CONJUNCTIVE USE OF SURFACE WATER AND GROUNDWATER FOR SUSTAINABLE AGRICULTURAL PRODUCTION

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Acknowledgments

I am thankful for the advice I received in preparing this document. I hope the resulting document and software will significantly benefit field personnel. The attached software is the most powerful tool that I know of for optimal field-level management of stream-aquifer systems.

I appreciate the valuable guidance of Fernando Chanduvi and Arumugam Kandiah, I am grateful for the aid of Lyman Willardson concerning salinity management, and suggestions by Jack and Andy Keller, Ron Bliesner, Rick Allen and Wynn Walker. I am grateful for the superb graphic and document preparation support of Colleen Gnehm and have immensely enjoyed working with Shengjun Wu in developing and testing CONJUS and preparing its user's manual and sample problems.

I own any shortcomings of the document and software. Users are welcome to offer suggestions concerning how both can be improved.

Executive Summary

This report is a guide to aid conjunctive water use in stream-aquifer systems. Conjunctive use refers to coordinated use of groundwater and surface water resources. This document discusses principles useful for field engineers and agricultural experts. It presents hydrologic and hydraulic principles and quantitative and qualitative parameters relating to sustainable conjunctive use. It provides tools and software.

Conjunctive use is more complicated than using a single water resource. Using either resource can affect availability of the other in time and space. Often, conjunctive water managers must attempt to satisfy increasing water needs while reconciling conflicting water user goals and legal/regulatory or societal systems.

Attempting to assure sustainable crop production via conjunctive use involves water quantity and quality issues. Assuring a sustainable regional water supply often requires carefully managing both groundwater and surface water. It requires considering water quality and its affect on soil, crop and environment. It requires adapting to existing water laws and practices or possibly helping their change.

This report also provides a simple computer program useful for these tasks. The program contains simple simulation models (here abbreviated S models) based upon analytical equations. The program also contains a general purpose simulation/optimization (S/O) model useful for many physical and institutional systems. This flexible S/O model permits using data of varied sophistication, including output from numerical finite difference or finite element simulation models used on site.

S and S/O models differ in purpose and utility. A S model is designed to simulate how a physical system will respond to stimulus (groundwater extraction or injection, stream stage change, recharge and others). This is an important role because one cannot optimize management of a complicated system unless one can predict how the system will respond to management.

A S/O model is designed to compute the best water management strategy[1] for a user-specified management problem. A modeler can use a S/O model to <u>calculate</u> a better management strategy than he can usually <u>develop</u> using a normal simulation (S) model. The difference in meaning between calculate and develop is significant.

Although normal simulation models have frequently been used to develop water management strategies, the best strategy developed in that way is usually not really optimal. It is only the best choice from among the strategies assumed and tested by the modeler. On the other hand, a strategy computed by a S/O model can be truly optimal--mathematically the best of an infinite number of possible strategies that will all satisfy the restrictions of a particular scenario. Personal experience has shown improvements of 10-40% in water management strategies between those developed using S/O models versus S models alone.

The report includes CONjunctive use Utility Software (CONJUS)--a S/O model that can address physical systems ranging widely in complexity. With little effort the user can

[1] A 'strategy' is a set of spatially and temporally distributed water fluxes that can be directly managed by man. For example, a conjunctive use strategy might be a set of groundwater pumping and surface water diversion rates. A wastewater loading strategy might be a set of rates of discharges to a stream. A loading and conjunctive use strategy addresses both issues. A management strategy is developed to address a particular management 'scenario' (posed management problem).

pose an optimization problem for a simple homogeneous stream-aquifer system. To calculate an optimal water management strategy CONJUS first computes influence coefficients based upon analytical expressions (such as the Theis Equation). At that point, the user can optionally replace the CONJUS-derived coefficients with more accurate influence coefficients computed using field data or more sophisticated numerical S models. This option is important because most systems are heterogeneous and nonuniform and are most accurately addressed by more exacting modeling. (However, providing numerical groundwater and surface flow simulation models and instruction in their calibration and use is beyond the scope of this report.)

After appropriate influence coefficients are selected, CONJUS performs optimization to calculate the mathematically best water use strategy for the user-specified problem. CONJUS-computed strategies are appropriate for many situations even though CONJUS assumes that the flow system can be described linearly. Through cycling, CONJUS can compute accurate pumping strategies even for heterogeneous and somewhat nonlinear systems.

CONJUS provides several objective functions to choose from. An objective function is a water management goal that the model user wants to maximize or minimize achievement of. CONJUS permits the user to impose a range of constraints that the computed optimal pumping strategy must satisfy. The constraints represent physical, environmental, legal or social restrictions. CONJUS is written in Visual Basic To simplify data organization and entry, and results post-processing, CONJUS and operates through Microsoft EXCEL. CONJUS provides tailored output plus the standard outputs of EXCEL, version 8 under Windows.

Sometimes legal and institutional arrangements do not permit implementing mathematically optimal water management strategies. At the least, the presented tools can help identify water management enhancements that might be desirable, rules permitting. The tools can certainly speed evaluation of, and create optimal water management strategies for, a wide range of problems regardless of legal environment.

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List of Symbols And Terms

a	=	perpendicular distance between observation point and line source, [L].
b^f	=	initial saturated thickness of fresh water overlying salt water in an aquifer, [L].
CF	=	consumed fraction, [dimensionless].
CropET	=	evapotranspiration by a crop, [L or L^3/T].
d	=	distance from surface water to well, [L].
E_e	=	effective efficiency, [dimensionless].
EC	=	electrical conductivity, [deciSiemens per metre at 25°m^{-1} , or millimhos per centimeter, mmho·cm^{-1}].
EC_{bw}	=	electrical conductivity of the blended water, [see EC].
EC_{caw}	=	EC of canal or surface water, [see EC].
EC_e	=	crop threshold electrical conductivity of a saturation extract from the soil, [see EC].
EC_{iw}	=	crop threshold electrical conductivity of irrigation water, [see EC].
EC_{frw}	=	electrical conductivity of the less saline (or fresh) water, [see EC].
EC_{gwx}	=	maximum groundwater salinity that can be used without causing water table rise, [see EC].
EC_{max}	=	greatest electrical conductivity the plant can sustain at the bottom of the root zone without yield reduction, [see EC].
EC_{saw}	=	electrical conductivity of the more saline water, [see EC].
ECR	=	electrical conductivity ratio, [dimensionless].
EF	=	evaporated fraction, [dimensionless].
erfc(d)	=	complementary error function of d.
ET	=	crop evapotranspiration (soil water depletion), [L or L^3/T].
h_f	=	height of fresh water surface above sea level at distance x from the coast, [L].
IE	=	irrigation efficiency (definition depends on situation), [dimensionless].
IE_i	=	irrigation efficiency (proportion of infiltrating irrigation water that is stored in the root zone), [dimensionless].
IW	=	total amount of irrigation water applied, [L or L^3/T].
IW_i	=	infiltration resulting from total applied irrigation water, [L^3].
k	=	integer identifying a stress period.
K_h	=	hydraulic conductivity, [LT^{-1}].
LF	=	leaching fraction, [dimensionless].
LF_{min}	=	smallest leaching fraction necessary to prevent salinization of the root zone, [dimensionless].
LR	=	leaching requirement, [dimensionless].
L^w	=	initial submerged well screen length, [L].
MAX_{gwx}	=	maximum amount of saline groundwater that can be blended with surface water without reducing crop yield or causing water table rise, [L^3].
MIN_{gw}	=	minimum groundwater that should be extracted to prevent water table rise, [L or L^3/T].
M^{ς}	=	number of stimuli being imposed by the management strategy.
N	=	number of stress periods during the management era.
NRF	=	nonreusable fraction, [dimensionless].
p	=	groundwater pumping rate, [L^3/T].
P_e	=	effective precipitation, [L or L^3/T].
$p_{\hat{e},k}$	=	pumping at well ê in stress period k, [L^3t^{-1}].
$p_{\hat{e}}^{ut}$	=	unit pumping rate used to compute influence coefficients describing effect of pumping well ê, [L^3T^{-1}].
P_i	=	infiltrating precipitation, [L^3].
p^{max}	=	maximum safe groundwater pumping rate, [L].
q	=	canal seepage loss per unit length [L^3/TL], or stream depletion rate [L^3k].

Q	=	net steady pumping rate, [L^3 /T]. the steady pumping rate minus the rate at which pumped water returns to the aquifer.
Q_D	=	irrigation diversion flowrate, [L^3 /T].
Q_{ET}	=	evapotranspiration flowrate, [L^3 /T].
Q_{EXP}	=	flowrate of water exported to outside the study area, [L^3 /T].
Q_f	=	fresh groundwater flow rate toward the coast per unit coast length, [L^3T^{-1}].
Q_{NR}	=	flowrate of nonreusable fraction, [L^3 /T].
Q_{RF}	=	flowrate of reusable fraction, [L^3 /T].
Q_s	=	total water diverted for irrigation.
Q_t	=	net volume pumped during time t, [L^3 /T].
r	=	distance between center of pumping well and head observation location, [L].
RF	=	reusable fraction, [dimensionless].
S	=	storativity, [dimensionless].
SAR	=	sodium adsorption ratio.
sdf	=	stream depletion factor, [T], = the time coordinate of the point at which v equals 28 percent of Qt on a curve that relates v and t.
$s_{ô,N}$	=	drawdown at observation point ô by the end of time period N.
s_t	=	drawdown resulting at time t, [L].
t	=	time of observation, [T].
t^{lag}	=	time lag between occurrence of maximum surface stage and maximum groundwater level, [T].
t^{per}	=	uniform stage fluctuation period, [T].
T_r	=	transmissivity, [L^2/T].
u	=	$\frac{r^2 S}{4T_r t}$, or $\left(a/(\Delta t T_r/S)^{0.5}\right)$ [dimensionless].
U_{ci}	=	crop consumptive use, [L or L^3/T].
U_e	=	effective water use, [L or L^3/T].
v	=	volume of stream depletion during time t, t_p, or $t_p + t_i$, [L^3 /T].
V	=	flow, [L or L^3/T].
V_e	=	effective flow, [L or L^3/T].
W(u)	=	well function for nonleaky isotropic artesian aquifer fully penetrated by well and constant discharge conditions (Theis, 1935), [dimensionless].
x_{of}	=	length of horizontal gap through which freshwater can escape to the sea, [L].
z	=	$d / (4tT_r/S)^{0.5}$.
z_s	=	depth below sea level of the fresh water-salt water interface, [L].
$\beta_{ô,ê,N-k+1}$	=	coefficient describing the change in potentiometric head at groundwater observation location ô by the end of period N caused by a unit change in stream stage ê during period k, [L].
$\delta^{\varsigma}_{ô,ê,N-k+1}$	=	influence coefficient, describing system response at point ô at the end of period N resulting from a stimulus of unit magnitude at location ê in period k, [dimension is that of the response].
ρ	=	density of fresh water, [ML^{-3}].
Δh or $\Delta h_{ô}$	=	change in head at a location, [L].
Δh^{grange}	=	range of groundwater level fluctuation due to surface water fluctuations, [L].
Δh^{smax}	=	maximum periodic surface water level rise (half the distance between peak and trough), [L].
$\Delta\rho$	=	difference in density of fresh water versus salt water, [ML^{-3}]; and
Δt	=	duration of period of uniform hydraulic stress, [T].
$\Delta\sigma_k$	=	instantaneous surface-water stage rise at time t = k Δt, [L].
$\Delta\sigma^{ut}_{ê}$	=	magnitude of unit change in stream stage ê used to develop the β influence coefficients, [L].
$\varsigma_{ê,k}$	=	magnitude of stimulus imposed by management at location ê during period k, [dimension is that of the stimulus].

$\varsigma_{\hat{e}}^{ut}$ = unit magnitude stimulus imposed at location ê used to compute influence coefficients, [dimension is that of the stimulus].

$\Psi_{\hat{o},N}$ = value of state variable ô at the end of period N resulting from implementing the optimal water management strategy, [dimensions depend upon the particular variable].

$\Psi^{non}_{\hat{o},N}$ = value of state variable ô at the end of period N if the optimal water management strategy is not implemented, [dimensions depend upon the particular variable].

$\Upsilon_{\hat{o},N}$ = change in state variable value due directly to the water management strategy either input to or computed by CONJUS, [dimensions depend upon the particular variable].

Definitions

anisotropy: A condition in which aquifer properties can differ with direction.

aquifer: Geologic formation(s), or parts of formation(s) that contain sufficiently saturated and permeable material to yield significant water to wells and/or springs.

bound: An upper or lower limit imposed on a variable in an optimization problem and S/O model.

cone of depression: The cone-like depression of the potentiometric surface that results near a well that extracts water.

confined or artesian aquifer: An aquifer in which the head (potentiometric surface) is above the top of the aquifer formation.

conjunctive use: The coordinated or combined use of groundwater and surface water, especially when the two water resources are in hydraulic connection.

constraint: A condition that must be satisfied by the optimal water management strategy computed by S/O model. Constraint can refer to bounds on variables and other conditions.

consumptive use: Activities in which the use of water causes a loss in the quantity of water originally supplied.

decision variable: Within an S/O model, a decision variable is one that water managers assumedly can directly control. CONJUS examples include groundwater extraction or recharge rate, and stream stage.

diversion: The transfer of water from a stream to some other location.

goal programming optimization: Solution of an optimization problem having a linear or nonlinear goal programming objective function.

homogeneous: The property of uniformity. A homogeneous material has identical hydrologic properties everywhere.

hydraulic conductivity: The hydraulic conductivity of a homogeneous and isotropic porous material is the volume of water at the existing kinematic viscosity that will move per unit time under a unit hydraulic gradient through a unit area measured perpendicular to the flow direction (Lohman and others, 1972). [LT^{-1}]

hydraulic diffusivity: Aquifer transmissivity divided by storativity. [$L^2 T^{-1}$]

hydraulic gradient: The change in static head per unit of distance in a specified direction. [dimensionless]

influence coefficient: Physical system response at a specified location to a unit hydraulic stimulus occurring at a specified location.

isotropy: A condition in which all aquifer properties are the same in all directions.

linear optimization: Solution of an optimization problem having only linear objective function and constraint equations.

line source: A line source has the shape of a one dimensional line. It provides deep percolation that recharges an aquifer.

objective function: An equation that an S/O model tries to maximize or minimize the value of. When using CONJUS, the user-specified objective function defines the primary water management goal.

optimization problem: A management problem for which an S/O model will compute an optimal management strategy. An optimization problem includes an objective function, decision and state variables, bounds on the variables, and constraints.

potentiometric surface: A surface representing the static head in an aquifer. It is synonymous with the piezometric surface in a confined aquifer or the water table in an unconfined aquifer.

quadratic optimization: Solution of an optimization problem having a quadratic objective function and linear constraint equations.

recharge: Water that enters an aquifer.

saturated thickness: The vertical thickness of the saturated zone of an aquifer. [L]

saturated zone: The part of an aquifer in which, ideally, all voids are filled with water under pressure that is greater than atmospheric.

simulation model (abbreviated as S model): In this report a S model is a computer model that can predict physical system response to the hydraulic stimuli and boundary conditions assumed and input by the user.

simulation/optimization model (abbreviated as S/O model): An S/O model can calculate the best water management strategy (set of hydraulic stimuli) for a specified water management problem. The user specifies an objective function, the constraints to be employed and bounds on decision and state variables. The S/O model includes both simulation and mathematical optimization algorithms.

state variable: A state variable is one that describes physical system response to hydraulic stimuli or to decision variables. CONJUS state variables include aquifer head and stream depletion rate and volume.

steady flow: The flow existing when the magnitude and direction of velocity of groundwater flow are constant in time.

storage coefficient or storativity: The volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head (Lohman and others, 1972). [dimensionless]

transmissivity: The rate at which water of the prevailing kinematic viscosity is transmitted through a unit width of the aquifer under a unit hydraulic gradient (Lohman and others, 1972). It is the product of hydraulic conductivity and aquifer saturated thickness. [$L^2 T^{-1}$]

unconfined aquifer: An aquifer in which the head lies beneath the top of the aquifer formation.

water table: The surface in an unconfined aquifer at which the pressure is atmospheric.

I. Introduction

Crop production sustainability can be enhanced by improving use of all available water resources, i.e. more comprehensively managing the hydrologic system. Coordinated management of surface waters and groundwater is termed conjunctive use.

Figure I-1 illustrates typical liquid-phase flows of concern in field and water management district level conjunctive water use. It indicates that both groundwater and diverted surface waters might support irrigation. It shows that return flows from irrigation augment surface water and groundwater flows. One infers that both the rate and the quality of the flows is important.

Improved sustainable crop production is needed to satisfy increasing food requirements. Generally this requires irrigation. Some areas are not producing at all or as much as they could because of inadequate water quantity or quality. Other irrigated areas are being degraded because of inadequate drainage.

Several historic conditions contribute to the problems. Lack of resources (time, money, information) sometimes forces piece-wise development. A water distribution or irrigation project might be developed without sufficient drainage provisions if one cannot predict when a drainage system will be needed. It might be impossible to realistically evaluate surface water/groundwater interactions before distribution systems are in place.

Surface water and groundwater use systems are frequently implemented separately. Because surface waters are readily seen, they have usually been managed before ground waters. Sometimes there is no groundwater in an area before surface water is diverted to the area.

Leaky canal systems are sometimes constructed without preparing for the long-term resulting aquifer recharge and rising groundwater. Similarly, the potential effects of deep percolation from irrigation on groundwater levels are occasionally ignored until significant problems result. In both cases, high groundwater levels and inadequate drainage have caused water logging, salinization, soil degradation and reduced crop yield.

Changes in surface water diversions have reduced availability of groundwater recharged by the initial distribution system. Increased groundwater use has depleted surface waters by increasing stream/aquifer seepage.[2]

Water use practice/efficiency changes have sometimes unintentionally affected water supply via return flow. Actions promoting improved irrigation application efficiency[3] might help one user, while harming a downstream user dependent on return flow to groundwater or surface water.

Also contributing to difficulties is misunderstanding by decision-makers (DMs) and legal authorities concerning surface-water/groundwater interactions. The resulting laws and regulations (L/R) have caused the two resources to be treated independently.

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2 Here, the term stream-aquifer seepage includes all water moving between a surface water body and an aquifer.

3 Irrigation application efficiency refers to that proportion of the applied irrigation water that becomes stored within the root zone.

Lawmakers and decision makers (DMs) sometimes instruct water managers to maximize use of both groundwater and surface water. This poses a challenge if both resources are interacting hydraulically. One cannot simultaneously maximize achievement of two conflicting goals--the zeroeth law of operations research Morel-Seytoux, 1975). A management strategy that maximizes groundwater use is generally not the same strategy that maximizes surface water use.

Efforts to apply the best separate management practice to each resource in a common area can lead to conflict, especially if different organizations manage groundwater and surface water. On the aquifer scale, maximizing sustained groundwater yield involves maximizing capture (i.e. maximizing recharges and minimizing undesirable discharges). Since some recharges are from surface waters and some discharges might be to surface waters, maximizing sustained groundwater yield can reduce surface water availability. On the other hand, maximizing surface water use often requires lining canals to reduce seepage, and aquifer recharge.

It is possible to maximize total water use--the sum of all groundwater plus all surface water use. This is a reasonable conjunctive use goal. To best assure sustainability of regional water supplies, the total hydrologic system should be managed as if groundwater and surface water are the same resource. This goal is in harmony with the reality that groundwater and surface waters interact. A molecule that is surface water in one location might be groundwater later and someplace else. The opposite is also true.

To the extent practicable, the process for developing a conjunctive water management strategy for sustainable crop development should include:

- taking appropriate advantage of the different characteristics of the two resources--surface water's potentially rapid movement and groundwater's slow movement and inexpensive conveyance and storage. Groundwater offers the greater security against drought.
- considering the system-wide effects of water use practice changes that affect return flows. Sometimes low upstream irrigation application efficiencies can result in increased seasonal water availability because of groundwater lag time.
- reusing water as often as practicable to achieve goals. The multiplier effect of multiple use-cycles can cause total applied water to exceed total inflows to the system plus net storage losses.
- assuring the sustained availability of groundwater of adequate quantity and quality. This requires avoiding unacceptably large extraction rates and might require artificial groundwater recharge. It requires considering return flow quantity and quality and stream-aquifer seepage.
- assuring sustainable soil productivity. This might require blending waters of different qualities at different times, as well as appropriate irrigation and drainage practices.
- evaluating tradeoffs between conflicting goals so the best compromise water management strategy can be implemented. This is best done via simulation/optimization (S/O) model.
- modifying or developing rules to allow appropriate water management. Sometimes water laws need to be changed to permit conjunctive water management.

This report discusses principles and basic techniques supporting the above actions, except for water rules modification. The report also provides software permitting a computer user to develop optimal site-specific water use strategies. The software is applicable for problems ranging widely in scope. The better the site information, the better the resulting management strategy. The intent is to provide all water managers with tools normally used only in fairly-well equipped facilities.

II. Principles and Tools: Selecting and Achieving Management Goals

II.1. Introduction to Water Flow and Quality Considerations

Implementing conjunctive water management strategies should first involve identifying the objectives and constraints that define strategy acceptability. Sample objectives include maximizing crop production, maximizing delivered water, maximizing net economic return, and others. Sample constraints include avoiding excessive stream dewatering, assuring adequate downstream streamflow quantity and quality, and avoiding excessively high groundwater levels, among others.

In the design or strategy creation process, the sooner one defines the major objective(s) and constraints, the better the process proceeds. This is especially true when irrigation competes with other users for available water. The implication is that different objectives and constraints are important to the different users. In such cases tradeoffs should be evaluated before water management development plans or strategies are selected.

The intent of this chapter is to help one understand how to select management goals and constraints. Goals differ depending upon the scale of the problem. Individual users having perhaps only one well and one diversion might be concerned with using as much of their legal water right as possible without causing undesirable consequences in their soils and on their property. A single user is probably going to be primarily concerned with maximizing his own beneficial use. As problem hydrologic complexity and the numbers of users and decision-makers (DMs) increase, management problem formulation becomes more challenging. At the national, regional and basin scales, planners try to assure the sustained availability of water of adequate quantity and quality for many users and circumstances. They are more concerned with total system management. Irrigation district manager goals usually lie somewhere in between those of the individual user and the basin manager.

Potential water management strategies should be evaluated and contrasted concerning how well they satisfy management goals and constraints. That is why early development of the criteria of acceptability is important. Disagreements inevitably occur between DMs at different scales. A water management strategy that is optimal for a DM at one scale is unlikely to be optimal for a DM at another scale. This results from the differing flow speeds of groundwater and surface water, and water quality effects. Many hydrologic consequences of a DM's actions might not occur within his area or period of responsibility. DMs might also be responsible to more than one type of water user.

