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# AUTOCORRELATION-BASED ESTIMATE OF PARTICLE IMAGE DENSITY IN PARTICLE IMAGE VELOCIMETRY

by

Scott O. Warner

## A thesis submitted in partial fulfillment of the requirements for the degree

of

## MASTER OF SCIENCE

in

Mechanical Engineering

Approved:

Dr. Barton L. Smith Major Professor Dr. Heng Ban Committee Member

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UTAH STATE UNIVERSITY Logan, Utah

2012

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## Abstract

Autocorrelation-Based Estimate of Particle Image Density in Particle Image Velocimetry

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Scott O. Warner, Master of Science Utah State University, 2012

Major Professor: Dr. Barton L. Smith Department: Mechanical and Aerospace Engineering

In Particle Image Velocimetry (PIV), the number of particle images per interrogation region, or particle image density, impacts the strength of the correlation and, as a result, the number of valid vectors and the measurement uncertainty. Therefore, any a-priori estimate of the accuracy and uncertainty of PIV requires knowledge of the particle image density. An autocorrelation-based method for estimating the local, instantaneous, particle image density is presented. Synthetic images were used to develop an empirical relationship based on how the autocorrelation peak magnitude varies with particle image density, particle image diameter, illumination intensity, interrogation region size, and background noise.

This relationship was then tested using images from two experimental setups with different seeding densities and flow media. The experimental results were compared to image densities obtained through using a local maximum method as well as manual particle counts and are found to be robust. The effect of varying particle image intensities was also investigated and is found to affect the particle image density.

(89 pages)

## **Public Abstract**

Autocorrelation-Based Estimate of Particle Image Density in Particle Image Velocimetry

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Scott O. Warner, Master of Science Utah State University, 2012

Major Professor: Dr. Barton L. Smith Department: Mechanical and Aerospace Engineering

Particle Image Velocimetry (PIV) is an optical method for measuring the speed of liquids and gases. As part of PIV, the flow is seeded with small particles. Images of the particles are captured at time intervals and the movement of the particles between images is used to calculate the speed and direction of the fluid flow. The ability of PIV to accurately measure the flow velocity is a function of many parameters including the particle image density, or number of particles contained within an image. Therefore, a knowledge of the particle image density can be used to estimate the accuracy and uncertainty of the PIV measurements.

A method for estimating the particle image density is presented. With the use of synthetic (computer generated) images, a formula was developed that relates the particle image density to a function called an autocorrelation, as well as the particle image diameter, average particle intensity, and the interrogation region size. The relationship was then tested on PIV images from two experimental setups. Particle image density estimates from the experimental images were compared to results acquired by using a local maximum method as well as manual particle counts and are found to be robust. The effect of varying the particle image intensity was also investigated and found to affect the particle image density. To my wife, Rachel, who supported me each step of the way.

## Acknowledgments

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Scott O. Warner

# Contents

														Р	age
Ab	strac	:t													iii
Pu	blic 4	Abstrac	ct		••••										$\mathbf{iv}$
Ac	know	ledgme	ents								• • • •				$\mathbf{vi}$
$\mathbf{Lis}$	t of 7	Tables													ix
$\mathbf{Lis}$	t of l	Figures													$\mathbf{x}$
No	tatio	n													$\mathbf{xiv}$
Ac	ronyı	ms													xvi
1	<b>Intro</b> 1.1 1.2	oduction           Particle           Uncerta           1.2.1           I.2.2           I           1.2.3           1.2.4	n Image Veloc inty in PIV Particle Imag Displacement Sub-Pixel Int Particle Imag	imetry ge Diameto Gradient erpolation ge Density	er	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	· · · · ·	· · · ·	· · · ·		· · · · ·	· · · · · · · · · · · · ·	· · · · · ·	<b>1</b> 1 2 2 3 4 4
<b>2</b>	Obje	ctives .												•••	7
3	Appr 3.1 3.2 3.3 3.4 3.5	roach . Relative Average Particle Particle Image P 3.5.1 I 3.5.2 I	Autocorrela particle inte Image Diam Image Dens Preprocessing Background mage Norma	tion Peak ensity eter Estim ity Estima Image Ren alization .	: Height  nation ation .  noval .	· · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · ·	· · · · ·	· · · ·			· · · ·	· · · · · · · · · · · · · · · ·		8 8 10 12 13 14 14 16
4	Synt: 4.1 4.2	hetic Iı Syntheti Experim 4.2.1 I 4.2.2 A	mage Gene ic Image Gen nental Setups Laminar Jet Aquarium	ration an           neration .	nd Exp   	erim(   	ental • • • • • • • • • • • •	Set:	ups .	· · · ·	· · · ·	· · · ·	· · · · · · · · · ·		<ol> <li>18</li> <li>19</li> <li>19</li> <li>20</li> </ol>

<b>5</b>	Res	ults		5
	5.1	Empiri	ical Relationship	25
	5.2	Test w	ith Synthetic Images	3
		5.2.1	Effect of Density	3
		5.2.2	Effect of Diameter	6
		5.2.3	Effect of Noise	8
	5.3	Experi	mental Verification	0
		5.3.1	Laminar Jet Tests	2
		5.3.2	Aquarium Tests	.3
		5.3.3	Effect of Particle Image Density 4	4
		5.3.4	Effect of Image Intensity	7
6	Con	clusior	ns & Future Work	9
	6.1	Conclu	asions	-9
	6.2	Future	Work $\ldots \ldots 5$	1
Re	eferei	nces		3
AĮ	open	$\mathbf{dix}$		6

viii

# List of Tables

Table		Page
4.1	The properties of Potters hollow microspheres used as seed. <sup>1</sup> Density is measured by gas displacement pycnometer. <sup>2</sup> Bulk Density is the weight as measured in a container and includes the interstitial air. <sup>3</sup> Data represents percent volume distribution measured using laser light scatter technique	21
5.1	Synthetic image parameter space	25
5.2	The coefficients for Eq. 5.1 determined using a least-squares fit and the corresponding goodness of fit value	27
5.3	The mean absolute errors $\overline{\epsilon}$ and the mean relative errors $\overline{\eta}$ for the ABD method obtained for each interrogation size. Errors were obtained by averaging results from the synthetic images used to develop Eq. 5.2	33
5.4	The mean and standard deviation of noise from two PIV cameras. $\ldots$ .	38
5.5	The samples of hollow glass spheres mixed into the water of the aquarium experiment. The mass of each sample was measured using an analytical balance	45

# List of Figures

gure		Page
1.1	A Particle Image Velocimetry System [7]. A double-pulsed laser illuminates seeding particles that are distributed in a flow. A CCD camera captures images of the seeding particles separated by discrete time intervals. The images are then subdivided into interrogation regions and processed using a cross-correlation-based algorithm to produce a velocity vector field	2
1.2	A demonstration of the effects of displacement gradients on the correlation peak. The correlation of synthetic images with constant parameters show the difference between applying (a) no gradient and (b) a 0.2 pixels/pixel gradient	nt. 4
3.1	The relative height of the autocorrelation peak $R_h$ as a function of (a) $N_I$ , (b) $d_{\tau}$ , (c) $A$ , and (d) $I_P$ . These figures were generated by autocorrelating synthetic images with known particle image diameters, densities, and inten- sities.	9
3.2	Preforming an autocorrelation using Eq. 3.3 produces (a) an autocorrelation map. A cross section of the autocorrelation map (b) shows the highest and lowest value of the autocorrelation map, $R_P$ and $R_{min}$ repectively. The difference between these values is the relative autocorrelation peak height.	11
3.3	The maximum particle intensity depends on the sub-pixel location of the particle. A particle with a 0.25 pixel offset (a) has a reduced maximum pixel intensity and a larger standard deviation in surrounding pixels. A particle that is pixel centered (b) provides the best estimate of the actual maximum particle intensity. If a particle is pixel centered, the four adjacent pixels will have nearly identical intensity values and a low standard deviation	12
3.4	A 3-point Gaussian is applied to the cross section of the correlation peak using the three largest values	13
3.5	Particle image density estimation using a the local maximum method to locate and count local maximums (particles) in synthetic images with known particle image diameters and image densities.	15
3.6	Demonstration of the impact of background removal: (a) the original image (b) the background image based on the local minimum value from each pixel, and (c) the processed image.	16
4.1	The schematic for the lamnar jet setup. The air drawn in by the blower is seeded, conditioned, and then exits to the measurement region	19

4.2	Data is taken at the channel exit. The camera is positioned perpendicular to the flow and normal to the laser plane	20
4.3	The schematics for the aquarium setup. Views include (a) an overhead and (b) side view of the setup. The camera is positioned normal to the laser plane.	22
4.4	Effect of laser intensity and camera aperture on the estimated particle image density. Three apertures are depicted: (a) large, (b) medium and (c) small. For each case, four values of total seed mass m are depicted. The black and red shades represent the camera noise floor and saturation level, respectively.	23
5.1	Averaged values of the particle image diameter, relative autocorrelation peak height, interrogation area, and average particle intensity obtained from syn- thetic images are plotted as a function of the true particle image density. Values for $R_h$ , $A$ , and $\overline{I_P}$ are combined through multiplication in order to reduce the 5-D surface to a 3-D surface for viewing purposes	26
5.2	The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average $N_I$ from 1000 synthetic images with known densities. A 128 × 128 interrogation region was used.	29
5.3	The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average $N_I$ from 1000 synthetic images with known densities. A $256 \times 256$ interrogation region was used.	30
5.4	The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average $N_I$ from 1000 synthetic images with known densities. A $64 \times 64$ interrogation region was used.	31
5.5	The relative and absolute errors of the average particle image density calcu- lated using the ABD method. Estimated results are based on the average $N_I$ from 1000 synthetic images with known densities. A $32 \times 32$ interrogation region was used.	32
5.6	The effect of particle image density on the particle image density estimation. Synthetic images were used with known densities, particle image diameter of 3 pixels, and no noise. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.	34
5.7	Precision uncertainty of the particle image density estimation results in Fig. 5.6. The following values represent the 95% confidence interval $(1.96s_x/\sqrt{N})$ for each interrogation region size.	35

xi

5.8	The effect of particle image diameter on the particle image density estimation. Synthetic images with a particle image density of 40 particles per $32 \times 32$ region and no noise were used. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.	36
5.9	Precision uncertainty of the particle image density estimation results in Fig. 5.6. The following values represent the 95% confidence interval, $1.96s_x/\sqrt{N}$ .	37
5.10	Effect of introducing noise into the synthetic images on the particle image density estimation for a $32 \times 32$ interrogation region. The noise generated simulates the noise introduced using two different types of cameras	39
5.11	The effect of noise on the precision uncertainty of the density estimation results in Fig. 5.10	40
5.12	Effect of introducing noise into the synthetic images on the particle image density estimation for a $128 \times 128$ interrogation region. The noise generated simulates the noise introduced using two different types of cameras	41
5.13	The effect of noise on the precision uncertainty of the density estimation results in Fig. 5.12	41
5.14	Effect of introducing noise into the synthetic images on the particle image density estimation for a $128 \times 128$ interrogation region when background image subtraction is not used. The noise generated simulates the noise introduced using two different types of cameras. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.	42
5.15	Interrogation regions $(32 \times 32 \text{ pixels})$ from images acquired using the jet setup. The particle image density was adjusted by increasing the volume flow rate (SLPM) into a Laskin Nozzle.	43
5.16	Particle image density estimates as a function of the mass flow rate to the seeder for the rectangular jet case. Results were acquired implementing the autocorrelation-based density method (solid lines and symbols) as well as the local maximum method and manually counting particles from several interrogation regions. Error bars represent the 95% confidence interval	44
5.17	Images with varying densities acquired from the aquarium setup. The density was increased by adding known masses of hollow glass spheres. Images shown are for the $f/22$ case	45

xii

5.18	Estimated seeding density as a function of the total mass of seed particles	
	added to the water. Values are shown using three different particle image	
	diameters. The solid lines with the solid symbols represent the average parti-	
	cle image density calculated using the autocorrelation-based density method.	
	The dashed and dotted lines with the open symbols represent the average	
	particle image density estimates from the local maximum (LM) method and	
	manual counting (MC), respectively.	46

5.19 Estimated seeding density as a function of the total mass of seed particles added to the water. Values are shown using three different image intensities. The image intensity was adjusted by using different *f*-numbers while maintaining a constant laser intensity. The solid lines with the solid symbols represent the average particle image density calculated using the autocorrelation-based density method. The dashed and dotted lines with the open symbols represent the average particle image density estimates from the local maximum (LM) method and manual counting (MC), respectively.

