variable-head cell, recharge in a constant-head cell, total recharge in a constant-head subsystem, or total interflow in a stream/aquifer subsystem. If the variable is representative of drawdown, the value of the variable is equal to the drawdown at that cell. If however, the variable represents one of the slack variables, pumping, recharge, or total reach response, the given value of the variable is equal to the difference between the flux and the lower limit on that flux. For example, if the value of the variable representing pumping in cell 9 is equal to zero, than the actual value of pumping is computed as P-PMIN=Ø or P=PMIN.

The second output line for each decision variable lists the constrained derivative of the objective function with respect to that particular decision variable, and the maximum change in the value of that decision variable for which the constrained derivative is applicable. The constrained derivative indicates the change in the value of the objective function for a unit change in the value of the decision variable. The maximum change in the decision variable indicates the range within which the constrained derivative applies, assuming that the decision variable is changed in the direction which improves the value of the objective function. A unit change in the decision variable in which the absolute value of the constrained derivative is the largest has the most effect on the value of the objective function.

The third output line for each decision variable indicates the state variable which becomes tight when the decision variable is changed by the maximum amount indicated in the second line. The variable numbering system applied to these state variables is common throughout the output and will be encountered again. For this reason it is now explained in detail.

The total number of variables, including slack variables, is equal to NVAR+K where K is the total number of inequality constraints as defined previously. The total number of variables is also expressed as NVAR + NVAR + KCH + NCHSUB + NSUB. The first NVAR variables are numbered according to the cell numbering system and represent the drawdown in cells 1 to NVAR. The next NVAR variables represent the groundwater pumping in cells 1 to NVAR and are designated by the integers NVAR+1 to 2*NVAR. The next KCH variables are those corresponding to the recharge in individually constrained constant-head cells and are numbered from 2*NVAR+1 to 2*NVAR+KCH. (The individually constrained constant-head cells are located as described in the constant-head cell numbering system.) The following NCHSUB variables are those representing the total recharge in each of the constant-head cell subsystems. These variables are numbered from 2*NVAR+KCH+1 to 2*NVAR+KCH+NCHSUB. The final NSUB variables represent total interflow from the stream/aquifer subsystems and are numbered from 2*NVAR+KCH+NCHSUB+1 to 2*NVAR+KCH+NCHSUB+NSUB.

Unit 6

Unit 6 is designated by QPTHOR to receive the output from the optimization process. This output listing begins by indicating the number of variables in the objective function, NVAR, and the total number of inequality constraints, K. The information that follows is the upper and lower limit on each variable, including slack variables, and the value of each variable used as an initial feasible solution. The value used as an initial feasible solution must be within or equal to the upper or lower bounds. The variable numbering system is that as explained previously.

The initial value of the objective function is indicated followed by the optimal value of all system variables. Recall that the values listed for the slack variables are the differences between the flux and the lower limit on the flux. Finally, the optimal value of the objective function and the number of iterations required is output.

Before performing the optimization, all input values are modified such that all volumes are represented in units of millions of cubic feet. This is done in order that the matrices of coefficients are all of an order of one. The values of variables and limits listed in this output are therefore in units of millions of cubic feet and must be divided by 0.04356 to change them to acre feet.

Unit IMAP

The information on this unit is a listing of some input and output values on a cell by cell basis in map format. If IW = 1, the listing begins with a map of the cell numbering system followed by a map of the constant head cell numbering system. These maps are useful in locating a particular variable by i,j coordinates. Initial input elevations (current or target), corresponding drawdowns, bounds on the variables and initial saturated thicknesses are then printed in a map format.

The next parts of this output file are formulated in subroutine INFOUT and contain two main portions 1) information about the initial feasible solution and 2) information associated with the optimal solution. When ISUS is equal to Ø or 3, that is when subroutine GSIMEQ is called to generate the initial feasible solution, the first portion shows the results associated with drawdowns calculated by that subroutine. The second portion in this case contains the optimal solution either from LPMIN (ISUS = Ø) or from TARGET (ISUS = 3). When ISUS = 2, the first portion contains information from the initial solution supplied by the user and the second portion shows the results of optimization by TARGET. When ISUS = 1, the first portion shows the results from optimization in LPMIN and the second portion contains the results from TARGET.

APPENDIX C

Table C.1

Table C.1 Fractions of Western Boundary Cells Enclosed Within The Boeuf-Tensas Area (to the east of the eastern divide of the Bayou Bartholomew Watershed)

I	J	Fraction of Cell Within Boeuf-Tensas Area
9	7	0.30
10	7	0.20
11	8	1.00
12	8	0.25
13	9	0.30
14	9	0.25
15	10	0.50
16	11	0.50
17	11	0.10
18	12	0.50
19	13	0.25
20	13	0.30
21	13	0.30
22	13	0.25
23	13	0.30
24	13	0.30
25	13	0.30
26	13	0.40
27	13	0.70
28	13	0.90
29	13	0.85
30	12	0.40
31	12	0.70
32	12	0.85
33	12	0.90
34	11	0.25
35	11	0.25