When evaluating potential water management strategies it is helpful to consider how water flows and quality are/will be affected. Clearly, different terms are important depending on the scale of the study area. Historically, the term "irrigation efficiency" has been used to denote how effectively water was applied[4]. This has been useful for evaluating field system performance but is not the most appropriate criterion for evaluating large conjunctive use systems (Jensen, 1993; Willardson et al., 1994; Allen et al., 1996 a, b) because it ignores the importance of return flow to water reuse.

Better ways of describing water use disposition are needed to discuss usefulness for multiple users of conjunctive use systems. Clemmens et al (1995) discussed irrigation water

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[4] Several different equations describe irrigation efficiency. One divides the total irrigation amount into the amount of the irrigation water that ends up stored in the root zone after drainage by gravity, but before evapotranspiration. Another definition includes water needed for leaching in the numerator.

disposition as beneficial, nonbeneficial, consumptive and nonconsumptive. Allen et al., (1996 b) enhanced the disposition categories as shown in Figure II-1.

Figure II-1, Row 1 shows that what has historically be considered beneficial uses includes crop evapotranspiration/protection, water incorporated into saleable products and water used for salt control leaching. The proportion that this makes up of the total water diverted for irrigation (Q_D) is sometimes termed irrigation efficiency. (IE).

Figure II-1, Row 2 identifies flows commonly considered to be nonbeneficial. However, the term 'nonbeneficial' is not always appropriate for Row 2 flows. Water planning now more commonly includes important ecosystem and water quality concerns. Recognition of return flow effects on valuable ecosystems and water quality suggests that revision into Figure II-2 is desirable for some situations. Rows 2 and 3 of Figure II-2 are derived from Row 2 of Figure II-1. Question marks beside some of the Rows 2 and 3 flows indicate those that might or might not be desirable, depending on the situation.

Figure II-2, Columns A thru C identify the proportions of the diverted water that are evaporated, consumed, nonreusable and reusable. The fractions denoting disposition of water diverted or extracted for a particular purpose are defined in Figure II-3. These are useful to those who must consider the effect of water reuse. They include evaporated fraction (EF), nonreusable fraction (NRF), consumed fraction (CF) and reusable fraction (RF).

In conjunctive water management one wants to make the best use of all available water resources. On a basin scale, a tactic might be to reuse water as many times as possible before it is unalterably lost. In a long, steep watershed, for example, it is often desirable for water to pass several times through the cycle of: being diverted and applied, percolating, becoming return flow, and again being diverted. Keller and Keller (1995) defined these as multiple use-cycle systems.

For a particular study area, water that is lost or consumed includes flows in Columns A and B in Figure II-2. Water might have been applied and percolated several times (i.e., be a column C flow) before moving into columns A or B and being lost from the area and/or consumed. Which Figure II-2 cell water appears in, and when it appears, can depend upon the time scale as well as the area under consideration.

Because water might be used more than once before finally being released, the total applied to fields during a year might exceed the amount annually diverted to the area (Q_D) plus net removal from storage. David Seckler termed this a water multiplier effect (Keller et al, 1990). Figures II-1 and II-2 help provide a transition between historic irrigation field-oriented and newer multi-user conjunctive-use approaches.

In a single use-cycle system (or part of system) there is limited potential for water reuse. Most single use-cycle systems are those where it is too expensive to recapture or relift the water, or it enters a salt sink, or it becomes too polluted for economical reuse. In these uni-cycle systems, the classical goal of maximizing irrigation efficiency (IE) might be appropriate because water not beneficially used in the first cycle is lost.

Generally a large-scale conjunctive water manager seeks to multiply water use; reduce undesirable discharges and increase recharges as practicable. Phreatophyte use might or might not be an undesirable discharge, depending on the ecosystem constituency. (Some of the body politic want to protect plants that provide habitat for endangered animal species, or are themselves endangered.) The manager will seek to avoid unacceptable degradation or enhance quality of return flow. He will not use classical irrigation efficiency as the criteria to evaluate upper basin irrigation designs[5]. He is more interested in slowing water flow and assuring its availability as groundwater during periods of low surface water flows.

Assume the primary conjunctive water management objective(s) is to maximize sustainable food production or economic benefit, subject to adequately achieving other goals and satisfying constraints. Attendant goals might be to assure all water users are treated equitably within existing water rights and laws, or assure adequate production reliability. In order to achieve sustainability or production objectives, one might be tempted to state his goals as to: reduce or prevent waterlogging and salinization, increase crop yields, maintain or enhance delivery system capacities, reduce energy pumping costs, reduce pesticide and nutrient application and/or leaching, reduce soil erosion, reduce undesirable flows, increase desirable flows, reduce or prevent unacceptable water quality consequences, maintain fisheries and ecosystems, and satisfy other goals.

Problems can arise if a water manager seeks to achieve one of the lesser goals without carefully considering the consequences. One example occurs if management seeks to improve irrigation application efficiency on a large scale without evaluating the effect on and of return flows. In many areas, low CFs (often attributed to low irrigation efficiencies) help create and support important wetlands, recharge aquifers, and dampen streamflow extremes. Low CFs sometimes help increase utilizable water supplies, increase hydro power production, and improve navigation, habitat and recreation. In some areas, low agricultural CFs upstream provide, via return flow, much of the river water intentionally permitted for ecological reasons to flow to the ocean.

On the other hand, increasing the CF and reducing the RF can be beneficial. Such acts can reduce: nutrient and pesticide leaching, NRF to saline water bodies, waterlogging, energy consumption, and irrigation / drainage / conveyance system requirements. These acts can also increase water availability for other users, and increase irrigated areas. In summary, high or low NRFs and CFs can be desirable depending upon the situation and the perspective of the evaluator.

Using the presented water disposition terminology for all water users (not merely agriculture) aids designing, evaluating and selecting conjunctive water management strategies. Sometimes it is wise to compare potential water management strategies using their respective resulting $CF \cdot Q$. Some of the CF might be outside a DM's control, yet might still result from his management. Whether strategy evaluation should consider all resulting CF, regardless of whose control it is under, is a site-specific or regional decision. Depending on the situation, a particular water user might be considered responsible for all or part of the $CF \cdot Q$.

Water management strategy evaluation should also consider the effect of management on RF water quality. For a situation in which RF water quality differs from that of the original inflow, Keller and Keller (1995) define an "effective" efficiency (E_e). Useful for irrigated agriculture, E_e incorporates impacts of RF salinization on downstream users into the efficiency term.

$$E_e = \frac{U_{ci}}{U_e} = \frac{U_{ci}}{V_{eI} - V_{eO}} = \frac{CropET - P_e}{(1 - LR_I)\, V_I - (1 - LR_O)\, V_O} \qquad \text{(Eq. II-1)}$$

[5] As generally defined (Clemmens et al., 1995), $IE \cdot Q_D$ includes some Q_{RF} and omits some Q_{NR}.

where subscripts I and O denote inflow and outflow, respectively;

U_{ci} = crop consumptive use, [L or L^3/T];
U_e = effective water use, [L or L^3/T];
V = flow, [L or L^3/T];
V_e = effective flow, [L or L^3/T];
CropET = evapotranspiration by a crop, [L or L^3/T];
P_e = effective precipitation, [L or L^3/T]; and
LR = leaching requirement.

The effect of multiplying a flowrate by (1-LR) is to represent how much of a flow, V, would be needed if its salt content were not so great. That is akin to converting the flow into a clean water equivalent. If a one cms flow has a LR of .3, the clean-water equivalent of that flow is 0.7 cms.

To consider the worth of return flows from different management strategies, some sort of scaling of flows to denote their quality or the cost of utilizing them is desirable. One approach is to multiply return flows by an influent equivalence factor that denotes the degradation of the effective returned fraction (Q_{RF}) in terms of the quality of the initial diversion.

$$Q_{RFE} = Q_{RF} \left(\frac{1 - LR_O}{1 - LR_I} \right) \qquad \text{(Eq. II-2)}$$

where Q_{RFE} = the effective return flow, [L or L^3/T].

The scaling approach can be applied to water intended for any particular downstream use, by considering the value of the water for that use (Eq. II-3).

$$Q_{RFE} = Q_{RF} \left(\frac{value_O}{value_I} \right) \qquad \text{(Eq. II-3)}$$

where value $_O$ and value $_I$ = monetary values of the return flow and inflow, respectively, [$].

II.2. Introduction to Optimization

Assume a situation in which a landowner wants to know the most he can totally pump from three wells without causing unacceptable streamflow depletion or drawdowns at any of the wells by the end of four days of pumping. The Figure II-4 map illustrates stream and well locations.

Assume the wells and stream fully penetrate a semi-infinite aquifer. The potentiometric surface is initially at steady state at 100 m elevation in both stream and aquifer. Aquifer hydraulic conductivity is 6 m/d, storativity is 0.001 and saturated thickness is 50 m.

Assume the landowner must pump exactly 600 m^3d^{-1} (cmd) for all four days from well 3, but he does not know how much to pump from wells 1 and 2. Assume the utilized pumping rates will be unchanging during the four-day period. Assume the landowner does not want to cause the stream depletion rate to exceed 1400 m^3d^{-1} by the end of the four days, and he does not want drawdown at any of the wells to exceed 5 m by the end of that period.

This problem has only two variables for which we want to determine optimal values (p_1 and p_2). However, to illustrate how we can also consider the effect of the known pumping at well 3, we include it in what we term the objective function (Eq. II-4).

$$\textbf{maximize } Z = \sum_{\hat{e}=1}^{M^p=3} p_{\hat{e}}$$

(Eq. II-4)

where:

Z = value of the objective function [units same as included variables];
M^p = number of wells for which extraction pumping is being optimized; and
$p_{\hat{e}}$ = pumping rate at well ê, [L^3/T].

This expression states that we want to determine those values of steady pumping rates which, summed together, will yield the greatest total. If that were the only equation we had to consider, one would want to pump and infinite amount from each well. However, physical realism and additional management restrictions must also be satisfied. Therefore we want to determine the optimal pumping strategy, subject to the following constraints.

subject to:

P_1,P_2	$\geq$	0.0	(Eq. II-5)
P_3	$=$	600	(Eq. II-6)
s_1,s_2,s_3	$\leq$	3	(Eq. II-7)
Δq^f	$\leq$	1400	(Eq. II-8)

where:

$s_{\hat{o}}$ = drawdown at location ô resulting from pumping strategy; [L];
Δq^f = change in streamflow resulting from pumping strategy; [L^3/T].

Equation II-5 indicates that pumping extraction is considered positive in sign and that no injection is permitted in wells 1 and 2. No upper limit is placed on how much one can pump from either of those wells. Equation II-6 specifies the pumping rate of well 3.

Equation II-7 describes the requirement that drawdown not exceed 3 m in any observation location. In this example each observation location is also a pumping (excitation) location. Equation II-8 describes the requirement that the streamflow depletion rate not exceed 1400 m^3d^{-1} by the end of the pumping period.

In order to develop a pumping strategy that satisfies Eqs. II-7 and II-8, we must employ equations that relate pumping rate with drawdown and streamflow depletion. Here we assume the physical system can be treated as if it is linear (we assume that pumping does not significantly change the assumed physical system parameters, or the stream stage).

Replacing Equations II-7 and II-8 with linear expressions relating physical system response to pumping, yields the following equations.

$$\delta_{1,1}(P_1/1) + \delta_{1,2}(P_2/1) + \delta_{1,3}(P_3/1) \leq 3 \qquad \text{(Eq. II-9)}$$
$$\delta_{2,1}(P_1/1) + \delta_{2,2}(P_2/1) + \delta_{2,3}(P_3/1) \leq 3 \qquad \text{(Eq. II-10)}$$
$$\delta_{3,1}(P_1/1) + \delta_{3,2}(P_2/1) + \delta_{3,3}(P_3/1) \leq 3 \qquad \text{(Eq. II-11)}$$
$$\beta_{1,1}(P_1/1) + \beta_{1,2}(P_2/1) + \beta_{1,3}(P_3/1) \leq 1400 \qquad \text{(Eq. II-12)}$$

where:

$\delta_{\hat{o},\hat{e}}$ = influence coefficient describing the drawdown at location ô at the end of four days of unit pumping at location ê, [L]; (Here the unit pumping equals 1 L^3/T. If the unit pumping equaled 5 L^3/T, the denominator of each fraction would be 5 instead of 1);

$\beta_{\hat{o},\hat{e}}$ = influence coefficient describing streamflow depletion rate from stream ô at the end of four days of unit pumping at location ê, [L^3/T].

Table II-1 contains a matrix of influence coefficients needed for Equations II-9 to II-12. It is developed by determining the drawdown or streamflow depletion resulting from extracting 1 cmd of groundwater at the respective wells (when only one well pumps at a time). These drawdowns are shown in Table II-1 and represent influence coefficients.

The upper left value in Table II-1 indicates that pumping 1 cmd for four days from well 1 would cause 0.00407 m drawdown just outside the well 1 casing. The upper right value indicates that pumping the same rate from well 3 would cause 0.00023 m drawdown in well 1.

Equations II-9 through 12 are used because we are assuming that the optimal pumping strategy will not significantly affect system properties. Thus, as with a confined aquifer in which transmissivity does not change with pumping, properties of linear systems apply. From the multiplicative property, pumping 2 cmd at any well individually would cause twice the drawdown shown in Table II-1.

From the additive property, total drawdown at any well equals the sum of the drawdowns caused at that well by pumping at each of the three wells. Therefore, pumping at 1 cmd from all three wells simultaneously would cause drawdown at well 1 to equal the sum of the values in row 1 (0.00548 m).

Equation II-9 describes the drawdown that will occur at well 1 as a result of pumping at all three wells simultaneously, but perhaps at different rates. Equations II-10 and II-11 are analogous for wells 2 and 3. Equation II-12 reflects the effect of pumping on stream depletion. In summary, these equations employ the multiplicative property in that the drawdown at a well due to pumping at another well is proportional to the influence coefficient relating those two wells's response and stimulus. These equations employ the additive property in that total drawdown at a well is the sum of the drawdowns caused by pumping at each well individually.

The management problem for which we want to obtain an optimal pumping strategy consists of objective function Equation II-4, and constraint equations II-5, 6, and 9 through 12. Equation II-4 indicates that we want to maximize total pumping (In other situations, we might want to minimize the value of the objective function).

Figure II-5 shows how to determine the optimal solution graphically. Lines on the figure correspond to the constraint equations presented as equality constraints. The x and y axis, respectively represent solutions to Equation II-5, at which p_1 and p_2 equal zero. The set of feasible values to Equation II-5 for both p_1 and p_2 includes all points above the p_1 axis and to the right of the p_2 axis.

Equations II-9 through II-12 prevent drawdown or depletion from exceeding specified limits. Those system responses increase as pumping increases. Therefore, the feasible solution space lies to the left and below the lines representing those equations. The lines are developed by substituting 600 for p_3 and 0.0 for either p_1 or p_2.

The feasible solution space is the set of all possible combinations of p_1 and p_2 that satisfies all the constraint equations. It lies in the lower left corner bounded primarily by the p_1 and p_2 axes, and the lines depicting the stream depletion and well 1 drawdown constraints. The well 3 drawdown constraint also bounds the upper left portion of the feasible solution space.

Figure II-5 shows iso-lines of objective function value. Each iso-line consists of the set of p_1 and p_2 values that yield a selected total value. The orientation of the iso-lines confirms that a strategy that maximizes total pumping is going to be as much toward the upper right of the feasible solution space as possible.

To identify the optimal solution by inspecting the figure, we look at intersections of constraint lines that lie most to the upper right (i.e. at vertices of the feasible solution space). We find the greatest combination of p_1 and p_2 to be 805 m^3d^{-1} ,(p_1 =590 m^3d^{-1} and p_2 = 391 m^3d^{-1}). The value of p_3 is of course 600 m^3d^{-1} . Optimal total pumping determined by addition is 1,581 m^3d^{-1} , the value of the objective function.

The optimal pumping strategy lies at the intersection of the well 1 drawdown and stream depletion constraint lines. This means the specified pumping rates will cause computed drawdown at well 1 to be precisely 3 meters, and the stream depletion rate to be precisely 1400 m^3d^{-1}. These two constraints are termed 'tight' constraints because they prevent the optimal pumping value from being greater. The other two drawdown constraints and the lower bounds on pumping are considered 'loose' constraints or bounds because they do not prevent the optimal pumping strategy from being better.

We can easily solve this problem graphically in two dimensions because we are only trying to optimize the values of two variables. Had there been three unknown pumping rates we would have had to use a three-dimensional drawing. The more pumping rates needing simultaneous optimization, the more dimensions to the problem.

Many computer programs that perform optimization check the vertices automatically and identify the best solution that satisfies all constraints. Other algorithms use different techniques, but the idea of automation is clearly desirable. Determining optimal solutions for management problems by hand rapidly becomes untractable.

The CONJUS software provided with this report computes optimal water management strategies directly without requiring graphical analysis (Figure II-6). To do this CONJUS first computes influence coefficients. Figure II-7 illustrates CONJUS output for the above problem. It shows total and individual optimal pumping rates, resulting heads (initial head minus drawdown), and resulting stream depletion rate. Units are consistent with the input data--meters and days for length and time.

In Fig. II-7, particularly note the two tight constraints. These are head or drawdown at well 1 (97 or 3 m respectively) and streamflow depletion rate (1400 $m^3\ d^{-1}$). Fig. II-7 also shows the total streamflow depletion volume by the end of the four days. Depletion volume is computed and can be constrained, even though it was not constrained in this problem.

CONJUS can be used for simulation if one wants to evaluate the consequences of some assumed management strategy, rather than calculating an optimal strategy. Note that p_3 was specified in the sample problem. If p_1 and p_2 had also been specified, the posed problem would not have been an optimization problem. It would have merely been a simulation problem. CONJUS can be used to solve simulation problems simply by not giving it any freedom (i.e. by specifying both lower and upper bounds to be precisely the same and to be the desired values for all decision variables). In the above example, CONJUS simulated the effect of pumping a known rate at well 3 and superimposed the effect of pumping optimal rates for the other two wells.

Figure II-6 shows a flowchart to guide the user in CONJUS use. First the user inputs data describing the management problem and site. Then he instructs CONJUS to compute steady or time-varying influence coefficients (CONJUS does this using analytical equations described in Chapter III). For cases in which the CONJUS-computed influence coefficients are inappropriate, CONJUS permits the user to provide his own influence coefficients.

Next, the user can choose whether to accept default weights assigned by CONJUS to variables in the objective function. In the illustrative problem, a unit of pumping from each well was considered of equal importance in the objective function. In other words, in the objective function pumping from each well had a weight of one-- the CONJUS default weight value (for simplicity, the weights are not shown in Equation II-4).

Then the user can instruct CONJUS to compute an optimal pumping strategy. CONJUS responds by building the necessary optimization problem equations, such as Eqs. II-4 through 6, and 9 through 12. It uses the influence coefficient approach illustrated above. Then CONJUS applies linear or nonlinear optimization algorithms to calculate the optimal strategy.

CONJUS is termed a simulation/optimization (S/O) model. Differences between normal simulation (S) models and S/O models are highlighted in Table II-2. The major difference is that S models require the user to input a proposed management strategy. S/O models compute the optimal management strategy for the posed management problem. [6] Of

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REMAX is the most powerful available S/O model for managing stream-aquifer systems (Peralta and Aly, 1997). It has constraints for managing groundwater, surface water and seepage variables. MODMAN, a groundwater S/O model, has constraints for managing groundwater variables (Greenwald, 1998).

course, as stated above, a S/O model can also be used as a simulation model if one does not provide freedom in selecting decision variable values[7].

Some model differences listed in Table II-2 (e.g. fixed boundary flows) are most applicable to S/O models that have more detailed simulation abilities than CONJUS. Powerful S/O models such as REMAX generally employ finite difference or finite element simulators to develop influence coefficients.

With sufficient calibrated data, REMAX can address complex nonhomogeneous stream aquifer systems. REMAX users are usually experienced groundwater modelers.

CONJUS is more appropriate for use by field engineers who might not have much time or data to calibrate models such as REMAX. Using a poorly calibrated sophisticated model can be worse than using a simpler model. CONJUS has relatively simple simulation capabilities because it relies upon analytical solutions to describe system responses to stimuli. The analytical solutions assume homogenous systems and the absence of irregular boundary conditions. Sophocleous et al. (1995) discuss suitability of analytical versus numerical models for predicting streamflow depletion due to groundwater pumping.

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Variables that managers can control directly are often termed 'decision' variables. These include pumping and diversion rates. Variables describing system state responses to those management stimuli are termed 'state' variables. Example state variables are groundwater head and streamflow depletion.

III. Principles and Tools: Field Level

III.1. Introduction

Analytical expressions analogous to those of heat flow (Carslaw and Jaeger, 1959) have long been used to predict aquifer head response to hydraulic stimuli. These equations can also be used to estimate aquifer properties from observed hydrologic responses to stimuli. Analytical equations can be very useful and applied relatively easily. Some useful analytical equations are included here, although many others exist. Drawbacks to using these equations involve the attendant simplifying physical system assumptions.

III.2. Introduction to CONJUS Simulation

A goal of Chapter III is to provide technical tools for improving conjunctive water management. The tools include analytical equations and a software package to ease their use. The provided CONJUS computer program uses the following general expression (Eq. III-1) to predict state variable values. State variables include variables that respond to management, such as head and groundwater flow. Decision variables include variables the value of which are directly selected by management, such as groundwater extraction rates. Decision variables are directly managed stimuli of the physical system.

$$^{non}{}_{\hat{o},N} + \Upsilon_{\hat{o},N} = \Psi^{non}{}_{\hat{o},N} + \sum_{\hat{e}=1}^{M^{\varsigma}} \sum_{k=1}^{N} \delta^{\varsigma}{}_{\hat{o},\hat{e},N-}$$

(Eq. III-1)

++where:

$\Psi_{\hat{o},N}$ = value of state variable ô at the end of period N resulting from implementing the optimal water management strategy, [dimensions depend upon the particular variable]

$\Psi^{non}{}_{\hat{o},N}$ = value of state variable ô at the end of period N if the optimal water management strategy is not implemented, [dimensions depend upon the particular variable]

$\Upsilon_{\hat{o},N}$ = change in state variable value due directly to the water management strategy either input to or computed by CONJUS, [dimensions depend upon the particular variable].

M^{ς} = Number of stimuli being imposed by the management strategy.

N = Number of stress periods during the management era.

$\varsigma_{\hat{e},k}$ = magnitude of stimulus imposed by management at location ê during period k.

$\varsigma_{\hat{e}}^{ut}$ = unit magnitude stimulus imposed at location ê used to compute influence coefficients, [dimension is that of the stimulus];

$\delta^{\varsigma}{}_{\hat{o},\hat{e},N-k+1}$ = influence coefficient, describing system response at point ô at the end of period N resulting from a stimulus of unit magnitude at location ê in period k, [dimension is that of the response];

Note that the second term of the right hand side (RHS) of Eq III-1 is analogous to the last unnumbered equation of Section III.2.5. In Section III.2.5 one well is pumping for three periods.

