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MANAGEMENT OF IRRIGATION AND DRAINAGE SYSTEMS:
INTEGRATED PERSPECTIVES

Proceedings of the 1993 NATIONAL CONFERENCE ON IRRIGATION AND DRAINAGE ENGINEERING
Park City, Utah
July 21-23, 1993

Sponsored by the
Irrigation and Drainage Division
of the American Society of Civil Engineers

Edited by Richard G. Allen ***and Christopher M.U. Neale***

Published by the
American Society of Civil Engineers
345 East 47th Street
New York, New York 10017-2398

OPTIMIZING CONJUNCTIVE WATER USE IN A DYNAMIC STREAM-AQUIFER SYSTEM WITH US/REMAX

Getachew Belaineh[1] *and Richard C. Peralta,*[2] *A.M. ASCE*

ABSTRACT

Long-term water management planning models frequently use large time steps and must employ fairly crude assumptions (such as average climatic conditions, etc.). Managing stream aquifer systems during a dry season requires using finer discretization in time and space. Presented is a computer model, US/REMAX, developed by Utah State University personnel for aiding best management of stream-aquifer systems for both long and short eras.

The model computes strategies for optimally allocating surface and ground water resources in time and space. For a water supply problem the model can maximize the sum of delivered surface and ground water. For an environmental protection problem the model can minimize total groundwater pumping (extraction plus injection) needed to capture contaminant plume. The model can provide optimal steady-state or time variant solutions. Weighting coefficients can be used in the objective function: (1) to emphasize substitution of surface water diversion for groundwater pumping or vice-versa, (2) or to achieve linear economic optimization.

[1]Graduate student, Dept. of Biological and Irrigation Eng., Utah State University, Logan, Utah.

[2]Professor, Dept. of Biological and Irrigation Eng., Utah State University, Logan,Utah.

INTRODUCTION

The usefulness of computer models as tools for solving water resource management problems is widely recognized. In this study a microcomputer based simulation/optimization(s/o) model US/REMAX is presented (Peralta et al.,1993). US/REMAX includes both simulation and optimization capabilities. It simulates system responses to unit groundwater pumping (extraction or injection) and river diversions. It organizes operations research type of optimization problems and calculates optimal strategies for a wide range of management problems. The optimal strategies are based upon user selected management criteria.

LITERATURE REVIEW

Linear programming models have been applied to a wide variety of groundwater or conjunctive water management problems. Most models have assumed system linearity. This has permitted use of linear influence coefficients, having additive and multiplicative properties, to describe system response to excitation. Such coefficients have also been termed discrete kernels or technological functions.

Numerical or analytical methods have been employed to compute influence coefficients in time and space. Usually numerical methods are required to calculate influences at cells and analytical methods are used for computing at points (wells). Maddock (1972) showed how to compute algebraic technological functions from analytical expressions for a two-dimensional homogenous aquifer. Verdin et al. (1981) developed a Fortran program which generates discrete kernels for points and cells in a two-dimensional heterogenous aquifer. Peralta et al.(1990) developed and used coefficients for both cells and wells.

Many s/o models based on the use of influence coefficients have been reported by numerous researchers, (Bredehoeft and Young, 1970; Lefkoff and Gorelick; 1987; Peralta et al., 1988; among others).

Most coefficients relate ground water pumping to aquifer head. Coefficients that describe the effect of diversion on the system are not widely used. This is due partially to the fact that models for simulating dynamic-aquifer systems are uncommon.To some extent it might also be due to the nonlinearity of the head-discharge relationship. When Peralta et al. (1988) addressed such a system they bounded stream stage to achieve system quasi linearity within the range of stages being considered.

Of the reviewed literature, the UTAC model (Peralta et al., 1990) reported the ability to optimize diversion and groundwater pumping while constraining stream flow and aquifer head. However, UTAC did not use influence coefficients for stream flow and was not easily transferable. Here the transferable US/REMAX s/o model addresses the same issue more efficiently than UTAC. US/REMAX calculates optimal time-varying conjunctive use strategies.

