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Unsteady Computational Fluid Dynamics (CFD) Validation and Uncertainty Quantification for a Confined Bank of Cylinders Using Particle Image Velocimetry (PIV)

Brandon M. Wilson
Utah State University

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UNSTEADY COMPUTATIONAL FLUID DYNAMICS (CFD) VALIDATION AND UNCERTAINTY QUANTIFICATION FOR A CONFINED BANK OF CYLINDERS USING PARTICLE IMAGE VELOCIMETRY (PIV)

by

Brandon M. Wilson

A dissertation submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Mechanical Engineering

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2012
Abstract

Unsteady Computational Fluid Dynamics (CFD) Validation and Uncertainty Quantification for a Confined Bank of Cylinders Using Particle Image Velocimetry (PIV)

by

Brandon M. Wilson, Doctor of Philosophy
Utah State University, 2012

Major Professor: Dr. Barton L. Smith
Department: Mechanical and Aerospace Engineering

A CFD validation study of unsteady flow through a confined bank of cylinders is performed to demonstrate validation of modeling and simulation (M&S) for time-varying quantities. Additionally, this dissertation assesses appropriate unsteady system response quantities (SRQ’s) and proper estimates of the numerical and experimental uncertainty. CFD simulations include the Unsteady Reynolds-Averaged Navier-Stokes (URANS) \( k-\omega \) model and two variations of the Detached Eddy Simulation (DES) model. Data is acquired using a non-intrusive measurement technique, called particle image velocimetry (PIV), coupled with time-varying pressure measurements along the facility walls. These data are used to validate the numerical results. The numerical simulations are designed to closely resemble the experimental facility boundary and inlet conditions and be geometrically identical.

Requirements for validation studies include 1) non-invasive measurement techniques, such as PIV, and 2) accurate quantification of the measurement uncertainty. However, accurate - and even adequate - local, instantaneous PIV uncertainty quantification is often difficult and ambiguous, particularly for higher-order terms (e.g. temporal variations and turbulence fluctuations). Thus, the effects of four PIV error sources on velocity and turbulence statistics are assessed and demonstrated within this study: particle image size,
particle image displacement, particle image density, and gradients within the flow. Results of this uncertainty study are found to accurately estimate the uncertainty from these error sources and are extended to generate PIV uncertainty estimates for the validation study.

Unsteady spatial and temporal validation SRQ's, such as frequency, fluctuations, autocorrelations, and correlations, are assessed to determine their effectiveness as validation quantities in the bank of cylinders experiment. Some of the models accurately predicted frequencies present in the velocity, bulk velocity, and pressure SRQ's for cylinders 1, 4, and 5. However, additional, non-physical frequencies are predicted for the remaining cylinders from all models. As expected, the temporal behavior of the DES was generally far superior to that of the URANS model. While some models are capable of predicting fluctuation amplitudes, and autocorrelation and correlation time-scales throughout the facility for most SRQ's, this is generally not observed for the remaining models. A grid convergence study shows typical global quantities (such as pressure losses) converge well while temporal quantities converge poorly for the same grids.
Public Abstract

Unsteady Computational Fluid Dynamics (CFD) Validation and Uncertainty Quantification for a Confined Bank of Cylinders Using Particle Image Velocimetry (PIV)

Brandon M. Wilson

Computational modeling is of particular interest to science and engineering for the improvement of design and development of products and research of physical phenomena. However, confidence in a computational model must be validated prior to its application through comparison to experimental data. The nuclear power industry has interest in the application of computational modeling to plant design, safety, and development for the increased understanding of heat transfer and fluid dynamics. Fluid dynamics, particularly time-varying phenomena within the reactor core, has a strong effect on heat (energy) transfer and transient accident scenarios of a nuclear power plant. While this work was funded by Idaho National Labs, this research is also applicable to engineering design of heat exchangers. This research began May of 2009.

The research within this dissertation demonstrates the validation of various time-varying quantities predicted by computational fluid dynamics models (CFD) with the use of experimental measurements. Several CFD turbulence models are implemented and experimental fluid velocity and pressure data are acquired using non-invasive measurement techniques for flow through a channel containing cylinders (confined cylinders). The CFD and experimental geometries and inlet and boundary conditions are identical and model the geometries found within a nuclear reactor core. Validation of the CFD models is assessed and it is found that while some models can accurately predict the time-varying quantities considered, the majority demonstrate deficiencies. Higher fidelity models typically predict these quantities with increased accuracy. The uncertainty for both the computational and experimental results are estimated and discussed.

Uncertainty estimation of both CFD and experimental results is required for meaningful validation. However, uncertainty from the non-invasive velocity measurement technique, particle image velocimetry (PIV), is inadequate for validation purposes, particularly random time-varying flow fluctuations (turbulence). Within this dissertation, the uncertainty from four dominate error sources are demonstrated, calculated, and analyzed for PIV. This method of estimating instantaneous, mean, and fluctuation uncertainty for PIV is demonstrated with significant accuracy and is applied to the confined cylinders experimental results for the validation study.
Acknowledgments

I would like to acknowledge my friends and colleagues at the Experimental Fluid Dynamics Laboratory, particularly Dr. Barton Smith for his dedication and insight provided in preparation and completion of my dissertation and education. I also want to thank Ben Timmins, Jeff Harris, and Scott Warner for the research and time given to PIV uncertainty and the bank of cylinders experiment necessary to accomplish this research.

I would also like to acknowledge my wife, Chelsie Wilson, for her patience and support during the long days and weeks prior to completion of the research and writing of this dissertation.

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Notation

**Lower-case Roman**

- \( b_{ue} \): The systematic uncertainty on the instantaneous velocity.
- \( b_{\pi e} \): The bias or systematic uncertainty on the time-averaged velocity.
- \( b_{\phi e} \): The systematic uncertainty on the instantaneous SRQ.
- \( b_{\phi e} \): The bias or systematic uncertainty on the time-averaged SRQ.
- \( c_{reqd} \): A predefined confidence requirement for validation by Rebba and Mahadevan.
- \( d_p \): The actual particle image diameter in PIV measurements.
- \( d_s \): The diffraction limited spot diameter.
- \( du/dy \): The flow gradients in PIV measurements.
- \( d_{\rho} \): The particle image density in PIV measurements.
- \( d_{r} \): The particle image diameter in PIV measurements.
- \( e_{a} \): The approximate relative error for the fine grids.
- \( f \): The measurement data acquisition rate or frequency.
- \( f_{exact} \): The exact solution to the numerical model.
- \( f_s \): The focal length of the PIV image.
- \( f_1 \): The solution the numerical model on a refined grid size.
- \( f_2 \): The solution the numerical model on a coarse grid size.
- \( f^\# \): The \( f \)-number (inversely related to aperture size) of a lens.
- \( g(t) \): A fluctuating quantity used to calculate the autocorrelation coefficient.
- \( g'(t) \): A fluctuating quantity at a different time used to calculate the autocorrelation coefficient.
- \( h \): The grid size used in the numerical model.
- \( h_{coarse} \): The grid size for the coarse mesh.
The grid size for the fine mesh.

The final interrogation area size for PIV measurements.

The jet half-height for the PIV uncertainty experiment.

The minor loss factor across the cylinder array.

The number of cylinders used to calculate the minor loss factor.

The precision uncertainty on the time-averaged velocity.

The precision uncertainty on the time-averaged SRQ.

The rate at which the error in the model solutions converge to zero.

The random uncertainty on the instantaneous velocity.

The random uncertainty on the instantaneous SRQ.

The random uncertainty on the time-averaged SRQ.

The positive random uncertainty on the time-averaged fluctuation SRQ.

The negative random uncertainty on the time-averaged fluctuation SRQ.

The standard deviation of the random error standard deviation for the Monte Carlo simulation.

The standard deviation of the random error of the Monte Carlo simulation.

The experimental standard deviation of the instantaneous SRQ.

The standard deviation of the random measurement error.

The variance of the random measurement error.

The variance of the random measurement error in the $x$-direction.

The variance of the random measurement error in the $y$-direction.

The time term.

The $t$-statistic from the $t$-distribution.

The initial time limit of a specified time interval.

The final time limit of a specified time interval.

The $t$-statistic from the $t$-distribution with a 95% confidence interval.

The time-averaged component of the velocity SRQ ($u(t)$).

The bulk velocity at the maximum area for the cylinder array.
The time-averaged bulk velocity at the minimum area of the cylinder array.

The component of velocity in the x-direction.

The component of velocity in the y-direction.

The maximum time-averaged or centerline velocity for the PIV uncertainty experiment.

The experimental random error on the instantaneous velocity.

The total fluctuating component of the measured instantaneous velocity.

The instantaneous, time-dependent velocity SRQ.

The turbulent fluctuating component of the velocity SRQ \((u(t))\).

The non-turbulent variational component of the velocity SRQ \((u(t))\).

The time averaged random measurement error component of the velocity SRQ \((u(t))\).

The time averaged total fluctuating component of the measured velocity SRQ \((u(t))\).

The time averaged turbulent fluctuating component of the velocity SRQ \((u(t))\).

The span-wise width of the cylinder array.

The stream-wise direction through the cylinder array facility.

The cross-stream direction through the cylinder array facility.

The wall coordinates for the CFD model boundary layer thickness.

The span-wise direction through the cylinder array facility.

**Upper-case Roman**

\(C\) The comprehensive error factor as proposed by Sprague and Geers.

\(C_{\text{DES}}\) A constant parameter for the DES model.

C.I. The defined confidence interval for the uncertainty.

\(D\) The cylinder diameter of the cylinder array.

\(D_a\) The aperture diameter of the lens.
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<td>$E$</td>
<td>The difference between the true and numerical SRQs.</td>
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<td>$\dot{E}$</td>
<td>The difference between the experimental and numerical SRQs.</td>
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<td>$F_2$</td>
<td>The blending function distinguishing DES models.</td>
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<td>$F_{\text{DES}}$</td>
<td>The shielding function for the DES model.</td>
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<td>$F_s$</td>
<td>The factor of safety for the numerical uncertainty.</td>
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<td>$\text{GCI}$</td>
<td>The Grid Convergence Index, or numerical uncertainty.</td>
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<td>$H$</td>
<td>The span-wise height of the cylinder array.</td>
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<td>$I$</td>
<td>The turbulence intensity at the inlet of the cylinder array.</td>
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<td>$L$</td>
<td>The stream-wise length of the cylinder array.</td>
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<td>$L_t$</td>
<td>The turbulent length scale for the DES model.</td>
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<td>$M$</td>
<td>The magnitude error factor as proposed by Sprague and Geers.</td>
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<tr>
<td>$M_s$</td>
<td>The magnification of the PIV image.</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of experimental realizations within a dataset.</td>
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<td>$P$</td>
<td>The phase error factor as proposed by Sprague and Geers.</td>
</tr>
<tr>
<td>$P(t)$</td>
<td>The correlation coefficient proposed by Tennekes and Lumley.</td>
</tr>
<tr>
<td>$P/D$</td>
<td>The pitch ratio within the cylinder array.</td>
</tr>
<tr>
<td>$R$</td>
<td>The model reliability as defined by Rebba and Mahadevan.</td>
</tr>
<tr>
<td>$\text{Re}_D$</td>
<td>The Reynolds number for the cylinder array.</td>
</tr>
<tr>
<td>$\text{Re}<em>{h</em>{1/2}}$</td>
<td>The Reynolds number for the PIV uncertainty jet experiment.</td>
</tr>
<tr>
<td>$St$</td>
<td>The Strouhal frequency number for the cylinder array.</td>
</tr>
<tr>
<td>$St_1$</td>
<td>The first harmonic frequency for the Strouhal frequency number in the cylinder array.</td>
</tr>
<tr>
<td>$St_2$</td>
<td>The second harmonic frequency for the Strouhal frequency number in the cylinder array.</td>
</tr>
<tr>
<td>$St_3$</td>
<td>The third harmonic frequency for the Strouhal frequency number in the cylinder array.</td>
</tr>
<tr>
<td>$U_{\text{reqd}}$</td>
<td>The programmatic validation requirement.</td>
</tr>
<tr>
<td>$U_{u_e}$</td>
<td>The total uncertainty estimate on the instantaneous velocity.</td>
</tr>
</tbody>
</table>
\( U_{\bar{u}_e} \) The total uncertainty estimate on the time-averaged velocity.

\( U_{\hat{u}_e} \) The positive uncertainty estimate on the total fluctuating component of velocity.

\( U_{-\hat{u}_e} \) The negative uncertainty estimate on the total fluctuating component of velocity.

\( U_{\hat{u}_x} \) The total uncertainty estimate on the measured fluctuating component of \( x \)-velocity.

\( U_{\hat{u}_y} \) The total uncertainty estimate on the total fluctuating component of \( y \)-velocity.

\( U_{V,C.I.} \) The validation uncertainty with a defined confidence interval.

\( U_{\phi_e} \) The total uncertainty estimate on the instantaneous SRQ.

\( U_{\bar{\phi}_e} \) The total uncertainty estimate on the time-averaged SRQ.

\( U_{\phi_{in}} \) The model input uncertainty on the numerical SRQ.

\( U_{\phi_{in-e}} \) The combination of experimental and model input uncertainty on the numerical SRQ.

\( U_{\phi_n} \) The numerical uncertainty on the numerical SRQ.

\( U_{\phi_V} \) The validation uncertainty on the numerical SRQ.

\( U_\xi \) Uncertainty on the model input variables affecting the model solution.

\( U_{\bar{\phi}_e/\phi_e} \) The total uncertainty estimate on the time-averaged SRQ.

\( V \) The validation metric as proposed by Oberkampf and Trucano.

\( Y_k \) The modified dissipation term for turbulent kinetic energy in the DES model.

**Lower-case Greek**

\( \alpha \) The error between two grid solutions.

\( \alpha_{21} \) The error between the second and first refined grid solutions.

\( \alpha_{32} \) The error between the third and second refined grid solutions.

\( \beta_{ue} \) The experimental systematic error on the instantaneous velocity.

\( \beta_{\phi e} \) The experimental systematic error on the instantaneous SRQ.
\( \delta_{BL} \) The boundary layer thickness.
\( \delta_{m} \) The true model error.
\( \delta_{t} \) The time-step used for the CFD models.
\( \delta_{\phi_e} \) The experimental error on the instantaneous SRQ.
\( \delta_{\bar{\phi}_e} \) The experimental error on the time-averaged SRQ.
\( \delta_{\phi_{c\phi_e}} \) The experimental error on the measured fluctuations SRQ.
\( \epsilon_{\phi_e} \) The experimental random error on the instantaneous SRQ.
\( \epsilon_{DES} \) The new dissipation rate for the DES model.
\( \epsilon_{reqd} \) A predefined accuracy limit required for validation proposed by Rebba and Mahadevan.
\( \bar{\epsilon}_{\phi_e} \) The time-averaged random error on the instantaneous SRQ.
\( \phi \) The “true” value of the SRQ of interest.
\( \bar{\phi} \) The “true” mean component of the instantaneous SRQ.
\( \phi' \) The “true” fluctuating component of the instantaneous SRQ.
\( \epsilon_{\phi_e} \) The experimental random error on the instantaneous SRQ.
\( \hat{\phi}_e \) The combined fluctuations on the instantaneous SRQ \((\phi' + \epsilon_{\phi_e})\).
\( \bar{\phi}_e \) The experimental mean of the time-averaged SRQ.
\( \phi_e \) The SRQ measured in the experiment.
\( \phi_m \) The SRQ predicted from the numerical model.
\( \phi_1 \) The SRQ predicted from the numerical model on the coarse grid.
\( \phi_2 \) The SRQ predicted from the numerical model on the medium grid.
\( \phi_3 \) The SRQ predicted from the numerical model on the refined grid.
\( \gamma \) The ratio of the grid size for discretized solutions.
\( \gamma_{21} \) The ratio of the grid size for the second and first discretized solutions.
\( \gamma_{32} \) The ratio of the grid size for the third and second discretized solutions.
\( \lambda \) The wavelength of the laser light sheet.
\( \mu \) The mean of the parent population or “true” SRQ.
\( \mu_f \) The dynamic viscosity of the fluid.
\( \mu_t \) The turbulent viscosity for the CFD models.

\( \nu \) The degrees of freedom (number of samples) for the \( t \)-distribution.

\( \nu_f \) The kinematic viscosity of the fluid.

\( \rho(t) \) The autocorrelation coefficient proposed by Tennekes and Lumley.

\( \rho_f \) The density of the fluid.

\( \sigma \) The standard deviation of the parent population or “true” SRQ.

\( \sigma_{\text{corr}} \) The peak width (one standard deviation) of the autocorrelation.

\( \tau \) The time difference in the autocorrelation or convolution.

\( \xi \) Model input variables affecting the model solution.

**Upper-case Greek**

\( \Delta_{\text{max}} \) The local grid spacing for the DES model.

\( \Delta p \) The pressure drop across the cylinder array.

\( \Delta t \) The time interval between images for PIV measurements.

\( \Delta x \) The particle image displacement in PIV measurements.

\( \Phi \) The Gaussian cumulative distribution function.

\( \text{P}(t) \) The correlation coefficient proposed by Tennekes and Lumley.
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIAA</td>
<td>American Institute of Aeronautics and Astronautics</td>
</tr>
<tr>
<td>ASME</td>
<td>American Society of Mechanical Engineers</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>C.I.</td>
<td>Confidence Interval</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide-Semiconductor</td>
</tr>
<tr>
<td>DES</td>
<td>Detached Eddy Simulation</td>
</tr>
<tr>
<td>DES-B</td>
<td>Detached Eddy Simulation - Blending</td>
</tr>
<tr>
<td>DES-NB</td>
<td>Detached Eddy Simulation - No Blending</td>
</tr>
<tr>
<td>DES-P</td>
<td>Detached Eddy Simulation - Perturbations</td>
</tr>
<tr>
<td>DoD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transforms</td>
</tr>
<tr>
<td>GCI</td>
<td>Grid Convergence Index</td>
</tr>
<tr>
<td>IA</td>
<td>Interrogation Area</td>
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<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
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<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
</tr>
<tr>
<td>LIF</td>
<td>Laser Induced Fluorescence</td>
</tr>
<tr>
<td>M&amp;S</td>
<td>Modeling and Simulation</td>
</tr>
<tr>
<td>NEA</td>
<td>Nuclear Energy Agency</td>
</tr>
<tr>
<td>NE-KAMS</td>
<td>Nuclear Energy - Knowledge base for Advanced Modeling and Simulation</td>
</tr>
<tr>
<td>pdf</td>
<td>probability distribution function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<tr>
<td>RANS</td>
<td>Reynolds-Averaged Navier Stokes</td>
</tr>
<tr>
<td>RPC</td>
<td>Robust Phase Correlation</td>
</tr>
<tr>
<td>SCC</td>
<td>Standard Cross Correlation</td>
</tr>
<tr>
<td>SRQ</td>
<td>System Response Quantity</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>UQ</td>
<td>Uncertainty Quantification</td>
</tr>
<tr>
<td>URANS</td>
<td>Unsteady Reynolds-Averaged Navier Stokes</td>
</tr>
<tr>
<td>VHTR</td>
<td>Very High Temperature Reactor</td>
</tr>
<tr>
<td>V&amp;V</td>
<td>Verification and Validation</td>
</tr>
</tbody>
</table>
Chapter 1
Introduction

Over the last 40 years, numerical modeling and simulations (M&S), particularly computational fluid dynamics (CFD), have evolved from simplistic 0-D or 1-D models capable of offering limited calculations and data to complex, coupled multi-physics, and multidimensional models predicting accurate solutions of physical phenomena. These advances in M&S have established it as a primary tool in engineering research, design, and development. M&S offers several advantages over testing and experiments in the engineering process:

1. Although computationally costly, M&S is often less expensive than experiments.

2. Specific physical phenomena may be better understood through M&S by restricting and controlling irrelevant phenomena (e.g., radiation, buoyancy, etc.). These phenomena are difficult to restrict or quantify in experimental measurements.

3. Unlike the limitations of experiments, M&S offers high-fidelity temporal and spatial data over entire flow fields.

4. Simulation conditions and properties are easily modified and controlled allowing physical sensitivities to be studied.

Although the engineering process may be enhanced by M&S optimizing engineering research, design, and development a cooperative role between M&S and experimentation must be adopted. Benefits obtained through the synergetic use of M&S and experimentation are outlined by [1], such as, shorter design and test processes, reduction of uncertainties, and improved fidelity. Sargent [2] presents three stages of experimentation in M&S: developing the numerical model, validation of the model, and performing experiments with the
validated model. With the continuous improvements in computational technology, a more dominant role of M&S in all phases of engineering can be anticipated.

While M&S does have advantages over experimentation, it also presents deficiencies. Numerical models contain assumptions embedded within the governing equations that may not be applicable to particular situations. Other weaknesses include “bugs” within software and user errors, such as inaccurate definitions of initial and boundary conditions and inadequate grid refinement. These shortcomings expose vulnerabilities in the implementation of M&S to all engineering situations and are of particular interest to high-risk settings such as commercial aerospace optimization and nuclear reactor operation and design. Quantifying the capabilities of M&S to model specific applications presents the largest obstacle numerical simulations face for engineering research, design, and development purposes.

The goal of the emerging field of verification and validation (V&V) is to determine the accuracy and applicability of M&S to particular situations. Simply put, V&V is a two-stage process consisting of determining if 1) the code, in fact, solves the model correctly (verification) and 2) the solutions to the model represent the appropriate physical phenomena (validation); more formal definitions are provided by the Department of Defense (DoD) [3:10] and accepted by many organizations (e.g. AIAA [4] and ASME [5,6]):

**Verification**: The process of determining that a model implementation accurately represents the developer’s conceptual description of the model and the solution to the model.

**Validation**: The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

While verification is equally essential, validation is the primary aim of the research contained within this dissertation. To maintain consistency with the AIAA and ASME standards, the formal definition of validation given by the DoD will be used throughout this dissertation.
1.1 Validation

The most fundamental mechanism of understanding the physical world derives from experimental measurements, and as such, experiments present the foremost method of quantitatively comparing numerical simulations to physical phenomena. Using experimental measurements as the primary comparison tool for validation is strongly encouraged by the validation community; only during unreasonable circumstances, such as nuclear weapons testing, should other means be used to validate M&S. The reasoning for using experimental data in validation is expressed by Oberkampf [7,14]:

This strategy does not assume that the experimental measurements are more accurate than the computational results. The strategy only asserts that experimental measurements are the most faithful reflections of reality for the purposes of validation.

Considering both experimental and numerical results contain errors, a complete analysis of the experimental uncertainties and numerical errors must be performed [7]. Roache [8] attests “experimental data are necessary for validation,” however, much more is required of the data, including quality, quantity, and uncertainty estimates. Section 1.2 provides a complete review of experimental and numerical uncertainty quantification.

The objectives of M&S within a specific engineering application will always differ from other situations; this could be due to the risk or complexity involved, precision required, budget allocated, or quantities desired. Boundary conditions, inlet profiles, initial conditions, and geometry are also unlikely to be similar for each application. The level of validation required for each situation will also vary. Thus, as validation of M&S is developed, the circumstances and requirements of the specific program must be considered. A general, all-encompassing validation strategy cannot be developed.

1.1.1 Validation Experiments

From this perspective, it is easily concluded that M&S cannot be universally validated but validated only for the distinct applications of the model or simulation [9,10]. In
Fig. 1.1: Diagram of the tiered approach for validation experiments proposed by Oberkampf and Trucano. This system divides a complex system into progressively simpler experiments: 1) complete system (complex), 2) subsystem, 3) benchmark cases, 4) unit problems (basic).

To extend the validated range for a specific numerical model, validation experiments encompassing multiple phenomena and regimes must be designed. Oberkampf and Trucano [11] recommend a tiered approach to validation (shown in Fig. 1.1), in which a complex system is divided into progressively simpler experiments: 1) complete system (complex), 2) subsystem, 3) benchmark cases, 4) unit problems (basic). Unit problems and benchmark cases usually consist of validation of fundamental phenomena such as flow over a backstep or convection over a flat plate.

This tiered approach should be implemented for many reasons. Due to complexity, it is often difficult to acquire comprehensive descriptions and measurements of all the phenomena within the complete system tier. In addition, discrete non-physical model errors may combine to generate erroneous numerical and experimental agreement. However, unit problems and benchmark cases allow acquisition of higher fidelity measurements for indi-
vidual phenomenon while eliminating other effects. Also, similar phenomena exist for many complete system models, and when applicable, validation results from benchmark cases and unit problems describing this similar phenomena may be used for multiple complete system models. This leads to a later discussion in Section 1.1.2 concerning databases or repositories dedicated to the validation experiments for unit problems and benchmark cases.

The design and purpose of validation experiments, therefore, is fundamentally different than traditional experiments. While traditional experiments are designed for fundamental understanding, model improvement, or system and component testing, the purpose of validation experiments is to quantitatively determine the validity of a numerical model. This is achieved via acquisition of high-fidelity, accurate measurements of the physical phenomena and comparison of this data to the numerical model simulations \[12\]. This requires the same phenomena be modeled by both the validation experiment and the numerical simulation. This includes boundary, initial, inlet, and outflow conditions and model geometries, which must be accurately measured from the experimental model and provided to the numerical model.

To promote favorable validation practices, six requirements of validation experiments have been developed by researchers at Sandia National Laboratories \[11\]:

1. The validation experiment should be jointly designed by experimentalists and code developers/users.

2. The validation experiment should contain all relevant physics within it, including boundary, initial, inlet and outflow conditions required by the code.

3. The validation experiment should emphasize the synergism between experimental and computational approaches.

4. The experimental and computational results should be obtained independently.

5. The hierarchy of computational difficulty should be specified.

6. The validation experiment should be designed to accurately estimate the bias and precision uncertainties.
Similar guidelines are proposed by Babuska and Oden [13] with the additional condition of reproducibility of experimental results. Additional requirements are proposed for a completely-described experiment by Wilson [9] and Wilson et al. [14] and validation-level experiments and data sets by Lee and Bauer [10]. For the latter case, Lee and Bauer [10] supplement the requirements from Sandia National Laboratories with two elements:

7. The validation experiment should use non-invasive, high precision instruments, such as PIV, LDA, and LIF.

8. The validation experiment should contain fully documented and detailed measurements of the boundary and initial conditions and as-built geometry of the experiment.

Although the motivation and needs of each validation experiment will differ, concrete validation-level data is achievable by adherence to these eight requirements.

Validation of numerical M&S requires the comparison of appropriate quantities representing the requirements of the particular use of the model; these quantities are often called system response quantities (SRQ’s). As mentioned previously, the purpose of M&S will be different for each situation and this requires the development of appropriate SRQ’s. For example, the loss coefficient across a bank of cylinders may be a sufficient SRQ to determine the loss through a hydraulic system, yet inadequate for determining the fluid-elastic vibrations on the cylinders. The latter case may require a more complex SRQ such as the dominant frequency; however, it would be meaningless to validate the hydraulic system to a dominant frequency level. Careful consideration of application requirements must be contemplated when developing validation SRQ’s.