APPENDIX D

LISTING OF PROGRAM SSTARS

```
C ***********************************************************
C *          SSTAR5 WATER MANAGEMENT MODEL:                  *
C *               MAXIMUM PUMPING AND                        *
C *      QUADRATIC GOAL-PROGRAMMING OBJECTIVE FUNCTIONS      *
C *          DEVELOPED : UNDER THE DIRECTION OF              *
C *        R.C. PERALTA, BY THE 1982-1985 STAFF OF           *
C *         WATER RESOURCES MANAGEMENT LABORATORY            *
C *          AGRICULTURAL ENGINEERING DEPARTMENT             *
C *       UNIV. OF ARKANSAS, FAYETTEVILLE AR, 72701          *
C *                 (501)  575-2351                          *
C *        UNDER FUNDING BY THE CORPS OF ENGINEERS           *
C *         AND THE AGRICULTURAL ENGINEERING DEPT.           *
C ***********************************************************
C *                                                          *
C *NOTE: SUBROUTINE QPTHOR IS A SLIGHTLY MODIFIED VERSION OF *
C *      THE PROGRAM DEVELOPED BY VERDIN ET AL, 1981. QPTHOR *
C *      IS NOT TO BE EXTRACTED FROM THIS PROGRAM FOR SEPRATE*
C *      USE WITHOUT PERMISSION FROM H.J. MOREL-SEYTOUX,     *
C *      DEPT, OF CIVIL ENG., COLORADO STATE, FORT COLLINS   *
C *      CO, 80523.                                          *
C *                                                          *
C ***********************************************************
C *
C *  FOR ARRAY MODIFICATIONS CHANGE THE FOLLOWING:
C *   (IMAX) : MAXIMUM NUMBER OF COLUMNS
C *   (VAR)  : TOTAL NUMBER OF VARIABLE CELLS
C *   (NKT)  : TOTAL NUMBER OF CONSTRAINTS
C *   (NPK)  : (VAR) + (NKT)
C *   (IM,JM): IMAX, JMAX
C *   (NCH)  : TOTAL NUMBER OF CH CELLS
C *   (NC)   : NUMBER OF CH SUBSYSTEMS
C *   (NCL)  : TOTAL NUMBER OF CELLS
C *   (NS)   : NUMBER OF S/A SUBSYSTEMS.
C *   (NS+NC)   :  THE TOTAL NUMBER OF SUBSYSTEMS.
C *   (NKT,NPK) :
C *   (NKT,VAR) :
C *   (NKT,NKT) :
C *   (VAR,VAR) :
C ******************************************************
C *  MAIN PROGRAM: DIRECTS ACTION TO OTHER SUBROUTINES *
C ******************************************************
```

```
      SUBROUTINE NUMBER
C ***********************************************************************
C *  ASSIGNS A INTEGRAL VALUE TO EACH I,J, COORDINATE NCELL(I,J)=1,NVAR*
C *  CONSTANT HEAD CELLS HAVE A NCELL(I,J) = Ø. AND A NCHN(I,J)=1,NCH  *
C *  NCHN(I,J)= Ø FOR ALL NON- CONSTANT HEAD CELLS.                    *
C ***********************************************************************
C
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/ONE/ICELL,NVAR,NCH,NGA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
C
      NUMCH=Ø
      NUMC=Ø
      DO 1ØØ I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 1ØØ J=JBEGIN,JSTOP
      DO 2ØØ L=1,NCH
       IF(ICH(L).EQ.I.AND.JCH(L).EQ.J)GOTO 3ØØ
2ØØ   CONTINUE
      NUMC=NUMC+1
      NCELL(I,J)=NUMC
      NCHN(I,J)=Ø
      GOTO 1ØØ
3ØØ   CONTINUE
C  I,J IS A CONSTANT HEAD CELL
      NCELL(I,J)=Ø
      NUMCH=NUMCH+1
      NCHN(I,J)=NUMCH
1ØØ   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TSAVG
C ******************************************************************
C *  CALCULATES FINITE DIFFERENCE TRANSMISSIVITIES USING GEOMETRIC AVE*
C ******************************************************************
C
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
C
      DO 100 I=ISTART,IMAX
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
      DO 100 J=JBEGIN,JSTOP
      IF(I.EQ.IMAX)GOTO 90
      IF(J.GT.JEND(I+1))GOTO 90
      IF(J.LT.JSTART(I+1))GOTO 90
      DTR(I,J)=SQRT(T(I,J)*T(I+1,J))
      GOTO 100
90    DTR(I,J)=0.0
100   CONTINUE
C
      DO 200 I=ISTART,IMAX
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
      DO 200 J=JBEGIN,JSTOP
      IF(J.EQ.JSTOP)GOTO 190
      DTU(I,J)=SQRT(T(I,J)*T(I,J+1))
      GOTO 200
190   DTU(I,J)=0.0
200   CONTINUE
C     DO 150 I=ISTART,IMAX
C       JBEGIN=JSTART(I)
C       JSTOP=JEND(I)
C     DO 150 J=JBEGIN,JSTOP
C       XDTU=DTU(I,J)*10.
C       IDTU=XDTU
C       XDTU=IDTU
C       DTU(I,J)=XDTU/10.
C         XDTR=DTR(I,J)*10.
C         IDTR=XDTR
C         XDTR=IDTR
C         DTR(I,J)=XDTR/10.
150   CONTINUE
C
      DO 300 I=ISTART,IMAX
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
      DO 300 J=JBEGIN,JSTOP
        T(I,J)=0.0
       IF(I.EQ.ISTART)GOTO 220
       IF(J.GT.JEND(I-1))GOTO 220
       IF(J.LT.JSTART(I-1))GOTO 220
        T(I,J)=T(I,J)+DTR(I-1,J)
220    CONTINUE
       IF(I.EQ.IMAX)GOTO 230
       IF(J.LT.JSTART(I+1))GOTO 230
       IF(J.GT.JEND(I+1))GOTO 230
        T(I,J)=T(I,J)+DTR(I,J)
230    CONTINUE
```

```
       IF(J.EQ.JSTOP)GOTO 240
        T(I,J)=T(I,J)+DTU(I,J)
240    CONTINUE
       IF(J.EQ.JBEGIN)GOTO 245
        T(I,J)=T(I,J)+DTU(I,J-1)
245    CONTINUE
300   CONTINUE
C
      ITOUT=7
C      WRITE(ITOUT,2000)
C     DO 400 I=ISTART,IMAX
C      JBEGIN=JSTART(I)
C      JSTOP=JEND(I)
C     DO 400 J=JBEGIN,JSTOP
C      WRITE(ITOUT,1000)I,J,DTR(I,J),DTU(I,J),T(I,J)
400   CONTINUE
1000  FORMAT(1X,I2,1X,I2,2X,3F10.2)
2000  FORMAT(1X,' I   J          DTR         DTU         T')
      RETURN
      END
C
```

```
      SUBROUTINE MAP (DATA)
C ****************************************************
C *  TAKES GIVEN DATA AND WRITES IT OUT IN MAP FORMAT *
C ****************************************************
C
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      DIMENSION DATA(35,22)
C
C     KTOP=JMAX+1
10     CONTINUE
      DO 100 I=ISTART,IMAX
      DO 100 J=1,JMAX
C      IF(IDIFF.GT.JMAX)GOTO 20
      K=KTOP-J
20     CONTINUE
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
C      IF(IDIFF.GT.JMAX)GOTO 30
C     IF(K.LT.JBEGIN.OR.K.GT.JSTOP)DATA(I,K)=1.ØE+22
3Ø     CONTINUE
      IF(J.LT.JBEGIN.OR.J.GT.JSTOP)DATA(I,J)=1.ØE+22
1ØØ   CONTINUE
C      IF(IDIFF.GT.JMAX)GOTO 4Ø
C     DO 2ØØ J=1,JMAX
C     K=KTOP-J
C     GOTO 5Ø
4Ø     CONTINUE
      DO 2ØØ I=ISTART,IMAX
5Ø     CONTINUE
C     IF(IDIFF.LE.JMAX) WRITE(9,1ØØØ)(DATA(I,K),I=ISTART,IMAX)
       WRITE(9,1ØØØ)(DATA(I,J),J=1,JMAX)
2ØØ   CONTINUE
1ØØØ  FORMAT(1X,21F6.Ø,/)
      RETURN
      END
C
```

```
      SUBROUTINE COEF3
C   ********************************************************************
C   *  FORMULATES CONSTRAINTS FOR EACH CONSTANT HEAD CELL.             *
C   *  IF THERE ARE NO RECHARGE CONSTRAINTS, THIS SUBROUTINE IS SKIPPED.*
C   ********************************************************************
C
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),JCF(62)
     *,ISW(376)
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(1),CHSMAX(1)
C
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
C THERE WILL BE NO EQUALITY CONSTRAINTS IN THIS SET.
C
      DIMENSION SCR(35,22)
       ITOUT=7
C  SET VARIABLE SCRATCH DRAWDOWNS TO ZERO
      DO 100 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 100 J=JBEGIN,JSTOP
       SCR(I,J)=0.0
        DO 100 LL=1,NCH
         IF(ICH(LL).EQ.I.AND.JCH(LL).EQ.J)SCR(I,J)=SI(I,J)
100   CONTINUE
C
C  SINGULAR CH CONSTRAINTS.
       NK=NVAR
      DO 200 LLCH=1,NCH
       LCH=LLCH+NVAR
      ADD=0.0
      I=ICH(LLCH)
      J=JCH(LLCH)
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
       IF(NCHSUB.EQ.0)GOTO 202
       IF(ISW(LCH).GT.0)GOTO 200
202   CONTINUE
      NK=NK+1
      ADD=ADD-SCR(I,J)*T(I,J)
C
       IF(I.GE.IMAX)GOTO 66
       IF(J.GT.JEND(I+1))GOTO 66
       IF(J.LT.JSTART(I+1))GOTO 66
        M=NCELL(I+1,J)
        IF(M.EQ.0)GOTO 65
         AA(NK,M)=(-DTR(I,J))
65     CONTINUE
        ADD=ADD+DTR(I,J)*SCR(I+1,J)
66     CONTINUE
C
       IF(J.GE.JSTOP)GOTO 76
        M=NCELL(I,J+1)
        IF(M.EQ.0)GOTO 75
         AA(NK,M)=(-DTU(I,J))
75      CONTINUE
         ADD=ADD+DTU(I,J)*SCR(I,J+1)
76      CONTINUE
C
       IF(I.LE.ISTART)GOTO 86
       IF(J.GT.JEND(I-1))GOTO 86
       IF(J.LT.JSTART(I-1))GOTO 86
        M=NCELL(I-1,J)
        IF(M.EQ.0)GOTO 85
         AA(NK,M)=(-DTR(I-1,J))
85      CONTINUE
```

```
          ADD=ADD+DTR(I-1,J)*SCR(I-1,J)
86       CONTINUE
C
        IF(J.LE.JBEGIN)GOTO 96
         M=NCELL(I,J-1)
         IF(M.EQ.0)GOTO 95
          AA(NK,M)=(-DTU(I,J-1))
95       CONTINUE
          ADD=ADD+DTU(I,J-1)*SCR(I,J-1)
96      CONTINUE
C
            R(NK)=ADD+APS(I,J)*(SST(I,J)-SCR(I,J))+PMIN(I,J)+ACCRN(I,J)
200     CONTINUE
C
        IF(NCHSUB.EQ.0)RETURN
C  SUBSYSTEM CH CELL CONSTRAINTS
        DO 300 L=1,NCHSUB
         NK=NVAR+KCH+L
         ADD=0.0
        DO 400 LLCH=1,NCH
         I=ICH(LLCH)
         J=JCH(LLCH)
         JBEGIN=JSTART(I)
         JSTOP=JEND(I)
         LCH=LLCH+NVAR
         IF(ICF(LLCH).EQ.0)GOTO 400
         IF(ICF(LLCH).NE.L)GOTO 400
        ADD=ADD-SCR(I,J)*T(I,J)+PMIN(I,J)
C
        IF(I.GE.IMAX)GOTO 466
        IF(J.GT.JEND(I+1))GOTO 466
        IF(J.LT.JSTART(I+1))GOTO 466
         M=NCELL(I+1,J)
         IF(M.EQ.0)GOTO 465
        AA(NK,M)=(-DTR(I,J))
465     CONTINUE
        ADD=ADD+DTR(I,J)*SCR(I+1,J)
466     CONTINUE
C
        IF(J.GE.JSTOP)GOTO 476
         M=NCELL(I,J+1)
         IF(M.EQ.0)GOTO 475
        AA(NK,M)=(-DTU(I,J))
475     CONTINUE
        ADD=ADD+DTU(I,J)*SCR(I,J+1)
476     CONTINUE
C
        IF(I.LE.ISTART)GOTO 486
        IF(J.GT.JEND(I-1))GOTO 486
        IF(J.LT.JSTART(I-1))GOTO 486
         M=NCELL(I-1,J)
         IF(M.EQ.0)GOTO 485
        AA(NK,M)=(-DTR(I-1,J))
485     CONTINUE
         ADD=ADD+DTR(I-1,J)*SCR(I-1,J)
486     CONTINUE
C
        IF(J.LE.JBEGIN)GOTO 496
         M=NCELL(I,J-1)
         IF(M.EQ.0)GOTO 495
        AA(NK,M)=(-DTU(I,J-1))
495     CONTINUE
        ADD=ADD+DTU(I,J-1)*SCR(I,J-1)
496     CONTINUE
C
400     CONTINUE
        R(NK)=ADD+CHSMIN(L)
300     CONTINUE
C
        RETURN
        END
C
```

```
      SUBROUTINE COEF1
C *************************************************************
C *  DETERMINES COEFFICIENTS OF CONSTRAINTS ON VARIABLE CELLS *
C *  ALL ARE INEQUALITY CONSTRAINTS.  XU(L)=XL(L) ON CF CELLS * -
C *************************************************************
C
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/CHUNK  1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/CHUNK  2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
C
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/QUA 1/Q(327,327)
C
      DIMENSION SCR(35,22)
      ITOUT=7
      ISCR=10
C SET SCRATCH DRAWDOWNS TO ZERO
      DO 5 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 5 J=JBEGIN,JSTOP
        SCR(I,J)=0.0
         DO 5 L=1,NCH
          IF(ICH(L).EQ.I.AND.JCH(L).EQ.J)SCR(I,J)=SI(I,J)
5      CONTINUE
C
      DO 100 L=ISTART,IMAX
      JBEGIN=JSTART(L)
      JSTOP=JEND(L)
      DO 100 M=JBEGIN,JSTOP
       I=NCELL(L,M)
       IF(NCELL(L,M).LE.0)GOTO 100
         IF(I.GT.NVAR)GOTO 100
         IF(L.LE.ISTART)GOTO 15
       J=NCELL(L-1,M)
       IF(J-0)15,15,10
10      CONTINUE
      AA(I,J)=(-DTR(L-1,M))
15      CONTINUE
       IF(L.GE.IMAX)GOTO 25
       J=NCELL(L+1,M)
       IF(J-0) 25,25,20
20      CONTINUE
      AA(I,J)=(-DTR(L,M))
25      CONTINUE
       IF(M.LE.JBEGIN)GOTO 35
      J=NCELL(L,M-1)
      IF(J-0) 35,35,30
30    CONTINUE
      AA(I,J)=(-DTU(L,M-1))
35     CONTINUE
      IF(M.GE.JSTOP)GOTO 45
      J=NCELL(L,M+1)
      IF(J-0) 45,45,40
40      CONTINUE