MODEL FORMULATION AND ASSUMPTIONS

Consider an area relying on an aquifer and a stream for its water supply. The objective is to maximize the water provided from both surface diversion and pumping wells to meet water demand in time and space. Let $g_{\bar{e}}(k)$ be the rate of water pumped from cell $\bar{e}$ during time period k (extraction is negative). Let $d_{\bar{e}}(k)$ be the rate of water diverted at diversion point $\bar{e}$ during time period k. The actual objective function minimizes the sum of negative pumping minus positive diversion.The effect is Eq.1.

$$\text{MAX:} \quad \sum_{k=1}^{n} \sum_{\bar{e}=1}^{M^{\bar{e}}} [c_{\bar{e}}^{g}(k) \; |g_{\bar{e}}(k)| \; + \; c_{\bar{e}}^{d}(k) \; d_{\bar{e}}(k)] \qquad (1)$$

In Equation 1 the weighting coefficients, $c_{\bar{e}}^{d}(k)$ and $c_{\bar{e}}^{g}(k)$, are all equal to one. All decision variables will have identical impact on the objective value. A decision variable can be de-emphasized by using a coefficient smaller than that used for other decision variables. By making a coefficient equal to zero, the variable becomes ineffective in the objective function.

The objective function in equation 1 can be optimized subject, for example to legal, economic, environmental, and social limitations or constraints. The aquifer head ($h_{\bar{o}}(k)$) must remain within a permissible range at each pumping cell.

Influence coefficients are used in equations describing head. Let $\delta^{h}_{\bar{o},\bar{e}}(n-k+1)$ and $\beta^{h}_{\bar{o},\bar{e}}(n-k+1)$ be influence coefficients describing the effect of groundwater pumping and stream diversion, respectively, at cell $\bar{e}$ in stress period k, on aquifer head at observation cell $\bar{o}$ by the end of period n. Equation 2 is the general expression used to constrain the potentiometric surface elevation at pumping cells, where $h_{\bar{o}}^{non}(k)$ and $h_{\bar{o}}(k)$ are nonoptimal and optimal heads of the system at control cells and $h^{L}_{\bar{o},n}$ is the lower bound on aquifer head.

$$h_{\bar{o},n}^{non} + \sum_{k=1}^{n} \sum_{\bar{e}=1}^{M^{\bar{e}}} [(\delta^{h}_{\bar{o},\bar{e}}(n-k+1)) \frac{g_{\bar{e}}(k)}{g_{\bar{e}}^{ut}} + (\beta^{h}_{\bar{o},\bar{e}}(n-k+1)) \frac{d_{\bar{o},\bar{e}}(k)}{d_{\bar{e}}^{ut}}] \geq h^{L}_{\bar{o},n} \qquad (2)$$

Similarly, stream diversion in any time period must not cause the minimum flow requirement for downstream legal users to be violated. Flow in the stream is a function of stream diversion and groundwater pumping. Let $f_{\bar{o}}(k)$ be reach outflow and $\delta^{f}_{\bar{o},\bar{e}}(n-k+1)$ and $\beta^{f}_{\bar{o},\bar{e}}(n-k+1)$ be influence coefficients describing the effect of groundwater pumping and stream diversion respectively, at cell $\bar{e}$ in stress period k, on stream reach outflow at cell $\bar{o}$ by the end of period n. The utilized constraint at stream flow control points is shown in equation 3. The notation $f^{L}_{\bar{o},n}$ represents the minimum acceptable flow rate.

$$f_{\bar{o},n}^{non} + \sum_{k=1}^{n} \sum_{\bar{e}=1}^{M^{\bar{e}}} [(\delta^{f}_{\bar{o},\bar{e}}(n-k+1)) \frac{g_{\bar{e}}(k)}{g_{\bar{e}}^{ut}} + (\beta^{f}_{\bar{o},\bar{e}}(n-k+1)) \frac{d_{\bar{e}}(k)}{d_{\bar{e}}^{ut}}] \geq f^{L}_{\bar{o},n} \qquad (3)$$

Equations 4 and 5 define upper and lower bounds on pumping.

$$g_{\bar{e}}^{L}(k) \leq g_{\bar{e}}(k) \leq g_{\bar{e}}^{U}(k) \qquad (4)$$

$$d_{\bar{e}}^{L}(k) \leq d_{\bar{e}}(k) \leq d_{\bar{e}}^{U}(k) \qquad (5)$$

Constraints can also be imposed on the sum of pumping or diversion from groups of cells. For example, let M_{ρ}^{pg} be the total number of pumping cells in pumping group ρ, M_{ν}^{dg} the number diversion reaches in diversion group ν and M_{λ}^{pdg} be the total number of cells and diversion in group λ. Equations 6 and 7 describe upper and lower bounds on summations of pumping and diversions respectively. Equation 8 defines bounds on the sum of pumping and diversion in the other group (Again, absulute values are used, for clarity).