Similar to validation experiments, a tiered hierarchy should be considered when considering validation SRQ’s (Oberkampf and Roy [15]). The hierarchy ranges from integrated statistics (averaged SRQ’s) to SRQ derivatives, shown in Fig. 1.2. Integrated SRQ’s consist of quantities such as bulk velocity, minor-loss factors, and time-means, while shear, acceleration, and vorticity are higher-order SRQ derivatives. Generally, this hierarchy is established due to the difficulties required in both measuring and simulating these quantities. The level of validation attained corresponds to the hierarchy of SRQ’s validated.
Graphical comparisons of SRQ’s are frequently used to assess M&S validation. Oberkampf and Barone [16], however, note this method does not account for the M&S and experimental uncertainties and provides little in the way of quantitative assessment. Instead, more statistical means of comparison must be used, such as confidence intervals, hypothesis testing, and validation metrics. These methods allow quantitative measures of SRQ’s validation levels and require accurate assessment of M&S and experimental SRQ uncertainties. Following a general outline of experimental and M&S uncertainty quantification in Section 1.2, suggested statistical validation metrics for quantifying SRQ validation will be discussed in Section 1.3.

1.1.2 Validation Databases

Validation of numerical M&S using experimental measurements is a widely accepted practice in V&V. However, the current dearth of validation-level experiments and datasets presents a daunting obstacle for numerical validation. Causes for this include the increased cost and time required in acquiring experimental measurements, particularly validation-level datasets and the inability of sharing proprietary datasets.
To promote the increased use of M&S in engineering, the responsibility of validation-level dataset acquisition must be shared by the M&S community. One cannot expect a M&S facility to acquire sufficient quantities of appropriate experimental measurements for its validation purposes. Additionally, benchmark and unit level validation experiments may apply to several complete system cases (see Fig. 1.2).

While authors often describe their experimental data as a validation database (e.g. [17] [18]), this data is often difficult to access. A compilation of shareable validation-level data within one easily-accessible database does not exist. Lee and Mousseau [19] propose a database dedicated to the sharing and storage of experimental and M&S datasets for the use of M&S validation. This database, recognized as Nuclear Energy - Knowledge base for Advanced Modeling and Simulation (NE-KAMS), has four purposes regarding V&V: to develop and implement

- Standards, requirements and best practices for performing V&V assessments of M&S,
- Standard and procedures for the evaluation and classification of experimental and numerical benchmark data for use in V&V,
- Quality-assessed, web-accessible databases for V&V benchmark data, and
- A knowledge base of search-able information, documents and data related to V&V and uncertainty quantification (UQ) of M&S.

Such a database promotes the application of M&S to engineering research, design, and development through improved access of validation-level experiments and datasets for V&V.

NE-KAMS maintains requirements for submission and third-party rankings of validation data completeness to promote sufficient confidence among the validation community. Rankings are given for six attributes of a validation-level experiment:

1. Experimental Facility,
2. Analogy instrumentation and signal processing,
3. Boundary and initial conditions,

4. Fluid and material properties of the walls,

5. Test conditions, and


These attributes quantify input information required by the numerical model, measurement of experimental SRQ’s predicted by the model, and methodology of measurement acquisition by the experiment. The documentation of these attributes is assigned a level of completeness (4 levels) based on requirements designated by NE-KAMS. Improved detail and documentation is required to achieve higher completion levels. Consequently, experimental cost increases for each dataset as levels of completeness are improved. Further documentation regarding the requirements for validation data within the NE-KAMS database is contained within [19].

1.2 Uncertainty

Both experimentation and numerical simulations attempt to measure or predict an SRQ of interest; however, the results from both of these methods differ from the “true” quantity $\phi$ by some error. An accurate estimate of the error within a validation study is critical to obtaining a meaningful conclusion. Estimates of the error (uncertainties) within a validation study may be broken into three components: experimental data, input parameters, and numerical simulations [6]. Of these, the latter two components (input parameters and numerical simulations) affect errors within the numerical SRQ $\phi_m$, while the experimental SRQ $\phi_e$ is only affected by the experimental data. The process of quantifying the numerical and experimental uncertainty is fundamentally different and, consequently, will be discussed separately.
1.2.1 Experimental Uncertainty

Although every measurement system and experimental facility observes different dominant error sources, the method of estimating the size of these errors can be generalized. Measurement error magnitudes and directions are generally unknown and uncertainty estimates are used to statistically quantify the magnitude of these errors. Uncertainty estimates of measurement error effects on instantaneous, mean, and fluctuations of any time-varying quantity are derived and explained in Appendix A.

As in Appendix A, the total uncertainty can be decomposed into systematic uncertainties (unvarying uncertainties or uncertainties that affect each measured value similarly) and random uncertainties (randomly distributed uncertainties that affect each instantaneous measurement differently). Sources of systematic and random uncertainty in particle image velocimetry (PIV) will be discussed in Section 1.5. The generalized uncertainty estimates for instantaneous, mean, and fluctuation measurements will be presented within this section.

Instantaneous measurements are affected by both systematic $b_{\phi_e}$ and random $r_{\phi_e}$ uncertainties. The total uncertainty on the instantaneous quantity can be expressed as the root-sum-square of all the uncertainties:

$$U_{\phi_e} = \sqrt{b_{\phi_e}^2 + r_{\phi_e}^2},$$  \hspace{1cm} (1.1)

with a consistent confidence interval applied to both uncertainty types.

Often the mean value and standard deviation are used as validation SRQ’s or inputs to numerical simulations. The discrete forms of these quantities are given in the following equation:

$$\bar{\phi}_e = \frac{1}{N} \sum_{i=1}^{N} \phi_{e,i}^m$$  \hspace{1cm} and  \hspace{1cm} $$s_{\phi_e} = \frac{1}{N-1} \sum_{i=1}^{N} (\phi_{e,i} - \bar{\phi}_e)^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\hat{\phi}_{e,i})^2.$$  \hspace{1cm} (1.2)

with $N$ being the number of measurement samples acquired. Due to the critical use of these quantities, accurately quantifying the errors on these terms is very important.
Magnitudes and directions of errors are typically unknown and must be estimated using uncertainty estimates. An uncertainty estimate quantifies a range with a specified confidence interval that contains the error magnitudes. A summary of uncertainty estimates on time-varying quantities (i.e. means and fluctuations) is presented for clarification. The derivation of systematic and random error effects on mean and fluctuating quantities are given in Appendix A and can be referenced for more thorough descriptions.

The time-averaged uncertainty on the mean quantity $U_{\overline{\phi_e}}$ is obtained via the root sum of all systematic and random uncertainties:

$$U_{\overline{\phi_e}} = \sqrt{\sum_{i=1}^{k} b_{\phi_e,i}^2 + \left( t_{C.I.,\nu} \frac{s_{\overline{\phi_e}}}{\sqrt{N}} \right)^2}, \quad (1.3)$$

where $b_{\phi_e}$ denotes the individual systematic uncertainties on the mean. The first term accounts for the systematic errors on the mean quantity, while the mean affects of random errors - or precision uncertainties $r_{\overline{\phi_e}}$ - are estimated with the second term. The variable $t_{C.I.,\nu}$ is the $t$-statistic from the $t$-distribution and is dependent on the confidence interval C.I. and the degrees of freedom $\nu$ or number of samples acquired. When a sufficient number of samples ($N \geq 20$ samples) are attained, the $t$-distribution can be represented using the normal distribution. Commonly, a 95% confidence interval is used meaning there is 95% confidence the parent mean $\mu$ is within the interval $\overline{\phi_e} \pm r_{\overline{\phi_e}}$. For Gaussian statistics, a 95% confidence interval the corresponding $t$-value is given as $t_{C.I.} = t_{95\%} \approx 1.96$. It should be noted that, when using Eqn. 1.3, the random and systematic uncertainties should be quantified at the same confidence interval.

While the effect of the systematic and random measurement errors on the mean quantity is well understood, the manner in which they affect the time-averaged fluctuations is less clear. Once again, uncertainties on time-averaged measured fluctuations are demonstrated in Appendix A where it is shown that only random errors influence the time-averaged measured fluctuation size. While systematic errors do not contribute to time-averaged fluctuation uncertainties, random uncertainties elevate converged time-averaged measured fluctuation statistics. Thus, time-averaged fluctuation uncertainties within this study are
one-sided and negative. Filtered data or measurements with improper measurement response will attenuate time-averaged measured fluctuations; however, the measurement response in this study is sufficient to reduce this effect and will not be considered within this study.

The magnitude of the fluctuating errors on these directional components is calculated through an uncertainty root sum square of the random, instantaneous measurement uncertainties. Total time-averaged measured fluctuation uncertainty is two-directional, but the random errors only elevate the uncertainty giving two directional (positive and negative) time-averaged measured fluctuation uncertainty equations:

\[
U^+_{\phi_e \phi_e} = \frac{\phi_e \phi_e}{(N - 1)} \approx \delta^+_{\phi_e \phi_e}, \quad \text{and} \quad (1.4)
\]

\[
U^-_{\phi_e \phi_e} = \sqrt{\left(\frac{\phi_e \phi_e}{(N - 1)}\right)^2 + \frac{1}{N} \sum_{i=1}^{N} (r_{\phi_e,i})^4} \approx \delta^-_{\phi_e \phi_e}. \quad (1.5)
\]

where \(r_{\phi_e}\) is the maximum of the random uncertainty with a 95% confidence.

The traditional approach above is the most widely accepted method to quantify experimental uncertainty; however, Aeschliman and Oberkampf [20] and Oberkampf and Trucano [11] propose a statistical method accepted in other fields, yet rarely utilized in wind tunnel testing. This procedure makes use of symmetry planes of the flow and model. For example, a perfect wind tunnel test section would have uniform flow with an infinite number of symmetry planes, while a perfect model often has at least one plane of symmetry. Comparison of measurements (and repeated measurements) obtained at carefully devised locations on these planes of symmetry are used to quantify the measurement variance within the experiment. Comparison of repeated run measurements quantifies instrument uncertainty, while comparisons of measurements on flow and model symmetry planes provide estimates of uncertainty on flow field non-uniformities and model geometries, respectively. A detailed example is provided by Oberkampf and Aeschliman [21].

Oberkampf and Trucano [11] note that while random uncertainties are easily evaluated using the traditional method of uncertainty quantification, systematic uncertainties are
often underestimated. They also demonstrate systematic uncertainties, such as flow field non-uniformities are often the largest uncertainties in a wind tunnel experiment. This new technique offers improvements over the traditional methods by directly measuring these systematic uncertainties. While investigation of this method remains primarily with wind tunnel testing, this technique can be applied not only to other fluid dynamics experimental facilities, but measurement techniques, such as PIV.

Experimental results are often calculated from multiple variables or fundamental measurements. A simple example is the calculation of the Reynolds number, which is a function of three variables: velocity, length scale, and kinematic viscosity. Uncertainties in the fundamental measurements propagate into the uncertainty of the calculated value. Two methods of propagating this uncertainty are available: the Taylor Series method and the Monte-Carlo method [22].

1.2.2 Modeling Uncertainty

Like experimental uncertainty, M&S uncertainty is composed of two sources. The first component, numerical uncertainty $U_{\phi_{n}}$, is generated from numerical error sources (e.g. iteration convergence, model discretization, artificial dissipation, and time step size), while the second component, model input uncertainties $U_{\phi_{in}}$, are caused by uncertainty in the simulation inputs (e.g. model geometry, initial and boundary conditions, fluid properties) [6,23].

**Numerical Uncertainty**

The conventional methods of quantifying numerical uncertainty derive from Richardson Extrapolation [24,25], which demonstrates the behavior of the error in a discretized numerical model to decrease asymptotically as the grid size $h$ is decreased. Using this behavior, Richardson developed a formula to extrapolate the model’s exact solution $f_{\text{exact}}$ using two discretized solutions ($f_{1}$ and $f_{2}$):
\[ f_{\text{exact}} = f_1 + \frac{(f_1 + f_2)}{\gamma^2 - 1}; \]

where \( \gamma \) is the ratio of the grid size for the first and second discretized solutions.

One may consider using Richardson Extrapolation to calculate the exact solution and eliminate most numerical uncertainty; however, Roache [26] points out that, by doing this, exact model solutions may not maintain property conservation (e.g., mass, momentum, etc.). Richardson [25] also notes that this method does not apply to higher derivatives of the solution, which decreases the numerical accuracy on higher-order derivatives obtained through Richardson Extrapolation. For these reasons it is suggested that Richardson Extrapolation not be used to extrapolate the exact model solution [26].

Although exact model solutions should not be calculated via Richardson Extrapolation, it is apparent this method may be used to approximate the error between the discretized solution and exact model solution. The Grid Convergence Index (GCI) [27] is the most widely accepted method of using Richardson Extrapolation to quantify the uncertainty on a numerical solution.

The rate at which the error on the model solutions converge to zero is called the apparent order or convergence rate \( q \) of the method. Although the theoretical order may be obtained through a Taylor series analysis of the model method, the apparent order is dependent on the model solutions being analyzed. Richardson Extrapolation is based on model solutions with sufficiently refined grids that are within the monotonic, asymptotic convergence range. Insufficiently refined grids may produce convergence rates that are not monotonic and oscillating. Numerical models also experience artificial diffusion and round-off errors not accounted for in the calculation of the theoretical order. These and remaining sources, may cause apparent orders to be smaller and even larger than the theoretical order. While some argue GCI is applicable only to solutions in the monotonic, asymptotic convergence range [28], Celik et al. [27] allows numerical uncertainty on oscillating convergent solutions to be quantified using GCI with the cautionary note to solution users that the solutions may have oscillatory convergence.
GCI requires model solutions on three increasingly refined grids with a grid ratio of \( \gamma \geq 1.3 \). Preferably, the grid ratios will be equal (i.e. \( \gamma_{32} = \gamma_{21} \), where the subscripts designate the \( k^{th} \) grid), however, this is not required. The numerical uncertainty and rate of convergence for a solution parameter \( \phi_m \) determined important to the simulation (e.g. drag, bulk velocity, or frequency) is quantified. The apparent order of the numerical model is calculated from the following three equations based on the grid ratio and solution parameters:

\[
q = \frac{1}{\ln(\gamma_{21})} \left| \ln \left( \frac{\alpha_{32}}{\alpha_{21}} \right) + c(q) \right|, \quad (1.7)
\]

\[
c(q) = \ln \left( \frac{\gamma_{q}^{21} - s}{\gamma_{32}^{q} - s} \right), \quad (1.8)
\]

\[
s = \text{sign} \left( \frac{\alpha_{32}}{\alpha_{21}} \right), \quad (1.9)
\]

where the subscripts once again designate the \( k^{th} \) grid and \( \alpha \) is the error between grid solution parameters (e.g. \( \alpha_{32} = \phi_3 - \phi_2 \)). Celik et al. notes that \( c(q) \) is equal to zero when \( \gamma_{32} = \gamma_{21} \). For situations where this is not the case, the apparent order must be calculated iteratively.

The GCI and numerical uncertainty for the fine grid is quantified using the relation,

\[
U_{\phi_n} = \text{GCI}_{\text{fine}} 21 = \frac{Fs \cdot e_{a}^{21}}{\gamma_{21}^{q} - 1}, \quad (1.10)
\]

where \( e_{a}^{21} \) is the approximate relative error for the fine grids, expressed as \( e_{a}^{21} = \left| \frac{\phi_1 - \phi_2}{\phi_1} \right| \) and \( Fs \) is the factor of safety. The recommended factor of safety [6] of \( Fs = 1.25 \) should represent the 95% uncertainty of the solution parameter. Variations to the approach discussed above have been considered, including Least Squares methods to approximating the GCI [29].

Standardized methods of computing time-discretization uncertainties have yet to be developed, although suggestions have been offered by Roache [26].
Model Input Uncertainty

A compilation of methods used to quantify uncertainty based on model inputs has been discussed and analyzed by Walters and Huyse [30]. These methods include interval analysis, Moment method or sensitivity derivatives, Monte Carlo estimates, and Polynomial Chaos and will be discussed within this section. Additionally, Martins et al. [31] discuss the efficiency of using the adjoint method to calculate the sensitivity derivatives over the interval analysis and Moment methods. For all methods, an output variable or solution parameter $\phi_m$ is considered to be a function of several model input variables (i.e. $\phi_m = f(\xi_1, \xi_2, ..., \xi_k)$). These inputs can range from thermodynamic properties, boundary and initial conditions, and model closure coefficients. These methods estimate the propagation of uncertainty from input errors on the output variable to obtain the model input uncertainty $U_{\phi_{in}}$.

The first of these methods, interval analysis, determines the interval of all possible outcomes based on the model’s mathematical operations on the intervals covering all possible input variables. While this may be the simplest method to implement, it will only provide maximum or worst-case uncertainty bounds on the output. This method also assumes the mathematical expressions within the model are equivalent for both point values and intervals. This is often not appropriate as pointed out by [30]. In addition, round-off error becomes significant to interval analysis when iterative loops are contained within the numerical method - as is often the case for CFD. Overall, while this method may provide a rough, computationally inexpensive estimate of model input uncertainty, lack of accuracy and robustness demands the use of other methods.

The strategy for calculating model input uncertainties using the Moment Method is derived from the first-order Taylor series expansions of the output variable with respect to the input variables. The output variable uncertainty $U_{\phi_{in}}$ is then a function of the sensitivity of the output variable to the model inputs ($\frac{\partial \phi_m}{\partial \xi_i}$) and input variable uncertainties $U_{\xi_i}$:

$$U_{\phi_{in}} = \sqrt{\sum_{i=1}^{k} \left( \frac{\partial \phi_m}{\partial \xi_i} \right)^2 U_{\xi_i}^2}.$$  (1.11)
This method is known by various names, with the primary difference being the confidence interval on the input and output uncertainties. While the Moment Method [30], implements a one standard deviation or 68% confidence interval, the sensitivity coefficient method [6] uses a 95% interval. It is advantageous to maintain matching confidence intervals for experimental and numerical uncertainties.

In keeping with the validation experiment guidelines proposed by [11], the input variable uncertainty $U_\xi$ should be obtained from the experimental measurements [6]. Calculating the sensitivity coefficients ($\frac{\partial \phi_m}{\partial \xi_i}$) may not be as straightforward as estimating the input variable uncertainties. While several methods to calculate sensitivity coefficients exist, the most realizable is through finite differences in parameter space [6]. This method does not require source code access, however, each sensitivity coefficient requires additional numerical solutions with the different corresponding input variables. Two numerical solutions with the corresponding input variable varied (i.e. $\xi_i + \Delta \xi_i$ and $\xi_i - \Delta \xi_i$) are required for a second-order finite difference equation for calculating the sensitivity coefficients:

$$\frac{\partial \phi_m}{\partial \xi_i} \approx \frac{\phi_m(\xi_1, \xi_2, ..., \xi_i + \Delta \xi_i, ..., \xi_k) - \phi_m(\xi_1, \xi_2, ..., \xi_i - \Delta \xi_i, ..., \xi_k)}{2\Delta \xi_i}. \quad (1.12)$$

Although the computational cost to calculate the model input uncertainty through sensitivity coefficients increases with the number of model inputs, it remains an easily implementable option.

For large quantities of model inputs, the adjoint method may be used more efficiently than the finite difference methods discussed earlier. The finite difference methods (interval analysis and Moment method) require the sensitivities to be calculated for each model input. However, as demonstrated by Martins et al. [31] the adjoint method is independent of the number of model inputs. The adjoint equation is recomputed for each design function, which is equal to the number of SRQ’s considered. Therefore, the adjoint method, for cases of large quantities of model inputs, is much more efficient than the finite-difference methods. When the number of SRQ’s increases, the adjoint method reduces it’s computational efficiency.
When computation time is not a concern, the Monte-Carlo method may be used to obtain extremely accurate solutions. To implement this method, the probability distribution functions (pdf) for all input variables must be estimated. Random input variables are “drawn” from each input variable pdf and used to obtain a model solution. This step is iterated to form an estimate of the model solution pdf. When the pdf for all the input variables are perfectly known and a sufficient number of simulations are calculated (often near 10,000), the Monte-Carlo method will provide exact distributions of the model input uncertainty [30]. From the model solution pdf the model input uncertainty within a specified confidence interval may be calculated.

Polynomial Chaos has been implemented for years in structural mechanics, however, is fairly new to fluid dynamics. This method estimates the model input uncertainty by transferring the output variable $\phi_m$ into stochastic space. The use of this method has been studied extensively by Mathelin et al. [32], Mathelin and Hussaini [33], Lucor et al. [34], and Knio and Le Maître [35].

1.2.3 Validation Uncertainty

Within a validation study, the experimental and numerical SRQ’s often contain correlated sources, such identical data reduction equations to obtain SRQ’s or experimentally measured boundary and inlet conditions with correlated error sources to the experimental SRQ. ASME [6] suggests accounting for these correlations with a validation uncertainty, $U_{\phi_V}$, which is a combination of the experimental, $U_{\phi_e}$, numerical, $U_{\phi_n}$, and input $U_{\phi_in}$ uncertainties. Numerical accuracy, and therefore numerical uncertainty, is affected by the numerical model and grid convergence and is independent of experimental uncertainty. The validation uncertainty can then be expressed as the numerical uncertainty $U_{\phi_n}$ and a combination of experimental and model input uncertainties $U_{\phi_{in-e}}$:

$$U_{\phi_V}^2 = U_{\phi_n}^2 + U_{\phi_{in-e}}^2. \quad (1.13)$$
The correlated affects are incorporated into $u_{\phi_{in-e}}$ by a sensitivity analysis of the uncertainty of the difference in the experiment and model:

$$U_{\phi_{in-e}}^2 = \sum_{i=1}^{k} \left[ \frac{\partial \phi_{in}}{\partial \xi_i} - \frac{\partial \phi_{e}}{\partial \xi_i} \right]^2 U_{\xi_i}^2,$$

where, once again, $\xi_i$ is a sensitivity variable and $U_{\xi_i}$ is the uncertainty in that variable.

The Monte-Carlo method may replace the sensitivity analysis in Eqn. 1.14 to calculate the validation uncertainty. When experimental and simulation SRQ's are uncorrelated, the validation uncertainty becomes the summation of the total uncertainties:

$$U_{\phi V}^2 = U_{\phi_n}^2 + U_{\phi_{in}}^2 + U_{\phi_e}^2.$$

1.3 Validation Quantification

A complete assessment of M&S validation should not only compare the measured or predicted SRQ but must also consider uncertainty within these SRQs. This often requires the use of statistical metrics. While graphical assessment allows subjective decision-making, more rigorous, statistical approaches, e.g. hypothesis testing and confidence intervals, provide objective validation criteria [2]. Oberkampf and Barone [16] recommend including several features within a validation metric; a metric should

1. include an estimate of numerical uncertainty in the SRQ, unless previously determined to be small,

2. be a quantitative evaluation of predictive accuracy of the SRQ,

3. include estimates of uncertainty from post-processing of experimental data,

4. include an estimate of experimental uncertainty in the SRQ,

5. be dependent on the number of experimental measurements acquired of the SRQ,

6. exclude indications of the level of adequacy between experimental and M&S results.
Formulating a validation metric that includes all of these suggestions may be difficult and even limiting; however, a combination of a few of these recommendations can generate objective and quantitative metrics.

Stern et al. [23] pioneered the direction of validation metrics by introducing metrics that incorporated all experimental and M&S uncertainty. Validation is achieved through comparison of the model comparison error (the difference between experimental and model SRQ’s, $\tilde{E} = \phi_m - \overline{\phi_e}$), uncorrelated validation uncertainty (Eqn. 1.15), and the programmatic validation requirement (uncertainty limit required by the application, $U_{\text{reqd}}$). Validation occurs at the programmatic level when $\tilde{E} \leq U_{\text{reqd}}$ and $U_{\phi_V} \leq U_{\text{reqd}}$ and at the validation uncertainty level when $\tilde{E} \leq U_{\phi_V}$ because the comparison error is within the validation uncertainty. This metric only provides a pass/fail criteria to assess validation.

The metric proposed by Stern et al. [23] is also suggested by ASME [6], with the inclusion of correlated validation uncertainty as calculated in Eqn. 1.14 instead of Eqn. 1.15. It is also proposed that an assumed distribution of all the errors may be used to determine an interval in which the model error $\delta_m$ is within the model comparison error $\tilde{E}$. A factor, $t_{C.I.,\nu}$, based on the confidence interval from the assumed error distribution can provide an uncertainty with confidence C.I. that contains the model error. This is expressed by the equation,

$$U_{\phi_V,C.I.} = t_{C.I.,\nu} U_{\phi_V}.$$  \hspace{1cm} (1.16)

At this time it is noted that Eqn. 1.16 is only appropriate if the uncertainties used to calculate the validation uncertainty have a 68% confidence interval or 1$\sigma$ distribution.

Oberkampf and Trucano [11] counter this validation metric approach for three reasons: 1) numerical error is treated as a statistical quantity, which it is not, 2) the validation uncertainty $U_{\phi_V}$ does not account for modeling assumptions, and 3) the ability to achieve validation at the validation uncertainty level increases as experimental and M&S uncertainties increase. However, they propose a validation metric $V$ that quantifies the level of agreement between experiment and simulation ranging from perfect agreement ($V = 1$) to
no agreement $V = 0$. This validation metric for one SRQ is expressed as,

$$V = 1 - \tanh \left( \frac{|\phi_m - \phi_e|}{\phi_e} \right) + \int_{-\infty}^{+\infty} \frac{s_{\phi_e}}{\sqrt{N}} \left| \frac{z}{\phi_e} \right| f(z) \, dz,$$

(1.17)

where $\phi_m$ is the model SRQ and $\phi_e$ is the mean experimental SRQ, $n$ is the number of measurements acquired, $s_{\phi_e}$ is the standard deviation of the experimental SRQ, and $f(z)$ is the probability density function for the student’s t-distribution. When one measurement is made, the second term (the integral of $f(z)$) is zero. As mentioned previously, this validation metric yields continuous values from no agreement to perfect agreement or $[0, 1]$.

This metric may be extended as a continuous function along the line from $[0, L]$; this requires sufficient experimental data be acquired along the line to obtain an adequate interpolation, $\phi_e(x)$. This gives a single metric to quantify the agreement along the entire line:

$$V = 1 - \frac{1}{L} \int_0^L \tanh \left( \frac{\phi_m(x) - \phi_e(x)}{\phi_e(x)} \right) + \int_{-\infty}^{+\infty} \frac{s_{\phi_e}(x)}{\sqrt{N}} \left| \frac{z}{\phi_e(x)} \right| f(z) \, dz \, dx.$$

(1.18)

While this metric provides a quantifiable level of validation as opposed to the binary (i.e. pass/fail) criteria proposed by Stern et al. [23], it also comes with several limitations. This method generates a continuous function of validation values from $[0, 1]$; however, the significance of the values between total agreement and no agreement are left to the users judgment, allowing subjective opinions into the validation assessment. Oberkampf and Trucano [11] also assume the model does not exhibit chaotic phenomena and numerical uncertainties are negligible. Additional research could consider including numerical uncertainty into Equations 1.17 and 1.19.