For CONJUS to employ the above equation, the user must input $\Psi^{non}_{\hat{o},N}$, the nonoptimal value of that state variable that would exist if the optimal water management strategy were not implemented. The user obtains $\Psi^{non}_{\hat{o},N}$ either from field experience or from numerical simulation model. In the simplest case, the system is initially at steady-state and would remain that way unless disturbed. In that case the 'nonoptimal heads' for all periods are the initial steady-state heads. For a system not at steady-state, the 'nonoptimal heads' are the transient aquifer heads that would exist if an optimal pumping strategy were not implemented. If the CONJUS user inputs these nonoptimal heads and either inputs nor computes a pumping strategy, CONJUS calculates the heads resulting from subtracting the computed drawdowns ($\Upsilon_{\hat{o},N}$) from those nonoptimal heads.

The right hand side (RHS) summation terms within Equation III-1 constitute a general superposition expression to describe system state response ($\Upsilon_{\hat{o},N}$), to the optimal water management strategy (consisting of a set of stimuli that can exist at different locations). The RHS assumes all stress periods are of equal duration; and that the physical system can be adequately described linearly, although cycling might be necessary to achieve that. Reilly et al., (1987) discuss applying superposition to groundwater hydraulics.

The CONJUS user must also input the magnitude of the unit stimulus and sufficient physical information for computing influence coefficients, $\delta^{\varsigma}_{\hat{o},\hat{e}}$ which describe the effect of management decisions on the physical system.

Appendix B discusses how influence coefficients are computed. The intent is to permit the user to adapt CONJUS for his own system, if the simplifying assumptions of CONJUS are insufficiently accurate.

III.3. Estimating Aquifer Head Response to Pumping from a Well of Small Diameter (Theis Equation)

This nonequilibrium equation is frequently used to compute transient head response to steady pumping in a confined aquifer (Figure III-1). Under some conditions it is also suitable for an unconfined aquifer. Assumptions are:

- confined aquifer is homogeneous and isotropic and overlain and underlain by aquicludes;.
- aquifer is infinite and unaffected by boundaries;
- transmissivity is constant and unaffected by pumping;
- flow is horizontal;
- well fully penetrates the aquifer;
- radial flow.

$$s_t = \frac{p}{4\pi T_r} W(u) \qquad \text{(Eq. III-2)}$$

where:

s_t = drawdown resulting at time t, [L];
p = groundwater pumping rate, [L^3/T];
T_r = transmissivity, [L^2/T];
r = distance between center of pumping well and head observation location, [L];
S = storativity, [dimensionless];
W(u) = well function for nonleaky isotropic artesian aquifer fully penetrated by well and constant discharge conditions (Theis, 1935), [dimensionless];

$$= \int_u^\infty \frac{e^{-u}}{u}\, du. \quad \text{(See Figure III-2.)};$$

$$u = \frac{r^2\, S}{4 T_r t}, \quad \text{[dimensionless]}.$$

Note that as long as aquifer parameters (T and S) don't change, this equation is linear. For a particular time and distance (t and r, respectively) drawdown (s) is linearly proportional to pumping rate (p). This characteristic applies to many analytical expressions commonly used in hydrogeology. As long as all assumptions of the equation are satisfied, the addressed systems are linear.

III.4. Stream Depletion Response to Groundwater Pumping

The below expressions are based on work by Jenkins (1968a,b) summarizing earlier work by several authors (Glover and Balmer, 1954; Theis, 1941; and Hantush, 1965). These describe: the rate of streamflow depletion caused by pumping at any time after pumping commences; and total volume of depletion at a specified time. Through superposition, the effect of time-varying pumping can be determined. Streamflow depletion is either the reduction of groundwater flow to the stream, or increase in flow from stream to aquifer. Merely by changing signs, the expressions can also show the effect on streamflow of injection.

Figure III-3 illustrates an ideally applicable physical system. Assumptions are:

- Aquifer is isotropic, homogeneous and semi-infinite in extent.
- Transmissivity does not change with time. Drawdown in an unconfined aquifer is assumed to be small compared with the saturated thickness.
- Temperature in the aquifer and the stream are constant and approximately the same.
- The straight stream fully penetrates the aquifer.
- Water is released instantaneously from storage.
- The well is screened in the full aquifer saturated thickness
- Pumping rate is steady.
- Groundwater pumping does not affect aquifer head at the line source stream, although it does affect groundwater hydraulic gradient toward the stream.

$$sdf = a\,\frac{s}{t}$$

(Eq. III-3)

$$\frac{q}{Q} = 1 - \mathrm{erf}\left(\sqrt{\frac{sdf}{4t}}\right)$$

(Eq. III-4)

$$\frac{v}{Qt} = \left(\frac{sdf}{2t} + 1\right)\ \mathrm{erfc}\left(\frac{\sqrt{sdf}}{4t}\right) - \left(\frac{\sqrt{sdf}}{4t}\right)\frac{2}{\sqrt{\pi}}\ \exp\left(-\frac{sdf}{4t}\right)$$

(Eq. III-5)

where:

sdf	=	stream depletion factor, [T], = the time coordinate of the point at which v equals 28 percent of Qt on a curve that relates v and t;
t	=	time of observation, [T];
v	=	volume of stream depletion during time t, t_p, or $t_{p+} t_i$, [L^3 /T];
Q	=	net steady pumping rate, [L^3 /T]; the steady pumping rate minus the rate at which pumped water returns to the aquifer;
Qt	=	net volume pumped during time t, [L^3 /T]; and

$$\mathrm{erfc}(d) = \frac{2}{\sqrt{\pi}} \int_d^{\infty} e^{-t^*t}\, dt$$

One can estimate the value of sdf via Equation III-3, or from field observation. Once sdf is known, Figure III-4 aids determining q and v (without having to resort directly to Equations III-4 and 5, respectively) during a period of steady groundwater pumping. Estimates obtained when using these figures are usually within 10-20 percent of the mathematically correct value (depending on the user). For precision one can use CONJUS.

Figure III-4 and Equations III-4 and III-5 are valid if all ideal assumptions are satisfied. In the field, one can empirically determine sdf to integrate the effects of non- ideal stream/aquifer hydraulic connection, stream meanders, distance to stream, areal variation in aquifer parameters, and less permeable barriers.

Differences from the idealized conditions can cause actual stream depletion to be less or greater than estimated values. The following differences affect sdf.

- Water table lowering in a flood plain might mean that much water that would otherwise be transpired will be captured. This is similar to intercepting another recharge boundary. Thus, less stream water will be depleted by pumping.
- If the zone of influence of the pumped well intercepts a less permeable boundary, there will be greater stream depletion than otherwise.
- If the water table drops below the bottom of the streambed, depletion depends upon stream stage, streambed area, and unsaturated vertical conductance, and is not described accurately by these equations.
- Decreasing T causes decreasing stream depletion if pumping is constant.
- Because of viscosity changes, stream depletion is somewhat less when temperatures are low than when they are high.

It takes a while (lag time) for pumping to affect the stream. The resulting residual effect means that it might take a long time for the depletion volume to equal the volume pumped. These two volumes will be ultimately be equal only if the stream is the sole recharge source. If irrigation return flow and precipitation occur, the depletion volume might never equal the pumped volume, i.e. residual effects are reduced.

Figures III-5 and III-6 are useful for estimating q and v respectively at times after steady pumping has ceased. Superposition can also be used to describe the effect of pumping multiple wells. As with the Theis equation, one can develop influence coefficients and then superposition to determine depletion rate and total depletion volume after any point in time. CONJUS and REMAX use this approach.

III.5. Aquifer Head Response to Surface Water and Recharge Influences

III.5.1. Aquifer Head Response to Line Source (Unsaturated Canal Leakage)

Several different stream-aquifer situations can exist. The aquifer water table might be above the base of the stream. In that case saturated flow will occur between the aquifer and the surface water body. Flow will be in the direction of decreasing head, whether from aquifer to stream or vice-versa. On the other hand, if the aquifer head is beneath the base of the surface water body, water will flow from the surface body downward under unsaturated conditions.

Here we consider a system as shown in Figure III-7. If one can estimate the average canal seepage loss per unit length, one can estimate the increase in groundwater level that will result with time at a specified distance from the canal. Following Glover (1978), we assume half of the seepage flows to each side of the canal. Example IV-D in Chapter IV illustrates equation use.

$$\frac{\overline{\pi t\ T_r/S}}{2\pi T_r} \quad \left(\frac{a}{\sqrt{4t\ T_r/S}}\right) \quad \sqrt{\pi}\left[\frac{e^{-z^2}}{z\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}} \int \right.$$

(Eq. III-6)

where:

Δρ = change in head at a location, a distance "a" from the line source, [L];
q = canal seepage loss per unit length [L^3/TL];
a = perpendicular distance between observation point and line source;
z = $a / (4tT_r/S)^{0.5}$

Directly beneath the line source at a equals zero, Eq. III-6 becomes:

$$\Delta h_{\hat{o}} = \frac{q\sqrt{4\pi t\ T_r/S}}{2\pi T_r}$$

(Eq. III-7)

Using heads at two observation wells placed parallel to the flow direction lets one compute hydraulic gradient. One can then use Darcy's Law to estimate groundwater flow.

III.5.2. Aquifer Head Response to Stream-stage Changes

In this section assume a physical system similar to that of Section III.4.--stream and aquifer are in saturated fully penetrated connection. Here we assume the well is for observation only and does not pump. However, the stream stage can change with time at the beginning of any stress period. The analytical equation describing aquifer head response to stream stage is here modified for computing influence coefficients needed by Equation III-8.

The following superposition expression describes time varying groundwater head response ($\Delta h_{\hat{o},N}$) to transient changes in stream stage ê , ($\Delta\sigma_{\hat{e},k}$).

$$\Delta h_{\hat{o},N} = \sum_{k=1}^{N} \beta_{\hat{o},\hat{e},N\text{-}k+1} \left(\frac{\Delta\sigma_{\hat{e},k}}{\Delta\sigma_{\hat{e}}^{ut}} \right) \qquad \text{(Eq. III-8)}$$

$$\beta_{\hat{o},\hat{e},N\text{-}k+1} = \left(\operatorname{erfc} \frac{a}{2\sqrt{T_r\ (N-k+1)\ \Delta t/S}} \right) - \left(\operatorname{erfc} \frac{a}{2\sqrt{T_r\ (N-k)\ \Delta t/S}} \right)$$

where:

$\beta_{\hat{o},\hat{e},N\text{-}k+1}$ = coefficient describing the change in potentiometric head at groundwater observation location ô by the end of period N caused by a unit change in stream stage ê during period k, [L]

$\Delta\sigma^{ut}{}_{\hat{e}}$ = magnitude of unit change in stream stage ê used to develop the β influence coefficients, [L]

III.5.3 Aquifer Head Response to Recharge over a Rectangular Area

Percolation from irrigated fields, recharge basins and surface water bodies recharges underlying unconfined aquifers. Hantush (1967) developed the following equation to describe the attendant water table mounding (Fig III-8).

$$\begin{aligned} h_{x,y,t} - H = \frac{v_a t}{4f} \{ & F[(W/2 + x)n, (L/2 + y)n] \\ & + F[(W/2 + x)n, (L/2 - y)n] \\ & + F[(W/2 - x)n, (L/2 + y)n] \\ & + F[(W/2 - x)n, (L/2 + y)n] \end{aligned}$$

(Eq. III-9)

where:

$h_{x,y,t}$	=	height of water table above impermeable layer at x, y, and time t (Figure III-8)
H	=	original height of water table above impermeable layer
v_a	=	arrival rate at water table of water from infiltration basin
t	=	time since start of recharge
f	=	fillable porosity ($1 > f > 0$)
L	=	length of recharge basin (in y direction)
W	=	width of recharge basin (in x direction)
n	=	$(4tT/f)^{-1/2}$
$F(\alpha, \beta)$	=	$\int_0^1 \mathrm{erf}\ (\alpha\tau^{-1/2}) \cdot \mathrm{erf}\ (\beta\tau^{-1/2})\ d\tau$ (Many textbooks contain tables of this function, which was tabulated by Hantush.)

III.6. Water Quality Considerations

III.6.1. Introduction

Irrigating with poor quality water can potentially cause specific ion toxicity, infiltration or salinity problems, and can reduce crop production. Not all available water has quality suitable for unrestricted use for irrigation. Ayers and Wescot (1985) provide guidelines for assessing water suitability based on specific ions, alkalinity and salinity. Several references provide guidance for sustaining agricultural production using water of different qualities (Ayers and Wescot (1985), Pescod, 1992; Rao et al, 1994).

The most common problems result from using waters having excessive sodium (alkali waters) or high salt concentrations (saline waters). Several schemes exist for classifying water quality and/or suitability for irrigation (Ayers and Wescot, 1985; Gupta, 1994). Generally these classifications are based on sodium adsorption ratio (SAR) to define alkalinity hazard and electrical conductivity (EC) to define salinity hazard.

Sometimes, different water sources in a given area have significantly different qualities. The groundwater might be more saline than the surface water, or vice versa. Saline groundwater might underlie better quality groundwater, or vice versa. The different quality waters might have differing availabilities during the year. Under some conditions, combining waters to modify salt concentration might effectively increase the supply of useable water.

Subsequent subsections provide equations relating to adjustment of water quality by mixing and blending. Some of these equations can be used to develop bounds or constraints for management use within CONJUS. Examples include: an upper limit on the amount of saline groundwater that can be blended with canal water to irrigate a particular crop; and an upper limit on groundwater pumping from fresh water overlying saline groundwater to prevent salt water intrusion into the fresh groundwater .

III.6.2. Using Saline Water and Blending Waters of Differing Qualities

Normally one leaches excess salts from the crop root zone by applying extra irrigation water (a leaching requirement, LR, expressed as a fraction). Results of this leaching include some increased salinity in underlying shallow groundwater and increased groundwater elevations. Reuse of groundwater or artificial drainage can be employed to control groundwater level rise. The potential for reuse of shallow groundwater depends on the groundwater quality and the salt tolerance of the crops being grown.

Figure III-9 shows threshold tolerances of crops to salinity. A threshold tolerance is a limit at which irrigation with saline water will not cause crop yield loss. The table identifies values of:

(a) the threshold electrical conductivity of a saturation extract from the soil (EC_e). This is the basic salinity parameter for classifying crop tolerance to salt.
(b) the allowable threshold irrigation water salinity (EC_{iw}), and
(c) the greatest electrical conductivity the plant can tolerate in the soil at the bottom of the root zone, without yield reduction (EC_{max}).

The values in Figure III-9 were experimentally obtained using water of constant concentration throughout an irrigation season. Under normal irrigation conditions, applying irrigation water having a concentration of EC_{iw} will result in the root zone saturation extract becoming EC_e. If one uses irrigation water of greater EC, the resulting soil water saturation extract salinity will exceed the EC_e value in column 1 and crop yield might be reduced.

The maximum EC of irrigation water that should be used for irrigating particular crops throughout a growing season is EC_{iw} (column 2). In conjunctive use situations the EC of one water supply might exceed EC_{iw} , but the EC of a second supply might not. In that case, two different water use approaches are possible (Rhoades, 1988). One might use the fresh and saline waters at different times (Rhoades and Dinar, 1990; Hoffman et al, 1990). The fresh water can be used for crop germination because many plants are most sensitive to salinity during early growth stages. The more saline water can then be used during later irrigations when the plants are more tolerant to salinity. Unfortunately, threshold EC values for partial-season use of saline water are not generally available.

An alternative to using different water qualities at different times is to blend the fresh and saline waters for all irrigations so that the resulting water is not excessively saline (Dinar et al., 1986; Hoffman et al,1990; Peralta et al, 1990). The concentration of the resulting blended water is computed by Equation III-10. Example IV-Ha) in Chapter 4 illustrates equation use.

where:

EC_{bw} = electrical conductivity of the blended water [deciSiemens per metre at 25°, or millimhos per centimeter, $mmho \cdot cm^{-1}$];
Ec_{frw} = electrical conductivity of the less saline (or fresh) water;
Ec_{saw} = electrical conductivity of the more saline water;
V_{frw} = volume of less saline water, $[L^3]$; and

$$EC_{bw} = \frac{EC_{frw} V_{frw} + EC_{saw} V_{saw}}{(V_{frw} + V_{saw})} \qquad \text{Eq. (III-10)}$$

V_{saw} = volume of more saline water, $[L^3]$.

As long as the EC of the blended irrigation water does not exceed EC_{iw} (Fig III-9) it will probably not reduce crop yields. Ayers and Wescot (1985) show the percentage yield reductions expected to result from irrigating with water of EC values exceeding ECIW.

III.6.3. Anticipating Water Table Rise Caused by Irrigating with Saline Groundwater

Rising groundwater levels commonly threaten the sustainability of agriculture in irrigated areas. Root zone salinization and crop yield reduction result when the water table is in or near the root zone. The most accurate way of predicting when and where groundwater levels will threaten the root zone is by use of calibrated numerical groundwater models. The next most accurate method for predicting water table elevation involves using analytical equations on a local scale. For example, Hantush (1967) prepared analytical expressions for estimating groundwater level rise resulting from uniform recharge occurring over rectangular and circular areas (Section III.5.3). Walton (1970) and Bouwer (1978) discuss this and other equations describing aquifer response to recharge.

An alternative to using a numerical model or an analytical equation is the one-dimensional reconnaissance approach presented by Willardson (1990). Problem IV-Hb)and c) illustrate his approach, which begins by defining the total irrigation water amount needed.

$$IW_i = ((ET - P_i)\ (1 + (LF / (IE_i - LF))))\ /\ IE_i \qquad \text{(Eq. III-11)}$$

where:
IW_i = total applied irrigation water (not including surface runoff), $[L^3]$;
ET = evapotranspiration (soil water depletion), $[L^3]$;
P_i = infiltrating precipitation, $[L^3]$;
IE_i = irrigation efficiency or stored fraction (proportion of infiltrating irrigation water that is stored in the root zone); and
LF = leaching fraction.

A simple mass balance approach can be used to estimate the minimum allowable leaching fraction (Hoffman et al, 1990):
where:

LF_{min} = smallest leaching fraction necessary to prevent salinization of the root zone, [dimensionless]; and
EC_{max} = greatest electrical conductivity the plant can sustain at the bottom of the root zone without yield reduction, [deciSiemens per metre].

It is possible to roughly estimate the least amount of groundwater that should be extracted to prevent groundwater level rise (Eq. III-13). This equation assumes that one is extracting groundwater from beneath the area one is irrigating, and that there is no surface runoff. The equation also ignores lateral groundwater movement that might occur after percolating water reaches the water table .

$$MIN_{gw} = IW_i + P_i - ET \qquad \text{(Eq. III-13)}$$

where:

MIN_{gw} = the least amount of groundwater that must be extracted to prevent groundwater levels from rising, $[L^3]$. This groundwater can be used directly for irrigation if its salt concentration does not exceed the EC_{iw} of the crop being irrigated.

A dilemma results if the groundwater is extremely saline. The more saline the groundwater the more freshwater needs to be blended with it to cause the resulting EC of the blended water to be less than EC_{iw}, and the greater will be the LF required to prevent development of excess salinity in the root zone. For high groundwater EC, the amount of leaching water percolating to groundwater will exceed the amount of groundwater that can be safely used, and groundwater levels will rise.

Willardson (1990) presented the following approach to estimate the maximum EC of groundwater that could be blended with less saline water while preventing water table rise. The approach continues the above one-dimensional analysis.

$$EC_{gwx} = ((IW_i)(EC_{iw} - EC_{caw}) / MIN_{gw}) + EC_{caw} \qquad \text{(Eq. III-14)}$$

where:

Ec_{gwx} = maximum groundwater salinity that can be blended with canal or surface water without causing groundwater level rise.
EC_{caw} = EC of canal or surface water.

The maximum amount of the saline groundwater that can be blended with the surface water without reducing crop yield or causing water table rise is:

$$MAX_{gwx} = (IW_i)(ECR) \qquad \text{(Eq. III-15)}$$

where:

$$ECR = (_{ECIW} - EC_{caw}) / (EC_{gwx} - EC_{caw}) \qquad \text{(Eq. III-16)}$$

= the maximum proportion of the saline groundwater that can be included in the blended water without causing so much deep percolation that the groundwater level will rise, [dimensionless].

The above expressions can be used with any consistent set of units over any time period and apply for continuous use of blended water. They can also apply for alternating conjunctive use of surface and groundwater, as long as no single irrigation is with water having a toxic salt concentration. These equations provide guidance on how to prevent excessive groundwater level rise while irrigating with conjunctively used water and preventing root zone salinization.

III.6.4. Salt Water Upconing Beneath a Well

This section describes how to determine the rate at which groundwater can be pumped from an individual well without causing unacceptable salt-water upconing, if saline groundwater underlies a layer of fresh groundwater. Situations as shown in Figure III-10 exist naturally and in locations where seepage from canals or natural precipitation have caused a zone of fresh water to overlie naturally existing saline groundwater or seawater. Equation III-17 describes the groundwater pumping rate that will not cause the salt water to rise more than one third the initial distance from the salt water interface to the bottom of the well screen, $(b^f - L^w)/3$ (McWhorter and Sunada, 1977).

$$p^{max} = \frac{2\pi}{3} b \, (bD^{fv} - L^w)^2 \, \frac{\Delta\rho}{\rho} \, K \qquad \text{(Eq. III-17)}$$

where:

p^{max}	=	maximum safe groundwater pumping rate, [L];
b^f	=	initial saturated thickness of fresh groundwater overlying saline groundwater, [L];
L^w	=	initial submerged well screen length, [L];
$\Delta\rho$	=	salt water density minus fresh water density, $[ML^{-3}]$; and
ρ	=	density of fresh water, $[ML^{-3}]$.

III.6.5. Salt Water Intrusion in a Coastal Unconfined Aquifer

Assume an unconfined aquifer discharges fresh water to the sea at a rate Q per unit length of outcrop (Figure III-11). The elevation of the water table above sea level at a specified distance (x) from the coast is:

$$h_f = \left(\frac{2Q_f \Delta\rho}{K_h(\rho + \Delta\rho)} \, x \right)^{½} = \left(\frac{2Q_f \Delta\rho}{K_h \rho} x \right)^{½} \qquad \text{(Eq. III-18)}$$

where:

h_f	=	height of fresh water surface above sea level at a distance x from the coast, [L];
Q_f	=	rate of fresh groundwater flow toward the coast per unit length of coast, $[L^3T^{-1}]$;
K_h	=	hydraulic conductivity, $[LT^{-1}]$.

Glover (1959) provided the following to predict the location of the salt water-freshwater interface.

$$z_s^2 = \frac{2\rho Q_f}{\Delta\rho K_h} x + \left(\frac{\rho Q_f}{\Delta\rho K_h}\right)^2 \qquad \text{(Eq. III-19)}$$

Where

z_s = depth below sea level of the fresh water-salt water interface, [L].