$$(\Sigma g(k))_{\rho}^{L} \leq \sum_{\bar{e}=1}^{M_{\rho}^{pg}} (g_{\bar{e}}(k)) \leq (\Sigma g(k))_{\rho}^{U} \qquad (6)$$

for $\bar{e} \in pg\rho$

$$(\Sigma d(k))_{\nu}^{L} \leq \sum_{\bar{e}=1}^{M_{\nu}^{dg}} (d_{\bar{e}}(k)) \leq (\Sigma d(k))_{\nu}^{U} \qquad (7)$$

for $\bar{e} \in dg\nu$

$$(\Sigma |g_{\bar{e}}(k)| + d(k))_{\lambda}^{L} \leq \sum_{\bar{e}=1}^{M_{\lambda}^{pdg}} |g_{\bar{e}}(k)| + d_{\bar{e}}(k) \leq (\Sigma |g_{\bar{e}}(k)| + d_{\bar{e}}(k))_{\lambda}^{U} \qquad (8)$$

for $\bar{e} \in pdg\lambda$

Here superscripts U and L denote upper and lower limits respectively, and $pg\rho$, $dg\nu$, $pdg\lambda$ represent sets of groups of excitation cells.

APPLICATION AND RESULTS

Simulation and pre-optimization. US/REMAX reads data and computes system responses to stimuli. These include non-optimal potentiometric heads, reach outflows and river stages. Next, the responses of the system that result from unit excitations (pumping or diversion) are computed. Differences between responses to nonoptimal stimuli and nonoptimal stimuli plus unit pulses are termed influence coefficients. Thus, the model can compute steady or unsteady-state coefficients for aquifer head, river reach outflow, and river stage.

To illustrate model applications a small hypothetical study area having a single layer aquifer and a stream is considered (Figure 1). The stream and aquifer are in hydraulic connection. They are surrounded by no-flow boundaries. The only flows to the system are vertical recharge due to rainfall and the stream inflow.

In this study four sets of influence coefficients are computed: (1) change in aquifer head due to pumping (extraction), (2) change in stream flow due to pumping, (3) change in head due to stream diversion, and (4) change in stream flow due to diversion. These influence coefficients are subsequently used in constraint Equations 2 and 3 via superposition. Two control locations for observing aquifer responses, and two other control locations for observing stream responses are used. When generating influence coefficients the system is excited with a known pulse during one stress period and responses are observed during that and subsequent stress periods. A negative influence coefficient describes a decrease in head or flow. Table 1 presents influence coefficients describing the effect of pumping and diversion on stream flow.

Table 1. Influence coefficients describing the effect of pumping and diversion on stream flow. (obs = observation location, and stm = stimulus location).

The effect of groundwater pumping (pulse of -100,000 m^3/day during week 1) on stream flow:

obs.stm	Time period (weeks) 1	2	3	4	5	6	7	8
1.1	-8287	-8610	-7460	-6333	-5345	-4507	-3801	-3210
1.2	0	0	0	0	0	0	0	0
2.1	-7619	-8290	-7097	-6052	-5115	-4316	-3642	-3081
2.2	-8551	-10480	-10120	-9138	-8049	-7019	-6099	-5294

The effect of stream diversion (pulse of 200,000 m^3/day during week 1) on stream flow:

obs.stm	Time period (weeks) 1	2	3	4	5	6	7	8
1.1	-175900	760	183	71	44	34	29	24
1.2	1	0	0	0	0	0	0	0
2.1	-160500	-3150	-1114	-535	-334	-247	-199	-167
2.2	-172000	-980	-617	-348	-218	-156	-124	-103

Notice that the effect of groundwater pumping on streamflow did not peak during the first week. This reflects the time-lag effect (Figure 2).