      AA(I,J)=(-DTU(L,M))
45     CONTINUE
      J=NCELL(L,M)
      AA(I,J)=T(L,M)+APS(L,M)
```

```
C RIGHT SIDE OF CONSTRAINT ONE.
      R(I)=SCR(L-1,M)*DTR(L-1,M)+SCR(L+1,M)*DTR(L,M)+SCR(L,M-1)
     C*DTU(L,M-1)+SCR(L,M+1)*DTU(L,M)+SST(L,M)*APS(L,M)+ACCRN(L,M)
     C+PMIN(L,M)
1ØØ   CONTINUE
C
C     DO 2ØØ I=1,ICELL
C     I=17Ø
C      WRITE(ITOUT,2ØØØ)I,R(I),AA(I,I)
C     DO 2ØØ J=1,NVAR
C      WRITE(ITOUT,1ØØØ)I,J,AA(I,J)
2ØØ   CONTINUE
C
1ØØØ  FORMAT(3X,I3,1X,I3,1X,E12.6)
2ØØØ  FORMAT(1X,'RIGHT SIDE ',I3,' = ',E12.6,' AA(I,I)=',E12.6)
      RETURN
      END
C
```

```
      SUBROUTINE SWCON
C *********************************************************************
C * FORMULATES ADDITIONAL CONSTRAINTS FOR LIMITING TOTAL USE OF
C * INTERFLOW IN A STREAM AQUIFER CELL.  EACH GROUP IS IDENTIFIED BY
C * THE ISW INDEX.
C *********************************************************************
C
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(1),CHSMAX(1)
C
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
C
      ITOUT=7
C INITIALIZE
      DO 5 NN=1,NSUB
       NEX=NVAR+KCH+NCHSUB+NN
       IF(IRCH.EQ.Ø)NEX=NVAR+NN
       R(NEX)=Ø.Ø
      DO 5 L=1,NVAR
       AA(NEX,L)=Ø.Ø
5     CONTINUE
C
      DO 1ØØ NN=1,NSUB
       NEX=NVAR+KCH+NCHSUB+NN
       IF(IRCH.EQ.Ø)NEX=NVAR+NN
       ADD=Ø.Ø
      DO 2ØØ I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 2ØØ J=JBEGIN,JSTOP
       L=NCELL(I,J)
       IF(L.EQ.Ø)GOTO 2ØØ
       IF(ISA(L).NE.NN)GOTO 2ØØ
C
       AA(NEX,L)=(-1.Ø)*APS(I,J)
       ADD=ADD-APS(I,J)*SST(I,J)
2ØØ    CONTINUE
        R(NEX)=SWMIN(NN)+ADD
1ØØ    CONTINUE
C
       RETURN
       END
C
```

```
      SUBROUTINE LPMIN
C ***********************************************************
C *  SETS UP OBJECTIVE FUNCTION TO MAXIMIZE TOTAL GW PUMPING *
C ***********************************************************
C
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(1),CHSMAX(1)
C
      COMMON/FASTI 1/IPR,IREAD,Y
      COMMON/KONST 7/ACC
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),X0(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 7/IPRINT
      COMMON/KONST 1/IFREQ
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/CONST 3/EPSY,EPSV,EPSCO,EPSD
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/POUT/IOUT
C
      DIMENSION FACTOR(327)
C
       ITOUT=7
C
       L3=NVAR*2
C
      WRITE(ITOUT,8787)KCH
      WRITE(ITOUT,8786)NCHSUB,NSUB
8786  FORMAT(1X,'NCHSUB = ',I5,' NSUB = ',I5)
8787  FORMAT(1X,'KCH = ',I5)
      DO 27 I=1,NCHSUB
       L4=NVAR*2+KCH+I
       XU(L4)=CHSMAX(I)-CHSMIN(I)
       XL(L4)=0.0
       CA(L4)=0.0
       ITYPE(L4)=1
27    CONTINUE
C
      DO 60 I=1,NSUB
       L5=NVAR*2+KCH+NCHSUB+I
       XU(L5)=SWMAX(I)-SWMIN(I)
       XL(L5)=0.0
       CA(L5)=0.0
       ITYPE(L5)=1
60    CONTINUE
C
      DO 100 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 100 J=JBEGIN,JSTOP
       L=NCELL(I,J)
       L2=L+NVAR
       IF(L.GT.0)GOTO 43
       L=NVAR+NCHN(I,J)
       IF(ISW(L).EQ.0)L3=L3+1
          XU(L3)=ACCRX(I,J)-ACCRN(I,J)
C         WRITE(ITOUT,1200)L3,XU(L3),ACCRX(I,J)
          CA(L3)=0.0
          ITYPE(L3)=1
          XL(L3)=0.0
          GOTO 100
43    CONTINUE
```

```
            FACTOR(L)=0.0
            TMIN=PMIN(I,J)
            TMAX=PMAX(I,J)
            TMAX=0.0
            TMIN=0.0
            XO(L2)=0.0
C
C  UPPER AND LOWER LIMITS (L2) ON GW PUMPING NOT ON TOTAL DISCHARGE.
          XU(L2)=PMAX(I,J)-PMIN(I,J)
          XL(L2)=0.0
          XL(L)=0.0
          XO(L2)=0.0
          IF(XL(L).GT.XU(L))XL(L)=XU(L)
          ITYPE(L)=1
          ITYPE(L2)=1
          CA(L2)=0.00
          X(L)=SI(I,J)
200     CONTINUE
C
        IF(ISW(L).EQ.0)GOTO 100
C
        IF(I.GE.IMAX)GOTO 10
         M=NCELL(I+1,J)
         IF(M.LE.0)GOTO 8
         TEST=CA(M)
         CA(M)=TEST+DTR(I,J)
8          CONTINUE
         CA(L)=CA(L)+DTR(I,J)*(-1.0)
10      CONTINUE
         IF(J.GE.JSTOP)GOTO 20
          M=NCELL(I,J+1)
          IF(M.LE.0)GOTO 18
          TEST=CA(M)
          CA(M)=TEST+DTU(I,J)
18          CONTINUE
          CA(L)=CA(L)+DTU(I,J)*(-1.0)
20      CONTINUE
         IF(I.LE.ISTART)GOTO 30
          M=NCELL(I-1,J)
          IF(M.LE.0)GOTO 28
          TEST=CA(M)
          CA(M)=TEST+DTR(I-1,J)
28          CONTINUE
          CA(L)=CA(L)+DTR(I-1,J)*(-1.0)
30      CONTINUE
         IF(J.LE.JBEGIN)GOTO 40
          M=NCELL(I,J-1)
          IF(M.LE.0)GOTO 38
          TEST=CA(M)
          CA(M)=TEST+DTU(I,J-1)
38          CONTINUE
          CA(L)=CA(L)+DTU(I,J-1)*(-1.0)
40      CONTINUE
          CA(L)=CA(L)-APS(I,J)
100     CONTINUE
C
C  FIND INITIAL FEASIBLE SOLUTION
        CALL GSIMEQ (FACTOR)
C
       LP=1
       EPSY=0.0
       NIMAX=390
        N=NVAR
        NF=0
        K=NVAR+KCH+NCHSUB+NSUB
         IF(IRCH.EQ.0)K=NVAR+NSUB
        KE=0
C
```

```
C   TEST THE LEAST PUMPING SOLUTION TO SEE IF ANY BOUNDS ARE VIOLATED.
      DO 150 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 150 J=JBEGIN,JSTOP
       L=NCELL(I,J)
       IF(L.LE.0)GOTO 150
        IF(XL(L).LE.XO(L))GOTO 110
         TEST=XL(L)-XO(L)
         XL(L)=XO(L)
         WRITE(ITOUT,1000)I,J,TEST
110     CONTINUE
        IF(XU(L).GE.XO(L))GOTO 120
         TEST=XO(L)-XU(L)
          TEST2=XU(L)
         XU(L)=XO(L)
         WRITE(ITOUT,2000)I,J,TEST,TEST2
120     CONTINUE
150     CONTINUE
C
C   PERFORM OPTIMIZATION
       CALL QPTHOR
C
1000    FORMAT(/,1X,'LOWER BOUND FOR CELL ',I2,1X,I2,' DECREASED BY ',
     *F10.2)
1200   FORMAT(1X,I5,2X,F10.2,2X,F10.2)
2000   FORMAT(/,1X,'UPPER BOUND FOR CELL ',I2,1X,I2,' INCREASED BY ',
     *F10.2,1X,'WAS = ',F10.2)
4000   FORMAT(1X,'FACTOR(',I3,') = ',F5.3)
       RETURN
       END
C
```

```
      SUBROUTINE GSIMEQ(FACTOR)
C ****************************************************************
C *  SOLVES FOR A GIVEN % OF PUMPING UNDER CONSTRAINT ONE (COEF1) *
C ****************************************************************
C
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(1),CHSMAX(1)
C
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/KONST 7/ACC
C
      DIMENSION FACTOR(327)
      DIMENSION TV(327),SCR(327),SCR2(327)
C
      ITOUT=7
      IMAP=9
      ISCR=10
      NIT=1000
C INITIATE STARTING SOLUTION
      DO 135 I2=ISTART,IMAX
       JBEGIN=JSTART(I2)
       JSTOP=JEND(I2)
      DO 135 J2=JBEGIN,JSTOP
       L=NCELL(I2,J2)
        IF(L.LE.0)GOTO 135
         SCR(L)=PMAX(I2,J2)-PMIN(I2,J2)
         IF(ISUS.EQ.3) X(L)=SI(I2,J2)
135   CONTINUE
C
      DO 100 IT=1,NIT
       DO 300 I=1,NVAR
C LARGEST ELEMENT SHOULD BE ON DIAGNAL- POS.DEF. MATRIX.
        JMX=I
C LARGEST ELEMENT IS (I,JMX) IN ROW (I)
         ADD=0.0
         DO 120 JJ=1,NVAR
          IF(JJ.EQ.JMX)GOTO 120
          ADD=ADD+AA(I,JJ)*X(JJ)
120      CONTINUE
C
C CALCULATE X(NEW) BY CONSTRAINT 1 AND FACTOR.
        RS=R(I)-ADD+FACTOR(I)*SCR(I)
         X(JMX)=RS/AA(I,JMX)
300      CONTINUE
C      WRITE(ITOUT,1212)IT,X(1)
1212   FORMAT(I5,2X,'X(1)= ',F10.2)
1234   CONTINUE
C
C ERROR CHECK
        SUM=0
         DO 350 J=1,NVAR
          SUM=SUM+X(J)
          TV(J)=0.0
         DO 350 JJ=1,NVAR
          TV(J)=TV(J)+AA(J,JJ)*X(JJ)
350      CONTINUE
C  COMPUTE ACCURACY AS DONE BY QPTHOR TO INSURE FEASIBILITY.
      AVEX=SUM/NVAR
      EPD=ABS(ACC*AVEX)+0.0000001
      DO 450 J=1,NVAR
       TEST=TV(J)-R(J)-FACTOR(J)*SCR(J)
        IF(TEST.LT.-EPD)GOTO 400
        IF(ABS(TEST).GT.EPD)GOTO 400
450   CONTINUE
```

```
       GOTO 600
400    CONTINUE
       X(J)=X(J)+TEST/AVEX
C      WRITE(ITOUT,2211)IT,J,EPD,TEST
2211   FORMAT(2I5,' EPD = ',F12.4,' TEST = ',F12.8)
100    CONTINUE
C
C NUMBER OF ITERATIONS COMPLETE
601      WRITE(ITOUT,2000)
C ALL MEET ERROR CRITERIA.
600      WRITE(ITOUT,3000)IT
         WRITE(ITOUT,3001)EPD
C
      DO 700 L=1,NVAR
      XO(L)=X(L)
      SCR(L)=X(L)
700   CONTINUE
C
      IF(IRCH.EQ.0)GOTO 111
C  CHECK RECHARGE CONSTRAINTS.
        ITEMP=KCH+NCHSUB+NSUB
       DO 800 L=1,ITEMP
        L2=NVAR+L
        ADD=0.0
         DO 850 J=1,NVAR
          ADD=ADD+AA(L2,J)*X(J)
850      CONTINUE
        IF(ADD.GT.R(L2))GOTO 800
        DIFF=(R(L2)-ADD)/.04356
         WRITE(ITOUT,5000)L2,DIFF
         R(L2)=ADD
800     CONTINUE
C
111     IF(ISUS.NE.0)GOTO 112
        WRITE(IMAP,4000)
        CALL INFOUT (SCR)
112     CONTINUE
C
2000  FORMAT(1X,'MAXIMUM NUMBER OF ITERATIONS EXCEEDED IN GSIMEQ',/)
3000  FORMAT(1X,'ITERATIONS = ',I5,' IN SUBROUTINE GSIMEQ',/)
3001  FORMAT(/,'**** EPD = ',E16.8,/)
4000  FORMAT(/,1X,'OUTPUT FROM I.F.S.',/)
5000  FORMAT(1X,'RECHARGE CONSTRAINT ',I3,' EXCEEDED BY ',E12.6)
5100  FORMAT(1X,'UPPER LIMIT ON RECHARGE VARIABLE ',I3,' EXCEEDED BY ',
     *E12.6)
      RETURN
      END
C
```

```
      SUBROUTINE INFOUT(SS)
C ************************************************************
C * OUTPUTS IN MAP FORM GENERAL INFORMATION ABOUT OPTIMAL *
C * SOLUTION.                                              *
C ************************************************************
C
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/AQUIF/TOP(376),BOT(376)
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(1),CHSMAX(1)
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/OPOUT/QG(35,22),SSOP(35,22)
      COMMON/AAA/TAR(35,22)
C
      DIMENSION SS(327),SCR(35,22),SCR2(35,22),SCR3(35,22)
      DIMENSION RT(5),CH(35,22)
C
      IMAP=9
C INITIALIZE
      IF(NCHSUB.EQ.Ø)GOTO 5
      DO 4 L=1,NCHSUB
       RT(L)=Ø.Ø
4     CONTINUE
5     CONTINUE
      IF(NSUB.EQ.Ø)GOTO 8
      DO 12 L=1,NSUB
       L2=L+NCHSUB
       RT(L2)=Ø.Ø
12    CONTINUE
8     CONTINUE
C  FILL TWO DIMENSIONAL DRAWDOWN ARRAY
      DO 1ØØ I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 1ØØ J=JBEGIN,JSTOP
       SSOP(I,J)=SI(I,J)
        L=NCELL(I,J)
        IF(L.EQ.Ø)GOTO 9Ø
       SSOP(I,J)=SS(L)
9Ø     CONTINUE
       QG(I,J)=Ø.Ø
C  COMPUTE ELEVATIONS
       SCR(I,J)=3ØØ.-SSOP(I,J)
       IF(ISUS.NE.Ø)CH(I,J)=SCR(I,J)-TAR(I,J)
1ØØ   CONTINUE
C
C  WRITE OUT DRAWDOWNS
      WRITE(IMAP,1ØØØ)
      CALL MAP (SSOP)
C  WRITE OUT ELEVATIONS
      WRITE(IMAP,2ØØØ)
      CALL MAP(SCR)
C  WRITE DIFFERENCE BETWEEN OPTIMAL AND TARGET ELEVATIONS
      IF(ISUS.NE.Ø)WRITE(IMAP,2021)
      IF(ISUS.NE.Ø)CALL MAP(CH)
C
C COMPUTE GROUNDWATER PUMPING
       TGW=Ø.Ø
       TACR=Ø.Ø
      DO 2ØØ I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 2ØØ J=JBEGIN,JSTOP
      L=NCELL(I,J)
```

```
      ADD=T(I,J)*SSOP(I,J)
      IF(I.LE.ISTART) GOTO 220
      IF(J.LT.JSTART(I-1))GOTO 220
      IF(J.GT.JEND(I-1))GOTO 220
      ADD=ADD-DTR(I-1,J)*SSOP(I-1,J)
220   CONTINUE
C
      IF(J.LE.JBEGIN)GOTO 240
      ADD=ADD-DTU(I,J-1)*SSOP(I,J-1)
240   CONTINUE
C
      IF(J.GE.JSTOP)GOTO 260
      ADD=ADD-DTU(I,J)*SSOP(I,J+1)
260   CONTINUE
C
      IF(I.GE.IMAX)GOTO 280
      IF(J.GT.JEND(I+1))GOTO 280
      IF(J.LT.JSTART(I+1))GOTO 280
      ADD=ADD-DTR(I,J)*SSOP(I+1,J)
280   CONTINUE
C  ASSIGN VALUE TO TOTAL EXCITATION.
      SCR(I,J)=ADD/.043560
C
      ACR=ACCRN(I,J)
      IF(L.EQ.0)ACR=ADD-PMIN(I,J)-APS(I,J)*(SST(I,J)-SSOP(I,J))
      QG(I,J)=ADD-ACR-APS(I,J)*(SST(I,J)-SSOP(I,J))
      QG(I,J)=QG(I,J)/.043560
C  ASSIGN VALUE TO TOTAL ACCRETION
      SCR2(I,J)=ACR/.043560
C
      DO 282 LSUB=1,NCHSUB