# Notation

# Roman symbols

0	
a	gradient parameter
A	interrogation domain area
$D_a$	aperture diameter
$d_D$	correlation peak width
$D_I$	interrogation region size
$d_{\tau}$	particle image diameter
$d_p$	actual particle diameter
$d_s$	diffraction-limited spot diameter
f	focal length
$f^{\#}$	ratio of $f$ to $D_a$
$IA_1$	interrogation area
$I_{max}$	minimum image intensity
$I_{min}$	maximum image intensity threshold
$\overline{I_p}$	average intensity of the pixel centered particles
${\mathcal J}$	laser intensity
M	image magnification
n	integer value
N	number of image density measurements
$N_I$	particle image density
$N_{I,true}$	true particle image density of the synthetic images
$R_h$	relative autocorrelation peak height
$R_{min}$	minimum value of the autocorrelation
$R_p$	highest autocorrelation peak height
R(r,s)	discrete autocorrelation

# Greek symbols

- $\epsilon \qquad \text{absolute error} \qquad$
- $\eta$  relative error
- $\lambda$  light sheet wavelength
- $\mu_p$  mean particle intensity
- $\sigma_p$  standard deviation of the particle intensities
- $\sigma$  standard deviation of the Gaussian fit

# Acronyms

- ABD Autocorrelation-Based Density
- CFD Computational Fluid Dynamics
- DWO discrete window offsets
- EM Empricial Density
- FFT Fast Fourier Transform
- LM Local maximum
- MC Manual Count
- PIV Particle Image Velocimetry

## Chapter 1

## Introduction

Particle image velocimetry (PIV) is a non-intrusive measurement technique used in experimental fluid mechanics to acquire spatially resolved velocity fields. PIV is capable of producing two- or three-component velocity fields which other flow measurement techniques, such as hot wire anemometry and acoustic Doppler velocimetry, cannot. PIV is applied to a wide variety of flow problems, ranging from measuring flows in turbomachinery [1] to determining Reynolds stresses in artificial heart valves [2], and is often used as a validation technique for numerical simulations, including Computational Fluid Dynamics (CFD).

As with any measurement technique, it is important to be aware of the parameters that contribute to the measurement uncertainty. Being able to estimate these values allows for a better understanding of how a given parameter effects the measurement and provides a means of quantifying the uncertainty [3]. The remainder of this chapter discusses the fundamentals of PIV and the parameters that contribute to its uncertainty.

#### 1.1 Particle Image Velocimetry

The method of PIV involves introducing neutrally buoyant seeding particles in a flow field and illuminating them with a double-pulsed laser sheet. Image-pairs of the illuminated particles are acquired at discrete time intervals using a high-speed CCD camera. The images are then subdivided into interrogation regions and processed using a cross-correlation-based algorithm to obtain the associated particle displacements and, thus, velocity fields. A diagram of the PIV method is shown in Fig. 1.1. Articles by Prasad [4], Adrian [5], and Westerweel [6] provide a comprehensive description of the technique.

Since its beginning, PIV has undergone substantial improvements in measurement accuracy and reliability [8]. The early technique of analog film recording and evaluation have



Fig. 1.1: A Particle Image Velocimetry System [7]. A double-pulsed laser illuminates seeding particles that are distributed in a flow. A CCD camera captures images of the seeding particles separated by discrete time intervals. The images are then subdivided into interrogation regions and processed using a cross-correlation-based algorithm to produce a velocity vector field.

been replaced with digital camera techniques. The introduction of sub-pixel interpolation resulted in accuracy values reported as low as 0.1 pixel units [9]. The use of adaptive evaluation algorithms, such as discrete window offsets (DWO) [10] and iterative image deformation methods [11] have been shown to reduce uncertainty levels. A comprehensive report of the historical development of PIV is provided by Adrian [8].

#### 1.2 Uncertainty in PIV

The uncertainty in PIV measurements is a function of many parameters [12]. These parameters include, but are not limited to, particle image diameter, displacement gradients, sub-pixel displacements, and particle image density. Each of these parameters are discussed in the following sections.

#### 1.2.1 Particle Image Diameter

The particle image diameter,  $d_{\tau}$ , is the diameter (in pixel units) of the particle as it

appears in the recording medium and is proportional to the width of the correlation peak [13,14]. The correlation peak can be generated by either implementing a cross-correlation or an autocorrelation. The cross-correlation method involves correlating two separate images together, while an autocorrelation involves correlating a single image with itself. Many different values have been suggested for optimum particle image diameter ranging from just under 2 pixel units [15] to over 6 pixel units [16]. In general, the optimum particle image diameter ranges from 2 to 4 pixel units [17]. The variation in optimum diameter stems from the method used to estimate the sub-pixel displacements (e.g. three-point Gaussian, three-point parabolic, and centroid fits).

When particle images become too small ( $\leq 1$  pixel) the uncertainty can increase due to "peak locking," or pixel locking. Peak locking results in displacements that are biased toward integer pixel values [12]. The presence of peak locking can be detected by making a histogram plot of PIV displacements. If peak locking is present, the histogram will have distinct peaks at integer displacement values. Image preprocessing can reduce the peak locking effect.

The uncertainty also increases when particles become too large (for a fixed camera resolution and 3-point sub-pixel estimation method). As the particle image diameters increase, the correlation peak becomes more wide, and the center of the correlation peak (used to estimate sub-pixel displacement) becomes more difficult to locate [17].

#### 1.2.2 Displacement Gradient

Displacement gradients, or flow velocity gradients, diminish the amplitude of the correlation peak and broaden its width. As the correlation peak decreases and broadens, the peak becomes less detectable, which results in increased uncertainty. The effect of the displacement gradient on the correlation peak can be seen in Fig. 1.2, where Fig. 1.2a and Fig. 1.2b are cross-correlation maps obtained from synthetic images with no displacement gradient and a 0.2 pixels/pixel displacement gradient, respectively.

The displacement gradient can be obtained using various techniques [18], including finite difference methods [19], biquadratic polynomial with least-squares interpolation [20],



Fig. 1.2: A demonstration of the effects of displacement gradients on the correlation peak. The correlation of synthetic images with constant parameters show the difference between applying (a) no gradient and (b) a 0.2 pixels/pixel gradient.

and radial basis functions [21, 22].

#### 1.2.3 Sub-Pixel Interpolation

Originally, the resolution of PIV measurements was limited to integer pixel values. To improve the resolution and accuracy, sub-pixel interpolation methods are used in PIV. These methods fit a curve to the correlation peak profile and estimate the location of the correlation peak.

Many methods exist for estimating the sub-pixel displacement, including the peak centroid method [23], Gaussian interpolation [9], sinc interpolation [24], and polynomial interpolation [25]. It has been suggested that the optimal sub-pixel fit method may be a function of the particle image diameter [26]. The uncertainty of the sub-pixel displacement depends on the interpolation method used and how well it fits the correlation peak profile.

#### 1.2.4 Particle Image Density

Another important parameter that effects the PIV measurement uncertainty is the particle image density,  $N_I$ . The particle image density is the mean number of particles per interrogation region. It is recommended to have  $N_I$  greater than 10 particles [27]. The PIV

measurement uncertainty is impacted by  $N_I$  in at least two ways:

- 1. Increasing the number of particle image pairs within an interrogation window increases the probability of a valid displacement vector [28] and decreases the measurement uncertainty [12, 29].
- 2. In regions of shear, small  $N_I$  can result in increased random uncertainty [3].

Due to its influence on uncertainty, it is important to know the local instantaneous particle image density. The particle image density may vary in space and time to due to variations in seeding levels or illumination issues. An estimation of the particle image density can provide the ability to rapidly scan a PIV dataset for these issues, determine the quality of a PIV dataset, and eliminate bad images prior to vector computation.

Few methods for estimating  $N_I$  are available. For sufficiently low  $N_I$ , one may attempt to simply count the number of particle images by hand. However, the task is time consuming and subject to user bias. It is also difficult to account for dim and overlapping particles within an interrogation domain.

Another method of determining  $N_I$  involves using a local maximum routine to find particles. A threshold is needed to separate low level peaks generated by noise and those created by actual particles. This method is also unable to account for overlapping particles and tends to underestimate  $N_I$  as the particle image density increases.

Previously, as part of a method for estimating the instantaneous local uncertainty, an estimate of the particle image density  $N_I$  has been made by applying a binary threshold to an interrogation domain, summing the binary values of the image, and then dividing by the approximate pixel area of a single particle [3]. This method was applied to computer generated PIV images called "synthetic images." This density estimate is not robust in that it requires a correction factor and threshold value unique to each image set and it also cannot account for overlapping particles.

The autocorrelation peak magnitude is proportional to the particle image density, particle image diameter, average particle image intensity, and the size of the interrogation domain. With the use of synthetic images and estimates of the particle image diameter and average particle image intensity, an empirical relationship between these parameters was determined; and a method to estimate the particle image density was developed.

The next chapter lists the objectives of this research. In chapter 3, the methods for calculating the autocorrelation peak magnitude, particle image diameter, and average particle intensity are discussed along with preprocessing techniques. Chapter 4 covers synthetic image generation and both experimental setups. Chapter 5 discusses the development of the empirical equation used in the autocorrelation-based density (ABD) method followed by the results of the ABD method on synthetic images with noise as well as images from the two experimental setups.

# Chapter 2

# **Objectives**

The objectives of the research are as follows:

- Develop a method for estimating the average particle intensity to be used in the autocorrelation-based density (ABD) method.
- Generate synthetic images that contain various particle image diameters, densities, and intensities.
- Develop an empirical relationship relating the particle image density to the correlation peak height, particle image diameter, average intensity, and interrogation region size.
- Determine the effect of noise on method by introducing synthetic noise derived from low speed and high speed PIV cameras.
- Determine the effect of particle image intensity on the particle image density estimation by varying the lens aperture.
- Estimate the particle image density of the experimental data using the ABD method and compare results those obtained by the local-maximum method and manual particle counting.

## Chapter 3

# Approach

The relative height of the autocorrelation peak  $R_h$  for an interrogation region in a PIV image is a function of particle image diameter  $d_{\tau}$ , particle image density  $N_I$ , interrogation domain area A, and average particle intensity  $\overline{I_p}$ . As each of these parameters increase, so does the height of the autocorrelation peak (see Fig. 3.1). In other words,

$$R_h = f(d_\tau, N_I, A, \overline{I_p}). \tag{3.1}$$

A method is presented to extract the particle image density from the autocorrelation peak height by quantifying the effects from the other contributing parameters. Before the density estimation takes place, noise due to the image background is removed and the images are normalized. Each of the parameters in Eq. 3.1 are discussed in detail, with exception to A, as its calculation is trivial, followed by the pre-processing techniques used.

#### 3.1 Relative Autocorrelation Peak Height

An autocorrelation map for a single interrogation area  $IA_1$  can be generated by computing the discrete autocorrelation

$$R(r,s) = \sum_{i=0}^{D_I - 1} \sum_{j=0}^{D_I - 1} IA_1(i,j) IA_1(i+r,j+s)$$
(3.2)

where  $D_I$  is the interrogation cell size and  $r, s = -D_I/2, ..., D_I/2 - 1$  [30].

Alternatively, a frequency domain based correlation may be used in place of Eq. 3.2 by applying the Wiener-Khinchin theorem [12]. Using this theorem, the autocorrelation is



Fig. 3.1: The relative height of the autocorrelation peak  $R_h$  as a function of (a)  $N_I$ , (b)  $d_{\tau}$ , (c) A, and (d)  $I_P$ . These figures were generated by autocorrelating synthetic images with known particle image diameters, densities, and intensities.

computed with Fourier transforms as

$$R(r,s) = \operatorname{Re}\left[FFT^{-1}\left\{FFT^{*}\left(IA_{1}\right)FFT\left(IA_{1}\right)\right\}\right]$$
(3.3)

where FFT denotes a Fast Fourier Transform, \* denotes the complex conjugate and the Re operator returns the real part of the complex number [12]. The use of FFTs in Eq. 3.3 to compute the autocorrelation requires less computation time compared to the discrete method in Eq. 3.2 [30]. In order to use Eq. 3.3, the interrogation size must be  $D_I = 2^n$ , where n is an integer value.

As a result of using Eq. 3.3 to correlate  $IA_1$  with itself, an autocorrelation map is generated such as shown in Fig. 3.2a. The relative peak height  $R_h$  is defined as

$$R_h = R_p - R_{min}.\tag{3.4}$$

where  $R_p$  is the height of the highest correlation peak and  $R_{min}$  is the lowest value in the correlation plane (Fig. 3.2b).  $R_h$  is calculated relative to the correlation background to help copmpensate for a general image background level or background noise.

#### 3.2 Average particle intensity

Since the autocorrelation peak magnitude depends on the average particle image intensity, a means to quantify this value based on the images is required. The average particle intensity is calculated by first locating obvious particles using a function that locates local maximum values in the image. Since the pixel intensity is an average of the light intensity incident on the pixel, the maximum intensity of a particle is highest and closest to the true maximum intensity when the particle is centered on a pixel. Therefore, the average particle intensity estimate was based on the particle images that were well aligned with the pixel grid. Figure 3.3 shows the difference in the particle image intensity for an off-center particle image and the preferred pixel-centered particle.

To determine if a particle (represented by a local maximum) is pixel-centered, the standard deviation of the intensity of the four adjacent pixels is computed. If the standard deviation is beneath a specified threshold value, it is deemed pixel-centered and contributes to the average intensity calculation. All of the values of the pixel-centered particles are averaged together to estimate the average particle intensity that contributes to the particle



Fig. 3.2: Preforming an autocorrelation using Eq. 3.3 produces (a) an autocorrelation map. A cross section of the autocorrelation map (b) shows the highest and lowest value of the autocorrelation map,  $R_P$  and  $R_{min}$  repectively. The difference between these values is the relative autocorrelation peak height.

image density calculation.

The average particle intensity estimation method is based solely on local maximum intensities and not the average intensity of the actual individual particles. When two or



Fig. 3.3: The maximum particle intensity depends on the sub-pixel location of the particle. A particle with a 0.25 pixel offset (a) has a reduced maximum pixel intensity and a larger standard deviation in surrounding pixels. A particle that is pixel centered (b) provides the best estimate of the actual maximum particle intensity. If a particle is pixel centered, the four adjacent pixels will have nearly identical intensity values and a low standard deviation.

more particles overlap, the intensities of the particles are summed together, which will increase the average particle intensity estimate. As a results the average particle intensity estimate will tend to overestimate the true average particle intensity as the density increases.