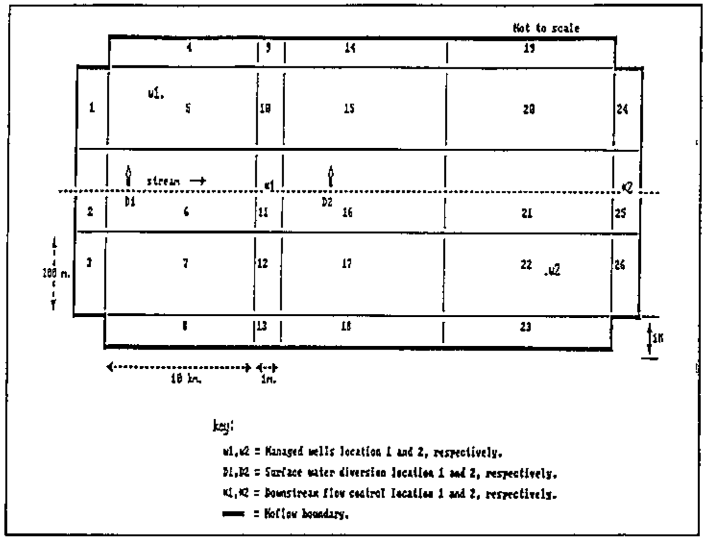

Figure 1. Hypothetical study area and cell indices.

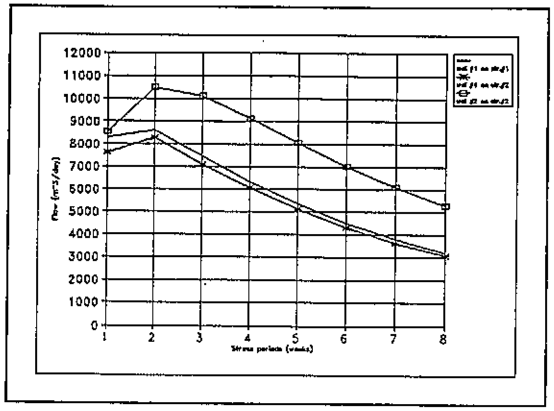

Figure 2. Figure showing the time-lagged effect of pumping on stream flow.

Table 2 Summary of the scenarios#.

OPTIMUTILIZED BOUNDS AND CONSTRAINTS										OPTIMAL STRATEGIES		
1	2	3	4	5	6	7	8	9	10	11	12	13
scenario	bound	f_1 (10^6 m^3/d)	f_2 (10^6 m^3/d)	h (m)	g (10^6 m^3/d) per pump cell	d (106 m^3/d) per div. cell	Σg (10^6 m^3/d) per pair of pump cell	Σd (10^6 m^3/d) per pair of div. cell	Σg+d (10^6 m^3/d) per each of four cells	g (10^6 m^3/d) time avg. per pump cell	d (10^6 m^3/d) time avg. per div. cell	Σg+d (10^6 m^3/d) time avg. per all cells
1	upp low	0.15*	0.10*	55.00 48.00	0.40* 0.00*	0.50* 0.00*				0.061	0.221	0.564
2	upp low	0.15*	0.10*	55.00* 50.00*	0.40* 0.00*	0.50* 0.00				0.026	0.235	0.522
3	upp low	0.15*	0.10*	55.00* 50.00*	0.40* 0.00*	3.00 0.00				0.014	0.266	0.560
4	upp low	0.25*	0.20*	55.00* 50.00*	0.40* 0.00*	3.00 0.00				0.019	0.197	0.433
5a	upp low	0.15*	0.10*	55.00* 49.00*	0.40* 0.00	0.50* 0.00	0.20* 0.00			0.025	0.237	0.524
5b	upp low	0.25*	0.20*	55.00* 50.00*	0.40* 0.00*	1.00* 0.00		1.00* 0.00		0.026	0.183	0.418
5c	upp low	0.07*	0.05*	55.00* 50.00*	0.40* 0.00*	1.00* 0.00			0.40* 0.00	0.009	0.189	0.396

For simplicity extraction is shown as a positive value.
* Tight constraints for at least one cell for at least one stress period.

Optimization. Once the influence coefficients have been generated and saved, optimal solutions subject to acceptable system responses can be computed for a wide range of scenarios. Here seven different management scenarios are tested. The first 10 columns of Table 2 summarize bounds and constraints used within the scenarios. The last three columns describe the resulting computed strategies.

The management objective is to maximize the water delivered by diversions from two points and by withdrawing groundwater (pumping) from two cells. The last three columns of Table 2 summarize the time average rates computed to be optimal for all 8 weeks (of course US/REMAX actually computes time varying rates for each stimulus location). Values in columns 11 and 12 are obtained by averaging the total values for each variable over the number of variables and stress period. The time average daily delivery rate is in column 13. This is determined by adding column 11 multiplied by the number of pumping cells to column 12 multiplied by the number of diversion points. To compute the total volume of water delivered during eight weeks one multiplies the column 13 value by 56, (7 days/week * 8 weeks).