       IF(ICF(L).NE.LSUB)GOTO 282
       RT(LSUB)= RT(LSUB)+SCR2(I,J)
282   CONTINUE
C
      TACR=TACR+SCR2(I,J)
      TGW=TGW+QG(I,J)
C COMPUTE % GW PUMPING
      SCR3(I,J)=0.0
      IF(PMAX(I,J).EQ.0.0)GOTO 200
      SCR3(I,J)=(QG(I,J)*0.043560)/PMAX(I,J)*100.
200  CONTINUE
C
C  WRITE OUT GROUNDWATER PUMPING
     WRITE(IMAP,3000)
     CALL MAP(QG)
     WRITE(IMAP,3100)TGW
C WRITE OUT % GW PUMPING
     WRITE(IMAP,5500)
     CALL MAP(SCR3)
C  WRITE OUT ACCRETION (-RECHARGE)
     WRITE(IMAP,4100)
     CALL MAP (SCR2)
     WRITE(IMAP,4300) TACR
      IF(NCHSUB.EQ.0)GOTO 290
      DO 292 LSUB=1,NCHSUB
       WRITE(IMAP,4310)LSUB,RT(LSUB)
292   CONTINUE
290   CONTINUE
C  WRITE OUT TOTAL EXCITATION
     WRITE(IMAP,4000)
     CALL MAP (SCR)
C
C COMPUTE % OF MAX ACCRETION AND S/A RESPONSE.
      TSA=0.0
     DO 300 I=ISTART,IMAX
      JBEGIN=JSTART(I)
      JSTOP=JEND(I)
     DO 300 J=JBEGIN,JSTOP
      L=NCELL(I,J)
```

```
      IF(L.EQ.0)L=NVAR+NCHN(I,J)
      SCR(I,J)=0.0
      IF(L.GT.NVAR.AND.ISW(L).EQ.1)GOTO 305
      IF(ACCRN(I,J).EQ.0.0)GOTO 305
      SCR(I,J)=(SCR2(I,J)*.043560)/ACCRN(I,J)*100.
305   CONTINUE
C S/A RESPONSE
C     IF(NSA.EQ.0)GOTO 310
      SCR2(I,J)=APS(I,J)*(SST(I,J)-SSOP(I,J))/0.043560
C
      DO 312 LSUB=1,NSUB
       L2=LSUB+NCHSUB
       IF(ISA(L).NE.LSUB)GOTO 312
       RT(L2)=RT(L2)+SCR2(I,J)
312   CONTINUE
C
      TSA=TSA+SCR2(I,J)
      SCR3(I,J)=0.0
      IF(L.EQ.0)GOTO 310
      SCR3(I,J)=300.-SST(I,J)
      IF(ISA(L).EQ.0)SCR3(I,J)=0.0
310   CONTINUE
C  COMPUTE SATURATED THICKNESS
      IF(L.EQ.0)L=NVAR+NCHN(I,J)
       TTOP=TOP(L)
       EL=300.-SSOP(I,J)
       IF(EL.LT.TTOP)TTOP=EL
       XM(I,J)=TTOP-BOT(L)
300   CONTINUE
C
      IF(NSA.EQ.0)GOTO 350
C WRITE OUT STREAM/AQUIFER RESPONSE
      WRITE(IMAP,4200)
      CALL MAP (SCR2)
      WRITE(IMAP,4400)TSA
       IF(NSUB.EQ.0)GOTO 340
       DO 344 LSUB=1,NSUB
       L2=NCHSUB+LSUB
        WRITE(IMAP,4410)LSUB,RT(L2)
344    CONTINUE
340    CONTINUE
C WRITE OUT STREAM ELEVATIONS
      WRITE(IMAP,5600)
      CALL MAP (SCR3)
350   CONTINUE
C WRITE OUT % ACCRETION
      WRITE(IMAP,5000)
      CALL MAP(SCR)
C WRITE OUT SATURATED THICKNESS
      WRITE(IMAP,6000)
      CALL MAP (XM)
C
1000  FORMAT(/,6X,'DRAWDOWN VALUES FROM 300. DATUM.(FT)',/)
2000  FORMAT('1',/,6X,'ELEVATIONS.(FT)',/)
2021  FORMAT('1',/,6X,'DIFFERENCE(OPTIMUM-TARGET) IN ELEVATIONS(FT)',/)
3000  FORMAT('1',/,6X,'GROUNDWATER PUMPING IN ACRE-FEET',/)
3100  FORMAT(/,6X,'TOTAL REGIONAL GROUNDWATER PUMPING = ',E12.6)
4000  FORMAT('1',/,6X,'TOTAL EXCITATION IN ACRE-FEET',/)
4100  FORMAT('1',/,6X,'RECHARGE IN ACRE-FEET',/)
4200  FORMAT('1',/,6X,'STREAM/AQUIFER RESPONSE IN ACRE-FEET',/)
4300  FORMAT(/,6X,'TOTAL REGIONAL RECHARGE = ',E12.6)
4310  FORMAT(/,6X,'TOTAL RECHARGE IS CH SUBSYSTEM ',I3,' = ',G12.4)
4400  FORMAT(/,6X,'TOTAL REGIONAL S/A RESPONSE = ',E12.6)
4410  FORMAT(/,6X,'TOTAL INTERFLOW IN S/A SUBSYSTEM ',I3,' = ',G12.4)
5000  FORMAT('1',/,6X,'RECHARGE AS % OF MAX. ALLOWABLE RECHARGE',/)
5500  FORMAT('1',/,6X,'GW PUMPING AS % OF MAX ALLOWABLE PUMPING',/)
5600  FORMAT('1',/,6X,'STREAM ELEVATIONS',/)
6000  FORMAT('1',/,6X,'SATURATED THICKNESS (FT)',/)
      RETURN
      END
C
```

```
      SUBROUTINE SENSE
C *****************************************************************
C *  PERFORMS SENSITIVITY ANALYSIS ON OPTIMAL RESULTS             *
C *  OUTPUTS CONSTRAINED DERIVATIVES OF OBJECTIVE FUNCTION        *
C *  WITH RESPECT  TO THE DECISION VARIABLES.                     *
C *  COMPUTES AND OUTPUTS MAXIMUM CHANGE IN A DECISION VARIABLE   *
C *****************************************************************
     COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
     COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
     COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
    *,ISW(376)
     COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
    *V(327)
     COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
     COMMON/QUA 1/Q(327,327)
     COMMON/CONST 8/KOUNT,NIMAX,LP
     COMMON/CONST 1/N,NF,K,KE
     COMMON/BLOCK 3/NS(356),ND(327),NN(683)
C
C D-ARRAY HOLDS PARTIAL OF STATES WITH RESPECT TO DECISIONS
C V-ARRAY HOLDS PARTIAL OF OBJECTIVE FUNCTION W.R.T. DECISIONS
     ITOUT=7
     N9=N-KE
     DDMAX=0.0
     N2=N*2
     N3=N*2+KCH
     N4=N*2+KCH+NCHSUB
     ICEL2=ICELL
C
     WRITE(ITOUT,1000)
C
     DO 800 JJMX=1,NVAR
      LD=ND(JJMX)
      DDMIN=10.E+10
       IF(V(JJMX).EQ.0.0)GOTO 660
       IF(V(JJMX).LT.0.0)GOTO 650
C FIRST PARTIAL IS POSITIVE:
      IMIN=1
      AMIN=10.0E+10
C
      DO 100 I=1,ICEL2
       IF(D(I,JJMX).LE.0.0)GOTO 200
       XO(I)=(X(NS(I))-XL(NS(I)))/D(I,JJMX)
       IF(XO(I).GE.AMIN)GOTO 200
        AMIN=XO(I)
        IMIN=I
200    CONTINUE
100   CONTINUE
      DELS=AMIN
C
     IUMIN=1
     UMIN=10.0E+10
      DO 300 I=1,ICEL2
       IF(D(I,JJMX).GE.0.0)GOTO 350
       XO(I)=(X(NS(I))-XU(NS(I)))/D(I,JJMX)
        IF(XO(I).GT.UMIN)GOTO 350
      UMIN=XO(I)
      IUMIN=I
350  CONTINUE
300  CONTINUE
      DELSS=UMIN
C
C  FIND MIN(DELS,DELSS)
     DDMIN=DELS
     IF(DELSS.LT.DDMIN)DDMIN=DELSS
      GOTO 660
C
650  CONTINUE
C ************* V(JJMX) < 0
     IMIN=1
     AAMIN=10.0E+10
      DO 500 I=1,ICEL2
       IF(D(I,JJMX).GE.0.0)GOTO 550
```

```
        XO(I)=(XL(NS(I))-X(NS(I)))/D(I,JJMX)
         IF(XO(I).GT.AAMIN)GOTO 550
       AAMIN=ABS(XO(I))
       IMIN=I
550   CONTINUE
500   CONTINUE
       DELS=AAMIN
C
      IUMIN=I
      UMIN=10.E+10
       DO 700 I=1,ICEL2
        IF(D(I,JJMX).LE.0.0)GOTO 750
        XO(I)=(XU(NS(I))-X(NS(I)))/D(I,JJMX)
         IF(XO(I).GT.UMIN)GOTO 750
       UMIN=XO(I)
       IUMIN=I
750   CONTINUE
700   CONTINUE
       DELSS=UMIN
C
C FIND MIN(DELS,DELSS)
640   CONTINUE
      DDMIN=DELS
       IF(DELSS.LT.DDMIN)DDMIN=DELSS
C
660   CONTINUE
      IF(LD.GT.N)GOTO 510
       V(JJMX)=V(JJMAX)/.0435560
        XXX=X(ND(JJMX))
       WRITE(ITOUT,2000)LD,XXX,V(JJMX),DDMIN
        GOTO 850
510   CONTINUE
      IF(LD.GT.N2)GOTO 520
       LLD=LD-N
       XXX=X(ND(JJMX))/.043560
       WRITE(ITOUT,3000)LLD,XXX,V(JJMX),DDMIN
        GOTO 850
520   CONTINUE
      IF(LD.GT.N3)GOTO 530
       LLD=LD-N2
       XXX=X(ND(JJMX))/.043560
       WRITE(ITOUT,4000)LLD,XXX,V(JJMX),DDMIN
        GOTO 850
530   CONTINUE
      IF(LD.GT.N4)GOTO 540
      LLD=LD-N3
      XXX=X(ND(JJMX))/.043560
      WRITE(ITOUT,5000)LLD,XXX,V(JJMX),DDMIN
        GOTO 850
540   CONTINUE
      LLD=LD-N4
      XXX=X(ND(JJMX))/.043560
      WRITE(ITOUT,6000)LLD,XXX,V(JJMX),DDMIN
850   CONTINUE
       IF(DDMIN.EQ.DELS)WRITE(ITOUT,7200) NS(IMIN)
       IF(DDMIN.EQ.DELSS)WRITE(ITOUT,7400)NS(IUMIN)
800   CONTINUE
C
1000  FORMAT(/,1X,'FINAL DECISION VARIABLES, VALUE, DY/DX, DXMAX'
     *,/)
2000  FORMAT(1X,'DRAWDOWN IN CELL ',I3,' VALUE = ',E9.3,' FT.',/,2X,
     *'DY/DX = ',E9.3,' AC-FT/FT.  MAX DX = ',E9.3)
3000  FORMAT(1X,'PUMPING IN CELL ',I3,' VALUE = ',E9.3,' AC-FT.',/,2X,
     *'DY/DX = ',E9.3,' AC-FT/AC-FT.  MAX DX = ',E9.3)
4000  FORMAT(1X,'RECHARGE IN CH CELL ',I3,' VALUE = ',E9.3,' AC-FT.',
     */,2X,'DY/DX = ',E9.3,' AC-FT/AC-FT.  MAX DX = ',E9.3)
5000  FORMAT(1X,'RECHARGE IN CH SUBSYSTEM ',I3,' VALUE = ',E9.3,
     *' AC-FT.',/,2X,'DY/DX = ',E9.3,' AC-FT/AC-FT.  MAX DX = ',E9.3)
6000  FORMAT(1X,'RESPONSE IN S/A SUBSYSTEM ',I3,' VALUE = ',E9.3,
     *' AC-FT.',/,2X,'DY/DX = ',E9.3,' AC-FT/AC-FT.  MAX DX = ',E9.3)
7200  FORMAT(2X,'THE STATE X(NS(IMIN)) GOES TO LOWER LIMIT. NS(IMIN)= ',
     *I5)
7400  FORMAT(2X,'THE STATE X(NS(IUMIN)) GOES TO UPPER LIMIT. NS(IUMIN)=
     *',I5)
      RETURN
      END
```

```
      SUBROUTINE READIN
C*************************************************************************
C
C     THIS SUBROUTINE DOES THE FOLLOWING :
C              1.  READS AND PRINTS INPUT DATA
C              2.  FORMULATE QUADRATIC PROGRAMMING PROBLEM IN STANDARD FO
C*************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/FASTI 1/IPR,IREAD,Y
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 2/N1,N2,N3,N4,N5,N6
      COMMON/KONST 1/IFREQ
      COMMON/CONST 3/EPSY,EPSV,EPSCO,EPSD
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/POUT/IOUT
      COMMON/KONST 7/ACC
      COMMON/CONST 7/IPRINT
      COMMON/GLOB 1/IGLOB,IGMAX,IG
C*************************************************************************
C
C     REMEMBER TO CHANGE LIMITS IN LIMIT-CHECK AND -MESSAGE BELOW
C
C*************************************************************************
C
      IF(IGLOB.EQ.2) GO TO 9901
      IF(.NOT.IREAD.EQ.1) GO TO 311
 9999 FORMAT('-'/T5,120(1H*),/'-')
      READ 100, N,NF,K,KE,IGMAX
  100 FORMAT(5I4)
C*************************************************************************
C
C     FOR DIMENSION CHANGE, INCREASE LIMITS ON N AND K IN THE TWO SUBSEQ
C     STATEMENTS, AND IN THE FORMAT 97 MESSAGE
C
311   CONTINUE
C     IF(K.GT.60.OR.N.GE.61.OR.N+K-KE.GT.120.OR.N-KE.GT.60)PRINT97
97    FORMAT(1H1,T5,'LIMIT EXCEEDED  K=60  N=60   N+K-KE=120   N-KE=60')
C     IF(K.GT.60.OR.N.GE.61.OR.N+K-KE.GT.120.OR.N-KE.GT.60)  STOP
C
C*************************************************************************
C
      IF(.NOT.IREAD.EQ.1) GO TO 9901
      READ 101,(CA(I),ITYPE(I),I=1,N)
  101 FORMAT(8(G8.0,I2))
      DO 11 I=1,N
   11 READ 103,(Q(I,J),J=1,N)
      DO 10 I=1,K
   10 READ 102, (AA(I,J),J=1,N)
  102 FORMAT(10G8.0)
      READ 103,(R(I),I=1,K)
  103 FORMAT(10G8.0)
      READ 104,(XU(I),I=1,N)
      READ 104,(XL(I),I=1,N)
      READ 104,(XO(I),I=1,N)
  104 FORMAT (10G8.0)
      READ 105, EPSY,ACC,NIMAX,LP
  105 FORMAT(2G8.0,3I3)
      READ 106, IPRINT,IFREQ,IOUT
 106  FORMAT(3I2)
      IG=0
      IGLOB=0
 9901 CONTINUE
C
```

```
C ********************************************************************
      SUBROUTINE QPTHOR
C
C********************************************************************
C
C  COMMENTS WERE DELETED BECAUSE NUMBER WAS UNACCEPTABLE TO IBM
C  FORTRAN COMPILER.  LOOK IN TEST FORTRAN ON DISK RP24012 192 FOR
C  ORIGINAL COMMENTS.
C
C********************************************************************
C
      LOGICAL KT,IVPOS
      COMMON/CON 1/NSTAR
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      ICASE1=0
      ICASE2=0
C
C     READ AND PRINT INPUT DATA
C     FORMULATE AND PRINT STANDARD FORMULATION
C
      CALL READIN
C
C     PARTITION VARIABLES, AND PRINT TABLE OF CORRESPONDENCE
C
      CALL PART
C
C      PARTITION AND PRINT MATRICES OF COEFFICIENTS
C
      CALL PART AA
C
C     CALCULATE DELTA COEFFICIENTS
C
      CALL IDELTA
C
C     CALCULATE CONSTRAINED DERIVATIVES
C
      CALL CONDER
C
C     INITIALIZE ITERATION COUNTER
C
      NSTAR=0
      KOUNT=0
   10 CONTINUE
C
C      FIND NUMERICALLY LARGEST CONSTRAINT DERIVATIVE
C     IF POSITIVE, IVPOS=.TRUE., IF NEGATIVE, IVPOS=.FALSE.
C     CHECK KUHN-TUCKER CONDITIONS, AND PRINT OPTIMAL SOLUTION IF SATISF
C
      CALL MAXV(IVPOS,JMAX,KT)
      IF(KT) GO TO 15
C
C     FIND HOW MUTCH THE VARIABEL CAN CHANGE AND WHAT IS THE RESTRICTION
C
      IF(IVPOS) CALL CASEA1(ICASE1,JMAX,IMIN,IUMIN,DELD,DELV,DELS,DELSS)
      IF(.NOT.IVPOS) CALL CASEA2(ICASE2,JMAX,IMIN,IUMIN,DELD,DELV,DELS,
     $DELSS)
C
C     CARRY OUT THE ITERATION
C