#### 3.3 Particle Image Diameter Estimation

The particle image diameter  $d_{\tau}$  and the width of the displacement-correlation peak  $d_D$ are related by

$$d_D \cong \sqrt{2d_\tau^2 + \frac{4}{3}a^2},$$
 (3.5)

where the gradient parameter a can be neglected when  $d_D$  is obtained through autocorrelation [17]. The autocorrelation peak width is commonly calculated using the  $e^{-2}$  width, which is four times the standard deviation for a Gaussian distribution. In order to find the standard deviation from the autocorrelation peak, a 3-point Gaussian fit [12] is applied to a cross section of the peak (Fig. 3.4).



Fig. 3.4: A 3-point Gaussian is applied to the cross section of the correlation peak using the three largest values.

Having obtained the standard deviation, and thereby the autocorrelation peak width, equation 3.5 is solved directly for the particle image diameter

$$d_{\tau} \cong 2\sqrt{2}\sigma,\tag{3.6}$$

where  $\sigma$  is the standard deviation of the Gaussian fit.

#### 3.4 Particle Image Density Estimation

Under favorable conditions, the particle image density can be approximated by counting particles within an interrogation region. As previously mentioned, the task is time consuming and subject to user bias. Also, it is difficult to estimate the number of particles when particles have low intensity or are overlapping.

The counting method for estimating the particle image density can also be automated by locating and counting local maxima above a specified threshold intensity. This is done by comparing the intensity of each pixel to the intensity of its 8 nearest neighbors. If the center pixel intensity is larger than its neighboring pixels, then it is deemed a local maximum and, therefore, contributes to the particle image density.

This local maximum method is able to provide somewhat accurate results when particle image diameter and density are low. As the particle image diameter and density increase, more particle images overlap, which decreases the number of local maximums and results in an under estimate of the particle image density. To demonstrate this trend, synthetic images were generated with known particle image diameters and image densities, the densities were estimated using the local maximum method. The results are shown in Fig. 3.5. As the density and diameter increase, the estimated value of the density decreases and the error increases.

#### 3.5 Image Preprocessing

One advantage of using digitally acquired images is the ability to remove non-ideal aspects. Under ideal circumstances images would contain brightly and uniformly illuminated particles against a perfectly dark background. However, this idealized scenario is rarely achieved in the experimental world. Generally, the image background is not perfectly dark due to background noise and particles may vary in intensity or have low contrast. This section will discuss methods for removing background noise and enhancing image contrast.

#### 3.5.1 Background Image Removal

Background noise can limit the ability to estimate the particle image diameter and particle image density. In some situations, noise may also cause error in the PIV measurements. Background noise results from many things, including the zero-level noise of the camera sensor, environmental lighting, non-uniform lighting, and laser reflections from



Fig. 3.5: Particle image density estimation using a the local maximum method to locate and count local maximums (particles) in synthetic images with known particle image diameters and image densities.

stationary objects in the flow [31]. Noise from such sources should be avoided; however, this is not always possible. Alternatively, preprocessing can be use to remove non-ideal aspects from the images and is generally beneficial [32, 33]. One technique used to remove noise is to subtract a background image from the original image.

Some of the most common methods of determining the background image include:

- 1. Recording an image of an illuminated flow without tracer particles.
- 2. Using the average or local minimum of all images within the image set [34, 35].
- 3. Applying a low-pass/uniform or a median filter [17].

In the present work, the local minimum method was used to generate the background image. The entire image set was analyzed, and the minimum value for each pixel location was used for the background [26]. Fig. 3.6 shows an original image, the calculated background using this approach, and the image with the background subtracted. Without the use of background removal, stationary objects and ambient lighting contribute to the density estimate, resulting in significantly inflated results.



Fig. 3.6: Demonstration of the impact of background removal: (a) the original image (b) the background image based on the local minimum value from each pixel, and (c) the processed image.

#### 3.5.2 Image Normalization

The range of pixel intensities within an image set may vary due to the laser intensity, lens focal number, and bit depth of the camera sensor. By adjusting these parameters, particle intensities can range from values below the noise floor to the saturation threshold. Local variations in intensity can occur due to variations in the size of the tracer particles, nonuniform lighting, and reflections [17]. Preconditioning methods are available to optimize the contrast levels and provide uniform intensity across the full image. Some of these methods include histogram equalization and min-max filtering [17].

The presence of unequal illumination between image sets, and interrogation regions within those sets, can result in inaccurate estimates of the density. Therefore, the images are normalized to ensure a consistent range in pixel intensities. Normalization occurs by subtracting off the smallest intensity value  $I_{min}$  in the image and then dividing by  $I_{max}$ , where

$$I_{max} = \mu_p + 4\sigma_p, \tag{3.7}$$

and where  $\mu_p$  and  $\sigma_p$  are the mean and standard deviation of the particle intensities respectively. Any pixel intensity greater than  $I_{max}$  is set to  $I_{max}$  in order to remove the effects of particles with intensities far from the mean. As a result, images from different cameras with different bit-depths, appear more similar and provide more consistent density estimates.

## Chapter 4

## Synthetic Image Generation and Experimental Setups

The autocorrelation-based density method was developed using synthetic images with known particle image diameters, image densities, particle image intensities, and interrogation region sizes. The method was then verified using images from two experimental setups. This chapter discusses the synthetic image generator and the experimental setups used to supply images to develop and verify the autocorrelation-based density method.

#### 4.1 Synthetic Image Generation

The particle image density estimation method that has been developed employs an empirical relationship between the autocorrelation peak height and the other parameters previously discussed to estimate the particle image density. The relationship was developed using synthetic data with known particle image diameter, density, and intensity. The synthetic image generator that was used is described by Timmins *et al.* [3]. The simulated particles are randomly distributed throughout the interrogation domain area and within the width of the light sheet. The intensity distributions of the particles are represented by Gaussian functions and the particle image diameter is four times the standard deviation of the Gaussian distribution. The laser sheet intensity distribution is Gaussian and the maximum intensity of any given particle is determined by the particle's position within the light sheet. Flow properties, such as displacement and shear, have no effect on the autocorrelation and are not accounted for.

In order to determine the effect of noise on the density estimation, two levels of background noise were added to the synthetic images to simulate the noise generated by the cameras used in acquiring PIV measurements. To approximate the background noise, 100 images pairs were acquired using two different cameras with the lens caps on. The cameras that were used are the PCO Sensicam QE 12-bit  $1376 \times 1040$  CCD and the Photron Fast-Cam APX RS 10-bit CMOS camera. From these images, the mean and standard deviation of the noise intensity were calculated and applied to the synthetic images.

#### 4.2 Experimental Setups

In order to determine the robustness of the autocorrelation-based density method, testing the method on actual experimental data is important. This was done by applying the method on data from two separate experimental setups with different seeding particles and flow media. The image densities were varried by introducing more seed into the fluid. Each experimenta setup is discussed in detail in the following sections.

#### 4.2.1 Laminar Jet

The first experimental setup consisted of a high-Reynolds-number, large shear, laminar rectangular jet submerged in ambient air (Fig. 4.2). Images were acquired using the PCO camera described above with a 105 mm lens and a New Wave dual-cavity 50 mJ / pulse Nd:YAG laser. This system was controlled using DaVis 7.2 from LaVision [7] and seeded with olive-oil droplets formed in a Laskin nozzle and added at the blower inlet.



Fig. 4.1: The schematic for the lamnar jet setup. The air drawn in by the blower is seeded, conditioned, and then exits to the measurement region.

Six evenly spaced values of the volume flow rate through a Laskin nozzle were used to adjust the seeding density and, thereby, the particle image density. The values through


Fig. 4.2: Data is taken at the channel exit. The camera is positioned perpendicular to the flow and normal to the laser plane.

the Laskin nozzle ranged from 200 to 450 standard liters per minute (SLPM) in increments of 50 SLPM. The input air volume flow rate and the output seed mass flow rate have been shown to behave somewhat linearly [36]. For each case, 1000 images were acquired and analyzed using the autocorrelation-based density method. Density estimates were also made by applying the local maximum method discussed earler, as well as manually counting particles from several interrogation regions.

# 4.2.2 Aquarium

The second experimental setup consisted of a 10-gallon aquarium filled with water and seeded using Potters 110P8 hollow glass microspheres, which have the characteristics shown in Table 4.1. The same laser, lens, and camera were used in conjunction with DaVis 7.2 to acquire the images. A schematic of the aquarium setup is found in Fig. 4.3. As part of this experiment, the effect of particle image diameter and image intensity on the density estimation was investigated.

The particle image diameter can be varied by adjusting the f-number (or aperture) of the lens. The particle image diameter varies as a function of the f-number through the following approximation [12]:

$$d_{\tau} = \sqrt{(Md_p)^2 + d_s^2},$$
(4.1)

Table 4.1: The properties of Potters hollow microspheres used as seed. <sup>1</sup>Density is measured by gas displacement pycnometer. <sup>2</sup>Bulk Density is the weight as measured in a container and includes the interstitial air. <sup>3</sup>Data represents percent volume distribution measured using laser light scatter technique.

Properties of Potters 110P8 Microspheres		
Density <sup>1</sup> , $g/cc$	$1.10\pm0.05$	
Bulk Density <sup>2</sup> , $g/cc$	0.49	
Size Distribution <sup>3</sup> ( $\mu$ m)		
10%	5	
50%	10	
90%	21	
97%	25	

where M denotes the magnification of the image,  $d_p$  is the actual particle diameter, and  $d_s$  is the diffraction-limited spot diameter,

$$d_s = 2.44 \,(1+M) \, f^\# \lambda, \tag{4.2}$$

where  $\lambda$  is the wavelength of the light sheet, and  $f^{\#}$  is the ratio of the lens focal length, f, and aperture diameter,  $D_a$ .

The amount of light that reaches the CCD array of the camera is a function of the laser intensity and the aperture of the lens. By reducing the camera lens aperture while maintaining a constant laser intensity, the effective amount of light incident on the CCD array of the camera will decrease. This decrease in the amount of light captured by the camera will result in fewer recognizable particle images.

To better illustrate why this decrease particle image density occurs, consider seeding particles that have a size distribution that is Gaussian (the exact shape of the distribution is not important to this discussion). The intensity of the light reflected from the particles is a strong function of particle diameter [12,17] and their location in the Gaussian-profiled light sheet. Therefore, the intensities of the reflections off the particles will also have a wide distribution. Meanwhile, the camera sensor can only capture particle intensities that are above its floor value. A decrease in the image intensity, weather due to the laser intensity or lens aperture size, will result more particles dropping below the noise floor.



Fig. 4.3: The schematics for the aquarium setup. Views include (a) an overhead and (b) side view of the setup. The camera is positioned normal to the laser plane.

The situation is illustrated in Fig. 4.4. As the aperture size decreases (Fig. 4.4a to Fig. 4.4c), for a fixed laser intensity and number of actual particles, the average particle reflection intensity decreases and the reflections from smaller particles, as well as particles near the edge of the laser sheet, are lost in the noise floor. Therefore, even though the number of particles inside the field of view remains the same, the particle image density

(from the camera's point of view, which is what is important to PIV) decreases. Clearly, the extent to which this is observed will depend on the width of the particle size distribution.



Fig. 4.4: Effect of laser intensity and camera aperture on the estimated particle image density. Three apertures are depicted: (a) large, (b) medium and (c) small. For each case, four values of total seed mass m are depicted. The black and red shades represent the camera noise floor and saturation level, respectively.

The area under any curve and outside of the shaded regions in Fig. 4.4 represents the number of visible particles in the interrogation region. When the amount of seed is doubled, the density will double. As more seed is added, the density will continue to increase linearly, but with different slopes for each case. This trend may also occur for a case with a constant aperture and decreasing laser intensity.

Five sets of data, each with twelve density levels, were acquired using a PCO Sensicam QE 12-bit 1376  $\times$  1040 CCD with a Nikon 105mm f/2.8D AF Micro-Nikkor Lens. The f-number (which is inversely proportional to aperture size) and laser intensity were varied in each set to determine the effect of particle image diameter and image intensity on the density estimation.

The first three image sets had f-numbers of 11, 16, and 22. The laser intensity  $\mathcal{J}$  for the first three sets was adjusted such that few particles reached the camera saturation level. The final two image sets, f/16 ( $\mathcal{J}_{f11}$ ) and f/22 ( $\mathcal{J}_{f11}$ ), were acquired using the higher two f-numbers while using the same laser intensity used for the f = 11 image set.

The seeding density was varied by adding known masses of seed to the flow medium. The mass of each seed sample was measured using a highly accurate analytical balance. For each measured mass of seed, 500 images were acquired and analyzed using the autocorrelationbased density method. Again, density estimates were obtained by applying the local maximum method, as well as manually counting particles from several interrogation regions.

# Chapter 5

# Results

This chapter discussed the derivation of the empirical equation for estimating the particle image density. The autocorrelation-based density (ABD) method is then applied to synthetic data with known parameters in order to demonstrate the effect of particle image diameter and particle image density on the ABD estimate. The effect of noise on the ABD estimate is then investigated by adding artificial noise to the synthetic images. Lastly, the ABD method is applied to experimental data from the two different experimental setups. Results from the experimental data are discussed, including the effect of varying the particle image diameter and image intensity.