In a later paper, Oberkampf and Barone [16] conclude that while the above-mentioned metric combines features 2, 3, and 4 into one quantity, this may not be the best approach in a validation metric. An additional metric is proposed in which statistical confidence intervals from the experimental measurements are used to compare experimental and numerical
uncertainty in a quantitative manner. This metric can be applied to a single SRQ or a set of SRQ’s as a function of the input variable \( x \). The single SRQ metric is a simplification of the functional SRQ metric and only the functional SRQ validation metric will be demonstrated:

\[
\tilde{E}(x) - t_{C.I.,\nu} \frac{s_{\phi_e}(x)}{\sqrt{N}} \leq E(x) \leq \tilde{E}(x) + t_{C.I.,\nu} \frac{s_{\phi_e}(x)}{\sqrt{N}},
\]

where \( E \) is the true error of the numerical model and \( \tilde{E} \) is the estimated error calculated by the difference in model and experimental SRQ’s (i.e. \( \tilde{E}(x) = \phi_m(x) - \phi_e \)).

This metric uses the experimental uncertainty as the exclusive method of uncertainty quantification. Oberkampf and Barone \[16\] note this metric is designed for deterministic computational results in which the numerical uncertainty is small. While this may not be adequate for most validation studies, particularly high-risk systems, this formulation has potential to be extended to combine experimental and numerical uncertainties. This metric has similar defects as that observed in the metric proposed by Stern \et al. \[23\], in which, as the experimental uncertainty increases, the ability to validate a particular numerical model increases. This can be countered by defining an appropriate accuracy requirement and acquiring a sufficient amount of experimental data to drive the uncertainty levels within this limit.

The use of accuracy limits within a validation metric have been investigated by Rebba and Mahadevan \[36\] using Bayesian hypothesis testing. They extend beyond the traditional Bayesian approaches of \( p \)-value and interval-based hypothesis testing to define a validation metric which indicates the model reliability. If the SRQ follows a normal distribution the model reliability may be calculated as,

\[
R = \Phi \left[ \frac{\sqrt{N} (\alpha_{\text{reqd}} - |\phi_e - \phi_m|)}{s_{\phi_e}} \right] - \Phi \left[ \frac{\sqrt{N} (-\alpha_{\text{reqd}} - |\phi_e - \phi_m|)}{s_{\phi_e}} \right],
\]

with \( \Phi \) denoting the normal cumulative distribution function and \( \alpha_{\text{reqd}} \) is the predefined accuracy limit. The validation metric may be thought of as the probability that the difference
\( \phi_m - \overline{\phi}_e \) is within the accuracy limit \( \alpha \) and ranges from 0 \( \leq r \leq 1 \). The model prediction of the SRQ is accepted only if \( R \geq c_{\text{reqd}} \) where \( c_{\text{reqd}} \) is a predetermined confidence requirement. Once again, this metric provides a quantitative metric to assess the accuracy of the model but an adequate confidence requirement must first be chosen to determine validation. Rebba and Mahadevan \cite{36} state that numerical uncertainties can be treated as uncertain quantities; however, the procedure for accounting for numerical uncertainty and experimental systematic uncertainties is not outlined. This metric may also be extended into multivariate input variables.

Validation metrics quantifying magnitude and phase components of unsteady and transient simulations have also been proposed by Geers \cite{37} and Sprague and Geers \cite{38}. The magnitude \( M \), phase \( P \), and comprehensive \( C \) error factors are used to quantify the respective transient components of model and experimental time-dependent SRQ’s:

\[
M = \sqrt{\frac{\Upsilon_{m,m}}{\Upsilon_{e,e}}} - 1, \\
P = \frac{1}{\pi} \cos^{-1} \left( \frac{\Upsilon_{m,e}}{\sqrt{\Upsilon_{m,m} \Upsilon_{e,e}}} \right), \text{ and} \\
C = \sqrt{M^2 + P^2}
\]  

where

\[
\Upsilon_{m,m} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \phi_m^2(t) \, dt, \\
\Upsilon_{e,e} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \phi_e^2(t) \, dt, \text{ and} \\
\Upsilon_{m,e} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \phi_m(t) \phi_e(t) \, dt.
\]  

In the previous equations, \( t_2 \) and \( t_1 \) define the time interval of interest for the given SRQ. Aspects of these metrics make them ideal for transient validation; \( M \) is independent of phase response, while \( P \) is unaffected by SRQ magnitude. These metrics are also unaffected by
uncertainty in polarity and the magnitude error factor is unaffected by uncertainty in the
time variable; this is not the case for the phase error factor \[37\]. Although the polarity
and time uncertainty may not influence the metrics, other modeling and experimental un-
certainty sources are not accounted for in the metric calculation. Geer \[37\] also uses the
average of \(M\), \(P\), and \(C\) to obtain a validation metric capable of describing the global
temporal response of the model.

The metrics introduced by Geer \[37\] and Sprague and Geer \[38\] provide excellent mea-
sures of the transient capabilities of a numerical model. Although these metrics cannot be
directly applied to random, unsteady simulations (e.g. vortex shedding and flow-induced
vibrations), they may be extended to these circumstances in a phase-averaged sense. Cross-
correlations of experimental and model SRQ’s may also be used to quantify their phase
relation and was originally introduced by Geer \[37\]. Additional methods may also be
derived to develop validation metrics for frequency-response SRQ’s. A demonstration of
transient-response validation metrics is provided by Schwer \[39\].

Of the metrics proposed, incomplete quantification of numerical and experimental un-
certainty seems to be common among all metrics. Improvements on these metrics require
accounting for numerical uncertainty and experimental systematic uncertainties. Of course,
when numerical uncertainty is shown to be small these metrics may provide ideal quantities
to compare numerical and experimental validation data. For all the metrics proposed, the
level of accuracy required must be specified to obtain numerical validation. Appropriate
validation metrics will vary with each study and consideration of application requirements
must be assessed when developing metrics for a validation study.

1.4 Validation of CFD

Applications of CFD M&S in industry include atmospheric climate predictions, devel-
opment of biomedical devices, tidal and ocean energy collection, vehicle drag reduction, heat
transfer within nuclear and fossil-fueled power plants, and turbo-machinery. It is reasonable
to predict CFD will continue to establish itself as a primary engineering tool in the future.
However, as discussed earlier, V&V of CFD M&S is essential to its continued development.
While many of the applications of CFD may be low-risk and require moderate validation, rigorous validation of CFD’s predictive capabilities for local metrics must be assessed for higher risk situations. Universal validation of these quantities is unattainable and often must be quantified at the level of the specific application.

This motivates, not only the acquisition of experimental data for CFD simulations, but also the development of requirements for validation-level experiments. The intent of this dissertation is to acquire validation-level experimental measurements meeting the NE-KAMS validation database and previously mentioned validation experiment requirements and, simultaneously, demonstrate correct procedures for validation-level data acquisition. The development and demonstration of validation SRQ’s is also be presented. The validation experiment consists of unsteady flow through a bank of cylinders in which unsteady validation SRQ’s will be demonstrated and recommended. Demonstrations of PIV error sources on mean and fluctuating components will also be conducted and uncertainty estimates for the previously described validation measurements and simulations are quantified.

Although much attention has been given to the time-averaged validation of CFD simulations, failure and accident scenarios often involve large transients and unsteady flow, thus requiring temporal validation. Transients and unsteady phenomenon are easily modeled within CFD; however, the current practice of validating these simulations is achieved through time-averaged measurements, instantaneous comparisons, and results at significant discrete times, such as the final steady state. To the author’s knowledge, little consideration has been directed toward the validation of the processes involved in unsteady simulations.

While the meaning of validation in the time-averaged sense is not difficult to grasp, one must consider what is to be expected from an acceptable unsteady simulation. Unsteady, instantaneous, turbulent velocity measurements may be thought of as three velocity components, \( u(t) = \bar{u} + u^*(t) + u'(t) \), where \( \bar{u} \) is the time-average velocity, \( u^* \) is a non-turbulent variation (e.g. vortex shedding) which often varies “slowly” in time, and \( u' \) is a turbulent fluctuation. The time-varying components (i.e. \( u^* \) and \( u' \)) are composed of a spectra of frequencies (or time-scales) and amplitudes. The manner in which these time-varying com-
ponents are modeled differs between each CFD model and affects the ability of each model to accurately calculate unsteady SRQ’s.

Unsteady Reynolds Averaged Navier Stokes (URANS) simulations do not calculate turbulent fluctuations and consist solely of $\pi + u^*(t)$. Any variation in time is, by definition, due to factors other than turbulence. Under the RANS assumptions, the time-scale of predicted non-turbulent variations $u^*(t)$ should be larger than the averaged-out turbulent fluctuations $u'(t)$ \[40\]. Therefore, URANS calculates the non-turbulent time-variations of any flow quantity and the total variations should be expected to be smaller in amplitude than the experimental variations. This should be more evident at smaller time-scales as turbulent fluctuations become more dominant. For flows purely dominated by turbulent fluctuations, URANS models are insufficient for temporal validation of turbulent fluctuations.

Large Eddy Simulations (LES) compute the large-scale turbulent motions through the Filtered Navier Stokes Equations while modeling the smaller scales. Therefore, while large-scale variations should agree experimentally, attenuation of variations among intermediate- and small-scales should be observed. This is due to the filtering of smaller scales from the Filtered Navier Stokes Equations. Detached Eddy Simulation (DES) blends solutions from URANS simulations in the boundary layer and LES outside of these regions and, consequently, modeling of large and small time-scale variations will be affected by the LES filter in the core of the flow and URANS exclusion of turbulent fluctuations in the boundary layer.

So, while it is not realistic to expect all simulations to predict time-variations of similar amplitude to an experiment, specifically among the turbulent fluctuations, one could hope to obtain accurate information on the non-turbulent variational behavior of an experiment. This is particularly applicable to systems dominated by non-turbulent variations (such as, vortex shedding and bistable regimes), in which it can be expected that URANS, LES, and DES models predict the large-scale, non-turbulent, amplitudes and frequencies. In nuclear applications, such frequencies may be important examining fluid-structure interactions, re-suspension of dust, and time-varying pressure loads during accidents.
While local and global, spatial SRQ’s are often used in the validation of unsteady numerical models, validation of temporal SRQ’s is often not addressed. This may be for many reasons: temporal data from experimental and numerical models are difficult to compare, experimental data are often time-averaged measurements or acquired at time-scales too large to resolve temporal features, or the scale of the unsteady fluctuations may be determined to be too small \([41]\). However, transient and unsteady applications often contain temporal features that may be used to assess numerical accuracy (\emph{e.g.} frequency, amplitude, and phase of vortex shedding) and time-scales (\emph{e.g.} autocorrelation coefficient \([42]\)).

One example of a previous unsteady validation is the thermal mixing temperature oscillations in a T-junction. Temperature fluctuations were measured and calculated using CFD by Westin \textit{et al.} \([43]\). The unsteady numerical and experimental results were compared at various locations in the T-junction by plotting these oscillations through time. However, no attempt was made to assess the model’s ability to correctly predict the dominant frequencies present in the flow. Estimation of the Taylor micro scale was estimated from the autocorrelation of the LES calculation, but was not compared to the experimental work. To the authors’ knowledge, the current state of unsteady validation remains at this level of fidelity and does not extend to essential unsteady SRQ’s.

Extensive experimental and numerical work is available for representative global and local, spatial and temporal flow quantities within arrays of cylinders. Experimental measurements and numerical simulations of unsteady pressure quantities within a bank of cylinders are introduced by Mahon and Meskell \([44,45]\). Global, spatial quantities (\emph{e.g.} drag and lift coefficients) and local, spatial and temporal quantities (\emph{e.g.} surface pressure distribution and histograms of pressure time traces) are demonstrated. Bistable pressure distribution modes along the cylinder are observed by Mahon and Meskell \([44,45]\) and these will be discussed later.

Local, temporal quantities (Strouhal number) were experimentally investigated by Polak and Weaver \([46]\) and numerically simulated by Hassan \textit{et al.} \([47]\) for several pitch ratios and Reynolds numbers. The Strouhal number \(St\) is given by,
\[ St = \frac{u_{\text{max}}}{D}. \] 

(1.23)

and the Reynolds number is given by

\[ \text{Re}_D = \frac{u_{\text{max}} D}{\nu_f}. \] 

(1.24)

where \( D \) is the cylinder diameter and length scale, \( u_{\text{max}} \) denotes the time-averaged bulk velocity at the minimum area, and \( \nu_f \) is the kinematic viscosity of the fluid. Scholten and Murray [48] also investigate the frequency and phase spectra and coherence of frequency signals from LDA measurements for three cylinders within a tube bundle. The majority of experimental and numerical work consists of a tube bundle facility with several flow paths.

1.5 Application of PIV Uncertainty to Validation

Engineering, research, and design increasingly relies on numerical modeling and simulation (M&S), particularly computational fluid dynamics (CFD), as a tool to improve design efficiency and safety. One obstacle to implementing M&S for the advancement of engineering is validation. The CFD Best Practice Guidelines [49] from the NEA states that validation of nuclear M&S must be attained by comparison to experimental data; alternative guidelines are suggested by Lee and Bauer [10] in which CFD should be validated with validation-level experiments and datasets. These guidelines state that validation-level datasets should be acquired using non-intrusive measurement techniques and provide a complete uncertainty analysis of the validation metrics.

Particle Image Velocimetry (PIV) is a powerful, full-field, non-intrusive velocity measurement technique for single phase and some two-phase flows. While other non-intrusive techniques exist (such as Laser Doppler Anemometry), no other provides the full velocity field at an instant in time.

A common, but incorrect, practice for quantifying PIV uncertainty is to estimate a maximum uncertainty for the flow field and reference this value as the measurement uncertainty to the entire field. This will inevitably overestimate the uncertainty in most of the
flow field, rendering the data unsuitable for CFD validation. Two efforts have been made recently to calculate these uncertainties locally. Timmins et al. [50, 51] have studied the effects of particle image density, diameter, displacement, and flow shear and rotation and the correlation between these sources to determine the PIV uncertainty for each vector (in space and time) in a measurement set. They reported that these four parameters could each generate bias and fluctuations in the measurement, similar to the results of Raffel et al. [52]. A broader approach with less emphasis on uncertainty from the PIV algorithm was recently presented by Lazar et al. [53] but found uncertainty values that were much smaller.

Simplified analytical expressions for PIV error effects on PIV measurements are also derived by Westerweel [54], Westerweel [55], and Adrian and Westerweel [56]. While useful in determining an estimate of the magnitudes of individual error sources, measurement uncertainty from PIV error sources are strongly dependent on the PIV algorithm implemented. An algorithm specific uncertainty quantification, such as that proposed by Timmins et al. [50, 51], is necessary to provide adequate confidence in most CFD validation scenarios.

Most flows of interest contain regions with variable levels of uncertainty throughout the measurement field. M&S validation requires accurate assessments of the localized uncertainty. For example, consider a jet exit. This flow contains regions of high shear at the edges of the jet. These generate large amounts of “noise” in the PIV measurement [51, 52], while smaller uncertainties are located within the core region of the jet where low shear and nearly uniform profiles are observed. While the effect of this noise on the time-averaged velocity can be eliminated by increasing the number of samples acquired, its impact on turbulent statistics, which are also important to CFD validation, cannot. Under this and similar situations, a global uncertainty would give an unrealistic measure of the full measurement field.

The application of PIV to M&S validation requires accurate measurement of velocity statistics. These are composed of two components: time-averaged velocity and fluctuations or variance. While uncertainties on the mean due to velocity fluctuations and PIV noise may be reduced by acquiring increased quantities of data, measurements of the fluctuation
levels will always increase with the introduction of measurement error. This phenomena will be discussed later, but this fluctuation error must be quantified prior to using PIV data for validation purposes. In many situations, PIV data is used to provide inlet conditions to CFD models or global and local quantities. The data acquired ranges from mean velocities to turbulence statistics. Turbulence statistics are measured by the fluctuations within the measurement and, as such, are susceptible to elevated fluctuation levels due to measurement error. It is impossible to validate a CFD simulation that is expected to predict these elevated fluctuation or turbulence levels.

Measured velocity statistics (means and fluctuations) are affected differently by errors on the measured data, where an error has both direction and magnitude. The effects of systematic and random measurement errors on velocity statistics are demonstrated mathematically in Appendix A and follow the methods of uncertainty quantification given in Section 1.2. In these previous sections, uncertainty calculations are shown for a generalized measurement quantity \( \phi_e \); herein, the measurement quantity of interest, velocity, \( u \), will be exchanged with \( \phi_e \).

Systematic and random errors on the instantaneous velocity measurement are estimated by systematic \( b_{ue} \) and random \( r_{ue} \) uncertainties. The uncertainty on the instantaneous velocity is estimated as in Appendix A:

\[
U_{ue} = \sqrt{b_{ue}^2 + r_{ue}^2}.
\] (1.25)

Both instantaneous systematic and random uncertainties affect the mean statistics (demonstrated in Appendix A) and quantification of these errors are well known. The uncertainty on the time-averaged velocity is given in Eqn. A.9:

\[
U_{\bar{u}e} = \sqrt{\sum_{i=1}^{k} b_{\bar{u}e,i}^2 + \left( t_{C.I.\nu} \frac{s_{\bar{u}e}}{\sqrt{N}} \right)^2}.
\] (1.26)

where \( b_{\bar{u}e,i} \) are the bias uncertainties on the time-averaged velocity and the second term is the precision uncertainty \( p_{\bar{u}e} \). It can quickly be seen that the influence of random uncertainties
on the time-averaged velocity will decrease by increasing the number of samples $N$ at a rate of $1/\sqrt{N}$.

As demonstrated in Appendix A, this is not the case for the time-averaged measured fluctuations $\hat{u}_e \hat{u}_e$: the time-averaged measured fluctuations will always be elevated due to random measurement errors. The estimate of random measurement error effects on time-averaged measured fluctuations is calculated by converting Eqn. 1.5:

\[
U_+^{\hat{u}_e \hat{u}_e} = \frac{\bar{u}_e \hat{u}_e}{(N - 1)}, \quad \text{and}
\]

\[
U_-^{\hat{u}_e \hat{u}_e} = \sqrt{\left(\frac{\bar{u}_e \hat{u}_e}{(N - 1)}\right)^2 + \frac{1}{N} \sum_{i=1}^{N} (r_{u_e,i})^4}.
\]

Similar expansions can be obtained for time-averaged velocity components in the $x$- and $y$-directions. When converting velocity components to velocity magnitude, the uncertainty on an instantaneous magnitude measurement is calculated as a combination of uncertainties on the instantaneous directional components:

\[
U_{\hat{u}_e \hat{u}_e} \approx U_{\hat{u}_x \hat{u}_x} + U_{\hat{u}_y \hat{u}_y} + 2 \left(U_{\hat{u}_x \hat{u}_x} U_{\hat{u}_y \hat{u}_y}\right)^{1/2},
\]

where $U_{\hat{u}_x \hat{u}_x}$ and $U_{\hat{u}_y \hat{u}_y}$ are the uncertainties on the velocity fluctuations in the $x$ and $y$ directions.

Herein, time-averaged measured fluctuations will be referred to as either measured fluctuations unless otherwise specified.
Chapter 2

Purpose and Objectives

The purpose of this dissertation is to perform a high-fidelity validation study composed of both experimental and numerical research that meets the validation experiment requirements proposed by Sandia National Laboratory [11] and Lee and Bauer [10]. These requirements include complete experimental model descriptions including boundary, initial, and inflow conditions, the use of non-invasive, high precision instrumentation, complete quantification of both experimental and numerical uncertainties. The purpose of the validation study is to assess the ability of CFD turbulence models to accurately simulate temporal quantities within a bank of cylinders. The facility for this unsteady bank of cylinders validation experiment is given in 3.2.

Additionally, to adequately quantify the uncertainty within the PIV measurements for this validation experiment, the unsteady bank of cylinders experiment is supplemented by a PIV uncertainty study. This study demonstrates several of the dominant PIV error sources observed within the bank of cylinders experiment and quantifies the uncertainty on both the time-averaged velocity and turbulent fluctuations.

Although two distinct projects are presented (i.e. 1) validation of flow through a bank of cylinders and 2) demonstration of PIV error sources on time-averaged velocity and turbulent intensity), both projects are required to fulfill the purposes of this dissertation. Both sections will ensure a complete validation study satisfying the requirements in Section 1.1.1.

Seeing as the projects are different, differing objectives are required from each section. The objectives supporting the achievement of this dissertation (validation study of unsteady flow through a bank of cylinders) are presented in two sections:
2.1 Effects of PIV Error Sources on Time-Averaged Velocity and Turbulent Fluctuations

By assessing these PIV error sources it is proposed to accomplish the following objectives:

- Acquire high-fidelity PIV measurements in which particular error sources are emphasized: particle image displacement, particle image size, particle image density, and gradients within the flow.
- Demonstrate the effect of error fluctuations to overestimate turbulent fluctuations.
- Estimate uncertainty on the mean and fluctuating components of velocity for each error source.
- Apply uncertainty analysis to PIV data from validation experiments.

2.2 Validation of Flow through a Bank of Cylinders

The objectives for the cylinder array validation study include,

- Measure the velocity field at the mid-plane of the facility at locations up- and downstream of each cylinder using time-resolved PIV.
- Acquire point measurements of pressure at the wall for all full cylinders and synchronized with the PIV measurements.
- Simulate the experimental model using unsteady CFD turbulence models (i.e. URANS \(k-\omega\) model and Detached Eddy Simulation (DES) model).
- Develop and analyze global and local, spatial and temporal validation SRQ’s for unsteady validation, such as dominant frequencies, correlations, velocity distributions, and loss factors.
- Identify advantages and defects of unsteady validation SRQ’s and appropriate situations in which they may apply,
• Quantify uncertainty on experimental and numerical results and SRQ’s using results from the PIV error sources research.
Chapter 3

Experimental Facilities and Numerical Models

The intentions of this work will be accomplished within two separate studies. These are contained within Sections 3.1 and 3.2. Within these sections, the proposed plan, including measurement acquisition and numerical modeling, and facilities necessary to accomplish the objectives listed in Chapter 2 are outlined.

3.1 Effects of PIV Error Sources on Time-Averaged Velocity and Turbulent Intensity

While the effects of error sources on PIV results have been assessed using synthetic images, little data exists from using real PIV images. As discussed previously, PIV error sources will generate random fluctuating errors around the mean and bias errors from the mean. The fluctuating errors also contribute to the variance or turbulence intensity quantities producing overestimated turbulence measurements by PIV. The proposed PIV error sources assessed for this dissertation include particle image displacement, particle image size, particle image density, and gradients within the flow.

3.1.1 Experimental Facility and Plan

Demonstration of these error sources is achieved using a high Reynolds number, large shear, laminar rectangular jet (Fig. 3.1). The facilities consist of 4 components: 1) blower, 2) diffuser and flow conditioning, 3) contraction, and 4) jet exit/measurement region. The contraction of the jet provides a high Reynolds number laminar flow with regions of high shear at the profile edges and uniform flow at the profile center. These features are critical for demonstrating the individual effects of the PIV error sources by maintaining the turbulent fluctuations at smaller magnitudes than the error sources.
The jet half-height was $h_{1/2} = 4.76$ mm and the Reynolds number for the rectangular jet are expressed as in Eqn. 1.24 with the length scale being the jet half height $h_{1/2}$ and the maximum time-averaged or centerline velocity $\bar{u}_{y=0}$ used as the velocity scale:

$$\text{Re}_{h_{1/2}} = \frac{\bar{u}_{y=0}h_{1/2}}{\nu_f},$$

(3.1) corresponding to $\text{Re} = 2.57 \times 10^3$. Both the jet and lab environment are at similar atmospheric conditions. The mean atmospheric air properties at the Experimental Fluid Dynamics Lab (EFDL) based on local barometric pressure, humidity, and temperature are $\rho_f = 1.0048 \pm 0.00325$ kg/m$^3$ and $\mu_f = 1.88 \times 10^{-5} \pm 1.94 \times 10^{-8}$ kg/m-s. Atmospheric quantities are monitored during acquisition of the data to ensure similar conditions for both hot wire and PIV measurements.

The blower draws air from the lab environment and into the diffuser section of the experimental model. At this point the fluid enters a flow control region made from foam to dampen any oscillations and smooth the flow. A two-piece contraction follows, in which the downstream end forms the jet exit. The half-height at the exit is $h_{1/2} = 4.76$ mm and has a depth of $8h_{1/2}$. The unique contraction is designed to provide a nearly uniform laminar profile at the jet exit for Reynolds numbers of up to $\text{Re} = 60,000$ with approximately 70% of the jet profile within 0.5% of the maximum time-averaged velocity.
Fig. 3.2: The time-averaged velocity and turbulence profiles of the laminar jet. The jet exhibits regions of high shear at the jet edges and uniform laminar flow at the jet core. Non-uniformities between jet edges have negligible effect on the PIV uncertainties within this study.

To exploit the random measurement errors, $s_{\text{true}}^2$, it is necessary to minimize the “true” fluctuations, $\overline{u^3u'}$, within the jet. Laminar flow within the jet core is indicated by turbulent fluctuations $(\overline{u'u'})^{1/2}/\overline{u} = 0$ within the jet core below 3%. The jet profile at the exit (shown in Fig. 3.2 and measured using a hot wire) exhibits laminar flow with regions of high shear at the jet edges and uniform flow in the jet core. One side of the jet exit terminates upstream of the other, leading to asymmetric fluctuation profiles at the jet edges. Noting that the PIV error sources are the interest of this paper and not the jet profile, this asymmetry has no impact on the present study.
Velocity profiles at the jet exit were measured using a single wire hot wire and particle image velocimetry (PIV). Although acquired separately, both hot wire and PIV measurements were made in the same plane at the center of the jet. Hot wire measurements were acquired using Labview 10, a single, coated wire probe, and a TSI IFA 100 Hot Wire Anemometer. A response of $12\mu s$ was observed in the hot wire signal. The hot wire was attached to a stepper-motor-operated traverse (accurate within $\pm 5\mu m$) and was used to position the hot wire at discrete locations along the jet profile. At each location, $N = 262144$ measurements were obtained and a sampling rate of $f = 10,000$ Hz. Jet profile measurements were acquired at two locations downstream of the exit: $x = 0.1h_{1/2}$ and $1.25h_{1/2}$. In this manner, two accurate and resolved profiles of the flow were measured with different magnitudes of shear.