      IF(ICASE1.EQ.1) CALL CASEB1(JMAX,DELD,1)
      IF(ICASE1.EQ.2.OR.ICASE2.EQ.2) CALL CASEB2(JMAX,DELV)
      IF(ICASE1.EQ.3.OR.ICASE2.EQ.3) CALL CASEB3(JMAX,IMIN,DELS,3)
      IF(ICASE2.EQ.4) CALL CASEB1(JMAX,DELD,4)
      IF(ICASE1.EQ.5.OR.ICASE2.EQ.5) CALL CASEB3(JMAX,IUMIN,DELSS,5)
   15 CONTINUE
      IF(.NOT.KT) GO TO 20
      RETURN
   20 CONTINUE
C
C     ....KT CONDITIONS ARE NOT SATISFIED....
C
      ICASE1=0
      ICASE2=0
      GO TO 10
      END
C
```

```
C       ....DEFINE UPPER BOUNDS OF UNBOUNDED VARIABLES....
C       ....AND LOVER BOUND OF FREE VARIABELS (ARTIFICIAL)...
        DO 110 I=1,N
        IF(ITYPE(I).EQ.Ø) XL(I)=-1ØE1Ø
  11Ø   IF(XU(I).EQ.Ø.Ø) XU(I)=1Ø.E1Ø
C
C
C       ....DEFINE TOLERANCE PARAMETERS....
        II=Ø
        SUM=Ø.Ø
        DO 96 I=1,N
        SUM=SUM+ABS(CA(I))
        IF(ABS(CA(I)).NE.Ø.) II=II+1
        DO 96 J=1,N
        IF(Q(I,J).NE.Ø.) II=II+1
96      SUM=SUM+ABS(Q(I,J))
        AVERC=SUM/II
        EPSV=ABS(ACC*AVERC) +Ø.ØØØØØØØØØ1
C
        II=Ø
        SUM=Ø.
        DO 95 I=1,N
        SUM=SUM+ABS(XO(I))
        IF(ABS(XO(I)).NE.Ø.) II=II+1
95      CONTINUE
        AVERX=SUM/II
        EPSD=ABS(ACC*AVERX) +Ø.ØØØØØØ1
        N1=N+1
        N2=N+K
        N3=N+KE
        N4=KE+1
        N5=N+KE+1
        N6=(N+K)-(NF+KE)
        N7=N+K-KE
        N9=N-KE
C
C       CHECK WETHER THE PROBLEM IS L.P. OR Q.P.
C
        IF(LP.NE.2) GO TO 436
        DO 321 I=1,N
        DO 321 J=1,N
        IF(Q(I,J).NE.Ø.) LP=Ø
        IF(LP.EQ.Ø) GO TO 436
321     CONTINUE
C
C       ....PRINT INPUT DATA....
C
436     IF(IPR.NE.1) GO TO 7
        PRINT 9999
        PRINT 9998
        IF(IGMAX.EQ.Ø) PRINT 7272
        IF(IGMAX.GT.Ø) PRINT 7273,IGMAX
        IF(LP.GT.Ø) PRINT 322
322     FORMAT('-',T5,'THE PROBLEM IS LINEAR PROGRAMMING PROBLEM')
 7272   FORMAT('-',T5,'CONVEX PROGRAMMING PROCEDURE ONLY')
 7273   FORMAT('-',T5,'CONVEX PROGRAMMING + SEARCH PROCEDURE',/T5,'MAXIMUM
     1  NUMBER OF SEARCH ITERATIONS =',I4)
 9998   FORMAT(T5,'QUADRATIC PROGRAMMING')
 8888   FORMAT('-',T5,12Ø(1H-),/'Ø')
        PRINT 9999
        PRINT 199
  199   FORMAT(1H1,T5,'INPUT DATA')
        PRINT 8888
```

```
      PRINT 200,N,K
  200 FORMAT('-'/'0',T5,'TOTAL NUMBER OF ORIGINAL VARIABLES',T40,'N=',
     1T50,I3/'0',T5,'TOTAL NUMBER OF CONSTRAINTS',T40,'K=',T50,I3)
      PRINT 8888
      PRINT 201
  201 FORMAT('-'/'0',T5,'INDEX',T28,'COEFFICIENTS',T45,'TYPE OF',
     1' VARIABLE',T65,'INITIAL SOLUTION',T85,'UPPER BOUNDS',T105,
     2'LOWER BOUNDS')
      PRINT 202
  202 FORMAT('-',T7,'I',T31,'C(I)',T49,'ITYPE(I)',T71,'XO(I)',T90,
     2'XU(I)',T109,'XL(I)')
  193 FORMAT('-',T5,I3,T40,T53,T67,G12.5,T85,E12.5,T105,E12.5)
        PRINT 210,  (I,CA(I),ITYPE(I),XO(I),XU(I),XL(I),I=1,N)
          IN2=N+1
C       PRINT 193,  (I,XO(I),XU(I),XL(I),I=IN2,N7)
  210 FORMAT('-',T5,I3,T28,G12.5,T52,I1,T67,G12.5,T85,E12.5,T105,E12.5)
C     PRINT 8888
C     PRINT 211
  211 FORMAT('-',T7,'I',T28,'COEFFICIENTS Q(I,J)')
C     DO 21 I=1,N
C  21 PRINT 212, I,(Q(I,J),J=1,N)
  212 FORMAT('-',T5,I3,(/T28,8G12.5))
C     PRINT 8888
  237 FORMAT('-'/'0')
C     PRINT 203
C     DO 20 I=1,K
C  20 PRINT 204, I,R(I),(AA(I,J),J=1,N)
  203 FORMAT(1X,T5,'CONSTRAINT',T30,'RIGHT HAND',T70,'COEFFICIENT MATRIX
     $ OF CONSTRAINTS'/T5,'NUMBER',T28,'SIDE'/T8,'I',T34,'R(I)',T86,
     $'A(I,J)')
  204 FORMAT(1H0,T6,I3,T29,G12.5,(/T45,7G12.5))
      PRINT 8888
7     CONTINUE
 1111 CONTINUE
C
C     INPUT DATA ARE NOW READ AND PRINTED
C     FORMULATE QUADRATIC PROGRAMMING PROBLEM IN STANDARD FORM
C
C     DEFINE AUGMENTED COEFFICIENT VECTOR OF OBJECTIVE FUNCTION, CA(I).
C
      DO 31 I=N1,N7
      ITYPE(I)=1
   31 CA(I)=0.0
C     DEFINE AUGMENTED COEFFICIENT MATRIX AA(I,J)
      DO 40 I=1,K
      DO 42 J=N1,N7
      AA(I,J)=0.
      IF(I-KE.EQ.J-N) AA(I,J)=-1.
   42 CONTINUE
   40 CONTINUE
```

```
C
C     CALCULATE SLACK VARIABLES ASSOCIATED WOTH INEQUALITY CONSTRAINTS


C     DEFINE ARRAY OF VARIABLES, ORIGINAL AND SLACK
C
C     ORIGINAL VARIABLES
C
      DO 50 I=1,N
   50 X(I)=XO(I)
C        5
C     SLACK VARIABLES, INEQUALITY CONSTRAINTS
C
      DO 70 I=1,K
      SUM=0.
      DO 71 J=1,N
71    SUM=SUM+AA(I,J)*X(J)
      IK=N+I
      XXXXX=SUM-R(I)
      IF(I.GT.KE.AND.XXXXX.LT.-EPSD) GO TO 331
      IF(I.LE.KE.AND.ABS(XXXXX).GT.EPSD) GO TO 331
      IF(I.LE.KE) GO TO 70
      X(N+I-KE)=XXXXX
C     XL(N+I-KE)=0.
C     XU(N+I-KE)=10.E10
         IF(X(IK).LT.XL(IK))XL(IK)=X(IK)
         IF(X(IK).GT.XU(IK))XU(IK)=X(IK)
   70 CONTINUE
      PRINT 210,(I,CA(I),ITYPE(I),X(I),XU(I),XL(I),I=IN2,N7)
C
      RETURN
331   PRINT 332,I
332   FORMAT('-'/'0',T5,' THE INITIAL SOLUTION IS NOT FEASIBLE',
     1/'0',T6,'THE FIRST VIOLATED CONSTRAINTS IS NR',I3)
333   CONTINUE
      PRINT 335, IK
335   FORMAT(1X,'VARIABLE NUMBER ',I3,' IS LESS THAN LOWER BOUND')
334   CONTINUE
      PRINT 336, IK
336   FORMAT(1X,'VARIABLE NUMBER ',I3,' IS GREATER THAN UPPER BOUND')
      STOP
      END
```

```
      SUBROUTINE PART
C
C***********************************************************************
C     THIS SUBROUTINE MAKES THE PARTITION OF X(I) INTO :
C             1.  K STATE VARIABLES, NS(I), FREE + BOUNDED
C             2.  N-KE DECISION VARIABLES, ND(J), ZEROS + BOUNDED
C***********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 2/N1,N2,N3,N4,N5,N6
      COMMON/CONST 7/IPRINT
      COMMON/FASTI 1/IPR,IREAD,Y
      N9=N-KE
      N7=N+K-KE
C
C     SELECT FREE VARIABLES TO BE STATE VARIABLES
C     DEFINE ARRAY OF BOUNDED VARIABLES,NN(J)
C
      J=1
      I=1
      DO 110 L=1,N
      IF(ITYPE(L).EQ.0) NS(I)=L
      IF(ITYPE(L).EQ.0) I=I+1
      IF(ITYPE(L).GE.1) NN(J)=L
      IF(ITYPE(L).GE.1) J=J+1
  110 CONTINUE
      DO 111 L=N1,N7
      NN(J)=L
      J=J+1
  111 CONTINUE
C
C     SELECT BOUNDED VARIABELS FAREST FROM THEIR LIMITS TO BE  STATE
      DO 10 I=1,N7
10    XO(I)=X(I)
      ISS=NF+1
      IF(ISS.GT.K) GO TO 37
      DO 11 IS=ISS,K
      XMAX=0
      MAX=0
      DO 12 L=1,N6
      CC=AMIN1(XU(NN(L))-XO(NN(L)),XO(NN(L))-XL(NN(L)))
      IF(CC.GE.XMAX) MAX=L
      IF(CC.GE.XMAX) XMAX=CC
12    CONTINUE
       IF(IS.LE.N)MAX=IS
       IF(IS.GT.N)MAX=IS+N
      NS(IS)=NN(MAX)
      XO(NN(MAX))=-10E12
11    CONTINUE
```

```
C
C     SELECT REMAINING VARIABLES TO BE DECISION VARIABLES
C
C     SELECT REMAINING NON.NEG VARIABLES TO BE DECISION VARIABLES
37    J=1
      DO 121 L=1,N6
      IF(.NOT.XO(NN(L)).EQ.-10E12) ND(J)=NN(L)
      IF(.NOT.XO(NN(L)).EQ.-10E12) J=J+1
  121 CONTINUE
C
C     PRINT TABLE OF CORRESPONDENCE
C
      IF(IPRINT.EQ.1) GO TO 2607
      PRINT 400
  400 FORMAT(1H1,T5,'TABLE OF CORRESPONDANCE',/'-')
      PRINT 401
  401 FORMAT(T5,'STATE VARIABLES',/'0',T5,'NS(I)',T30,'X(NS(I))')
      PRINT 402,(NS(I),X(NS(I)),I=1,K)
  402 FORMAT(T7,I3,T31,G12.5)
      PRINT 403
  403 FORMAT('-',T5,'DECISION VARIABLES',/'0',T5,'ND(J)',T30,
     1'X(ND(J))')
      PRINT 404,(ND(J),X(ND(J)),J=1,N9)
  404 FORMAT(T7,I3,T31,G12.5)
 2607 CONTINUE
C
C     CALCULATE INITIAL VALUE OF OBJECTIVE FUNCTION
C
      IF(IPR.NE.1) GO TO 7
      Y=0.0
      DO 1100 I=1,N
 1100 Y=Y+CA(I)*X(I)
      YY=0.0
      DO 2677 I=1,N
      DO 2677 J=1,N
      YY=YY+Q(I,J)*X(I)*X(J)
 2677 CONTINUE
      Y=Y+0.5*YY
C
C     PRINT INITIAL VALUE OF OBJECTIVE FUNCTION
C
      PRINT 1103,Y
 1103 FORMAT('-'/T5,'INITIAL VALUE OF OBJECTIVE FUNCTION,Y=',T50,
     1G12.5/'-')
      PRINT 9999
 9999 FORMAT('-'/T5,120(1H*),/'-')
7     CONTINUE
      RETURN
      END
C
```

```
      SUBROUTINE PART AA
C
C***********************************************************************
C      THIS SUBROUTINE MAKES THE PARTITION OF AA(I,J) INTO
C         1. THE K*K COEFFICIENT MATRIX B(I,J) OF STATE VARIABLES
C         2. THE K*N-KE COEFFICIENT MATRIX D(I,J) OF DECISION VARIABLES
C***********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 2/N1,N2,N3,N4,N5,N6
      COMMON/CONST 7/IPRINT
      N9=N-KE
C
      DO 10 I=1,K
      DO 10 J=1,K
10    B(J,I)=AA(J,NS(I))
      DO 20 I=1,N9
      DO 20 J=1,K
20    D(J,I)=-AA(J,ND(I))
C
C     PRINT PARTITIONED MATRICES B(I,J) AND C(I,J)
C
      IF(IPRINT.EQ.1) GO TO 2607
      IF(IPRINT.EQ.0)GOTO 2607
      PRINT 500
  500 FORMAT(1H1,T5,'COEFFICIENT MATRIX OF STATE VARIABLES B(I,JS)'
     1,/'0')
      PRINT 503
  503 FORMAT(T7,'I',T80,'B(I,JS)')
      DO 501 I=1,K
  501 PRINT 502, I, (B(I,JS),JS=1,K)
  502 FORMAT(1H0,T5,I3,(/T29,8G12.5))
      PRINT 505
  505 FORMAT('-'/T5,
     1'COEFFICIENT MATRIX OF DECISION VARIABLES D(I,JD)',/'0')
      PRINT 506
  506 FORMAT(T7,'I',T80,'D(I,JD)')
      DO 507 I=1,K
  507 PRINT 508, I, (D(I,JD),JD=1,N9)
  508 FORMAT(1H0,T5,I3,(/T29,8G12.5))
 2607 CONTINUE
C
C
      RETURN
      END
```

```
      SUBROUTINE IDELTA
C
C*********************************************************************
C     THIS SUBROUTINE CALCULATES THE DELTA COEFFICIENTS BY GAUSS ELIMINA
C     IF NEW PARIITION IS NEEDED SUBROUTINE NEWPAR IS CALLED
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C*********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 7/IPRINT
      COMMON/CONST 1/N,NF,K,KE
C
      EPSA=0.00000001
      N9=N-KE
      I=1
1     II=I+1
C*****FIND LARGEST ELEMENT IN COLUMN I*****
99    IMAX=I
      BMAX=ABS(B(I,I))
      IF(BMAX.GT.EPSA) GO TO 196
      DO 10 L=II,K
      IF(ABS(B(L,I)).GT.BMAX) IMAX=L
      IF(ABS(B(L,I)).GT.BMAX) BMAX=ABS(B(IMAX,I))
   10 CONTINUE
C*****LARGEST ELEMNT IN COLUMN I=B(IMAX,I)*****
      IF(BMAX.LE.EPSA) CALL NEWPAR(I)
      IF(BMAX.LE.EPSA) GO TO 99
C*****INTERCHANGE ROWS,MAKING LARGEST ELEMENT THE PIVOT*****
      DO 11 J=I,K
      XX=B(I,J)
      B(I,J)=B(IMAX,J)
      B(IMAX,J)=XX
   11 CONTINUE
      DO 12 J=1,N9
      XX=D(I,J)
      D(I,J)=D(IMAX,J)
      D(IMAX,J)=XX
   12 CONTINUE
C*****PERFORM GAUSS OPERATIONS*****
196   DO 13 L=II,K
      IF(ABS(B(L,I)).LT.EPSA) GO TO 13
      B(L,I)=B(L,I)/B(I,I)
      DO 14 J=II,K
      B(L,J)=B(L,J)-B(L,I)*B(I,J)
14    IF(ABS(B(L,J)).LE.EPSA) B(L,J)=0.
      DO 15 J=1,N9
   15 D(L,J)=D(L,J)-B(L,I)*D(I,J)
   13 CONTINUE
```

```
      I=I+1
      IF(I.LT.K) GO TO 1
C*****MATRIX IS NOW UPPER TRIANGULAR, AND I=K*****
      IF(ABS(B(K,K)).LE.EPSA) CALL NEWPAR(I)
C*****PERFORM GAUSS OPERATIONS BACKWARDS*****
    2 CONTINUE
      I2=I-1
      DO 21 L=1,I2
      LL=I-L
      B(LL,I)=B(LL,I)/B(I,I)
      IF(ABS(B(LL,I)).LE.EPSA) GO TO 21
      DO 22 J=1,N9
   22 D(LL,J)=D(LL,J)-B(LL,I)*D(I,J)
   21 CONTINUE
      I=I-1
      IF(I.GE.2) GO TO 2
C*****DIVIDE BY DIAGONAL ELEMENT*****
      DO 23 I=1,K
      DO 23 J=1,N9
      D(I,J)=D(I,J)/B(I,I)
23    IF(ABS(D(I,J)).LE.EPSA)D(I,J)=Ø.
C     ....PRINT DELTA(I,J)....
      IF(IPRINT.EQ.1) GO TO 2607
      IF(IPRINT.EQ.Ø)GOTO 2607
      PRINT 32
   32 FORMAT('-'/'Ø',T5,'DELTA COEFFICIENTS')
      DO 33 I=1,K
   33 PRINT 34, I,(D(I,J),J=1,N9)
   34 FORMAT(1HØ,T5,I3,(/T1Ø,1ØG12.5))
 2607 CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE NEWPAR(I)
C
C***************************************************************************
C     THIS SUBROUTINE MAKES NEW PARTITION BETVEEN STATE VARIABEL I AND O
C     EACH IN FOLLOVING ORDER.
C     1.  STATE VARIABEL WITH HIGHER NUMBER
C     2.  DECISION NOT ON ITS BOUNDARY