Figure III-11 shows that freshwater can escape to the sea through a horizontal gap of length x_{of}, [L].

$$x_{of} = -\frac{\rho Q_f}{2 \Delta \rho K_h} \qquad \text{Eq. III-20}$$

III.7. Aquifer Property Estimation

III.7.1. Introduction and General Approach

Many of the analytical expressions listed in earlier sections can be used to estimate aquifer parameters. This requires that head data be available. If doing this manually, one commonly estimates T_r/S, the aquifer diffusivity. This is simpler and sometimes almost as useful as trying to estimate T_r and S separately. Two equations that can be used in this way follow.

III.7.2. Estimating Aquifer Parameters by Observing Aquifer Head Response to Stream Stage Changes

Assume a stream and aquifer in saturated fully penetrated connection. The stream stage can change instantaneously at the end of any stress period, i.e. at times denoted by $k\Delta t$. (This differs from Section III.5.2 in which the stage is assumed to exist for the entire duration of a stress period.) The change in groundwater level, $\Delta h_{\delta,N}$ at time $N\Delta t$ after a series of surface water level changes can be estimated by Equation III-21 (Pinder et al, 1969). The same equation can be used to estimate aquifer parameters, such as diffusivity. Chapter IV, problem IV-J illustrates use of this expression within CONJUS.

$$\Delta h_{\delta,N} = \sum_{k=1}^{N} \Delta\sigma_k \left(\mathrm{erfc} \; \frac{u}{2\sqrt{N-k}} \right)$$

(Eq. III-21)

$\Delta\sigma_k$ = instantaneous surface-water stage rise at time $t = k\Delta t$, [L];
Δt = uniform duration of period of stage change (stress period length), [T].
u = $a / (\Delta t T_r/S)^{0.5}$
erfc = complementary error function.

III.7.3. Aquifer Head Response to Periodic Surface Water Fluctuations

When surface water levels fluctuate sinusoidally and with a common magnitude, a corresponding fluctuation results in an intersecting aquifer (Ferris, 1951). The following expression can be used to predict groundwater levels. Conversely, it can be used to estimate (T_r/S) from observed heads. Example problem IV-E in Chapter IV illustrates usage.

where:

$$\Delta h^{grange} = 2\Delta h^{smax} e^{-a\sqrt{\frac{\pi S}{t^{per} T_r}}}$$

(Eq. III-22)

Δh^{grange} = range in groundwater level fluctuation [L];
Δh^{smax} = maximum surface water level rise (half the distance between peak and trough) [L];
a = distance from surface water to well [L];
t^{per} = uniform stage fluctuation period, [T];

The duration of the time between when the max surface water elevation occurs and the max groundwater elevation occurs is:

$$t^{lag} = a\sqrt{\frac{t^{per} S}{4\pi T_r}}$$ (Eq. III-23)

t^{lag} = time lag between occurrence of maximum surface stage and maximum groundwater level [T].

III. 8. Optimization in CONJUS

III.8.1. Introduction

Section II.2 and Table II-2 indicate that the mathematical formulation of an optimization problem includes constraints (mainly describing physical system response to stimuli), lower and upper bounds on variables (mainly describing restrictions on management strategy acceptability) and an objective function.

To describe physical response to stimuli, CONJUS includes analytical equations shown in Sections III.3-5and III.8. Variables included in these expressions are: groundwater pumping (extraction or injection), groundwater head, streamflow depletion rate, streamflow depletion volume, intentional change in stream stage, and recharge from line and rectangular sources. CONJUS also explicitly includes as variables: surface water diversion from and return flow to a stream. In addition, CONJUS permits the formulation of simulation equations to represent many other algebraic equations, such as the water quality blending equation of Section III.6.2. Using these constraint equations and variables, CONJUS can develop optimal water management strategies for the situations described in Options A-E of Table A-1.

CONJUS requires the user enter lower and upper bounds on acceptable values for the above variables. If there is no reason to impose a particularly restrictive limit, one can use a very uninhibiting value. For example, if one wants to prevent too much stream depletion (due to pumping), but does not care how little stream depletion one causes, the lower bound on stream depletion should be zero. The upper bound on stream depletion should be the greatest value considered acceptable.

Normally, variables that managers can directly control (such as groundwater pumping or the water level in a nonleaky canal) are termed decision variables. Variables that describe how the physical system responds to stimuli are normally termed state variables, because they describe the state of the system.

III.8.2. Objective Function Options

III.8.2.1. Introduction

The objective function describes the dominant management goal. When developing an optimal management strategy, one wants a strategy that maximizes or minimizes the value of the objective function. For management flexibility, one wants to be able to select from a range of possible objective functions.

To permit the user to address as wide a range of management goals as practicable, CONJUS employs a three-component objective function. This organization permits one to build a composite objective function equation that includes linear, quadratic and nonlinear goal-programming terms.

The basic objective function is:

$$Z = W_1 Z_1 + W_2 Z_2 + W_3 Z_3 \qquad \text{(Eq. III-24)}$$

where:

Z is total objective value;

Z_1, Z_2 and Z_3 are the values of the linear, quadratic and goal programming objective components, respectively;

W_1, W_2 and W_3 are the weights assigned to linear, quadratic and goal programming objective components, respectively.

III.8.2.2. Linear Objective Component

$$Z_1 = \sum_{k=1}^{K} [\sum_{\hat{e}=1}^{M^{ep}} C^{ep}(\hat{e},k) p^{e}(\hat{e},k) + \sum_{\hat{e}=1}^{M^{ip}} C^{ip}(\hat{e},k) p^{i}(\hat{e},k) + \sum_{\hat{e}=1}^{M^{sd}} C^{sd}(\hat{e},k) d^{e}(\hat{e},k) + \sum_{\hat{e}=1}^{M^{rf}} C^{rf}(\hat{e},k) r^{i}(\hat{e},k)] \qquad \text{(Eq. III-25)}$$

where:

K = total number of stress periods;

k = stress period;

ê = extraction and injection pumping location;

C^{ep} (ê, k), C^{ip}(ê, k), C^{ep} (ê, k) and C^{ip}(ê, k) = coefficients for extraction and injection pumping, diversion from and return flow to stream, respectively ;

p^{e}(ê, k), p^{i}(ê, k), d(ê, k) and r(ê, k) = extraction and injection pumping, diversion and return flow rates, respectively. All are positive in sign;

M^{ep}, M^{ip}, M^{sd} and M^{rf} = total numbers of extraction and injection pumping wells, diversions and return flows, respectively.

III.8.2.3. Quadratic Objective Component

$$
\begin{aligned}
Z_2 = \sum_{k=1}^{K} \{ & \sum_{\hat{e}=1}^{M^{ep}} C^{epq}(\hat{e},k)\, p^{e}(\hat{e},k)\,[GSELEV(\hat{e}) - h^{e}(\hat{e},k)] \\
& + \sum_{\hat{e},=1}^{M^{ip}} C^{ipq}(\hat{e},k)\, p^{i}(\hat{e},k)\,[GSELEV(\hat{e}) - h^{i}(\hat{e},k)] \\
& + \sum_{\hat{e},=1}^{M^{sd}} C^{sdq}(\hat{e},k)\, d^{i}(\hat{e},k)\,[GSELEV(\hat{e}) - h^{d}(\hat{e},k)] \\
& + \sum_{\hat{e},=1}^{M^{rf}} C^{rfq}(\hat{e},k)\, r^{i}(\hat{e},k)\,[GSELEV(\hat{e}) - h^{r}(\hat{e},k)] \}
\end{aligned}
$$

(Eq. III-26)

where:

C^{epq}, C^{ipq},C^{sdq} and C^{rfq}= quadratic coefficients for extraction and injection pumping, surface water diversion and return flow, respectively;
GSELEV(ê) = ground surface elevation;
h(ê, k) = groundwater potentiometric surface.

III.8.2.4. Goal Programming Component

III.8.2.4.1. Introduction. CONJUS permits one to try to maximize the degree to which one achieves goals concerning groundwater pumping, cumulative pumping, groundwater head, and stream depletion rate and volume. Not achieving a goal (target) is termed a deviation from goal achievement. For flexibility, CONJUS permits one to select the degree to which one penalizes such deviations. Deviations can be penalized all equally using linear coefficients or nonlinearly using exponents. In the below goal programming objective options, deviations can be penalized to the n^{th} power.

III.8.2.4.2. The first option is used for the goal variables: stream depletion rate, stream depletion volume or cumulative pumping volume. Each is represented by generic variable V.

$$Z_3 = \sum_{k=1}^{K} C^{gv}(k)[V_1(k) - GOAL(k)]^n \qquad \text{(Eq. III-27a)}$$

where:

$V_1(k)$ = the user specified generic goal variable (stream depletion rate or volume or cumulative pumping volume);

n = power to which deviation from GOAL(k) is raised;

GOAL(k) = goal value for generic variable V at period k;

$C^{gv}(k)$ = Coefficient applied to the deviation from the goal.

III.8.2.4.3. Option two applies to groundwater pumping, surface water diversion and return flow.

Where:

$GOAL^{ep}$, $GOAL^{ip}$, $GOAL^{d}$, and $GOAL^{r}$ = goals for extraction, injection, diversion and return flow, respectively;
C^{gep}, C^{gip}, C^{gd}, and C^{gr} = coefficients assigned to the deviations GOALs.

III.8.2.4.4. The third option applies to groundwater head:

$$\begin{aligned} Z3 = \sum_{k=1}^{K} \{ & \sum_{\hat{e}=1}^{M^{ep}} C^{geh}(\hat{e},k)[h^{e}(\hat{e},k) - GOAL^{eh}(\hat{e},k)]^{n} \\ & + \sum_{\hat{e}=1}^{M^{ih}} C^{gih}(\hat{e},k)[h^{i}(\hat{e},k) - GOAL^{ih}(\hat{e},k)]^{n} \\ & + \sum_{\hat{o}=1}^{M^{oh}} C^{goh}(\hat{o},k)[h^{o}(\hat{o},k) - GOAL^{oh}(\hat{o},k)]^{n} \\ & + \sum_{\hat{e}=1}^{M^{sd}} C^{gdh}(\hat{e},k)[h^{d}(\hat{e},k) - GOAL^{dh}(\hat{e},k)]^{n} \\ & + \sum_{\hat{e}=1}^{M^{rf}} C^{grh}(\hat{e},k)[h^{r}(\hat{e},k) - GOAL^{rh}(\hat{e},k)]^{n} \} \end{aligned} \quad \text{(Eq. III-27c)}$$

where:

M^{oh} = total number of observation wells at which there cannot be pumping;
$C^{geh}, C^{gih}, C^{g\hat{o}h}, C^{gdh}, C^{grh}$ = coefficients assigned to deviations from respective head goals at extraction, injection and observation wells, and surface water diversions and return flows, respectively.

III.8.2.4.5. The fourth and fifth options apply to minimax or maximin optimization for any goal variable addressed in Equations III-25a-c.

$$Z3 = \min \{\max[c^{gv}(1)(V_2(1) - GOAL(1))^n, ..., c^{gv}(K)(V_2(K) - GOAL(K))^n]\} \quad \text{(Eq. III-27d)}$$

$$Z3 = \max \{\min[C^{gv}(1)(V_2(1) - GOAL(1))^n, ..., C^{gv}(K)(V_2(K) - GOAL(K))^n]\} \quad \text{Eq. III-27e)}$$

where:

GOAL(k) = goal value for generic variable V_2, period k;
$C^{gv}(k)$ = Coefficient applied to the deviation from the goal.
$V_2(k)$ = the user specified generic goal variable (any variable listed in Equations III-25a through III-25c);
n = power to which deviation from GOAL(k) is raised;
K = total number of stress periods.

Equation III-25e is nonlinear, meaning that it is difficult to know how close a computed optimal solution is to the global optimal solution. The most common approach to trying to get close to a globally optimal solution is to make numerous optimizations, each using a significantly different initial guess of the optimal solution. Example IV-G in Chapter IV shows how different initial guesses can yield different computed optimal solutions.

IV. Solved Problems

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Example IV-A (Reference, Section III.4; Determining:

- maximum duration of a single pumping era while avoiding unacceptable stream depletion.
- stream depletion rate and volume during and after pumping.
- maximum pumping rate that will not cause undesirable stream depletion and aquifer head change.)

Given: Well ê has a 15.24 cm radius and is located 2,500 m from a stream that fully penetrates a 10 m thick aquifer. Aquifer hydraulic conductivity is 100 md^{-1} (1.157 ms^{-1}) and storativity is 0.1. Assume that pumping will not cause stream stage to change significantly. Observation well ô is located halfway between well ê and the stream.

• IV-A.1.a) Wanted: Determine how many days well ê can pump at 0.03 $m^3\,s^{-1}$ without causing stream depletion to exceed $1.5{\cdot}10^{-3}\ m^3\,s^{-1}$ during the pumping period.

Solution: To solve this problem using provided equations and figures first determine sdf using Eq. III-3.∴

a	=	2,500 m	
Q	=	0.03 $m^3\,s^{-1}$	= 2,592 $m^3 d^{-1}$
S	=	0.1	
T_r	=	(10 m) (100 md^{-1}) = 1,000 $m^2\,d^{-1}$	
T/S	=	10,000 md^{-1}	

From Equation III-3, sdf = a^2S/T = $(2{,}500\ m)^2(0.1)$ / 1,000 $m^2\,d^{-1}$= 625 d
q/Q = $(1.5{\cdot}\ 10^{-3}\ m^3\ s^{-1})/(0.03\ m^3\ s^{-1})$ = 0.05

From Figure III-4, Curve A; for q/Q = 0.05, t/sdf = 0.13 = t / 625 d

∴ t = 81 days

Begin CONJUS input by specifying a single time period, one extraction and one observation well in Option A and select Option B to address a stream-aquifer system (Fig. IV-1, page 1). After resizing the rest of the input array, input the rest of the data as shown in the Figure. To use CONJUS for simulation rather than for optimization, merely fix the upper and lower bounds of the pumping rate at the same value 0.03 $m^3\,s^{-1}$ (2592 $m^3\,d^{-1}$). This prevents CONJUS from using a different pumping rate.

Additional Problem Information: aquifer storativity; hydraulic conductivity; duration of unit period at bottom of Fig IV-1, p. 1. After invoking Step 3 on the Main Sheet, input detailed Optimization Information. Because you are using CONJUS for simulation rather than optimization, any of the three options of the Optimization Information tab are acceptable. Linear programming is the appropriate Solver Option, and no Special Constraints are needed.

CONJUS runs quickly, producing the output of Fig. IV-1, p. 3. Note that outputs such as stream depletion rate and volume are listed as optimal values. They are indeed optimal, but for such a tightly constrained problem that only those shown values are possible.

Figure IV-1 shows input and output if an 81 day unit period is selected. Choosing a longer period causes the computed stream depletion rate to exceed 0.03 m^3 s^{-1} (129.6 m^3 d^{-1}) by the end of the period. For a problem such as this, repeated CONJUS implementations are needed in order to determine that 81 days is the longest acceptable duration..

CONJUS also computes heads just outside the well casing of the extraction well, head at the additional observation well, and stream flow depletion rate and total volume at the end of 81 days.

- IV-A.1.b) <u>Wanted:</u> Determine total stream depletion volume during the period determined in part a).

<u>Solution:</u> To address using Fig. III-4:

Qt = (2,592 m^3 d^{-1})(81 d) = 209,952 m^3
From Curve B; for t/sdf = 0.13, v/Qt = 0.013
v = (0.013)(Qt) = (0.013)(209,952 m^3) $\therefore$ <u>v = 2,729 m^3</u>

Note that after 81 days of pumping, the volume of stream depletion is only about one percent of the groundwater pumped.

CONJUS automatically computes the stream depletion volume when addressing the previous problem. The 2,690 m^3 reported (bottom of Fig. IV-1, p. 3) is similar to that estimated via Fig. III-4. CONJUS is theoretically accurate. Estimating values from the figures is sometimes difficult and inaccurate, with errors commonly ranging from 10-20 percent.

- IV-A.1.c) <u>Wanted:</u> Assume pumping ceases when the stream depletion rate equals $1.5 \cdot 10^{-3}$ m^3 s^{-1}. Determine the stream depletion rate 30 days after pumping stopped.

<u>Solution:</u> In the manual approach, employ values from part (a),

sdf = 625 days, t_p/sdf = 0.13, and t_p = 81 days
t_i = 30 days
(t_p+t_i)/sdf = (81 d + 30 d)/625 d = 0.178

From Figure III-5, for t_p/sdf = 0.13 and (t_p+t_i)/sdf = 0.178; q/Q = 0.09

q = (0.09)Q = (0.09)(0.03 m^3 s^{-1}) $\therefore$ <u>q = $2.7 \cdot 10^{-3}$ m^3 s^{-1}</u> (<u>233 m^3 d^{-1}</u>)

You must rerun CONJUS using much the same information as above. However, employ two time periods of different duration. (For all except a two-period problem CONJUS requires that all stress periods be of equal duration). Significant input shown in Fig. IV-2 includes:

General Information: one extraction well.

- Extraction Well Information: x coordinate and upper and lower pumping bounds set at 0.03 m^3 s^{-1} (2592 m^3 d^{-1}) for period 1 and 0 for period 2.
-. Additional Problem Information: duration of unit period 1 and 2 are 81 and 30 days, respectively.

CONJUS results include heads at pumping and observation wells and depletion rates and total volumes by the end of each of the two periods (Fig IV-2).

- IV-A.1.d) Wanted: Determine the total volume of stream depletion 30 days after pumping ceased.

Solution: To do this manually, begin with Figure III-6.

For t_p/sdf = 0.13 and (t_p+t_i)/sdf = 0.178; v/(Qsdf) = 0.005
$v = (0.005)Q(sdf) = (0.005)(0.03\ m^3\ s^{-1})(625\ d)(86{,}400\ s\ d^{-1})$
$\therefore$ $\underline{v = 8{,}100\ m^3}$

CONJUS computes a total volume of depletion of 8,220 m^3 (Figure IV-2). Note that even by 30 days after pumping ceased, the stream depletion volume equals less than four percent of what was pumped.

- IV-A.1.e) Wanted: Determine the greatest stream depletion rate and when it will occur, in response to pumping well ê at 0.03 $m^3\ s^{-1}$ for the period determined in part a).

Solution: From Figure III-5, for an interpolated curve of t_p/sdf = 0.13; the greatest q/Q is about 0.106.

Maximum q = (0.106)Q = $(0.106)(0.03\ m^3\ s^{-1}) = 3.18\cdot 10^{-3}\ m^3\ s^{-1}$.
From the same point on Fig. III-5, the maximum q occurs when (t_p+t_i)/sdf = 0.25

$(81\ d + t_i) = (0.25)(625\ d)$, $\therefore$ $\underline{t_i = 75\ \text{days}}$

To solve this using CONJUS, one systematically runs simulations for different period 2 durations. Direct comparison will identify which duration (to the nearest day) causes the greatest depletion rate. This does not take long.

- IV-A.2.a) Wanted: Assume that you want to pump for 81 days at some maximum rate and then will stop pumping for 30 days. First, determine the maximum steady pumping that, if continued for 81 days, will not cause stream depletion by the end of that time to exceed $2\cdot 10^{-3}\ m^3\ s^{-1}$.

Also, determine the following for a moment right at the end of the 81 days of maximum pumping:

a) the stream depletion rate,
b) total volume of stream flow depletion to that moment,
c) the drawdown just outside the well ê casing, and
d) the drawdown at well ô.

Solution: The easiest way to solve this problem set is via CONJUS (set up for two stress periods of 81 and 30 days, respectively). Important input data shown in Fig. IV-3 not discussed previously includes:

Extraction Well Information: x coordinate, well radius, lower pumping bounds set at 0, and no upper pumping bound.

Additional Problem Information: 81 day duration of unit period 1.

Stream Depletion Constraints: 172.8 m^3 d^{-1} upper limit on stream depletion during period 1.

Figure IV-3 shows CONJUS output including:

a) maximum steady pumping rate = 3,490 m^3 d^{-1} .
b) stream depletion rate by end of period 1 = 172.8 m^3 d^{-1} .
c) total volume of stream flow depletion by end of period 1 = 3,623 m^3 d^{-1}.
d) drawdown just outside the well ê casing by end of period 1 = 100 - 94.95 = 5.05 m.
e) drawdown at well ô by end of period 1 = 100 - 99.84 = 0.16 m.

• IV-A.3. <u>Wanted:</u> Assume the well pumps for 81 days at the maximum rate determined in part 2, and then stops pumping for 30 days. Determine the same variable values for the moment 30 days after ceasing pumping.

<u>Solution:</u> The CONJUS optimization run for problem IV-A.2 solves this problem by merely forcing the upper and lower bounds on pumping in period 2 to equal zero. Depletion by the end of period 2 is unbounded. Fig. IV-3 identifies that:

a) stream flow depletion rate at the end of period 2 = 321.5 m^3 d^{-1} .
b) total volume of stream flow depletion by the end of period 2 = 11,068 m^3.
c) drawdown just outside the well ê casing at the end of period 2 = 100 - 99.64 = 0.36 m.
d) drawdown at well ô at the end of period 2 = 100 - 99.82 = 0.18 m.

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Example IV-B (Reference, Section III-IV; Siting a well to avoid unacceptable stream depletion)

Given: A well is needed to provide 0.02 $m^3 s^{-1}$ continuously during a 182-day summer each year. The well will tap an alluvial aquifer near a stream. Based on legal water rights, the total stream depletion volume by the end of 182 days should not exceed 3,000 m^3. The aquifer recharges completely each winter. Aquifer thickness is 10 m, hydraulic conductivity is 20 m d^{-1} , T_r is 200 $m^2 d^{-1}$ and S is 0.15.

Wanted: What is the smallest acceptable distance between the well and the stream.

Solution: The manual approach employs Figure III-4.

$Q = 0.02\ m^3 s^{-1} = 1{,}728\ m^3 d^{-1}$

$t_p = 182$ d

$v = 3{,}000\ m^3$

$T_r/S = (200\ m^2 d^{-1})/(0.15) = 1{,}300\ m^2 d^{-1}$

$v/(Q\ t_p) = (3{,}000\ m^3)/(1{,}728\ m^3 d^{-1} \cdot 182\ d) = 0.0954$

From Fig III-4, curve B; for $v/(Q\ t_p) = 0.0954$, ; $t_p/sdf = 0.34$

From Equation III-3; $sdf = a^2S/T_r$, so $0.34 = t_pT_r/\ a^2S$

$a = \{t_p(T_r/S)/(0.34)\}^{0.5} = \{(182\ d)(1{,}300\ m^2 d^{-1})/(0.34)\}^{0.5}$ $\therefore$ $\underline{a = 834\ m}$

To do this with CONJUS requires repeated optimizations employing different distances between well and stream. The final answer would usually be more precise than that obtained graphically.