PIV measurements were acquired in the same plane as the hot wire profile measurements using a 12-bit, $1376 \times 1040$ pixel Imager Intense CCD camera and a New Wave dual-cavity 50 mJ/pulse Nd:YAG laser. This system was controlled with DaVis 7.2 from LaVision and seeded using oil droplets uniformly distributed at the blower inlet. To obtain converged fluctuation statistics, $N = 1000$ images were acquired at an average of $f = 3$ Hz. The spatial resolution for these images were $0.0108 \text{mm/pixel}$. 
PIV images were processed using two PIV algorithms: DaVis 7.2 from LaVision \cite{57} and PRANA from Virginia Tech \cite{58,59}. Images processed using the DaVis algorithm were processed with square interrogations regions with an initial window size of 32×32 and with two consecutive passes at 16×16 using a 2D standard cross-correlation (SCC) algorithm. All interrogation regions were overlapped by 50%. Results were then post-processed using four parameters \cite{57}: an allowable pixel range (vectors displacing more than 15 pixels are rejected), correlation peak ratio, neighboring vectors median filter, and small groups (spurious vectors in groups smaller than 5 vectors are thrown out). Image processing from the PRANA algorithm used the same processing and post-processing settings used in \cite{51} using the robust phase correlation (RPC) method \cite{58,59}. Both DaVis and PRANA results are obtained for interrogation regions of $h_{IA} = 16$ pixels across.

Given the accuracy of the slider stages, images of the hot wire at multiple known locations were acquired and used to calibrate the PIV images. The hot-wire location was accurately known from the hot wire sensor images. Profiles from the PIV data were extracted at the hot wire profile locations (as measured from the hot wire images) for direct comparison to the hot-wire results.

### 3.1.2 Comparison of PIV and Hot Wire Results

A single wire hot wire is sensitive to both the instantaneous, stream-wise $u_x$ and span-wise $u_y$ velocity components, and, accordingly, if equally sensitive to both directions, the hot wire output is simply the magnitude of the instantaneous velocity in the $x$ and $y$ directions. Using a TSI hot wire calibrator, the sensitivity to both instantaneous velocity components was demonstrated to be within a relative difference of 5% (see Appendix B). However, PIV data measures both instantaneous velocity components individually and a method of comparing magnitudes of velocity means and fluctuations is needed. While this is simple for the time-averaged velocity magnitude ($u^2 = u_x^2 + u_y^2$), the relation to combine the fluctuations is not as straight forward. The relationship between the two component fluctuations and the fluctuation magnitude is shown in Appendix B to be:
\[
\overline{u'u'} = \left[ \overline{u'_x u'_x} + \overline{u'_y u'_y} \right],
\]

(3.2)

where \( \overline{u'_x u'_x} \) and \( \overline{u'_y u'_y} \) are the velocity fluctuations in the \( x \) and \( y \) directions and the fluctuations in the velocity magnitude are denoted by \( \overline{u'u'} \). The hotwire probe sensitivity in the stream-wise and span-wise velocity components was accurate within 2.5\%. The hotwire uncertainty is much smaller than the turbulent fluctuations and is therefore denoted by the “true”, turbulent fluctuations \( \overline{u'u'} \) as opposed to \( \overline{u'u'} \).

PIV results are given in dimensions of pixels for pixel displacement (velocity) and spatial coordinates and pixels/pixel for gradients to remain consistent with [51]. All hotwire velocities are also converted to pixels (using the PIV spatial calibration constant and time interval between images, \( \Delta t \)) to allow for direct comparisons. Hotwire measurements were accurate to within \( \pm 5\% \).

### 3.1.3 PIV Uncertainty Estimation

Timmins et al. [51] discuss several factors causing PIV uncertainty, including particle image density, diameter, and displacement, and flow gradients. Local, instantaneous uncertainties due to these sources were determined using synthetic PIV images. From these synthetic images, a 4-D uncertainty surface of the correlated errors from particle image density, diameter, and displacement, and flow gradients is generated. It is essential to note that accurate results from this 4-D uncertainty map applies only to the specific algorithm used by Timmins et al. [51].

Using robust methods of measuring particle image density, diameter, and density and flow gradients directly from the PIV images and results, local, instantaneous uncertainties are interpolated from the 4D uncertainty surface. The methods used to quantify the range of local error sources will be discussed later in the paper.

Each of the error sources mentioned above generate an elevated magnitude of total time-averaged measured fluctuations \( \overline{u'u'} \) above the “true”, turbulent fluctuations \( \overline{u'u'} \). This effect is particularly apparent when the true fluctuations are small (due to the total mea-
sured fluctuations being the root sum of the actual fluctuations and the PIV measurement uncertainty). Reference to the “true” turbulent fluctuations are measured from the hot wire measurements for all cases. As shown in Appendix A the time-averaged measured fluctuations will always over-predict the true fluctuations and therefore be one-sided and negative. An estimate of the “true” fluctuations can be obtained through the difference of the measured fluctuations and total random measured fluctuation uncertainty:

\[
\overline{u'w'} \approx \overline{\hat{u}} - \overline{U_{ue}}. \tag{3.3}
\]

A time-averaged fluctuation uncertainty estimate \(U_{ue}^{\pm}\) of the time-averaged measured fluctuation error \(s_{ue}^2\) is automatically calculated using the methods shown in Appendix A with the 95% instantaneous random uncertainties \(r_{ue}\) from Timmins et al. [50, 51].

Uncertainties on the total time-averaged measured fluctuations are calculated from instantaneous random measurement errors \(r_{ue}\) estimated by Timmins et al. [50, 51] for the PRANA algorithm (to which they are specific) and DaVis algorithm. No uncertainty information on the four PIV uncertainty contributors studied currently exists for the DaVis algorithm. While the uncertainties from Timmins et al. [50, 51] are applied to both algorithms, the uncertainties do not directly apply to the DaVis algorithm and this dissertation will demonstrate the inadequacies using algorithm-general uncertainty estimates.

3.2 Validation of Unsteady Flow Through a Bank of Cylinders

3.2.1 Experimental Facility and Plan

While the study of temporal quantities within banks of cylinders is not new, a compilation of unsteady validation SRQ’s, along with a demonstration of their advantages and applications, is not readily available. The intent of this study is not to demonstrate new developments in fundamental experimental research and numerical models, but to demonstrate a high-fidelity temporal validation-level study for particular CFD turbulence models. This work has four purposes:
1. demonstrate specific unsteady, spatial and temporal SRQ’s that may be used for unsteady validation,

2. identify advantages and defects of unsteady validation SRQ’s and appropriate situations in which they may apply,

3. present the ability of particular CFD turbulence models to adequately predict unsteady experimental validation SRQ’s, and

4. provide validation-level experimental data for the use of spatial and temporal validation studies.

As this work is not meant to provide new fundamental knowledge concerning flow through bundles of cylinders, agreement with previous work will be referenced and discussed but not directly compared. Keeping consistency with the requirements of validation studies, this validation will be accomplished through comparison of the experimental measurements and numerical simulations acquired during this study.

The facility for the unsteady bank of cylinders experiment (used previously for steady validation experiments [60]) consists of three sections shown in Fig. 3.4: inlet contraction, test section, and an outlet which connects to the downstream blower. The inlet has a 1.4:1 contraction in the span-wise and a 3:1 contraction in the cross-stream directions. A single bank of 112 wires/cm screen located within the contraction is used to minimize non-uniformities in the mean and fluctuating velocity profiles at the test section inlet. Flow leaving the test section is drawn through two perforated plates oriented perpendicular to the flow and inside the outlet. The perforated plates suppress separation and pulsations generated by the blower. The frequency-controlled centrifugal blower is located downstream of the outlet, connected to the outlet by a flexible duct, and draws atmospheric air through the facility. Blower speeds are corrected for atmospheric variations between datasets to maintain the target Reynolds number (Re = 40,000).

The test section is designed to resemble the lower plenum of a very high temperature reactor (VHTR) as shown in the schematic of Fig. 3.4. The test section is an \( L = 89.7 \text{ cm} \).
channel in the stream-wise \((x)\) direction, \(w = 8.53\) cm in the cross-stream \((y)\) direction, and \(H = 34.8\) cm in span-wise \((z)\) direction. The channel contains an array of cylinders and half-cylinders \((D = 5.03\) cm\) which mimic an infinite array of cylinders arranged on equilateral triangles. Five center cylinders are placed on the cross-stream centerline with a stream-wise distance of 14.8 cm between cylinders. Cylinder 4 is made from polished polycarbonate tube, providing measurement access to all sides of the cylinder. Four sets of half-cylinders are placed directly between the centerline cylinders on the cross-stream or channel edges. While this model does not exactly duplicate actual reactor geometry, it will generate the relevant physics, which is the aim of a validation study.

Velocity measurements were obtained using particle image velocimetry (PIV) consisting of low and high speed systems from LaVision. The low speed system consisted of a 12-bit, \(1376 \times 1040\)-pixel Imager Intense CCD camera and a New Wave dual-cavity 50-mJ Nd:YAG lasers. The high speed system was composed of a 10-bit, \(1024 \times 1024\)-pixel Fastcam CMOS high-frame-rate camera and a Photonics ND:YLF 20-mJ single cavity laser. Both systems were controlled with DaVis 7.2 from LaVision and uniformly seeded using oil droplets entering the test section at the inlet. Images were processed with interrogations regions with an initial size of \(32\times32\) and two consecutive passes at \(16\times16\) using a 2D SCC algorithm.
All interrogation regions were overlapped by 50%. Results were then post-processed using four parameters [57]: an allowable pixel range (vectors displacing more than 15 pixels are rejected), correlation peak ratio, neighboring vectors median filter, and small groups (spurious vectors in groups smaller than 5 vectors are thrown out).

Velocity measurements were made upstream and downstream of each cylinder on the $x$-$y$ plane (the downstream measurement plane for cylinder 3 is shown in Fig. 3.5). The laser sheet entered the facility from the transparent polycarbonate cross-stream side and was terminated on the far opaque side. The cameras were mounted above the test section and both the low speed and high speed lasers were located in front of the test section. The cameras and lasers were attached to independent traverse systems.

Instantaneous velocity time traces were extracted at specific locations from each measurement plane (shown in Fig. 3.5) for use as spatial validation SRQ’s for the CFD results. Post-processing of the spatial SRQ’s allowed the calculation of temporal validation SRQ’s. Validation SRQ’s include mean bulk and local velocities, velocity distributions, dominant frequencies, pressure-velocity and velocity-velocity spatial correlations, and autocorrelations.

Low speed measurements were obtained on $x$-$y$ planes at $z = 0.69D, 1.5D, 3.45D, 5.4D,$ and $6.21D$. At these locations, $N = 850$ images at an average rate of $f = 2.5$ Hz were acquired to determine the time-averaged statistics of the flow and the desired flow Reynolds number (Re= 40,000). The resolution of these images were $64.5 \mu m/pixel$. Strong 3-D motions were present within the facility, particularly downstream of cylinder 2. The low-speed measurements also provided reassurance that the time-average flow was uniform through the span of the channel, allowing measurement near the span-wise centerline ($z = 3.64D$) using the high-speed system. Ten sets of $N = 1024$ time-resolved images were acquired at $f = 1500$ Hz on the $x$-$y$ plane at $z = 3.64D$ with the high-speed system. The resolution of these images was $88.23 \mu m/pixel$.

Pressure taps of diameter 1.59 mm were drilled into the back wall of the facility (shown by arrows in Fig. 3.4). All taps were placed at the span-wise centerline ($z = 3.45D$)
Fig. 3.5: Numerical and experimental measurement trace locations. Instantaneous velocity traces are located on the $z = 3.64D$ plane, while instantaneous pressure traces are on the span-wise centerline, $z = 3.45D$ plane. Pressure measurements and CFD pressure traces (designated by ‘o’) were located on the upper wall on the span-wise center behind the full cylinders for both experimental and numerical data. A PIV measurement plane (denoted in green) is downstream of cylinder 3. Instantaneous velocity trace locations extracted up- and downstream of each cylinder from both experimental and numerical results; examples of instantaneous velocity trace locations are shown as ‘×’. Instantaneous bulk velocity traces (designated by red lines) were acquired up- and downstream of each cylinder (examples of downstream bulk traces are designated by red lines).

and were at the same stream-wise position as the axis of the full cylinders. The pressure measurements were made using five Endevco pressure transducers with a range of 1 psi and sensitivity of about 175 mV/psi. The resonant frequency of the pressure transducers was 55,000 Hz. Pressure measurements were acquired using a data acquisition system from National Instruments using two approaches: 1) individual measurements at 10,000 Hz for approximately 180 s and 2) measurements synchronized with the high-frame-rate PIV system at 1500 Hz. Interference on the pressure transducer signal was introduced by thermal expansion waves from the high-speed laser and required that the high-speed PIV measurements be located on the $z = 3.64D$ plane.

### 3.2.2 Numerical Model

Reynolds-Averaged Navier-Stokes two-equation $k$-$\omega$ [40] and Detached Eddy Simulation models available in the general purpose CFD code FLUENT [61] were used to solve the Navier-Stokes equations for the unsteady flow. The $k$-$\omega$ model included corrections
for viscous damping and shear flow spreading, and was integrated to the wall. The DES approach is essentially a hybrid model that employs a RANS model within boundary layers, and an LES model in the core of the flow. This allows for decreased computational times when compared with a full LES calculation since the near wall region need not be as finely meshed. In the present work, the DES model was based on an unsteady SST $k$-$\omega$ model with low Reynolds number corrections in the near-wall region. Relative to the Menter $[62]$ SST $k$-$\omega$ model, the DES implementation of the model uses a modified dissipation term, $Y_k$, for the turbulent kinetic energy (Menter $[63]$). In particular

$$Y_k = \rho f_{\beta^*} k \omega f_{\beta^*}, \quad (3.4)$$

where $f_{\beta^*}$ is given as:

$$f_{\beta^*} = \max \left( \frac{L_t}{C_{DES} \Delta_{max}} (1 - F_{DES}), 1 \right). \quad (3.5)$$

Here, $C_{DES}$ is given a value of 0.61, $\Delta_{max}$ is the maximum local grid spacing, and $F_{DES}$ is the shielding function which will be discussed later. The turbulent length scale is given as:

$$L_t = \frac{k}{\beta^* \omega}. \quad (3.6)$$

The DES model distinguishes the LES region from the RANS region by the minimum of the grid spacing ($C_{DES} \Delta_{max}$) and turbulent length scale $L_t$. This minimum value is used to calculate the dissipation rate $\epsilon_{DES}$ using the equation,

$$\epsilon_{DES} = \frac{k^{3/2}}{\min (L_t, C_{DES} \Delta_{max})}. \quad (3.7)$$

This new dissipation rate replaces the dissipation rate term in the turbulent kinetic energy $k$ transport equation $[61]$.

Artificial flow separation between the $k$-$\omega$ region and the LES grid refinement region is inherent in the formulation of the DES model; this may occur when the grid spacing is less than the boundary layer thickness, or $\Delta_{max} < \delta_{BL}$. As proposed by Menter $[63]$ and
Spalart et al. [64], the SST $k$-$\omega$ model uses blending functions to combine solutions of the $k$-$\omega$ model within the boundary layer and $k$-$\epsilon$ model (modified to be compatible with the $k$-$\omega$ model) in the far field. The SST $k$-$\omega$ based DES model provides the ability to shield the boundary layer from the DES limiter by the definition of the SST-$k$-$\omega$ blending functions $F_{DES}$.

Results for two DES models are calculated: no blending ($F_{DES} = 0$) and delayed blending ($F_{DES} = F_2$) and are later referred to as DES-NB and DES-B, respectively. The SST $k$-$\omega$ blending function, $F_2$, is defined as,

$$F_2 = \tanh \left( \Phi_2^2 \right),$$

where $\Phi_2$ is defined as

$$\Phi_2 = \max \left[ \frac{2 \sqrt{k}}{0.09 \omega y}, \frac{500 \mu_f}{\rho_f y^2 \omega} \right].$$

The DES models used a bounded second-order central differencing method, while the $k$-$\omega$ models used the QUICK scheme to discretize the momentum terms; turbulence and dissipation terms were interpolated to the cell faces using the QUICK scheme. Pressure-velocity coupling was achieved using the SIMPLEC method. Temporal terms employed a second-order implicit time formulation. Iterative convergence for each time-step was achieved by reducing the normalized residuals of all the discretized transport equations within five orders of magnitude, after which the solution advanced to the next time step. Solutions were calculated at time-steps of $10^{-4}$ s. Time-step convergence was also assessed and will be discussed later.

The computational domain matched the experimental configuration. The mesh and geometry of the mid-plane at cylinders 3 and 4 appear in Fig. 3.6. The numerical model and experimental test section are dimensionally equivalent. The test section is composed of nine wall regions: two vertical side walls joined by four half cylinders, two horizontal walls, and five full cylinders located on the cross-stream centerline. The DES and $k$-$\omega$
turbulence models were applied to this three dimensional computational domain for a single mesh consisting of 3,098,000 cells with grid clustering toward the walls to ensure that $y^+ \leq 1$. The flow enters the $k$-$\omega$, DES-NB, and DES-B numerical models at the channel inlet with uniform profiles: velocity ($u_{\text{bulk}} = 5.967 \text{ m/s}$), turbulence intensity ($I = 1.5\%$) and turbulence viscosity ratio ($\mu_t/\mu_f = 10$). Two additional solutions were obtained by applying perturbations from a random fluctuating vortex field at the inlet (180 vortices at the inlet) \cite{61} of the parent models (DES-NB, and DES-B). Insignificant differences between the perturbation and parent models were often observed and are typically not shown in figures. The DES-B with perturbations model will be referenced as DES-P. Flow exiting the numerical model leaves with normal derivatives set to zero at the outlet.

Solutions from the above numerical models were calculated and compared to the experimental results. Comparisons include dominant frequencies present in the instantaneous
pressure and velocity signals at each cylinder and velocity distributions averaged over the
time. Validation SRQ’s for both the coarse and fine $k$-$\omega$ models are compared to deter-
mine grid convergence. Converged transient solutions of the $k$-$\omega$ model were also calculated
from a refined mesh (6,196,000 cells) and a coarse mesh (1,758,250 cells); these additional
solutions were used to determine grid convergence of the $k$-$\omega$ model solution. Grid con-
vergence was assessed for all validation parameters presented in this dissertation using the
GCI method [65].

3.2.3 Numerical Uncertainty

Numerical uncertainty on the proposed global and local validation SRQ’s was assessed
for both temporal and spatial convergence using time-step and grid convergence studies.
The influence of truncation errors (grid convergence) on the results obtained with the $k$-$\omega$
turbulence model was assessed by solving the problem using model discretization of 1.8,
3.1, and 6.2 million cells. Grid convergence for all validation SRQ’s considered have been
examined and examples of grid convergence for global and local, spatial and temporal vali-
dation SRQ’s are discussed below. These include the standard deviations of time-variations
in bulk velocity, amplitudes and frequencies for temporal varying pressure and velocity
SRQ’s through Fourier transforms, time-scales of velocity features calculated from autocor-
relations, similarities between two time-varying quantities through correlations, and minor
loss factors. Locations for these data are shown in Fig. 3.5. We note that the influence of
truncation errors for the LES-based DES models is more difficult to determine, due primar-
ily to the implementation of subgrid-scale models. In fact, Durbin and Medic state, “No
accuracy criterion for LES exists at present. In particular, the idea of grid independence is
not applicable to LES” [66:247].

Quantification of grid convergence observed in the $k$-$\omega$ model is accomplished using the
Grid Convergence Index (GCI) as outlined by Celik et al. in [65]. This method requires three
mesh sizes of increasing size (in this dissertation: 1.758, 3.098, and 6.196 million cells). The
GCI is a measure of the relative discretization uncertainty (with 95% confidence) for the
given SRQ based on the solutions and the rate of convergence, $q$, obtained from the refined
meshes. The relative discretization uncertainty for the majority (60%) of the validation SRQ’s were below 55% and the level of uncertainty is strongly dependent on the validation SRQ considered. The effects of unsteady, nonlinear modeling on grid convergence among the temporal SRQ’s merits more discussion; consequently, these effects on the numerical uncertainty values for individual validation SRQ’s are covered in detail in Section 4.2.9.

3.2.4 Validation Quantification

Although several validation metrics have been proposed for steady CFD simulations (as discussed in Section 1.3), use of these metrics are extremely limited by unsteady flows and numerical uncertainty. As CFD validation is application-specific, programmatic uncertainty levels must be specified prior to assessment and a pass/fail assessment is achieved. Increased research is required for the development of appropriate general CFD validation metrics.

Considering the limitations of current validation metrics and the intentions of this research did not have a specific application in mind, establishment of an arbitrary validation metric could mislead readers into forming inappropriate validation conclusions. Thus, to provide a more general assessment, the experimental and numerical results are given for each unsteady spatial and temporal SRQ along with the corresponding uncertainties. The reader is allowed to develop application-specific validation metrics and obtain a more appropriate validation assessment using the results presented within this paper.
Chapter 4
Results and Discussion

4.1 PIV Uncertainty Demonstration and Results

The effects of each error source (i.e. particle image density $d_{\rho}$, diameter $d_{\tau}$, and displacement $\Delta x$ and flow gradients $du/dy$) on the measured time-averaged velocity and fluctuations are demonstrated. The experimental data acquired is designed to emphasize individual uncertainty sources and individual error sources are varied systematically. Values of error sources are given as a reference in Table 4.1 along with the average unvaried error sources in Table 4.2. Best practices are implemented to decrease the contribution of error from other sources. Due to the nature of the jet (Fig. 3.2), gradient errors and individual error effects are combined at the jet edges for all cases. A sufficient number of measurements will be acquired for each experimental dataset to provide converged time-averaged velocity and turbulence statistics. All data was acquired at the $x = 0.1h_{1/2}$ location except the $du/dy = 0.09$ pixels/pixel error source case which was located at $x = 1.25h_{1/2}$.

The automatic uncertainty algorithm proposed by Timmins et al. [50,51] is indifferent

Table 4.1: Values of error source for all datasets. * The particle image displacement of $\Delta x = 1.1$ is repeated for sub-pixel and super-pixel displacements.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Datasets Acquired</th>
<th>Number of Datasets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($d_{\rho}$, particles/pixel$^2$)</td>
<td>0.002, 0.011, 0.022, 0.031, 0.038, 0.039</td>
<td>6</td>
</tr>
<tr>
<td>Diameter ($d_{\tau}$, pixels)</td>
<td>2.5, 2.6, 2.9, 3.1, 3.8, 4.5, 4.7, 7.4</td>
<td>8</td>
</tr>
<tr>
<td>Displacement ($\Delta x$, pixels)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sub-pixel</td>
<td>0.5, 0.7, 0.9, 1.1, 6.4, 8.6, 10.7, 15.9</td>
<td>4*</td>
</tr>
<tr>
<td>super-pixel</td>
<td>1.1, 2.2, 3.2, 4.3, 5.3, 9*</td>
<td></td>
</tr>
<tr>
<td>Gradients ($du/dy$, pixels/pixel)</td>
<td>0.07, 0.09, 0.14</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4.2: Average values of unvaried error source for all datasets.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>$d_p$</th>
<th>$d_r$</th>
<th>$\Delta x$</th>
<th>$du/dy$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>N/A</td>
<td>2.7 pixels</td>
<td>10.1 pixels</td>
<td>0.07 pixels/pixel</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.021 particles/pixel$^2$</td>
<td>N/A</td>
<td>10.1 pixels</td>
<td>0.07 pixels/pixel</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.021 particles/pixel$^2$</td>
<td>2.7 pixels</td>
<td>N/A</td>
<td>0.07 pixels/pixel</td>
</tr>
<tr>
<td>Gradients</td>
<td>0.021 particles/pixel$^2$</td>
<td>2.7 pixels</td>
<td>10.1 pixels</td>
<td>N/A</td>
</tr>
</tbody>
</table>

To regions of multiple error sources and will account for the known error sources at all locations. In these regions, only the left edge of the profile (the lower side of the jet in Fig. 3.2) will be shown to improve clarity. Individual error source effects are observed at the centerline, while individual and gradient effects are demonstrated at the jet edge. To demonstrate the effect of individual uncertainty sources on the time-averaged measured fluctuations, the time-averaged measured fluctuations along the centerline of the jet are plotted as a function of the uncertainty source for each case.

4.1.1 Particle Image Displacement Errors

The time interval, $\Delta t$, between consecutive images determines the particle image displacement $\Delta x$ for an image set and is distinguished by two groups: sub-pixel and super-pixel cases. The dominant source of error from particle image displacement is from the PIV algorithm’s ability to resolve sub-pixel displacement. Consequently, significant bias and random errors from small particle image displacements near a single pixel are observed [67]. Increased error is expected for sub-pixel displacements at (or near) the same order of magnitude as the particle image displacement.

As the instantaneous velocity or spatial dynamic range ($\Delta x$) is increased, the error from sub-pixel displacement decreases relative to $\Delta x$. This is strongly evident in the sub-pixel cases (Fig. 4.1) as sub-pixel errors generate a strong bias on the time-averaged velocity, which is reduced as the dynamic range is increased. While the sub-pixel cases begin to converge on the mean velocities, time-averaged measured fluctuations are dramatically elevated for all sub-pixel cases. As these elevations are too large to clearly demonstrate, they are only shown for the super-pixel cases.
Fig. 4.1: Profile of the time-averaged velocity magnitude, $\bar{u}$, using the hot wire and PIV algorithms (DaVis and PRANA) with $16 \times 16$ pixel interrogation regions for varying sub-pixel displacements: $\Delta x \approx 0.5, 0.7, 0.9, 1.1, 2.1,$ and $3.2$ pixels.
For all super-pixel cases (Fig. 4.2), the time-averaged velocity magnitude agrees well with the hot-wire result (within 2%). However, significant error can still be seen in the total measured fluctuations except when the displacements are very large (Fig. 4.3). Measured fluctuations are extremely elevated for the sub-pixel cases (not shown) and do not provide additional information beyond the super-pixel cases. Elevated measured fluctuation levels are most dominant near small particle image displacements and converge to the turbulent fluctuation (hot wire) measurements as the dynamic range is increased. While particle image displacements of $\Delta x \geq h_{IA}/8$ are sufficient to provide optimal time-averaged velocity measurements when sufficient data is acquired, total measured fluctuations require particle image displacements greater than half the interrogation region size ($\Delta x \geq h_{IA}/2$). Elevated measured fluctuations for the $\Delta x = 15.9$ pixel case using DaVis 7.2 result from larger average particle image sizes for this case.

Uncertainty levels for DaVis and PRANA are designated with uncertainty bars in Fig. 4.1 and 4.2 and thin dashed lines in Fig. 4.3. Reference to the turbulent fluctuations are provided by hot wire fluctuations and uncertainties should account for the difference between hot wire and PIV results. The automatic uncertainty calculation method accounts for the measurement error on the time-averaged velocity and elevated measured fluctuation levels at individual (jet core) and combined (jet edge) error source locations for the PRANA method. While the PIV results are not significantly affected at the jet edge for the time-averaged velocity, increasingly elevated measured fluctuations are observed. The PRANA uncertainty at these locations, however, still accounts for the correlated and combined error effects. The DaVis uncertainty under-estimates the measurement error at the jet centerline but is usually adequate at the jet edge.