C     3.  DECISION ON ITS BOUNDARY
C
C     THIS SUBROUTINE ALSO CONSTRUCTS NEW COLUMS IN THE B AND D MATRIXES
C     GAUSSIAN ELIMINATION PROCESS CAN BE CONTINUED IN COLUMN I
C***************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 7/IPRINT
      COMMON/CONST 1/N,NF,K,KE
C
      EPSA=Ø.ØØØØØØØ1
      N9=N-KE
C
C     FIND IF STATE VARIABEL OF HIGHER NUMBER HAS NONZERO ELEMENT
C
      II=I+1
      IF(I.EQ.K) GO TO 19
      DO 16 J=II,K
      DO 17 L=I,K
      IF(ABS(B(L,J)).GT.EPSA)NSS=J
      IF(ABS(B(L,J)).GT.EPSA) GO TO 18
17    CONTINUE
16    CONTINUE
      GO TO 19
C
C     CHANGE PARTITION OF STATE VARIABELS I AND NSS
C
18    CONTINUE
      IF(ITYPE(NS(I)).EQ.Ø.AND.ITYPE(NS(NSS)).NE.Ø) NN(N+K-KE)=7
C****THIS IS TO INDICATE THAT NOW THE NF FIRST STATE ARE NOT ALL FREE VA
      JJ=NS(I)
      NS(I)=NS(NSS)
      NS(NSS)=JJ
      IF(IPRINT.NE.1) PRINT 81,I,NSS
81    FORMAT('-'/'-','CHANGE PARTITION BETWEEN STATE VARIABELS ',2I5)
C
C     CHANGE TWO COLUMNS IN B MATRIX
C
      DO 2Ø J=1,K
      XO(J)=B(J,I)
      B(J,I)=B(J,NSS)
      B(J,NSS)=XO(J)
2Ø    CONTINUE
      RETURN
C
```

```
C
C      TRY TO FIND NONZERO COLUMNS AMONG NONZERO DECISIONS VARIABELS
C
19     CONTINUE
       DO 22 J=1,N9
       IF(ABS(X(ND(J))-XL(ND(J))).LE.EPSA.OR.ABS(XU(ND(J))-X(ND(J))).LE.E
      $PSA) GO TO 22
       DO 23 L=I,K
       IF(ABS(D(L,J)).GT.EPSA) LD=J
       IF(ABS(D(L,J)).GT.EPSA) GO TO 30
23     CONTINUE
22     CONTINUE
C
C      TRY TO FIND NONZERO COLUMS WITHIN ZERO DECISIONS VARIABELS
C
       DO 25 J=1,N9
       IF(.NOT.ABS(X(ND(J))-XL(ND(J))).LE.EPSA.OR.ABS(XU(ND(J))-X(ND(J)))
      $.LE.EPSA) GO TO 25
       DO 26 L=I,K
       IF(ABS(D(L,J)).GT.EPSA) LD=J
       IF(ABS(D(L,J)).GT.EPSA) GO TO 30
26     CONTINUE
25     CONTINUE
100    PRINT 77
77     FORMAT('0',T5,' SOMETHING IS WRONG NO EQUATIONS OF STATE CAN BE',
      1' OBTAINED')
       STOP
C
C      CHANGE PARTITION BETWEEN STATE I AND DECISION LD
C
30     CONTINUE
       JJ=NS(I)
       NS(I)=ND(LD)
       ND(LD)=JJ
       IF(IPRINT.NE.1)PRINT 88,I,LD
88     FORMAT('-'/'-'/'-','CHANGE PARTITION BETWEEN STATE ',I3,
      1' AND DECISION ',I3)
C
C      CHANGE TWO COLUMNS IN B AND D MATRIXES
C
        DO 33 J=1,K
       XO(J)=B(J,I)
       B(J,I)=-D(J,LD)
       D(J,LD)=-XO(J)
33     CONTINUE
C
C      IF THE STATE VARIABEL WAS FREE VARIABEL MAKE IT BOUNDED  WHEN IT B
C      DECISION VARIABEL.
C
       IF(ITYPE(ND(LD)).EQ.1) RETURN
       XU(ND(LD))=10.E10
       XL(ND(LD))=-10.E10
       NN(N+K-KE)=7
       RETURN
       END
```

```
      SUBROUTINE CONDER
C
C*************************************************************************
C     THIS SUBROUTINE CALCULATES THE CONSTRAINED DERIVATIVES OF THE OBJE
C     FUNCTION WITH RESPECT TO THE NON NEGATIVE DECISION VARIABLES
C     IF THE PROBLEM IS L.P. THE CODE USES DIFFERENT FORMULA TO CALCULAT
C*************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/QUA 1/Q(327,327)
      COMMON/CONST 7/IPRINT
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 3/EPSY,EPSV,EPSCO,EPSD
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C
      N9=N-KE
      IF(LP.NE.Ø) GO TO 4ØØ
      DO 16Ø J=1,N9
      V(J)=Ø.
      DO 1Ø I=1,K
      WA=Ø.
      IF(NS(I).GT.N) GO TO 1Ø
      IF(D(I,J).EQ.Ø.) GO TO 23
      DO 11 IR=1,K
      IF(NS(IR).GT.N) GO TO 11
      WA=WA+Q(NS(I),NS(IR))*X(NS(IR))
11    CONTINUE
      DO 12 IT=1,N9
      IF(ND(IT).GT.N) GO TO 12
      WA=WA+Q(NS(I),ND(IT))*X(ND(IT))
   12 CONTINUE
      WA=WA+CA(NS(I))
      V(J)=V(J)+WA*D(I,J)
23    IF(ND(J).GT.N) GO TO 1Ø
      V(J)=V(J) +Q(NS(I),ND(J))*X(NS(I))
1Ø    CONTINUE
      IF(ND(J).GT.N) GO TO 21
      DO 13 IT=1,N9
      IF(ND(IT).GT.N) GO TO 13
      V(J)=V(J)+Q(ND(IT),ND(J))*X(ND(IT))
13    CONTINUE
21    V(J)=V(J)+CA(ND(J))
      IF(ABS(V(J)).LT.1.E-9) V(J)=Ø.Ø
  16Ø CONTINUE
      GO TO 4Ø1
```

```
C
C      CALCULATE  V(J) BY L.P. FORMULA
C
400    CONTINUE
       DO 402 J=1,N9
       V(J)=CA(ND(J))
       DO 403 I=1,K
403    V(J)=V(J)+CA(NS(I))*D(I,J)
402    IF(ABS(V(J)).LE.10.E-10) V(J)=0.
C
C      DEFINE ALL THE TAU COFFICINTS AS ZERO IN L.P. CASE
C
       DO 404 J=1,N9
       DO 404 I=1,N9
404    B(I,J)=0.
C
C      PRINT CONSTRAINED DERIVATIVES
C
401    IF(IPRINT.EQ.1) GO TO 2607
       PRINT 900
  900 FORMAT('-'/T5,'CONSTRAINT DERIVATIVES V(J)')
       PRINT 901
  901 FORMAT(1H0,/'0',T5,'INDEX OF DECISION VARIABLE',T40,
     1'CONSTRAINT      DERIVATIVE'/'0',T17,'J',T48,'V(J)',/'-')
       PRINT 902, (J,V(J),J=1,N9)
  902 FORMAT(T15,I3,T44,G12.5)
2607   RETURN
       END
```

```
      SUBROUTINE MAXV(IVPOS,JMAX,KT)
C
C*************************************************************************
C     THIS SUBROUTINE FINDS THE NUMERICALLY LARGEST CONSTRAINED DERIVAT(
C     IF V(J) IS POSITIVE, IVPOS=.TRUE.
C     IF V(J) IS NEGATIVE, IVPOS=.FALSE.
C
C     THIS SUBROUTINE ALSO TECKS THE KUHN-TUCKER CONDITIONS
C        1.  KT-CONDITIONS SATISFIED : RETURN KT=.TRUE.
C        2.  KT-CONDITIONS NOT SATISFIED : RETURN KT=.FALSE.
C
C     IF THE OPTIMUM IS REACHED,I.E. IF KT=.TRUE., THE OPTIMAL SOLUTION
C     TED
C
C*************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 7/IPRINT
      COMMON/ENTER/NEW
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 3/EPSY,EPSV,EPSCO,EPSD
      LOGICAL IVPOS
      COMMON/QUA 1/Q(327,327)
      DIMENSION YSTAR(50)
      COMMON/FASTI 1/IPR,IREAD,Y
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CON 1/NSTAR
      COMMON/CONST 8/KOUNT,NIMAX,LP
      LOGICAL KT
      N9=N-KE
      N7=N+K-KE
C
C     ....COUNT ITERATIONS....
      KOUNT=KOUNT+1
      IF(KOUNT.GT.NIMAX) PRINT 1109
 1109 FORMAT('-'/T5,'MAXIMUM NUMBER OF ITERATIONS EXCEEDED,',
     1' STOP, THE PRINTED OPTIMAL SOLUTION IS NOT CORRECT',
     2' US IT AS AN NEW INITIAL SOL')
 8888 FORMAT('-',T5,120(1H-),/'0')
      KT=.TRUE.
      IF(KOUNT.GT.NIMAX) GO TO 541
C
C     FIND THE NUMERICAL LARGEST FEASABLE CONSTRAINED DERIVATIVE TO CHAN
C     IF NONE IS FEASABLE (JMAX=0) THE KUHN-TUCKER CONDITIONS ARE SATISF
C
77    JMAX=0
      VMAX=0.0
      DO 1200 J=1,N9
      IF(ABS(V(J)).LT.EPSV) GO TO 1200
      IF(V(J).GT.EPSV.AND.X(ND(J))-XL(ND(J)).LT.EPSD) GO TO 1200
      IF(V(J).LT.(-EPSV).AND.X(ND(J)).GT.(XU(ND(J))-EPSD)) GO TO 1200
      IF(ABS(V(J)).GT.ABS(VMAX)) GO TO 1201
      GO TO 1200
 1201 CONTINUE
      VMAX=V(J)
      JMAX=J
 1200 CONTINUE
      IF(JMAX.EQ.0) GO TO 541
C
C     ....CALCULATE TAU FOR JMAX....
      NEW=1
      CALL NEWVAL(JMAX,IMIN)
C     END OF LOOP, VMAX=V(JMAX) AND JMAX DETERMINED
      IF(VMAX.GT.0.0) IVPOS=.TRUE.
      IF(VMAX.LT.0.0) IVPOS=.FALSE.
      IP=ND(JMAX)
      KT=.FALSE.
```

```
C
C     PRINT MAXIMUM CONSTRAINED DERIVATIVE, AND VARIABLE TO BE CHANGED
C
      IF(IPRINT.EQ.1) GO TO 2607
      PRINT 1211, VMAX
 1211 FORMAT(T5,'NUMERICALLY LARGEST CONSTRAINT DERIVATIVE, VMAX=',T60,
     $G12.5,//)
      PRINT 1213, IP
 1213 FORMAT(T5,'VARIABLE TO BE CHANGED:X(IP)=X(ND(JMAX)),IP=',T66,I3,/)
      PRINT 1215, JMAX
 1215 FORMAT('0',T46,'JMAX=',T66,I3,/)
      PRINT 1216, X(IP)
 1216 FORMAT(T46,'X(IP)=',T60,G12.5)
      IF(IVPOS)PRINT 1214
      IF(.NOT.IVPOS)PRINT 1219
 1214 FORMAT('0',T5,'IF V(JMAX) IS POSITIVE, IVPOS=.TRUE.',/T5,
     $'IF V(JMAX) IS NEGATIVE, IVPOS=.FALSE.',/'0',T5,'IVPOS=',T66,
     $'.TRUE.')
 1219 FORMAT('0',T5,'IF V(JMAX) IS POSITIVE, IVPOS=.TRUE.',/T5,
     $'IF V(JMAX) IS NEGATIVE, IVPOS=.FALSE.',/'0',T5,'IVPOS=',T66,
     $'.FALSE.')
 2607 CONTINUE
C
C      CALCULATE VALUE OF OBJECTIVE FUNCTION
      IF(EPSY.EQ.0.0) GO TO 7
541   CONTINUE
      Y=0.0
      DO 1100 I=1,N
 1100 Y=Y+CA(I)*X(I)
      YY=0.0
      DO 2677 I=1,N
      DO 2677 J=1,N
      YY=YY+Q(I,J)*X(I)*X(J)
 2677 CONTINUE
      Y=Y+0.5*YY
C
C     ....IF THE CHANGE OF Y IN FIVE  ITERATIONS IS LESS THAN EPSY :
C
      IF(KT) GO TO 10
      YSTAR(NSTAR)=Y
      IF(NSTAR.LE.5) GO TO 10
      DELTAY=ABS(YSTAR(NSTAR)-YSTAR(NSTAR-5))
      IF(DELTAY.LE.EPSY) KT=.TRUE.
      IF(DELTAY.LE.EPSY) PRINT 20
   20 FORMAT('-'/T5,'LITTLE CHANGE IN OBJECTIVE VALUE IN FIVE',
     1' ITERATIONS - STOP',/'-')
   10 CONTINUE
7     CONTINUE
      NSTAR=NSTAR+1
C
      IF(.NOT.KT) RETURN
C
C     PRINT OPTIMAL SOLUTION
C
      IF(IPR.NE.1) GO TO 8
      PRINT 1101
 1101 FORMAT('-'/'0',T5,'OPTIMAL SOLUTION',/'-',T5,'I',T10,'X(I)')
      PRINT 1102,(I,X(I),I=1,N7)
 1102 FORMAT(1H0,T3,I3,T6,G12.5)
      PRINT 9999
      PRINT 1103, Y
 1103 FORMAT('-'/T5,'MINIMUM VALUE OF OBJECTIVE FUNCTION,Y=',
     1T50,G12.5)
      PRINT 9999
      PRINT 1104, KOUNT
 1104 FORMAT('-'/T5,'NUMBER OF ITERATIONS =',T30,I3)
      PRINT 9999
 9999 FORMAT('-'/T5,120(1H*),/'-')
8     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NEWVAL(JMAX,IMIN)
C
C*************************************************************************
C     THIS SUBROUTINE CALCULATES NEW DELTAS AND ONE SET OF TAU COEFFICIE
C     WHEN CALLED FROM SUBROUTUNE MAX
C     IN L.P. CASE IT DOES NOT CALCULATE TAU COEFFICIENTS.
C*************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 7/IPRINT
      COMMON/CONST 2/N1,N2,N3,N4,N5,N6
      COMMON/ENTER/NEW
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      DIMENSION Z(683)
C     THE VECTOR XO(N+K) IS USED FOR LOCALISED STORAGE
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C
      IF(NEW.EQ.1) GO TO 2607
      N9=N-KE
C     NEW DELTA COEEFICIENTS
C
      DO 2000 I=1,K
      Z(I)=D(I,JMAX)
 2000 CONTINUE
      DO 1999 J=1,N9
      XO(J)=D(IMIN,J)
 1999 CONTINUE
      DELTRP=D(IMIN,JMAX)
      DO 2001 I=1,K
      IF(Z(I).EQ.0.) GO TO 2001
      DO 2002 J=1,N9
      D(I,J)=D(I,J)-Z(I)*XO(J)/DELTRP
      IF(ABS(D(I,J)).LE.1.E-8) D(I,J)=0.
 2002 CONTINUE
 2001 CONTINUE
      DO 2009 J=1,N9
      D(IMIN,J)=-XO(J)/DELTRP
 2009 CONTINUE
      DO 2003 I=1,K
      D(I,JMAX)=Z(I)/DELTRP
 2003 CONTINUE
      D(IMIN,JMAX)=1.0/DELTRP
```

```
C
C      PRINT NEW DELTA COEFFICIENTS
       IF(IPRINT.EQ.1) GO TO 2607
       IF(IPRINT.EQ.Ø)GOTO 2607
C
       PRINT 2ØØ4
 2ØØ4 FORMAT('-'/T5,'NEW COEFFICIENTS DELTA(I,J)',/'-')
       PRINT 2ØØ5
 2ØØ5 FORMAT(T5,'INDEX',T75,'COEFFICIENTS'/'Ø',T7,'I',T76,
      1'DELTA(I,J)'/'Ø')
       DO 2ØØ6 I=1,K
       PRINT 2ØØ7, I,(D(I,J),J=1,N9)
 2ØØ7 FORMAT(1HØ,T5,I3,(/T4Ø,7G12.5))
 2ØØ6 CONTINUE
 26Ø7 CONTINUE
       IF(NEW.EQ.Ø) RETURN
C
       IF(IGLOB.EQ.1) RETURN
C
C
C      ....CALCULATE NEW TAU COEFFICIENTS FOR ONE STATE VARIABEL ONLY....
C
       IF(LP.GT.Ø) GO TO 752
       IT=JMAX
       N9=N-KE
       DO 14 J=1,N9
       TA=Ø.
       DO 15 I=1,K
       IF(NS(I).GT.N) GO TO 15
       IF(D(I,J).EQ.Ø.) GO TO 21
       TAA=Ø.
       DO 16 IR=1,K
       IF(NS(IR).GT.N) GO TO 16
       IF(Q(NS(I),NS(IR)).NE.Ø) TAA=TAA+Q(NS(I),NS(IR))*D(IR,IT)
16     CONTINUE
       TA=TA+TAA*D(I,J)
       IF(ND(IT).LE.N)TA=TA+Q(NS(I),ND(IT))*D(I,J)
21     IF(ND(J).GT.N) GO TO 15
       TA=TA +Q(NS(I),ND(J))*D(I,IT)
15     CONTINUE
       B(J,IT)=TA
       IF(ND(J).LE.N.AND.ND(IT).LE.N)   B(J,IT)=B(J,IT)+Q(ND(J),ND(IT))
       IF(ABS(B(J,IT)).LE.1.E-8) B(J,IT)=Ø.
14     CONTINUE
C
C      ....PRINT TAU COEFFICIENTS....
       IF(IPRINT.EQ.1) GO TO 26Ø8
       IF(IPRINT.EQ.Ø)GOTO 26Ø8
       PRINT 897