# 5.1 Empirical Relationship

The ABD method for estimating the particle image density is based on relationship between the autocorrelation peak height and the other parameters discussed previously. The relationship was developed using synthetic data with known particle image diameter and particle image density. The parameter space for the synthetic images used are shown in Table 5.1. While the particle image diameter and density were specifically specified, the average particle intensity was allowed to vary as a function of the other parameters. As the particle image diameter and density increase, so does the number of overlapping particles and, therefore, the estimated average particle intensity.

Synthetic Image Parameter	Lower Limit	Upper Limit	Step size
Particle Image Diameter (pixels)	1.5	8.0	0.5
Particle Image Density (particles/ $32 \times 32$ )	5	80	5

Table 5.1: Synthetic image parameter space.

For each combination of parameters, 1000 interrogation sized synthetic images were generated. Interrogation sizes included  $32 \times 32$ ,  $64 \times 64$ ,  $128 \times 128$ , and  $256 \times 256$ . The relative autocorrelation peak height, average particle intensity, and particle image diameter were calculated for each synthetic image as previously described. Values for these parameters were averaged and are shown in Fig. 5.1 where the particle image density  $N_I$  is a function of the particle image diameter  $d_{\tau}$  and the multiplication of  $R_h$ , A, and  $\overline{I_P}$ .



Fig. 5.1: Averaged values of the particle image diameter, relative autocorrelation peak height, interrogation area, and average particle intensity obtained from synthetic images are plotted as a function of the true particle image density. Values for  $R_h$ , A, and  $\overline{I_P}$  are combined through multiplication in order to reduce the 5-D surface to a 3-D surface for viewing purposes.

The relationship between the parameters and the particle image density was modeled using a power-law function of the form

$$N_I \cong a \left( R_h \right)^b \left( d_\tau \right)^c \left( \overline{I_p} \right)^d \left( A \right)^e, \tag{5.1}$$

where the coefficients a, b, c, d, and e were determined using a nonlinear least-squares fit with a trust-region algorithm. Values for the coefficients are found in Table 5.2 with their corresponding goodness of fit value.

Table 5.2: The coefficients for Eq. 5.1 determined using a least-squares fit and the corresponding goodness of fit value.

a	b	c	d	e	R-square
178	1.40	-2.03	-2.05	-1.42	0.993

Applying the coefficients from Table 5.2 to Eq. 5.1 yields

$$N_I \cong 178 \frac{(R_h)^{1.40}}{(d_\tau)^{2.03} \left(\overline{I_p}\right)^{2.05} (A)^{1.42}}.$$
(5.2)

All of the synthetic images defined by the parameter space were processed using Eq. 5.2. For each parameter set, the mean particle image density  $\overline{N_I}$  was calculated as

$$\overline{N_I} = \frac{1}{N} \sum_{i=1}^{N} (N_I)_i \tag{5.3}$$

where N is the number of measurements  $(N_I)_i$ . The absolute error  $\epsilon$  and the relative error  $\eta$ between the mean particle image density values and the actual density values for a  $128 \times 128$ interrogation region are shown in Fig 5.2a-b. The absolute and relative errors are defined as

$$\epsilon = \left| N_{I,true} - \overline{N_I} \right| \tag{5.4}$$

and

$$\eta = \frac{\left|N_{I,true} - \overline{N_{I}}\right|}{\left|N_{I,true}\right|} \tag{5.5}$$

where  $N_{I,true}$  is the actual density of the synthetic images.

While using a  $128 \times 128$  interrogation region, the ABD method produces a maximum absolute error of 4.5 particles at a particle image density of 80 particles/( $32 \times 32$ ) and a particle image diameter of 1.5 pixels which corresponds to a relative error of 5.6%. The maximum relative error of 44.1% occurs at a particle image density of 5 particles and a 2.0 pixel particle image diameter with a corresponding absolute error of 2.2 particles/ $(32 \times 32)$  region.

For the  $256 \times 256$  interrogation region, the absolute error (Fig. 5.3a) reaches a maximum of 4.8 particles/ $(32 \times 32)$  at an image density of 80 particles/ $(32 \times 32)$  and particle image diameter of 1.5 pixels which corresponds to a relative error of 6.0%. The relative error (Fig. 5.3b) reaches a maximum of 47.0% at a particle image density of 5 particles/ $(32 \times$ 32) and particle image diameter of 2 pixels which corresponds to an absolute error of 2.3 particles/ $(32 \times 32)$ .

When using a  $64 \times 64$  interrogation region, the absolute error (Fig. 5.4a) reaches a maximum of 5.2 particles/( $32 \times 32$ ) at a particle image density of 80 particles/( $32 \times 32$ ) and particle image diameter of 1.5 pixels which corresponds to a relative error of 6.4%. The relative error (Fig. 5.4b) reaches a maximum of 40.6% at a particle image density of 5 particles/( $32 \times 32$ ) and particle image diameter of 2 pixels which corresponds to an absolute error of 2.0 particles/( $32 \times 32$ ).

Lastly, a  $32 \times 32$  interrogation region provides a maximum absolute error of 6.9 particles/( $32 \times 32$ ) at a particle image density of 80 particles/( $32 \times 32$ ) and particle image diameter of 1.5 pixels which corresponds to a relative error of 8.6% (Fig. 5.5a). The maximum relative error for a  $32 \times 32$  interrogation region is 31.9% at a particle image density of 5 particles/( $32 \times 32$ ) and particle image diameter of 2 pixels which corresponds to an absolute error of 1.6 particles/( $32 \times 32$ ) (Fig. 5.5b).

The average absolute and relative errors for each interrogation region size are shown in Table 5.3. The lowest errors on average are obtained by using the  $64 \times 64$  interrogation region. For all cases, the relative error is largest at an image density of 5 particles/( $32 \times 32$ ) and a particle image diameter of 2 pixels. The absolute error at this location is relatively small, however, the value for particle image density is below the value recommended for PIV and should be avoided. For all cases the aboslute error is highest at a particle image density of 80 particles and a particle image diameter of 1.5 pixels.



Fig. 5.2: The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average  $N_I$  from 1000 synthetic images with known densities. A 128 × 128 interrogation region was used.



Fig. 5.3: The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average  $N_I$  from 1000 synthetic images with known densities. A 256 × 256 interrogation region was used.



Fig. 5.4: The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average  $N_I$  from 1000 synthetic images with known densities. A  $64 \times 64$  interrogation region was used.



Fig. 5.5: The relative and absolute errors of the average particle image density calculated using the ABD method. Estimated results are based on the average  $N_I$  from 1000 synthetic images with known densities. A  $32 \times 32$  interrogation region was used.

Table 5.3: The mean absolute errors  $\overline{\epsilon}$  and the mean relative errors  $\overline{\eta}$  for the ABD method obtained for each interrogation size. Errors were obtained by averaging results from the synthetic images used to develop Eq. 5.2

A	$\overline{\epsilon} \text{ (particles}/(32 \times 32))$	$\overline{\eta}~(\%~)$
$32 \times 32$	1.91	7.13
$64 \times 64$	1.09	4.50
$128 \times 128$	1.24	5.31
$256 \times 256$	1.57	6.67

#### 5.2 Test with Synthetic Images

While the autocorrelation-based density method was developed using synthetic data, testing it further on synthetic data makes it possible to determine how precision uncertainty varies with the given image parameters and to investigate the ABD method's sensitivity to image noise. The autocorrelation-based density method previously discussed was coded into MATLAB and is shown in the Apendix. The ABD method was applied to synthetic images with known particle image diameters and densities. The parameter space used may be found in Table 5.1.

The results for the following cases were obtained by calculating the particle image density for 100 interrogation regions using both the autocorrelation-based density method and the local maximum method previously discussed. The sample mean  $\overline{N_I}$ , defined in Eq. 5.3, and sample standard deviation  $s_{N_I}$ , defined by

$$s_{N_{I}} = \left[\frac{1}{N-1} \sum_{i=1}^{N} \left( (N_{I})_{i} - \overline{N_{I}} \right)^{2} \right]^{1/2},$$
(5.6)

were calculated for each case. The mean particle image density was plotted for each case and the standard deviation was used to calculate uncertainty bands. All uncertainty bands contained in the following figures represent the 95% confidence interval  $(1.96 s_{N_I} / \sqrt{N} [37])$ of the mean particle image density.

## 5.2.1 Effect of Density

In the first case, synthetic images were generated with varying densities ranging from

5 to 80 particles per  $32 \times 32$  interrogation region, particle image diameters of 3 pixels, and no noise. Figure 5.6 shows the calculated particle image density as a function of the actual particle image density. The particle image density was calculated using the ABD method as well as the local maximum method for interrogation sizes of  $32 \times 32$ ,  $64 \times 64$ ,  $128 \times 128$ , and  $256 \times 256$ .



Fig. 5.6: The effect of particle image density on the particle image density estimation. Synthetic images were used with known densities, particle image diameter of 3 pixels, and no noise. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.

The ABD method provides similar averaged results for all interrogation sizes. The local maximum method underestimates the particle image density as the actual density increases due to an increase in overlapping particle images. Increasing the density also increases the the precision uncertainty (Fig. 5.7). However, as the interrogation region size increases, the effect of increased density on the precision uncertainty diminishes due to spatial averaging. Larger interrogation regions allow for more spatial averaging and less variation in the estimated particle image density.



Fig. 5.7: Precision uncertainty of the particle image density estimation results in Fig. 5.6. The following values represent the 95% confidence interval  $(1.96s_x/\sqrt{N})$  for each interrogation region size.

# 5.2.2 Effect of Diameter

The particle image diameter also affects the performance of the ABD method as shown in Fig. 5.8. Images were generated with varying particle image diameters ranging from 1.5 to 8 pixels, constant density of 40 particles per  $32 \times 32$  region, and no synthetic noise. Again the particle image density was calculated using the autocorrelation-based density method as well as the local maximum method for interrogation sizes of  $32 \times 32$ ,  $64 \times 64$ ,  $128 \times 128$ , and  $256 \times 256$ .



Fig. 5.8: The effect of particle image diameter on the particle image density estimation. Synthetic images with a particle image density of 40 particles per  $32 \times 32$  region and no noise were used. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.

The mean ABD estimates vary between interrogation sizes due to the goodness of the nonlinear least squares fit. In general, the particle image density estimates decrease with increased interrogation size. For all cases the ABD estimates are significantly more accurate than the results obtained by the local maximum method. The local maximum method underestimates the particle image density as the particle image diameter increases. Increasing the particle image diameter increases the amount of particle image overlap and decreases the number of distinct particle images. For all interrogation sizes, the precision uncertainty remains fairly constant as the diameter increases (Fig. 5.9). However, as the interrogation region size increases, the effect of diameter on the precision uncertainty diminishes due to spatial averaging.



Fig. 5.9: Precision uncertainty of the particle image density estimation results in Fig. 5.6. The following values represent the 95% confidence interval,  $1.96s_x/\sqrt{N}$ .

# 5.2.3 Effect of Noise

Two levels of background noise were added to the synthetic images to simulate the noise generated by the PCO Sensicam and Photron FastCam cameras with lens caps in place. The mean and standard deviation of the noise intensity was calculated from 100 images pairs and are found in Table 5.4. The artificial noise was applied to synthetic images randomly using a normal distribution. The synthetic images had particle images with 3-pixel diameters. The average image densities for 1000 images were calculated with the autocorrelation-based density method and the local maximum method using  $32 \times 32$  and  $128 \times 128$  interrogation regions.

Table 5.4: The mean and standard deviation of noise from two PIV cameras.

Camera	Bit Depth	Mean	Standard Deviation
PCO Sensicam	12-bit	41.411	2.441
FastCam APX RS	10-bit	2.902	14.541

The results in Fig. 5.10 show that the particle image density estimation is mostly unaffected by the synthetic background noise for a  $32 \times 32$  interrogation region. As the density increases the particle image density estimates from the camera 2 noise synthetic images drop below the particle image density estimates from the images with no noise and camera 1 noise. The same trend is seen in the precision uncertainty shown in Fig. 5.11.

The results in Fig. 5.12 show that the density estimation is mostly unaffected by the synthetic background noise for a  $128 \times 128$  interrogation region and produces similar results for all cases. However, as the particle image density increases, the precision uncertainty increases similarly for the no noise and camera 1 noise cases (Fig. 5.13), while the precision uncertainty for the camera 2 noise case somewhat drops off.

Overall, the addition of noise to the synthetic images has little effect on the autocorrelationbased density method. Most of the negative effects of the synthetic noise are removed by the background image removal process discussed earlier in this paper. In order to demonstrate the effect of background image subtraction, the image densities of the synthetic images



Fig. 5.10: Effect of introducing noise into the synthetic images on the particle image density estimation for a  $32 \times 32$  interrogation region. The noise generated simulates the noise introduced using two different types of cameras.

with noise from camera 1 and 2 were reprocessed without background removal. Results from both the autocorrelation-based density method and the local maximum method for a  $128 \times 128$  interrogation region are shown in Fig. 5.14.

Without the use of background subtraction, both the autocorrelation-based density method and the local maximum method are unable to provide acceptable estimates of the particle image density. At low image densities both methods severely over estimate the particle image density due to the random fluctuations of the noise. For the local max method, the random noise fluctuations form peaks, or "false particles," that contribute to the particle image density. These false particles have a relatively low intensity compared to true particles. At low image densities the false particles dominate the number of true particles



Fig. 5.11: The effect of noise on the precision uncertainty of the density estimation results in Fig. 5.10.

and, therefore, reduce the average intensity which, in turn, causes the autocorrelation-based density method to overestimate the particle image density. As the density increases, the particle image density estimates begin to be dominated by true particle images (rather than false particles), and the slopes of the noisy data converge to the slope of the no noise particle image density estimates. Background image subtraction should always be performed when using the autocorrelation-based density method to estimate the particle image density.