Along the centerline, the dominant uncertainty source is due to particle displacement, while the shear region trend demonstrates combined effects from flow gradients and particle image displacements. In Fig. 4.4, measured fluctuations are shown at a) the shear region and b) the jet edge. The individual particle image displacement errors are a strong function of particle image displacement for both the PRANA and DaVis algorithms when compared
Fig. 4.2: Profile of the time-averaged velocity magnitude, $\bar{u}$, using the hot wire and PIV algorithms (DaVis and PRANA) with $16 \times 16$ pixel interrogation regions for varying pixel displacements: $\Delta x \approx 1.1, 2.1, 3.2, 4.3, 5.3, 6.4, 8.6, 10.7, \text{ and } 15.9$ pixels.
Fig. 4.3: Profile of the measured hot wire turbulent fluctuations, $\overline{u'u'}$, and measured fluctuations, $\overline{uu}$, using the DaVis and PRANA algorithms with $16 \times 16$ pixel interrogation regions for varying super-pixel displacements: $\Delta x \approx 1.1, 2.1, 3.2, 4.3, 5.3, 6.4, 8.6, 10.7, \text{ and } 15.9\text{ pixels.}$
Fig. 4.4: Hot wire turbulent fluctuations, $u' u'$, and measured fluctuations $\hat{u} \hat{u}$ from DaVis and PRANA algorithm measurements for varying particle displacements, $\Delta x$ at the a) shear region and b) jet centerline. Fluctuation levels are overpredicted for small particle displacements, however, at a particle image displacement of $\Delta x \approx 4$ these are within the uncertainty limits.

to the hot wire results. As the spatial dynamic range increases, the size of the measurement errors decrease relative to the total particle image displacement of the measurement.

At the jet edge, the effects of flow gradients are combined with particle image displacement errors. In this region, errors are dominated by flow gradient errors for particle image displacements larger than $\Delta x > 1$. A trend similar to the jet centerline is observed at sub-pixel displacements indicating dominance by the particle image displacement errors for these cases. Uncertainties are accurately quantified at the jet edge and for most super-pixel cases ($\Delta x > 3$) at the jet centerline.
4.1.2 Particle Image Diameter Errors

An easily managed, yet often disregarded, contributor to PIV measurement error is particle image diameter $d_\tau$: defined as the particle diameter (in pixels) as seen by the camera sensor. Theoretically, particle image diameter is strongly affected by the image diffraction through the lens. Understanding this, with an ideal lens, the 95% confidence particle image diameter should vary with the $f^\#$ through the equation [56]:

$$d_\tau = \sqrt{M_s^2 d_p^2 + d_s^2},$$  \hspace{1cm} (4.1)

where $M_s$ denotes the magnification of the image, $d_p$ is the actual particle diameter, and $d_s$ is the diffraction-limited spot diameter defined through,

$$d_s = 2.44 (1 + M_s) f^\# \lambda. \hspace{1cm} (4.2)$$

In the previous equation, $\lambda$ is the wavelength of the light sheet and $f^\#$ is the ratio of the lens focal length, $f_s$, and aperture diameter, $D_a$.

Particle image diameter errors were controlled by varying the lens $f^\#$. Uniform particle image densities and intensities were controlled through modifications in laser power and seed quantities for all datasets. A control image at the smallest $f^\#$ ($f^\# = 2.8$) was acquired and densities and intensities were visually matched for subsequent image sets. Verification of seed density and image intensity was achieved by monitoring the mean and standard deviation of intensities over the entire image.

At small $f^\#$ (large apertures) typical lenses experience aberrations which increase the particle image diameter. However, the real particle image diameter as seen by a camera sensor (including “true” particle size, image diffraction, and lens aberrations) is directly related to the correlation peak width of an autocorrelation and can be estimated from the correlation peak width [56]:

$$d_\tau = 2\sqrt{2}\sigma_{corr}. \hspace{1cm} (4.3)$$
Fig. 4.5: Comparison of real particle image diameters and ideal diffraction-limited particle image diameters as a function of the $f^\#$ for the particle image diameter error dataset in this dissertation. Real particle image diameters are obtained through the autocorrelation peak width and correspond to diameters of $d_r = 2.5, 2.6, 2.9, 3.1, 3.8, 4.5, 4.7, \text{ and } 7.4$ pixels. Real particle image diameters in these datasets differ from the theoretical diameters due to image aberrations beginning at an $f^\# = 11$ (demonstrated in Fig. 4.5). This is most evident for the $f^\# = 2.8$ case with a dramatic increase in particle image diameter. The minimum particle image diameter possible for the given experimental facility corresponds to $d_r \approx 2.6$ and $2.5$ pixels for the $f^\# = 4$, and $5.6$ cases. Aberrations cause images at small $f^\#$ to resemble out of focus particle images and will introduce additional error components.

Small errors are observed on the time-averaged velocity profiles (Fig. 4.6) with negligible differences seen for all particle sizes. This, however, is not the case for the measured fluctuations, in which most particle image size cases observed elevated measured fluctuations above the “true” fluctuations. Uncertainty estimates correctly account for the fluctuation...
measurement errors for most cases. The uncertainty is under-estimated for particle images strongly affected by lens aberrations \( (d_\tau = 4.5 \text{ and } 2.6 \text{ pixels}) \) and a portion of the measurement error may be attributed to out-of-focus particles generated by aberrations. For the largest particle image diameter \( (d_\tau = 7.4 \text{ pixels}) \) the average particle image diameter is outside the uncertainty surface from Timmins et al. \cite{50, 51} and uses the uncertainty for a particle image diameter of \( d_\tau = 5.0 \) pixels. When the error sources are within the uncertainty surface generated by Timmins et al. \cite{50, 51}, the uncertainty surface is capable of accounting for fluctuation measurement error in both the jet core and jet edge regions for PRANA. DaVis uncertainties are often under-estimated at all centerline locations, but accurate at the combined shear regions.

Individual (jet centerline) and combined error effects (jet edge) are shown in Fig. 4.8. Elevated measured fluctuations are observed for increased particle image diameters corresponding to theoretical trends \cite{52, 56}. Raffel et al. \cite{52} and Adrian and Westerweel \cite{56} demonstrate optimal particle image diameters at \( 2.0 \leq d_\tau \leq 2.5 \) pixels. In this study the minimum error is observed at particles near \( d_\tau = 2.9 \) and 3.1 pixels (shown in Fig. 4.9). Differences in optimal particle image diameters may be attributed to particle image aberrations within the lens for this experimental setup. Lens aberrations prohibit particle image sizes smaller than \( d_\tau = 2.5 \) pixels, preventing determination of an optimal particle image size, and increasing measurement error in small physical particle images \( (f\# \leq 5.6) \). In the combined error source region - jet edge region denoted by a) - the shear strongly dominates measured fluctuation errors. Once again, the largest particle image diameter case \( (d_\tau = 7.4 \text{ pixels}) \) is outside the uncertainty surface and uses the nearest uncertainty surface location to estimate the uncertainty.

4.1.3 Particle Image Density Errors

Particle image density is defined by the average number of visible particles within each interrogation region. This is estimated within the automatic uncertainty algorithm using an intensity threshold based on the mean and standard deviation of image intensity. The local particle image density is calculated by dividing the total number of pixels above the
Fig. 4.6: Profile of the time-averaged velocity magnitude, $\bar{u}$, using the hot wire and PIV algorithms (DaVis and PRANA) with $16 \times 16$ pixel interrogation regions for varying particle image sizes: $d_r = 2.5, 2.6, 2.9, 3.1, 3.8, 4.5, 4.7,$ and $7.4$ pixels.
Fig. 4.7: Profile of the measured hot wire turbulent fluctuations, $u'u'$, and measured fluctuations, $\tilde{u}\tilde{u}$, using the DaVis and PRANA algorithms with 16 × 16 pixel interrogation regions for varying particle image sizes: $d_r = 2.5, 2.6, 2.9, 3.1, 3.8, 4.5, 4.7,$ and $7.4$ pixels.
Fig. 4.8: Hot wire turbulent fluctuations, $\overline{u'u'}$, and measured fluctuations $\hat{u}u$ from DaVis and PRANA algorithm measurements for varying particle image sizes, $d_\tau$ at the a) shear region and b) jet centerline. Fluctuation levels are elevated for large particle sizes. The optimal particle image size is found to be near $d_\tau = 2.9$, and 3.1 pixels. Cases marked by an * denote particles sizes strongly influence by lens aberrations. The $d_\tau = 7.4$ pixel case (marked by !) was outside the 4D uncertainty surface and used the uncertainty for $d_\tau = 5.0$ pixels.
Fig. 4.9: The effect of particle image diameter \( d_\tau \) on random error as demonstrated by Raffel et al., in which an optimal particle size of \( d_\tau \approx 2.5 \) is observed. Timmins et al. demonstrate an optimal particle size near \( d_\tau \approx 2.9 \).

The effects of particle image density on PIV measurements are studied using particle image densities of \( d_\rho \approx 0.002, 0.011, 0.022, 0.031, 0.038, \) and \( 0.039 \) particles/pixel\(^2\). At the most dense case \((d_\rho \approx 0.039 \text{ particles/pixel}^2)\), particle images began to blend and to be composed of multiple particles.

Both time-averaged velocities and measured fluctuations are negligibly affected by particle image density errors for most cases (Fig. 4.10, 4.11, and 4.12); however, at low particle image densities, insufficient quantities of particles begin to introduce measurement noise or error on the fluctuation levels. Fluctuation measurement errors increase for the \( d_\rho \approx 0.039 \) particles/pixel\(^2\) case. While the individual effects of particle image density on measurement error at the jet centerline are often negligible, the correlated effects of density and gradients are evident near the jet edges, as evidenced by elevated measurement fluctuations as the particle image density decreases.
Fig. 4.10: Profile of the time-averaged velocity magnitude, $\bar{u}$, using the hot wire and PIV algorithms (DaVis and PRANA) with 16 × 16 pixel interrogation regions for varying particle image densities: $d_\rho \approx 0.002, 0.011, 0.022, 0.031, 0.038$, and 0.039 particles/pixel$^2$. 
Fig. 4.11: Profile of the measured hot wire turbulent fluctuations, $u'u'$, and measured fluctuations, $u\bar{u}$, using the DaVis and PRANA algorithms with $16 \times 16$ pixel interrogation regions for varying particle image densities: $d_p \approx 0.002$, 0.011, 0.022, 0.031, 0.038, and 0.039 particles/pixel$^2$. 
Fig. 4.12: Hot wire turbulent fluctuations, $\overline{u'}u'$, and measured fluctuations $\hat{u}_m$ from DaVis and PRANA algorithm measurements for varying particle image densities, $d_\rho$ at the a) shear region and b) jet centerline. Fluctuation levels are relatively constant among particle image densities with a shear-density correlation observed in the shear region. The case marked by an * denotes particle image densities that may be under-predicted due to multiple particles looking like a single particle. The $d_\rho = 0.03$ particles/pixel$^2$ case (marked by !) was outside the 4D uncertainty surface and used the nearest density to calculate uncertainty.
4.1.4 Flow Gradient Errors

Of the error sources studied by Timmins et al. [51], the largest uncertainties were found to be due to instantaneous velocity gradients (or shear) in the flow. This is also demonstrated during the previous sections (Sections 4.1.1-4.1.3) by significantly elevated fluctuation levels at the jet edge. Although shear will introduce error to all PIV algorithms, the magnitude of measurement error between algorithms will likely differ. Flow gradients introduce error into a PIV measurement in two ways: increasing the correlation peak shape and size and introducing random, instantaneous biases from non-uniform particle image densities. While increasing the particle image density can reduce the latter, the measurement will always be affected by the correlation peak shape. For flows with shear, the correlation peak width will increase in the direction of the shear, decreasing the algorithms ability to locate the center of the peak. Algorithms including window deformation can reduce the effects of error due to gradients on the correlation peak shape.

Difficulties in measuring instantaneous, local velocity gradients often require the use of the time-averaged velocity gradients to assess gradients within PIV images. Finite differencing methods often introduce large noise components on the instantaneous, local gradient calculations; however, local mean gradients (based off the time-averaged velocity profile) tend to under-predict instantaneous, local velocity gradients. Consequently, time-averaged gradients under-estimate measurement uncertainties due to the non-linear relationship between measurement error and gradients. This is of particular interest in turbulent flows, where a zero mean gradient may exist, but the flow may observe large local, instantaneous turbulent gradients. The instantaneous, local gradient measurement uncertainties in this study are estimated automatically using the high accuracy, low noise gradient estimator introduced by Karri et al. [68].

Data were acquired for three different cases in which the time-averaged gradient levels at the jet edges were observed to be approximately $du/dy \approx 0.07, 0.09$ and 0.14 pixels/pixel and zero in the jet core. The latter cases were acquired at an upstream ($du/dy \approx 0.14$ pixels/pixel) and downstream ($du/dy \approx 0.09$ pixels/pixel) location of a laminar jet with a
maximum time-averaged velocity of $u_{\text{max}} = 35.1$ m/s, while the maximum jet velocity of the first case ($du/dy \approx 0.07$ pixels/pixel) corresponds to $u_{\text{max}} = 10.1$ m/s.

Hot-wire, DaVis, and PRANA profiles of the time-averaged velocity $u$ are shown in Fig. 4.13 for the shear level cases of 0.07, 0.09 and 0.14 pixels/pixel. For all cases, accurate measurements of the time-averaged velocity magnitude are obtained. Velocities at the jet centerline compare well between hot wire and PIV measurements. In the high shear region, the PIV time-averaged velocity profiles tend to be smeared or smoothed due to PIV averaging over the interrogation region. This effect, however, is small and has little impact on the mean profiles.

Profiles of the fluctuations, $\overline{\hat{u}u}$ for all three shear cases are also shown in Figs. 4.13. These profiles consist of a core region with no shear and small velocity fluctuations and the jet edge regions that contain varying levels of shear. In the jet edge region of large gradients, elevated levels of measured fluctuations are expected from gradient errors. As the gradient increases, the DaVis and PRANA algorithms begin to deviate from the hot wire results and experience elevated fluctuation levels. Uncertainty estimates on the measured fluctuations caused by the flow gradients show the time-averaged measured fluctuation uncertainty $U_{\hat{u}u/e}$ from the PIV results are composed primarily of uncertainties from gradients within the flow.

The $du/dy \approx 0.09$ pixels/pixel or downstream case introduces an interesting phenomena: increased measurement error with decreased flow gradients. This is generated by the addition of vortical structure and three-dimensional flow gradients. The gradients at this location are also, naturally, more turbulent and instantaneous gradients can be much larger than the time-averaged gradient. This motivates the quantification of PIV uncertainty at an instantaneous level and, later, propagating these uncertainties into the time-averaged results, as is accomplished using the method in this dissertation. Even with the inclusion of these extra phenomena, the automatic uncertainty method is capable of correctly estimating the elevated portion of total fluctuations.

For all cases, the difference in measured fluctuations $\overline{\hat{u}u}$ from time-averaged measured fluctuation uncertainty $U_{\hat{u}u/e}$ agrees well with the hot wire results or turbulent fluctua-
Fig. 4.13: Cross stream profiles of the time-averaged velocity magnitude, $\overline{u}$, and magnitude fluctuations, $\overline{u^2}$, for gradients of $du/dy \approx 0.07, 0.09$ and $0.14$ pixels/pixel from the hot wire, Davis, and PRANA measurements.
tions $\overline{u'w'}$ for the PRANA algorithm. Once again, the time-averaged measured fluctuation uncertainty $U_{\hat{u}'\hat{u}'}$ is only applicable for the PRANA algorithm. Fig. 4.13 easily demonstrates the potential for different dominant error sources from different algorithms. For the $du/dy = 0.07$ and 0.14 pixels/pixel cases, the flow gradients are strongly one-dimensional, reducing the fluctuation error observed from DaVis; however, for the downstream case ($du/dy = 0.09$ pixels/pixel) vortical structures begin to occur and flow gradients are no longer dominant in the $y$-direction, increasing measured fluctuation levels by DaVis. The opposite trend is observed in the PRANA results and is attributed to the lack of window deformation within the version of PRANA used by Timmins et al. [51].

4.2 Cylinder Array Validation Results

Low frame-rate measurements were obtained on the downstream side of cylinder 4 at five heights ($z$ values) described above. These measurements spanned the middle 80% of channel height. These were used to determine the required blower speed to achieve the desired time-averaged bulk velocity (consequently, Reynolds number) and calculate time-averaged statistics. Data were acquired at three Reynolds numbers ($Re \approx 34,000, 39,000, \text{ and } 44,000$) to determine the settings needed to provide $Re = 40,000$. Temporal fluctuations in the instantaneous bulk velocity were observed in each of these measurements. After ensuring that these fluctuations were due to our spatial averaging methods, we determined that the standard deviation of the time-averaged bulk velocity was consistent for each measurement height. We interpret these fluctuations as temporal variation in the bulk flow at each $z$ station. In other words, at any instant, the instantaneous, spatially-averaged bulk velocity varies in the $z$ direction. However, the time-averaged bulk velocity at each measurement location was found to be within 2% difference of that required for $Re = 40,000$.

After determining the time-averaged flow was uniform spatially, data were acquired at $Re = 40,000$ in the $x$-$y$ plane (see Fig. 3.4) using both the low and high-frame-rate PIV systems. Atmospheric pressure (and thus fluid density) variations were measured between datasets and corrections in the blower time-averaged velocity were made to provide the target Reynolds number, $Re = 40,000$. The low frame-rate data were acquired at
\[ H = 3.45D \] (instantaneous and time-averaged velocity and fluctuation fields are shown in Fig. 4.14-4.16). The Reynolds number was calculated using the maximum time-averaged bulk velocity \( u_{\text{max}} \) found at the minimum cross-section and the diameter of the cylinder \( D \) for the velocity and length scales, respectively.

High frame-rate PIV measurements and CFD velocity time-traces were acquired on the \( x\)-\( y \) plane at \( z = 3.64D \). This location is about 0.2\( D \) lower than the centerline due to interference from the laser sheet with the pressure sensor located at the transparent cylinder 4. Laser pulses striking the far surface of the channel were found to generate thermal expansion waves that appeared as sinusoidal noise on the pressure signal. This noise was present while the laser was within 0.15\( D \) of the centerline. A schematic of the PIV measurement planes is shown in Fig. 3.5. High-frame-rate or time-resolved measurements were oriented on the \( x\)-\( y \) plane for the upstream and downstream locations of all full cylinders. Discussion of temporal SRQ’s are made for coupled high-speed PIV and pressure measurements.

### 4.2.1 Uncertainty of PIV Results

Uncertainties for the spatially local, instantaneous velocity measurements were calculated from the technique and uncertainty surface recommended by Timmins et al. While these results are applicable to the PRANA robust phase correlation, an estimate of the instantaneous uncertainty magnitudes for DaVis 7.2 is shown in Fig. 4.14.

As would be expected, strong local variations are observed in the measurement uncertainty on an instantaneous measurement. Regions of increased uncertainty are located at downstream sides of cylinders (enhanced turbulence, 3-D motions, and mixing) and regions of high shear. Uncertainty stripes are also observed near cylinder 1 as the flow accelerates toward the smaller cross-section. At this location, the normally dominant sources of error (e.g. shear) are small and the nearly temporally uniform flow allows sub-pixel estimation errors to be observed. The stripes are generated by the flow accelerating between integer pixel displacements.

Using the method discussed in Section 4.1, local, mean and fluctuation uncertainties were obtained from the local, instantaneous uncertainties. Mean velocities, fluctuations,
Fig. 4.14: The instantaneous velocity vectors colored by the instantaneous velocity magnitude (left), bulk normalized uncertainty (center), and the locally normalized uncertainty (right) for the experimental model.
and uncertainties are given in Fig. 4.15 and 4.16. Time-averaged velocity uncertainties range from 2.5% to 15% of the maximum time-averaged bulk velocity and 2.5% to 22% of the local velocities. Within these figures, the uncertainties are enhanced at downstream cylinders, shear regions, and vortex shedding locations. Once again, sub-pixel uncertainty stripes are evident near cylinder 1 within the time-averaged velocity results. The average local uncertainty on the time-averaged velocity is near 5% near cylinder 1 and increases to 7.5% downstream of cylinder 2.

Normalized total measured fluctuations are generally near 1.2% of the maximum bulk velocity beginning from cylinder 2 (Fig. 4.16). The uncertainty for the measured fluctuations range from 0.25% to 1.25% of the maximum bulk velocity. This corresponds with roughly 55% of the measured fluctuation amplitude. Sub-pixel uncertainty stripes are not as dominant in the fluctuation uncertainty as that observed by the instantaneous and time-averaged velocities.

Uncertainties (calculated from Timmins et al. [50,51] and using Eqn. 1.26) are also given for both the instantaneous, local measurements and time-averaged, local measurements (Appendix D with representative examples given in Fig. 4.17 - 4.20). Local uncertainties (dashed lines) are obtained from instantaneous measurements, while time-averaged uncertainties (solid gray lines) are based on the root-sum-square of the average instantaneous uncertainties and random variation uncertainties. Put simply, the time-averaged uncertainties should contain at least 95% of the instantaneous measurements. When the time-averaged uncertainties contain more than 95% of the instantaneous measurement traces, the instantaneous uncertainties dominate the random uncertainties from fluctuations (i.e. turbulence and temporal variations). Likewise, when only 95% of the instantaneous data is contained by the time-averaged uncertainty bars, the instantaneous uncertainties are negligible.

Uncertainties rapidly increase for instantaneous velocity traces (shown in Fig. 4.17 and 4.18) as the flow evolves through the facility and becomes more chaotic. Random uncertainties dominate the time-averaged uncertainty at locations downstream of cylinder 2. Upstream of this position, both instantaneous and random uncertainties contribute to
Fig. 4.15: The time-averaged velocity vectors colored by the time-averaged velocity magnitude (left), bulk normalized mean uncertainty (center), and the locally normalized mean uncertainty (right) for the experimental model.
Fig. 4.16: The time-averaged velocity vectors colored by velocity fluctuations (left), bulk normalized fluctuation uncertainty (center), and the locally normalized fluctuation uncertainty (right) for the experimental model.
Fig. 4.17: The experimental uncertainty for the instantaneous velocity time-traces, $U_{u_e}$, and time-averaged velocity, $U_{\bar{u}_e}$, on the centerline and upstream side of each cylinder for the experimental model.

the total time-averaged uncertainties. This is particularly evident upstream of cylinder 1. For all instantaneous velocity traces, the instantaneous uncertainties are typically within 7.5% of the local velocity.

The instantaneous bulk velocity traces (Fig. 4.19 and 4.20) are spatially-averaged and attenuate the random variations, increasing the affects of instantaneous measurements as evidenced by the large time-averaged confidence intervals near cylinder 1. Instantaneous uncertainties for the bulk velocity traces are near 10% for downstream trace locations.
Fig. 4.18: The experimental uncertainty for the instantaneous velocity time-traces, $U_{u_e}$, and time-averaged velocity, $U_{\bar{u}_e}$, on the far and downstream side of each cylinder for the experimental model.
Fig. 4.19: The experimental uncertainty for instantaneous bulk velocity time-traces at the upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. 4.20: The experimental uncertainty for instantaneous bulk velocity time-traces at the downstream side of each cylinder for the experimental and numerical ($k-\omega$, DES-NB, and DES-B) models.
Fig. 4.21: Temporal drag variation on the five cylinders according to DES. To improve clarity, the drag value for cylinders 3, 4, and 5 are displaced upward by 0.5, 1.0 and 1.5, respectively. The traces for cylinders 1 and 2 are not displaced.

4.2.2 Instantaneous Time-Trace SRQ’s

The discussion of the results begins by examining the drag on each cylinder predicted by unperturbed DES-NB, as these results will illuminate the remainder of the results. Measurement of this SRQ through velocity fields would require two stereo high speed PIV systems and therefore no experimental results are provided.

The drag variation with time on all five cylinders is shown in Fig. 4.21. The variation in time of drag (and any temporal SRQ) can be compared using both fluctuation amplitude and frequency. Cylinders 2-5 have a similar mean drag; however, for clarity, the drag signal
for cylinders 3, 4, and 5 are displaced upward by 0.5, 1.0 and 1.5, respectively. It is striking that the drag on cylinder 1 (which is not displaced) is nearly steady and significantly larger than the rest. Cylinder 2 exhibits a mean drag less than half of cylinder 1 with very large frequency amplitudes. The largest frequency present, which is likely the shedding frequency of cylinder 1, corresponds to a Strouhal number near 0.5. A sub-harmonic frequency corresponding to St = 0.25 is also clearly present. The highest frequency appears to diminish near the center of the array before reemerging at cylinder 5. As Fig. 4.21 demonstrates, time traces of temporal varying SRQ’s provide only qualitative and subjective comparisons; critical analysis requires more robust temporal validation metrics of autocorrelated and correlated time-scales and fluctuation amplitude and frequency.

Time traces for instantaneous velocity (at locations shown by ‘×’ in Fig. 3.5 repeated at up- and downstream locations of each cylinder) and pressure (shown by ‘○’ in Fig. 3.5) point SRQ’s were recorded during the unsteady CFD simulations. These locations consist of five pressure points along the centerline of the channel and on the opaque wall to the side of each full cylinder. While, instantaneous velocity traces were removed sufficiently from the boundary layer to be in the LES region of the DES models, pressure traces were within the boundary layer and, therefore, modeled using the $k-\omega$ model. Instantaneous pressure and velocity were measured at identical locations as the numerical traces from the pressure transducers and extracted from the PIV results, respectively.

Both experimental and numerical time-traces demonstrate similar trends (Fig. 4.22) with small turbulent fluctuations upstream of the cylinders increasing in amplitude at downstream locations. Additional time-trace SRQ’s are given in Appendix C. The uncertainty (gray lines) on the experimental mean (based on instantaneous uncertainties and 95% confidence bounds) is provided with the time-trace data. It can be seen, that although varying trends are observed among the CFD and experimental models, the numerical results lie within the experimental uncertainty bounds. This, however, does not provide a quantitative measure of the fluctuation amplitudes.
Fig. 4.22: The instantaneous velocity time-traces on the far and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
4.2.3 Trace Distribution and Statistics SRQ’s

A more representative method of comparing the fluctuation amplitudes can be observed from the statistics of the local time-traces. The first statistic considered is the time-average of the traces (given in Fig. 4.23-4.26 for several trace locations) and is calculated from Eqn. [1.2]. As the flow evolves downstream, the experimental mean, local time-traces experience large fluctuation differences between up- and downstream cylinder locations. Large time-averaged velocity magnitudes are found at locations downstream of a full cylinder, while cylinder upstream locations observed a decreased magnitude trend for traces acquired at cylinder edges (Fig. 4.23 and 4.25). An opposite tendency is found for traces along the cylinder centerlines (Fig. 4.24) with large magnitudes for upstream traces and decreased means at the downstream locations.