  897 FORMAT('-'/T5,'CONTROL:TAU(J,IT)'/'Ø',T7,'J',T24,'TAU(J,IT)'/'Ø')
       PRINT 895,(B(J,IT),J=1,N9)
  895 FORMAT('Ø',T5,I3,(/T2Ø,8G12.5))
 26Ø8 CONTINUE
C
752    NEW=Ø
```

```
C
      SUBROUTINE CASEA1(ICASE1,JMAX,IMIN,IUMIN,DELD,DELV,DELS,DELSS)
C
C************************************************************************
C     ....THIS SUBROUTINE DETERMINES WHETHER:....
C         A DECISION VARIABLE GOES TO ITS LOVER LIMITS ICASE1=1
C         A CONSTRAINED DERIVATIVE GOES TO ZERO : ICASE1=2
C         A STATE VARIABLE GOES TO ITS LOVER LIMITS ICASE1=3
C         A STATE VARIABEL GOES TO ITS UPPER LIMITS  ICASE1=5
C************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CON 1/NSTAR
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
      COMMON/CONST 7/IPRINT
C     THE VECTOR XO(N+K) IS USED FOR LOCALIZED STORAGE
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C
      IF(IGLOB.EQ.1) GO TO 2607
      IF(IPRINT.EQ.1) GO TO 2607
      PRINT 1729
 1729 FORMAT('-'/T5,'CASEA1 : VMAX(J)=V(JMAX) IS POSITIVE')
      PRINT 19, JMAX
   19 FORMAT(1H0,T5,'JMAX=',T10,I3)
 2607 CONTINUE
C
C     ....THE MINIMUM OF DELD,DELV,DELS,DELSS DETERMINES WHERE TO GO
C
C     ....DEFINE DELD....
      DELD=X(ND(JMAX))-XL(ND(JMAX))
C
C     ....CALCULATE DELS....
      IMIN=1
      AMIN=10.E10
      N8=NF+1
      IF(NN(N+K-KE).EQ.7) N8=1
      DO 1302 I=N8,K
      IF(D(I,JMAX).LE.0.0) GO TO 1305
      XO(I)=(X(NS(I))-XL(NS(I)))/D(I,JMAX)
      IF(XO(I).LT.AMIN) GO TO 1303
      GO TO 1305
 1303 CONTINUE
      AMIN=XO(I)
      IMIN=I
 1305 CONTINUE
 1302 CONTINUE
C     END OF LOOP : AMIN=XO(IMIN) AND IMIN DEYERMINED
C
      DELS=AMIN
```

```
C
C      ....CALCULATE DELSS....
       IUMIN=1
       UMIN=10.E10
       DO 30 I=N8,K
       IF(D(I,JMAX).GE.0.0) GO TO 31
       XO(I)=(X(NS(I))-XU(NS(I)))/D(I,JMAX)
       IF(XO(I).LT.UMIN) GO TO 32
       GO TO 31
   32  CONTINUE
       UMIN=XO(I)
       IUMIN=I
   31  CONTINUE
   30  CONTINUE
C
C      ....UMIN=XO(IUMIN) IS NOW DETERMINED....
C
       DELSS=UMIN
C
       IF(IGLOB.EQ.1) GO TO 2610
       IF((DELS.EQ.0.0.OR.DELSS.EQ.0.0).AND.IPRINT.NE.1) PRINT 8127
 8127  FORMAT('-',T5,'DEGENERATE CASE',/'0')
       IF(DELS.EQ.0.0.OR.DELSS.EQ.0.0) NSTAR=NSTAR-1
C
C      ....CALCULATE DELV....
       IF(B(JMAX,JMAX).LE.0.000) DELV=10.E11
       IF(B(JMAX,JMAX).GT.0.0) DELV=V(JMAX)/B(JMAX,JMAX)
C
 2610  CONTINUE
C      ....FIND MIN(DELD,DELV,DELS,DELSS)....
       IF(IGLOB.EQ.1) DELV=10.E11
       DDMIN=AMIN1(DELD,DELV,DELS,DELSS)
       IF(DDMIN.EQ.DELD) ICASE1=1
       IF(DDMIN.EQ.DELV) ICASE1=2
       IF(DDMIN.EQ.DELS) ICASE1=3
       IF(DDMIN.EQ.DELSS) ICASE1=5
C
C      ....PRINT FINDINGS....
       IF(IPRINT.EQ.1) GO TO 2611
       PRINT 103,DELD,DELV,DELS,DDMIN
  103  FORMAT('-'/T5,'MAXIMUM POSSIBLE CHANGE OF THE DECISION',
      1' X(ND(JMAX)):',/'0',T5,'DELD=',G12.5,2X,'DELV=',G12.5,2X,
      2'DELS=',G12.5,2X,'DDMIN=',G12.5)
       IF(DDMIN.EQ.DELD) PRINT 100,JMAX
  100  FORMAT('-'/T5,'CASE B1 : THE DECISION X(ND(JMAX)) GOES TO ZERO',
      $/'0',T5,'JMAX=',T10,I3,T15,'ICASE1=1')
       IF(DDMIN.EQ.DELV) PRINT 101,JMAX
  101  FORMAT('-'/T5,'CASE B2 :THE CONSTRAINED DERIVATIVE',
      1' V(JMAX) GOES TO ZERO'/'0',T5,'JMAX=',T10,I3,T15,'ICASE1=2')
       IF(DDMIN.EQ.DELS) PRINT 102,IMIN
  102  FORMAT('-'/T5,'CASE B3 : THE STATE X(NS(IMIN)) GOES TO ZERO',
      1/'0',T5,'IMIN=',T10,I3,T15,'ICASE1=3')
       IF(DDMIN.EQ.DELSS) PRINT 33, IUMIN
   33  FORMAT('-'/T5,'CASEB5 : THE STATE X(NS(IUMIN))',
      1' REACHES ITS UPPER LIMIT'/'0',T5,'IUMIN=',T10,I3,T15,'ICASE1=5')
C
 2611  RETURN
       END
```

```
C
      SUBROUTINE CASEA2(ICASE2,JMAX,IIMIN,IUMIN,DELD,DELV,DELS,DELSS)
C
C***********************************************************************
C     ....THIS SUBROUTINE DETERMINES WHETHER :....
C         A DECISION REACHES ITS UPPER LIMITS ICASE2=4
C         A CONSTRAINED DERIVATIVE GOES TO ZERO : ICASE2=2
C         A STATE GOES TO ITS LOVER LIMITS  ICASE2=3
C         A STATE GOES TO ITS UPPER LIMITS  ICAE2=5
C***********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CONST 1/N,NF,K,KE
      COMMON/KONST 1/IFREQ
      COMMON/CON 1/NSTAR
      COMMON/CONST 7/IPRINT
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C     THE VECTOR XO(N+K) IS USED FOR LOCALIZED STORAGE
C
      IF(IGLOB.EQ.1) GO TO 2607
      IF(IPRINT.EQ.1) GO TO 2607
      PRINT 210
  210 FORMAT('-'/T5,'CASE A2 : V(JMAX) IS NEGATIVE')
      PRINT 211, JMAX
  211 FORMAT('-',T5,'JMAX=',T10,I3)
 2607 CONTINUE
C
C     ....THE MINIMUM OF DELD,DELV,DELS,DELSS DETERMINES WHERE TO GO
C
C     ....DEFINE DELD....
      DELD=XU(ND(JMAX))-X(ND(JMAX))
C
C     ....CALCULATE DELS....
C     FOR NEGATIVE DELTA(I,JMAX), AND NON-NEG STATE VARIABLES, THE MINIM
C     ABS(X(NS(I))/DELTA(I,JMAX)) IS DETERMINED
C
      N8=NF+1
      IF(NN(N+K-KE).EQ.7) N8=1
      IIMIN=1
      AAMIN=10.E10
      DO 1900 I=N8,K
      IF(D(I,JMAX).GE.0.0) GO TO 1901
      XO(I)=(X(NS(I))-XL(NS(I)))/D(I,JMAX)
      IF(ABS(XO(I)).LT.AAMIN) GO TO 1902
      GO TO 1901
 1902 CONTINUE
      AAMIN=ABS(XO(I))
      IIMIN=I
 1901 CONTINUE
 1900 CONTINUE
C
C     END OF LOOP, AAMIN=ABS(XO(IIMIN)) AND IIMIN DETERMINED
      DELS=AAMIN
C
```

```
C     ....CALCULATE DELSS....
      IUMIN=1
      UMIN=10.E10
      DO 30 I=N8,K
      IF(D(I,JMAX).LE.0.0) GO TO 30
      XO(I)=(XU(NS(I))-X(NS(I)))/D(I,JMAX)
      IF(XO(I).LT.UMIN) GO TO 32
      GO TO 30
   32 CONTINUE
      UMIN=XO(I)
      IUMIN=I
   30 CONTINUE
C
C     ....UMIN=XO(IUMIN) IS NOW DETERMINED....
      DELSS=UMIN
      IF(IGLOB.EQ.1) GO TO 2610
C
      IF((DELS.EQ.0.0.OR.DELSS.EQ.0.0).AND.IPRINT.NE.1) PRINT 8127
 8127 FORMAT('-',T5,'DEGENERATE CASE',/'0')
      IF(DELS.EQ.0.0.OR.DELSS.EQ.0.0) NSTAR=NSTAR-1
C
C     ....CALCULATE DELV....
      IF(B(JMAX,JMAX).LE.0.000) DELV=10.E11
      IF(B(JMAX,JMAX).GT.0.0) DELV=ABS(V(JMAX)/B(JMAX,JMAX))
C
 2610 CONTINUE
C     ....FIND MIN(DELD,DELV,DELS,DELSS)....
      IF(IGLOB.EQ.1) DELV=10.E11
      DDMIN=AMIN1(DELD,DELV,DELS,DELSS)
      IF(DDMIN.EQ.DELV) ICASE2=2
      IF(DDMIN.EQ.DELD) ICASE2=4
      IF(DDMIN.EQ.DELS) ICASE2=3
      IF(DDMIN.EQ.DELSS) ICASE2=5
C
C     ....PRINT FINDINGS....
      IF(IPRINT.EQ.1) GO TO 2608
      PRINT 222, DELV,DELS,DDMIN
  222 FORMAT('-'/T5,'MAXIMUM POSSIBLE CHANGE OF THE DECISION',
     1' X(ND(JMAX)):'
     2/'0',T5,'DELV=',G12.5,2X,'DELS=',G12.5,2X,'DDMIN=',G12.5)
      IF(DDMIN.EQ.DELV) PRINT 220,JMAX
  220 FORMAT('-'/T5,'CASE B2 : THE CONSTRAINED DERIVATIVE',
     1' V(JMAX) GOES TO ZERO'/'0',T5,'JMAX=',T10,I3,T15,'ICASE2=2')
      IF(DDMIN.EQ.DELS) PRINT 221,IIMIN
  221 FORMAT('-'/T5,'CASE B3 : THE STATE X(NS(IIMIN)) GOES TO ZERO'
     1,/'0',T5,'IIMIN=',T10,I3,T15,'ICASE2=3')
      IF(DDMIN.EQ.DELD) PRINT 34,JMAX
   34 FORMAT('0'/T5,'CASEB4 : THE DECISION X(ND(JMAX))',
     1' REACHES ITS UPPER LIMIT'/'0',T5,'JMAX=',T10,I3,T15,'ICASE2=4')
      IF(DDMIN.EQ.DELSS) PRINT 33, IUMIN
   33 FORMAT('-'/T5,'CASEB5 : THE STATE X(NS(IUMIN))',
     1' REACHES ITS UPPER LIMIT'/'0',T5,'IUMIN=',T10,I3,T15,'ICASE2=5')
 2608 CONTINUE
C
C     ....RETURN NEGATIVE DELV AND DELS....
      DELD=-DELD
      DELV=-DELV
      DELS=-DELS
      DELSS=-DELSS
      RETURN
      END
```

```
      SUBROUTINE CASEB1(JMAX,DELD,IC)
C
C*********************************************************************
C     THIS SUBROUTINE HANDLES THE CASE A1,B1 AND A2,B4
C       1. SAME PARTITION AS PREVIOUSLY
C     2. DECISION VARIABEL GOES TO LOVER LIMITS  IC=1
C     3. DECISION VARIABEL GOES TO ITS UPPER LIMITS  IC=4
C*********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CONST 7/IPRINT
      COMMON/KONST 1/IFREQ
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/CONST 1/N,NF,K,KE
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C
      IF(IPRINT.NE.1.AND.IC.EQ.1) PRINT 1923,JMAX,XL(ND(JMAX))
1923  FORMAT('-'/,T5,'THE DECISION JMAX GOES TO ITS LOWER LIMITS ',/,
     1'    JMAX =',I3,'     LOWER BOUND= ',G8.5)
      IF(IPRINT.NE.1.AND.IC.EQ.4) PRINT 1924,JMAX,XU(ND(JMAX))
1924  FORMAT('-'/,T5,'THE DECISION JMAX GOES TO ITS UPPER LIMITS ',/,
     1'    JMAX =',I3,'     UPPER BOUND= ',G8.5)
C
C     CALCULATE NEW STATE VARIABLES
C
      DO 10 I=1,K
      X(NS(I))=X(NS(I))-D(I,JMAX)*DELD
   10 CONTINUE
C
C     DEFINE NEW DECISION VARIABLES

C
      X(ND(JMAX))=XL(ND(JMAX))
      IF(IC.EQ.4) X(ND(JMAX))=XU(ND(JMAX))
C
      IF(IGLOB.EQ.1) CALL PRINT(KOUNT,IFREQ)
      IF(IGLOB.EQ.1) RETURN
C     ....CALCULATE NEW CONSTRAINED DERIVATIVES....
      N9=N-KE
      DO 11 J=1,N9
      V(J)=V(J)-B(J,JMAX)*DELD
   11 CONTINUE
C
      CALL PRINT(KOUNT,IFREQ)
      RETURN
      END
```

```
C
      SUBROUTINE CASEB2(JMAX,DELV)
C
C*************************************************************************
C       ....THIS SUBROUTINE HANDLES CASE B2 :
C           A CONSTRAINED DERIVATIVE V(JMAX) GOES TO ZERO
C           THE PARTITION REMAINS UNCHANGED
C*************************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/CONST 7/IPRINT
      COMMON/KONST 1/IFREQ
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/CONST 1/N,NF,K,KE
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C
      IF(IPRINT.EQ.1) GO TO 2617
      PRINT 1923
 1923 FORMAT('-'/T5,'CASE B2 : THE CONSTRAINED DERIVATIVE',
     1'V(JMAX) GOES TO ZERO')
      PRINT 1924, JMAX
 1924 FORMAT('-'/,T5,'JMAX=',T10,I3)
 2617 CONTINUE
C
C     CALCULATE NEW STATE VARIABLES
C
      DO 10 I=1,K
      X(NS(I))=X(NS(I))-D(I,JMAX)*DELV
   10 CONTINUE
C
C     ....CALCULATE NEW DECISION VARIABLES.....
      X(ND(JMAX))=X(ND(JMAX))-DELV
C     ....CALCULATE NEW CONSTRAINED DERIVATIVES....
      N9=N-KE
      DO 11 J=1,N9
      V(J)=V(J)-B(J,JMAX)*DELV
   11 CONTINUE
C
      CALL PRINT(KOUNT,IFREQ)
      RETURN
      END
```

```
      SUBROUTINE CASEB3(JMAX,IMIN,DELS,IC)
C***********************************************************************
C     THIS SUBROUTINE HANDLES THE CASE
C     IF IC=3 THE DECISION JMAX IS CHANGED UNTIL THE STATE IMIN GOES TO
C     LOWER LIMITS
C     IF IC=5 THE DECISION JMAX IS CHANGED UNTIL THE STATE IMIN GOES TO
C     UPPER LIMITS
C     JMAX AND IMIN ARE SIMPLEXED AND NEV CONSTRAINED DERIVATIVES AND
C     DELTAS ARE CALCULATED
C***********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/KONST 1/IFREQ
      COMMON/CONST 7/IPRINT
C     THE MATRIX B( , ) STORES THE TAU( , ) COEFFICIENTS
C     THE MATRIX D( , ) STORES THE DELTA( , ) COEFFICIENTS
C
      IF(IPRINT.NE.1.AND.IC.EQ.3) PRINT 10,JMAX,IMIN
10    FORMAT('-'/,T5,' THE DECISION JMAX IS CHANGED UNTIL THE',
     1' STATE IMIN REACHES ITS LOWER LIMITS    JMAX=',I3,
     2'      IMIN=',I3)
      IF(IPRINT.NE.1.AND.IC.EQ.5) PRINT 11,JMAX,IMIN
11    FORMAT('-'/,T5,' THE DECISION JMAX IS CHANGED UNTIL THE',
     1' STATE IMIN REACHES ITS UPPER LIMITS    JMAX=',I3,
     2'      IMIN=',I3)
C
C     CHANGE PARTITION,SIMPLEXING THE DECISION X(IP) AND THE STATE X(IR)
C
      IP=ND(JMAX)
      IR=NS(IMIN)
      IF(IGLOB.EQ.1) GO TO 100
      NS(IMIN)=IP
      ND(JMAX)=IR
  100 CONTINUE
C
C     CALCULATE NEW STATE VARIABLES
C
      XOIP=X(IP)
      DO 1500 I=1,K
      X(NS(I))=X(NS(I))-D(I,JMAX)*DELS
 1500 CONTINUE
      X(IP)=XOIP-DELS
C