SUMMARY AND CONCLUSIONS

US/REMAX has a demonstrated capability for calculating optimal conjunctive water management strategies. it can maximize the sum of unsteady groundwater pumping and surface water diversions for a management period of multiple stress periods. Optimal strategies can be subjected to constraints on aquifer head, stream flow, river stage, and other management considerations. US/REMAX utilizes the response matrix method and linear systems theory. It is perfectly applicable to linear systems but can be applied to nonlinear systems also by cycling.

US/REMAX will assist decision makers and water managers in developing and selecting the best surface and ground water use strategy for a wide range of management problems.

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LLOYD C. FOWLER
385 VIA EL ENCANTADOR
SANTA BARBARA CA 93111

805-964-5430

26 October 1992

Getachew Belaineh
Richard C. Peralta
Department of Biological and Irrigation Engineering
Utah State University
Logan UT 84322-4105

Thank you for submitting an abstract of a paper on "Optimizing Conjunctive Water Use in a Dynamic Stream Aquifer System" to be presented at the National Conference on Irrigation and Drainage Engineering in Park City Utah, to be held 21-23 July 1993. Your paper has been accepted and will be part of a Session on "Ground Water Management."

Now is the time to begin to prepare your paper. It will be printed in the advance Proceedings of the Conference. ASCE headquarters has mailed, or will soon mail, you instructions regarding the preparation of your paper. The page limit will be eight pages, single spaced. A copy of the final draft of the paper must be given to Rick Allen, Program Chairman, located in your Department at Utah State University, by 15 January 1993. Please send me a copy at the same time. The paper will be reviewed and comments returned to you by 28 February 1993. Final camera ready papers must be received by Rick Allen prior to 6 April 1993.

I look forward to receipt of your paper and to your presentation at the Conference.

Lloyd C Fowler

Lloyd C. Fowler, P. E.
Ground Water Management
Session Contact

cc Rick Allen

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In Equation 1 the weighting coefficients, $c_{\bar{e}}^{d}(k)$ and $c_{\bar{e}}^{g}(k)$, are all equal to one. All decision variables will have identical impact on the objective value. A decision variable can be de-emphasized by using a coefficient smaller than that used for other decision variables. By making a coefficient equal to zero, the variable becomes ineffective in the objective function.

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$$h^{non}_{\bar{o},n} + \sum_{k=1}^{n} \sum_{\bar{e}=1}^{M^{\bar{e}}} [(\delta^{h}_{\bar{o},\bar{e}}(n-k+1)) \frac{g_{\bar{e}}(k)}{g^{ut}_{\bar{e}}} + (\beta^{h}_{\bar{o},\bar{e}}(n-k+1)) \frac{d_{\bar{o},\bar{e}}(k)}{d^{ut}_{\bar{e}}}] \geq h^{L}_{\bar{o},n} \quad \mathbf{(2)}$$

Similarly, stream diversion in any time period must not cause the minimum flow requirement for downstream legal users to be violated. Flow in the stream is a function of stream diversion and groundwater pumping. Let $f_{\bar{o}}(k)$ be reach outflow and $\delta^{f}_{\bar{o},\bar{e}}(n-k+1)$ and $\beta^{f}_{\bar{o},\bar{e}}(n-k+1)$ be influence coefficients describing the effect of groundwater pumping and stream diversion respectively, at cell $\bar{e}$ in stress period k, on stream reach outflow at cell $\bar{o}$ by the end of period n. The utilized constraint at stream flow control points is shown in equation 3. The notation $f^{L}_{\bar{o},n}$ represents the minimum acceptable flow rate.

$$f_{\bar{o},n}^{non} + \sum_{k=1}^{n} \sum_{\bar{e}=1}^{M^{\bar{e}}} [(\delta_{\bar{o},\bar{e}}^{f}(n-k+1)) \frac{g_{\bar{e}}(k)}{g_{\bar{e}}^{ut}} + (\beta_{\bar{o},\bar{e}}^{f}(n-k+1)) \frac{d_{\bar{e}}(k)}{d_{\bar{e}}^{ut}}] \geq f_{\bar{o},n}^{L} \quad (3)$$

Equations 4 and 5 define upper and lower bounds on pumping.