Both DES models and the $k$-$\omega$ model are capable of accurately predicting local time-averaged velocities upstream of cylinder 1. However, as the flow progresses downstream, the upstream/downstream fluctuation trend is much more extreme for all the CFD models than the experimental measurements. At downstream locations, the DES-NB model is often more accurate than the remaining models.

The numerical models accurately predict bulk velocities within the measurement uncertainty and similar to the experimental results as demonstrated in Fig. 4.26. While time-averaged bulk velocities vary slightly in the spatial evolution of the facility, the flow is both measured and predicted to be relatively uniform in the stream-wise direction on the centerline plane. Similar variations are observed within the numerical results for all models.

The standard deviation (an additional statistical quantity calculated by Eqn. [1.2]) measures the root-sum-square of the deviation of the instantaneous SRQs from the mean SRQ value. Using this definition, the standard deviation of the instantaneous bulk velocity is directly proportional to the sum of the amplitudes of the bulk velocity fluctuations and can be used as a means to measure their relative amplitude or spread. The fluctuation amplitudes (standard deviation) for several time-traces are given in Fig. 4.27-4.30.

Again, the inlet flow is steady, as expected. Quite surprisingly, large fluctuations are
Fig. 4.23: The mean of the instantaneous velocity time-traces at the upstream and downstream locations on the far side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Fig. 4.24: The mean of the instantaneous velocity time-traces at the upstream and downstream locations on the centerline of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. 4.25: The mean of the instantaneous velocity time-traces at the upstream and downstream locations on the near side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Fig. 4.26: The mean of the instantaneous bulk velocity time-traces at the upstream and downstream locations for each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
observed in the traces at locations downstream of cylinder 1. The experimental data at the
cylinder edges observes large fluctuation amplitudes at locations upstream of cylinders and
smaller fluctuations for cylinder downstream locations (Fig. 4.27 and 4.29). At the center-
line and instantaneous bulk velocity traces, the fluctuations grow until the flow reaches a
fully-developed fluctuation region. For the centerline traces this location corresponds to the
upstream side of cylinder 2, while the upstream side of cylinder 3 marks the instantaneous
bulk velocity initiation of fully-developed fluctuations as seen in Fig. 4.28 and 4.30 respect-
ively. The rapid growth rate of fluctuation amplitudes indicate strong 3-D motions behind
the cylinders followed by a fully-developed fluctuation region with no net downstream fluc-
tuation growth. Herein, the region of constant, fully-developed fluctuations will be referred
to as the fully-developed fluctuation region.

Similar fluctuation amplitudes are predicted by the DES-NB model for all traces lo-
cations except the centerline time trace. At this location, a fully-developed fluctuation
level is predicted downstream of cylinder 3, whereas the experimental results observe fully-
developed fluctuations downstream of cylinder 2. The fully-developed fluctuation region
of bulk velocity fluctuations is accurately predicted by the DES-NB model. The DES-NB
fluctuation amplitudes are typically within the experimental fluctuations measurement un-
certainty upstream of cylinder 1 and downstream of cylinder 2 for the edge trace locations
(Fig. 4.27 and 4.29). The cause for disagreement between cylinders 1 and 2 may be due
to inaccurate fully-developed fluctuation region predictions. Fluctuation amplitudes are
over-predicted for the instantaneous bulk velocity traces and under-predicted for centerline
traces.

For most trace locations, the $k$-$\omega$ and DES-B models consistently predict much smaller
fluctuations than those seen by both the DES-NB and experimental models, particularly
the centerline and instantaneous bulk velocity traces. Although the DES-B model does
not predict the experimental trend for the fully-developed fluctuation amplitudes, the $k$-$\omega$
model is capable of capturing the fully-developed fluctuation region downstream of cylinder
2. One may expect the fluctuations predicted by the URANS model to be smaller amplitude
Fig. 4.27: The standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the far side of each cylinder for the experimental and numerical (k-ω, DES-NB, and DES-B) models.

since turbulent fluctuations are modeled rather than simulated; note that spatial averaging the data should eliminate the impact of all but the largest turbulent structures.

Perturbations on the inlet velocity have very little effect on the predicted fluctuation amplitudes and trends for both DES models at all trace locations.

The instantaneous bulk velocity and pressure distributions for both the numerical and experimental models follow a Gaussian distribution making the standard deviation an excellent measure of fluctuation amplitude (or distribution width); however, instantaneous velocity traces at cylinder edges and upstream locations had non-Gaussian distributions. For distributions such as these, higher-order statistical quantities (e.g. kurtosis, skewness) can be used to further describe fluctuations. The SRQ trace probability distribution function or histogram can also be used to describe the distribution of fluctuations.

The probability distribution function (pdf) of the instantaneous experimental and numerical time traces (both URANS k-ω and DES models) for pressure, velocity, and bulk velocity time-traces are compared in Fig. 4.31-4.35. The total uncertainty width, based on the 95% confident measurement uncertainty and precision uncertainty using Eqn. 1.26 is also shown by gray bars. This magnitude of this uncertainty is the same as calculated in
Fig. 4.28: The standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the centerline of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Fig. 4.29: The standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the near side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. 4.30: The standard deviation of the instantaneous bulk velocity time-traces at the upstream and downstream locations for each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Section 4.2.1 but provides a different perspective of the uncertainty. Once again, 95% of the experimental data should lie within the uncertainty bands. When a greater percentage of data is bound by the uncertainty limits, the systematic uncertainties are the dominant sources of error. The DES models with different inlet flow conditions predict similar solutions for all validation SRQ’s and, therefore, to increase figure clarity, the DES-P model is not shown. Variations in the DES models due to inlet perturbations will only be addressed in the text when major differences are observed.

As mentioned earlier, Gaussian distributions are measured for all instantaneous bulk velocity traces (Fig. 4.31-4.32) with consistent fully-developed fluctuation regions near cylinder 2. The measurement uncertainty is dominated by the 95% confidence width for all instantaneous bulk velocity traces. Other than the DES-B model, the instantaneous bulk velocity distributions are within the measurement uncertainty widths; however, at upstream locations (upstream of cylinder 2), all CFD models predict bulk velocity fluctuations nearly Gaussian distributions with a smaller width or fluctuation amplitude than the measurement uncertainty widths.
Fig. 4.31: The distributions of instantaneous bulk velocity time-traces at the upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Downstream of cylinder 2, the DES-NB model experiences a rapid growth of fluctuations and this location marks the onset of the fully-developed fluctuation region. The DES-NB model predicts bulk velocity fluctuation distributions resembling the experimental results downstream of the fully-developed fluctuation region. While the $k$-$\omega$ model consistently under-predicts the bulk velocity distribution width, the DES-B model begins to observe distributions similar to the experimental results downstream of cylinder 3. Both the DES-B and $k$-$\omega$ models predict non-Gaussian distributions for most locations.

The DES-NB, DES-B, and $k$-$\omega$ models predict a bimodal distribution (jet switching) at cylinder 1 for the pressure traces (shown in Fig. 4.33). Less dominant jet-switching is also predicted by the DES-B and $k$-$\omega$ models for cylinders 2 and 3 and wider distributions are predicted by the DES-B model. A single modal distribution is observed for all the cylinders.
Fig. 4.32: The distributions of instantaneous bulk velocity time-traces at the downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
in the experimental pressure measurements; jet switching is never observed. The DES-NB model observes single mode Gaussian distributions with similar fluctuation widths as the experimental model for all cylinders downstream of cylinder 1.

Bimodal pressure distributions were observed both experimentally and numerically by Mahon and Meskell [44, 45] for a tube bundle consisting of several columns and rows of cylinders. They account for the bimodal distribution as a bi-stable flow regime called jet switching, in which the momentum of the fluid fluctuates from side to side as it passes the cylinder generating two flow regimes. The Mahon and Meskell [44] test section was comprised of multiple flow paths through the cylinder array, while the facility in this study had two. Jet switching in the experimental results may have been suppressed due to the narrow channel and two flow paths. This would also explain the lack of jet switching predicted by all models at downstream locations.

It is also noted that the pressure distribution widths for the experiment are similar to those from the DES-NB model, which are wider than the RANS results at all but cylinders 1 and 5. Given that the RANS results contain no turbulent fluctuations, and that DES contains non-turbulent and turbulent fluctuations, this is to be expected.

Non-Gaussian distributions are observed in both the experimental and numerical results for many local, instantaneous velocity traces. While the standard deviation will still describe a general width of the distribution function, information concerning the shape is lost. Instantaneous velocity trace distributions are shown for reference in Fig. 4.34 and 4.35 and in Appendix E. Experimental results measure single mode distributions skewed to the right or left. Instantaneous velocity trace distributions upstream of cylinder 1 and 2 are often narrower than the experimental results and bimodal distributions are once again predicted upstream of cylinder 1 by all CFD models. Skewed distributions are also predicted at several locations for the CFD models.

While the DES-NB velocity distributions do not resemble the measurement results as well as the instantaneous bulk velocity and pressure distributions, similar shapes and skewness is predicted. The DES-B and $k$-$\omega$ models under-predict the distribution widths for
Fig. 4.33: The distributions of instantaneous pressure along the wall along the span-wise centerline of all five cylinders for the experimental and numerical (k-ω, DES-NB, and DES-B) models.
most locations. DES-NB distribution accuracy is most apparent at jet edges and cylinder downstream locations. The decrease in distribution prediction accuracy for instantaneous velocity over instantaneous bulk velocity traces is attributed to less spatial averaging. Spatial and temporal averaging decreases the simulative difficulty of a predictive SRQ.

4.2.4 Trace Frequency Spectra SRQ’s

Not only does a temporally-validated model need to predict the fluctuation amplitudes but it should also be able simulate the frequencies of temporally evolving features such as vortex shedding. The average frequency spectra for each SRQ time-trace is calculated using the average frequency spectra of sets of $N = 512$ samples.
Fig. 4.35: The distributions of instantaneous velocity time-traces on the far and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
the measurement and simulation noise will be reduced, generating a smooth frequency spectra from which single frequencies corresponding to temporal features may be observed. The spectra is calculated using Fast Fourier Transforms (FFT).

Since the PIV measurements are limited to 1500 Hz and \( N = 1024 \) instantaneous velocity fields, pressure measurements were also acquired at the same frequency as the numerical time-step. The spectra of these signals for both the experimental pressure measurements and numerical pressure traces are compared in Fig. 4.36. Fluctuations can be compared using both amplitude and frequency. Of particular interest are the dominate large-scale frequencies resulting from non-turbulent variations, such as vortex shedding behind a cylinder, as these variations should be predicted from all numerical models. The turbulence modeling assumptions inherent in the DES and \( k-\omega \) models make frequency and amplitude spectra an inappropriate SRQ for validation of turbulent fluctuations.

At the location along side cylinder 1, the flow field lacks high frequency fluctuations and is dominated by a single frequency. The largest of these, near \( St = 0.5 \), is well predicted by all numerical models. This is the same frequency at which the drag varies and is likely the shedding frequency of the cylinder, which agrees with the results of Polak and Weaver [46] for the given pitch ratio \( P/D \). Although the experimental results show a slightly lower amplitude, the shedding frequency is well predicted by the DES simulations. A second peak at twice the shedding frequency is also apparent in the experiment and the numerical models, which is to be expected since pressure changes with the square of velocity. Interestingly, the DES-NB model predicts vortex pairing as evidenced by a sub-harmonic, while the experiment has a much broader response at half the shedding frequency.

The pressure at cylinders 2 and 3 in the experiment shows a very broad-band, high-frequency response with no dominant frequency. The attenuated frequency spectrum downstream of cylinder 1 is not observed by Polak and Weaver [46] and may be due to the confined cylinder bundle. The experimental facility in Polak and Weaver [46] contains many rows of cylinders with increased flow paths, inconsistent with the facility in this paper. The numerical models each predict very large peaks at several frequencies. At cylinder 3, the DES-NB
model pressure spectra again resembles the experiment, with broad band response and no dominant frequency (although the experimental and the DES-NB models have a single small peak near St = 1 and 2.5, respectively). The $k$-$\omega$ model predicts dominant frequencies at cylinder 3 not seen in the experiment. Polak and Weaver [46] also observed secondary frequencies at cylinders downstream of cylinder 1, however, Scholten and Murray [48] measure broad-band frequency spectra with dominant frequencies suppressed at downstream locations for heat flux and turbulence SRQ’s. These frequencies are also observed in the DES-B model, but are not seen in the DES-NB results. The very high frequencies in the numerical model are clearly attenuated due to the DES filter. The behavior of the experiment and the DES-NB model are essentially the same for cylinder 3 as cylinder 4. The DES-P model predicts much larger broadband noise at higher Strouhal numbers than the steady DES model, with the spectral power actually increasing near St = 10. For the first four cylinders, the DES-B model, resembles the predictions from the $k$-$\omega$ model more than the DES-NB model.

At cylinder 5, both the experiment and the numerical models show a smaller shedding frequency near St = 0.2. While the $k$-$\omega$ and DES-B models over and under predict, respectively, the amplitude of this frequency, the DES-NB not only predicts the amplitude but also has a similar response up to the DES filter. Additionally, at large frequencies (larger than the DES filter), the DES-NB model accurately predicts the broadband spectra amplitude at cylinders 3 and 4.

Similar results are seen in the power spectrum for the instantaneous velocity traces with examples shown in Fig. 4.37 and 4.38 and further spectra given in Appendix F. Velocity frequency spectra observe increased broadband noise than demonstrated by the pressure spectra, particularly at cylinder upstream locations. Upstream of cylinder 3, all CFD models predict harmonic and subharmonic frequencies with attenuated amplitudes. The DES-NB model accurately predict the broadband spectra amplitude at all locations downstream of cylinder 3.

Dominant frequencies are generally not observed for experimental velocity spectra at
Fig. 4.36: The average power spectra in the frequency domain for the instantaneous pressure along the far wall along the stream-wise centerline of all five cylinders for the experimental and numerical ($k$-ω, DES-NB, and DES-B) models.
cylinder upstream locations; however, at cylinder downstream locations, dominant frequencies are seen primarily in cylinders 1 and 5. Additional, non-physical frequencies upstream of cylinder 3 are predicted by the numerical models that are not observed in the measurements. The experimental dominant frequencies, amplitudes, and broadband noise magnitudes are accurately predicted by the DES-NB model at all cylinders for traces downstream of cylinder 2 up to the DES filter. Interestingly, the power spectrum for instantaneous velocity traces predicted by the DES-P model contains significantly less broadband noise than the pressure traces, and resembles the DES-B model.
Fig. 4.38: The average power spectra in the frequency domain for the instantaneous velocity traces on the far and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Dominant frequencies are removed from the spatial averaging of the experimental instantaneous bulk velocity traces 4.39 and 4.40 resulting in broadband bulk velocity spectra with rapid filtering of small scale turbulence fluctuations. While the CFD models are incapable of attenuating dominant frequencies upstream of cylinder 3, downstream of this position, the majority of dominant spectra are lost through enhance mixing in the DES-NB model. Extremely accurate predictions of the broadband spectra amplitudes are observed at these downstream locations. Both the DES-B and $k$-$\omega$ model under-predict broadband amplitudes and predict additional, non-physical shedding frequencies at all instantaneous bulk velocity trace locations. These additional frequencies are removed at cylinder 5 for the DES-B model.

4.2.5 Trace Autocorrelation SRQ’s

The autocorrelation coefficient $\rho(\tau)$ [42] in time was computed for all traces (Appendix G). The autocorrelation coefficient provides a quantitative approach to determine the autocorrelated time-scale of a particular flow feature. This coefficient is calculated as

$$\rho(\tau) = \frac{g(t)g(t')}{g^2},$$

where $g(t)$ and $g(t')$ are a fluctuating quantity at two different times and $\tau$ is the time difference $t - t'$ and $g(t)g(t')$ is the convolution of the two quantities (i.e. $g(t) * g(t')$). Autocorrelation coefficients in time are calculated for $\tau = 0.04s$ segments at each measurement location and these segments are averaged to obtain an average autocorrelation coefficient. The average autocorrelation coefficient ranges from correlated ($\rho(\tau) = 1$) to anti-correlated ($\rho(\tau) = -1$).

Autocorrelations for two representative trace locations are shown in Fig. 4.41 and 4.42 with additional locations given in Appendix G. Experimental velocity autocorrelations calculate small time-scales for locations upstream of cylinder 1 ($\tau \approx 0.01$ s), whereas downstream time-scales are near ($\tau \approx 0.0025$ s). Small oscillations from correlated to anti-correlated are observed downstream of cylinder 1 and 5, corresponding to vortex shedding.
Fig. 4.39: The average power spectra in the frequency domain for the instantaneous bulk velocity traces on the upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. 4.40: The average power spectra in the frequency domain for the instantaneous bulk velocity traces on the downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
downstream of these cylinders. However, cylinder upstream locations and cylinders 3, 4, and 5 do not observe any residual autocorrelated results past \((\tau \approx 0.005 \text{ s})\). Similar trends are observed for both instantaneous velocity and pressure experimental traces.

At cylinder 1, the DES-B, DES-NB, and \(k-\omega\) models predict oscillations in the autocorrelation (similar in frequency to the experimental results but larger in amplitude), which indicate periodic vortex shedding behind the cylinder. The DES-NB and DES-B models dampen the autocorrelated oscillations as the flow evolves downstream until it reaches an uncorrelated location. This location corresponds to cylinder 3 and 4 for the DES-NB and DES-B models, respectively. Downstream of this position, the models begin to predict autocorrelations similar to the experimental results. The autocorrelation time scales are slightly longer than the experimental time-scales, with the DES-NB model predicting more accurate time-scales. The \(k-\omega\) model predicts highly oscillatory autocorrelations for most cylinders and time-traces. Although all models predict much larger oscillations in the autocorrelation, the DES-NB model dampens these oscillations and predicts autocorrelation values much closer than the remaining models.

No significant differences are observed between the pressure and velocity autocorrelations and are not shown. However, perturbations added to the DES help destroy the periodicity of the pressure and more closely resemble the experiment.

The experimental spatially-averaged bulk velocity autocorrelations (Fig. 4.43 and 4.44) measure larger autocorrelation time-scales \((\tau \approx 0.015 \text{ s at cylinder 1 and } \tau \approx 0.005 \text{ s for the other cylinders})\) than the local, instantaneous velocity traces. Additionally, experimental bulk velocity autocorrelations do not experience autocorrelation oscillations. Similar dampened oscillations are also predicted by the DES-NB model downstream of the upstream side of cylinder 3 with analogous autocorrelation time-scales predicted at these locations. While the \(k-\omega\) and DES-B models predict oscillations at these locations, the DES-B and DES-NB models do not experience oscillations upstream of cylinder 1. However, these models over-predict the autocorrelation time-scale at the inlet.
Fig. 4.41: The average autocorrelation coefficient \( \rho(\tau) \) for instantaneous velocity traces on the centerline and upstream side of each cylinder for the experiment and numerical models (\( k-\omega \), DES-NB, and DES-B).
Fig. 4.42: The average autocorrelation coefficient $\rho(\tau)$ for instantaneous velocity traces on the far and downstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. 4.43: The average autocorrelation coefficient $\rho(\tau)$ for instantaneous bulk velocity traces on the upstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. 4.44: The average autocorrelation coefficient \( \rho(\tau) \) for instantaneous bulk velocity traces on the downstream side of each cylinder for the experiment and numerical models (\( k-\omega \), DES-NB, and DES-B).
4.2.6 Trace Correlation SRQ’s

Correlation coefficients, $P(\tau)$, in time between instantaneous velocity measurement pairs were also calculated as a validation SRQ to compare similarities mirrored on the centerline of the cylinders, such as jet switching, pressure-velocity coupling, and vortex shedding. This coefficient was calculated in a similar manner to the autocorrelation coefficient:

$$P(\tau) = \frac{g_i(t)g_j(t')}{g_ig_j},$$  \hspace{1cm} (4.5)

where $g_i(t)$ and $g_j(t')$ are fluctuating trace pairs at two different times and $\tau$ is once again the time difference $t - t'$.

The average correlation coefficient for several instantaneous velocity and pressure trace pairs are shown in Appendix H with representative results shown in Fig. 4.45-4.47. Little or no correlation is observed in the measurement signals for upstream sides of all cylinders; however, on the downstream side, slight correlated oscillations are measured at cylinders 1 and 5. These oscillations are indicative of vortex pairing downstream of cylinders 1 and 5. The uncorrelated trend is predicted accurately by the DES-NB model following the upstream side of cylinder 3. However, amplified correlated oscillations are apparent at cylinders 1 and 2.

Larger oscillations ranging from correlated to anti-correlated, caused by stronger vortex pairing tendencies, are once again predicted from the DES-B and $k$-$\omega$ models at all downstream locations for the correlation coefficient. Oscillatory correlations between the two measurement pairs are observed at cylinders 1 and 5 for the experimental model, with a smaller frequency at cylinder 5 (also observed in the velocity spectra in Section 4.2.4). While the oscillations predicted by the numerical models at these locations have similar frequencies, the amplitudes are larger in all cases except the DES-NB model at cylinder 5. At cylinder 3, 4, and 5, a reasonable representation of the correlation coefficient is predicted by the DES-NB.
Fig. 4.45: The average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near, upstream sides of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. 4.46: The average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near, downstream sides of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
The coupling of instantaneous pressure and velocity was also examined using the average correlation coefficient (Fig. 4.47). Similar trends to the vortex pairing correlations are observed with the exception of decreased correlation oscillations beginning at cylinder 3 for all CFD models.

4.2.7 Minor Loss Factor SRQ’s

It is also interesting to examine time-averaged global SRQ’s, such the minor loss factor

\[
k = \frac{\Delta p}{(0.5 \rho f u_{\text{max}}^2)},
\]  

(4.6)
Table 4.3: The minor loss factor for the experimental and numerical models ($k$-$\omega$, DES-NB, and DES-B). The steady RANS results from Hodson et al. are also given for the 2D $k$-$\epsilon$, $k$-$\omega$, and $v^2f$ models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Minor Loss Factor $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>0.239</td>
</tr>
<tr>
<td>$k$-$\omega$ (URANS)</td>
<td>0.220</td>
</tr>
<tr>
<td>DES-NB</td>
<td>0.224</td>
</tr>
<tr>
<td>DES-B</td>
<td>0.234</td>
</tr>
<tr>
<td>$k$-$\epsilon$ (Steady RANS)</td>
<td>0.23</td>
</tr>
<tr>
<td>$k$-$\omega$ (Steady RANS)</td>
<td>0.20</td>
</tr>
<tr>
<td>$v^2f$ (Steady RANS)</td>
<td>0.23</td>
</tr>
</tbody>
</table>

where $n$ is the number of cylinders (8 in this case). The experimental minor loss factor for this Reynolds number is $k = 0.239$. The DES-NB, DES-B, and $k$-$\omega$ models slightly under-predicted the minor loss factor (given in Table 4.3). Several 2-D steady RANS models studied earlier (Hodson et al. [69]) predicted values in the range $0.16 < k < 0.23$ are also shown in Table 4.3. Insignificant differences are observed between DES, URANS, and steady RANS models. Thus, prediction of minor loss factors and time-averaged pressure drop can be efficiently obtained using steady RANS models.

### 4.2.8 Inlet Profile Effects

The sensitivity of velocity and turbulence inlet profiles on CFD results was investigated for the DES-NB and $k$-$\omega$ models. Two variations of the models (uniform inlet and measured inlet) were simulated using the commercial software Star-CCM [70] from CD-Adapco. The models were implemented in the same manner as given in Section 3.2.2 with the only difference being the velocity and turbulence profiles. The uniform inlet profile cases match the previous simulations from FLUENT [61] and are prescribed a mean velocity values corresponding to the experimental facility. The measured inlet cases use non-uniform local profiles measured by PIV at the facilities inlet. The inlet profiles prescribed in the CFD models are shown in Fig. 4.48.

Comparison of SRQ’s from the uniform inlet and measured inlet cases demonstrated the sensitivity of these SRQ’s to the inlet conditions. SRQ locations are located on the
Fig. 4.48: The profiles of the stream-wise $u_x$ and cross-stream $u_y$ components of velocity and turbulence intensity $I$ as measured in the experimental facility. These profiles were used to define the inlet conditions for the measured inlet $k$-$\omega$ and DES models.
far side of the up- and downstream sides of the cylinders. Trace locations along with mean instantaneous velocity traces are shown in Fig. 4.49 with the standard deviation or fluctuation amplitude of the instantaneous velocity traces given in Fig. 4.50. Slight variations are observed in the mean velocity, but larger differences are predicted for the fluctuation amplitudes or standard deviations. The largest differences occur at up- and downstream locations of cylinder 4. Overall, the DES and $k$-$\omega$ models both predict similar means and fluctuation amplitudes.

Additionally, the frequency spectra through the facility was compared (Fig. 4.52 and 4.53). Similar dominant frequencies and spectra amplitudes are observed for all cylinders. Differences in high frequency spectra amplitudes are evident at cylinders 1 and 2 for the measured inlet cases and can be attributed to larger turbulence fluctuations due to the increased turbulence intensity profiles for these cases. However, as the mixing and 3D motions increase at downstream locations, this increased high frequency amplitude begins to diffuse and resemble the uniform inlet case. Particular agreement is evident for the downstream cylinders using the DES models.

Although slight differences occur between both the $k$-$\omega$ and DES models when uniform
Fig. 4.50: The standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the far side of each cylinder for the numerical (\(k-\omega\) and DES) models given uniform and measured inlet profiles.

and measured inlet profiles are applied to the CFD models, the amplitudes of fluctuations and frequency spectra agree with each other. Cylinder locations downstream of cylinder 2 tend to have increased agreement due to enhanced mixing and 3D motions. Mean and standard deviation amplitudes are similar throughout the facility. Due to the small SRQ differences, uniform inlet profile cases predict results similar to the measured inlet profile cases and may be accurately implemented to assess model validation.

### 4.2.9 Numerical Time-Step and Grid Convergence

Solutions were also calculated at a decreased time-step of \(\delta_t = 5 \times 10^{-5}\) s (half the previous value) to determine the sensitivity of the solution to time-step and time-step convergence. Dominant frequencies predicted by both time-step cases (e.g. \(\delta_t = 5 \times 10^{-5}\), and \(1 \times 10^{-4}\) s) are given in Fig. 4.54. The solution of the \(\delta_t = 5 \times 10^{-5}\) s case was obtained for a shorter temporal interval than the larger time-step cases. However, the same number of samples were used to calculate the average power spectra for both time-step cases. Solutions for both cases predict the same dominant frequencies with similar amplitudes, demonstrating time-step convergence between the two solutions at the individual frequency level. For
Fig. 4.51: The average power spectra in the frequency domain for the instantaneous velocity traces on the far and upstream side of each cylinder for the numerical (k-ω and DES) models given uniform and measured inlet profiles.
Fig. 4.52: The average power spectra in the frequency domain for the instantaneous velocity traces on the far and downstream side of each cylinder for the numerical ($k-\omega$ and DES) models given uniform and measured inlet profiles.
Fig. 4.53: The average power spectra in the frequency domain for the instantaneous pressure along the far wall along the stream-wise centerline of all five cylinders for the numerical ($k$-$\omega$ and DES) models given uniform and measured inlet profiles.
cylinders 1-4, much more broadband noise is observed near the dominant frequencies for the $\delta_t = 1 \times 10^{-4}$ s case than predicted by the smaller time-step and may be attributed to the increased temporal resolution in the $\delta_t = 1 \times 10^{-4}$ s case. However, amplitudes of the dominant frequencies located at cylinder 5 are attenuated for the $\delta_t = 5 \times 10^{-5}$ s case.