C     DEFINE NEW DECISION VARIABLES
C
      X(IR)=XL(IR)
      IF(IC.EQ.5) X(IR)=XU(IR)
C
      IF(IGLOB.EQ.1) CALL PRINT(KOUNT,IFREQ)
      IF(IGLOB.EQ.1) RETURN
C
C     ....CALCULATE CONSTRAINED DERIVATIVES....
      VOP=V(JMAX)
      N9=N-KE
      DO 13 J=1,N9
      DR=D(IMIN,J)/D(IMIN,JMAX)
      V(J)=V(J)-VOP*DR-DELS*(B(J,JMAX)-B(JMAX,JMAX)*DR)
   13 CONTINUE
      V(JMAX)=(VOP-B(JMAX,JMAX)*DELS)/D(IMIN,JMAX)
C
C     CALCULATE NEW DELTAS
C
      CALL NEWVAL(JMAX,IMIN)

      CALL PRINT(KOUNT,IFREQ)
      RETURN
```

```
C
      SUBROUTINE PRINT(KOUNT,IFREQ)
C
C***********************************************************************
C     THIS SUBROUTINE PRINTS TABLES OF CORRESPONDENCE, AND VALUES OF THE
C     VE FUNCTION
C     IF IPRINT=0 ALL SORTS OF DEBUGGING PRINTOUTS ARE PROVIDED
C     IF IPRINT=1 ONLY INPUT,TABLES OF CORRESPONDANCE, AND SOLUTION WILL
C     TED. FREQUENCY OF PRINTOUTS ARE DETERMINED BY IFREQ.
C           IFREQ=0  ONLY INPUT AND SOLUTION PRINTED
C           IFREQ=1  TABLE OF CORR PRINTED AT EACH LEVEL
C           IFREQ=5  TABLE OF CORR PRINTED AR EACH 5 LEVEL
C           IFREQ=10 TABLE OF CORR PRINTED AT EACH 10 LEVEL
C***********************************************************************
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/GLOB 2/XLOC(50),XBEST(50),Y
      COMMON/OPT/ IENTRY,ICOST,IOUT,NBIT,LPIT,ILP,LP,LPMAX,N0PMAX
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 7/IPRINT
      N9=N-KE
      IF(IGLOB.EQ.1.AND.IFREQ.EQ.1) GO TO 101
      IF(IGLOB.EQ.1.AND.IFREQ.EQ.0) GO TO 200
      IF(IFREQ.EQ.0) GO TO 100
      IF(IFREQ.EQ.1) GO TO 101
      IFIVE=0
      ITEN=0
      FIVER=FLOAT(KOUNT)/5.0
      FIVEI=FLOAT(KOUNT/5)
      TENR=FLOAT(KOUNT)/10.0
      TENI=FLOAT(KOUNT/10)
      IF(FIVER.GT.FIVEI*0.999.AND.FIVER.LT.FIVEI*1.001) IFIVE=1
      IF(TENR.GT.TENI*0.999.AND.TENR.LT.TENI*1.001) ITEN=1
      IF(IFREQ.EQ.5.AND.IFIVE.EQ.0) GO TO 100
      IF(IFREQ.EQ.10.AND.ITEN.EQ.0) GO TO 100
  101 CONTINUE
C     PRINT TABLE OF CORRESPONDENCE
C
      PRINT 1905
 1905 FORMAT(1H1,T5,'NEW TABLE OF CORRESPONDENCE'/'-')
      PRINT 1906
 1906 FORMAT(T5,'STATE VARIABLES'/'0',T8,'I',T10,'NS(I)',T17,
     1'X(NS(I))'/'0')
      PRINT 1907,(I,NS(I),X(NS(I)),I=1,K)
 1907 FORMAT(1H0,T6,I3,T11,I3,T15,G12.5)
      PRINT 1908
 1908 FORMAT('-'/T5,'DECISION VARIABLES'/'0',T8,'J',T10,'ND(J)',T17,
     1'X(ND(J))'/'0')
      PRINT 1909,(J,ND(J),X(ND(J)),J=1,N9)
 1909 FORMAT(1H0,T6,I3,T11,I3,T15,G12.5)
  200 CONTINUE
C
```

```
C
C     CALCULATE AND PRINT NEW VALUE OF OBJECTIVE FUNCTION
C
      IF(IPRINT.EQ.1.AND.IGLOB.NE.1) GO TO 2607
      Y=0.0
      DO 1910 I=1,N
      Y=Y+CA(I)*X(I)
 1910 CONTINUE
      YY=0.0
      DO 2677 I=1,N
      DO 2677 J=1,N
      YY=YY+Q(I,J)*X(I)*X(J)
 2677 CONTINUE
      Y=Y+0.5*YY
      IF(IOUT.NE.1) PRINT 1911,Y
 1911 FORMAT('-'/,T5,'NEW VALUE OF OBJECTIVE FUNCTION,Y=',T50,G12.5,/)
  100 CONTINUE
      IF(IGLOB.EQ.1) RETURN
C     PRINT CONSTRAINED DERIVATIVES
C
      IF(IPRINT.EQ.1) GO TO 2607
      PRINT 900
  900 FORMAT('-'/T5,'CONSTRAINED DERIVATIVES V(J)')
      PRINT 901
  901 FORMAT('-'/T5,'INDEX OF DECISION VARIABLE',T40,
     1'CONSTRAINT       DERIVATIVE'/'0',T17,'J',T48,'V(J)',/'-')
      N5=1
      PRINT 902, (J,V(J),J=1,N9)
  902 FORMAT(T15,I3,T44,G12.5)
 2607 CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE TARGET
C *************************************************
C * FORMULATES QUADRATIC GOAL-PROGRAMMING PROBLEM   *
C *************************************************
C
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/BUNCH 1/ISTART,IMAX,JMAX,JSTART(35),JEND(35)
      COMMON/BUNCH 3/NCELL(35,22),NCHN(35,22),ICH(62),JCH(62),ICF(62)
     *,ISW(376)
      COMMON/CHUNK 1/DTR(35,22),DTU(35,22),T(35,22)
      COMMON/CHUNK 2/ACCRX(35,22),ACCRN(35,22),PMAX(35,22),PMIN(35,22),
     *XM(35,22),SI(35,22)
      COMMON/STAQ/SST(35,22),APS(35,22),ISA(376)
      COMMON/SWC 1/NSUB,SWMIN(4),SWMAX(4),CHSMIN(3),CHSMAX(3)
      COMMON/AAA/TAR(35,22)
      COMMON/YAZ/SD(35,22),S2(384)
C
      COMMON/FASTI 1/IPR,IREAD,Y
      COMMON/KONST 7/ACC
      COMMON/BLOCK 1/CA(683),AA(384,683),R(384),B(384,384),D(384,327),
     *V(327)
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/BLOCK 3/NS(356),ND(327),NN(683)
      COMMON/QUA 1/Q(327,327)
      COMMON/CONST 1/N,NF,K,KE
      COMMON/CONST 7/IPRINT
      COMMON/KONST 1/IFREQ
      COMMON/CONST 8/KOUNT,NIMAX,LP
      COMMON/CONST 3/EPSY,EPSV,EPSCO,EPSD
      COMMON/GLOB 1/IGLOB,IGMAX,IG
      COMMON/POUT/IOUT
C
       DIMENSION SCR(35,22),FACTOR(327),SCR2(35,22)
C
      ISCR=10
      IQ=24
      ITOUT=7
      IMAP=9
C
C SET SCRATCH DRAWDOWNS TO ZERO
      DO 2 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 2 J=JBEGIN,JSTOP
        SCR(I,J)=0.0
        SCR2(I,J)=300.-TAR(I,J)
         DO 2 L=1,NCH
          IF(ICH(L).EQ.I.AND.JCH(L).EQ.J)SCR(I,J)=SI(I,J)
2      CONTINUE
C
C  READ IN SD VALUES FROM FT11F001
       DO 16 K1=1,ICELL
       READ(11,3000) I,J,XX
       SD(I,J)=XX
16     CONTINUE
C
      SUMSI=0.0
      CONST=0.0
C
       L3=NVAR*2
C
      WRITE(ITOUT,8787)KCH
      WRITE(ITOUT,8786)NCHSUB,NSUB
8786  FORMAT(1X,'NCHSUB = ',I5,' NSUB = ',I5)
8787  FORMAT(1X,'KCH = ',I5)
      DO 27 I=1,NCHSUB
       L4=NVAR*2+KCH+I
       XU(L4)=CHSMAX(I)-CHSMIN(I)
       XL(L4)=0.0
       CA(L4)=0.0
       ITYPE(L4)=1
27    CONTINUE
```

```
C
      DO 60 I=1,NSUB
       L5=NVAR*2+KCH+NCHSUB+I
       XU(L5)=SWMAX(I)-SWMIN(I)
       XL(L5)=0.0
       CA(L5)=0.0
       ITYPE(L5)=1
60    CONTINUE
C
      DO 100 I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 100 J=JBEGIN,JSTOP
       L=NCELL(I,J)
       L2=L+NVAR
       IF(L.GT.0)GOTO 43
       L=NVAR+NCHN(I,J)
       IF(ISW(L).EQ.0)L3=L3+1
          XU(L3)=ACCRX(I,J)-ACCRN(I,J)
C         WRITE(ITOUT,1200)L3,XU(L3),ACCRX(I,J)
          CA(L3)=0.0
          ITYPE(L3)=1
          XL(L3)=0.0
          GOTO 100
43    CONTINUE
          FACTOR(L)=0.0
C
C  UPPER AND LOWER LIMITS (L2) ON GW PUMPING NOT ON TOTAL DISCHARGE.
        XU(L2)=PMAX(I,J)-PMIN(I,J)
        XL(L2)=0.0
        XL(L)=0.0
        XO(L2)=0.0
        IF(XL(L).GT.XU(L))XL(L)=XU(L)
        ITYPE(L)=1
        ITYPE(L2)=1
        CA(L2)=0.00
C       X(L)=SI(I,J)
        S2(L)=SD(I,J)*SD(I,J)
C
       CA(L)=SCR2(I,J)*2.0*(-1.0)/S2(L)
        SUMSI=SUMSI+SCR2(I,J)*SCR2(I,J)
        CONST=CONST+SCR2(I,J)*SCR2(I,J)/S2(L)
C
100    CONTINUE
C
C  FILL Q-ARRAY WITH MATRIX OF QUADRATIC TERMS.
      DO 200 L=1,NVAR
      DO 200 M=1,NVAR
       Q(L,M)=0.0
       IF(L.EQ.M)Q(L,M)=2.0/S2(L)
200   CONTINUE
```

```
C
C CHECK IF THE Q MATRIX IS POSITIVE DEFINITE.
       IF(IPDM.EQ.1) CALL DETERM
C
       WRITE(ITOUT,2223)CONST
C
C                         INITIAL FEASIBLE SOLUTION
C IF ISUS=3 INITIAL SOLUTION IS GENERATED BY SUBROUTINE GSIMEQ
       IF(ISUS.EQ.3) CALL GSIMEQ(FACTOR)
       IF(ISUS.EQ.3) GOTO 456
      DO 234 I=1,NVAR
C  MAXIMUM SUSTAINED-YIELD (LPMIN OUTPUT) IS USED AS INITIAL SOLUTION.
      IF(ISUS.EQ.1) GOTO 123
C  INITIAL SOLUTION IS READ FROM AN INPUT FILE ON UNIT 25.
      IF(ISUS.EQ.2) READ(25,345) X(I)
123   XO(I)=X(I)
234   CONTINUE
345   FORMAT(15X,F15.7)
456   CONTINUE
C
      LP=Ø
      EPSY=Ø.Ø
      NIMAX=2ØØØ
       N=NVAR
       NF=Ø
       K=NVAR+KCH+NCHSUB+NSUB
        IF(IRCH.EQ.Ø)K=NVAR+NSUB
       KE=Ø
C
C  TEST THE INITIAL SOLUTION TO SEE IF ANY BOUNDS ARE VIOLATED.
      DO 15Ø I=ISTART,IMAX
       JBEGIN=JSTART(I)
       JSTOP=JEND(I)
      DO 15Ø J=JBEGIN,JSTOP
       L=NCELL(I,J)
       IF(L.LE.Ø)GOTO 15Ø
        IF(XL(L).LE.XO(L))GOTO 11Ø
         TEST=XL(L)-XO(L)
         XL(L)=XO(L)
         WRITE(ITOUT,1ØØØ)I,J,TEST
11Ø     CONTINUE
        IF(XU(L).GE.XO(L))GOTO 12Ø
         TEST=XO(L)-XU(L)
          TEST2=XU(L)
         XU(L)=XO(L)
         WRITE(ITOUT,2ØØØ)I,J,TEST,TEST2
12Ø     CONTINUE
15Ø     CONTINUE
C
C PRINT OUTPUT FROM INITIAL SOLUTION
        IF(ISUS.EQ.1) WRITE(IMAP,1Ø21)
        IF(ISUS.EQ.2) WRITE(IMAP,1Ø22)
        IF(ISUS.EQ.3) WRITE(IMAP,1Ø23)
        CALL INFOUT(X)
C
C  PERFORM OPTIMIZATION
       CALL QPTHOR
C
1ØØØ   FORMAT(/,1X,'LOWER BOUND FOR CELL ',I2,1X,I2,' DECREASED BY ',
     *F1Ø.2)
1Ø21  FORMAT('1',1X,'OUTPUT FROM INITIAL SOLUTION (MAX. PUMPING)',//)
1Ø22  FORMAT('1',1X,'OUTPUT FROM INITIAL SOLUTION (READ AS INPUT)',//)
1Ø23  FORMAT('1',1X,'OUTPUT FROM INITIAL SOLUTION (MIN. PUMPING)',//)
12ØØ  FORMAT(1X,I5,2X,F1Ø.2,2X,F1Ø.2)
2ØØØ  FORMAT(/,1X,'UPPER BOUND FOR CELL ',I2,1X,I2,' INCREASED BY ',
     *F1Ø.2,1X,'WAS = ',F1Ø.2)
2223  FORMAT(1X,'CONST = ',E12.6)
[illegible]  FORMAT(2I5,F1Ø.3)
[illegible]  FORMAT(1X,'FACTOR(',I3,') = ',F5.3)
       RETURN
       END
```

```
      SUBROUTINE CHECK
C ***********************************************************
C * CHECKS IF OPTIMAL SOLUTION VECTOR IS WITHIN ITS BOUNDS *
C ***********************************************************
C
      COMMON/BLOCK 2/X(683),ITYPE(683),XO(683),XU(683),XL(683)
      COMMON/CONST 1/N,NF,K,KE
C
      ITOUT=7
      IMAP=9
C
       WRITE(ITOUT,345)
       WRITE(ITOUT,456)
       NT=N+K-KE
       DO 2 I=1,NT
        IF(X(I).GE.XL(I).AND.X(I).LE.XU(I)) GOTO 1
        WRITE(ITOUT,123) XL(I),X(I),XU(I)
        GOTO 2
 1      WRITE(ITOUT,234) XL(I),X(I),XU(I)
 2     CONTINUE
       WRITE(ITOUT,567)
123    FORMAT(/,3F15.7,5X,'VIOLATED',/)
234    FORMAT(3F15.7)
345    FORMAT('1',//,5X,'CHECKING IF ANY BOUND IS  VIOLATED BY THE OPTIM
     *AL SOLUTION',//)
456    FORMAT(5X,'XL(I)',10X,'X(I)',11X,'XU(I)',/)
567    FORMAT(//)
       RETURN
       END
```

```
      SUBROUTINE DETERM
C ***********************************************************************
C * CHECKS MATRIX OF QUADRATIC TERMS Q(L,M) TO DETRMINE IF ITS A
C * POSITIVE DEFINITE MATRIX.  THE METHOD USED IS THE METHOD OF
C * PRINCIPLE MINORS- THE DETRMINANT OF EACH MINOR MUST BE .GE. Ø
C * TO BE SEMI-POS. DEF. AND .GT. Ø TO BE POS. DEF.
C ***********************************************************************
C
      COMMON/ONE/ICELL,NVAR,NCH,NSA,IRCH,ITER,NCHSUB,KCH,ISUS,ICHK,IPDM
      COMMON/QUA 1/Q(327,327)
C
      DIMENSION XX(315,315)
      ITOUT=7
      ICH=Ø
      WRITE(ITOUT,1ØØØ)
C
      DO 5Ø I=1,NVAR
      DO 5Ø J=1,NVAR
       XX(I,J)=Q(I,J)
5Ø    CONTINUE
C
       IF(XX(1,1).EQ.Ø.Ø)GOTO 3ØØ
       L=NVAR-1
       M=NVAR+1
      DO 1ØØ K=1,L
       KP1=K+1
      DO 2ØØ I=KP1,NVAR
       IM1=I-1
       IF(XX(I,I).EQ.Ø.Ø)ICH=1
       IF(XX(I,I).LE.Ø.Ø)GOTO 3ØØ
        XQ=XX(I,K)/XX(K,K)
        DO 2ØØ J=KP1,M
         XTEMP=XX(I,J)
         XX(I,J)=XTEMP-XQ*XX(K,J)
2ØØ     CONTINUE
       DO 1ØØ I=KP1,NVAR
        XX(I,K)=Ø.Ø
1ØØ    CONTINUE
C
      WRITE(ITOUT,2ØØØ)
      GOTO 4ØØ
3ØØ   CONTINUE
      IF(ICH.EQ.Ø)GOTO 35Ø
      WRITE(ITOUT,4ØØØ)
      GOTO 4ØØ
35Ø   CONTINUE
      WRITE(ITOUT,3ØØØ)IM1
4ØØ   CONTINUE
C
C     DO 5ØØ I=1,NVAR
C     WRITE(ITOUT,6ØØØ)I
C     WRITE(ITOUT,5ØØØ)(XX(I,J),J=1,NVAR)
5ØØ   CONTINUE
1ØØØ  FORMAT(/,1X,'MATRIX OF QUADRATIC TERMS, Q(L,M), IS ....')
2ØØØ  FORMAT(1X,'A POSITIVE DEFINITE MATRIX.',/)
3ØØØ  FORMAT(1X,'NOT A POSITIVE DEFINITE MATRIX DUE TO CELL ',I4,/)
4ØØØ  FORMAT(1X,'A SEMI-POSITIVE DEFINITE MATRIX.',/)
5ØØØ  FORMAT(8G1Ø.2)
6ØØØ  FORMAT(I3)
      RETURN
      END
```