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Example IV-C (Reference, Section III-IV; Managing multiseason groundwater pumping while avoiding unacceptable stream depletion)

Given: A cooperative wants to place a well 150 m near a stream and pump it at 0.06 $m^3 s^{-1}$ (5184 $m^3 d^{-1}$) continuously during a 122-day irrigation season beginning on the same date each year. However, the well must not cause more than 0.04 $m^3 s^{-1}$ (3456 $m^3 d^{-1}$) depletion during either the first or the second irrigation season. To ease CONJUS use, assume four stress periods, each of 365/3 or 121.7 days. Periods 1 and 4 will have the same pumping rate. There is no pumping during periods 2 and 3.

Aquifer thickness is 10 m, T_r is 600 $m^2 d^{-1}$ and S is .25.

a = 150 m
Q = 0.06 $m^3 s^{-1}$ (5184 $m^3 d^{-1}$)
q max permissible = 0.04 $m^3 s^{-1}$
t_p = 121.67 d
T_r/S = (600 $m^2 d^{-1}$ / 0 .25) = 2,400 $m^2 d^{-1}$

- IV-C.1. Wanted: Determine whether the stream depletion rate will exceed 0.04 $m^3 s^{-1}$ during either of the two irrigation seasons (periods 1 and 4).

Solution: In a manual approach, begin by determining what the depletion rate will be after 121.7 days of pumping.

sdf = $(150\ m)^2$ / (2,400 $m^2 d^{-1}$) = 9.4 days
t_p / sdf = (121.7 d)/ 9.4 d = 12.9
From Fig. III-4, (1-q/ Q) curve; for t_p / sdf = 12.9, (1-q/Q) = 0.15
q = (1 - 0.15)Q = 0.85 (0.06 $m^3 s^{-1}$)= 0.051 $m^3 s^{-1}$ (44.06 $m^3 d^{-1}$)
∴ stream depletion will exceed the limit during the first irrigation season if pump at 0.06 $m^3 s^{-1}$

Figure IV-4 shows CONJUS output for a four period run. Since pumping rates are specified for all four periods, this constitutes a simulation run. Stream depletion is not restricted. Bounding stream depletion to the desired rate while fixing groundwater pumping would leave no feasible solutions since the resulting depletion is 4,377 $m^3 d^{-1}$ at the end of period 1. The optimization algorithm would be unable to simultaneously satisfy all constraint equations and assigned pumping values if depletion was constrained to not exceed 3,456 $m^3 d^{-1}$ by the end of period 1.

- IV-C.2. Wanted: If the depletion rate does exceed the limit, determine to the nearest day when this will occur.

Solution: Using Fig III-4,
(q max permissible)/Q = (0.04 $m^3 s^{-1}$)/ 0.06 $m^3 s^{-1}$ = 0.67
From Fig. III-4, (1-q/ Q) curve; for (1-q/ Q)= 0.33, t/sdf = 2.3
t = 2.3 (sdf)= 2.3(9.4 d)=21.6 d
∴ stream depletion will exceed the limit during day 22 of the first irrigation season

Determining the time when exceedance occurs via CONJUS requires using an iterative approach. Input a unit period, examine the resulting depletion rate and modify the period duration accordingly. Soon a mathematically accurate estimate of 25 days for an acceptable period 1 duration will be identified. At the end of 25.5 days, the depletion rate reaches 3,463 m^3 d^{-1} . (25 and 26 days yield depletions of 3447 and 3479 m^3 d^{-1} , respectively). (Minimal data inputs include one period, one extraction well, yes for Option B, no for Options C-E, 0.25 storativity, 60 hydraulic conductivity, 25 day unit period, 150 and 0 extraction well coordinates, 10 aquifer thickness, arbitrary elevations for ground surface, initial head and nonoptimal head, n/a bounds on head, 5184 lower and upper bounds on pumping in period 1, (0,0) and (0,1) river definition coordinates, n or n/a bounds on stream depletion and cumulative pumping volume.

- IV-C.3. Wanted: Determine the greatest steady rate at which the well could be pumped during the two irrigation seasons in order not to exceed the depletion limit at the end of either season.

Solution: Initially, determine the maximum pumping that is acceptable with regards to stream depletion during the first season.

As above, t_p / sdf = (121.7 d)/ 9.4 d = 12.9
From Fig. III-4, (1-q/Q) curve; for t_p/sdf = 12.9, (1-q/Q) = 0.15, so q/Q = 0.85
Q = q maximum / 0.85 = (0.04 m^3 s^{-1})/0.85 = 0.047 m^3 s-1 (4060.8 m^3 d^{-1})

Now determine how significantly pumping at 0.047 m^3 s^{-1} during the first season affects the depletion rate at the end of the second pumping season (i.e. the residual effect). Since we are evaluating depletion some time after pumping ceased, one might expect to use Fig. III-5. However, the 12.9 t_p / sdf value exceeds that of any t_p / sdf curve in Fig. III-5. Therefore we will use Fig III-5 and superposition. The depletion rate after 486.7 days resulting from pumping (extraction) during the first 120 days is determined by summing: the depletion resulting from extracting groundwater for 486.7 days, plus the accretion resulting from injecting groundwater for the last 365 days.

$(t_{extraction})$/sdf = 486.7/9.4 = 51.8

From Fig. III-4, (1-q/Q) curve; for (t_p)/sdf = 51.8, (1-q/Q) = 0.078

$q_{depletion}$ = (1-0.078)Q = 0.922 (0.047 m^3 s-1) = 0.0433

$(t_{injection})$/sdf = 365/9.4 = 38.8

From Fig. III-4, (1-q/Q) curve; for (t_p)/sdf = 38.8, (1-q/Q) = 0.09

$q_{accretion}$ = (1-0.09)Q = 0.91(0.047 m^3 s-1) = 0.0428

The net stream flow depletion rate at the end of the second irrigation season due to pumping during the first irrigation season is (0.0433 - 0.0428) or 0.0005 $m^3\ s^{-1}$. Adding this to the result of pumping during the second season alone (0.04 $m^3\ s^{-1}$) will give a depletion very slightly larger than the intended 0.04 $m^3\ s^{-1}$ maximum permissible.

In order to not exceed a depletion rate of 0.04 $m^3\ s^{-1}$ at the end of the second irrigation season, one must pump less during season one. A result will be that depletion at the end of period one will be slightly less than the maximum acceptable rate.

To use CONJUS to compute the maximum acceptable pumping rate that could be used in both seasons input the following. Use 4 periods, one extraction well, Option B, 0.25 storativity, 60 conductivity, 121.67 unit period, (150,0) well coordinates, 10 aquifer thickness, reasonable well radius, ground surface and head elevations, a midrange unit pumping (1000 for example), no bounds on head, 0.0 lower bounds on pumping, no upper bounds on pumping in periods 1 and 4, 0.0 upper bounds on pumping in periods 2 and 3, (0,0) and (0,1) stream coordinates, 0.0 lower bound on stream depletion rates, no bounds on stream depletion volume, 3456 upper bound on stream depletion rate in all periods, no bounds on cumulative pumping. In addition, after pushing button 3 on the main menu to perform the optimization, select the special constraints tab, and select the option that forces pumping to be equal for all periods having n/a on the lower or upper bound on pumping.

The computed optimal pumping is 4035.3 for periods 1 and 4, and the resulting stream depletion rates are 3407.4, 182, 81 and 3456.0 for periods 1 to 4, respectively.

- IV-C.4. Wanted: Develop a maximum pumping strategy that has different pumping rates in the two irrigation seasons and satisfies the depletion rate constraint at the end of each irrigation season.

Solution: The easiest way to address this problem is via CONJUS. This requires using almost all the same input as problem IV-C.4. The only difference is that for this problem you should not employ the special constraint of forcing the period 1 and 4 pumping rates to be equal. Without that extra pumping constraint, CONJUS is able to provide a little more total pumping--4,093, 0, 0, and 4,035 $m^3\ d^{-1}$ for periods 1 through 4, respectively. Resulting stream depletion rates are 3,456 , 185, 83 , and 3,456 at the end of the same four periods.

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Example IV-D (Reference III.5.1; Determining aquifer head change due to a line source of recharge)

Given: When flowing full, a canal constructed in alluvial sediments in a river valley is expected to leak at the rate of one cubic foot per second per mile of length. The homogeneous aquifer is 18.28 m (60 ft) thick, has a storativity of 0.16 and a hydraulic conductivity of 0.0012 m/s (0.004 ft/s). A well exists 402.44 m (1320 ft) from the canal. Initially there is no water in the canal, and the underlying groundwater is at steady-state conditions. At time=0.0 assume the canal is put into operation and begins flowing full. Knowns include: $q = 1\ ft^{3}\ s^{-1}\ mile^{-1} = 0.000189394\ ft^{2}s^{-1}$ and a = 1320 ft.

Wanted:
Estimate how much the water table will rise at the well by time = 180 days.

Solution: In the manual approach, employ Equation III-6.

$T_r = (60)(0.004) = 0.24\ ft^{2}\ s$
$T_r / S = 0.24\ ft^{2}\ s / 0.16 = 1.5\ ft^{2}\ s$
$t = 180 \bullet 24 \bullet 60 \bullet 60 = 1.5552000\ 10^{7}\ s$
$z = 1320\ ft / \{ 4 \bullet 1.5552000\ 10^{7}\ s \bullet 1.5\ ft^{2}\ s^{-1})^{0.5} = 0.13665\ ft^{-1}$

The error function value is 5.7166 Glover (1978). Substituting the above values into Eq. III-6 yields Δh = about 1.68ft.

To employ CONJUS, invoke step 1 on the main sheet and bring up the data input sheet. Assume a single stress period, and declare that one extraction well exists (recall that CONJUS requires that one pumping well be declared, even if it does not pump). Select option D, which permits computing aquifer head response to a line source. After resizing the input arrays, enter the appropriate data in all white cells. Insert 0.0, n/a or a letter such as n as the upper and lower bounds on pumping at the well, and on cumulative pumping.(Recall that you can disable any constraint by entering a letter or string of letters in the related cell.) Remember to use consistent units, such as feet and seconds.

CONJUS computes a head at 101.67 ft, i.e. a 1.67 ft rise in head at the well by the end of 180 days.

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Example IV-E (Reference III.5.2; Predicting aquifer head change due to periodic changes in a surface water body.)

Given: Water levels in a large freshwater body fluctuate sinusoidally. Potentiometric heads in an adjacent unconfined aquifer respond with regular fluctuations, after a time lag. The amplitude or half range of the stage fluctuation of the surface water body level is 1 m, and the period of the fluctuation is 12 hours. A 30 cm radius well is located 50 m from the shore. Aquifer transmissivity is 300 $m^2\ d^{-1}$ and storativity is 0.1. The well does not pump in problems IV-E.1 and 2. It does pump in problem IV-E.3.

- IV-E.1. Wanted: Determine the lag time between the highest surface water stage and the highest potentiometric head in the well.

Solution:

Δh^{smax} = 1 m
a = 50 m
S/T_r = $(0.1)/300\ m^2\ d^{-1} = 3 \cdot 10^{-4}\ d\ m^{-2}$
P = 12 hr = 0.5 d

From Eq. III-23, $t^{lag} = a\ \{PS/4\pi T_r\}^{0.5}$

$= 50\ m\ \{(0.5\ d)(.1)/(4 \cdot 3.1416 \cdot 300\ m^2\ d^{-1})\}^{0.5}$
$= 50\ (1.326 \cdot 10^{-5})^{0.5} = 0.1821\ d$

- IV-E.2. Wanted: Determine the range of groundwater fluctuations at the well resulting from surface water fluctuations.

From Eq. III-22 $\Delta h^{grange} = 2\ \Delta h^{smax}\ (e^{r1})$, where $r1 = -a\ (\pi S/PT_r)^{0.5}$

rl $= -50\ m\ (3.1416 \cdot 0.1\ /0.5\ d\ 300\ m^2\ d^{-1})^{0.5} = -50\ (2.1 \cdot 10^{-3})^{0.5}$
$= -2.2882$
$\Delta h^{grange} = 2\ \Delta h^{smax}\ (e^{r1}) = 2 \cdot 1 \cdot (0.1014) = 0.20\ m$

This Δh_{gw} is the maximum amplitude of the groundwater wave from trough to peak because Δh_{sw} is one-half the amplitude of the full range of the surface water fluctuation.

- IV-E.3. Wanted: Assume the mean groundwater elevation at the well is 95 m above sea level if the well is not pumping. The groundwater level fluctuates around that value due to the surface water oscillations. Assume the well then begins pumping continuously at 325 $m^3\ d^{-1}$. What is the lowest water table elevation one would then expect just outside the well casing? (Assume that pumping from the well will not affect the river stage and that drawdown from the well will not extend beyond the river.)

Solution: This can be solved using CONJUS and its ability to accept as input heads expected to occur without the effect of optimal water management. Here one first determines that the lowest head that will result at the well without pumping is 94.9 m (i.e. 95 - 0.2/2). This is input as the nonoptimal head at the end of period 1. Other CONJUS inputs include: one time period, 1 extraction well, Option B (including stream and image well), 0.1 storativity, 10 m/d conductivity, unit pumping period long enough to assure steady-state drawdown is achieved (for example 60 d), (50,50) well location, 30 m aquifer thickness, 325 $m^3\ d^{-1}$ unit pumping and lower and upper bounds on pumping, initial water level of 95 m, end-of-period 1 water level of 94.9, and a stream located along the y axis (at x=0).

CONJUS automatically subtracts from the 94.9 m nonoptimal head the drawdown resulting from the assumed pumping. Because this drawdown just outside the well casing is 1 m, the lowest head expected due to regular fluctuations and steady pumping is 93.9 m (i.e. 94.9 - 1.0).

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Example IV-F (Reference III.5.2; Predicting aquifer head change due to managed stage changes in fully hydraulically-connected canal)

Given: A fully penetrating well is located 50 m from a canal that fully penetrates a confined aquifer. Well diameter is 20 cm. Aquifer hydraulic conductivity is 100 m d^{-1}, initial saturated thickness is 20 m, and storativity is 0.2. Heads in canal and aquifer are initially at equilibrium at 10.0 m MSL. Water levels in the canal change at the beginning of each week and persist for a week. During weeks 1-4, canal water levels are 11 m, 10.5 m, 10 m and 11.5 m.

- F.1. Wanted: Assume there is no pumping from the well. Predict the well heads at the end of weeks 1-4.

Solution: This problem is much more easily solved using CONJUS than by hand. In this case CONJUS will be used to simulate system response to an input water management strategy. The imposed constraints (bounds) will not permit it to compute a strategy that differs from what is input.

In the CONJUS main sheet declare four stress periods and a single extraction well (Option A) and select Option C. After resizing the input data sheet, input the appropriate data. It is easiest to use units of meters and days. The duration of the unit stress period is 7 days. Because Option C forbids employing wells that actually pump, employ 0.0 as the lower and upper bounds on pumping and cumulative pumping.

Data input includes: Storativity = 0.2; conductivity = 25 m d^{-1}; period duration = 7 days; well x location = 50 m; y location is arbitrary (for example 100); aquifer thickness = 80 m; well radius = 0.1 m; ground surface elevation is arbitrary (e.g. 20 m); unit pumping is arbitrary since the well will not pump (use 1 m^3 d^{-1}); initial and non optimal heads are 10 m since that is the equilibrium water level; head lower and upper bounds are not used (n); stream (x1,y1) and (x2,y2) coordinates are selected maintain the appropriate distance from the well, for example (0, 0) and (0, 200); unit stream stage should be in the range of what will occur (e.g. 1 m); and minimum and maximum stream stage changes should be identical for a particular period, but should differ by period according to the planned management (e.g. 1, 0.5, 0.0, and 1.5 m respectively for periods 1-4).

Using the above input data, CONJUS predicts that head at the well will be 10.89m, 10.48m, 10.03m and 11.36m by the end of periods 1-4 respectively.

- F.2. Wanted: Assume that pumping from the 20 cm diameter well will not significantly affect stream stage, but that stream stage is changing as described in Problem IV-F.1. Determine head just outside the well at the end of weeks 1-4 if the well is pumping $2 \cdot 10^{-2}$ m^3 s^{-1} for the entire period beginning at time zero.

Solution: When trying to apply multiple analytical solutions to the same area, one should be careful that the equation assumptions do not conflict and become violated. For this reason, CONJUS does not explicitly permit pumping at wells when predicting aquifer head change due to stream stage changes. However, one can still use CONJUS to address Problem IV-F.2. To do this one must combine the results of two separate CONJUS implementations, and assume simultaneous acceptability of both sets of assumptions. An illustration follows.

The first CONJUS implementation is that of Problem IV-F.1. It employs the assumptions of Options A (without using groundwater pumping) and C. The second CONJUS implementation employs Options A and B (pumping from a well in a stream-aquifer system, and using image wells to cause zero drawdown at the stream).

The approach is as follows. We use the well heads resulting from Implementation 1 as the nonoptimal heads for Implementation 2. In Implementation 2 CONJUS will subtract from those heads the drawdown resulting from pumping from the well. In this way CONJUS superimposes the separate effects of the stream and the well to estimate a combined effect. Combining both implementations is theoretically acceptable if, in the field, drawdown due to pumping does not extend on the other side of the stream from that of the pumping well, and if pumping will not significantly affect stream stage. Of course, all the other assumptions of aquifer homogeneity and extent and horizontal flow must also be acceptable.

Use the same data input for this implementation as for Implementation 1, with the following exceptions. Use 10m for the initial head; 10.89, 10.48, 10.03 and 11.36 as the nonoptimal heads at the ends of periods 1-4, respectively; 172.8 $m^3\, d^{-1}$ ($2 \cdot 10^{-2}\, m^3\, s^{-1}$) as the unit pumping and lower and upper bounds on pumping for the respective periods; and n or n/a for all bounds on stream depletion and cumulative pumping.

CONJUS computes well heads of 8.51, 7.97, 7.46 and 8.76 m, for the four respective periods. These are the heads resulting from both stream stage changes and pumping. Of course, the pumping strategy could be time varying if desired.

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Example IV-G (Reference, Section III.5.3; Determining how high the water table will rise beneath a recharge basin).

Given: A rectangular basin (100 m by 50 m) is located above an unconfined aquifer. The steady-state water table elevation is 10 m. An impervious barrier underlies the aquifer at an elevation of 0 m. Assume the basin is filled with water within one day and water begins seeping downward. By the time the percolating water reaches the water table (assume this is day zero) its infiltration rate is 17 cm d^{-1}. Aquifer hydraulic conductivity is 4.896 m d^{-1}, transmissivity is 0.034 m^2 min^{-1} and storativity is 0.12

Wanted: Determine the elevation of the water table directly beneath the center of the recharge basin 15 days after the infiltrating water reaches the water table.

Solution: The easiest way to solve this is with CONJUS. Specify one stress period, employ one extraction well in Option A, and invoke Option E. After resizing, specify the well coordinates to be the same as you input for the center of the recharge basin, (200,200) for example. Enter aquifer parameters in meters and days, and specify a 15 day unit stress period. Specify a 10 m aquifer thickness, an arbitrary well radius (e.g. 0.1 m); a 30 m ground surface elevation; initial and nonoptimal period 1 water table elevations of 10 m, n/a for the unnecessary bounds on head, and 0.0 for lower and upper bounds on pumping and cumulative pumping. Specify zero angle of rotation, a length of 100 m, a width of 50 m, 0.17 m d^{-1} for the unit, lower bounds and upper bounds on infiltration rate, and n for bounds on cumulative pumping.

This CONJUS solution takes about 15 minutes on a Pentium II (400 Mhtz). CONJUS predicts that the water level beneath the center of the recharge basin will be at an elevation of 14.2 m by the end of the 15 day period. This corresponds to a rise of 4.2 m. Note that CONJUS can easily be used to compute the maximum deep percolation rate that will not cause the water level to rise more than some specified elevation.

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Example IV-H. (Reference, Section III.6; Blending waters of different qualities and preventing groundwater table rise due to irrigation and leaching)

a) Given:
Assume cantaloupe is to be irrigated. Therefore, 1.5 dS $^{-1}$ is the maximum allowable electrical conductivity of the irrigation water to be used (Fig III-9). The ECs of available canal and groundwater are 0.3 and 2.8 dS $^{-1}$, respectively. The total flowrate that must be delivered to the project site is 5 m^3s^{-1}. There is insufficient canal water to satisfy this need, so groundwater must be blended with the canal water

Wanted: Determine the maximum amount of groundwater that can be blended with canal water in order to provide 5 m^3s^{-1} of acceptable quality blended water, and how much canal water should be used.

Solution:

$Ec_{bw} = EC_{iw}$ of Figure III-9 = 1.5 dS $^{-1}$
$EC_{frw} = 0.3$ dS $^{-1}$
$EC_{saw} = 2.8$ dS $^{-1}$
$(V_{frw} + V_{saw}) = 5\ m^3s^{-1}$

From Eq. III-10, determine V_{saw}

$Ec_{bw} = (EC_{frw}\ V_{frw} + EC_{saw}\ V_{saw}) / (V_{frw} + V_{saw})$
$1.5\ dS^{-1} = \{(0.3\ dS^{-1})(5\ m^3s^{-1} - V_{saw}\ m^3s^{-1}) + (2.8\ dS^{-1})(V_{saw}\ m^3s^{-1})\}/(5\ m^3s^{-1})$
$V_{saw} = 2.4\ m^3s^{-1}$ (Note this is 48% of the total)
$V_{frw} = 5 - 2.4 = 2.6\ m^3s^{-1}$

b) Given: Assume total annual evapotranspiration is 90 cm, infiltrating precipitation is 25 cm, and irrigation efficiency is 0.7. Assume groundwater used for irrigation will be extracted from beneath the area being irrigated.

Wanted: Using the one-dimensional approach of Willardson, estimate the least amount of groundwater (as a volume per unit area) that should be extracted to prevent groundwater level rise.

Solution:
$ET = 0.90$ m
$P_i = 0.25$ m
$IE_i = 0.7$

First determine the leaching fraction.

From Fig. III-9, for cantaloupe, $EC_{max} = 3.2$ dS m^{-1}
From Eq. III-12, $LF_{min} = EC_{iw} / EC_{max} = 1.5 / 3.2 = 0.47$
From Eq. III-11, determine the total irrigation water needed.

$IW_i = \{(ET - P_i)\ (1 + (LF / (IE_i - LF)))\} / IE_i$
$= (0.90\ m - 0.25\ m)\ (1 + (0.47 / (0.7 - 0.47))) / 0.7 = (0.65)(3.0)/0.7$
$= 2.8$ m

From Equation III-13, the least amount of groundwater that must be extracted beneath the area experiencing deep percolation of irrigation water, in order to prevent water table rise, is:

$MIN_{gw} = IW_i + P_i - ET = 2.8\ m + 0.25\ m - 0.90\ m = 2.2\ m.$
If less than 2.2 m of groundwater is removed, groundwater levels will rise.

c)<u>Wanted:</u> Determine the maximum groundwater salinity that can be blended with 0.3 dS m^{-1} canal water while preventing water table rise.