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1.2	0	0	0	0	0	0	0	0
2.1	-7619	-8290	-7097	-6052	-5115	-4316	-3642	-3081
2.2	-8551	-10480	-10120	-9138	-8049	-7019	-6099	-5294

The effect of stream diversion (pulse of 200,000 m^3/day during week 1) on stream flow:

obs.stm	Time period (weeks) 1	2	3	4	5	6	7	8
1.1	-175900	760	183	71	44	34	29	24
1.2	1	0	0	0	0	0	0	0
2.1	-160500	-3150	-1114	-535	-334	-247	-199	-167
2.2	-172000	-980	-617	-348	-218	-156	-124	-103

Notice that the effect of groundwater pumping on streamflow did not peak during the first week. This reflects the time-lag effect (Figure 2).

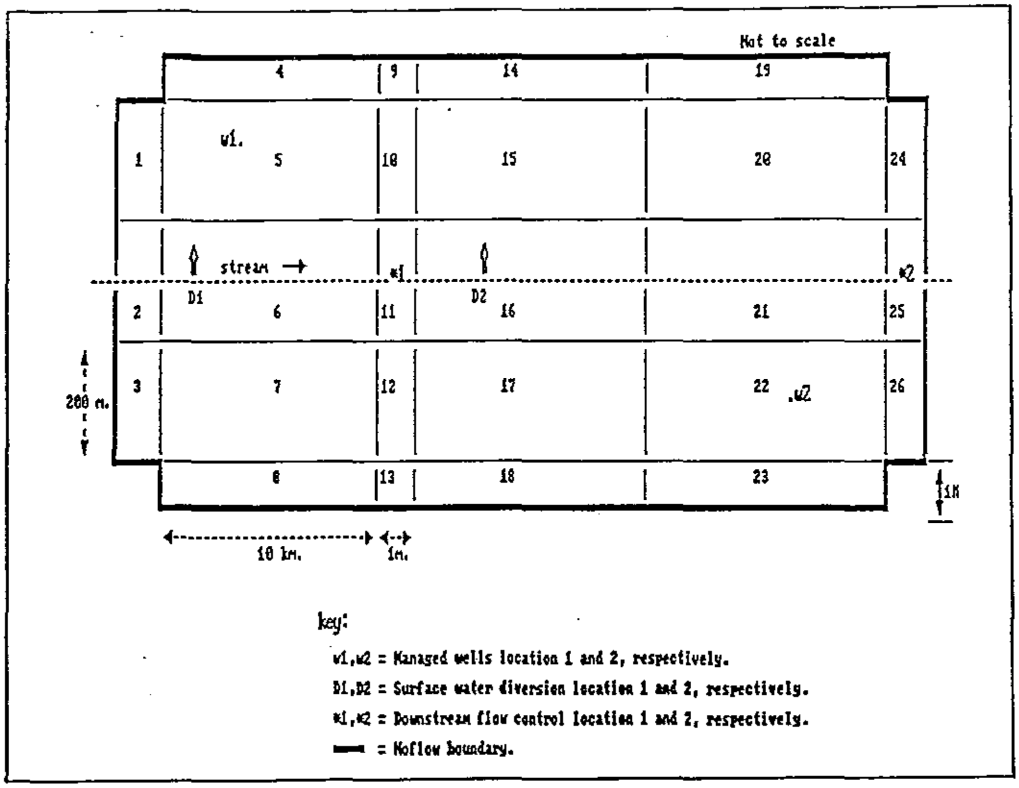

Figure 1. Hypothetical study area and cell indices.