#### 5.3 Experimental Verification

Experimental verification of the synthetic image results requires a PIV setup with adjustable particle image density. This requirement was realized by incrementally increasing the amount of seed particles within the fluid. Images with increasing density were collected



Fig. 5.12: Effect of introducing noise into the synthetic images on the particle image density estimation for a  $128 \times 128$  interrogation region. The noise generated simulates the noise introduced using two different types of cameras.



Fig. 5.13: The effect of noise on the precision uncertainty of the density estimation results in Fig. 5.12.



Fig. 5.14: Effect of introducing noise into the synthetic images on the particle image density estimation for a  $128 \times 128$  interrogation region when background image subtraction is not used. The noise generated simulates the noise introduced using two different types of cameras. The particle image density was estimated using the autocorrelation-based density (ABD) and local maximum (LM) methods.

from two separate experimental setups with different flow media and seed particles.

#### 5.3.1 Laminar Jet Tests

The first experimental setup was a submerged rectangular jet in ambient air seeded with olive-oil droplets as previously described in section 4.2.1. For each of the six evenlyspaced values of the volume flow rate through the Laskin nozzle, 1000 images were acquired and analyzed using the ABD method. Interrogation regions  $(32 \times 32 \text{ pixels})$  from the images acquired using the laminar jet setup are shown in Fig. 5.15. Particle image density estimates were also made using the local maximum method and by manually counting particles from several interrogation regions. The average of the manually counted densities with their accompanying 95% confidence intervals are plotted along side the results from the ABD and local maximum methods in Fig. 5.16. Confidence intervals for the ABD method and local maximum method are not shown as they are insignificant for such a large sample size.



Fig. 5.15: Interrogation regions  $(32 \times 32 \text{ pixels})$  from images acquired using the jet setup. The particle image density was adjusted by increasing the volume flow rate (SLPM) into a Laskin Nozzle.

The results show a nearly linear relationship between flow rate into the Laskin nozzle and the estimated particle image density as anticipated. The local maximum method and manual count results resemble the ABD results until the particle density exceeds 40 particles, after which the local maximum results and manual count start to fall below the ABD methods results. This difference in the results is expected. As the particle image density increases, particle images become lost due to particle overlap. In effect, the rate of particle image density per increase in flow rate decreases for the methods based on local maximums.

#### 5.3.2 Aquarium Tests

The second experimental setup, described in section 4.2.2, consisted of an aquarium filled with water and seeded with hollow glass spheres. Known masses of seeding particles were added to increase  $N_I$ . The purpose of this experiment was to further explore the autocorrelation-based density method's ability to estimate the particle image density when subject to variations in particle image density and image intensity.

Twelve values of particle image density were obtained by adding measured masses of seed, whose values are shown in Table 5.5, to the aquarium setup. In attempt to evenly distribute the particles and avoid clumping, the seed samples were first blended into a



Fig. 5.16: Particle image density estimates as a function of the mass flow rate to the seeder for the rectangular jet case. Results were acquired implementing the autocorrelation-based density method (solid lines and symbols) as well as the local maximum method and manually counting particles from several interrogation regions. Error bars represent the 95% confidence interval.

portion of the flow media being added to the remaining fluid within the aquarium. Sets of 500 images were acquired for each diameter and density. Interrogation regions  $(32 \times 32$ pixels) from the images acquired using the aquarium setup are shown in Fig. 5.17.

#### 5.3.3 Effect of Particle Image Density

The first aim of this experiment was to determine the effect of the particle image diameter on the autocorrelation-based density method. The diameter was altered by by adjusting the f-number (or aperture) of the lens. Using f-numbers of 11, 16, and 22, produced particle image diameters of 2.4, 3.0, and 4.0 pixels, respectively.

As the f-numbers increases, the aperture decreases, and the amount of light captured by the camera decreases. To compensate for the variation in captured light, the laser intensity

Sample	Mass (g)	Total Mass (g)
1	0.02037	0.02037
2	0.03016	0.05053
3	0.04959	0.10012
4	0.04962	0.14974
5	0.05012	0.19986
6	0.05009	0.24995
7	0.15047	0.40042
8	0.20017	0.60059
9	0.20141	0.80200
10	0.20017	1.00217
11	0.39960	1.40177
12	0.39823	1.80000

Table 5.5: The samples of hollow glass spheres mixed into the water of the aquarium experiment. The mass of each sample was measured using an analytical balance.



Fig. 5.17: Images with varying densities acquired from the aquarium setup. The density was increased by adding known masses of hollow glass spheres. Images shown are for the f/22 case.

was adjusted in order to obtain similar average image intensities and few saturated particle images for each f-number. The average particle image density values estimated using the autocorrelation-based density method, local maximum method, and manual counting are shown in Fig. 5.18.

The results in Fig. 5.18 show a linear relationship between the cumulative mass of the seeding particles and the density estimated by the ABD method. The data for f/11



Fig. 5.18: Estimated seeding density as a function of the total mass of seed particles added to the water. Values are shown using three different particle image diameters. The solid lines with the solid symbols represent the average particle image density calculated using the autocorrelation-based density method. The dashed and dotted lines with the open symbols represent the average particle image density estimates from the local maximum (LM) method and manual counting (MC), respectively.

and f/16 are very similar because the average image intensities were nearly identical. The maximum available laser intensity for the f/22 case was not sufficient to compensate for the decrease in aperture, and thus the intensity of the images was noticeably decreased, leading to a slight loss in recognizable particles and a slight decrease in slope.

The density estimates obtained by the local maximum method and manually counting particles follow the same linear trend at lower densities. As the density increases, the local maximum and manual counting methods are unable to account for overlapping particles and, therefore, underestimates the particle image density. The difference between the local maximum particle image density estimates is due to the increase in particle image diameter. As the diameter increases, more particle overlap occurs, which reduces the particle image density estimate.

## 5.3.4 Effect of Image Intensity

In the second part of the this experiment, the effect of image intensity was investigated. The image intensity was varied by adjusting the f-number while maintaining a constant laser intensity  $\mathcal{J}$ . Two image sets were acquired  $(f/16 \ (\mathcal{J}_{f11}) \ \text{and} \ f/22 \ (\mathcal{J}_{f11}))$  with the same laser intensity used for the f/11 image set. The average particle image density values estimated using the autocorrelation-based density method, local maximum method, and manual counting are shown in Fig. 5.19.

Although the slopes for each f-number is different, the results in Fig. 5.19 still show a linear relationship between the cumulative mass of the seeding particles and the density estimated by the ABD method. For smaller apertures (larger f numbers) the particle image density is smaller as less light reaches the CCD sensor. As before, the local maximum method and manually counting are unable to maintain a linear particle image density relationship as the mass of seed increases due to particle overlap.



Fig. 5.19: Estimated seeding density as a function of the total mass of seed particles added to the water. Values are shown using three different image intensities. The image intensity was adjusted by using different f-numbers while maintaining a constant laser intensity. The solid lines with the solid symbols represent the average particle image density calculated using the autocorrelation-based density method. The dashed and dotted lines with the open symbols represent the average particle image density estimates from the local maximum (LM) method and manual counting (MC), respectively.

# Chapter 6 Conclusions & Future Work

# 6.1 Conclusions

An autocorrelation-based (ABD) method for estimating the particle image density has been develop. The ABD method is based on the relationship between the relative autocorrelation peak magnitude and the parameters that contribute to its magnitude, namely the particle image diameter, particle image density, interrogation region size, and average particle intensity. The magnitude of each of these parameters was found to be directly proportional to the autocorrelation peak magnitude. Established methods for quantifying the relative autocorrelation peak height and particle image diameter, as well as a new method for estimating the average particle intensity were presented.

Synthetic images with know image parameters were generated. The known image parameters were estimated using the methods previously discussed. A least squares fit was implemented to develop an empirical relationship between the known particle image density and the estimated parameters from interrogation regions of size  $32 \times 32$ ,  $64 \times 64$ ,  $128 \times 128$ , and  $256 \times 256$  pixels. The error between the known particle image density and the estimated particle image density from the autocorrelation-based density method was presented. The mean errors were minimized when using the  $64 \times 64$  interrogation region size having mean absolute and relative errors of 1.09 particles/ $(32 \times 32)$  and 4.50%, respectively.

The ABD method was tested further on synthetic images in order to investigate the effect of image parameters on the precision uncertainty and the effect of noise on the particle image density estimate. In general the precision uncertainty decreases with interrogation size, increases with particle image density, and relatively unaffected by particle image diameter. Two levels of noise were added to the synthetic images based on the noise generated by PIV cameras. The effect of the synthetic noise was found to have little influence on

the particle image density estimation and precision uncertainty. The effect of background image subtraction on noisy data was also investigated. In the absence of background image subtraction, the error of the particle image density estimates from the noisy data was highly elevated for both the ABD method and the local maximum method. The measurement error was especially high at low particle image density.

The ABD method was then applied to experimental data acquired from two experimental setups. The first setup consisted of a laminar rectangular jet submerged in air and seeded with olive oil droplets. The density of the flow was increased by increasing the volume flow rate to the Laskin Nozzle. The particle image density was estimated with the ABD method, the local maximum method, and by manually counting particles. As anticipated, results from the ABD method provided a near linear increase in particle image density while the estimates of the other two methods eventually leveled off as the volume flow rate increased.

The second experimental setup consisted of an aquarium filled with water. The density was increased by adding known amounts of seed to the medium. As part of this experiment, the effect of particle image diameter and image intensity was investigated by adjusting the f-number and laser intensity. As the particle image diameter increased little change was observed in the particle image density estimates from the ABD and local maximum methods. The image intensity, however, was found to affect the particle image density estimation substantially. As the image intensity decreased (due to an increase in f-number) so did the slope of particle image density results. It is suggested that this difference in the particle image density is due, at least in part, to the amount of light captured by the camera. As the aperture decreases, the intensity of light captured by the camera decreases, which results in dim particles becoming more dim and becoming lost amidst the noise floor.

For both experimental studies, the ABD method was able to provide a near linear increase in particle image density as a function of volume flow rate (for the laminar jet) and mass (for the aquarium) as expected. The local maximum method was able to match the ABD method when the particle image density was low, however, as the density increased the slop of the particle image density estimate began to decay as a result of particle overlap. Similar results were seen by manually counting particle images. The consistent linear results obtained with the autocorrelation-based density method demonstrates its ability to account for overlapping particles.

## 6.2 Future Work

Future work into the development of the autocorrelation-based estimate of the particle image density may include developing a different method to estimate the average particle intensity, using an alternate image normalization technique, and refining and expanding the image parameter space.

The average particle intensity estimation used in the ABD method is based on a local maximum method. When calculating the average particle intensity no distinction is made between individual and overlapping particles. When particles overlap their intensities sum, which results in an overestimation of the average particle intensity. The development of an alternate method to estimate the average particle intensity that is able to account for particle overlap may provide more accurate results. Alternatively, the influence of the average particle intensity on the ABD method may possibly be removed or lessened through the implementation of a different image normalization method.

The normalizing method used in the image intensity estimation subtracts the lowest intensity from the entire image and then normalizes the image intensity with respect to the highest intensity values within a given interrogation region. Other methods for image normalization are available, including a min-max filtering method presented by Adrian [17]. The min-max filter is a nonlinear filter that, in effect, determines the upper and lower envelope on the image signal. The lower and upper envelops are smoothed using a uniform filter. The lower envelop is subtracted from the image and the upper envelop is used to normalize the image. As a result the image particles have a more uniform intensity level across the entire image and between image sets. The min-max method for normalizing the image particle intensity may remove the need to quantify the average particle intensity, or at least decrease the average particle intensities influence on the ABD method. Finally, refining and expanding the image parameter space may provide the means to generate a more accurate fit capable of providing higher accuracy results over a wider range of densities. An expansion of the particle image diameter may be done; however, most PIV algorithms implement 3-point fits to estimate the sub-pixel displacements which perform poorly on particles with large image diameters.