<u>Solution:</u>

From Eq. III-14 and Figure III-9,

$$EC_{gwx} = ((IW_i)(EC_{iw} - EC_{caw}) / MIN_{gw}) + EC_{caw}$$
$$= ((2.8)(1.5 - 0.3) / 2.2) + 0.3 = 1.8\ dS\ m^{-1}$$

Because EC_{gwx} is less than that of the groundwater (i.e. EC_{saw} or 2.8 dS m^{-1}) water tables will rise unless more groundwater is removed from the area than can be safely applied to the crop. This conclusion can also be reached from the answers to parts a) and b). Since 2.2 m of required groundwater extraction exceeds 48% of the 2.4 m irrigation requirement, more groundwater must be removed from the area through a water disposal drainage system.

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Example IV-I. (Reference, Section III.6; Coastal salt water intrusion)

Given: A company is considering buying some land along the coast for possible agricultural development. The land is underlain by a 60 m thick unconfined aquifer that outcrops to the ocean. Currently, the aquifer discharges $2.0 \cdot 10^{-5}$ m^3 s^{-1} per meter of coastline to the ocean. The company is considering constructing an unlined canal that would carry fresh water which would recharge the aquifer. The canal would parallel the straight coastline. The groundwater level is far enough below the ground surface that one would consider the canal as a line source. Assumedly, at least one half of any recharge from the canal would flow toward the coast.

> Assume z = 60 m; $K = 1.0 \cdot 10^{-4}$ m^3 s^{-1} ; $\rho = 1$ g cm^{-3} ; $\Delta\rho = 0.025$ g cm^{-3} ;and $Q_f = 2.0 \cdot 10^{-5}$ m^3 s^{-1} per meter.

Wanted: IV-Ia). Estimate how far inland from the coast is the toe of the salt water wedge. IV-Ib). Estimate what that distance would ultimately be if total canal seepage were $0.25 \cdot 10^{-5}$ m^3 s^{-1} per meter length.

Solution:

IV-Ia). To ease Equation III-19 use, first define $M = (\rho/\Delta\rho\ K)$

$$= (1)/\{(0.025)(1.0 \cdot 10^{-4})\} = 4.0 \cdot 10^{5}$$

$$L = x = \{1/(2(M\ Q_f))\}(z^2 - (Q_f M)^2)$$

$$= \{1/(2(4.0 \cdot 10^{5})(2 \cdot 10^{-5}))\}\{60^2 - ((2 \cdot 10^{-5})(4.0 \cdot 10^{5})^2\}$$

$$= 0.0625\{3600-64\} = 221 \text{ m}$$

IV-Ib). Total flow to the coast will be $(2 + 0.125) \cdot 10^{-5}$

$$L = \{1/(2(4.0 \cdot 10^{5})(2.125 \cdot 10^{-5}))\}\{60^2 - ((2.125 \cdot 10^{-5})(4.0 \cdot 10^{5})^2\}$$

$$= 0.0588\{3600-72.25\} = 207 \text{ m}$$

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Example IV-J. (Reference, Section III.7; Aquifer property estimation)

Given: A 0.25 m radius well is located 50 m from a stream. Both well and stream fully penetrate a homogeneous 10 m aquifer having a 0.3 storativity. The initial equilibrium aquifer head is 10 m. The stream stage was suddenly increased by 1m and remained at the new elevation. By the end of 2 days, the head in the well had increased by 0.67 m.

Wanted: Estimate the aquifer hydraulic conductivity.

Solution: Use CONJUS Option C iteratively by assuming a conductivity, and then computing the head change at the well. After several such efforts, a conductivity of 100 m/d is used, yielding an aquifer head increase of 0.665 m.

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Example IV-K. (Reference, Section III-8; Minimizing size of reservoir needed to provide adequate irrigation water while avoiding unacceptable stream depletion)

Given: A farmer wants to irrigate using groundwater . His pumping well is 450 meters from a stream. The aquifer hydraulic conductivity is 20 m/day and storativity is 0.25. The aquifer thickness is 30 meters and the well radius is 0.3048 m. The initial water table is horizontal at 100 meters above sea level. The farmer needs 0.8 ha-meters of water per week for four weeks to satisfy his irrigation requirement, and is legally permitted to pump enough to satisfy his needs, as long as the stream depletion does not exceed 245 $m^3 d^{-1}$ at any time during the specified four-week groundwater pumping period. He does not want head just outside the well casing to drop below 97 m.

The farmer asked an assisting irrigation engineer how much stream depletion would result from pumping steadily at the 0.8 ha-m/wk (1142.8572 $m^3 d^{-1}$) rate. The CONJUS answer is 251 $m^3 d^{-1}$ by the end of week 4, exceeding the legal upper limit. The engineer also used CONJUS to compute the maximum steady pumping rate that would not cause unacceptable stream depletion. The 1115.4 $m^3 d^{-1}$ rate would yield a total cumulative pumping of 31,232 m^3 by the end of week 4.

The engineer then used CONJUS to minimize the total time-varying (transient) pumping that would satisfy both the 0.8 ha-m/wk lower bound and the 245 $m^3 d^{-1}$ upper bound. The computed optimal pumping strategy of 1751, 534, 1143 and 1170 $m^3 d^{-1}$ (for weeks 1-4) caused depletion rates of 25, 127, 181 and 245 $m^3 d^{-1}$, respectively; and pumped only 32,192 m^3 in total during the four week period (tight constraints were the 97 m head bound and the 245 $m^3 d^{-1}$ depletion bound). Unfortunately, by the end of week one this strategy would require a facility to store 4260 m^3 of water (12260 - 8000). This strategy does make use of the lag time between pumping and stream depletion, but probably does not do it as well as possible.

Wanted:

Determine the smallest reservoir size needed to store the extra groundwater pumping such that the farmer can satisfy his water need without harming downstream water rights.

Solution: An initial task is to determine whether any of the CONJUS objective functions (listed in Section III.8) can provide what is wanted. Restating the problem helps. To have some water to store in the pond (or tank) for later use, one must pump more than the crop needs during some weeks. The more water one must store at any time, the larger the pond must be. One does not want to store any more water than absolutely necessary, while assuring that cumulative pumping is never less than cumulative water need (in order to always satisfy water need).

Cumulative pumping is the decision (management) variable. The restated management problem is minimize the greatest weekly difference (exceedance) between cumulative pumping and cumulative water need; while using cumulative water need as the lower bound on cumulative pumping. Using Equation III-27d and a lower bound on cumulative pumping addresses this minimax (nonlinear) goal programming objective. Of course, one must also impose an upper bound on stream depletion.

This problem highlights the challenge of developing an optimal strategy for a nonlinear problem, and shows how precision and convergence criteria affect results. Optimization solvers always converge to the absolutely best (globally optimal) solution for linear optimization problems. However, when addressing nonlinear problems, solvers sometimes get stuck at locally optimal solutions.

To feel confident one has computed a solution that is globally or nearly globally optimal for a nonlinear optimization problem, one might have to perform many simulations, each beginning with a significantly different initial of the decision variables. The precision and convergence affect what the solver computes.

For this problem, we first show how the computed optimal solutions vary depending upon the initial guess, when CONJUS is using 0.001 precision and 0.2 convergence criteria.[8] Table IV-1, shows the initial guess (column 2), the computed optimal (minimum) reservoir size (column 3), and the time varying optimal pumping rates (column 4), for 14 situations each employing a different initial guess.

Column 5 shows the tight stream depletion rates computed for the end of period 4. These vary about 0.1 % (244.852775 to 245.060104 m3d-1). Note that rates exceeding 245 technically violate that imposed constraint, but are acceptable for practical purposes (the violation is due to the selected precision and convergence).

Of the displayed pumping strategies, that from Scenario 14 is the best. However, all the scenarios were developed using the same precision and convergence. It is useful to see how changing these criteria affect the optimization results–the more precision, the less impact of the initial guess.

Repeating those 14 optimizations using the same initial guesses, but more rigorous precision and convergence criteria, yields slightly different, but more stable answers. If using precision and convergence criteria of 0.00001 and 0.0002, respectively, there is much less variation in the computed minimum storage volume. Optimal storage volume computed for Scenarios 1,7-10, 13 and 14 is 4003.6 m3. Scenarios 2, 5, 6 and 11 yield 4003.0 m3. Scenarios 3, 4 and 12 yield 4003.4 and 4001.8 m3 respectively. This is much less than a 0.1 percent variation. Computed depletion rates vary even less. Scenario 14 still yields the best pumping strategy.

Table IV-1 and Figure IV-5 show that for all scenarios optimal pumping rates were lowest during period 2. This is consistent with the observation that the third period river depletion influence coefficient is the largest. Superposition Equation III-1, shows that pumping in period k=2 is multiplied by influence coefficient 3 (in this case, N-k+1 = 4-2+1) to describe its affect on the system by the end of period 4. Since the third period influence coefficient is the largest, CONJUS will pump as little as practicable during period 2. The site determines which influence coefficient is the largest.

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[8] In the CONJUS input sheet specify: four periods; one extraction well; and Option B. After resizing the input data area, enter: storativity = 0.25; conductivity = 20 m/ d; unit period = 7 d; (x, y) = (450,0); thickness = 30 m; ground surface elevation = n/a; unit pumping = 1000m^3 d^{-1} ; initial and all nonoptimal heads = 100 m (arbitrary); lower bound on head = 97 m; upper bound on head = n/a; lower and upper limits on pumping = 0 and n m^3 d^{-1} , respectively; x and y coordinates of points defining the stream = (0,0) and (0,1); lower and upper limits on river depletion = n/a and 245 m^3 d^{-1} respectively; no upper limit on cumulative pumping; and lower bounds on cumulative pumping = 8000, 16000, 24000, and 32000 for weeks 1-4, respectively.

Example L (Example of artificial injection in an unconfined aquifer)

Given: Assume a 0.3 m radius well fully penetrates a 10 m thick unconfined aquifer having an initial equilibrium water level of 7 m, a conductivity of 70 m/d and a storativity of 0.2.

Wanted: IV-La). Compute the greatest rate that can be injected into the well without causing water levels to rise closer than 1 m to the ground surface after 60 days of pumping. (The ground surface is 10 m above the aquifer base.)
IV-Lb). Resolve problem IV-La) if a stream exists 50 m away from the injection well. Also, determine the increase in stream flow rate and cumulative volume that result by the end of the pumping period.

Solution: IV-La). Use CONJUS with one stress period, one injection well, the unconfined aquifer option, and a 9 m upper limit on head at the injection well. (CONJUS will address no more than one pumping well and one stress period when using the nonlinear option.)

When using the nonlinear aquifer option, CONJUS first computes head response via the Theis equation (which is perfect for linear aquifers), and then adjusts that to unconfined aquifer conditions using an approach by Jacob (1944) to prepare an influence coefficient. During optimization, CONJUS cycles until it converges to the maximum pumping rate that will not cause more than 2 m of head increase at the well. For this problem, using this process yields a maximum pumping rate of 931 m^3d^{-1} .

In an unconfined aquifer, as injection mound height increases, transmissivity increases, and more water can flow away from the well. Therefore, an injection mound computed using the nonlinear option is lower than that computed using the linear option. Table IV-2 illustrates how injection mound height at the mound differs with injection rate for linear and nonlinear assumptions.

IV-Lb). Use the same CONJUS input data as problem IV-La) and the nonlinear aquifer assumption, but include use of option B and add a stream 50 m away from the well. Assuming a unit pumping of 931 m^3d^{-1} yields an influence coefficient of 2.51 m. Optimization results include a maximum injection rate of 725 m^3d^{-1} , streamflow increase rate of 672 m^3d^{-1} by the end of 60 days, and a streamflow volume increase of 37,468 m^3 by the end of 60 days. Note that because the stream is a constant head boundary, it prevents transmissivity from increasing. Thus the mound builds up more readily, and less pumping will cause a 2 m rise than if the stream were not present.

NOTE: During a low streamflow period you might want to extract water from the same well used for injection. To do this in CONJUS, site an extraction well at precisely the same location as the injection well. Set pumping bounds so the extraction well cannot pump during period 1, and the injection well cannot pump during period 2. Use the nonlinear solver. The challenge lies in knowing the most appropriate initial water level (initial saturated thickness) to use for the extraction well. It will be between the initial and final values of the injection well.

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Example M (Example of conjunctive use of extracted groundwater and diverted surface water)

Given: A farmer pumps from a groundwater well and diverts surface water from a stream passing through his property. He wants to maximize total water delivery to his crop during a two-month period, but must ensure that streamflow leaving his farm is sufficient for downstream users. Based upon the expected entering streamflow, he knows he should not deplete streamflow by rates exceeding 11,000 $m^3 d^{-1}$ (385,000 $ft^3 d^{-1}$) by the end day 30, or by more than 11,500 $m^3 d^{-1}$ (402,500 $ft^3 d^{-1}$) by the end of day 60. The maximum capacity of both well and diversion, individually, are 8,000 $m^3 d^{-1}$ (280,000 $ft^3 d^{-1}$). The maximum water that can be reasonably utilized totals 13,000 and 16,000 $m^3 d^{-1}$ (455,000 and 560,000 $ft^3 d^{-1}$), for the two respective months. Other project information includes:

- Stream runs from Northwest to Southeast (100,1000) to (800, 0)
- Diversion point is on the farm at (200,858)
- Groundwater well, of 0.2 m radius, is on the farm at (450, 850)
- Conductivity is 80 md^{-1}
- Ground surface is at elevation 45 m, and the groundwater surface is initially at equilibrium at elevation 40 m. Saturated thickness is 40 m.
- For sustainable production, (based on crop, soil, and salinity of the surface water and groundwater), at least 60 % of the water used during period 1 must be from the stream, and at least 48% of the total water delivered during the two periods combined must be from the stream. The period 1 constraint protects seeds during germination. The total planning horizon constraint provides adequate leaching to prevent salinity buildup in the root zone.

Wanted: Determine the maximum conjunctive water use strategy subject to constraints.

Solution: Employ CONJUS using Options A and B, one groundwater extraction well and one surface water diversion. Employ: two thirty day stress periods; an arbitrary unit pumping such as 8,000. Also, to represent:

- well and diversion capacities, use upper limits of 8,000 $m^3 d^{-1}$ in each period on extraction from the well and on diversion;
- sensitivity of seeds to germination, use lower limit of 0.6 on the ratio of {diversion/(diversion + pumping extraction)} in period 1
- need for suitable quality to prevent salinity buildup, use lower limit of 0.48 on the above diversion ratio for the total planning horizon,
- restrictions on streamflow depletion at the end of periods 1 and 2, use upper limits of 11,000 and 11,500 $m^3 d^{-1}$, respectively.
- to represent the maximum total reasonably utilizable water, employ upper limits of 13,000 and 16,000 $m^3 d^{-1}$ on the sum of pumping and diversion. Do this via the generic polynomial constraint (GPC) option. The GPC is accessible through the special constraints option that appears after the optimization button (step 3 on the main sheet) is pushed.

To form the GPC constraint for period 1, input a lower bound of 0 and an upper bound of 13,000; and change the default period 1 coefficient values from 0 to 1 for the extraction well and the diversion. To form the next GPC constraint (which is for period 2), do the same as for period one but input an upper bound of 16,000 and use coefficient values of 1 for period 2 (rather than for period 1) for the extraction well and diversion.

The computed optimal conjunctive use strategy provides 12,174 $m^3 d^{-1}$ seasonal average flow. Its components and system responses are shown below (in $m^3 d^{-1}$).

	Period 1	Period 2	Total Season
Groundwater pumping (GP)	4,774	8,000	

Surface water diversion (SD)	7,001	4,573	
Stream flow depletion	11,000	11,500	
GP + SD	11,774	12,573	
{SD/(SD + GP)}	0.6	0.36	0.48

Tight constraints are groundwater pumping in period 2, stream depletion in both periods, and the diversion ratio in period 1 and for the total season. Relaxing any constraint (i.e., decreasing the required proportion of surface water) will cause total conjunctively provided water to increase.

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V. Applications of S/O Models and CONJUS

Many papers and reports discuss applications of S/O modeling for optimizing groundwater or conjunctive water management. For water supply goals, most such applications employ either the response matrix method, the technique used within CONJUS and REMAX., or the embedding technique (Heidari, 1982; Gorelick, 1983; Willis and Yeh, 1987; Yazicigil and Misirli, 1995; Greenwald, 1998). Other techniques have also been used, but much less commonly for irrigation water management.

The embedding (EM) method involves including entire discretized (finite difference or finite element) flow equations as constraints (Kolterman, 1983). The embedding method is computationally more efficient for large scale sustained yield planning if the proportions of cells at which pumping is being optimized and requiring head constraint are large (Takahashi and Peralta, 1995). The response matrix (RM) method is preferable to the embedding approach for most transient management situations.

Representative applications of the EM method to optimizing management of stream-aquifer systems or conjunctive water management include: Peralta and Killian (1985), Datta and Peralta (1986, 1991), O'Mara and Duloy (1988), Peralta and Datta (1990; Peralta et al. (1991, 1995), and Gharbi and Peralta (1994).

Example applications of the RM method simulating stream-aquifer systems include: Morel-Seytoux and Daly (1975), Morel-Seytoux et. al. (1980a,b, 1985), Illangasekare and Morel-Seytoux (1982), and Morel-Seytoux and Zhang (1990). Sample RM applications for formally optimizing management of stream-aquifer systems or conjunctive water management include: Morel-Seytoux (1975), Danskin and Gorelick (1985), Reichard(1987), Chau (1988), Peralta et al. (1988), Willis and Finey (1989), Mueller and Male (1993), Kholgi et al. (1995), Belaineh et al (1999), and Basagaoglu and Marino (1999). Moosavi (1995) applied RM optimization to managing aquifers involving both salt and fresh water. Digney and Gillies (1995) developed optimal aquifer recharge strategies for injection wells. Barlow et al. (1996) developed optimal groundwater pumping strategies to prevent unacceptable groundwater level rise.

The most powerful response matrix method S/O model that currently available that includes constraints for managing stream-aquifer systems is REMAX. It has dozens of hard-coded objective functions and many constraints for tailoring management of stream-aquifer system water quality and quantity. REMAX uses finite difference flow and transport models to compute response functions (including influence coefficients) for water quantity and quality management. To develop influence coefficients, REMAX employs the MODFLOW groundwater flow (McDonald and Harbaugh, 1988) and STR stream routing (Prudhic, 1989) simulation models.

Sample applications of REMAX range from minimizing the cost of cleanup of an industrial contaminant plume to coordinating transient use of groundwater and diverted surface water, to determining maximum sustainable groundwater pumping strategies for large stream-aquifer systems. REMAX permits user input of special linear or nonlinear response functions, such as those that might be developed by applying statistical analysis to output from a surface water quality model such as QUAL2E (Brown and Barnwell, 1987; Ejaz and Peralta, 1995a,b), or a nonaqueous phase groundwater contaminant transport model (Cooper et al, 1995) .

A representative situation for applying S/O modeling is the 660 square-mile

Cache Valley, which lies in southeastern Idaho and northeastern Utah. The valley receives streamflow throughout the year from the surrounding mountains. The source of much of the water is snowmelt. In the higher reaches, water moves from the rivers into the aquifers. Closer to the valley floor, water moves from the aquifer into the rivers. In most of the valley, groundwater is pumped from the deeper aquifer layers. These layers are unconfined near the mountains but are confined in the valley floor. Within the valley, canals draw water from the rivers and distribute. Most water for domestic, municipal and industrial (M&I) use is obtained from the aquifer, as is much irrigation water.

Water needs are increasing in Cache Valley. However, increased groundwater extraction will result in reduced flow from aquifer to rivers. The rivers and their canals provide water for irrigation, important wildlife habitat, and for surface water users downstream. There is conflict between the goals of increasing groundwater use, maintaining existing surface water diversion rights, and maintaining existing wetlands.

After a multi-layer groundwater flow simulation model was calibrated for the area, REMAX was used to develop influence coefficients. REMAX was then used to develop optimal water management strategies for several Cache Valley scenarios. The scenarios generally shared the following characteristics or variations thereof.
- objective function is to maximize sustained yield groundwater pumping extraction, subject to preventing:

- heads near pumping wells from declining excessively (30 feet was the common bound);
- net flow from aquifer to river from decreasing excessively (10 % was most commonly used);
- flowing springs from becoming dry;
- the water table from dropping beneath the river bottom in reaches that currently have saturated hydraulic connection between aquifer and river;
- excessive development of new water distribution systems (thus often requiring that extracted water be reasonably utilizable near where it was extracted); and
- providing more water than can be used based on projected development.

Based on scenario result evaluations, current groundwater pumping can be sustainably increased by 40% without excessively harming surface water rights and availability. REMAX was also used to evaluate the possible consequences on the maximum sustained yield that would result from artificial aquifer storage and recovery (ASR). This scenario differed from the previous one in that a total of 21 cubic feet per second was injected in 21 selected finite difference cells. The maximum sustainable pumping strategy that resulted was slightly more than 21 cfs greater than the scenario without artificial injection. The slight excess resulted because, because, according to the simulation model, less water would be lost to undesirable evapotranspiration than previously.

Operationally, water would be injected into the aquifer in the spring, when surface water supplies are abundant. Based on the work with REMAX, careful recharge location selection will insure that the injected water can be extracted later, without undesirable loss.

Subsequently, CONJUS was used to evaluate how much water can be injected through wells in reasonable Cache Valley sites. First, sites were identified where: (i) nearby canals can provide a source of recharge water; (ii) the target aquifer layer is unconfined; (iii) hydraulic conductivity and initial saturated thickness is adequate to prevent excessive buildup of recharge mound; and (iv) unsaturated zone thickness is adequate for the recharge mound to build up without reaching the ground surface. Assumed were the nonlinear (unconfined aquifer) option; one 18 inch diameter well; storativity of 0.2, hydraulic conductivity of 50 ft per day; and ninety day injection period. CONJUS computed the maximum recharge possible, without causing water levels to reach the ground surface, for a representative range of initial saturated and unsaturated zone thicknesses. This information is helping selection of recharge sites.

CONJUS can be applied to help assure crop yield sustainability in a wide range of situations. In areas with scarce water resources, CONJUS can help assure maximize use of water from multiple sources. CONJUS can also help assure sustainability in areas experiencing high water tables.

High water tables cause waterlogging and salinity, and reduce crop yield and productive life of the irrigated area. Many irrigation projects worldwide require improved water management. For example, waterlogging and salinity problems have arisen in many coastal areas of Peru. However, as of 1997, the Peruvian Moche Valley did not have waterlogging problems. To help avoid future problems, Chanduvi and Vasquez (1997) used a finite difference simulation model to predict how alternative management practices would affect future groundwater levels. They described efforts needed for appropriate model usage–including data collection, volume balance determination, and model calibration and validation. They identified which management strategies would not cause waterlogging problems, and which might be more sustainable because of less reliance on imported surface water.