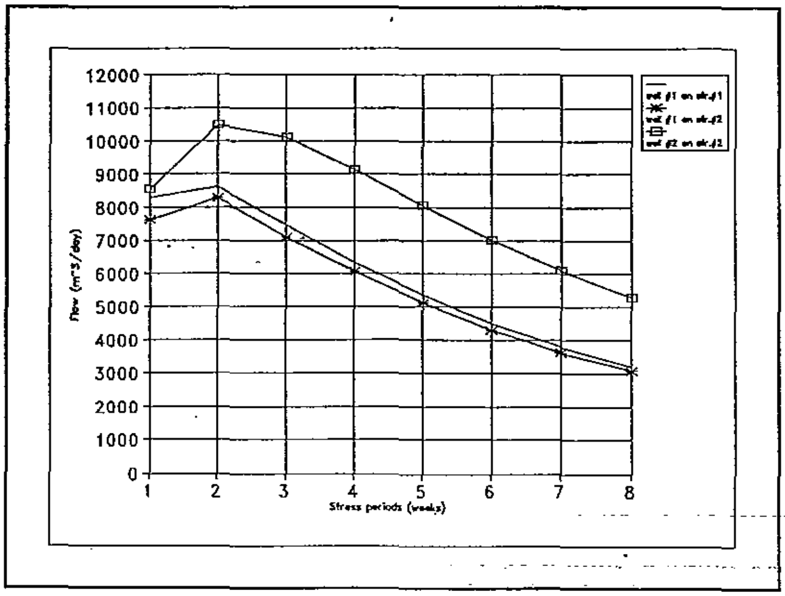

Figure 2. Figure showing the time-lagged effect of pumping on stream flow.

Table 2 Summary of the scenarios[#].

OPTIMUTILIZED BOUNDS AND CONSTRAINTS										OPTIMAL STRATEGIES		
1	2	3	4	5	6	7	8	9	10	11	12	13
scenario	bound	f_1 (10^6 m^3/d)	f_2 (10^6 m^3/d)	h (m)	g (10^6 m^3/d) per pump cell	d (106 m^3/d) per div. cell	Σg (10^6 m^3/d) per pair of pump cell	Σd (10^6 m^3/d) per pair of div. cell	Σg+d (10^6 m^3/d) per each of four cells	g (10^6 m^3/d) time avg. per pump cell	d (10^6 m^3/d) time avg. per div. cell	Σg+d (10^6 m^3/d) time avg. per all cells
1	upp low	0.15*	0.10*	55.00 48.00	0.40* 0.00*	0.50* 0.00*				0.061	0.221	0.564
2	upp low	0.15*	0.10*	55.00 50.00*	0.40 0.00*	0.50* 0.00				0.026	0.235	0.522
3	upp low	0.15*	0.10*	55.00 50.00*	0.40 0.00*	3.00 0.00				0.014	0.266	0.560
4	upp low	0.25*	0.20*	55.00 50.00*	0.40 0.00*	3.00 0.00				0.019	0.197	0.433
5a	upp low	0.15*	0.10*	55.00 49.00*	0.40* 0.00	0.50* 0.00	0.20* 0.00			0.025	0.237	0.524
5b	upp low	0.25*	0.20*	55.00 50.00*	0.40 0.00*	1.00* 0.00		1.00* 0.00		0.026	0.183	0.418
5c	upp low	0.07*	0.05*	55.00 50.00*	0.40 0.00*	1.00* 0.00			0.40* 0.00	0.009	0.189	0.396

For simplicity extraction is shown as a positive value.
* Tight constraints for at least one cell for at least one stress period.

Optimization. Once the influence coefficients have been generated and saved, optimal solutions subject to acceptable system responses can be computed for a wide range of scenarios. Here seven different management scenarios are tested. The first 10 columns of Table 2 summarize bounds and constraints used within the scenarios. The last three columns describe the resulting computed strategies.

The management objective is to maximize the water delivered by diversions from two points and by withdrawing groundwater (pumping) from two cells. The last three columns of Table 2 summarize the time average rates computed to be optimal for all 8 weeks (of course US/REMAX actually computes time varying rates for each stimulus location). Values in columns 11 and 12 are obtained by averaging the total values for each variable over the number of variables and stress period. The time average daily delivery rate is in column 13. This is determined by adding column 11 multiplied by the number of pumping cells to column 12 multiplied by the number of diversion points. To compute the total volume of water delivered during eight weeks one multiplies the column 13 value by 56, (7 days/week * 8 weeks).

SUMMARY AND CONCLUSIONS

US/REMAX has a demonstrated capability for calculating optimal conjunctive water management strategies. it can maximize the sum of unsteady groundwater pumping and surface water diversions for a management period of multiple stress periods. Optimal strategies can be subjected to constraints on aquifer head, stream flow, river stage, and other management considerations. US/REMAX utilizes the response matrix method and linear systems theory. It is perfectly applicable to linear systems but can be applied to nonlinear systems also by cycling.

US/REMAX will assist decision makers and water managers in developing and selecting the best surface and ground water use strategy for a wide range of management problems.

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Key words s/o, groundwater, conjunctive use,optimization,US/REMAX