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Appendix

This appendix contains the MATLAB code for the autocorrelation-based density method used to process data from synthetic and experimental data. A graphical user interface (GUI) is also available, but is not included.

```
1 %Version updated 10/9/12 PIVdiaden v.4.4a
2
  % USE THIS CODE BY RUNNING THE GUI 'PIVdiadenGUI.m' WITH
3
   % 'PIVdiadenGUI.fig' IN THE SAME LOCATION AS THIS CODE.
\mathbf{4}
5
   % TO USE THIS CODE WITHOUT THE GUI THE STRUCTURE 'Data' MUST
6
  % BE CREATED TO RUN 'PIVdiaden(Data)'.
7
  8{
8
        Data.imdirec = 'C:\Users\EFDL\Downloads\repivdiadencode';
9
        Data.imbase = 'B';
10
        Data.imzeros = '5';
11
        Data.imext = 'tif';
12
        Data.imcstep = '0';
13
        Data.imfstep = '1';
14
        Data.imfstart= '1';
15
        Data.imfend = '6';
16
        Data.imNum = '1';
17
        Data.DiaGS = '2';
18
        Data.DenGS = '2';
19
        Data.dens = '';
20
        Data.diam = '';
21
        Data.hardDens = '0';
22
        Data.hardDia = '0';
23
        Data.Plot = '0';
24
        Data.outbase = 'B00001';
25
        Data.direcout= 'C:\Users\EFDL\Desktop\';
26
        Data.par = '0';
27
        Data.parprocessors = '6';
28
        PIVdiaden(Data)
29
30 %
31
32 function PIVdiaden(Data)
33 % Uncertainty analysis tool based upon input structure "Data"
34 % generated using the PIVuncertainty.m and PIVuncertainty.fig GUI file.
35 %
36 % Required GUI inputs include:
       -Image files (including DaVis *.im7 files)
37 %
  % Data Structure includes:
38
39 %
        -imdirec - directory where images are located
40 %
        -imbase - base letter(s) of image (ie. 'B' for 'B00001.im7')
        -imzeros - number of number places in file(ie 'B00001.im7' has 5)
41 %
        -imext - extension on image file (ie. *.im7, *.png, etc.)
42 %
  8
        -imcstep - step from first image to second image
43
  90
        -imfstep - step between first and third images
44
45
  00
        -imfstart- first image number
46
  9
        -imfend - last image number
        -imNum - chose first or second image (ie. *.im7 file)
47 %
       -DiaGS - Diameter Grid size (ie. 128x128, 64x64)
48 %
        -DenGS - Density Grid size (ie. 128x128, 64x64)
49 %
```

```
- Image density if hard coded (leave as '' otherwise)
50 %
        -dens
               - Image diameter if hard coded (leave as '' otherwise)
        -diam
51 %
52 %
        -hardDens- Hard code density '0' = no, '1' = yes
        -hrdDia - Hard code diameter '0' = no, '1' = yes
53 %
        -Plot - Plot results '0' = no, '1' = yes
54 %
        -outbase - output file name (ie. B00001)
55 %
        -outdirec- directory where results are saved
56 %
57 %
        -par - run in parallel
58 %
        -parprocessors - number of processors to use
        -bkgd - minimum pixel value over all images
59 %
60 %
        -imgMean - mean pixel value over all images
        -imgStd - pixel standard deviation over all images
61 %
64
65 %Convert Grid Size selection to pixel values
66 if Data.DiaGS=='1'
       Data.GSdia=32;
67
68 elseif Data.DiaGS=='2'
       Data.GSdia=64;
69
70 elseif Data.DiaGS=='3'
       Data.GSdia=128;
71
72 elseif Data.DiaGS=='4'
       Data.GSdia=256;
73
74 end
75
76 if Data.DenGS=='1'
      Data.GSden=32;
77
78 elseif Data.DenGS=='2'
      Data.GSden=64;
79
80 elseif Data.DenGS=='3'
      Data.GSden=128;
81
82 elseif Data.DenGS=='4'
      Data.GSden=256;
83
84 end
85
86 %Set grid resolution to match interrogation size
87 Data.gridres=Data.GSden;
88
89 if Data.bkgd =='1'
90 %% Calculate BackGround:
91 %{
92
       Noise is introduced through stationary objects, reflections, and a
       non-zero backgrond. The background is calculated by comparing all
93
       images in the set and taking the minimum value for each pixel.
94
95 %
96 fprintf('\n------ Calculating Background Image ----
                                                              —\n')
97 %Image file location and base (ie. C:\Folder\File-prefix)
98 if ispc
       imbase=[Data.imdirec '\' Data.imbase];
99
100 else
       imbase=[Data.imdirec '/' Data.imbase];
101
102 end
103
104 %Image sets
```

```
I1 = str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
105
       Data.imfend);
   I2 = I1+str2double(Data.imcstep);
106
107
   if strcmp(Data.imext,'im7')==1 || strcmp(Data.imext,'IM7')==1
108
109
        %Load single image to determin image size
       Davis=readimx([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext],I1(1))...
110
           1);
        im2=double(Davis.Data(:,1:Davis.Ny))';
111
112 else
        %All other image files (Same method for DaVis *im7 files [above])
113
        im2=double(imread([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext],I2...
114
            (1))]));
115 end
116
   %Pre-allocate matrices for the background and for the mean/stdv
117
imgSum = zeros(size(im2));
imgSum2 = zeros(size(im2));
120 bkgd = ones([size(im2),2])*(2<sup>16</sup>);
121
   %Load every image pair
122
   for q=1:length(I1)
123
        if strcmp(Data.imext,'im7')==1 || strcmp(Data.imext,'IM7')==1
124
            %DaVis Image File
125
            Davis=readimx([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext],I2...
126
                (q))]);
            %Select image (DaVis image pairs are stored in a single matrix,
127
            %side by side)
128
            if Data.imNum == '1'
129
130
                im2=double(Davis.Data(:,1:Davis.Ny))';
            elseif Data.imNum == '2
131
132
                im2=double(Davis.Data(:,Davis.Ny+1:2*Davis.Ny))';
133
            elseif Data.imNum == '3
                im2=double(Davis.Data(:,2*Davis.Ny+1:3*Davis.Ny))';
134
            elseif Data.imNum == '4'
135
                im2=double(Davis.Data(:,3*Davis.Ny+1:4*Davis.Ny))';
136
            end
137
138
            %Assign current image to 2nd slot in matrix
139
140
            bkgd(:,:,2) = im2;
            %Compare the current min the the new image, keep lowest value
141
            bkgd(:,:,1) = min(bkgd,[],3);
142
143
            %Values for mean and stdv
            imgSum = imgSum + im2;
144
            imgSum2 = imgSum2 + im2.^2;
145
146
       else
            %All other image files (Same as DaVis *im7 files [above])
147
            if Data.imNum == '1'
148
            im2=double(imread([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext...
149
                ],I1(q))]));
            elseif Data.imNum == '2'
150
            im2=double(imread([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext...
151
                ],I2(q))]));
            end
152
153
            bkgd(:,:,2) = im2;
154
            bkgd(:,:,1) = min(bkgd,[],3);
```

```
imgSum = imgSum + im2;
155
            imgSum2 = imgSum2 + im2.^2;
156
157
        end
158 end
159 %Store background in Data structure for future use
160 Data.bkgrd = bkgd(:,:,1);
161 %Calculate and store mean
162 Data.imgMean = (imgSum./q);
163 %Calculate and store standard deviation using:
164 %http://en.wikipedia.org/wiki/Computational_formula_for_the_variance
165 Data.imgStd = sqrt(imgSum2./(q-1)-(q/(q-1))*((imgSum./q).^2));
   -8{
166
167 %Uncomment to make the following plots:
168
169 \text{ map} = \text{gray}(2^{10});
170 %Shows Original Image
171 figure(1)
172 imshow(im2,map)
173
174 %Shows time minimum background
175 figure(2)
176 imshow(Data.bkgrd,map)
177
178 %Shows Figure (1) without background
179 figure(3)
180 Temp2=im2-Data.bkgrd;
181 Temp2(Temp2<0)=0;
182 imshow(Temp2,map)
183
   8}
184 %End Calculage Backgroud
185 end
186
187 %% Run Jobs in Parallel
   8{
188
        If the "Run in Parallel" box is checked in the PIVuncertainty GUI,
189
        this section opens matlabpool which enables parallel processing.
190
        Images are devided between the total number of processors.
191
   8}
192
   if str2double(Data.par)
193
        mkdir('TempFolder')
194
        fprintf('\n--- Initializing Processor Cores for Parallel Job -----\n')
195
196
        poolopen=1;
197
        %Don't open more processors than there are image pairs
198
        if length(str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
199
            Data.imfend)) < str2double(Data.parprocessors)</pre>
            Data.parprocessors=num2str(length(str2double(Data.imfstart):...
200
                str2double(Data.imfstep):str2double(Data.imfend)));
201
        end
202
203
        trv
            %Open Matlab Pool
204
            matlabpool('open', 'local', Data.parprocessors);
205
206
        catch
207
            try
208
                %If First attepmt doesn't work, close and retry
```

```
matlabpool close
209
               matlabpool('open', 'local', Data.parprocessors);
210
           catch
211
212
               %If Second attempt doesn't work, continue with single processor
               beep
213
214
               disp('Error Running Job in Parallel - Defaulting to Single ...
                  Processor')
               poolopen=0;
215
                fprintf(' \ n-
                                   ——— Processing Dataset —
                                                                             ____\n...
216
                   ')
               Data.par = '0';
217
               diadenprocessing(Data)
218
                fprintf('______ Job Completed ______\n')
219
           end
220
       end
221
222
        if poolopen
            %Pool successfully open, divide sets
223
           I1=str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
224
               Data.imfend);
           I2=I1+str2double(Data.imcstep);
225
226
                fprintf('\n-
                                         - Processing Dataset ...
227
                                      -\n')
               spmd
228
                    verstr=version('-release');
229
                    if str2double(verstr(1:4))>=2010
230
                        Ildist=getLocalPart(codistributed(I1, codistributor('1d'...
231
                           ,2)));
232
                        I2dist=getLocalPart(codistributed(I2, codistributor('1d'...
                           ,2)));
                    else
233
                        Ildist=localPart(codistributed(I1, codistributor('1d',2),'...
234
                           convert'));
235
                        I2dist=localPart(codistributed(I2,codistributor('1d',2),'...
                           convert'));
236
                    end
                    diadenprocessing(Data,Ildist,I2dist);
237
               end
238
                fprintf('----
                                Job Completed _____\n...
239
                   ')
240
241
            if poolopen
242
               matlabpool close
           end
243
       end
244
        %Combine solution from the parallel processors
245
       DiadenCombine (Data)
246
   else
247
248
       %"Run in Parallel" not selected - Default to single processor
       fprintf('\n----- Processing Dataset -----
                                                                      —\n')
249
       Data.par = '0';
250
       diadenprocessing(Data)
251
       fprintf('_____
                               — Job Completed ———/n')
252
253 end
254
255
```

```
function diadenprocessing(Data, I1, I2)
256
257
258 tic
259 % Image file location/base (ie. C:\Folder\File-prefix) and
260 % output directory
261
   if ispc
        imbase=[Data.imdirec '\' Data.imbase];
262
       tempout = ['TempFolder\' Data.imbase];
263
       Data.outdirec= [Data.outdirec '\'];
264
   else
265
       imbase=[Data.imdirec '/' Data.imbase];
266
       tempout = ['TempFolder/' Data.imbase];
267
       Data.outdirec= [Data.outdirec '/'];
268
269
   end
270
   if nargin<3 %(Not runing in parallel)</pre>
271
272 %Image indices
   I1 = str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
273
       Data.imfend);
274 I2 = I1+str2double(Data.imcstep);
   end
275
276
277
   %% EVALUATE IMAGE PAIRS
278
   279
   for q=1:length(I1)
280
281
        %Load image pair and flip coordinates
282
283
        if strcmp(Data.imext,'im7')==1 || strcmp(Data.imext,'IM7')==1
284
            %DaVis Image File
           Davis=readimx([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext],I2...
285
               (q))]);
           if Data.imNum == '1'
286
                im2=double(Davis.Data(:,1:Davis.Ny))';
287
            elseif Data.imNum == '2'
288
                im2=double(Davis.Data(:,Davis.Ny+1:2*Davis.Ny))';
289
            elseif Data.imNum == '3'
290