Quantification of grid convergence observed in the $k$-$\omega$ model is accomplished on three mesh sizes (in our case $\approx 1.758$, 3.098, and 6.196 million cells) using the Grid Convergence Index (GCI) as outlined by Celik et al. in [65]. The use of unstructured meshes limits the ability to achieve uniform grid refinement; however, each mesh was designed to have
approximately double the number of cells than the coarser meshes with the mesh refinement being maintained as uniform as possible. As a measure of grid refinement, the grid refinement factor should be maintained at $\gamma = h_{\text{coarse}}/h_{\text{fine}} \geq 1.3$, where the average grid size for mesh being considered is denoted by $h$. For the global SRQ’s considered, the average grid size for the respective mesh is used, giving $\gamma \approx 1.52$.

In the GCI method, the apparent rate of convergence, $q$, is obtained through the expression,

$$q = \frac{1}{\ln \left( \gamma_{21} \right)} \left| \ln \left( \frac{\alpha_{32}}{\alpha_{21}} \right) + c(q) \right|,$$

where $\alpha_{32} = \phi_3 - \phi_2$ and $\alpha_{21} = \phi_2 - \phi_1$ with $\phi$ designating the validation SRQ and the subscripts 1, 2, and 3 denote SRQ from a particular mesh size from smallest to largest. Due to non-integer grid refinement, the GCI was calculated using the iterative process outlined in [65] with two additional equations:

$$c(q) = \ln \left( \frac{\gamma_{21}}{\gamma_{32}} \right) - s,$$

and

$$s = 1 \cdot \text{sign} \left( \frac{\alpha_{32}}{\alpha_{21}} \right).$$

The rate of convergence, $q$, measures the observed rate at which meshes of increasing refinement approach the model solution. Theoretically, the rate of convergence of a linear numerical model should match the order of accuracy of the model. However, observed convergence rates for nonlinear, time-dependent models may be inconsistent with the model’s order of accuracy. For this case, discretization of the $k$-ω model should be second order accurate temporally and spatially; however, the observed rate of convergence for the $k$-ω model varied between $0.3 \leq q \leq 12.6$ depending on the validation SRQ assessed with the majority of the results experiencing oscillatory convergence. This corresponds to GCI ranging from 0.01 to about 3.9 or relative discretization uncertainty of the numerical model SRQ’s of 1% to 390%. The relative discretization uncertainty was below 55% for
the majority of validation SRQ’s (60%). Individual GCI will be reported and discussed later. Using the method outlined by [65], discretization errors and GCI values increase and are inappropriate for validation SRQ’s “near zero”; for these $k$-$\omega$ model simulations, large GCI ($\approx 0.75$ or larger) are calculated for SRQ’s with magnitudes “near zero” due to larger relative differences between refined solutions.

Grid convergence among global SRQ’s (i.e. minor loss factor) was observed more frequently than among the downstream and local, temporal SRQ’s. Uncertainty on the minor loss factor predicted by the $k$-$\omega$ model was computed within $k = 0.234 \pm 0.032$, based on a GCI value of 0.14.

Although accurate solutions are often not observed in the local, temporal validation SRQ’s, solutions obtained on refined $k$-$\omega$ model meshes tend to approach the results observed from the experiments. The fluctuation amplitudes (as predicted by the $k$-$\omega$ models and shown in Fig. 4.55 and Appendix I) have a similar trend for each grid. The fluctuation amplitudes increase until cylinder 3 after which they remain similar in magnitude. The trend was similar for the experiments. For all except the cylinder 5 location, the fluctuation size increases as the mesh size increases. While all meshes predict fluctuation amplitudes nearly half the size of the experimental fluctuations, as the mesh size increases, the fluctuation amplitudes increase and begin to approach the experimental results. Normalized uncertainties for these SRQ’s were typically around 0.03 with a maximum of 0.077 upstream of cylinder 2 and a minimum of 0.001 upstream of cylinder 1.

Little variation is observed in the dominant frequencies predicted by the three $k$-$\omega$ meshes as shown in Fig. 4.56. Grid convergence results for the frequency SRQ’s were unable to be assessed due to inadequate frequency resolution from the fourier transform. While it is difficult to distinguish differences in dominant Strouhal numbers, the amplitude of these frequencies seems to decrease as the mesh is refined, better modeling the experimental results. In particular, the coarse and medium meshes predict harmonic frequencies downstream of cylinder 3 and 4 which are not observed in the fine mesh $k$-$\omega$ and experimental models. GCI for the amplitudes of the first three harmonics were assessed. Throughout the
Fig. 4.55: Standard deviation of the instantaneous bulk velocity at the upstream and downstream locations for each cylinder used for the grid convergence study. The experimental results are provided as a reference in light gray.

Table 4.4: The uncertainty of the amplitude for the first harmonic frequency for the instantaneous velocity traces downstream of each cylinder on the opaque wall side of the channel.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty (m/s)</td>
<td>±0.036</td>
<td>±0.562</td>
<td>±0.005</td>
<td>±3.348</td>
<td>±19.74</td>
</tr>
</tbody>
</table>

cylinder array, the average uncertainty for the amplitude was 0.99m/s ($St_1$), 0.11m/s ($St_2$), and 0.01m/s ($St_3$) for the first three harmonics denoted by ($St_1$), ($St_2$), and ($St_3$). The uncertainty on the frequency amplitudes increases at downstream locations; as an example, the uncertainty for the first harmonic for the instantaneous velocity traces downstream of each cylinder on the opaque wall side of the channel are given in Table 4.4.

While similar integral time-scales are observed for the autocorrelations for each mesh (shown in Fig. 4.57 and 4.58 with additional time-traces given in Appendix 4), the oscillations dampen as the mesh size increases providing a more representative solution to the experimental results. The average uncertainty on autocorrelation validation metrics increased at downstream locations, particularly locations near cylinders 3 and 4. Uncertainties for first peak within the autocorrelation for instantaneous velocity traces for up-
Fig. 4.56: The average power spectra in the frequency domain for the instantaneous velocity traces downstream of each cylinder on the opaque wall side used for the grid convergence study. The experimental results are provided as a reference in light gray.
Fig. 4.57: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity upstream of each cylinder near the transparent wall used for the grid convergence study. The experimental results are provided as a reference in light gray.

and downstream traces on the transparent wall side for each cylinder are provided in Table 4.5. Similar results were observed for the correlation time-scales (Appendix K). Improved resolution of turbulent structure and error propagation from upstream sources are causes for difference in refined mesh solutions. These will be discussed in more detail later.

Upstream validation SRQ’s tended to have higher rates of convergence than SRQ’s at downstream locations. This, perhaps, was due to the increased size of fluctuations present at downstream locations and the inability of the coarser meshes to resolve high-frequency motions. Small variations in mesh solutions at upstream locations also contribute to the differences in downstream error and rates of convergence. Downstream solutions evolve
Fig. 4.58: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity downstream of each cylinder near the transparent wall used for the grid convergence study. The experimental results are provided as a reference in light gray.

Table 4.5: Uncertainties for the first peak within the autocorrelation for instantaneous velocity traces for each cylinder on the transparent wall side of the channel.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream Uncertainty</td>
<td>$\pm 0.007$</td>
<td>$\pm 0.09$</td>
<td>$\pm 0.48$</td>
<td>$\pm 1.35$</td>
<td>$\pm 0.06$</td>
</tr>
<tr>
<td>Downstream Uncertainty</td>
<td>$\pm 0.006$</td>
<td>$\pm 0.36$</td>
<td>$\pm 0.72$</td>
<td>$\pm 1.12$</td>
<td>$\pm 0.06$</td>
</tr>
</tbody>
</table>
from nonlinear upstream conditions and any mesh related error in the upstream flow field propagates into the downstream solution, generating a different solution. The grids used on these models are adequately refined and allowed to evolve in time sufficiently to remove any initial solution effects; however, effects of the unsteady, nonlinear model and upstream error propagation into downstream solutions are observed in the grid convergence. Complications such as these are often observed for grid convergence studies of unsteady, nonlinear models, promoting improved methods for quantifying temporal accuracy. Methods of quantifying numerical uncertainty due to temporal discretization and for sub-grid models (i.e. DES and LES models) must also be developed.

Improved solution convergence is also observed for several cases downstream of cylinder 5. This may be caused by the introduction of a more dominant phenomena (vortex shedding behind a single cylinder) that the $k$-$\omega$ model is capable of easily resolving. This phenomena may be less sensitive to upstream solutions and error propagation, forcing an accurate grid converged solution at these locations.

4.2.10 Discussion of Temporal Validation SRQ’s

Several global and local SRQ’s are suggested for spatial and temporal validation of the array of cylinders in a channel. Application of the proposed validation SRQ’s to other validation experiments is possible and the advantages of each SRQ as a means of validation assessment is discussed. SRQ’s defining temporal variations include amplitude and frequency of temporal variations, and autocorrelation and correlation time-scales of flow features. These are analyzed through time traces, statistics (such as standard deviation), histograms, Fourier transforms, autocorrelations, and correlations.

Experimental and numerical amplitudes of non-turbulent time-varying quantities in the flow are compared using four SRQ’s: time traces, standard deviation of bulk velocities, histograms, and Fourier transforms. While time traces provide a visual comparison of amplitude and frequency, this SRQ remains qualitative and subjective suggesting the need for more robust SRQ’s. The standard deviation of the amplitudes provides a quantitative approach of comparison by assigning a value to the size of the amplitude; however, the
shape of the distribution is lost using this SRQ. As in the case of jet switching, numerical models may predict similar standard deviations as experimental results; however, this may be coincidental as distributions of the variation in the numerical and experimental results may be fundamentally different. Time-variation distributions using histograms are used to remedy this shortcoming. Higher order statistical SRQ (i.e. skew and kurtosis) may also be used as distributional validation SRQ’s.

Fourier transforms provide an alternative SRQ for which the distribution of amplitudes of a time-varying SRQ may be assessed. The amplitudes of each individual frequency and dominant frequencies present in a varying SRQ can be compared between numerical and experimental results. While this provides a powerful method to compare amplitude and frequency of large and intermediate scale temporal variations, the manner in which turbulent fluctuations are calculated must be taken into consideration as amplitude and frequency may be an inappropriate SRQ for some models at small temporal scales and turbulent fluctuations.

Autocorrelations and cross correlations are also analyzed as temporal validation SRQ’s and demonstrate impressive utility for comparison of autocorrelation and correlations time-scales of fluctuating features. Considering that larger-scale variations have more impact on autocorrelation and correlation results than small-scale variations, autocorrelations and correlations are advantageous over Fourier transforms in determining time-scales. However, unlike Fourier transforms, dominant frequencies and amplitudes of temporal variation are not assessed.

Special consideration of the nature of each numerical model and specific features contained in the flow must be accounted for when developing temporal validation SRQ’s.
Chapter 5
Conclusions

A validation study has been performed on a confined bank of cylinders with special attention given to temporal validation. Experimental results are obtained using low and high speed PIV systems along with time-varying pressure measurements along the test section walls. Two variations of the DES model (DES-NB and DES-B) and a URANS model ($k$-$\omega$ with three mesh sizes) are implemented and model the experimental model conditions as accurately as possible. Two cases for the inlet conditions (steady and vorticity perturbations) on the DES models are applied. Numerical and experimental results are compared to assess the CFD model. Grid convergence of the $k$-$\omega$ model is assessed.

5.1 PIV Uncertainty Conclusions

Acquisition of experimental results and implementation CFD models adhere to the validation guidelines proposed by Sandia National Laboratory [11] and Lee and Bauer [10]. These include the use of non-invasive measurement techniques, such as PIV, and quantification of all measurement uncertainties and numerical error. An additional study quantifying dominant PIV error sources (particle image density, displacement, and diameter and flow gradients) effects on instantaneous, mean, and fluctuating components of velocity is demonstrated to supplement the validation results.

PIV errors generated from four sources (1) particle image displacement, 2) particle image density, 3) particle image diameter, and 4) flow gradients) are shown using PIV and hot wire data of a planar jet generated from a laminar channel flow. The effects of PIV uncertainty on turbulent statistics, such as fluctuations, are also assessed. The instantaneous, local PIV uncertainty estimation technique proposed by Timmins et al. [51] is used to quantify instantaneous, local uncertainties. Two PIV algorithms are demonstrated (PRANA and
DaVis 7.2), and similar error trends would be expected from other PIV algorithms. Uncertainty quantification for other algorithms would require independent algorithm-specific estimates of uncertainty. The results improve the application of PIV to M&S validation by better quantifying uncertainties on mean statistics and exposing increased errors on measured fluctuations.

The four sources studied generate errors affecting the rate at which converged statistics are acquired. These errors fluctuate on both sides of the mean and, when sufficient amounts of data are obtained, a converged mean that agrees with the hot wire solution is measured. However, the random measurement errors increase the measured velocity fluctuations reported by the PIV measurement system. These effects are observed for all four cases studied. Of the these, velocity gradients are the largest contributor to PIV error. Error estimation using results from Timmins et al. agree with the errors observed by the PIV algorithm.

Although converged results occur for particle image density and most mean velocity measurements, the three remaining error sources demonstrate the potential of strongly elevating fluctuation levels. This is significant due to turbulence statistics and quantities being calculated from fluctuations. Converged measured fluctuation are observed for particle image displacements larger than $\Delta x \geq 8$ pixels for PRANA and $\Delta x \geq 10$ pixels for DaVis. Optimal fluctuation results were also demonstrated for smaller particle sizes with an optimum near $d_r \approx 2.9$ pixels. This differs from optimal particle image diameters of $d_r \approx 2$ pixels found in the literature due to the inability of the experimental facility to provide particle image diameters smaller than $d_r \approx 2.9$ pixels without significant lens aberrations.

The most significant source of error was found to be gradients within the flow, in which fluctuation levels were always elevated, particularly as the gradients increased. The method from Timmins et al. incorporated errors from instantaneous, turbulent flow gradients. Only considering time-averaged gradients and neglecting gradients from local turbulence would underestimate the elevated fluctuation levels. Flow gradients were also demonstrated to be the dominant source of error for all cases except small particle image
displacements ($\Delta x \leq 1.1$ pixels).

The automatic uncertainty quantification method introduced by Timmins et al. [51] was capable of accounting for the mean and fluctuation errors for most cases.

5.2 Flow through a Bank of Cylinders CFD Validation Conclusions

Time-resolved measurements along with simulated pressure and velocity signals demonstrate the ability of the DES-NB and DES-B model to roughly predict the amplitudes of velocity and pressure fluctuations. The URANS $k$-$\omega$ model predicts a much more unstable (and therefore periodic) flow with smaller fluctuation amplitudes in the flow. Although the amplitudes of the fluctuations are predicted well by the DES-B model, the flow tends to be more periodic and more similar to the $k$-$\omega$ model than the experimental results. Jet switching is predicted in all numerical models, however, this phenomenon is not observed in the experiment. The frequencies present due to vortex shedding and other unsteady phenomena are compared by a power spectra.

Both the DES and $k$-$\omega$ models accurately predict frequencies present in the first and fifth cylinders; however, frequencies not present in the second through fourth cylinders of the experimental model are observed in the numerical models. These frequencies are much smaller for the DES-NB and provide accurate predictions of the amplitude and frequency of the dominant Strouhal numbers. Autocorrelation and correlation coefficients between velocity and pressure traces are also calculated as other validation SRQ’s. Once again, the DES-B and $k$-$\omega$ model predicts flow much more periodic and with larger time-scales than the experimental model; in most cases, the DES-NB model predicts coefficients similar to those seen in the experiment.

Using the GCI method (proposed by Celik et al. [65]), grid convergence is analyzed for the $k$-$\omega$ model. For global SRQ’s, such as the minor loss coefficient and bulk velocity, increased grid convergence is observed, with discretization errors ranging from 0.05 to 0.7. Grid convergence among localized, temporal validation SRQ’s is difficult; this is also observed for downstream SRQ’s. This may be explained by variations in refined solutions at upstream locations evolving and propagating into the downstream solution. Although
the majority (60%) of GCI values for these validation SRQ’s lie below 55%, GCI vary from $0.03 \leq GCI \leq 4.10$. Considering grid convergence of downstream and localized, temporal validation SRQ’s from unsteady, nonlinear models is often difficult to achieve, improved methods for quantifying temporal accuracy are required.

In spite of the surprisingly good performance of the DES-NB model to predict point behavior in velocity and pressure in several locations, we find that its prediction of a steady global parameter (the minor loss factor) to be no better than those predicted with steady RANS models. Similarly, the DES-NB model also observes increased accuracy for higher-order validation SRQ’s (i.e. local frequencies and correlation coefficients); however, sufficiently accurate results are obtained from the DES-B and $k-\omega$ models. This promotes the importance of understanding the applications of the numerical model and the level of accuracy and validation SRQ’s required for model validation.

Several validation SRQ’s have been suggested and the advantages and disadvantages to each are discussed. Inadequate temporal validation assessment is achieved by time traces of time-varying SRQ’s and require more robust SRQ’s. While the spread of temporal variation amplitudes can be described by the standard deviation, shapes of amplitude distributions may are better quantified using histograms. Frequency and amplitude spectra of temporal variations are suitable for large and intermediate time-scales, however, irrelevant for small scale variations and turbulence fluctuations for most CFD models. Autocorrelations and correlations are capable of determining time-scales of specific flow features, self similarity, and similarities between two SRQ’s. Relevant temporal validation SRQ’s are dependent on experimental capabilities, flow features within the system, and turbulent modeling assumptions within the numerical model.

Improved methods of quantifying numerical uncertainty for spatial discretization of unsteady, nonlinear models, temporal discretization, and sub-grid models (i.e. DES and LES models) must be developed for proper validation assessment of temporal and unsteady models.
References


[54] Westerweel, J., “Theoretical analysis of the measurement precision in particle image velocimetry,” 


Appendices
Appendix A

Effect of Errors on Means and Fluctuations

Errors will vary between measurement system, experimental facility, and other factors; however, the method of estimating the uncertainty from these errors can be generalized and will be demonstrated within this Appendix. Of particular interest, this Appendix will derive uncertainties for fluctuating quantities, such as turbulence. While uncertainty estimates are fairly well understood for mean components, the uncertainty on fluctuating components is often misunderstood or ignored.

Any time-varying quantity $\phi$ can be decomposed into a mean component $\overline{\phi}$ and a fluctuating component $\phi'$: $\phi = \overline{\phi} + \phi'$. When this time-varying quantity is measured by any measurement system, systematic $\beta_{\phi_e}$ and random errors $\epsilon_{\phi_e}$ are introduced to the “true” measured quantity to make up the time-varying measured quantity $\phi_e$ [22]. These errors have both direction and magnitude and their impact on the measured time-varying quantity can be visualized in Fig. [A.1]

The expansion of this measured quantity will look as follows:

$$\phi_e = \overline{\phi} + \beta_{\phi_e} + \phi' + \epsilon_{\phi_e} = \overline{\phi} + \beta_{\phi_e} + \hat{\phi}_e,$$  \hspace{1cm} (A.1)

where the “true” and random measurement errors make up the measured fluctuating term ($\hat{\phi}_e = \phi' + \epsilon_{\phi_e} = \phi_e - \overline{\phi}_e$). The difference between the measured and “true” quantity make up the instantaneous error $\delta_{\phi_e}$ on the instantaneous measured quantity. For cases when the error on the instantaneous quantity is known the instantaneous error can be calculated from the sum of the systematic and random errors:

$$\delta_{\phi_e} = \phi_e - \phi = \beta_{\phi_e} + \epsilon_{\phi_e}.$$  \hspace{1cm} (A.2)
Fig. A.1: The components of a measured quantity, \( \phi_e \), composed of a mean component, \( \bar{\phi} \), a fluctuating component, \( \phi' \), systematic error, \( \beta_{\phi_e} \), and random error, \( \epsilon_{\phi_e} \). The fluctuating terms ("true" fluctuations and random error) make up the total measured fluctuations, \( \hat{\phi}_e \), while the measured mean, \( \bar{\phi}_e \), is composed of the "true" mean and systematic error. In this figure the "true" SRQ is shown in purple but the measured value is given in brown.

Generally, both magnitude and direction of the error are unknown and uncertainty estimates must be used to quantify the error range. Uncertainty quantification estimates the range (specified by a confidence interval, C.I.) that the actual error lies within. Similar to error, uncertainties are composed of bias \( b_{\phi_e} \) and random \( r_{\phi_e} \) uncertainties. The total uncertainty on the instantaneous quantity is expressed as,

\[
U_{\phi_e} = \sqrt{b_{\phi_e}^2 + r_{\phi_e}^2} \approx \delta_{\phi_e},
\]

where the bias and random uncertainties assure the error is probably within the given confidence interval. To maintain consistency with ASME, a 95% confidence interval is used within this dissertation.

Typically, time-averaged quantities \( \bar{\phi}_e \) or statistics are of more interest than instantaneous quantities \( \phi_e \). The mean value can be expressed in integral form or discretely:

\[
\bar{\phi}_e = \frac{1}{T} \int_0^T \phi_e dt \approx \frac{1}{N} \sum_{i=1}^N \phi_{e,i}.
\]
Due to the discrete nature of experimental measurements, the discrete form is most often used.

The effects of systematic and random errors on the mean quantity can be clearly demonstrated with the time-average of the instantaneous measurement. The time-average of the measured mean quantity is expressed as,

$$\overline{\phi_e} = \frac{1}{T} \int_0^T \left( \overline{\phi} + \beta_{\phi_e} + \phi' + \epsilon_{\phi_e} \right) dt.$$  

(A.5)

If the time-scale or number of samples $N$ is sufficiently large, the time-average of both “true” fluctuations $\phi'$ and random errors $\epsilon_{\phi_e}$ goes to zero. This gives a simplified expression for the time-averaged measurement error (where the time-averaged measurement error $\delta_{\phi_e}$ is the difference between the measured mean $\overline{\phi_e}$ and the “true” mean $\overline{\phi}$) as a function of systematic error only:

$$\delta_{\phi_e} = \overline{\phi_e} - \overline{\phi} = \beta_{\phi_e},$$  

(A.6)

where the bars over the variables denote time-averaged quantities.

Although simple, Eqn. (A.6) does not provide an uncertainty estimate. This is due to the time-averaging of the fluctuating components. While the systematic error $\beta_{\phi_e}$ remains constant when time-averaged, the ‘true” fluctuations $\phi'_e$ and random measurement errors $\epsilon_{\phi_e}$ will only reduce to zero when sufficiently long time-scales and number of samples are acquired. This is often not possible in an experimental setting and an insufficient number of data samples will generate an additional error term. This additional term is the precision uncertainty $p_{\phi_e}$:

$$p_{\phi_e} = \pm t_{C.I.,\nu}s_{\phi_e}/\sqrt{N},$$  

(A.7)

where $t_{C.I.,\nu}$ is the $t$-statistic describing the desired confidence interval and $s_{\phi_e}$ is the standard deviation of the samples. The precision uncertainty is statistically derived and requires independent measurement samples. As can be seen, the precision uncertainty reduces by
1/\sqrt{N}$ with the number of samples. Therefore, increasing the amount of data acquired will decrease the magnitude of the precision uncertainty. A sufficient number of samples is achieved when the precision uncertainty is reduced below or to the same magnitude as the systematic uncertainty.

Random uncertainties are generated by variations between individual measurements and described statistically; variations are centered around a mean value and defined by a probability distribution function (pdf). It is often assumed these fluctuations follow a normal or Gaussian distribution. When normally distributed, the distribution of the parent population of values may be described by the parent mean $\mu$ and standard deviation $\sigma$ \[22\].

In Eqn. A.7 the variable $t_{C.I.,\nu}$ is the $t$-statistic from the $t$-distribution and depends on the confidence interval C.I. and the degrees of freedom $\nu$ or number of independent samples acquired. When a sufficient number of samples are acquired, the $t$-distribution can be represented using the normal distribution. Per ASME guidelines \[6\], a 95% confidence interval is used, meaning there is 95% confidence the parent mean $\mu$ is within the interval $\overline{\phi}_e \pm p_{\overline{\phi}_e}$. For a 95% confidence interval the corresponding $t$-value is given as $t_{C.I.} = t_{95\%} \approx 1.96$ using a normal distribution \[6,22\].

For any measurement, a finite number of samples $N$ have been acquired and only an estimate of the parent mean and standard deviation may be calculated by a sample mean $\overline{\phi}_e$ and standard deviation $s_{\phi_e}$. The estimated sample mean is calculated by Eqn. A.4 while the sample standard deviation is calculated through the following equation,

$$s_{\phi_e} = \frac{1}{N-1} \sum_{i=1}^{N} (\phi_{e,i} - \overline{\phi}_e)^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\hat{\phi}_{e,i})^2.$$ \hspace{1cm} (A.8)

Expanding the uncertainty estimate of total error on the mean quantity to include precision uncertainty, the total uncertainty on a mean quantity is expressed via the root sum of all systematic and precision uncertainties:

$$U_{\overline{\phi}_e} = \sqrt{\sum_{i=1}^{k} b_{\phi_e,i}^2 + \left(t_{C.I.,\nu} \frac{s_{\phi_e}}{\sqrt{N}}\right)^2} \approx \delta_{\overline{\phi}_e},$$ \hspace{1cm} (A.9)
with $k$ denoting the number of systematic uncertainties on the mean. Bias on the mean is calculated using the root mean square of all instantaneous systematic uncertainties \[ b_{\sigma_e} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} b_{\phi_e,i}^2}. \] (A.10)

It should be noted that, when using Eqn. A.9, the random and systematic uncertainties should be quantified at the same confidence interval.