Worldwide, groundwater models have been calibrated for many irrigated areas. However, for many areas no models exist. In such cases, one can either use CONJUS to compute influence coefficients, or merely to organize the coefficient tables and then input coefficients derived from field observation. Thus, one can use CONJUS as a framework for general purpose conjunctive-use optimization. One extraction well in CONJUS might represent a group of wells in the field. One CONJUS injection well might represent a group of fields that recharge the aquifer. The CONJUS user can quantify the relationship between delivered irrigation water and surface return flow or subsurface streamflow increase–and input that relation to CONJUS.

CONJUS' generic polynomial constraints (GPC) permit formulating desired algebraic relationships between such variables. Thus CONJUS could be used to optimize some management decisions even for areas such as the Mahi-Kadana Irrigation Project in India's Gujarat state (Sakthivadivel and Gulati, 1997). That system includes a reservoir, more than 2700 km of canals and many wells. Many district management decisions there involve determining irrigation rotations and magnitudes.

S/O models that use linear and nonlinear programming optimization, such as CONJUS, are designed to develop optimal water management strategies for situations in which they can consider the entire planning horizon at one time. They are less suitable for determining optimal rotation schedules that might be based upon sequential if-then decisions. Nevertheless, CONJUS is suitable for predicting consequences of management or for optimizing management of parts of any district.

In the future, we hope to enhance the GPC so that it will better address stream water quality problems. A sample objective function might be to determine the maximum set of rates of loading nutrients to a stream, without causing downstream concentration to exceed a specified limit. To address this, one would use an external simulation model to develop a response function (Ejaz and Peralta, 1995a,b), that describes control point concentration in terms of streamflow, loadings and stream-aquifer leakage. With this function included as a GPC, CONJUS would be able to determine both optimal loading and conjunctive use strategies.

In summary, the future is bright for applications of S/O modeling for managing stream-aquifer systems. CONJUS is easy to use and very appropriate for addressing a wide range of problems. It can be useful at field and higher management levels. The REMAX software requires more data and user expertise, but can address very complex physical systems, and much larger project areas. Coordinating use of CONJUS and REMAX provides powerful planning and management capabilities.

VI. Summary

This publication provides guidance and tools to help field personnel optimize conjunctive water use. Here, conjunctive use involves managing a hydraulically linked stream and aquifer system. On a regional or basin scale, maximizing conjunctive use implies using and reusing water as many times as possible for intended water users, without causing unacceptable consequences. That effort usually requires simulation and optimization models not available to field personnel. The level of detail is sometimes too coarse to address problems of individual fields.

On the field scale, optimizing conjunctive use usually involves maximizing coordinated use from just a few groundwater wells and surface water diversions, (again subject to restrictions). Restrictions can address heads at individual wells and beneath individual fields or recharge basins. They can address the leaching requirement or salinity sensitivities of individual cropped fields. Optimizing field scale conjunctive use can help assure the water user does not harm the legal water rights of his neighbors.

Tools provided by this report include stand-alone equations and guidance, and the CONjunctive Use Software (CONJUS). Tool usage is illustrated in the Report body and in the CONJUS User's Manual provided as an appendix.

Some equations described in the report are not included within CONJUS. These include expressions describing: aquifer head fluctuations due to surface water stage fluctuations; salt water upconing beneath a pumping well, salt water intrusion into a coastal aquifer; determination of leaching fraction, and how much groundwater must be withdrawn from beneath irrigated fields to prevent rise in water table elevations beneath those fields.

CONJUS is a simulation/optimization (s/o) computer model that includes both simulation equations and formal mathematical optimization capabilities. The equations describing groundwater and stream levels and flows are primarily analytically-based and most suitable for relatively homogeneous physical systems.

CONJUS simulation abilities can aid estimating the values of aquifer parameters. CONJUS simulation expressions also predict the effects of:

- extraction or injection wells and stream diversions and return flows on aquifer head (groundwater level) and streamflow;
- managed stream stage change on aquifer head;
- line source of groundwater on aquifer head;
- recharge over a rectangular area on aquifer head;
- blending waters of different qualities on final water concentration.

Optimization problems are defined in terms of decision and state variables and include constraints and an objective function. Decision variables are usually those which a manager can control directly. State variables are usually system responses to management. A flow that is a decision variable in one situation might be a state variable in another situation (such as return flow). Variables in CONJUS include all those described above which it can simulate.

Constraint equations are expressions that relate decision and state variables and describe criteria of acceptability concerning the water management strategy. CONJUS employs linear and nonlinear constraints to describe ranges of heads, flows and concentrations that are acceptable to the water manager.

An objective function is an equation describing what a s/o model will maximize or minimize the value of. CONJUS formulates and computes optimal water management strategies using objective functions that include linear, quadratic and goal programming components.

CONJUS will compute optimal water management strategies for a wide range of problems. For example, it can show how to maximize coordinated use of groundwater and surface water:

- subject to sustainability (salinity) and stream depletion constraints;
- by optimal reservoir sizing; and
- by artificial aquifer recharge during periods of high streamflow.

By using CONJUS, field personnel can greatly enhance their ability to develop optimal conjunctive water use strategies to maximize sustainable crop production.

Table II-1. Influence coefficients $\delta_{\hat{o},\hat{e}}$ describing drawdown and stream depletion rate resulting after extracting 1 cmd of groundwater for four days at three wells, taken individually, (m).

	Pumping Well Number, ê		
Observation Well Number, ô	1	2	3
1	4.07238 E-03	1.18101 E-03	2.27248 E-04
2	1.18101 E-03	4.02364 E-03	2.46950 E-04
3	2.27248 E-04	2.46950 E-04	4.11655 E-03
Stream depletion rate	8.87073 E-01	8.97279 E-01	8.76885 E-01

Note: $\delta_{\hat{o},\hat{e}}$ is an influence coefficient describing response at location ô to stimulus at location ê

Table II-2. Partial comparison between inputs and outputs of Simulation and Simulation/Optimization (S/O) models[1]

Model Type	Input Values	Computed Values
Simulation (S)	Physical system parameters	
	Initial conditions	
	Some boundary flows	Some boundary flows
	Some boundary heads	Heads at 'variable' head cells
	Pumping Rates	
Simulation/ Optimization (S/O)	Physical system parameters	
	Initial conditions	
	Some boundary flows	Optimal boundary flows
	Some boundary heads	Optimal heads at 'variable' head cells
	Bounds on pumping, heads, & flows	Optimal pumping, heads, & flows
	Objective function (equation)	Objective function value

[1] Both types of models also require as input descriptors and parameters defining the physical system.

Table IV-1. Minimax storage reservoir problem: change in optimal pumping strategy due to change in initial guess of pumping strategy[1]

(1) (2) (3) (4) (5)

Scenario No.	Initial Guess of Pumping Rate for Each Period (m^3/day)				Objective Value (m^3)	Optimal Pumping Rate for Each Period (m^3/day)				Greatest Stream Depletion Rate and Period (m^3/day)
	1	2	3	4	Max Reservoir Vol.	1	2	3	4	
1	2000	570	1142	1142	4644.8	1806.4	552.9	1078.3	1128.9	244.852775 at the end of period 4
2[2]	0	0	0	0	4160.4	1737.2	548.5	1142.9	1159.6	244.999999 at the end of period 4
3	1000	570	1142	1142	4073.8	1724.8	560.9	1144.8	1140.9	245.000001 at the end of period 4
4	2000	600	1000	2000	4003.6	1714.8	570.9	1142.9	1142.9	244.999999 at the end of period 4
5	2000	600	0	2000						
6	0	570	1143	1143						
7	2000	1000	0	1142						
8	0	600	2000	2000	4001.5	1714.5	570.6	1143.5	1142.9	245.0000003 at the end of period 4
9	1800	500	1200	1000	3997.3	1713.9	573.2	1141.5	1142.8	245.016827 at the end of period 4
10	1000	100	100	100	3987.9	1712.6	573.2	1142.4	1143.3	245.000001 at the end of period 4
11	0	571	1142	1142	3958.6	1708.4	577.3	1142.0	1142.2	245.000002 at the end of period 4
12	0	0	2000	2000	3946.4	1706.6	579.1	1141.3	1144.4	245.000001 at the end of period 4
13	0	571	1143	1143	3914.8	1702.1	583.6	1141.1	1141.7	244.999998 at the end of period 4
14	2000	1000	0	0	3842.5	1691.8	593.9	1139.9	1144.5	245.060104 at the end of period 4

[1] The employed solver convergence is 0.002, and the precision is 0.001.
[2] The default initial guess is zero for all decision variables when selecting the 'Calculate Optimal Water Management Strategy' option on the CONJUS front page

Table IV-2. Effect of pumping on increase in head just outside well casing assuming linear and nonlinear aquifers (Problem IV-La).

Unit and Simulated Pumping	Linear Aquifer Head Increase	Nonlinear Aquifer Head Increase
($M^3 d^{-1}$)	(m)	(m)
500	1.23	1.14
920	2.26	1.98
931	2.29	2.00
940	2.31	2.02
1000	2.46	2.13

References

Allen, R.G., Willardson, L.S., and H.D. Frederiksen. 1996a. Water use fractions as substitutes for irrigation efficiency. J. Irrig. and Drain. Engrg., ASCE (sub).

Allen, R.G., Burt, C., Clemmens, A.J., and L. S. Willardson. 1996b. Water conservation definitions from a hydrologic viewpoint. Proceedings North American Water and Environment Congress, ASCE, Anaheim, CA, 6 p.

Ayers, R. S. 1977. Quality of water for irrigation. Journal of the Irrigation and Drainage Division, ASCE 103(IR2):135-154.

Ayers, R. S. and D. W. Wescot. 1985. Water quality for agriculture. Irrigation and Drainage Paper 29, Food and Agriculture of the United Nations. 97 p.

Barlow, P. M., Wagner, B. J. and K. Belitz. 1996. Pumping strategies for management of a shallow water table: the value of the simulation-optimization approach. Ground Water, 34(2):305-317.

Basagaoglu H. and M. Marino. 1999. Joint management of surface and ground water supplies. Ground Water 37(2):214-222.

Belaineh, G., Peralta, R. C. and T. Hughes. 1999. Journal of Water Resources Planning and Management. ASCE 125(3):154-161.

Bouwer, H. 1978. <u>Groundwater Hydrology</u>. McGraw-Hill, Inc., N. Y., N. Y.

Brown, L. C. and T. O. Barnwell. 1987. The enhanced stream water quality models QUAL2E and QUAL2E-UNCAS: Documentation and user's manual. US Environmental Protection Agency, Rep. EPA/600/3-87/007.

Chanduvi, F. and F. Vasquez, F. 1997. The use of mathematical models for the prevention and control of waterlogging in irrigation projects: the Moche valley case study. Food and Agriculture Organization of the United Nations, Internal document. 10 p.

Chao, T. S. 1988. Analysis of sustained ground-water withdrawals by the combined simulation-optimization approach. Groundwater GRWAAR, 26(4):454-463.

Clemmens, A.J. T.S. Strelkoff, and C.M. Burt. 1995. Defining efficiency and uniformity: problems and perspectives. Proc. First. Int. Conf. on Water Resources Engrg., ASCE, San Antonio, TX. p. 1521-1525.

Cooper, G., R. C. Peralta and J. J. Kaluarachchi. 1995. Optimizing separate phase light hydrocarbon recovery from contaminated unconfined aquifers. Advances in Water Resources, 21(5):339-350.

Danskin, W. R., and S. M. Gorelick. 1985. A policy evaluation tool: Management of a multiaquifer system using controlled stream recharge. Water Resources Research, 21(11), 1731-1747.

Datta, B. and R. C. Peralta. 1986. Interactive computer graphics-based multiobjective decision-making for regional groundwater management. Agricultural Water Management, 11(2):91-116.

Datta, B. and R. C. Peralta. 1991. Reconnaissance-level alternative optimal ground-water use strategies. Journal of Water Resources Planning and Management, 116(5):676-692.

Digney, J. E. and J. A. Gillies. 1995. Artificial recharge in Saskatchewan: current developments. Water Resources Bulletin, 31(1):33-42.

Dinar, A. J., Letey, J. and H. J. Vauz. 1986. Optimal ratios of saline and non-saline irrigation waters for crop production. Soil Sci. Soc. Amer. J. 50:440-443.

Ejaz, M. S. and R. C. Peralta. 1995a. Maximizing conjunctive use of surface and round water under surface water quality constraints. Advances in Water Resources, 18(2):67-75.

Ejaz, M. S. and R. C. Peralta. 1995b. Modeling for optimal management of agricultural and domestic wastewater loading to streams. Water Resources Research, 31(4):1087-1096.

Fenske, J. P. 1997. Interaction between surface water and groundwater. Draft Chapter 6 in U.S. Army Corps of Engineers Engineering Manual - Groundwater Hydrology.

Ferris, J. G. 1951. Cyclic fluctuations of water levels as a basis for determining aquifer transmissibility. Internation Association of Scientific Hydrology, pub. 33, p. 148-155.

Gharbi A. and R. C. Peralta. 1994. Integrated embedding optimization applied to Salt Lake Valley aquifers. Water Resources Research, 30(3):817-832.

Glover, R. E. 1978. Groundwater Hydraulics. Water Resources Publications, Highland Ranch, CO.

Glover, R. E., and C. G. Balmer. 1954. River depletion resulting from pumping a well near a river. American Geophysical Union Transactions, 35(3):468-470.

Gorelick, S. M. 1983. A review if distributed parameter groundwater management modeling methods, Water Resources Research, 19(2):305-319.

Greenwald, R. M. 1998. Documentation and user's guide: MODMAN, an optimization module for MODFLOW, version 4.0. HSI Geotrans, Freehold, N.J.

Gupta, R. K. 1994. Ground water quality for irrigation. In Salinity Management for Sustainable Agriculture. Eds. D. L. N. Rao, N. T. Singh, R. K. Gupta, N. K. Tyagi, Central Soil Salinity research Institute, Karnal, India. p89-101.

Hantush, M. S. 1965. Wells near streams with semipervious beds. Journal of Geophysical Research, v. 70(12): 2829-2838.

Hantush, M. S. 1967. Growth and decay of groundwater-mounds in response to uniform percolation. Water Resources Research v. 3, p. 227-234.

Heidari, M.. 1982. Application of linear system's theory and linear programming to ground water management in Kansas, Water Resources Bulletin, 18(6):1003-1012.

Hoffman, G. J., Rhoades, J. D., Letey, J., and F. Sheng. 1990. Salinity management. In Management of Farm Irrigation Systems. Eds. Hoffman, G. J., Howell, T. A. and Solomon,

K. H. American Society of Agricultural Engineers, St. Joseph, Michigan.p 667-715.

Illangasekare, T. H. and H. J. Morel-Seytoux. 1982. Stream-Aquifer influence coefficients as tools for simulation and management. Water Resources Research, 18(1):168-176.

Jacob, C. E. 1944. Notes on determining permeability by pumping tests under water-table conditions. U.S. Geological Survey mimeographed report.

Jenkins, C. T. 1968a. Computation of rate and volume of stream depletion by wells. In Techniques of Water-Resources Investigations of the United State Geological Survey, Book 4, Chapter D1. U.S. Geological Survey, Washington D.C.

Jenkins, C. T. 1968b. Techniques for computing rate and volume of stream depletion by wells, Ground Water, 66(2):37-46.

Jensen, M.E. 1993. Impacts of Irrigation and Drainage on the Environment. Fifth Gulhati Memorial Lecture. Proc. 15th ICID Congress. The Hague, Netherlands.

Keller, J., Peabody, N. S. III, Seckler, D. and D. Wichelns. 1990. Water policy innovations in California: Water resource management in a closing water system. Center for Economic Policy Studies Discussion Paper No. 2. Arlington, Virginia: Winrock International.

Keller, A.A. and J. Keller. 1995. Effective efficiency: a water use efficiency concept for allocating freshwater resources. Discussion paper 22, Center for Economic Policy Studies, Winrock Int., Arlington, VA. 19 p. also in USCID Newsletter, United States Committee on Irrigation and Drainage, Issue No. 71, Jan. 1995.

Kholgi, M., Razack, M., Mirabzadeh, M. and G. Porel. 1995. Multiobjective simulation-optimization model for conjunctive use of surface and groundwater management. Proceedings of Regional Conference on Water Resources Management, Eds S. F. Mousavi and M. Karamooz, Univ. of Technology, Isfahan, Iran, pp 657-666.

Koltermann, C. R. 1983. LP embedded simulation model for conjunctive use management optimization. Project OWRT A-103-NEV(1) completion report. Nevada Water Resources Center, Univ. of Nevada, Reno. 134 p.

Lohman, S. W. and others. 1972. Definitions of selected ground-water terms-revisions and conceptual refinements. Geological survey water-supply paper 1988. United States Government Printing Office, Washington, D. C. 21 p.

McDonald, M.G., and A. W. Harbaugh. 1988, A modular three-dimensional finite-difference ground-water flow model: U.S. Geological Survey Techniques of Water-Resources Investigations, book 6, chap. A1, 586 p.

McWhorter, D. B. and D. K. Sunada. 1977. Ground-water Hydrology and Hydraulics. Water Resources Publications, Highland Ranch, CO.

Moosavi, S. A. A. 1995. Management of salt-water upconing beneath a horizontal gallery. Proceedings of Regional Conference on Water Resources Management, Eds. S. F. Moosavi and M. Karamooz. Isfahan University of Technology, Isfahan, Iran. pp 243-264.

Morel-Seytoux, H. J. 1975. A simple case of conjunctive surface-ground-water management. Ground Water, v. 13(6):505-515.

Morel-Seytoux, H. J., and C. J. Daly. 1975. A discrete kernel generator for a stream-aquifer studies. Water Resources Research, 11(2), 253-260.

Morel-Seytoux H. J., and C. M. Zhang. 1990. The dynamics and mathematics of stream-aquifer-well interactions. Proceedings of Tenth Annual American Geophysical Union, Hydrology Days, April 10-12, 1990, Colorado State University, Fort Collins, Colorado.

Morel-Seytoux, H. J., T. Illangasekare, M. W. Bittinger, and N. A. Evans. 1980a. Potential Use of a stream-aquifer model for management of a river basin: case of the south platte river in Colorado. Progressive Water Technology, IAWPR/Pergamon Press Ltd, v. 13, p175-187.

Morel-Seytoux, H. J., G. Peters, R. Young, and T.Illangasekare. 1980b. Groundwater modeling for management. Paper presented at the International Symposium on Water Resource Systems. Water Resource Division Training Center, University of Roorkee, Roorkee, India.

Morel-Seytoux, H. J., J. I. Restrepo, and R. M. D. Rathnayake. 1985. S.A.M.S.O.N.: Executive Summary, Description of SAMSON Computer System, General Flow Chart, Flow Charts for Twelve Basic Blocks and Subroutine Documentations. HYDROWAR Reports Division, Hydrology Days Publications, report Number 85.5, Department of Civil Engineering, Colorado State University.

Mueller, F. A. and J. W. Male. 1993. A management model for specification of groundwater withdrawal permits. Water Resources Research, Vol. 29, no. 5. pp 1359-1368.

O'Mara, T. G, and J. H. Duloy. 1988. Modeling efficient conjunctive use of water in the Indus Basin. In Efficiency in Irrigation The Conjunctive Use of Surface and Groundwater Resources, G. T. O'Mara, Ed., A World Bank Symposium, 1818 H Street, N.W., Washington, D.C. 20433, U.S.A. pp 128-138.

Peralta, R. C. 1997. Optimal groundwater and conjunctive water management:BIE 735 Course Notes. Dept. Of Biological and Irrigation Engineering, Utah State University, Logan, Utah.

Peralta, R. C. and A. H. Aly. 1997. REMAX Users Manual, Systems Simulation/Optimization Laboratory, Software Engineering Division, Dept. of Biological and Irrigation Eng., Utah State University, Logan, Utah.

Peralta, R. C. and B. Datta. 1990. Reconnaissance-level alternative optimal groundwater use strategies. Water Res. Planning & Management, ASCE, 116(5):676-692.

Peralta, R. C., and P. J. Killian. 1985. Optimal regional potentiometric surface design: Least cost water supply/sustained groundwater yield, Trans. ASAE, 28(4), 1098-1107.

Peralta, R. C., Asghari, K., and R. Shulstad. 1991. SECTAR for economically optimal sustained ground-water yield planning. Journal of Irrigation and Drainage Engineering, v. 117(1):5-24.

Peralta, R. C., Cantiller, R. R. A., and J. E. Terry. 1995. Optimal large-scale conjunctive water-use planning: case study. Journal of Water Resources Planning and Management, v. 121(6):471-478.

Peralta, R. C., Gharbi, A., Willardson, L. S., and A. W. Peralta. 1990. Optimal conjunctive use of irrigation sources. In Management of Farm Irrigation Systems. Eds. Hoffman, G. J., Howell, T. A. and Solomon, K. H. American Society of Agricultural Engineers, St. Joseph, Michigan.p 428-458.

Peralta R.C., K. Kowalski, and R.R.A Cantiller. 1988. Maximizing reliable crop production in a dynamic stream/aquifer system. Transactions of the ASAE, vol.31, no.6. pp 1729-1736.

Pescod, M. B. 1992. Wastewater treatment and use in agriculture. Irrigation and Drainage Paper 47, Food and Agriculture Organization of the United Nations, Rome, Italy.

Pinder, G. F., Bredehoeft, J. D., and H. H. Cooper. 1969. Determination of aquifer diffusivity from aquifer response to fluctuations in river stage. Water Resources Research, v. 5(4): 850-855.

Prudic, D. E. 1989. Documentation of a computer program to simulate stream-aquifer relations using a modular,finite-difference, groundwater flow model, U.S.Geological Survey, open-file report 88-729.U.S. Government Printing Office, Washington, D.C.

Rao, D. L. N., Singh, N. T., Gupta, Raj K., and N. K. Tyagi., Eds. 1994. Salinity Management for Sustainable Agriculture. Central Soil Salinity Research Institute, Karnal, India.

Reichard, E. G. 1987. Hydrologic influences on the potential benefits of basinwide groundwaer management. Water Resources Research 23(1):77-91.

Reilly, T. E., O. L. Frank, and G. D. Bennett. 1987. The principle of superposition and its application in ground-water hydraulics, Techniques of water resources investigation of the United States Geological Survey. Chapter B6.

Rhoades, J.D. 1988. Intercepting, isolating and reusing drainage waters for irrigation to conserve water and protect water quality. Agricultural Water Management, 16(1989)37-57.

Rhoades, J. D. and A. Dinar. 1990. Reuse of agricultural drainage water to maximize the beneficial use of multiple water supplies for irrigation. In Economics and Management of Water and Drainage in Agriculture. San Juaquin Valley Drainage Program, Sacramento, CA.

Sakthivadivel, R. and O. T. Gulati. 1997. Assessing management interventions impact with IIMI's minimum set of comparative indicators: the case of Mahi-Kadana Project, India. Paper presented to the IIMI, ILRI, IHE and INCYTH - CRA - International Workshop on Irrigation Performance, 3-7 November 1997, Mendoza, Argentina. 16 p.