                im2=double(Davis.Data(:,2*Davis.Ny+1:3*Davis.Ny))';
291
            elseif Data.imNum == '4'
292
                im2=double(Davis.Data(:,3*Davis.Ny+1:4*Davis.Ny))';
293
            end
294
295
296
       else
           %All other image files
297
           if Data.imNum == '1'
298
           im2=double(imread([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext...
299
               ],I1(q))]));
            elseif Data.imNum == '2'
300
301
            im2=double(imread([imbase sprintf(['%0.' Data.imzeros 'i.' Data.imext...
               ],I2(q))]));
           end
302
       end
303
        %Subtract background and 20 to compensate for an low level noise
304
        %(Effects on the image densitsy are minimal (tipically 0.5% of max)
305
306
        if Data.bkgd =='1'
307
            im2 = im2-Data.bkgrd-20;
```

```
end
308
309
       %Any negative values set to zero
310
       im2(im2<0)=0;
311
312
313
       %Flip image
       im2=flipud(im2);
314
315
       %ESTIMATE THE FOLLOWING VALUES:
316
       8
         P Dia
                      - Particle image diameter
317
          P_Dens
                      - Image density
       2
318
                      - (dia_x/dia_y) to detect elliptical particles
       8
          DiaRatio
319
                      - Image density using peak counting (8-neighbor)
       2
          PeakDen
320
       2
                      - Image density using peak counting (20-neighbors)
321
          PeakDen2
       8
          Iavq
                      - Average particle intensity
322
323
       [P_Dia, P_Dens, DiaRatio, PeakDen PeakDen2 Iavq]=DiaDen(im2,Data);
324
325
       %Convert to single precision to save memory
326
       P_Dia=single(P_Dia);
       P_Dens=single(P_Dens);
327
       DiaRatio=single(DiaRatio);
328
       PeakDen=single(PeakDen);
329
       PeakDen2=single(PeakDen2);
330
       Iavg=single(Iavg);
331
332
       %If running in parallel, save individual files (combined ln 248)
333
       if str2double(Data.par)
334
           save([tempout sprintf(['%0.' Data.imzeros 'i.mat'],I1(q))],...
335
               'P_Dia', 'P_Dens', 'DiaRatio', 'PeakDen', 'PeakDen2', 'Iavg');
336
337
       else
       %Otherwise save results for each image to matrix
338
           avgI(:,:,q)=Iavg;
339
340
           Dia(:,:,q)=P_Dia;
           Den(:,:,q)=P_Dens;%/(32*32); %Convert to particles/pixel
341
           Rat(:,:,q)=DiaRatio;
342
343
          PkDen(:,:,q)=PeakDen;%/(32*32);
           PkDen2(:,:,q)=PeakDen2;%/(32*32);
344
       end
345
       %Output current step
346
       fprintf(['Completed (' num2str(q) '/' num2str(length(I1)) ') n'])
347
348
   end
   %If not running in parallel, continue to stats subfunction
349
350
   if Data.par == '0'
       DiaDenMeanStdev(Data, Dia, Den, Rat, PkDen, PkDen2, avgI)
351
352
   end
353
354
355
356
   357
   function [DIA PD R PkD PkD2 Iavg] = DiaDen(IM,Data)
358
359
   360
   % Code written to estimate the particle diameter and density for an
                                                                      00
361
362
   % image. A minimum grid size of 64x64 should be used. Results below
                                                                      8
363 % this grid size become significantly less accurate.
                                                                      00
```

```
8
364
   8
       Inputs:
                  IM - the full image
                                                                     8
365
   8
   00
                  Data- structure containing all other inputs
                                                                      8
366
                  dia - Diameters for each interrogation region
367
   00
       Output:
                                                                      %
                  PD - Particle Density for each interrogation region
368
   8
                                                                     8
                  R - Diameter ratio
369
  00
                  PkD - Peak counting (8-neigbors)
370
  8
                                                                      %
                                                                     00
                  PkD2- Peak counting (20-neighbors)
371
   8
                  Iavg- Average Particle intensity
                                                                     8
372
  8
   373
374
   [ysize xsize] = size(IM);
375
376 gridres = Data.gridres;
  S=[ysize xsize]./gridres;
377
378
379
   Begin Diameter Estimation
380 %
                                                                      %
382 if str2double(Data.hardDia)==0
383 k=0;
384 %Generate sub-images
385 GS = Data.GSdia; %Grid size (pix)
386 inc=GS/gridres;
   for i = 1:GS:floor(ysize/GS)*GS
387
       1=0;
388
       k = k + inc;
389
       for j = 1:GS:floor(xsize/GS)*GS
390
          l=l+inc;
391
          img=IM(i:i+(GS-1), j:j+(GS-1));
392
393
          [Diam, Ratio] = Diameter(imq, GS);
          DIA(k-inc+1:k,l-inc+1:l) = Diam;
394
          R(k-inc+1:k,l-inc+1:l) = Ratio;
395
396
       end
  end
397
398
   %Caluclate diameter for portion not withing the GSxGS regions
399
             ------x edge-
  %___
400
401 k=0;
  for i = 1:GS:floor(ysize/GS)*GS
402
       k = k + inc;
403
       j = xsize-GS+1:xsize;
404
405
       img=IM(i:i+(GS-1),j);
406
       [Diam, Ratio] = Diameter(img, GS);
       DIA(k-inc+1:k,l+1:ceil(xsize/gridres)) = Diam;
407
       R(k-inc+1:k,l+1:ceil(xsize/gridres)) = Ratio;
408
409 end
                ----y edge-
410
   8-
  i = ysize-GS+1:ysize;
411
412
  1 = 0:
  for j = 1:GS:floor(xsize/GS)*GS
413
       l=l+inc;
414
       img=IM(i,j:j+(GS-1));
415
       [Diam,Ratio] = Diameter(img,GS);
416
417
       DIA(k+1:ceil(ysize/gridres),l-inc+1:l) = Diam;
418
       R(k+1:ceil(ysize/gridres),l-inc+1:l) = Ratio;
419 end
```

```
420 %
                 -corner-
421 img=IM(ysize-GS+1:ysize, xsize-GS+1:xsize);
422 [Diam, Ratio] = Diameter(img, GS);
423 DIA(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = Diam;
424 R(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = Ratio;
425
426
   2
427
  else %Hard-Code box is checked
428
     DIA=str2double(Data.diam) *ones(ceil(S(1)), ceil(S(2)));
429
     R=ones(ceil(S(1)), ceil(S(2)));
430
431 end
  DIA(isnan(DIA))=0;
432
433
434
   435
   436
                          Estimate Density
  437
  438
440 if str2double(Data.hardDens) == 0
441
442 GS =Data.GSden;
443 inc=GS/gridres;
444 k=0;
445
   for i = 1:GS:floor(ysize/GS)*GS
446
      1=0;
447
      k = k + inc;
448
449
      for j = 1:GS:floor(xsize/GS)*GS
          l=l+inc;
450
451
          % Inerrogation region size (GS x GS)
          IA1=IM(i:i+(GS-1),j:j+(GS-1));
452
          % Particle diameter for region
453
          Dia = mean(mean(DIA(k-inc+1:k, l-inc+1:l)));
454
          % Calculate the Density
455
          [Dens pkDen pkDen2 avgI] = Density(IA1, Dia, GS, pkcut, thresh);
456
          % Store Solutions
457
          pd(k-inc+1:k,l-inc+1:l) = Dens;
458
459
          pkd(k-inc+1:k,l-inc+1:l) =pkDen;
          pkd2(k-inc+1:k,l-inc+1:l) =pkDen2;
460
          Iavg(k-inc+1:k,l-inc+1:l) =avgI;
461
462
      end
463 end
464
465
   8_
                —x edge—
      k=0;
466
   for i = 1:GS:floor(ysize/GS)*GS
467
      k = k + inc;
468
      IA1=IM(i:i+(GS-1), xsize-GS+1:xsize);
469
      Dia = mean(mean(DIA(k-inc+1:k,ceil(xsize/gridres)...
470
          -inc+1:ceil(xsize/gridres))));
471
      [Dens pkDen pkDen2 avgI] = Density(IA1, Dia, GS,pkcut,thresh);
472
473
      pd(k-inc+1:k,l+1:ceil(xsize/gridres)) = Dens;
474
      pkd(k-inc+1:k,l+1:ceil(xsize/gridres)) = pkDen;
475
      pkd2(k-inc+1:k,l+1:ceil(xsize/gridres)) = pkDen2;
```

```
Iavg(k-inc+1:k,l+1:ceil(xsize/gridres)) = avgI;
476
477
   end
478
                   —y edge-
479
       1=0;
480
481
   for j = 1:GS:floor(xsize/GS)*GS
482
       l=l+inc;
       IA1=IM(ysize-GS+1:ysize,j:j+(GS-1));
483
       Dia = mean(mean(DIA(ceil(ysize/gridres)...
484
           -inc+1:ceil(ysize/gridres),l-inc+1:l)));
485
       [Dens pkDen pkDen2 avgI] = Density(IA1, Dia, GS,pkcut,thresh);
486
       pd(k+1:ceil(ysize/gridres),l-inc+1:l) = Dens;
487
       pkd(k+1:ceil(ysize/gridres),l-inc+1:l) = pkDen;
488
       pkd2(k+1:ceil(ysize/gridres),l-inc+1:l) = pkDen2;
489
       Iavg(k+1:ceil(ysize/gridres),l-inc+1:l) = avgI;
490
   end
491
492
                 493
   IA1=IM(ysize-GS+1:ysize, xsize-GS+1:xsize);
494
   Dia = mean(mean(DIA(ceil(ysize/gridres)-inc+1:ceil(ysize/gridres),...
495
       ceil(xsize/gridres)-inc+1:ceil(xsize/gridres))));
496
   [Dens pkDen pkDen2 avgI] = Density(IA1,Dia,GS,pkcut,thresh);
497
  pd(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = Dens;
498
   pkd(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = pkDen;
499
   pkd2(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = pkDen2;
500
   Iavg(k+1:ceil(ysize/gridres),l+1:ceil(xsize/gridres)) = avgI;
501
502
503 pd (pd<0)=0;
504 pkd (pd<0)=0;
505 pkd2 (pd<0)=0;
506 PD=pd;
507 PkD=pkd;
508 PkD2=pkd2;
509 else %Hard Code box is checked
      PD=str2double(Data.dens)*ones(ceil(S(1)),ceil(S(2)));
510
      PkD=str2double(Data.dens) *ones(ceil(S(1)), ceil(S(2)));
511
      PkD2=str2double(Data.dens) *ones(ceil(S(1)), ceil(S(2)));
512
      Iavg=ones(ceil(S(1)), ceil(S(2)));
513
514 end
515
   % (un) comment to supress or plot images of diameter/density distribtions
516
517
   8{
518
   figure(10);surf(DIA,'DisplayName','DIA','edgecolor','none');
519 view(0,90);axis square;colorbar;title('Particle Image Diameter')
520
521 figure(11); surf(PD,'DisplayName','pd','edgecolor','none');
  view(0,90);axis square;colorbar;
522
  title('Particle Density')
523
524
   figure(12);map = gray(2<sup>8</sup>);imshow(flipud(IM),map);title('Raw Image')
525
526 %}
   527
528
529
530
531
```

```
532 function [Diam, Ratio] = Diameter(img, GS)
   % The relationship between the particle image diameter and the width of
533
534 % the correlation peak is discussed in Adrian (8.126)
535 %Subtract the minimum value of the image
536 img=img-min(img(:));
537 %Auto-correlate
538 A=fftn(img);
539 C2=real(ifftn(conj(A).*A));
540 %Correlation Peak at Center of image
541 C2(1:GS/2,1:GS/2)=rot90(C2(1:GS/2,1:GS/2),2);
542 C2(1:GS/2,GS/2+1:GS)=rot90(C2(1:GS/2,GS/2+1:GS),2);
543 C2(GS/2+1:GS,1:GS/2)=rot90(C2(GS/2+1:GS,1:GS/2),2);
544 C2(GS/2+1:GS,GS/2+1:GS)=rot90(C2(GS/2+1:GS,GS/2+1:GS),2);
   if max(C2(:))>0
545
        %Locate Correlation Maximum
546
547
       C2=C2-min(C2(:));
548
        [ymaxval yind] = max(C2,[],1);
        [~, xind] = max(ymaxval);
549
       yind = yind(xind);
550
551
        x=0.5:size(C2,1)-0.5;
       %Number of points used in Gauss Fit (N = val*2+1)
552
       val = 1;
553
       xData=x(yind-val:yind+val);
554
       yData=C2(yind-val:yind+val, xind);
555
        %Gauss Fit
556
          % [fitresult, ~] = fit( xData', yData, 'gauss1');
557
558
          % Diam = 2*fitresult.c1;
        %The function "fit" above requires curve fitting toolbox.
559
        %Alternate method:
560
561
       p=polyfit(xData',log(yData),2);
        sigma=sqrt(-1/(2*p(1)));
562
        %mu=p(2) *sigma<sup>2</sup>; A=exp(p(3)+mu<sup>2</sup>/(2*sigma<sup>2</sup>));
563
        %Particle image diameter in y-direction
564
       DiamY = 2*sqrt(2)*sigma;
565
566
       %In the Y direction
567
       y=0.5:size(C2,2)-0.5;
568
       yData2 = y(xind-val:xind+val);
569
       xData2 = C2(yind, xind-val:xind+val);
570
       p2=polyfit(yData2, log(xData2), 2);
571
        sigma2=sqrt(-1/(2*p2(1)));
572
573
        %Particle image diameter in x-direction
574
       DiamX = 2*sqrt(2)*sigma2;
        %Average particle image diameter
575
       Diam = (DiamX+DiamY)/2;
576
       %Calculate the diameter ratio
577
        if DiamY~=0
578
           Ratio=DiamX/DiamY;
579
580
        else
            Ratio=0;
581
       end
582
583
584 else
       Diam = 0; Ratio = 0;
585
586 end
587
```

```
588
589
590
592 function [Dens pkDen pkDen2 avgI] = Density(IA1, Dia, GS)
593 %If particle counting has (dens < pkcut), the value for particle
594 %ct is used
595 pkcut=3;
596 %(image intensities < thresh) are set to 0 (accounts for noise)
597 thresh=1;
598
599 IA3=IA1;
                                   % Store unalterd image
                                   % Makes for better solution
600 IA1=sqrt(IA1);
601
602 %Locate peaks
603 BWI = Peak8(IA3); %imregionalmax(IA1);
604 PeaksI = IA3.*BWI;
605 peaksI = PeaksI(PeaksI~=0);
606 %Take the average and standard deviation of peaks (for normalizing)