While the effects of systematic and random measurement errors on the mean quantity are well understood, the manner in which they affect the time-averaged fluctuations is complicated. Once again, error effects on the time-averaged fluctuations can be understood through the time-average of the square of the measured fluctuating term:

\[
\overline{\phi_e \phi_e} = \frac{1}{T} \int_0^T (\phi^' + \epsilon_{\phi_e})^2 dt = \frac{1}{T} \int_0^T (\phi^' \phi^' + \epsilon_{\phi_e} \epsilon_{\phi_e} + 2 \phi^' \epsilon_{\phi_e}) dt.
\] (A.11)

Assuming the `true` fluctuations $\phi^'$, and random measurement errors $\epsilon_{\phi_e}$ are uncorrelated, as is often the case, the cross-terms do not contribute to the time-averaged measured fluctuations. This is also true for a full expansion which can be made with the time-average of the full measured term $\overline{\phi_e^2}$. For the full expansion, cancellation of cross-terms and non-fluctuating terms will provide the same result as the previous equation. Eqn. A.11 may be simplified by understanding the time-averaged random measurement error term, $\overline{\epsilon_{\phi_e} \epsilon_{\phi_e}}$ is also the variance of the random measurement error $s_{\epsilon_{\phi_e}}^2$ where $s_{\epsilon_{\phi_e}}$ is just the standard deviation of the random measurement error. This simplifies Eqn. A.11 to the following form:

\[
\overline{\phi_e \phi_e} = \overline{\phi^' \phi^'} + s_{\epsilon_{\phi_e}}^2.
\] (A.12)
When the “true” fluctuations $\bar{\varphi}^\prime \varphi^\prime$ are subtracted from both sides, the measured fluctuation error ($\delta_{\phi_e \phi_e} = \bar{\phi}_e \bar{\phi}_e - \bar{\varphi}^\prime \bar{\varphi}^\prime$) is simplified to a function of random error fluctuations:

$$\delta_{\phi_e \phi_e} = \bar{\phi}_e \bar{\phi}_e - \bar{\varphi}^\prime \bar{\varphi}^\prime = s^2_{\phi_e \phi_e}. \quad (A.13)$$

Considering the variance of the random measurement error is always positive, it can be seen that the time-averaged measured fluctuation error is always elevated and in the negative direction. While this is typically true, measurement systems with an inadequate time-response will attenuate the “true” fluctuation signal and generate a positive time-averaged measured fluctuation error. This additional fluctuation error source is not considered within this study.

The size of the time-averaged measured fluctuation error $\delta_{\phi_e \phi_e}$ can be understood through the expansion of the time-averaged random error fluctuations. With any measured quantity, measurements are not continuous but acquired at discrete times, converting the time integral to a discrete summation:

$$\delta_{\phi_e \phi_e} = s^2_{\phi_e \phi_e} = \frac{1}{T} \int_0^T (s^2_{\phi_e \phi_e}) \, dt \approx \frac{1}{N} \sum_{i=1}^N (s^2_{\phi_e \phi_e,i}). \quad (A.14)$$

Often the magnitude of multiple components, such as velocity, is desired. When considering the fluctuations on the velocity magnitude, the random measurement errors $\epsilon_{\phi_e}$ are expanded into both random measurement error components ($s^2_{\epsilon_{\phi_e},x} = s^2_{\phi_{ex},x} \epsilon_{\phi_{ex}}$ and $s^2_{\epsilon_{\phi_e},y} = s^2_{\phi_{ey},y} \epsilon_{\phi_{ey}}$):

$$\epsilon_{\phi_e} \epsilon_{\phi_e} = (\epsilon_{\phi_{ex}} + \epsilon_{\phi_{ey}})^2 = s^2_{\epsilon_{\phi_e},x} + s^2_{\epsilon_{\phi_e},y} + 2 \epsilon_{\phi_{ex}} \epsilon_{\phi_{ey}}. \quad (A.15)$$

The last term in Eqn. (A.15) contains any correlated errors between the two components. Thus, if an estimate of the variance of the random measurement errors ($s^2_{\epsilon_{\phi_e},x}$ or $s^2_{\epsilon_{\phi_e},y}$) is
available, the total fluctuation uncertainty can be calculated.

Once again, the magnitude and direction of the instantaneous random measurement error $\epsilon_{\phi_e}$ and variance of the random measurement error $s_{\epsilon_{\phi_e}}^2$ is unknown and the time-averaged measured fluctuation error must be estimated. First, the instantaneous random uncertainty can be used to quantify the instantaneous random error (i.e. $\epsilon_{\phi_e} \approx r_{\phi_e}$) and the time-averaged measured fluctuation error is estimated with the time-averaged measured fluctuation uncertainty ($s_{\epsilon_{\phi_e}}^2 \approx r_{\phi_e \phi_e}$). Contrary to the uncertainty on the mean, the time-averaged measured fluctuation uncertainty is calculated through the root mean square of the squared instantaneous random uncertainties:

$$r_{\phi_e \phi_e} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (r_{\phi_e i} r_{\phi_e i})^2} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (r_{\phi_e i})^4}.$$  \tag{A.16}

While an explanation for using the root mean square as opposed to the traditional root sum square will not be given, the uncertainty calculation in Eqn. A.16 is demonstrated using a Monte Carlo simulation. As in Eqn. A.1, the measured quantity $\phi_e$ was composed of three components, a mean $\overline{\phi_e}$, “true” fluctuation $\phi'$, and random measurement error $\epsilon_{\phi_e}$. For this simulation, “true” fluctuation $\phi'$ realizations were drawn from a Gaussian distribution with a known standard deviation and random measurement errors $\epsilon_{\phi_e}$ were obtained from Gaussian distributions with known random standard deviations. The standard deviation $s_{\epsilon_{\phi_e}}$ of the random measurement error distribution was also normally distributed with a known standard deviation $s_{s_e}$ giving an instantaneous random measurement error standard deviation $s_{\epsilon_{\phi_e,i}}$. The term, $s_{s_e}$, can be thought of as the standard deviation of the random measurement error standard deviation. This is better demonstrated in Fig. A.2. This process was repeated to simulate $N = 10^7$ measurements or realizations.

The standard deviation $s_{\epsilon_{\phi_e,i}}$ of the instantaneous random measurement error was used to calculate the instantaneous random fluctuation uncertainties $r_{\phi_e}$ in Eqn. A.16:

$$r_{\phi_e} = (1.96s_{\epsilon_{\phi_e,i}})^2.$$  

The distribution of the difference between measured fluctuations and “true” fluctuations was used to calculate the confidence interval containing 95% of the instantaneous random measurement errors $\delta_{\phi_e \phi_e}$. This is the 95% confidence uncertainty
Fig. A.2: Demonstration of the distribution of random measurement error $\epsilon_{\phi_c}$ used for the Monte Carlo simulated uncertainty. The random measurement error is distributed normally with zero mean and a prescribed standard deviation $s_{\epsilon_{\phi_c}}$. To introduce variability to the distribution for all Monte Carlo realizations, the standard deviation of the random measurement error is also distributed with a normal distribution with a mean value of $s_{s_{\epsilon_{\phi_c}}}$ and the standard deviation $s_{s_{\epsilon_{\phi_c}}}$. This term can be thought of as the standard deviation of the random measurement error standard deviation ($s_{s_{\epsilon_{\phi_c}}}$).

on the random measurement fluctuations calculated by the Monte Carlo method $r_{\text{M.C.}}^{\phi_c\phi_c}$. The Taylor Series random measurement fluctuation uncertainty $r_{\text{T.S.}}^{\phi_c\phi_c}$ is calculated using Eqn. A.16. This process was repeated for several random measurement error standard deviation width to value ratios $s_{s_{\epsilon_{\phi_c}}}/s_{\epsilon_{\phi_c}}$ and are shown in Fig. A.3.

The magnitude of the Monte Carlo $r_{\text{M.C.}}^{\phi_c\phi_c}$ and Taylor Series $r_{\text{T.S.}}^{\phi_c\phi_c}$ random measurement fluctuations uncertainty shown in Fig. A.3 varies with the ratio of random measurement error standard deviation width to value ($s_{s_{\epsilon_{\phi_c}}}/s_{\epsilon_{\phi_c}}$); however, the relative difference remains constant regardless of the random measurement error. As in Fig. A.3, the uncertainty calculated in Eqn. A.16 estimates the Monte Carlo random measurement uncertainty within 10% for typical random measurement error standard deviation width to value ratios ($s_{s_{\epsilon_{\phi_c}}}/s_{\epsilon_{\phi_c}} \leq 0.5$). Typical random uncertainties are below this limit.

An important note is the direction of the uncertainty calculated using the Monte Carlo and Taylor Series methods. When using Eqn. A.16, the time-averaged measured fluctuation uncertainty $r_{\phi_c\phi_c}$ is two-directional (positive and negative). However, as demonstrated in Eqn. A.14, the error should be elevated and in the negative direction. While this is not
Fig. A.3: The 95% confidence uncertainty of the measurement fluctuations for the Monte Carlo method $r_{\hat{\phi}_e}^{MC}$ and the Taylor Series method $r_{\hat{\phi}_e}^{TS}$ (from Eqn. A.16) for several random measurement error standard deviation width to value ratios ($s_{\phi_e}/s_{\epsilon_{\phi_e}}$). The percent difference between the Monte Carlo and Taylor Series uncertainties is also given on the right axis. While the ratio of random measurement error to “true” fluctuations $s_{\epsilon_{\phi_e}}^2 / \phi'\phi'$ will change the magnitude of the error and uncertainty, the trend and percent difference values remain constant.
detected by the Taylor Series method, the Monte-Carlo method shows one-sided negative
time-averaged measured fluctuation uncertainties that are elevated above the “true” fluc-
tuations $\bar{\phi}$. Thus, it is important to remember that although two-sided uncertainties are
calculated from Eqn. A.16, it would only be appropriate to use the random uncertainty in
the negative direction. Eqn. A.16 can be rewritten to account for the one-sided random
measured fluctuation uncertainty:

$$
r_{+}^{\phi_{e} \phi_{e}} = 0, \text{ and}$$  
$$
r_{-}^{\phi_{e} \phi_{e}} = \left( \frac{1}{N} \sum_{i=1}^{N} (r_{\phi_{e} i})^4 \right)^{1/2}.$$

(A.17)  

As with the uncertainty on a mean quantity $\phi_{e}$, the finite number of acquired mea-
surements will affect the estimate of our measured fluctuations $\bar{\phi}_{e} \phi_{e}$. This uncertainty
component, like precision uncertainty on the mean, will decrease as the number of real-
izations $N$ increases [71]: $p_{\phi_{e} \phi_{e}} = \bar{\phi}_{e} \phi_{e} / (N - 1)$. Combining the time-averaged random
measured fluctuation uncertainty and fluctuation precision uncertainty through the root
sum square gives the total time-averaged measured fluctuation uncertainty $U_{\phi_{e} \phi_{e}}$:

$$
U_{+}^{\phi_{e} \phi_{e}} = \left( \frac{\bar{\phi}_{e} \phi_{e}}{(N - 1)} \right) \approx \delta_{+}^{\phi_{e} \phi_{e}}, \text{ and}
$$

$$
U_{-}^{\phi_{e} \phi_{e}} = \sqrt{\left( \frac{\bar{\phi}_{e} \phi_{e}}{(N - 1)} \right)^2 + \frac{1}{N} \sum_{i=1}^{N} (r_{\phi_{e} i})^4} \approx \delta_{-}^{\phi_{e} \phi_{e}}. 
$$

(A.19)  

(A.20)

The fluctuation precision uncertainty component is often much smaller than the time-
averaged random measured fluctuation uncertainty and sufficient quantities of data samples
can easily be acquired to significantly reduce the fluctuation precision uncertainty.

When concerned with individual components, like velocity, similar expansions can be
derived in a similar expansion. Effects on magnitudes (e.g. velocity) can be expanded from
directional components at the instantaneous uncertainty level using the equation:
\[ U_{\hat{\phi}_c \hat{\phi}_c} \approx U_{\hat{\phi}_{c_x} \hat{\phi}_{c_x}} + U_{\hat{\phi}_{c_y} \hat{\phi}_{c_y}} + 2 \left( U_{\hat{\phi}_{c_x} \hat{\phi}_{c_x}} U_{\hat{\phi}_{c_y} \hat{\phi}_{c_y}} \right)^{1/2}. \] (A.21)
Appendix B

Magnitudes of Fluctuations

A single wire hot wire is sensitive to both the streamwise, \( u_x \), and spanwise, \( u_y \), velocity components, and therefore its output is essentially the magnitude of the velocity in the \( x \) and \( y \) directions. We assume that \( u_z \) is very small in all of the cases presented here and that therefore \( u^2 = u_x^2 + u_y^2 \). This relationship is used to convert the PIV data into results comparable to the hot wire measurements. The hot wire probe must be equally sensitive to both components for this conversion to be accurate. The response of the single wire probe to velocity components normal to each other was assessed using a TSI hot wire calibrator that allows the probe to be rotated while maintaining the sensor position. This test yielded a difference in normal velocity components within 2.5\%. We can write that, for the hot wire sensor,

\[
(\overline{u} + u')^2 = (\overline{u}_x + u'_x)^2 + (\overline{u}_y + u'_y)^2, \tag{B.1}
\]

\[
(\overline{u}^2 + 2\overline{u}u' + u'^2) = [\overline{u}_x^2 + \overline{u}_y^2] + [u_x'^2 + u_y'^2] + [2\overline{u}_x u'_x + 2\overline{u}_y u'_y]. \tag{B.2}
\]

An informative way to consider these equations is that the left hand side is what is measured by the hot wire while the right hand side can be obtained from the PIV measurement. The time-average of the cross terms is zero. This provides a root-mean-square relationship for the mean velocity and fluctuations components,

\[
\overline{u^2} + \overline{u'^2} = [\overline{u}_x^2 + \overline{u}_y^2] + [u_x'^2 + u_y'^2]. \tag{B.3}
\]

Using this relationship, a direct comparison of PIV and hot wire measurements can be made. For simplicity, all variables discussed throughout will be time-averaged values and
the bars over the fluctuation variables will be dropped. This gives the magnitude statistics the relations,

\[ u'^2 = [u'^2_x + u'^2_y], \quad \text{(B.4)} \]

\[ u'u' = [u'_x u'_x + u'_y u'_y + 2]. \quad \text{(B.5)} \]

Using this form, PIV data and hot wire data are directly comparable.
Appendix C

Time-Traces for Local, Instantaneous Velocity SRQ’s

Instantaneous velocity, pressure, and bulk velocity time-traces were acquired at discrete locations up- and downstream of each cylinder. These time-traces were used to calculate local and global, temporal and spatial SRQ's designed for CFD validation. Local velocity time traces are shown for many cylinder trace locations (given in Fig. C.1-C.5).
Fig. C.1: The instantaneous velocity time-traces on the centerline and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Bulk velocity traces were also extracted between cylinders to observe bulk velocity variation through time. These are demonstrated for the spatially-averaged bulk velocity SRQ’s in Fig. C.6-C.7.
Fig. C.2: The instantaneous velocity time-traces on the near and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. C.3: The instantaneous velocity time-traces on the far and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. C.4: The instantaneous velocity time-traces on the centerline and downstream side of each cylinder for the experimental and numerical (k-ω, DES-NB, and DES-B) models.
Fig. C.5: The instantaneous velocity time-traces on the near and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. C.6: The instantaneous bulk velocity time-traces at the upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

\[ U \approx \frac{b^2}{\bar{u}} + \frac{p^2}{\bar{u}} \]
Fig. C.7: The instantaneous bulk velocity time-traces at the downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Appendix D

Uncertainty for Experimental Time-Traces on Local, Instantaneous Velocity SRQ’s

Uncertainties on the instantaneous velocity, pressure, and bulk velocity time-traces were acquired at discrete locations up- and downstream of each cylinder. Instantaneous uncertainties are given by dashed lines while the total uncertainty on the time-averaged SRQ is given by gray bars. Given a 95% confidence interval, 95% of the results should lie within the time-averaged uncertainty bars. Uncertainties for local velocity time traces are shown for many cylinder trace locations (given in Fig. D.1-D.4).
Fig. D.1: The experimental uncertainty for the instantaneous velocity time-traces on the far and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. D.2: The experimental uncertainty for the instantaneous velocity time-traces on the near and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. D.3: The experimental uncertainty for the instantaneous velocity time-traces on the centerline and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. D.4: The experimental uncertainty for the instantaneous velocity time-traces on the near and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

$$U_{\bar{u}_e} = \sqrt{b_{\bar{u}_e}^2 + p_{\bar{u}_e}^2}$$

$$U_{u_e} = \sqrt{b_{u_e}^2 + r_{u_e}^2}$$
Appendix E

Histograms for Local, Instantaneous Velocity SRQ’s

Extending beyond Gaussian statistics, higher-order statistical quantities, such as kurtosis and skewness, can be used to determine distribution shapes and widths. Given in Fig. E.1 - E.4 are the histograms or probability distribution functions for many of the instantaneous velocity, pressure, and bulk velocity time-traces.
Fig. E.1: The distributions of the instantaneous velocity time-traces on the far and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. E.2: The distributions of the instantaneous velocity time-traces on the near and upstream side of each cylinder for the experimental and numerical (\(k-\omega\), DES-NB, and DES-B) models.
Fig. E.3: The distributions of the instantaneous velocity time-traces on the centerline and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. E.4: The distributions of the instantaneous velocity time-traces on the near and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Appendix F

Frequency Spectra for Local, Instantaneous Velocity SRQ’s

Frequency spectra is also an important parameter in temporally evolving flows. This is particularly true for flows experiencing vortex shedding. Velocity, pressure, and bulk velocity frequency spectra are shown for all instantaneous velocity, pressure, and bulk velocity time traces in Fig. F.1 - F.11.
Fig. F.1: The average power spectra in the frequency domain for the instantaneous velocity traces on the far and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.2: The average power spectra in the frequency domain for the instantaneous velocity traces on the near and upstream side of each cylinder for the experimental and numerical (\(k-\omega\), DES-NB, and DES-B) models.
Fig. F.3: The average power spectra in the frequency domain for the instantaneous velocity traces on the centerline and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.4: The average power spectra in the frequency domain for the instantaneous velocity traces on the near and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.5: The average power spectra in the frequency domain for the instantaneous velocity traces in the far wake and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.6: The average power spectra in the frequency domain for the instantaneous velocity traces in the near wake and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.7: The average power spectra in the frequency domain for the instantaneous velocity traces on the far wall and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.8: The average power spectra in the frequency domain for the instantaneous velocity traces on the near wall and downstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.9: The average power spectra in the frequency domain for the instantaneous velocity traces on the far wall and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.10: The average power spectra in the frequency domain for the instantaneous velocity traces on the near wall and upstream side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. F.11: The average power spectra in the frequency domain for the instantaneous velocity traces on the near wall and streamwise centerline of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Appendix G

Autocorrelations and Self-Similar Time Scales for Local, Instantaneous Velocity SRQ’s

Time-scales of temporal features were calculated using the autocorrelation of instantaneous velocity, pressure, and bulk velocity time traces. The autocorrelation trends for experimental and numerical results are shown for all instantaneous time traces in Fig. G.1 - G.12.
Fig. G.1: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the far and upstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.2: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the near and upstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.3: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the centerline and downstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.4: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the near and downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. G.5: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces in the far wake and downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. G.6: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces in the near wake and downstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.7: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the far wall and downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. G.8: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the near wall and downstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.9: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the far wall and upstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.10: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the near wall and upstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. G.11: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous velocity traces on the near wall and streamwise centerline of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
Fig. G.12: The average autocorrelation coefficient $\rho(\tau)$ for the instantaneous wall pressure traces on the far wall side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Appendix H

Correlations and Correlated Time-Scales for Local, Instantaneous Velocity SRQ’s

Time-scales of paired phenomena (i.e. vortex pairing and pressure-velocity coupling) were calculated in a similar manner as the autocorrelation for instantaneous velocity, pressure, and bulk velocity time traces. The correlation trends for experimental and numerical results are shown for all instantaneous time traces in Fig. 4.45 - 4.47.
Fig. H.1: The average correlation coefficient $P(\tau)$ for the instantaneous velocity traces in the far and near wake, downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. H.2: The average correlation coefficient $P(\tau)$ for the instantaneous velocity traces on the far and near wall, downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. H.3: The average correlation coefficient $P(\tau)$ for the instantaneous velocity traces on the far and near wall, upstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Appendix I

GCI for Time-Average and Standard Deviation of
Time-Trace SRQ’s

The GCI for the mean and standard deviation statistics are given for instantaneous velocity, pressure, and bulk velocity time traces for the three $k$-$\omega$ mesh sizes. The correlation trends for experimental and numerical results are shown for all instantaneous time traces in Fig. I.1 - I.7.
Fig. I.1: The GCI for the mean of the instantaneous velocity time-traces at the upstream and downstream locations on the far side of each cylinder for the experimental and numerical (\(k-\omega\), DES-NB, and DES-B) models.

Fig. I.2: The GCI for the mean of the instantaneous velocity time-traces at the upstream and downstream locations on the centerline of each cylinder for the experimental and numerical (\(k-\omega\), DES-NB, and DES-B) models.
Fig. I.3: The GCI for the mean of the instantaneous velocity time-traces at the upstream and downstream locations on the near side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Fig. I.4: The GCI for the mean of the instantaneous bulk velocity time-traces at the upstream and downstream locations for each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. I.5: The GCI for the standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the far side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.

Fig. I.6: The GCI for the standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the centerline of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Fig. I.7: The GCI for the standard deviation of the instantaneous velocity time-traces at the upstream and downstream locations on the near side of each cylinder for the experimental and numerical ($k$-$\omega$, DES-NB, and DES-B) models.
Appendix J

Autocorrelations and Self-Similar Time Scales for Local, Instantaneous Velocity SRQ’s

GCI of the time-scales of temporal features were calculated using the autocorrelation of instantaneous velocity, pressure, and bulk velocity time traces for the three $k$-$\omega$ mesh sizes. The autocorrelation trends for experimental and numerical results are shown for all instantaneous time traces in Fig. J.2 - J.6.
Fig. J.1: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous velocity traces on the far and upstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. J.2: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous velocity traces on the centerline and upstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. J.3: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous velocity traces on the far and downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. J.4: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous velocity traces on the centerline and downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. J.5: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous bulk velocity traces on the upstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. J.6: The GCI for the average autocorrelation coefficient $\rho(\tau)$ for instantaneous bulk velocity traces on the downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Appendix K

Correlations and Correlated Time-Scales for Local, Instantaneous Velocity SRQ’s

The GCI for time-scales of paired phenomena (i.e. vortex pairing and pressure-velocity coupling) were calculated in a similar manner as the autocorrelation for instantaneous velocity, pressure, and bulk velocity time traces for the three $k$-$\omega$ mesh sizes. The correlation trends for experimental and numerical results are shown for all instantaneous time traces in Fig. K.1 - K.4.
Fig. K.1: The GCI for the average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near, upstream sides of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. K.2: The GCI for the average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near, downstream sides of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. K.3: The GCI for the average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near wall, downstream side of each cylinder for the experiment and numerical models ($k-\omega$, DES-NB, and DES-B).
Fig. K.4: The GCI for the average correlation coefficient $P(\tau)$ for instantaneous velocity traces on the far and near wall, upstream side of each cylinder for the experiment and numerical models ($k$-$\omega$, DES-NB, and DES-B).
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Awards

Graduate Researcher of the Year, Mechanical and Aerospace Engineering  
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Undergraduate Researcher of the Year, Mechanical and Aerospace Engineering  
2007

Research Experience

Graduate Research Assistant  
Experimental Fluid Dynamics Laboratory (EFDL), USU  
2008-Present

Advising Professor: Dr. Barton Smith

- Acquired thermal and fluid measurements using various measurement techniques for CFD validation: 2D and 3D PIV, hot-wire anemometry, thermocouples, and pressure transducers.
- Processed and acquired experimental data using Labview data acquisition software and data processing: analog filters and amplifiers.
- Designed experimental facilities to meet structural, thermal, and fluid constraints (deflection, thermal expansion, and flow regimes) using analytical and numerical means.
- Performed 3D steady and unsteady CFD turbulence simulations (URANS and DES) to be validated with experimental results.
- Created initial design, budget estimates and prepared preliminary research proposals for future projects, specifically: mixed convection wind tunnel and shear mixing.
- Revised experimental and facility designs and developed revised theories based on experimental and numerical results.
• Proposed research paths and projects with the intent to enhance the work produced within the lab.

• Presented conference publications of personal and colleagues research at refereed proceedings

**Graduate Intern**

Matched Index of Refraction Facility, INL  
Mentor: Dr. Hugh McIlroy

• Operated Matched Index of Refraction (MIR) tunnel and revised experimental model to acquire high-fidelity, fully-described PIV validation measurements.

• Measured flow characteristics (i.e. velocity, turbulent kinetic energy, and boundary and inlet conditions) for CFD validation codes using stereo PIV techniques.

**Undergraduate Researcher**  
2006-2008

Experimental Fluid Dynamics Laboratory, USU  
Advising Professor: Dr. Barton Smith

• Developed and implemented numerical algorithms to acquire, process, and analyze PIV measurements in Labview, FORTRAN, and MATLAB.

• Experimentally determined optimal flow conditions and characteristics for specific applications using 2D PIV techniques.

• Quantified and reduced error in various measurements such as, temperature, pressure, and velocity.

• Mapped temperature measurements of Utah Lake throughout the summer to assess CFD results.

• Designed experimental model and test apparatus for various experiments.

• Analyzed vectored flow stability by implementing a schleiren flow visualization system.

**Teaching Experience**

**Lecturer**  
2008

MAE 4400 - Thermal Fluids Laboratory, USU  
Advising Professor: Dr. Byard Wood

• Prepared and presented lectures on thermal/fluids fundamentals, experimental technique, uncertainty, and writing skills for 100 students.

• Designed laboratory experiments providing students with functional, hands-on thermal/fluids experience.

**Mentor**  
2010

Modeling, Experimentation, and Validation, INL  
School Director: Dr. Nam T. Dinh

• Directed student discussions concerning experimentation and validation within the nuclear community.
Specialized Skills

Experimental Experience

• 2D and 3D, steady and time-resolved PIV with multiple PIV packages, including one written personally.

• Hot wire anemometry, thermocouple measurements, and various pressure measurements techniques.

• Data acquisition, experimental timing, and control using Labview.

• Data processing and post-processing using analog filters, amplifiers, fourier transforms, and uncertainty analysis.

Numerical Modeling Experience

• 2D and 3D, steady and unsteady CFD using FLUENT.

• Turbulence modeling including URANS and DES amd model meshing and refinement.

• Finite Element modeling with FEMAP and IDEAS.

Computing Languages

• FORTRAN, C++, MATLAB, Labview.

• Parallel programming familiarity in FORTRAN and MATLAB.

Publications

Peer-Reviewed Journal Publications


Peer-Reviewed Conference Presentations and Papers
Wilson, B.M., Smith, B.L., “Errors in Mean and Fluctuating Velocity Due to PIV Bias and Precision Uncertainties,” Proceedings of the 14th International Topical Meeting on Nuclear Reactor Thermalhydraulics NURETH14, Toronto, Canada, 2011.


Non-Peer-Reviewed Conference Presentations and Papers


Theses and Dissertations

Wilson, B.M., “CFD Validation and Uncertainty Quantification using PIV” (PhD Dissertation), Utah State University, Logan, UT, 2012.


Peer Review Activities

1. Conference Reviewer for ASME Heat Transfer Division

2. Conference Reviewer for ASME Fluids Engineering Division