Sophocleous, M., Koussis, A., Martin, J. L. and S. P. Perkins. 1995. Evaluation of simplified stream-aquifer depletion models for water rights administration. Ground Water, 33(4):579-588.

Takahashi, S. and R. C. Peralta. 1995. Optimal perennial groundwater yield planning for complex nonlinear aquifers: methods and examples. Advances in Water Resources, 18:49-62.

Theis, C. V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. Transactions of the American Geophysical Union, 16:519-524.

Theis, C. V. 1941. The effect of a well on the flow of a nearby stream. American Geophysical Union Transactions, 22(3):734-738.

Walton, W. C. 1970. Groundwater Resource Evaluation. McGraw-Hill Book Co., New York, N.Y.

Willardson, L. S. 1990. An algorithm for conjunctive use of saline groundwater. In Proceedings of the Intl. Symp. on Land Drainage for Salinity Control in Arid and Semi-Arid Regions, 3:209-218. Ministry of Public Works and water Resources, Cairo, Egypt.

Willardson, L.S., Allen, R.G., and H.D. Frederiksen. 1994. Universal fractions for the elimination of irrigation efficiency. paper presented at the 13th Technical Conference of USCID, Denver, Colorado. Oct. 19-22, 1994.

Willis, R., Finney, B., and Z. Daoshuai. 1989. Water resources management in north China plain. Journal of Water Resources Planning and Management, 115(5):598-615.

Willis R. and W-G. Yeh. 1987. Groundwater systems planning and management. Prentice Hall Inc., Englewood Cliffs, New Jersey. 416 p.

Yazicigil, H. and F. Misirli. 1995. Groundwater management modeling with some applications. Proceedings, Regional Conference on Water Resources Management, Isfahan Univ. of Technology, Isfahan, Iran.

Appendix A

CONJUS User's Manual

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Appendix A - CONJUS Users Instructions

A.1. Introduction and Processing Overview

A.1.1. Introduction

CONJUS is powerful and easy-to-use software to aid field-level management of stream-aquifer systems. It is a user friendly spreadsheet model that combines simulation and optimization abilities. It is written in Visual Basic and accessed from Microsoft Excel. Microsoft Office 97 Excel must be fully loaded in order to run CONJUS. As soon as you open CONJUS it will automatically attempt to install the necessary solver.[1]

Spreadsheet models are sheet-based information systems. Different information types (such as input data, model-computed coefficients, and model output) are located in different Excel sheets. A sheet name identifies its contents and is located on the sheet tab at the bottom of the window.

CONJUS provides a password protection option. When exiting CONJUS, it asks if you want to establish a password. If you do so, you must supply that password when you next try to use CONJUS. If you forget the password you must reinstall CONJUS from the source diskette in order to use it again.

[1] To run CONJUS the appropriate files for three EXCEL 97'add-in features' must be available. These are solver.xla, funcres.xla, and solver32.dll. The error message "Compile error in hidden module: optimization" will appear if either *.xla file is not in the directory that Excel expects them to be in. A “Solver encountered an error value in a target or constraint cell” message indicates an inaccessible *.dll file. To correct the situation, close CONJUS and Excel. Sequentially invoke the Windows Start, Find, and Files options. Type the name of the add-in file you want to find, and push the Find Now button. Then copy the file to the directory containing CONJUS.XLS. Do this for all missing files. This process should only be needed the first time you use CONJUS from a particular directory.

A.1.2. Simulation Capabilities

CONJUS employs equations describing system response to wells, image wells, stream depletion, line source of ground water, stream stage change and field recharge. These abilities are useful for managing surface water and groundwater in hydraulically connected stream-aquifer systems. Because analytical equations are used, simple homogeneous physical systems are assumed. CONJUS also represents surface water diversion from and return flow to the stream.

Some CONJUS simulation features cannot be used together. Table A-1 indicates those system stimuli and responses that can be employed simultaneously. It summarizes the variables considered in each of five situational options. Note that image wells are automatically created by CONJUS and are used only for the situation in which both aquifer head and stream-depletion due to pumping are being simulated.

A.1.3. Optimization Capabilities

An optimization problem includes variables (decision and state), upper and lower limits (bounds) on those variables, constraint equations, and an objective function (an equation). Variables in CONJUS are the stimuli and responses mentioned above. Normally, decision variables are those which a manager can potentially control directly.

The most commonly used CONJUS decision variables are groundwater pumping, streamflow diversion, surface water return flow to a stream, and any combination of those. Other CONJUS decision variables are stream stage, line source recharge, and field recharge. (Although it might seem like a decision variable, cumulative groundwater pumping cannot be directly maximized or minimized like the other decision variables.) Table A-1 shows that within CONJUS you cannot simultaneously have groundwater pumping and stream stage change as variables. Other mutual exclusions are apparent in the upper (stimuli) portion of the table.

State variables represent the state of the physical system being modeled. CONJUS state variables include aquifer head and stream depletion rate and cumulative stream depletion volume. The user must input into CONJUS upper and lower limits on what he considers acceptable decision and state variable values.

CONJUS includes, as constraints, equations that link the stimuli and responses of Table A-1. CONJUS requires that the user specify the objective function that will drive the search for an optimal water management strategy. The objective function describes the dominant management goal–usually in terms of decision variables, but sometimes including state variables. (An example is to maximize the sum of extracted groundwater and stream diversions.)

When developing an optimal management strategy, one usually wants a strategy that maximizes or minimizes the value of the objective function. Sometimes, one wants to achieve a specific value of the objective function. For management flexibility, one wants to be able to select from a range of possible objective functions.

To permit the user to address as wide a range of management goals as practicable, CONJUS employs a three-component objective function. This organization permits one to build a composite objective function equation that includes linear, quadratic and nonlinear goal-programming terms (Table A-2).

A.1.4. Processing Overview

New users should begin on the front page (main sheet, Figure A-1) and follow the steps shown on the flowchart. Experienced users can begin at whatever step they prefer, as long as their choice does not violate the sequence logic of the Figure A-1 flowchart. For example, one cannot perform optimization until one has developed influence coefficients.

To begin addressing a new management problem the user should click the button for Step 1 on the flowchart. Alternatively, click the appropriate tab to go directly to 'input' sheet. After entering data concerning the general management problem being addressed, click on the top-left green button. This instructs CONJUS to format the arrays for subsequent data entry. The automatic sizing of input arrays helps reduce user mistakes. After entering all required input information, return to the 'main' sheet and invoke Step 2a.

CONJUS employs influence coefficients within subsequent simulation and optimization. In Step 2a CONJUS creates influence coefficient arrays and computes or assigns default influence coefficient values if so instructed. CONJUS also assigns default values of 1 for objective function weights, coefficients and goals. CONJUS does this automatically for all three types of objective function types that it addresses--linear, quadratic and goal programming.

Via Step 2b, the user can modify the influence coefficients that CONJUS provided. Via Step 2c, the user can modify goals, weights and coefficients used in the objective functions.

To compute an optimal water management strategy, click on the Step 3 button, 'Calculate Optimal Water Management Strategy'. The user can specify the optimization solver and options, and additional special constraints. Optimization results appear in the 'output' sheet.

A.2. Information and Guidance for CONJUS Use

A.2.1. Types of Sheets in CONJUS

'main' sheet (Fig A-1): This front page is the major CONJUS user interface. It includes a process flow chart for running CONJUS and information about developers and the links to their home pages.

'input' sheet (Fig A-2): The user must enter CONJUS input data here.

'*influence_coefficient*' sheet (Fig A-3): CONJUS places its computed and default influence coefficients here. The user can edit these if desired.

'linear' sheet (Fig A-4): The user must complete this if he wants to use linear optimization in the linear objective function. You should enter the linear objective component weight and the coefficients to be applied for all variables. The default value of weight and coefficients is one.

'quadratic' sheet (Fig A-5): The user must complete this if he wants to perform quadratic optimization. He should enter the quadratic objective component weight and the coefficients he desires to use with all quadratic objective function variables. The default value of weight and coefficients is 1.

'goal' sheet (Fig A-6): The user must complete this if he wants to use goal programming.

He should enter the desired variable goals and related weights, coefficients and goals for all stress periods included in the goal programming objective function. The default goal variable is pumping rate. The default value of weights and coefficients is 1.

'Generic polynomial constraint (GPC)' sheet (Fig A-7): Completing this sheet permits the user to formulate his own constraint using any algebraic combination of groundwater extraction and injection and surface water diversion and return flow. A GPC can be formulated for any stress period or for the entire planning horizon. The variables can individually be multiplied by user specified coefficients and raised to desired powers. They can then be combined with the other main decision variables using addition, subtraction, multiplication and division.

'output' sheet (Fig A-8): CONJUS places its computed output here.

A.2.2. Stress Period Length in CONJUS

A stress period is a period of uniform hydraulic stimulus. If the managed stimulus (pumping, stream stage change, etc.) changes with time, you must use more than one stress period, CONJUS can employ as many stress periods of uniform duration as desired (limited only by Excel ability). Because CONJUS employs equations that superimpose in time, stress periods must generally be of uniform duration. A special CONJUS feature (using tailored superposition equations) does permit using two periods of different duration, as long as the total management era is no longer than the two periods combined.

An example might help knowing what to enter for the 'unit period' in the CONJUS Input Sheet. For unit consistency, if hydraulic conductivity is given in meters per day, groundwater pumping must be in cubic meters per day. Assume one wants to determine a optimal pumping strategy that changes pumping rates no more often than weekly. In that case, the stress period (period of uniform hydraulic stress) is one week. One should enter '7' (days) as the unit period.

A.2.3. Running CONJUS

A.2.3.1. Step 1--Describing the Physical System to be Managed

Invoking Step 1 in the 'main' sheet moves you to the 'input' sheet to provide input data. Most input data are input in the 'input' sheet. Exceptions are influence coefficients, objective function component weights, objective variable coefficients and goal variables. CONJUS either calculates values for these exceptions or provides default values.

To address a specific water management problem, you must first complete the 'General Management Problem Information' data section. After inputting this data, click on the first green button at the top of the 'input' sheet. The message of Fig. A-9 will appear. CONJUS permits saving the current 'input' sheet using a different name before it changes the 'input' sheet for the new problem.[2] Then CONJUS sizes and formats the rest of the data input area to address the specified management problem. This helps prevent data input errors. (Section A.2.3.6 discusses subsequently revising the data input area.)

After CONJUS has sized the data input area, input the necessary data. You can only enter data into white cells, and must enter appropriate values in those cells. CONJUS colors cells (or rows) blue to indicate those which cannot accept data for the specified problem. After entering the data, view a map of the resulting study by clicking on the indicated button. Return to the main sheet by clicking on a 'Return to Main Sheet' button or the 'main' sheet tab at the bottom of the window.

A.2.3.2. Step 2a--Generating Influence Coefficients, Objective Component Weights and Variable

[2] Placing input values from a saved sheet to a new input sheet is fairly easy using copy and paste features, especially if one opens CONJUS in two different Excel implementations.

Coefficients and Goals

CONJUS uses influence coefficients and superposition equations to simulate physical system response to hydraulic stress. CONJUS uses this approach for both simulation and optimization.

Invoke Step 2a to 'Calculate Influence Coefficients and Default Weights'. Depending on your response to queries, CONJUS will compute influence coefficients, or merely provide appropriately sized arrays to which you can import influence coefficients you developed elsewhere (Fig A-10a).

If you wish, CONJUS prepares default objective weights and coefficients (Fig A-10b). These are needed for the first time you are addressing a particular management site and problem. (You can modify these weights and coefficients in subsequent Step 2b). Similarly, you can here instruct CONJUS to prepare default goal values for a specified goal variable (Fig A-10c). You should do this if you might (in Step 3) implement goal programming optimization.

After the first optimization for a management problem, the problem can be modified slightly without recomputing influence coefficients. For example, if you only change upper bounds or lower bounds in the 'input' sheet there is no need to recalculate influence coefficients before computing an optimal pumping strategy for the modified problem.

A.2.3.3. Step 2b--Manually Changing Influence Coefficient Values

The influence coefficient approach used in CONJUS employs linear systems theory and superposition in time and space. CONJUS computes the influence coefficients via analytical equations. CONJUS is completely appropriate for relatively homogenous systems adequately represented by the analytical equations.

Invoking Step 2b permits you to use more accurate influence coefficients (if you have an alternative way to develop them). Pressing the Step 2b button moves you to the 'influence_coefficient' sheet so you can edit the coefficients. You can also move to that sheet by clicking on the 'influence_coefficient' tab at the bottom of the window.

A.2.3.4. Step 2c--Manually Changing Objective Function Weights and Variable Goals and Coefficients

In Step 2a CONJUS implements default values of weights and coefficients for all three objective function types (linear, quadratic and goal programming). CONJUS stores the weights and coefficients in three sheets. For example, the 'linear' sheet contains values pertaining to the linear objective function type.

The weights and coefficients are important. When computing the optimal objective function value for a problem, CONJUS multiplies the objective function component by its assigned weight. In the same manner, each variable in the objective function is multiplied by its assigned coefficient.

All default weights and coefficient values equal one. To use other values you must edit the appropriate objective sheet. By careful weight selection you can employ any one or any combination of the mentioned objective function types.

To change the weights and coefficient values, click the Step 2c button of the flow chart. CONJUS will first move you to the 'linear' sheet, where you can edit the values. You can then move to any other objective type sheet by clicking the relevant button. After completing these sheets you are ready to perform the optimization via Step 3.

A.2.3.5. Step 3--Calculating an Optimal Water Management Strategy

A.2.3.5.1. Introduction. Clicking the button for Step 3 causes a window (Figure A-12a) to appear. This window solicits specifications concerning how you want CONJUS to perform optimization. The three tabs in Figure A-12a indicate three submenus at which you should provide information concerning objective function, solver and special constraints, respectively.

A.2.3.5.2. Objective Function Specifications (Figure A-12a). Here you specify whether you want CONJUS to develop a strategy that maximizes or minimizes the value of the objective function, or whether you want the objective function to equal some specific value.

You also specify the component(s) of your objective function (linear, quadratic and/or goal programming). To include more than one objective type in a multi-component objective function, click on more than one option. An example occurs when you want to minimize the cost of pumping some particular volume from a group of wells. The objective function can include linear terms describing the cost per unit pumping rate, and quadratic terms describing the cost per unit pumping of raising groundwater a unit distance.

A.2.3.5.3. Solver Options (Figure A-12b). Making a check in the check boxes (or entering values in the text boxes) shown below causes CONJUS to implement those instructions.

‘Use linear model to perform optimization’ checkbox. If you check this, CONJUS will use the linear solver. This is inappropriate if quadratic or goal programming to a power greater than 1 are chosen. If the linear solver has difficulty solving a problem (this sometimes happens even for linear problems), employ the nonlinear solver. See the Error Handling section for advice concerning when to select the nonlinear solver.

‘Precision’ textbox. The value entered here controls the precision of solutions. CONJUS uses the number you enter to determine whether the value of a constraint meets a target or satisfies a lower or upper bound. Precision must be indicated by a fractional number between 0 (zero) and 1. The greater the desired precision the more decimals needed. For example, 0.0001 is a greater precision than 0.01. Often, a greater precision requires more time to achieve a solution.

'Tolerance' textbox. Because CONJUS does not currently use integer constraints, this textbox is disabled.

'Convergence' textbox. The solver stops when the relative change in the target cell value is less than the number in the Convergence box for the previous five iterations. Convergence applies only to nonlinear problems and must be indicated by a fractional number between 0 (zero) and 1. The smaller the convergence value, the more time the Solver requires to reach a solution. For example, a 0.0001 convergence value usually requires more solution time than 0.01.

A.2.3.5.4. 'Special Constraints' (Figure A-12c):

‘Constrain objective function value’ checkbox. If you check this, the textbox 'Constraint on Objective' will appear. There you can enter the upper and/or lower bound(s) on the objective function value. (The objective function value is the Z of Equation III-22).

‘Force all variables to be non-negative (this will add additional constraints to those otherwise specified)’ checkbox. If you check this, CONJUS will use zero as the lower bound for all the decision variables.

‘Force pumping to be the same for all periods while obeying all other constraints’ checkbox. If you check this, CONJUS will try to force pumping to be the same for all periods but will permit pumping to be different for different wells. If the constraints conflict, CONJUS might be unable to find a feasible solution. In that case consider trying the next option.

‘Force pumping to be equal for all periods having “n” or “n/a” on the lower or upper bound on pumping, while obeying all other constraints’ checkbox. If you check this option, CONJUS will try to force pumping at all well(s) to be the same only for those periods having no upper bound or lower bound on the pumping. An "n/a"or “n” in a cell means that a particular constraint is not applied.

‘Generic Polynomial Constraint’ (GPC) checkbox. Selecting this option will cause Fig A-13 to appear. Clicking ‘OK’ (if you want to formulate a GPC) will cause Fig A-7 to appear. This table permits construction of any constraint that is an algebraic combination of groundwater extraction, injection, surface water diversion and return flow for any or all stress periods. Each such decision variable is multiplied by a user-selected coefficient and raised to a power. Default values of coefficients and powers equal 0 and 1, respectively. Thus, the user must change the coefficient to cause the variable to be included in the constraint. The mathematical operators of addition, subtraction, multiplication and division are used to relate the variables. The default operator is addition.

The user can display the mathematical form of the GPC constraint equation he has created. Fig A-14 illustrates a sample GPC constraint equation--(extraction at well 1 during period 1 plus 2 times stream depletion at period 1 plus 3 times period 2 extraction at well 1). You can add an additional component in the GPC by choosing ‘Add another component to this constraint’. The user can specify the arithmetic operation to be used to relate the two components

A.2.3.6. Post-optimization Actions

One might subsequently to change the numbers of wells, diversions, return flows or stress periods. To do this without losing all previously entered data go to the top of the input sheet; change the appropriate value on the first line; and then click the second green button from the left. Pushing this ‘revision’ button will add input data rows to the end of the previous input data blocks, or will delete rows from the end of existing blocks.[3] This is very convenient when increasing problem size.

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[3] All data on observation wells represents one data block. However, one must be careful when changing the number of decision variables because all data on extraction wells and diversions together represent one data block. All data on injection wells and return flows together represent another data block. Increasing the number of wells adds row(s) to the end of the block. Decreasing the number of wells removes row(s) from the block end. After pressing

The easient way to effectively decrease problem size (or reduce number of decision variables) without resizing data input blocks is to set the lower and upper bounds on the variable to zero.

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the second green button, one should verify that the input data is correct, and must sometimes copy old data to new rows.

A.2.3.7. Error Avoidance and Handling.

A.2.3.7.1. CONJUS provides detailed information to guide data entry and to diagnose data input errors. CONJUS also employs error messages from Excel. The following guidance helps avoid errors and deal with selected error messages.

A.2.3.7.2. After you change any input data, the absolutely safest procedure is to rerun all steps below that in the Main Sheet flowchart. However, in reality, you only need to rerun steps affected by any data change you might make. Examples concerning changes you might make in the input sheet are: (i) if you change data related to simulation (such as aquifer conductivity, well radius, or distance between wells), you need to recompute influence coefficients (Step 2a); and (ii) if you only change data related to optimization (such as a lower bound), you only need to rerun optimization (Step 3) and you do not need to recompute influence coefficients.

A.2.3.7.3. 'The conditions for Assume Linear Model are not Satisfied'. This error message

tells the user to select the nonlinear instead of the linear solver. The nonlinear solver is always needed if a nonlinear GPC is used, or if an employed goal programming constraint employs a power different than 1. Sometimes the nonlinear solver is also needed to address particular linear problems. Use the Excel Office Assistant (at the right end of the main menu bar). Use keyword 'Solver Trouble Shooting' for detailed information.

A.2.3.7.4. Use the Excel Office Assistant for a wide range of information about Excel messages and options.

A.2.3.6.5. You should not delete key sheets in CONJUS, deleting important sheets destroys CONJUS, and will require that you reinstall CONJUS. You can delete sheets you generated yourself.

Table A-1. CONJUS Stimulus / Response Options

		A	B	C	D	E
Stimuli specified by user	Pumping at well(s) [1]	✓	✓		✓	✓
	Diversion(s) from stream	✓	✓			
	Return flow(s) to stream	✓	✓			
	Stream stage change			✓		
	Line source recharge				✓	
	Field recharge					✓
Stimulus controlled by CONJUS	Pumping at image well(s)[2]		✓			
Response Computed by CONJUS	Aquifer head	✓	✓	✓	✓	✓
	Stream depletion rate and volume due to pumping		✓			

[1] Pumping includes extraction and injection

[2] CONJUS automatically determines all image well locations

Table A-2. CONJUS Optimization Options

	CONJUS maximizes or minimizes the value of the Objective Function, Z, Where $Z = W^1 Z^1 + W^2 Z^2 + W^3 Z^3$					
Action to be performed on Objective Function	Objective Function Component	Common Name (highest power of unknowns)	Solver Needed	Unknowns	Coefficients	Reference Eq. Number (in report body)
Maximize, minimize, or set equal to (unknowns x coefficients)	Z_1	Linear Programming (1)	Linear	Extraction pumping (p^e), Injection pumping, (p^i) Diversion (d), Return flow (r)	C^{ep} C^{ip} C^d C^r	III-23
"	Z_2	Quadratic Programming (2)	Nonlinear	p^e (GSELEV - h^e) p^i (GSELEV - h^i) d (GSELEV - h^d) r (GSELEV - h^r)	C^{epq} C^{ipq}	III-24
"	Z_3	Goal Prgm. (n)	"	$(V_1 - GOAL)^n$	C^{gv}	III-25a
"	"	"	"	$(p^e - GOAL^{ep})^n$ $(p^i - GOAL^{ip})^n$	C^{gep} C^{gip}	III-25b
"	"	"	"	$(h^e - GOAL^{eh})^n$ $(h^i - GOAL^{ih})^n$ $(h^o - GOAL^{oh})^n$ $(h^d - GOAL^{dh})^n$ $(h^r - GOAL^{rh})^n$	C^{geh} C^{gih} C^{goh} C^{gdh} C^{grh}	III-25c
Minimize the largest of a series of unknowns x coefficients	"	Minimax Goal Programming (n)	"	$(V_2 - GOAL)^n$	C^{gv}	III-25d
Maximize the smallest of a series of unknowns x	"	Maximin Goal Programming (n)	"	$(V_2 - GOAL)^n$	C^{gv}	III-25e

p^e, p^i, d and r= Extraction and injection pumping, surface water diversion, return flow, respectively.
h^e, h^i, h^d, h^r= Head at extraction and injection well (outside well casing), diversion & return flow locations, respectively. GSELEV = Ground surface elevation at respective flux locations.
V_1 = stream depletion rate, stream depletion volume, or cumulative pumping volume.
$V_2 = p^e, p^i, d, r, h^e, h^i, h^o, h^d, h^r$ or V_1