607 if isempty (peaksI)
608 imavg = 0;
609 \text{ imstd} = 0;
610 else
611 imavg = mean(peaksI);
612 imstd = std(peaksI);
613 end
614 %Values to normailze interrogation region
615 IMmin = sqrt(min(IA3(:)));
616 IMmax = sqrt(imavg+4*imstd);
617
618 %If the above value is larger than the maximum value in the image
619 if IMmax > max(IA1(:))
620
       IMmax = max(IA1(:));
621 end
622
623 IA1=IA1-IMmin;
                                   % Normalize image
624 IA1 = IA1 + 255 / (IMmax-IMmin);
                                   % Normalize image
625 IA1(isinf(IA1)|isnan(IA1))=0;
626 IA1(IA1>255)=255;
                                   % Any value above IMmax = IMmax
627 %-
628 %Autocorrelate
629 A=fftn(IA1);
630 C3=real(ifftn(conj(A).*A));
631 %Correlation Peak at Center of image
632 C3(1:GS/2,1:GS/2)=rot90(C3(1:GS/2,1:GS/2),2);
633 C3(1:GS/2,GS/2+1:GS)=rot90(C3(1:GS/2,GS/2+1:GS),2);
634 C3(GS/2+1:GS, 1:GS/2)=rot90(C3(GS/2+1:GS, 1:GS/2), 2);
635 C3(GS/2+1:GS,GS/2+1:GS)=rot90(C3(GS/2+1:GS,GS/2+1:GS),2);
636 %
637 %Count particles by finding local maximums
638 IA4 = IA1;
639 IA4(IA4<thresh)=0;
640 BW2 = Peak8(IA4);
641 Peaks2 = IA4.*BW2;
642 peaks2 = Peaks2 (Peaks2~=0);
643 pk2=length(peaks2);
```

```
644 pkDen = (32*32) * (pk2) / ((length(IA1)-2)^2);
645
646 %Consider using peaks2 from above instead of peaks for avg intensity
647 \quad BW = Peak(IA4);
648 Peaks = IA4.*BW;
649 peaks = Peaks(Peaks~=0);
650 pk3=length(peaks);
651 pkDen2 = (32*32) * (pk3) / ((length(IA1)-4)^2);
652
   653
   % AVERAGE INTENSITY
                                                                             2
654
        Calculate average intensity by taking peak locations and stdev of%
   2
655
         four points around them. If stdev is low then particle images is %
656
   8
         centered on a pixel and give a better representation of actual
657
   8
                                                                            8
        particle peak intensity
                                                                             8
658
   8
                                                                             .%
659
        %Pre-allocate
660
        I=zeros([size(IA1) 4]);
661
        %Create 3D matrix with 3rd dimension containing the four
662
663
        %surrounding points. (ie for pixel IA1(2,2) in image I(2,2,:)
        %contains: IA1(1,2) IA1(3,2) IA1(2,1) IA1(2,3)
664
        I(:,2:GS,1)=IA3(:,1:GS-1);
665
        I(:,1:GS-1,2)=IA3(:,2:GS);
666
        I(2:GS,:,3)=IA3(1:GS-1,:);
667
       I(1:GS-1,:,4)=IA3(2:GS,:);
668
        %Only keep values where peaks are located
669
670
        I(:,:,1)=(I(:,:,1)./IA3).*BW;
        I(:,:,2) = (I(:,:,2)./IA3).*BW;
671
672
        I(:,:,3) = (I(:,:,3)./IA3).*BW;
673
       I(:,:,4) = (I(:,:,4)./IA3).*BW;
        %Remove any Nan
674
       I(isnan(I))=0;
675
676
        %Compute standard deviation of four values surronding peaks
       Var=std(I,0,3);
677
        %Calculate the max of the four values surrounding peaks
678
679
        Imax=max(I,[],3);
        %Create mask
680
       Tmp1 = ones(size(IA1));
681
        %Remove saturate particles (ie peak value = surround value)
682
683
       Tmp1(IA3==Imax)=0;
        %Remove peaks where surround values have large standard deviations
684
        Tmp1(Var>=0.34)=0;
685
686
        %Remove values where there are no peaks
       Tmp1(BW == 0) = 0;
687
        %Apply mask
688
       Pks = Tmp1.*IA1;
689
        %Collect peak values
690
       Temp=Pks(Pks~=0);
691
692
        %Average Intensity:
693
       if size(Temp, 1)>1
694
       avgI = mean(Temp);
695
       elseif length(peaks)>=1
696
697
            pks=peaks(peaks>=25);
698
       avgI = mean(pks);
699
       else
```

```
avgI = max(IA3(:));
700
701
       end
   702
703
704 %Correlation Peak Value
705 CorrPeak=max(C3(:));
706
707 %Parameters for empirical density obtained through Least Squares fit
708 % Goodness of fit:
709 % R-square: 0.9999
710
711 a= 12.440818554697394;
712 b= 1.252907533572718;
713 c= -2.017253413241566;
714 d= -1.268448437457117;
715 e= -1.271518390712386;
716
717 %Calculate image density
718 Dens=a*(CorrPeak^b)*(Dia^c)*((GS*GS)^d)*(avgl^e);
719
720 %If density is below 3 particles, resort to peak counting.
   if pkDen < pkcut</pre>
721
       Dens = pkDen;
722
723 end
724 Dens(isnan(Dens))=0;
725
726
727
728
   729
   % Peak Finder
                                                                     8
       This function locates local maximums by comparing each location
                                                                     00
730
   8
       (i,j) to the surrounding 20 points
731 %
                                                                     2
732 %
                                                                     2
                            733 %
                                                                     %
734 %
                                                                     8
                            | - | - | - | - |
735 %
                         i |_ |_ x|__
                                                                     %
736 %
                                                                     0
                            | - | - | - | - | - |
737 😤
                                                                     2
                             | - | - | - |
738 %
                                                                     2
                                j
740 function Matrx = Peak(IMG)
741
742 ny = size(IMG, 2);
743 nx = size(IMG, 1);
744 Matrx = zeros(size(IMG));
745 for i = 3:nx-2
       for j = 3:ny-2
746
          if
                     IMG(i,j)>IMG(i,j-2) && IMG(i,j)>IMG(i,j-1)...
747
748
                  && IMG(i,j)>IMG(i,j+1) && IMG(i,j)> IMG(i,j+2)...
                  && IMG(i,j)>IMG(i-1,j-1) && IMG(i,j)> IMG(i-1,j)...
749
                  && IMG(i,j)>IMG(i-1,j+1) && IMG(i,j)> IMG(i-2,j)...
750
                  && IMG(i,j)>IMG(i+1,j-1) && IMG(i,j)> IMG(i+1,j)...
751
                  && IMG(i,j)>IMG(i+1,j+1) && IMG(i,j)> IMG(i+2,j)...
752
753
                  && IMG(i,j)>IMG(i-1,j-2) && IMG(i,j)> IMG(i+1,j-2)...
754
                  && IMG(i,j)>IMG(i+2,j-1) && IMG(i,j)> IMG(i+2,j+1)...
755
                  && IMG(i,j)> IMG(i-1,j+2)&& IMG(i,j)>IMG(i+1,j+2)...
```

```
&& IMG(i,j)> IMG(i-2,j+1)&& IMG(i,j)> IMG(i-2,j-1)
756
            Matrx(i, j) = 1;
757
         end
758
      end
759
  end
760
  761
762
763
764
  765
  % Peak Finder 8
                                                            8
766
      This function locates local maximums by comparing each location
                                                            %
  2
767
      (i,j) to the surrounding 8 points
                                                            %
768
   2
769
  8
                                                            8
                         | _ | _ | _ |
                                                            8
770
  8
771
   8
                        i i
                         _ X _
                                                            ÷
772
  8
                         |-|-|-|
                                                            %
                                                            0
773 %
                            j
775 function Matrx = Peak8(IMG)
776 ny = size(IMG, 2);
777 nx = size(IMG, 1);
  Matrx = zeros(size(IMG));
778
  for i = 2:nx-1
779
      for j = 2:ny-1
780
                  IMG(i,j) > IMG(i,j-1) \& IMG(i,j) > IMG(i,j+1)...
         i f
781
               && IMG(i,j)> IMG(i-1,j-1) && IMG(i,j)> IMG(i-1,j)...
782
               && IMG(i,j)> IMG(i-1,j+1) && IMG(i,j)> IMG(i+1,j-1)...
783
               &&IMG(i,j)> IMG(i+1,j) && IMG(i,j)> IMG(i+1,j+1)
784
            Matrx(i, j) = 1;
785
         end
786
787
      end
788
  end
  789
790
791
792
794 % When code PIVdiaden is run in parallel individual image solutions
                                                           2
  % are stored and then read in by this function. The individual results%
795
  % are stored in a temporary file that is deleted at the end of this
                                                           8
796
797
   % function.
                                                            0
798
   function DiadenCombine(Data)
799
800
  %Determine PC or Mac
801
  if ispc
802
      tempout = ['TempFolder\' Data.imbase];
803
804
      Data.outdirec= [Data.outdirec '\'];
  else
805
      tempout = ['TempFolder/' Data.imbase];
806
      Data.outdirec= [Data.outdirec '/'];
807
808
  end
809
810 %Number of Images processed
```

```
811 N = length(str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
      Data.imfend));
812
813 %Open first mat file to preallocate size
s14 fname = [tempout sprintf(['%0.' Data.imzeros 'i.mat'],str2double(...
      Data.imfstart))];
815 load(fname)
816 %Preallocate matrices
817 Dia = zeros(size(P_Dia,1), size(P_Dia,2), N, 'single');
818 Den = zeros(size(P_Dens,1),size(P_Dens,2),N,'single');
819 Rat = zeros(size(P_Dia,1), size(P_Dia,2), N, 'single');
820 PkD = zeros(size(P_Dens,1),size(P_Dens,2),N,'single');
821 PkD2 = zeros(size(P_Dens,1), size(P_Dens,2), N, 'single');
822 avgI = zeros(size(P_Dens,1),size(P_Dens,2),N,'single');
823 %Load all *.mat files and store in three dimensional array
824 n = 1;
825
  for i = str2double(Data.imfstart):str2double(Data.imfstep):str2double(...
      Data.imfend)
826
       fname = [tempout sprintf(['%0.' Data.imzeros 'i.mat'],i)];
827
       load(fname)
       Dia(:,:,n) = P_Dia;
828
       Den(:,:,n) = P_Dens;
829
       Rat(:,:,n) = DiaRatio;
830
       PkD(:,:,n) = PeakDen;
831
       PkD2(:,:,n) = PeakDen2;
832
       avgI(:,:,n) = Iavg;
833
       n = n + 1;
834
835 end
836 %Calculate Mean and Standard deviation
837 DiaDenMeanStdev (Data, Dia, Den, Rat, PkD, PkD2, avgI)
838 %Delete Temporary Folder
839 rmdir('TempFolder','s')
841
842
843
844
846 % This portion calculates the mean and standard deviation of the
                                                                    2
847 % entire image set. Used for both parallel and serial cases.
                                                                     2
   848
   function DiaDenMeanStdev(Data, Dia, Den, Rat, PkD, PkD2, avgI)
849
850 %Remove NaNs
851 Dia(isnan(Dia))=0;
852 Den(isnan(Den))=0;
853 Rat(isnan(Rat))=0;
854 PkD(isnan(PkD))=0;
855 PkD2(isnan(PkD2))=0;
856 avgI(isnan(avgI))=0;
857 %Calculate Mean and Standard Deviation
858 meanDia = mean(Dia,3);
859 meanDen = mean(Den,3);
860 meanRat = mean(Rat, 3);
861 meanPkD = mean(PkD,3);
meanPkD2 = mean(PkD2, 3);
863 meanIavg = mean(avgI,3);
```

```
864 sigDen=std(Den,0,3);
865 sigDia=std(Dia,0,3);
866 sigRat=std(Rat, 0, 3);
867 sigPkD=std(PkD,0,3);
ses sigPkD2=std(PkD2,0,3);
869 %Reduce solution to single value per interrogation region
870 ii = size(Dia,1);
871 jj = size(Dia,2);
872 VarDia = Data.GSdia/Data.gridres;
873 stdDia = sigDia(1:VarDia:ii,1:VarDia:jj);
874 muDia = meanDia(1:VarDia:ii,1:VarDia:jj);
875 Diam = Dia(1:VarDia:ii,1:VarDia:jj,:);
876 stdRat = sigRat(1:VarDia:ii,1:VarDia:jj);
877 muRat = meanRat(1:VarDia:ii,1:VarDia:jj);
   Ratio = Rat(1:VarDia:ii,1:VarDia:jj,:);
878
879
880 ii = size(Den, 1);
881 jj = size(Den,2);
882 VarDen = Data.GSden/Data.gridres;
883 muDen = meanDen(1:VarDen:ii,1:VarDen:jj);
stdDen = sigDen(1:VarDen:ii,1:VarDen:jj);
885 Dens = Den(1:VarDen:ii,1:VarDen:jj,:);
886 muPkD = meanPkD(1:VarDen:ii,1:VarDen:j);
stdPkD = sigPkD(1:VarDen:ii,1:VarDen:jj);
888 muPkD2 = meanPkD2(1:VarDen:ii,1:VarDen:jj);
889 stdPkD2 = sigPkD2(1:VarDen:ii,1:VarDen:jj);
         = PkD(1:VarDen:ii,1:VarDen:jj,:);
890 PkDen
891 PkDen2 = PkD2(1:VarDen:ii,1:VarDen:jj,:);
892 muIavg = meanIavg(1:VarDen:ii,1:VarDen:jj);
893 Iavq
           = avgI(1:VarDen:ii,1:VarDen:jj);
894 %Save Data to specified location
  save([Data.outdirec Data.outbase], 'Diam', 'Dens', 'Ratio', 'muDia',...
895
       'muDen', 'muRat', 'stdDia', 'stdDen', 'stdRat', 'muPkD', 'stdPkD',...
896
       'PkDen', 'muPkD2', 'stdPkD2', 'PkDen2', 'Iavg');
897
898
899 %Plot if desired
900 if Data.Plot == '1'
901 figure(100);
902 surf(double(meanDia),'DisplayName','DIA','edgecolor','none');
903 view(0,90); axis square; colorbar; title('Particle Image Diameter')
904 figure(101);
905 surf(double(meanDen), 'DisplayName', 'pd', 'edgecolor', 'none');
906 view(0,90);axis square;colorbar;
907 title('Particle Density')
908 end
```